18.355 FLUID DYNAMICS

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MW 2-3:30
Room 2-135

GRADING SCHEME

- 70% : 7 problem sets (group discussion encouraged)
- 30% : course project on subject of your choosing

There is **no required text** for the course, which will be based on the lecture notes; however, the following are recommended supporting material.

SUGGESTED TEXTS

Batchelor, G.K. **An Introduction to Fluid Dynamics**. Cambridge University Press

Acheson, D. J. **Elementary Fluid Dynamics**

A pair of comprehensive texts that present much of classical fluid dynamics in great depth.

Tritton, D.J. **Physical Fluid Dynamics**. Oxford University Press

A very readable and well-illustrated account of many fluid phenomena described from a physical perspective.

RELATED GOOD READS

McMahon, T.A. and Bonner, J.T. **On Size and Life**, Scientific American Books

An inspirational book describing the influence of geometrical scaling in biology. Includes an excellent introduction to dimensional analysis and some beautiful scaling arguments. A must read for those interested in biological fluid dynamics.

Simpson, J.E. **Gravity Currents**, Cambridge University Press

A clear concise description of gravity currents in their many guises with an outstanding collection of photos. A must read for those interested in geophysical and environmental fluid dynamics.

Van Dyke, M. **An album of Fluid Motion**

Samimy, Breuer, Leal and Steen, **A Gallery of Fluid Motion**

Collections of photographs of a variety of fluid motions.

DETAILED COURSE OUTLINE

Lecture 1

- motivation and course philosophy

Lectures 2-3: Dimensional Analysis and Scaling

- dimensional analysis and Buckingham's Theorem
- examples: frequency of a pendulum; Taylor's result for the atom bomb blast cloud; pressure inside a bubble; drag on a sphere
- dynamic similarity and its utility in experimental modeling
- scaling: physically informed dimensional analysis
- examples: the scaling of bouncing drops, birds in flight, rowboats
- demo: coalescence cascade

Lecture 4-5: Derivation of Navier Stokes Equations

- basic concepts: continuum hypothesis, Lagrangian/Eulerian descriptions
- Reynold's Transport Theorem
- conservation of mass and the continuity equation
- Cauchy's momentum equation
- the state of stress in a fluid: pressure and rate of strain tensor
- the stress tensor: definition and interpretation
- constitutive equations: Newtonian and non-Newtonian fluids
- demos: ferrofluids; custard powder; rod climbing

Lecture 6: Thermodynamics

- conservation of energy
- mechanical and thermal energy equations
- the heat equation

Lecture 7: Boundary Conditions

- no-slip and kinematic conditions
- derivation of the stress balance equation at an interface
- normal and tangential stress balance equtions
- demos: the water strider; soap boat, tears of wine, the beating heart

Lecture 8: Nondimensionalization of N-S equations and BCs

- dimensionless groups are those deduced from dimensional analysis
- important groups: Reynolds, Bond, Froude, Strouhal numbers

Lecture 9: Kinematics

- decompose velocity gradient into rate of strain and vorticity tensors
- circulation and Kelvin's Theorem
- vortex dynamics: examples of vortex stretching

Lecture 10-11: Unidirectional Flows - some exact solutions

- steady flows: Couette and Poiseuille flows
- unsteady flows: Rayleigh's first and second problems
  (motivate similarity solutions via dimensional analysis)
- start up flow in a pipe (solve via Sturm-Liouville theory)

Lecture 12: Invisicid Flow and Irrotational Viscous Flows

- Bernoulli's equation and its applications: flow through a weir; the Egyptian water clock; shape of a falling fluid jet
- the momentum integral: e.g. the expanding pipe junction
- demos: ping pong balls on a hair dryer; canoe in the wind
- the use of potential flow for describing irrotational flows
- irrotational flow past a sphere (generate soln from harmonic functions)
- drag calculation and D'Alembert's Paradox

