18.357: Lecture 3

Surface tension:

Capillary pressure, scaling, wetting

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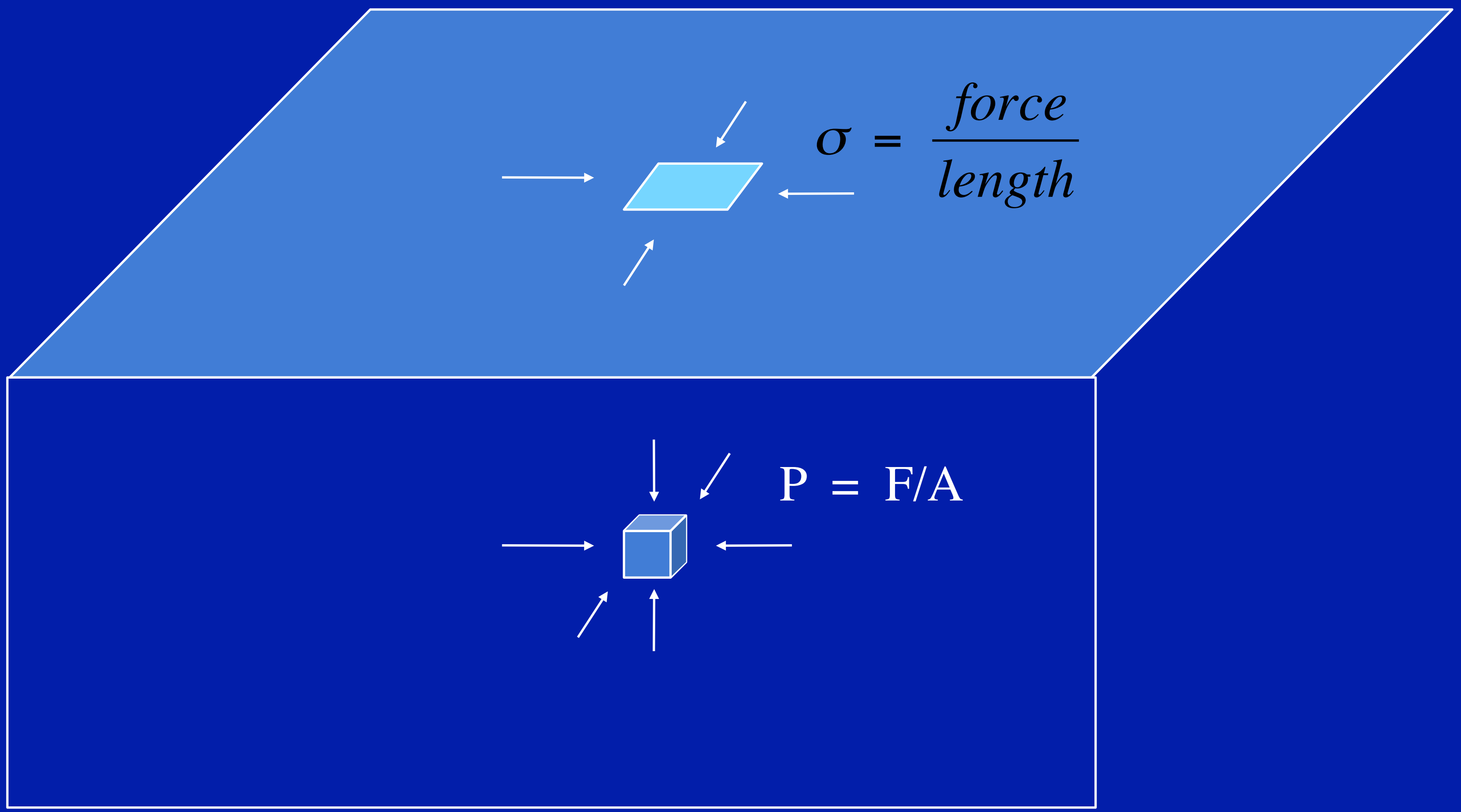
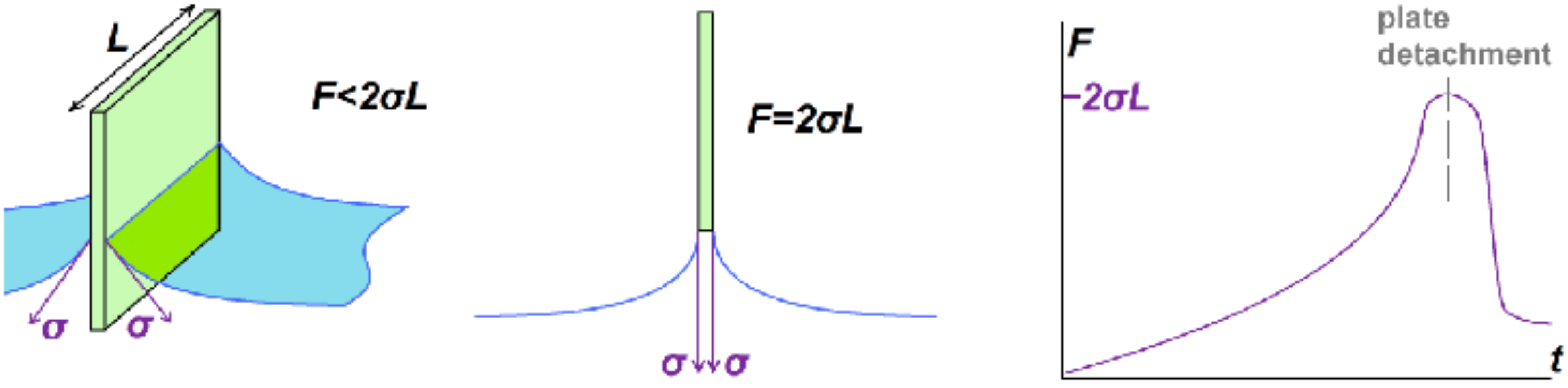
Surface tension: analogous to a negative surface pressure

