18.357: Lecture 8

Marangoni Flows I:

Thermocapillarity

John W. M. Bush

Department of Mathematics
MIT

Governing Equations

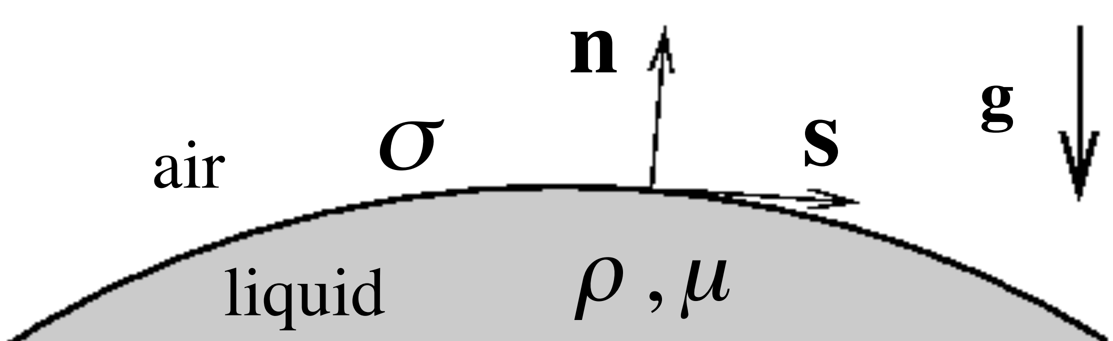
Navier-Stokes equations:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \rho\mathbf{g} + \mu\nabla^2\mathbf{u} \quad , \qquad \nabla\cdot\mathbf{u} = 0$$

Boundary Conditions

Normal stress: $$\mathbf{n}\cdot\mathbf{T}\cdot\mathbf{n}\,\big| = \sigma\,\nabla\cdot\mathbf{n}$$

Tangential stress: $$\mathbf{n}\cdot\mathbf{T}\cdot\mathbf{s}\,\big| = \nabla_s\sigma$$



Stress tensor

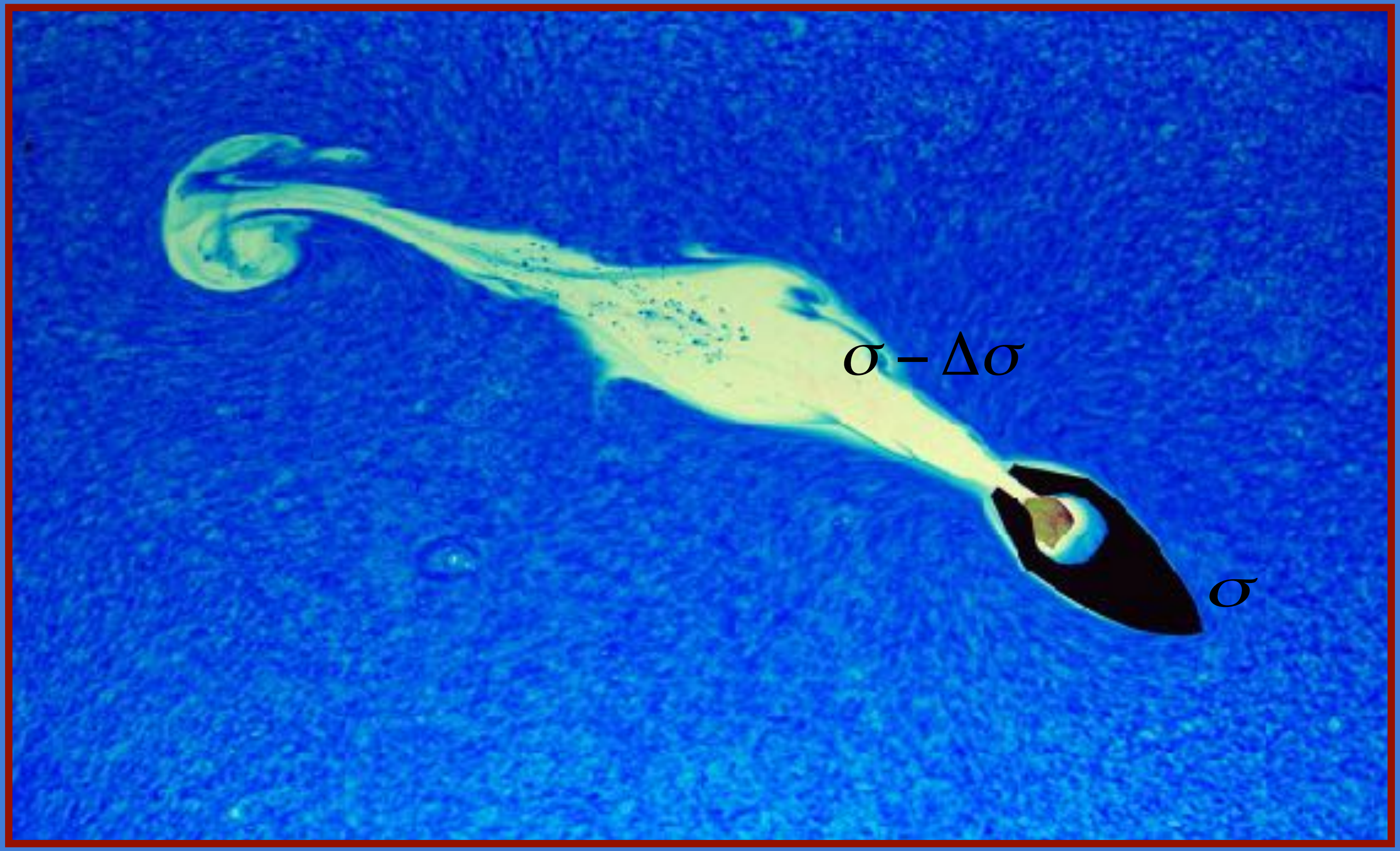
$$\mathbf{T} = -p\mathbf{I} + \mu\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right)$$

Marangoni Flows

- flows dominated by the influence of surface tension gradients

Recall tangential stress BC: $\Delta\, \mathbf{n}\cdot\mathbf{T}\cdot\mathbf{s} = \nabla\sigma$

- $\nabla\sigma$ may arise due to dependence of $\sigma(T, c, \Gamma)$



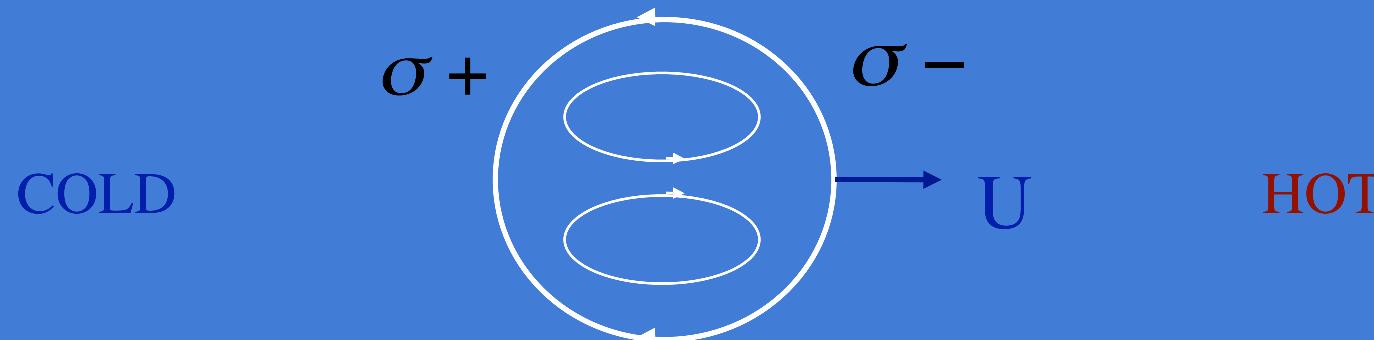
Thermocapillary Flows

- Marangoni flows induced by temperature gradients $\sigma(T)$
- Navier-Stokes coupled to heat equation through BCs

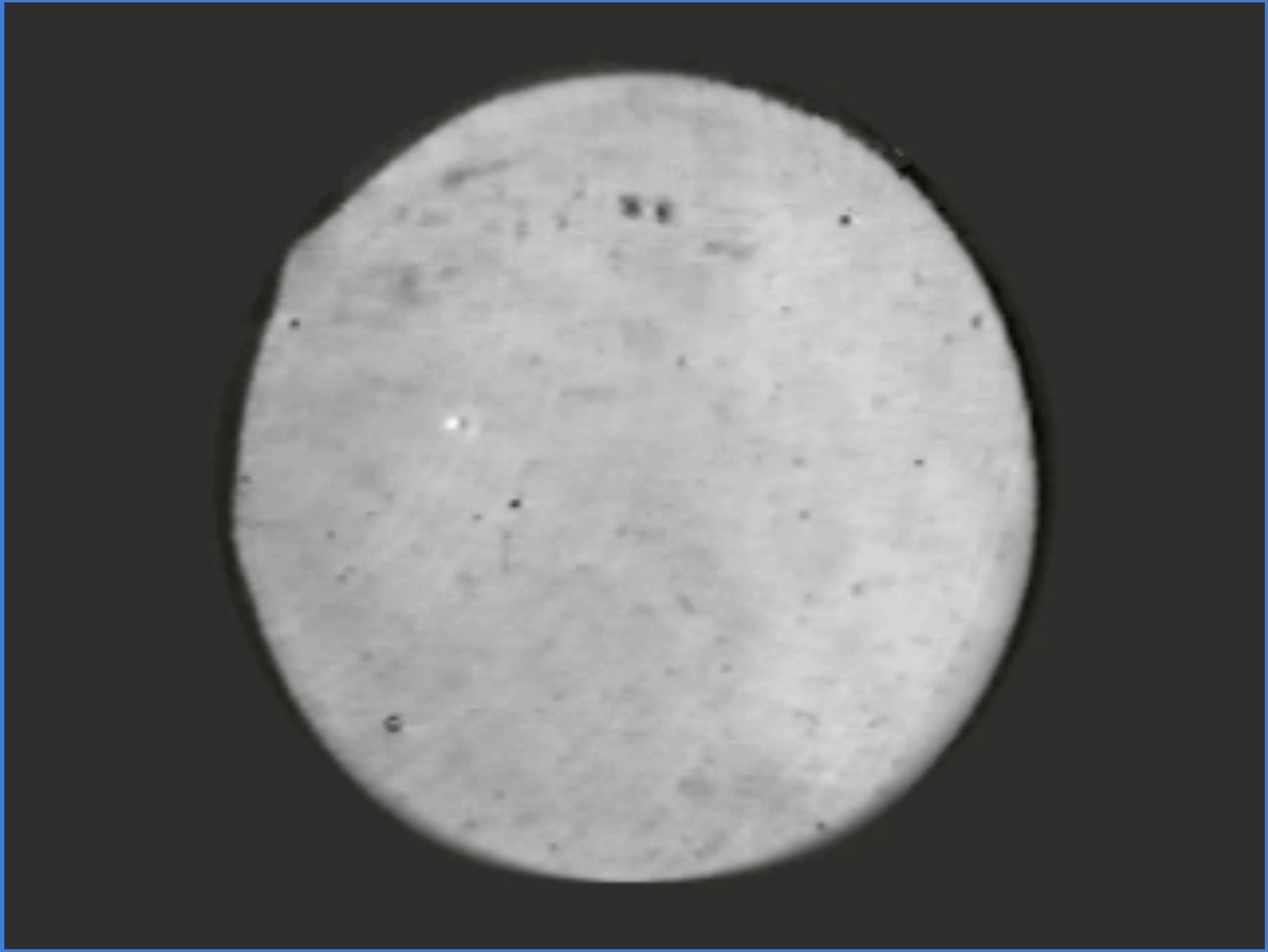
$$\frac{\partial T}{\partial t} + u \cdot \nabla T = \kappa \nabla^2 T$$

