

## 18.357 Interfacial Phenomena 2016

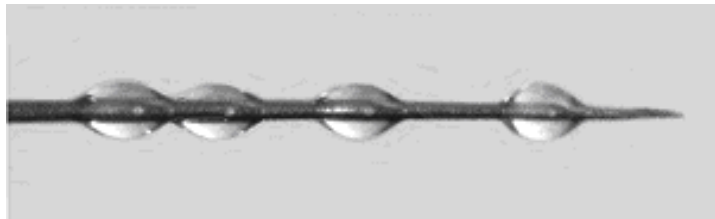
## Problem Set 2

Solve 5 of the 6 following problems. Each is worth 20 points.

1.

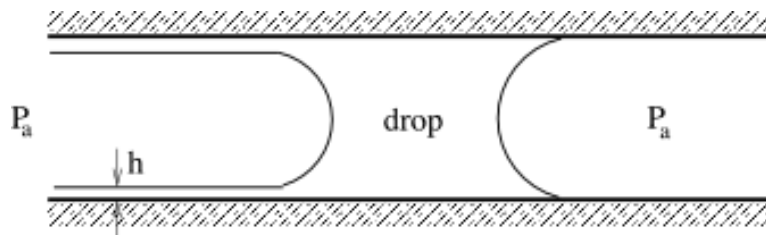
### More self-propulsion

- a) **Drops on a needle:** A drop of silicone oil is deposited on a conical copper wire (mean radius of  $250\ \mu\text{m}$ ), whose radius increases from the right to the left with a gradient  $dr/dz$  of the order of 1 %. Note that silicone oil totally wets copper. Four successive photographs are taken. The interval between two snapshots is about 1 second.



In which direction does the drop move ? Why? Why is the interval between two drops not constant? Might this be useful for cacti?

- b) **A fluid slug in a tube:** A fluid drop is placed in a tube and forced along the tube by air. The liquid totally wets the tube, so that a film (of thickness  $h$ ) is deposited on the wall in the wake of the drop. When the forcing is stopped, the pressure is atmospheric on both sides of the drop.

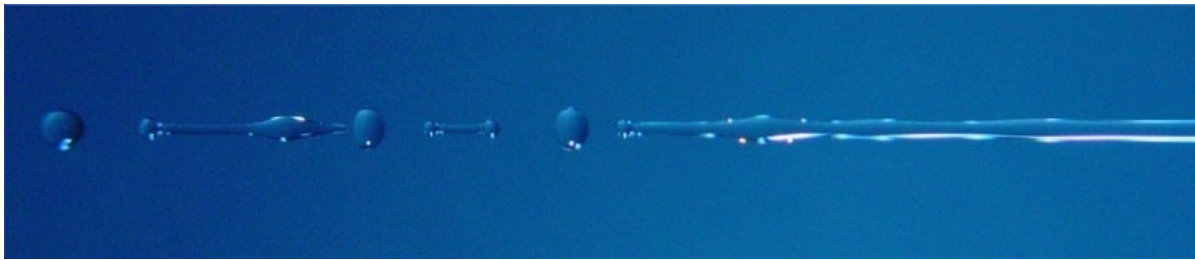


Does the slug move ? Calculate the force acting on it.

- b) **Self-propelling floater:** A cube of volume  $V = L^3$  floats at the air-water interface. Its density is the mean of air and water; thus, provided  $L$  is large relative to the capillary length, it will float with half its volume submerged. Assume that it floats with top and bottom faces horizontal. Explain how a lateral force can be generated by changing the chemistry of the body, that is, by having variable contact angles on the front and rear faces. Give rough estimates for the propulsion speed in the high and low  $Re$  limits. How do you expect the physical picture to change if the body is small relative to the capillary length?

## 2. Retraction of a fluid thread

Consider a fluid that initially assumes the form of a cylinder of radius  $a$  with hemispherical caps. Neglect the influence of gravity.



- a) How do you expect its shape to evolve? Why?
- b) In the limit where viscosity is negligible, deduce the rate of retraction of the thread.
- c) By comparing the retraction rate with the growth rate of the Rayleigh-Plateau instability, estimate under what circumstances the thread will retract into a single droplet.
- d) How will this physical picture change when the retraction and pinch-off are dominated by the influence of viscosity?

## 3. Viscous Rayleigh-Taylor instability

A fluid of density  $\rho$  and high viscosity  $\mu$  is emplaced on a flat plate, resulting in a layer thin relative to the capillary length. Suddenly, the plate is inverted, so that the fluid layer is hanging upside down and so subject to gravitational instability. Use lubrication theory to predict the fastest growing wavelength of instability, as well as the timescale of growth.



#### 4. Toroidal bubbles

Consider a toroidal air bubble with major radius  $R$ , minor radius  $r$ , volume  $V = 2 \pi^2 R r$  and area  $A = 4 \pi^2 R r$ , suspended in water.

- a) Calculate the curvature at an arbitrary point on the surface of the torus.
- b) By calculating the gradient with respect to  $R$  of its surface energy, calculate the force acting to collapse the torus into a sphere.
- c) Assuming that its collapse arises at high Reynolds number, estimate the rate of collapse in the limit of  $R \gg r$ .
- d) Dolphins are known to blow such bubbles, and to play with them for extended periods. Can you explain how they are stabilized; specifically, what sort of external flow would resist their instability and collapse?

#### 5. Bouncing drops

Consider a drop of diameter 1mm bouncing on a fluid bath at 50Hz, rising a few diameters above the interface and distorting the interface by approximately 100 microns on impact.

- a) By evaluating familiar dimensionless groups, estimate the relative magnitudes of the interfacial, inertial and hydrostatic forces that resist the intrusion of the drop into the bath.
- b) Deduce a model for the force imparted by the bath to the drop as a function of its intrusion depth by making the following simplifying assumptions: i) the drop does not deform during impact, ii) the interface conforms to precisely match the shape of the drop as the drop intrudes into the bath and is otherwise undeformed, iii) hydrostatic and inertial effects are negligible. If the interface behaves like a linear spring, calculate its spring constant. *Hint*: calculate the rate of increase of surface energy with intrusion depth. Discuss the shortcomings of this crude model on the basis of your estimates in part a).
- c) If inertia and gravity are indeed negligible, show that the interface shape beyond the drop's contact area must correspond to a catenoid to leading order.

#### 6. Photo essay

Take a photograph of a striking surface-tension related phenomenon. Describe and rationalize the phenomenon captured.