- gradients in surface tension necessarily drive surface motion



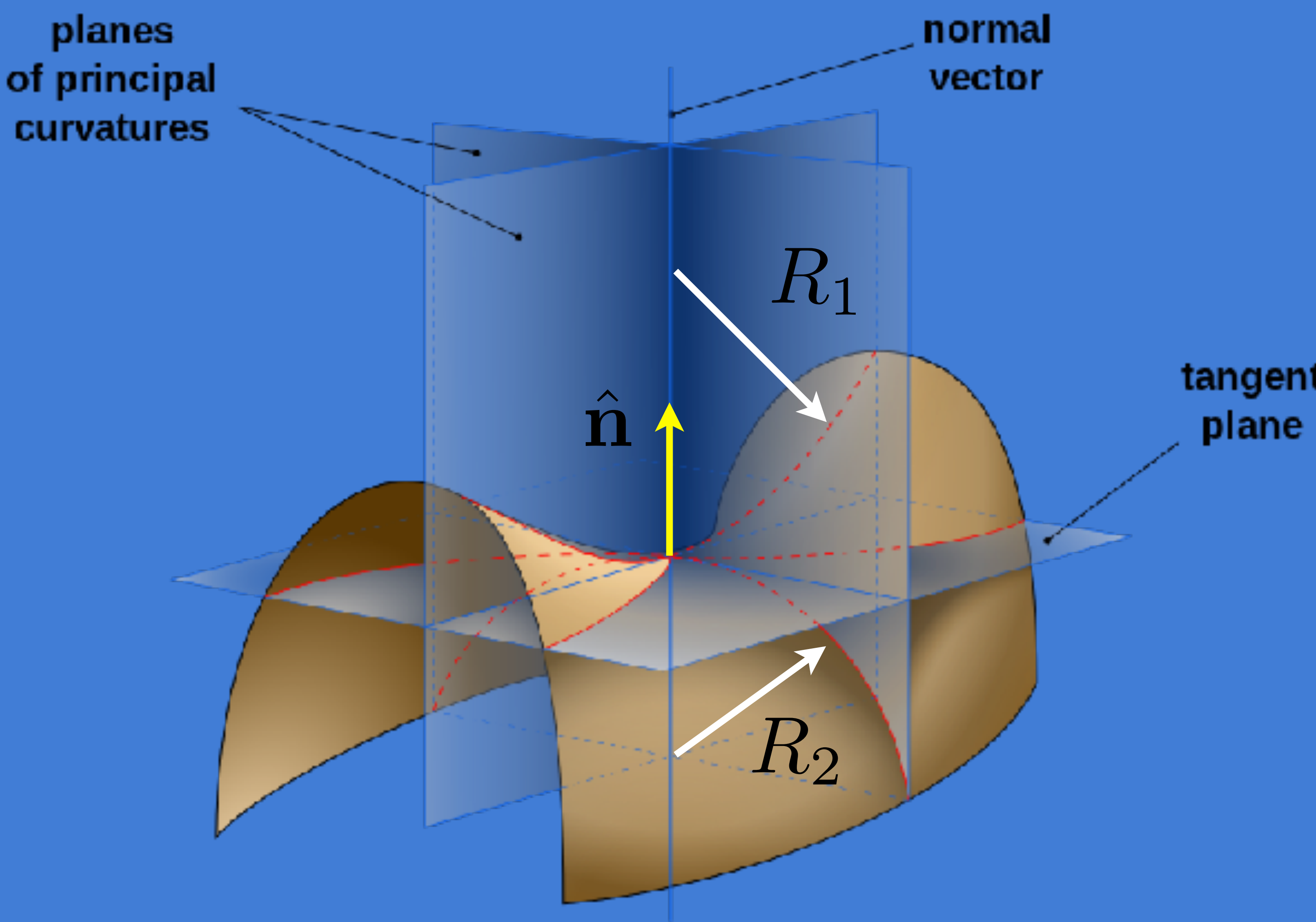
A simple way to measure surface tension



- measure the force required to withdraw a plate from a free surface

NOTES

Curvature



$$\nabla \cdot \hat{\mathbf{n}} = \frac{1}{R_1} + \frac{1}{R_2}$$

