

1. A sphere of radius a is half-submerged in water, and is forced to oscillate along a horizontal line with frequency ω and amplitude $2a$. Find the dimensionless parameters governing the system by:

- a) applying Buckingham's Theorem,
- b) nondimensionalizing the governing equations.

Assume that the air overlying the water is not dynamically significant.

2. Rationalize the energetics of some unidirectional flows seen in class.

a) *The shear viscometer*: a fluid layer of depth d in a circular domain of radius a is sheared from above by an applied torque τ . Show that the rate of work by the applied torque is equal to the rate of viscous dissipation.

a) *Poiseuille ($|p w \ddot{a}' z \ddot{o}|$) Flow*. A constant pressure gradient G is applied along the length of a straight circular pipe of radius R filled with a fluid of viscosity μ and density ρ . Show the rate of work done by the applied pressure gradient accounts for the rate of viscous dissipation.

c) *Oscillating flat plate beneath a semi-infinite domain*. Show that the time-averaged rate of work done by the surface stress is equal to the time-averaged rate of viscous dissipation. (Average both quantities over a single cycle.)

3. A viscous fluid with viscosity μ and density ρ completely fills an infinitely long cylinder of radius a that is rotating with constant angular velocity Ω about its long axis. At $t = 0$, the cylinder is suddenly brought to rest. Determine an exact solution for the resulting velocity profile by assuming that the flow is purely azimuthal (i.e. lies along circular streamlines).

4. *Stokes third problem*. Consider a semi-infinite body of fluid bounded below at $y = 0$ by an infinite plane solid boundary. For times $t < 0$, the fluid is quiescent; however, at $t = 0$ the boundary begins to move under the action of a constant force per unit area, or stress τ . Deduce an exact solution for the resulting velocity field.

HINT: the close relation to the Rayleigh problem treated in class suggests the use of a similarity solution; however, application of the constant force boundary condition indicates that the problem may not be entirely posed in terms of the similarity variable. Nevertheless, by reposing the problem in terms of the normal derivative of the velocity ($q = \frac{\partial u}{\partial y}$), one may deduce a similarity solution.

5. *The tears of wine*

a) Deduce an exact solution for the parallel flow forced by gravity and an opposing tangential stress τ applied at the surface of a layer of depth d inclined at an angle β relative to the vertical.

b) Describe the dependence of the flow form on the dimensionless parameter $\tau/(\rho g d \cos \beta)$.

c) Calculate the volume flux up the plane, the rate of work done by the surface stress, the rate of work done against gravity, and the total dissipation per unit area in the system.

d) The flow roughly corresponds to that on the side of a wine glass, which is responsible for the replenishment of the 'tears of wine'. In the wine glass, a vertical surface stress is generated by an evaporatively-forced gradient in alcohol concentration (i.e. lower alcohol concentration and thus higher surface tension in the thin film relative to the bulk). Assuming $d = 0.1\text{mm}$, $\beta = 0$ and $\tau = 10\text{ dynes/cm}$, estimate the number of tears (of approximate volume 0.05 cm^3) that you expect to fall per unit length per minute.