e.g. thermocapillary drop motion: drops migrate toward heat source

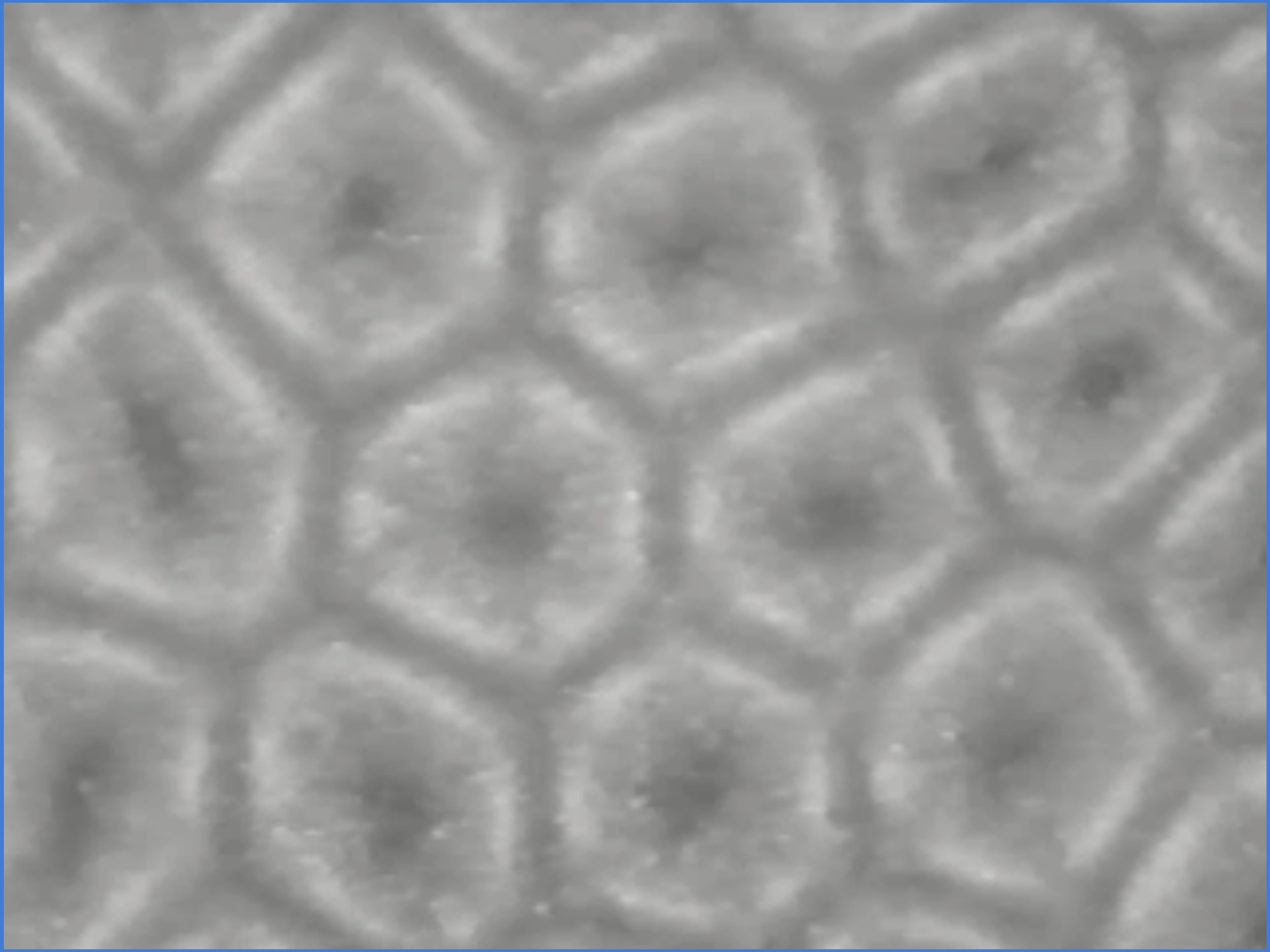
Young, Goldstein & Block (1962)



Convection in a plane layer

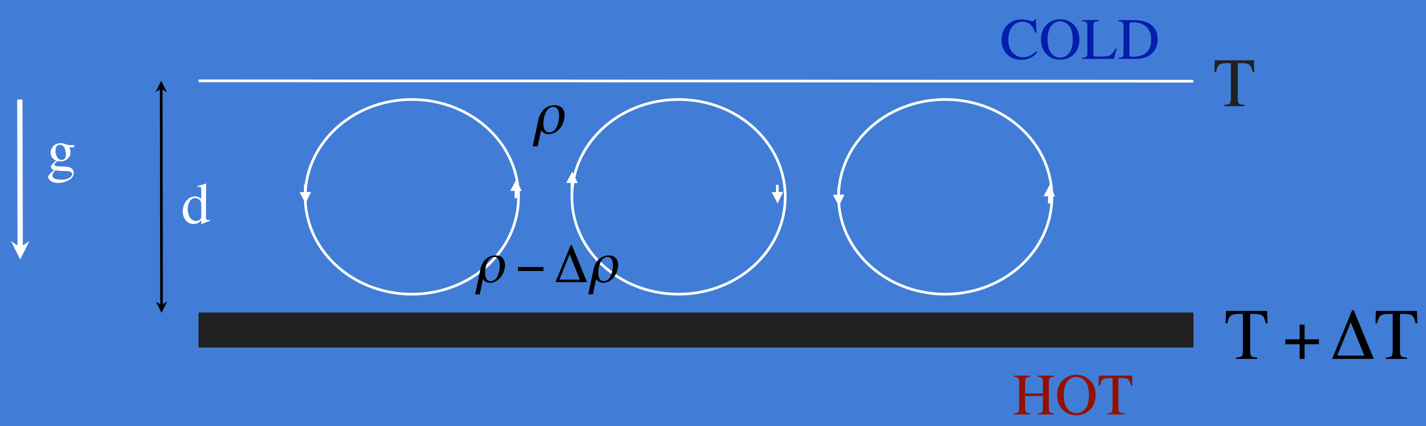


Convection in a plane layer



THERMAL CONVECTION

Rayleigh-Benard $\rho(T) = \rho_0\left[1 + \alpha(T - T_0)\right]$

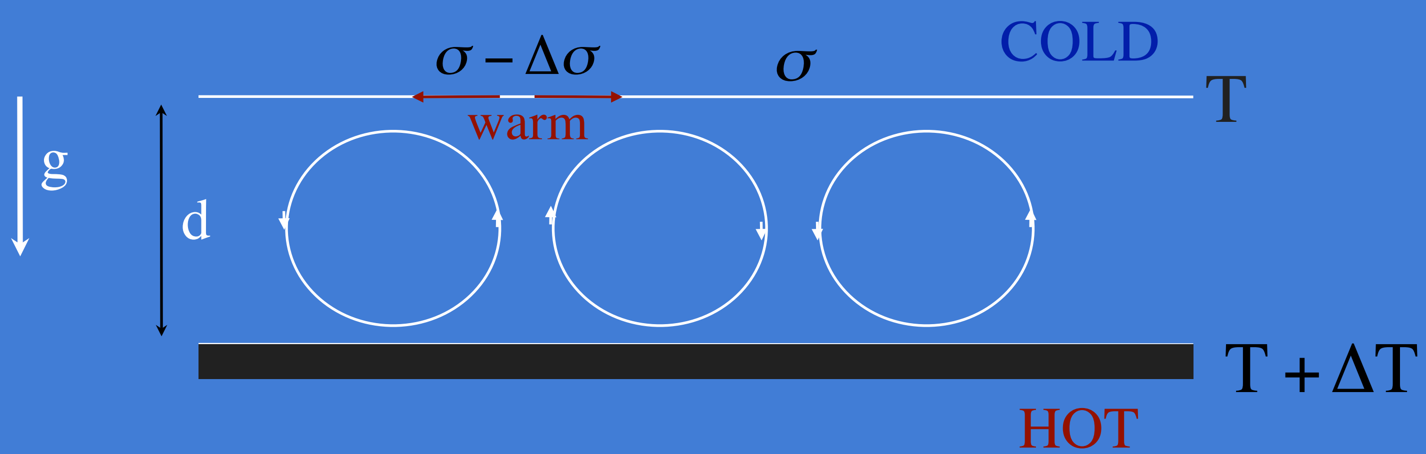


Stability prescribed by:

$$\boxed{Ra = \frac{g\alpha\Delta T d^3}{\kappa\nu}}$$

Rayleigh number

Marangoni-Benard $\sigma(T) = \sigma_0 - \Gamma(T - T_0)$



$$\boxed{Ma = \frac{\Gamma\Delta T d}{\kappa\mu}}$$

Marangoni number

Note: Marangoni convection dominates for thin films

18.357: Lecture 9

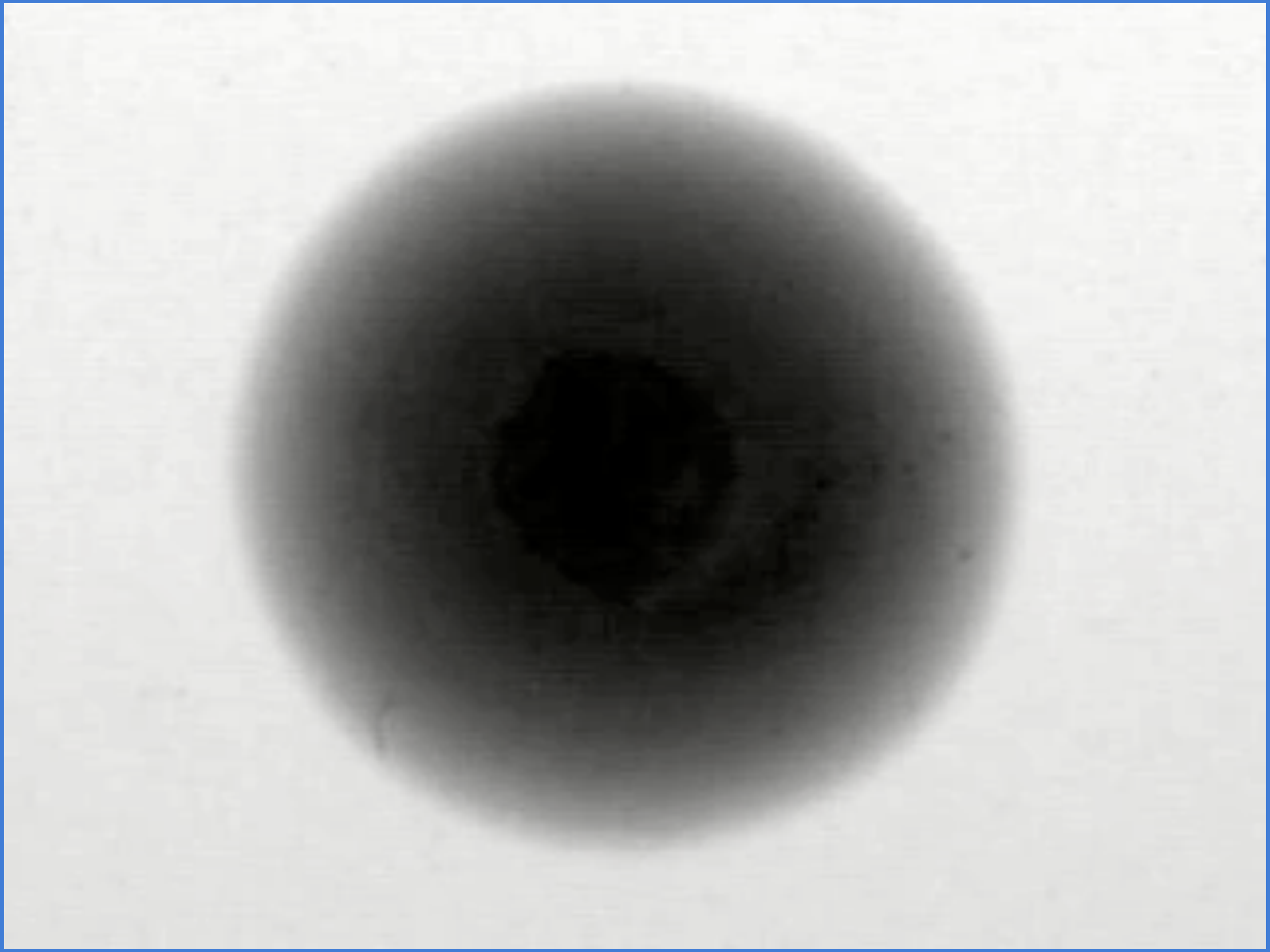
Marangoni Flows II:

Surfactants

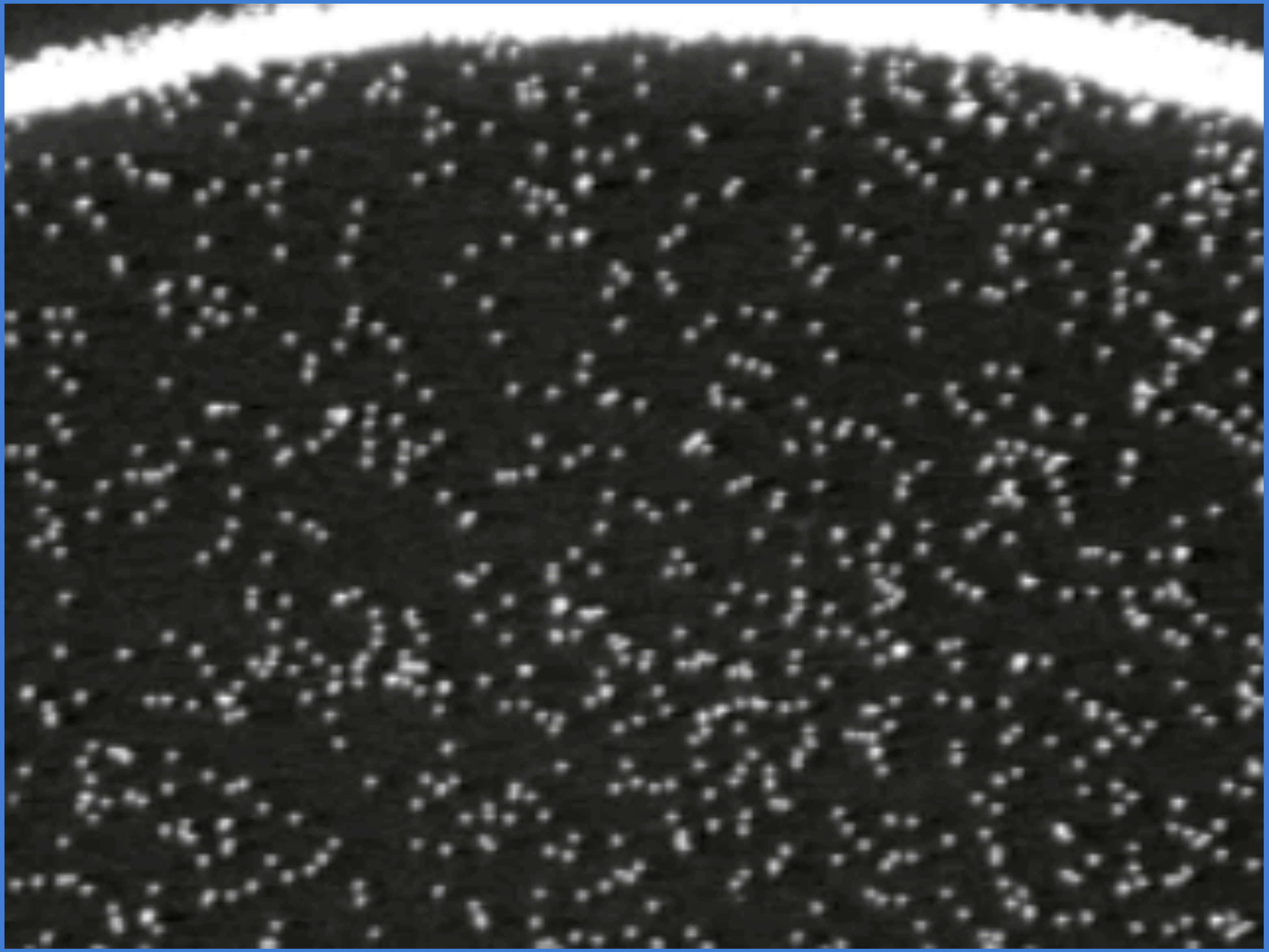
John W. M. Bush

Department of Mathematics
MIT

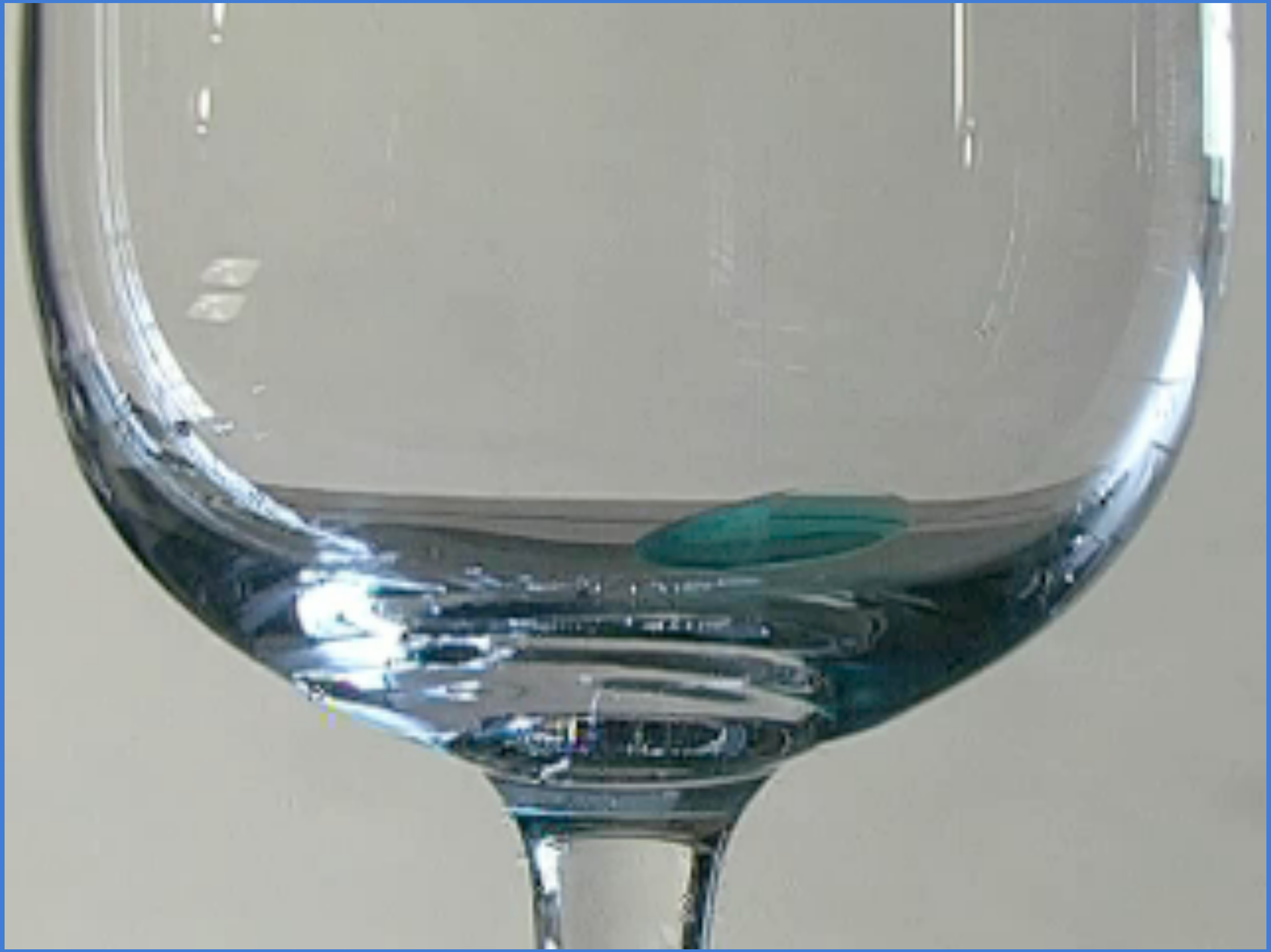
The flow in an evaporating coffee drop



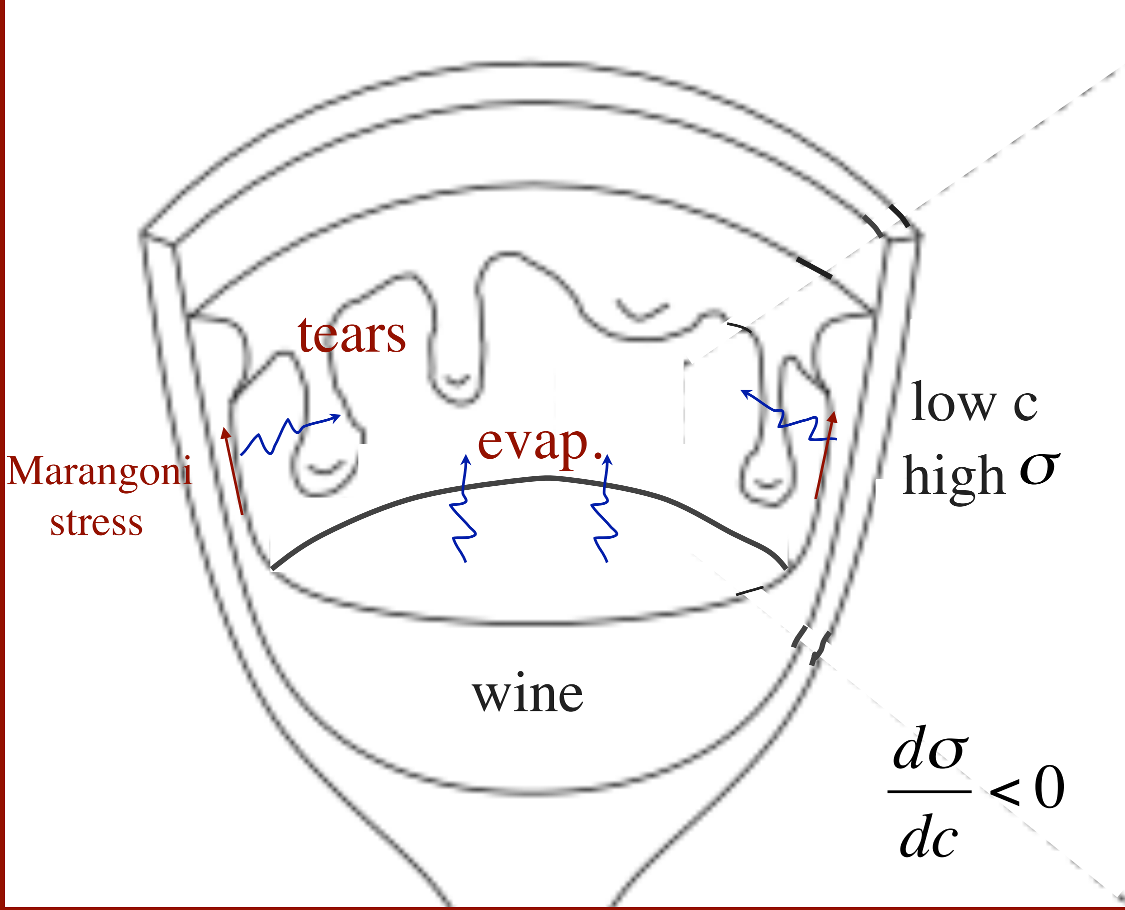
The flow in an evaporating coffee drop



The tears of wine



Note the nibbling tears…

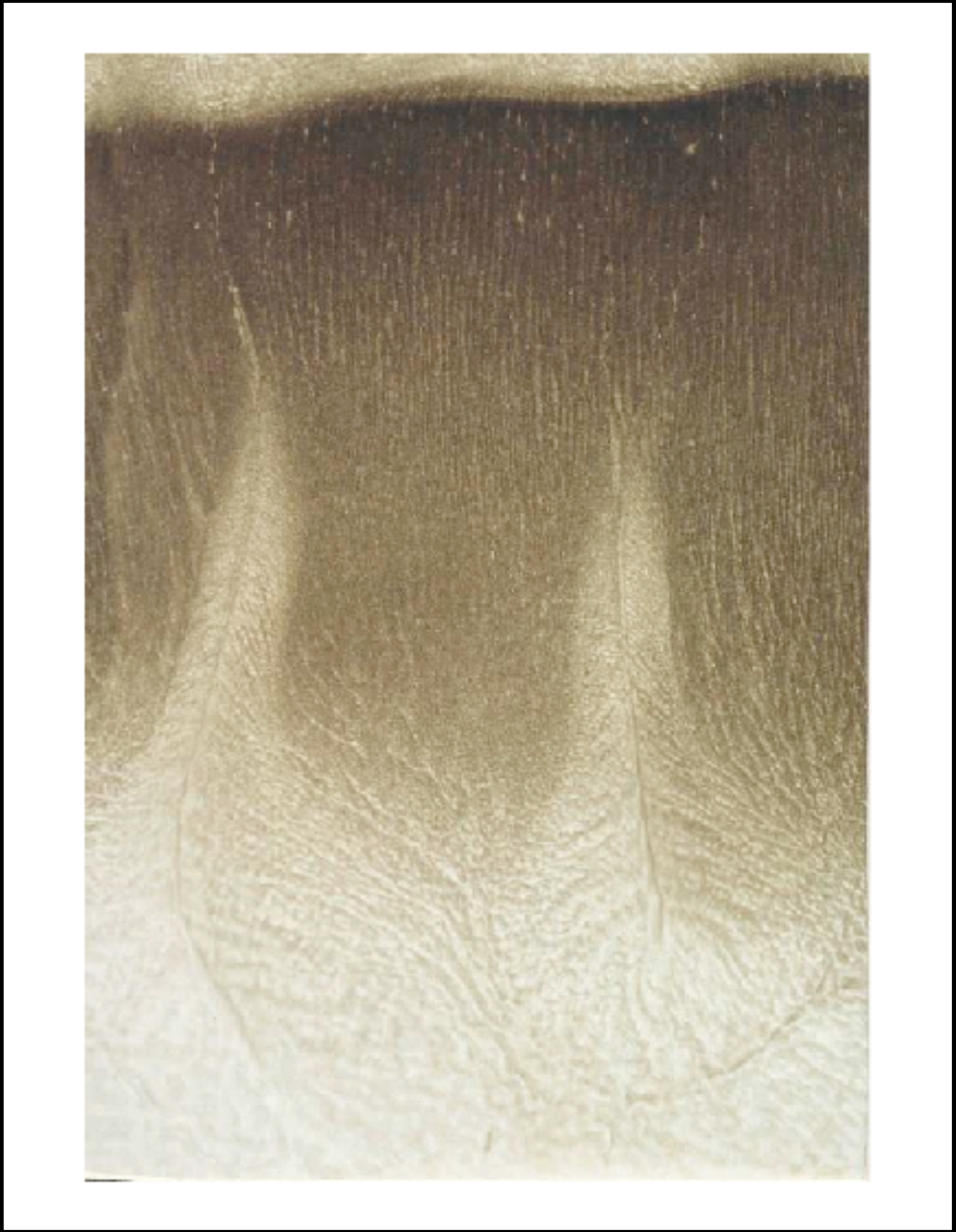


The Tears of Wine   (Thomson 1855)

"Who hath sorrow? Who hath woe? They that tarry long at the wine.
Look not though upon the strong red wine that moveth itself aright.
At the last it biteth like a serpent and stingeth like an adder."