where R_1 , R_2 are the principal radii of curvature

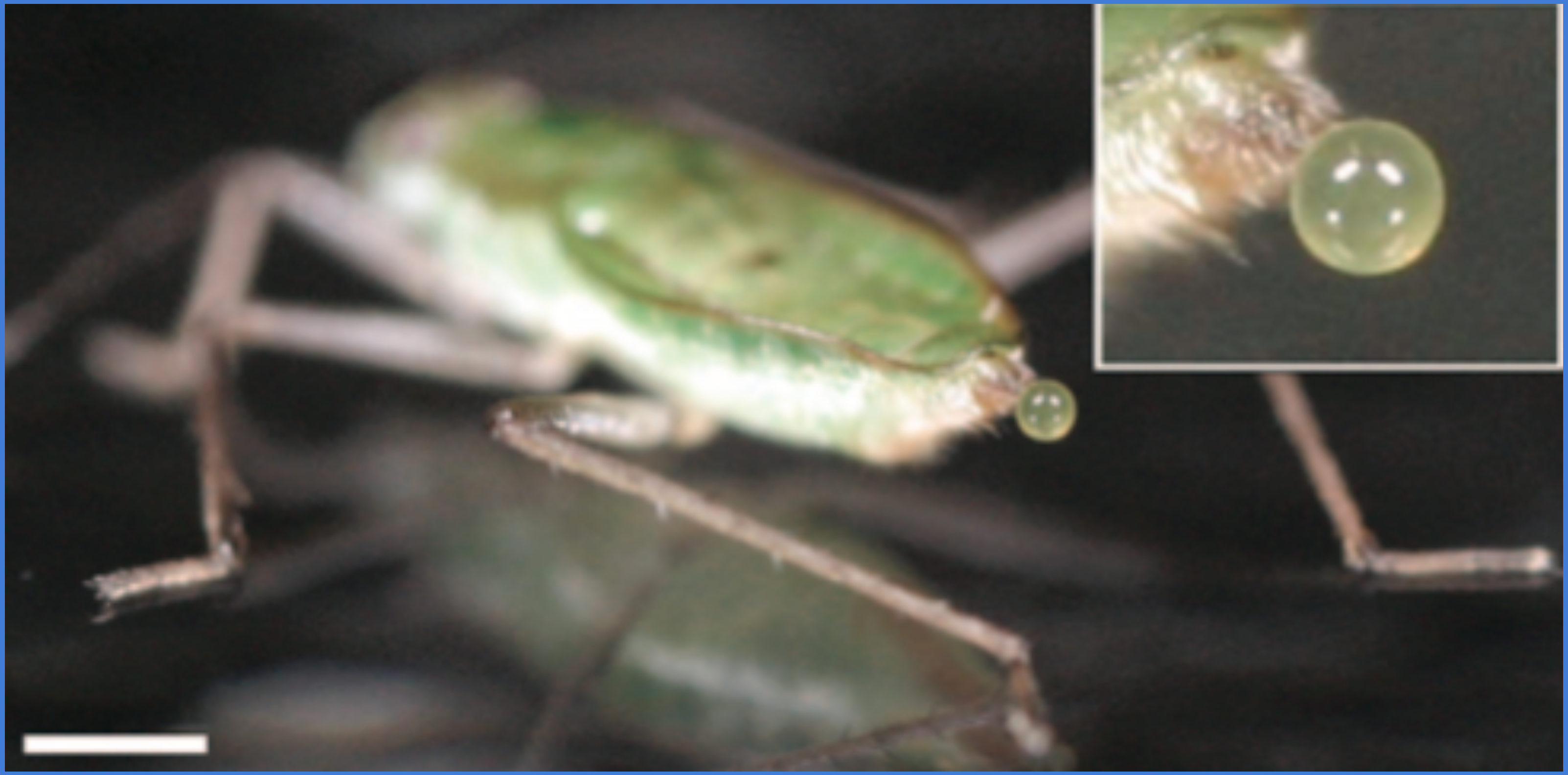
Curvature: $\nabla \cdot \hat{\mathbf{n}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ = 2/R for a sphere

Capillary pressure: $\Delta p = \dfrac{4\sigma}{R}$

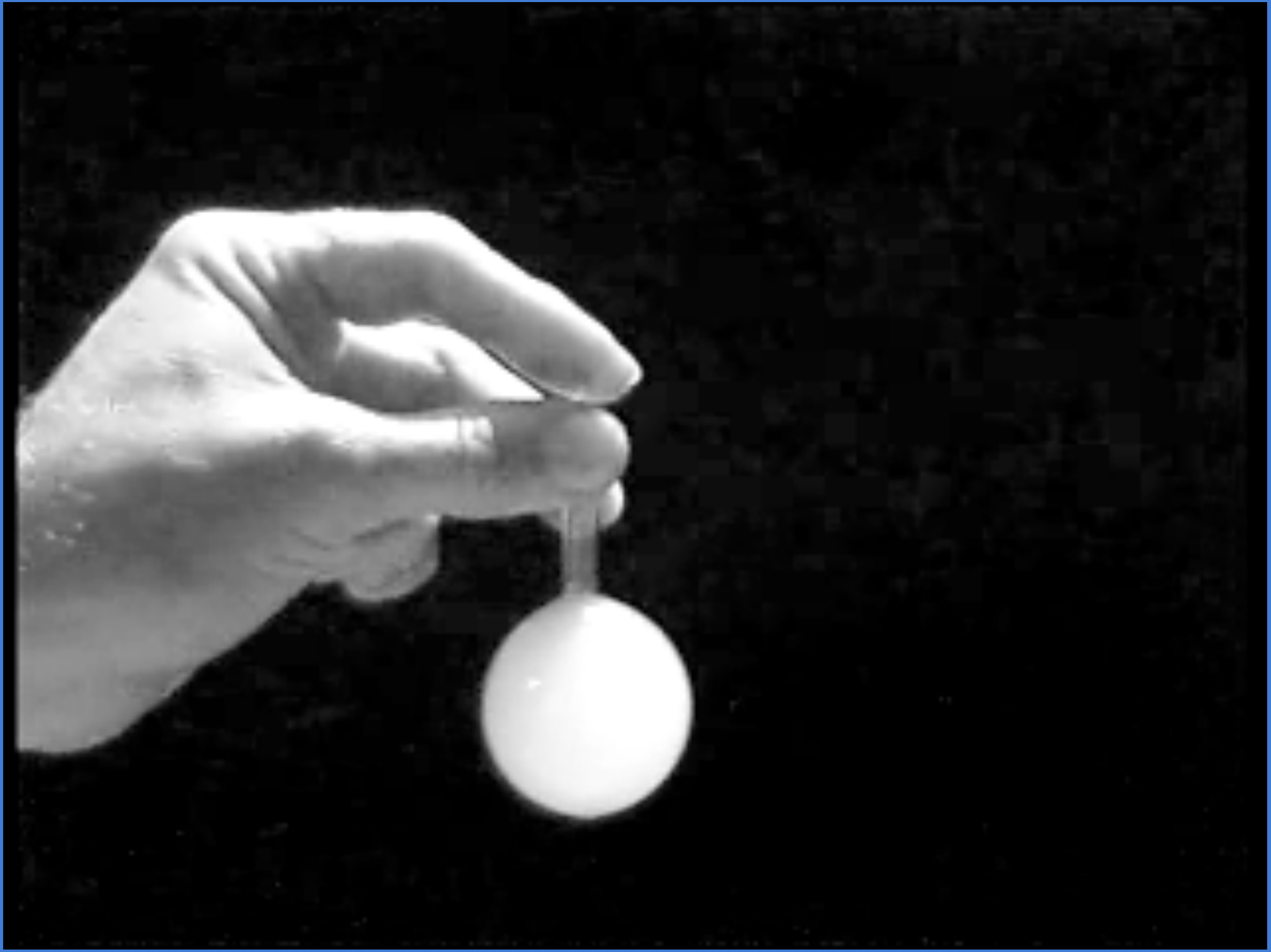
Which way does the air go?

