18.357: Lecture 8

Marangoni Flows I:
Thermocapillarity

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Governing Equations

Navier-Stokes equations:

\[ \rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{u} , \quad \nabla \cdot \mathbf{u} = 0 \]

Boundary Conditions

Normal stress:

\[ \mathbf{n} \cdot \mathbf{T} \cdot \mathbf{n} \mid = \sigma \nabla \cdot \mathbf{n} \]

Tangential stress:

\[ \mathbf{n} \cdot \mathbf{T} \cdot \mathbf{s} \mid = \nabla_s \sigma \]

Stress tensor

\[ \mathbf{T} = -p \mathbf{I} + \mu \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) \]


Marangoni Flows

- flows dominated by the influence of surface tension gradients

Recall tangential stress BC: \( \Delta n \cdot T \cdot s = \nabla \sigma \)

- \( \nabla \sigma \) may arise due to dependence of \( \sigma(T, c, \Gamma) \)
Thermocapillary Flows

- Marangoni flows induced by temperature gradients $\sigma(T)$
- Navier-Stokes coupled to heat equation through BCs

\[ \frac{\partial T}{\partial t} + u \cdot \nabla T = \kappa \nabla^2 T \]

E.g. thermocapillary drop motion: drops migrate toward heat source

Young, Goldstein & Block (1962)
Convection in a plane layer
Convection in a plane layer
THERMAL CONVECTION

Rayleigh-Benard

\[ \rho(T) = \rho_0[1 + \alpha(T - T_0)] \]

Stability prescribed by:

\[ Ra = \frac{g\alpha \Delta T d^3}{\kappa \nu} \]

Rayleigh number

Marangoni-Benard

\[ \sigma(T) = \sigma_0 - \Gamma(T - T_0) \]

\[ Ma = \frac{\Gamma \Delta T d}{\kappa \mu} \]

Marangoni number

Note: Marangoni convection dominates for thin films
The flow in an evaporating coffee drop
The flow in an evaporating coffee drop
The tears of wine

Note the nibbling tears…
The Tears of Wine  (Thomson 1855)

“Who hath sorrow? Who hath woe? They that tarry long at the wine. Look not though upon the strong red wine that moveth itself aright. At the last it biteth like a serpent and stingeth like an adder.”

- Proverbs 23: 29-32
The tear ducts of strong wine  (Hosoi & Bush 2001)

- layer marked by streamwise vortices, Marangoni convection rolls
Tears and Ducts

Surface Stress (dynes/cm²)

Film Thickness (cm)

Tears

No Tears

Liquors

Fortified Wines

Red Wines

White Wines

Sangria

Beer
Nibbling tears of wine
Surfactants: surface-active reagents

- molecules that find it energetically favourable to reside at an interface
e.g. commercial detergents

- generally act to reduce $\sigma$ locally, $\frac{d\sigma}{d\Gamma} < 0$ : may induce Marangoni flows
e.g. Soap boat

- beyond the CMC (critical micell concentration), there is no further dependence of $\sigma$ on $c$

micells shed by saturated interface, desorbed into bulk
**Surfactant properties**

**Diffusivity**
- prescribes the rate of diffusion, $D_s$, of a surfactant along an interface
- prescribes the rate of diffusion, $D_b$, of a surfactant within the bulk

**Solubility**
- prescribes the ease with which surfactant passes from the surface to the bulk
- an insoluble surfactant cannot dissolve into the bulk, must remain on surface

**Volutility**
- prescribes the ease with which surfactant sublimates/evaporates from surface
The evolution of a surfactant-laden interface

Since \( \sigma(\Gamma) \), N-S equations and BCs must be augmented by

**Surfactant evolution equation:**

\[
\frac{\partial \Gamma}{\partial t} + \nabla_s \cdot (\Gamma \mathbf{u}_s) + \Gamma (\nabla_s \cdot \mathbf{n})(\mathbf{u} \cdot \mathbf{n}) = J(\Gamma, C) + D_s \nabla^2_s \Gamma
\]

- advection
- surface expansion
- exchange with bulk
- surface diffusion
Surfactants: impart effective elasticity to contaminated interfaces through resisting flows with non-zero surface divergence.
**Surfactants:** impart effective elasticity to contaminated interfaces through resisting flows with non-zero surface divergence.
Clean interface = `slippery trampoline’

- resists deformation through generation of normal curvature pressures
- cannot generate traction on the interface

Surfactant-laden interface = trampoline

- resists surface deformation as does a clean interface
- can support tangential stresses via Marangoni elasticity
Marangoni propulsion

- lateral force may be generated by surface tension gradient

\[ F = \int_{C} \sigma s \, dl \]

Integrate around contact line

- motion driven by soap cannot be sustained in a closed contained
- motion may be sustained if driven by a volatile component (e.g. camphor)

e.g. water-walking insects, dispersal of pine needles
Nakata (2005)
Nakata (2006)
Marangoni Flows

- flows dominated by the influence of surface tension gradients
- $\nabla \sigma$ may arise due to dependence of $\sigma(T, c, \Gamma)$

The cocktail boat: fueled by alcohol

Video: Lisa Burton
Jose Andres
COMMUNICATION

Biomimicry and the culinary arts

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Abstract

We present the results of a recent collaboration between scientists, engineers and chefs. Two particular devices are developed, both inspired by natural phenomena reliant on surface tension. The cocktail boat is a drink accessory, a self-propelled edible boat powered by alcohol-induced surface tension gradients, whose propulsion mechanism is analogous to that employed by a class of water-walking insects. The floral pipette is a novel means of serving small volumes of fluid in an elegant fashion, an example of capillary origami modeled after a class of floating flowers. The biological inspiration and mechanics of these two devices are detailed, along with the process that led to their development and deployment.

(Some figures may appear in colour only in the online journal)

1. Introduction

“El gran libro, siempre obre I que cal esforçar se a llegir, és el de la Naturalesa.”

— Antoni Gaudi
donate those of gravity for fluid systems small relative to the capillary length \( \ell_c = \sqrt{\sigma / \rho g} \), where \( \rho \) represents the fluid density and \( g \) gravity. For air–water systems, the capillary length corresponds roughly to the size of a raindrop. Surface tension is thus an important player in the lives of
Soap films
Soap films

- stabilized against rupture by presence of surfactants

Draining soap film

- weight of film supported by Marangoni stress

\[ \rho g h(z) = 2 \frac{d\sigma}{d\Gamma} \frac{d\Gamma}{dz} = 2 \frac{d\sigma}{d\Gamma} \frac{d\Gamma}{dz} \]