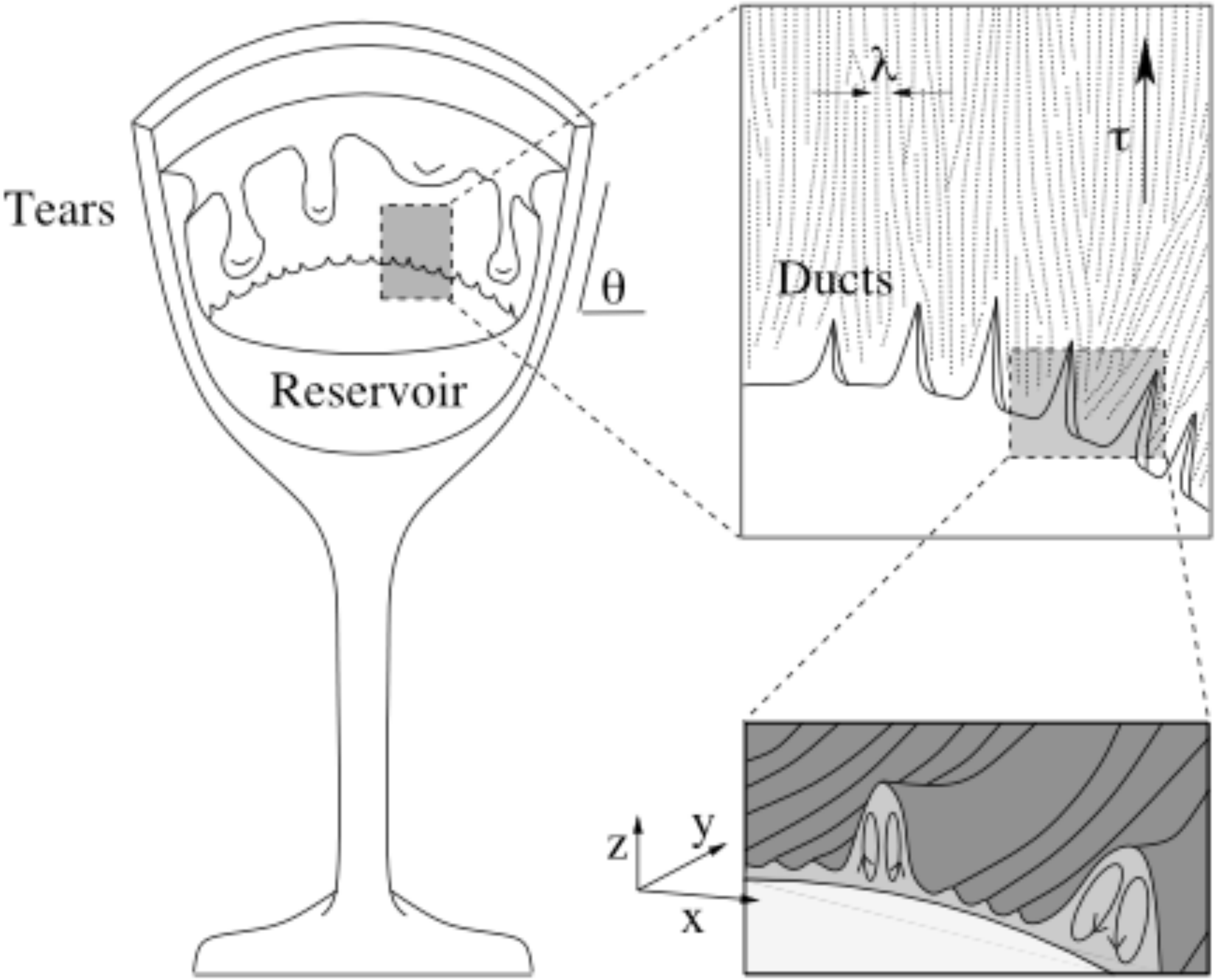
- Proverbs 23: 29-32

Tear line

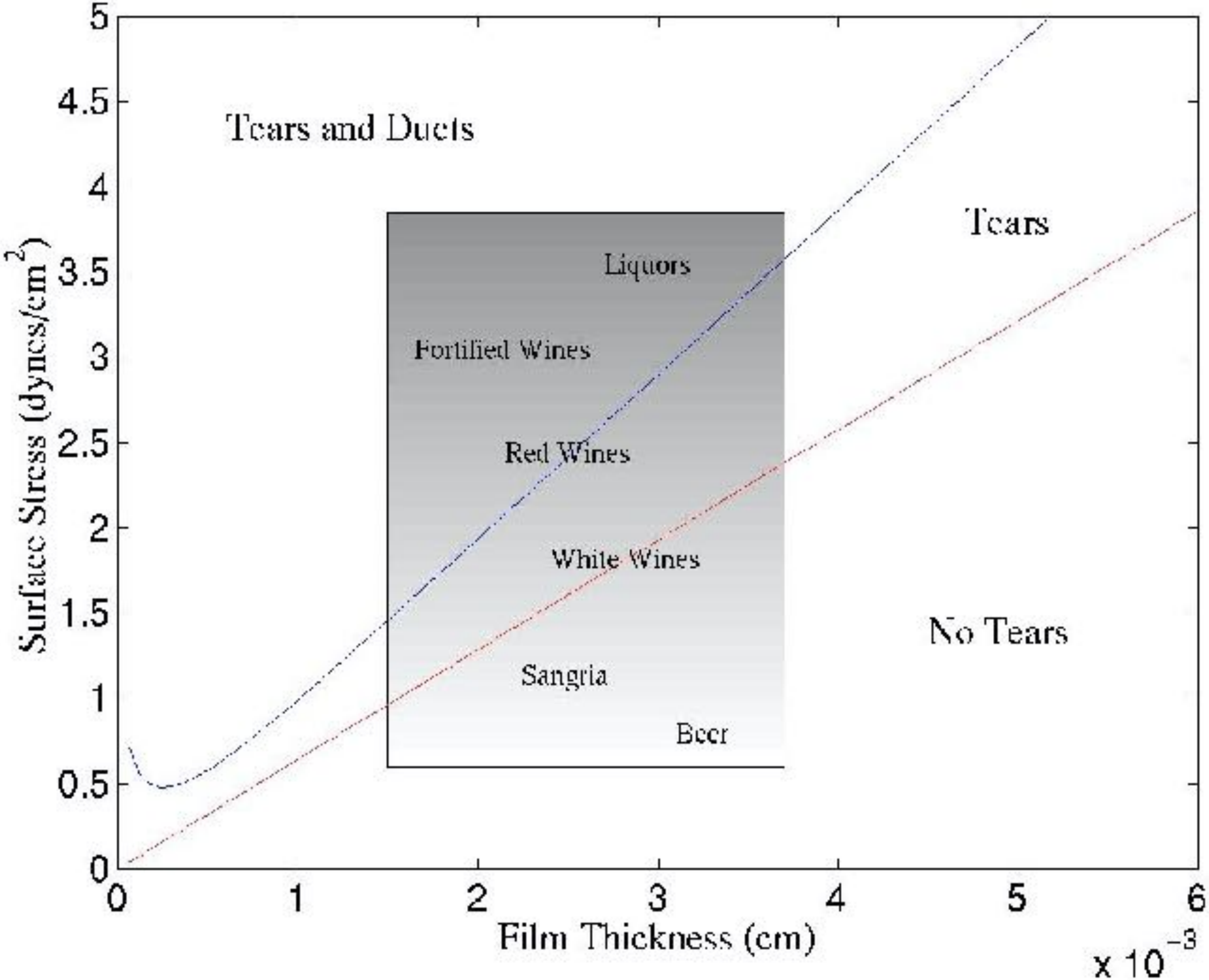


Reservoir

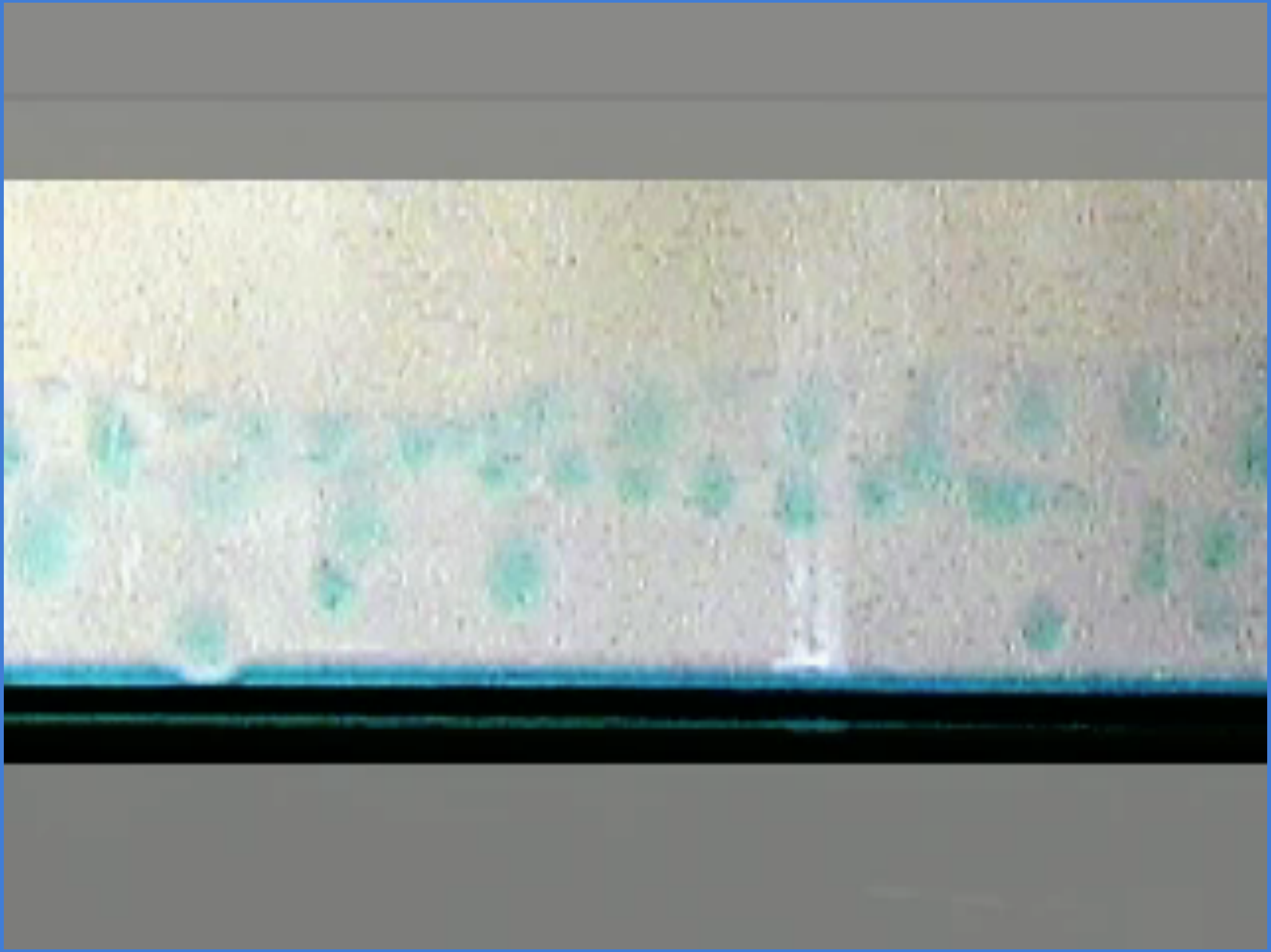
The tear ducts of strong wine (Hosoi & Bush 2001)