Who cares about surface tension?

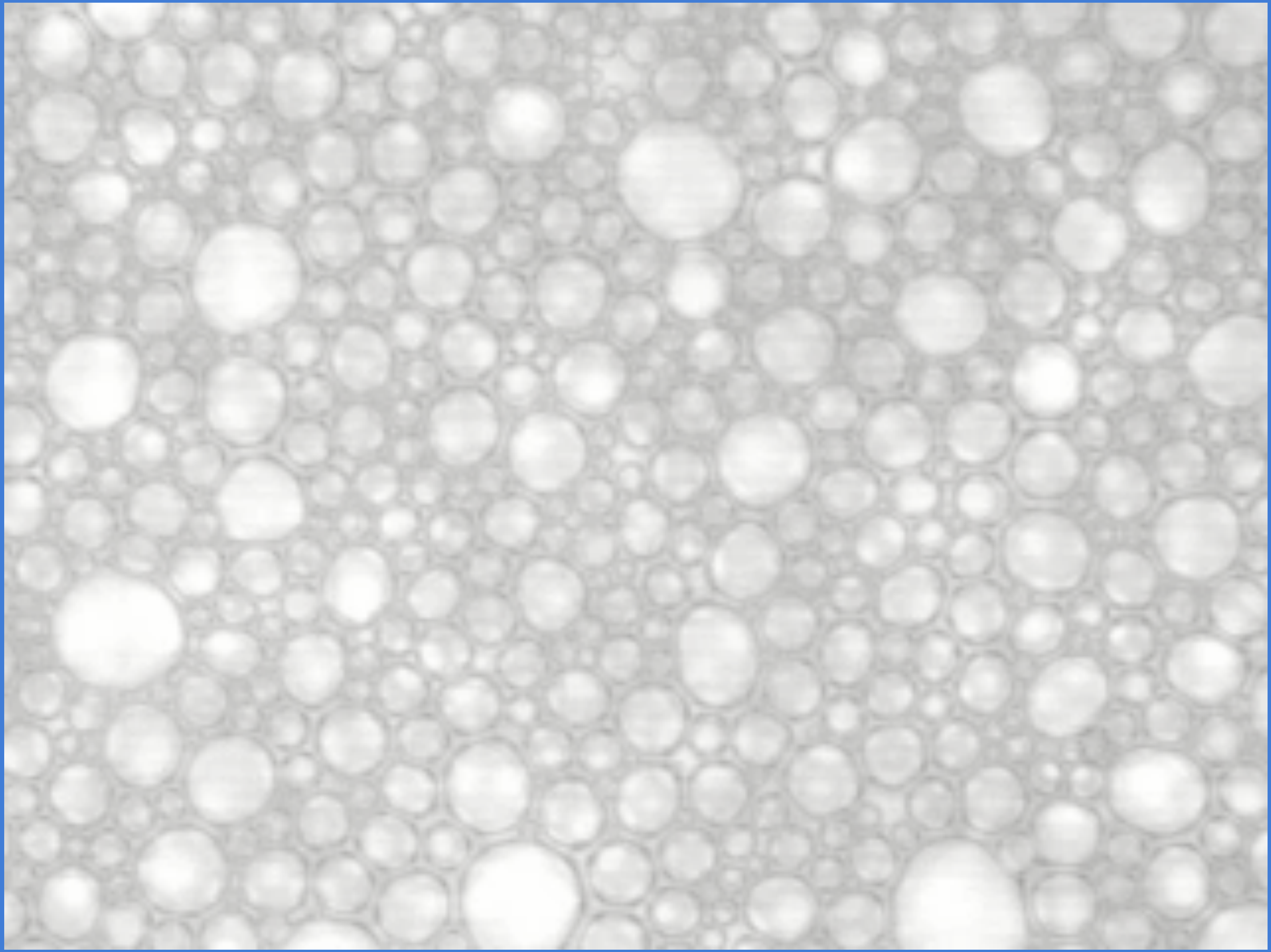
Capillary pressures in biology



1 mm



Ostwald ripening



The scaling of surface tension

DIMENSIONAL ANALYSIS

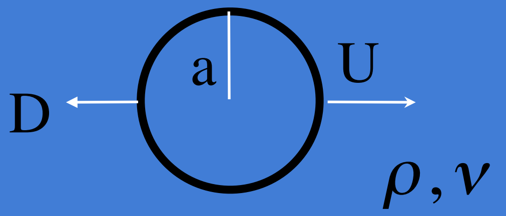
Fundamental Concept

The laws of Nature cannot depend on arbitrarily chosen system of units.
A system is most succinctly described in terms of dimensionless variables.

