Problem Set 6

18.355, Fall 2016
Due Wednesday Nov. 16

1. *The scaling of lava flows*

Deduce, via scaling arguments, the dependence on time of the lateral extent of a viscous gravity current of density ρ and viscosity μ generated by:

a) a constant volume V released from a point source,

b) a constant volume flux Q released from a point source,

c) a constant volume flux per unit length F released from a line source.

In all cases, ignore the dynamic influence of the ambient fluid.

2. *The spreading of a pancake*

a) Guided by your scaling result of problem 1a, find a similarity solution that describes the shape of a viscous gravity current generated by a constant volume V released from a point source.

b) Make an estimate of the time over which you expect your solution to be valid. Note that the approximations of lubrication theory are invalid at both early times, when fluid inertia is important, and in the later stages, when the curvature force (associated with surface tension) becomes important.

3. *Dip coating*

Suppose that a plate is withdrawn from a fluid bath, and that the fluid subsequently drains under the influence of gravity. Use the approximations of lubrication theory to show that the thickness $h(x,t)$ of the draining film at any height z satisfies

$$\frac{\partial h}{\partial t} = \frac{gh^2}{\nu}\frac{\partial h}{\partial z}$$

where z is the vertical distance from the contact line, and ν is the kinematic viscosity. Clearly state all assumptions.