- layer marked by streamwise vortices, Marangoni convection rolls


Tears and Ducts
Tears
No Tears
Liquors
Fortified Wines
Red Wines
White Wines
Sangria
Beer
Surface Stress (dynes/cm^2)
Film Thickness (cm)
x 10^{-3}
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
1
2
3
4
5
6

Nibbling tears of wine

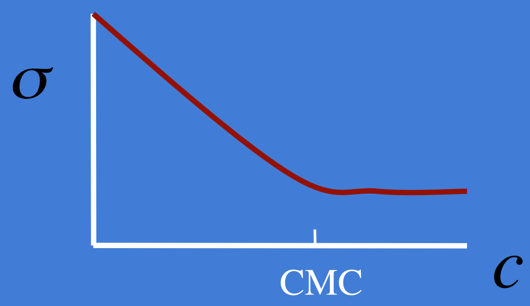


Surfactants: surface-active reagents

- molecules that find it energetically favourable to reside at an interface

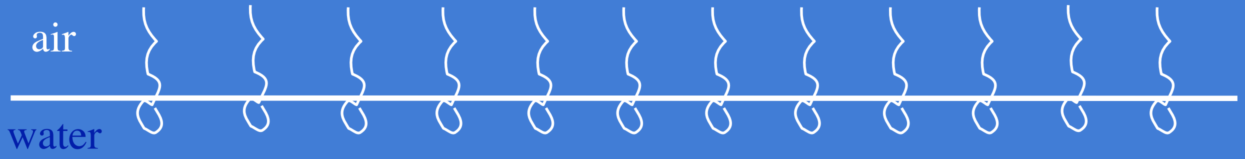
e.g. commercial detergents

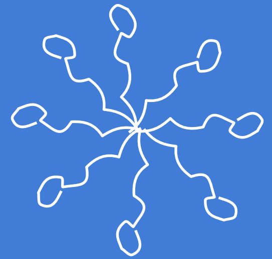
- generally act to reduce σ locally, $\dfrac{d\sigma}{d\Gamma} < 0$: may induce Marangoni flows



e.g. Soap boat

- beyond the CMC (critical micell concentration), there is no further dependence of σ on c

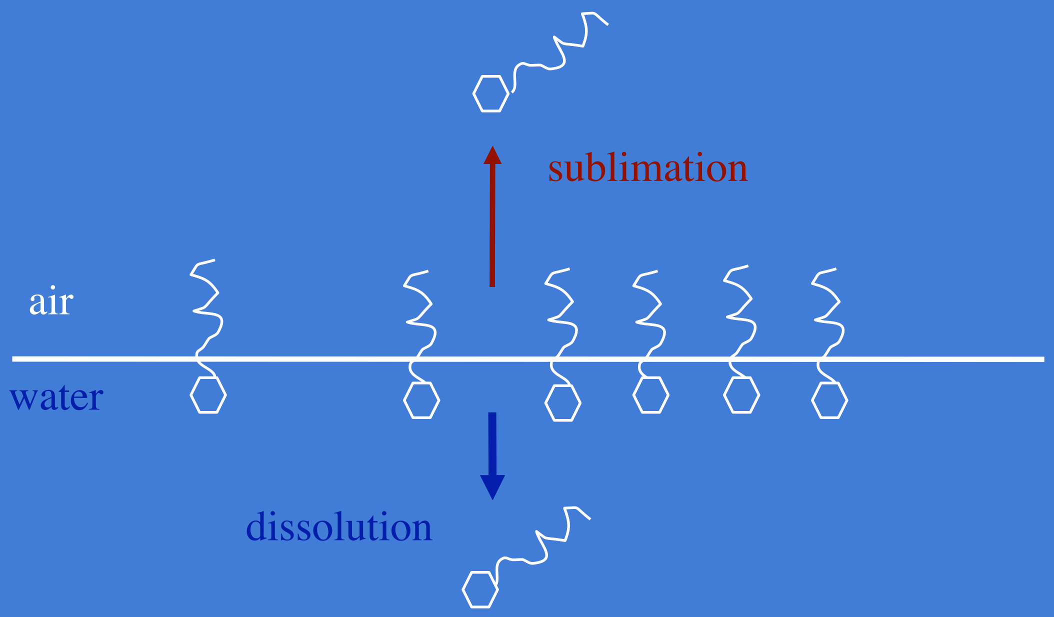




MICELL

micells shed by saturated interface, desorbed into bulk

Surfactant properties



Diffusivity

- prescribes the rate of diffusion, D_s , of a surfactant along an interface
- prescribes the rate of diffusion, D_b , of a surfactant within the bulk

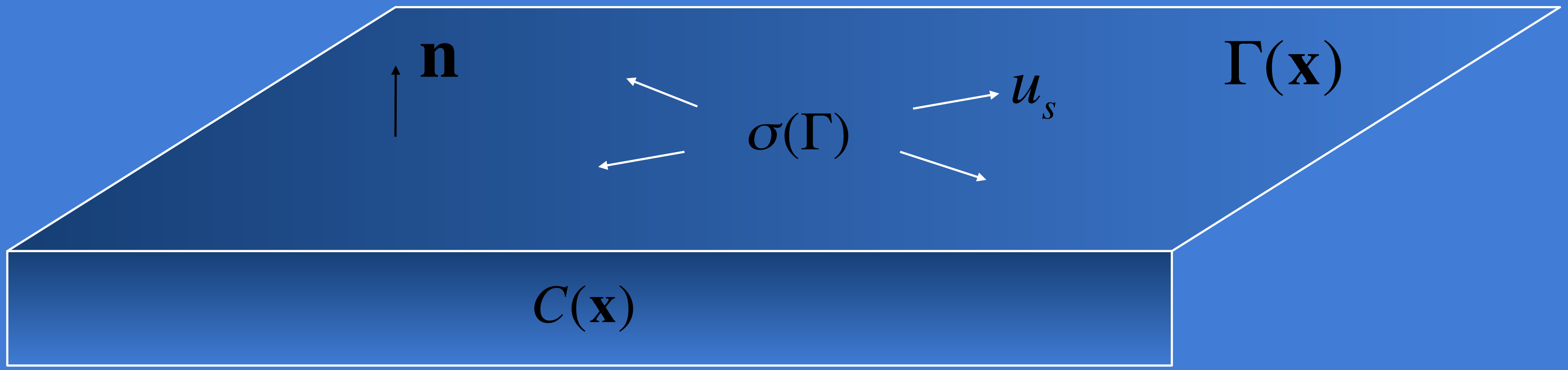
Solubility

- prescribes the ease with which surfactant passes from the surface to the bulk
- an insoluble surfactant cannot dissolve into the bulk, must remain on surface

Volatility

- prescribes the ease with which surfactant sublimates/evaporates from surface

The evolution of a surfactant-laden interface



Since $\sigma(\Gamma)$, N-S equations and BCs must be augmented by

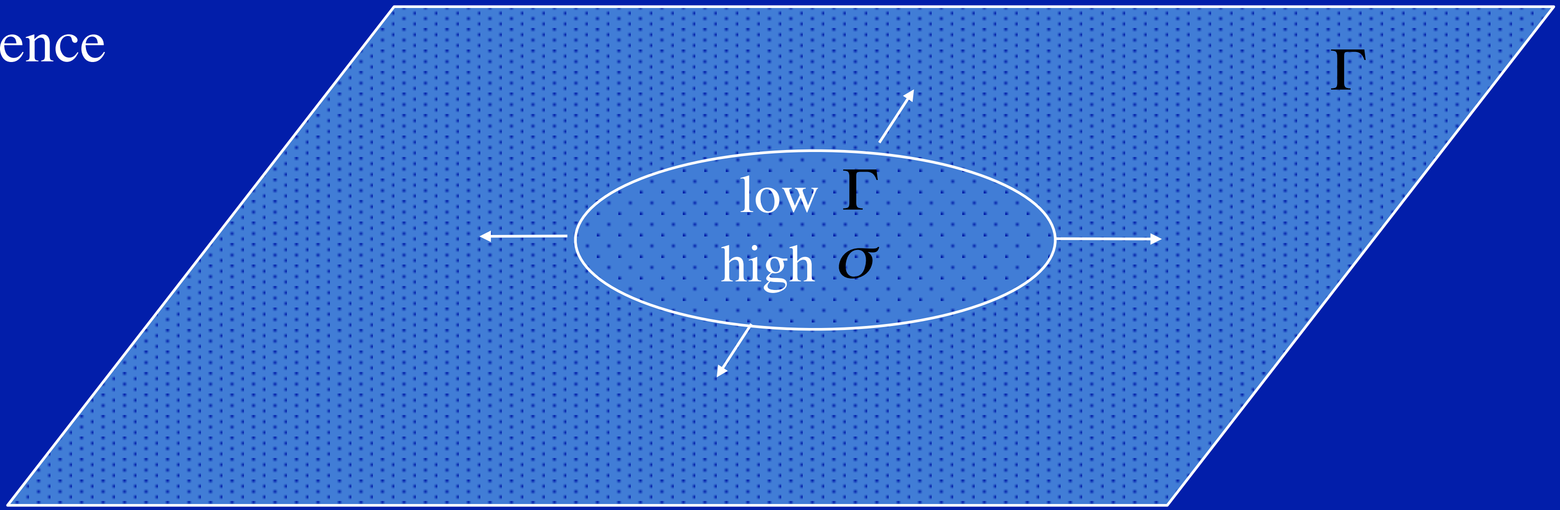
Surfactant evolution equation:

$$\boxed{\frac{\partial \Gamma}{\partial t} + \nabla_s \cdot \left(\Gamma u_s\right) + \Gamma\,(\nabla_s \cdot n)(u \cdot n) \;=\; J(\Gamma, C) \;+\; D_s \nabla_s^2 \Gamma}$$