Deduction of Dimensionless groups: **Buckingham's Theorem**

For a system with M physical variables (e.g. density, speed, length, viscosity) describable in terms of N fundamental units (e.g. mass, length, time, temperature), there are M - N dimensionless groups that govern the system.

E.g. Translation of a sphere



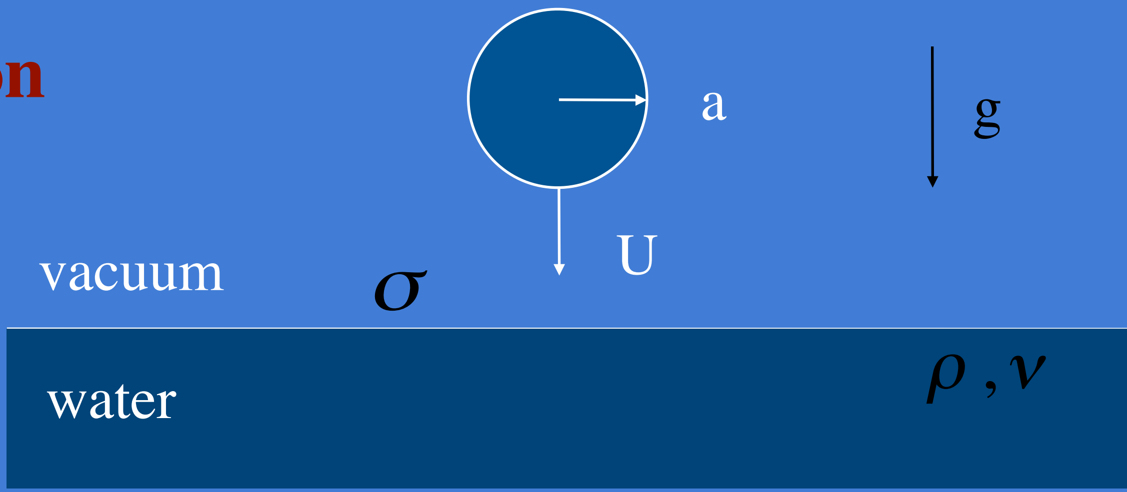
Physical variables: $U, \mathrm{a}, \nu, \rho, D \Rightarrow$ M = 5

Fundamental units: M , L , T $\Rightarrow$ N = 3

M - N = 2 dimensionless groups: $C_d = \dfrac{D}{\rho U^2}$, $\mathrm{Re} = \dfrac{U\mathrm{a}}{\nu}$

System uniquely determined by a single relation: $C_d = F(\mathrm{Re})$

The scaling of surface tension



$$W_e = \frac{\rho U^2 a}{\sigma} = \frac{\text{INERTIA}}{\text{CURVATURE}} = \text{Weber number}$$

$$C_a = \frac{\rho \nu U}{\sigma} = \frac{\text{VISCOSITY}}{\text{CURVATURE}} = \text{Capillary number}$$

$$B_o = \frac{\rho g a^2}{\sigma} = \frac{\text{GRAVITY}}{\text{CURVATURE}} = \text{Bond number}$$

Note: σ is dominant relative to gravity when $B_o < 1$

i.e. $a < \left(\frac{\sigma}{\rho g}\right)^{1/2} = \ell_c =$ capillary length ~ 2mm for air-water

When is surface tension important relative to gravity?

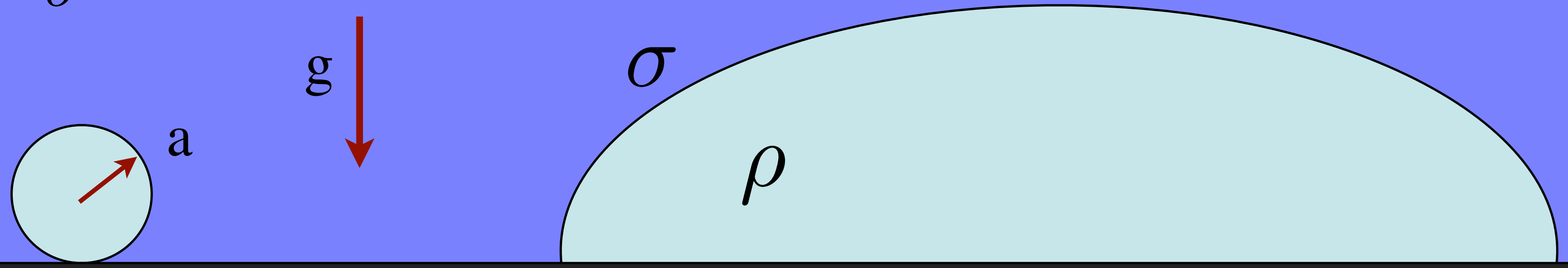
- when curvature pressures are large relative to hydrostatic:

Bond number: $$B_o = \frac{\rho g a}{\sigma / a} = \frac{\rho g a^2}{\sigma} < 1$$

i.e. for drops small relative to the capillary length:

$$\boxed{a < l_c = \left(\frac{\sigma}{\rho g}\right)^{1/2}} \sim$$ 2 mm for air-water ($\sigma = 70$ dynes/cm)

$B_o \ll 1$ $\qquad$ $B_o > 1$



Surface tension dominates the world of insects - and of microfluidics.