- prove that viscous forces vanish in a viscous irrotational flow
- hydrodynamic drag in an irrotational flow
- surface deformation for an irrotational vortex

Lecture 13: Unsteady Inviscid Flows

- water waves: derivation of dispersion relation
- special cases: gravity and capillary waves in deep and shallow water
- interpretation of waves generated by body moving at surface

Lecture 14: Instability of Superposed Sheared Fluids

- solution of linear problem via normal modes
- deduction of dispersion relation and Squires Theorem
- special cases: Rayleigh-Taylor and Kelvin-Helmholtz instabilities

Lecture 15: Translation of a body in 3D irrotational flow

- hydrodynamic forces on body: buoyancy, steady and acceleration forces
- most general form of D'Alembert's paradox: steady force vanishes
- express acceleration force in terms of added mass tensor
- example: acceleration from rest of a spherical bubble

Lecture 16: Potential Flow Theory

- the streamfunction and streamfunction-vorticity equation
- complex potentials for 2D irrotational flows
- simple applications: flow at a wall angle; sources and sinks; flow past a half body; flow past a cylinder with circulation
- analogies with electrostatics: method of images, superposition
- discussion of the limits of the potential flow description

Lecture 17: The Dynamics of Sports Balls

- hydrodynamic drag at high Reynolds numbers

- skin friction in flow past streamlined bodies
- form drag and separation in flow past bluff bodies
- the role of surface roughness on hydrodynamic drag
- the role of altitude and humidity on balls in flight
- the Magnus effect and its applications in sports and ballistics
- the reverse Magnus effect and its prominence in beach ball soccer
- applications: soccer, golf, ping pong, baseball, cricket
- Brazilian free kicks

Lecture 18-19: Lubrication Theory

- generic physical picture: derivation of governing equations, range of validity
- viscous gravity currents (motivate similarity soln from dimensional analysis)
- the squeeze film
- the thrust bearing: calculate normal and tangential stress on slider

Lecture 20-21: Stokes Flows

- steady and unsteady Stokes equations: range of validity
- general properties of Stokes flows: no inertia, quasi-steady, linearity, reversibility
- consequences of linearity for particle motion
- solving simple problems by application of time reversibility

Lecture 22-23: Vector Methods for Stokes Flows

- preliminaries: definition of true- and pseudo-vectors, harmonic functions
- the Hinch Method: express velocity and pressure fields in terms of harmonic pseudo- and real vector and scalar potentials; deduce soln by inspection
- Stokes flow past a sphere and drag calculation (compare to onerous streamfunction formulation)
- rotating sphere in an infinite fluid: flow and torque
- translation of a spherical drop: Hadamard-Rybczinski formula; discussion of influence of surfactants

- theromocapillary drop motion: calculate drop translation speed

Lecture 24: Stokes flow past a cylinder

- deduce sol'n via Hinch Method: note impossibility to satisfy BCs
- Stokes Paradox: interpret with a scaling argument
- resolution via the Oseen Method: linearization of inertial term; matched asymptotic expansions

Lecture 25: Flow in Porous Media

- definition of Darcy velocity and deduction of Darcy's Law
- flow past a cylindrical porosity anomaly via vector methods
- the Hele-Shaw cell as an experimental analogue of Darcy flow
- examples: gravity currents and bubbles in a Hele-Shaw cell

Lecture 26: Rotating Flows

- derivation of Navier-Stokes equations in a rotating frame
- vortex dynamics in a rotating frame
- statics in a rotating frame: shapes of bodies of revolution
- nondimensionalization of governing equations: Rossby and Ekman numbers
- geostrophic flow and the Taylor-Proudman theorem
- viscous boundary layers: the Ekman spiral
- axial particle motion in a rotating flow
- inertial, cyclostrophic and geostrophic vortices
- spin-down of inertial and geostrophic vortices
- demos: Taylor column, drop motion in a rotating tank