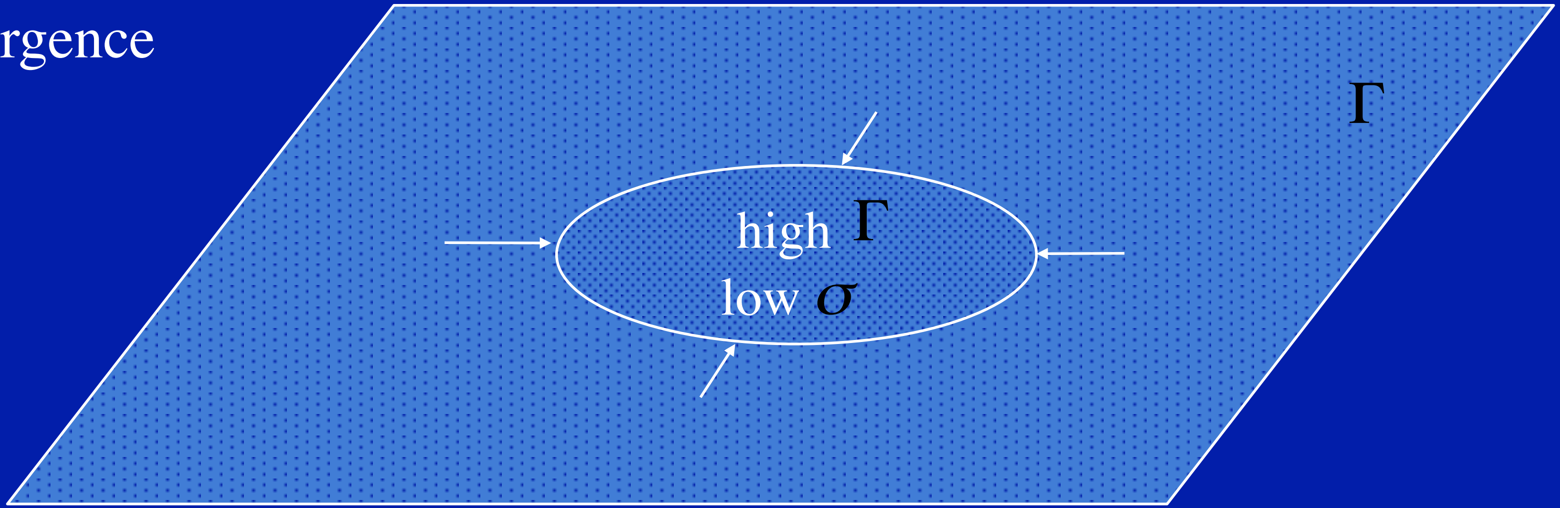
advection	surface expansion	exchange with bulk	surface diffusion

Surfactants: impart effective elasticity to contaminated interfaces through resisting flows with non-zero surface divergence

Surface divergence

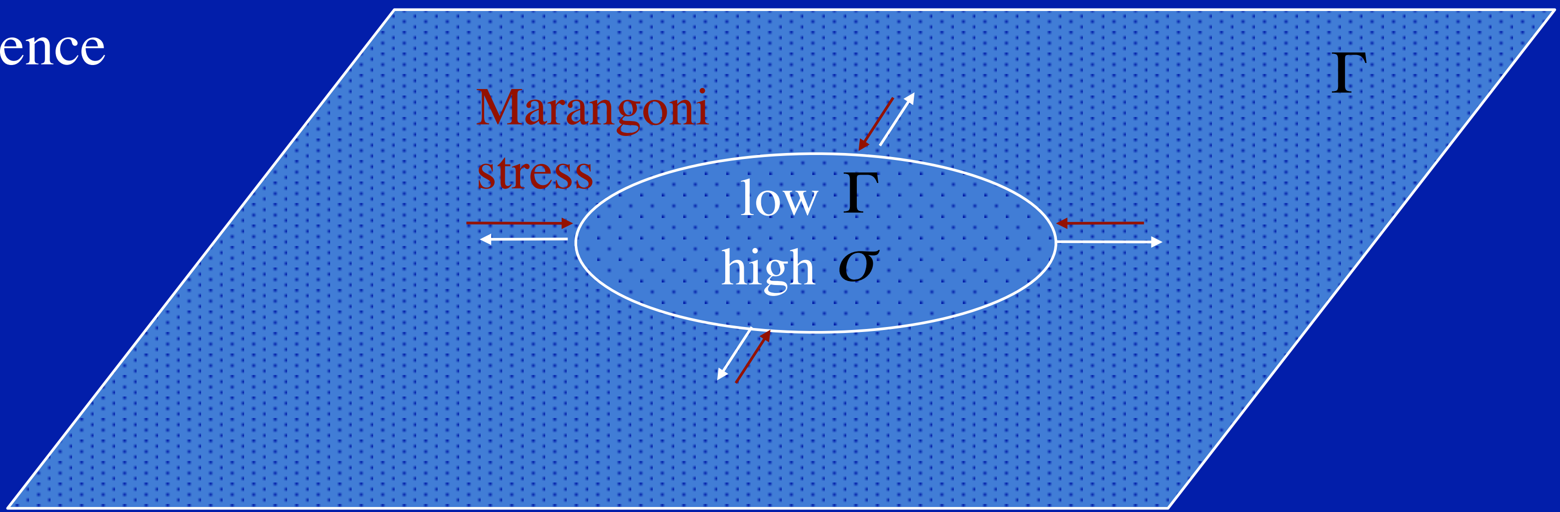


Surface convergence

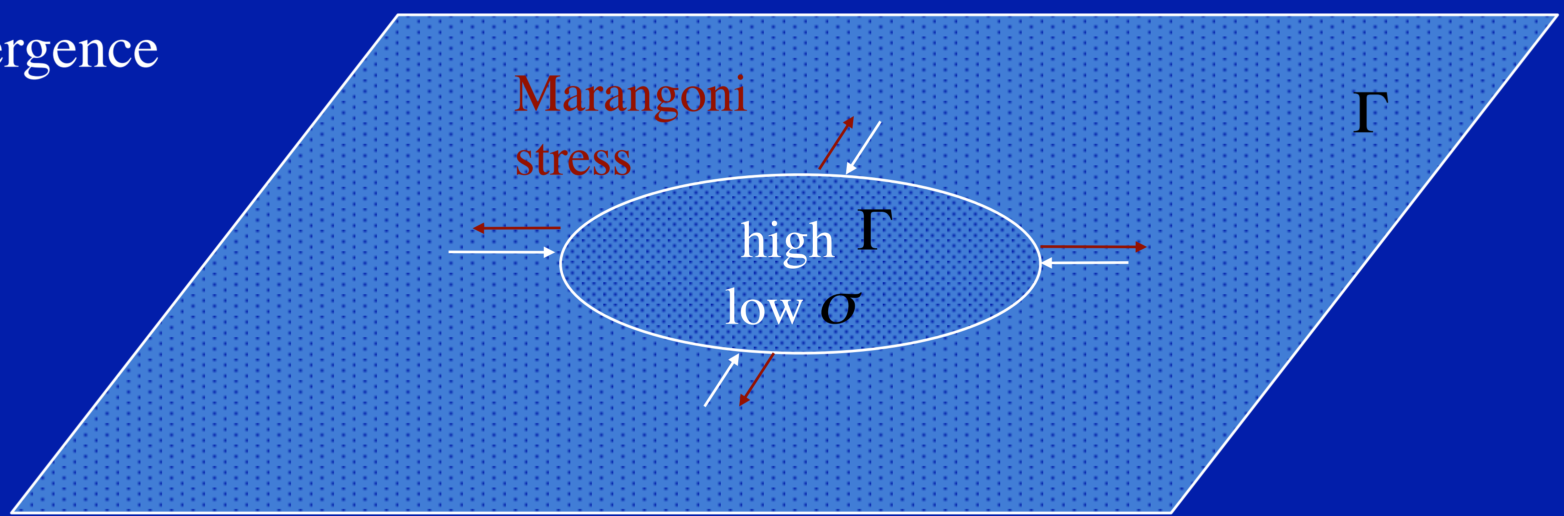


Surfactants: impart effective elasticity to contaminated interfaces through resisting flows with non-zero surface divergence

Surface divergence

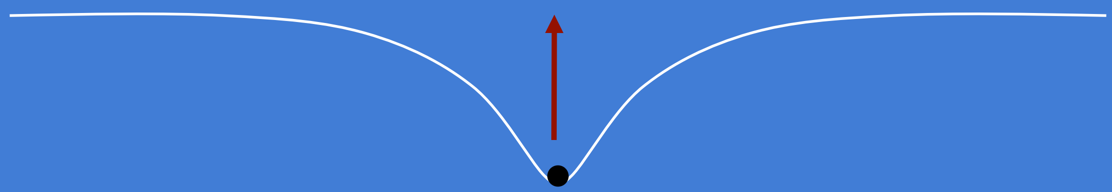


Surface convergence



Clean interface = `slippery trampoline'

- resists deformation through generation of normal curvature pressures
- cannot generate traction on the interface