Falling rain drops

Force balance:

$$\rho_a U^2 a^2 \sim \; M g = \tfrac{4}{3}\pi a^3 \rho g$$

Fall speed: $U \sim \sqrt{\dfrac{\rho\, ga}{\rho_a}}$

Drop integrity requires:

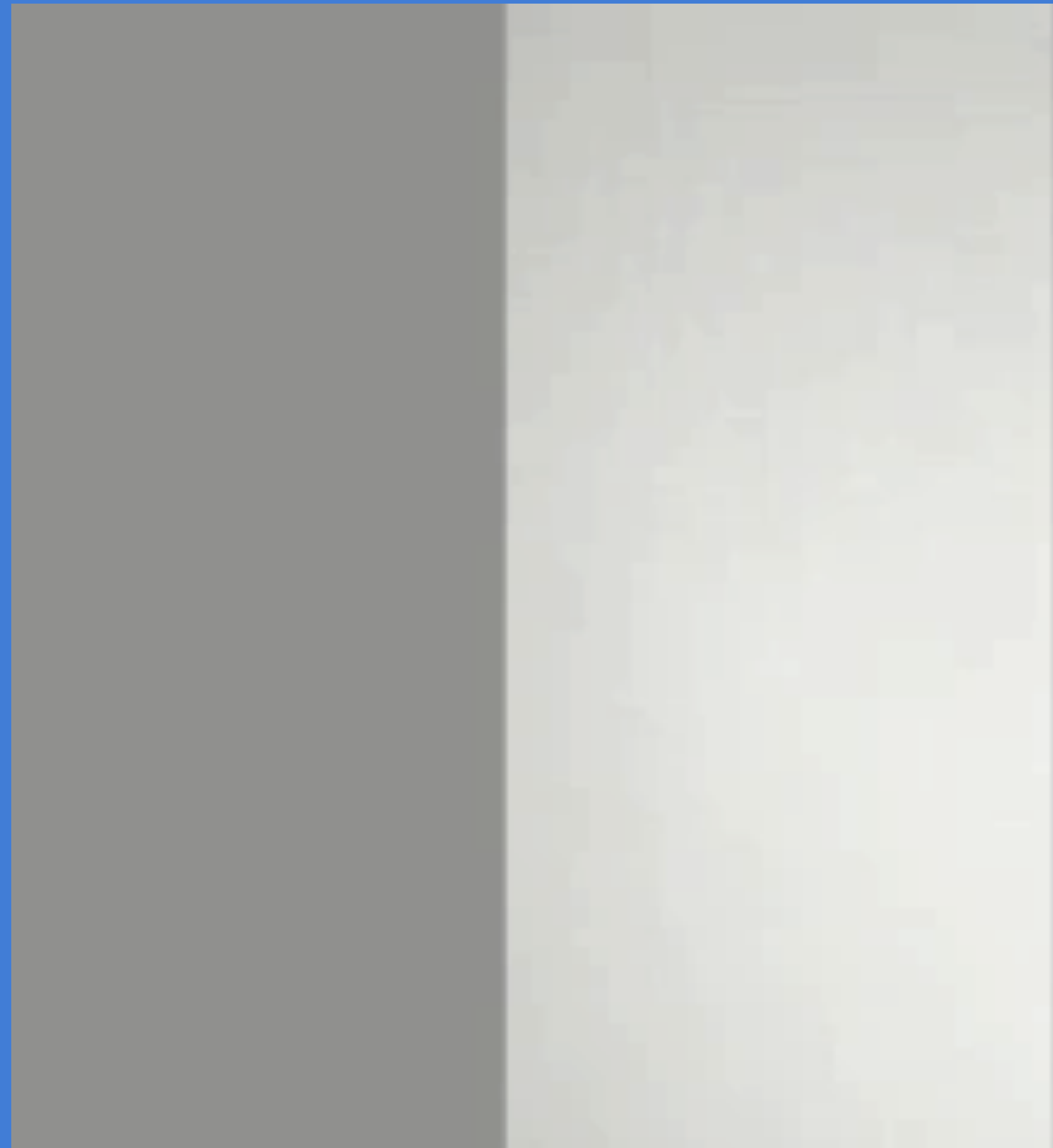
$$\rho_a U^2 \sim \rho g a < \sigma / a$$

Small drops

If a drop is small relative to the capillary length

$$a < \ell_c = \sqrt{\sigma / \rho g} \approx 2\text{mm} \;,$$

σ maintains it against the destabilizing influence of aerodynamic stresses.



Big drops

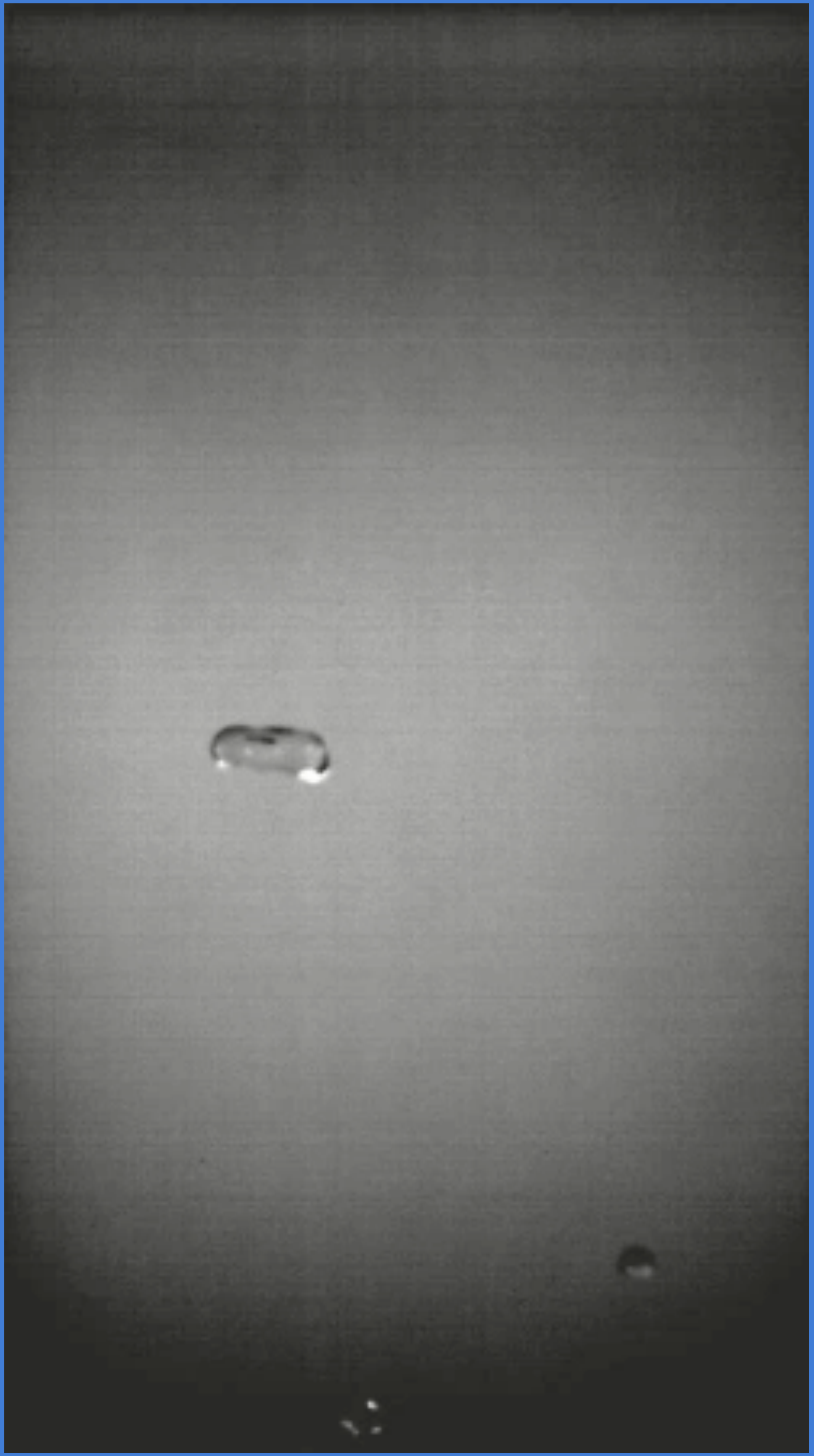
Drops larger than the capillary length ,

$$a > \ell_c \approx 2\text{mm}$$

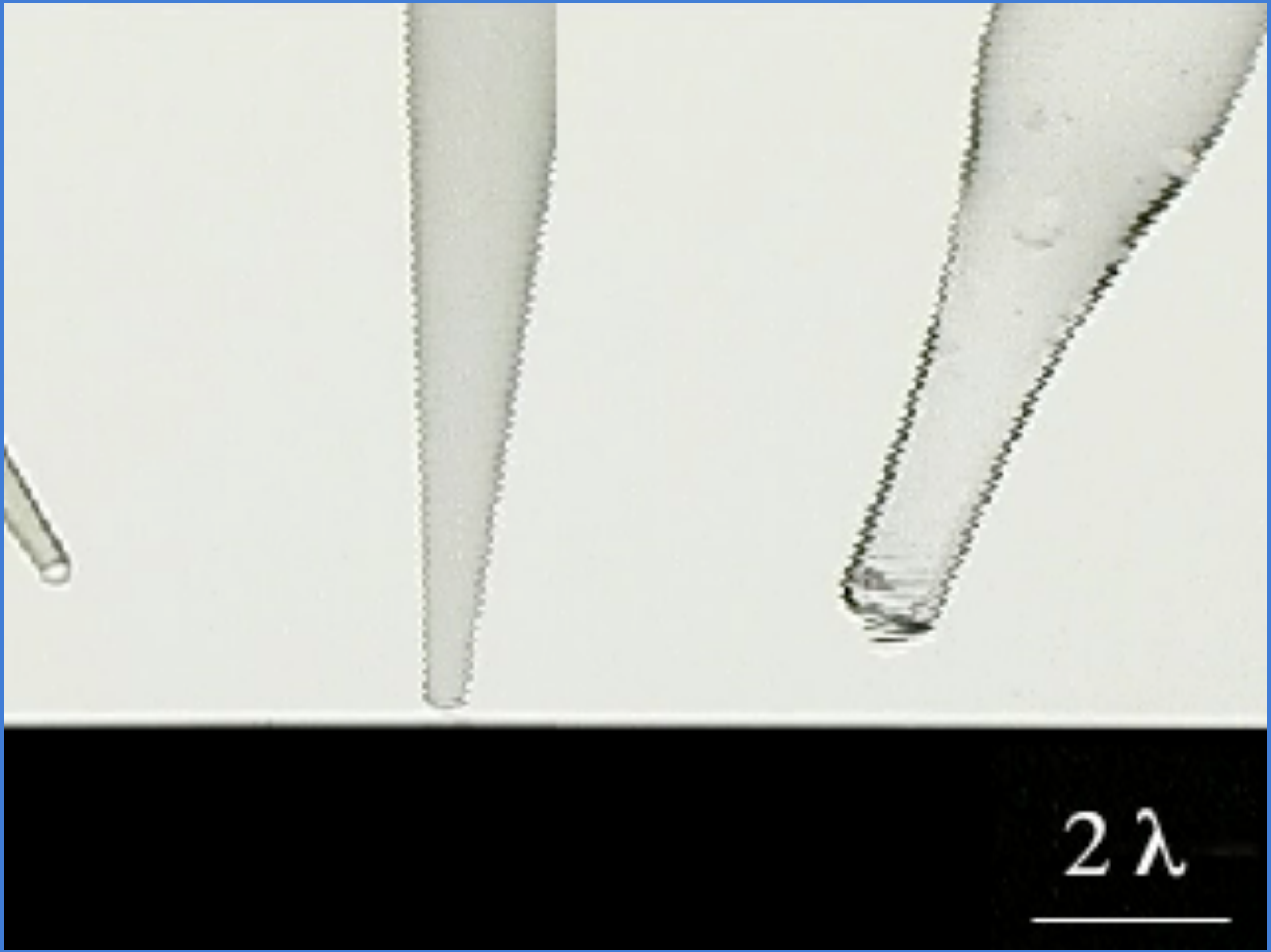
break up under the influence of aerodynamic stresses.