Surfactant-laden interface = trampoline

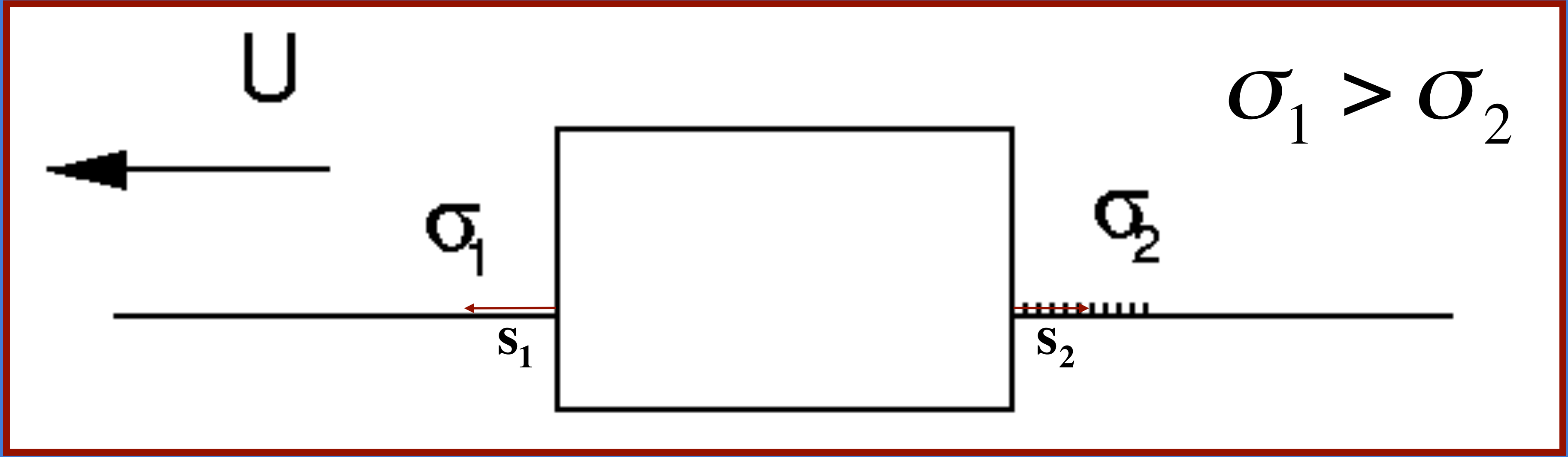
- resists surface deformation as does a clean interface
- can support tangential stresses via Marangoni elasticity

Marangoni propulsion

- lateral force may be generated by surface tension gradient

$$\boxed{F = \int_C \sigma \, \mathbf{s} \, d\ell}$$

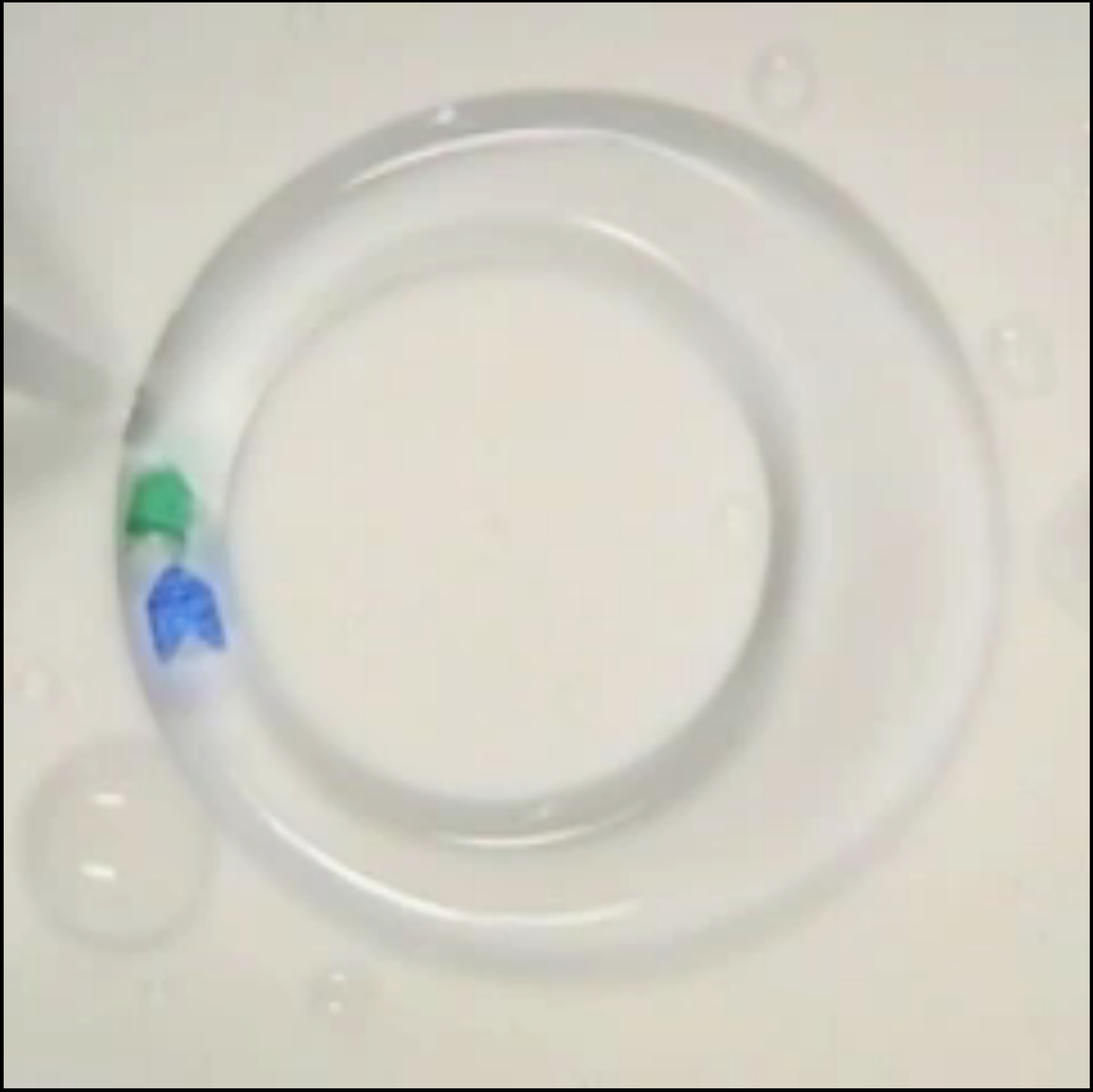
integrate around contact line



e.g. water-walking insects, dispersal of pine needles

- motion driven by soap cannot be sustained in a closed contained
- motion may be sustained if driven by a volatile component (e.g. camphor)

Nakata (2005)



Nakata (2006)

Marangoni Flows

- flows dominated by the influence of surface tension gradients
- $\nabla\sigma$ may arise due to dependence of $\sigma(T, c, \Gamma)$

Video:
Lisa Burton

The cocktail boat: fueled by alcohol

Jose Andres

IOP Publishing Bioinspiration & Biomimetics

Bioinsp. Biomim. 8 (2013) 044003 (6pp) doi:10.1088/1748-3182/8/4/044003

COMMUNICATION

Biomimicry and the culinary arts

Lisa J Burton[1], Nadia Cheng[1], César Vega[2], José Andrés[3] and John W M Bush[4]

[1] Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA, USA
[2] Mars, Inc., McLean, VA
[3] ThinkFoodGroup, Washington DC, USA
[4] Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA, USA

E-mail: bush@math.mit.edu

Received 30 May 2013
Accepted for publication 5 September 2013
Published 4 October 2013
Online at stacks.iop.org/BB/8/044003

Abstract
We present the results of a recent collaboration between scientists, engineers and chefs. Two particular devices are developed, both inspired by natural phenomena reliant on surface tension. The cocktail boat is a drink accessory, a self-propelled edible boat powered by alcohol-induced surface tension gradients, whose propulsion mechanism is analogous to that employed by a class of water-walking insects. The floral pipette is a novel means of serving small volumes of fluid in an elegant fashion, an example of capillary origami modeled after a class of floating flowers. The biological inspiration and mechanics of these two devices are detailed, along with the process that led to their development and deployment.

(Some figures may appear in colour only in the online journal)

1. Introduction

"El gran llibre, sempre obert I que cal esforçar se a llegir, és el de la Naturalesa."[5]

– Antoni Gaudi

dominate those of gravity for fluid systems small relative to the capillary length $\ell_c = \sqrt{\sigma/\rho g}$, where ρ represents the fluid density and g gravity. For air–water systems, the capillary length corresponds roughly to the size of a raindrop. Surface tension is thus an important player in the lives of

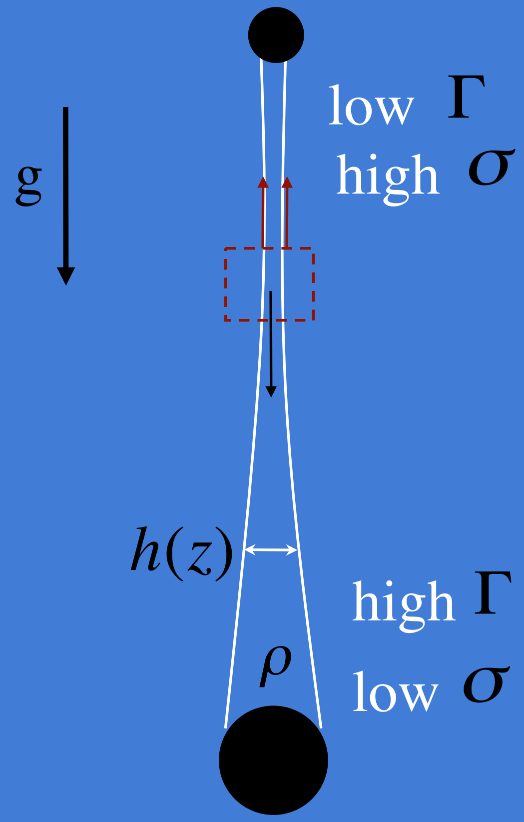
Soap films

Soap films

- stabilized against rupture by presence of surfactants

Draining soap film

- weight of film supported by Marangoni stress



$$\boxed{\rho g\, h(z) = 2\,\frac{d\sigma}{dz} = 2\,\frac{d\sigma}{d\Gamma}\,\frac{d\Gamma}{dz}}$$