The break-up yields drops with size of order:

$$\ell_c \approx 2\text{mm}$$



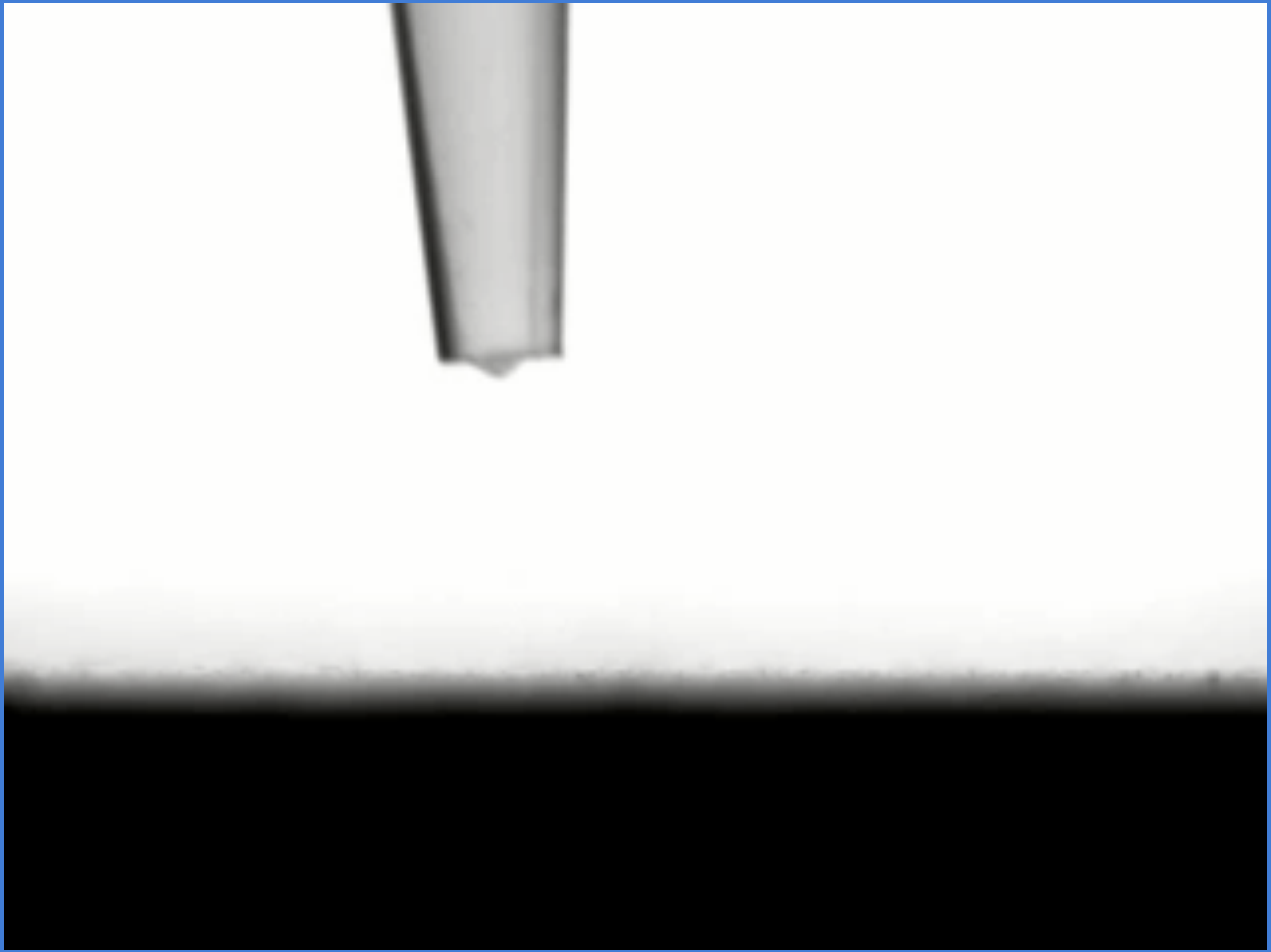
Puddles


2 λ

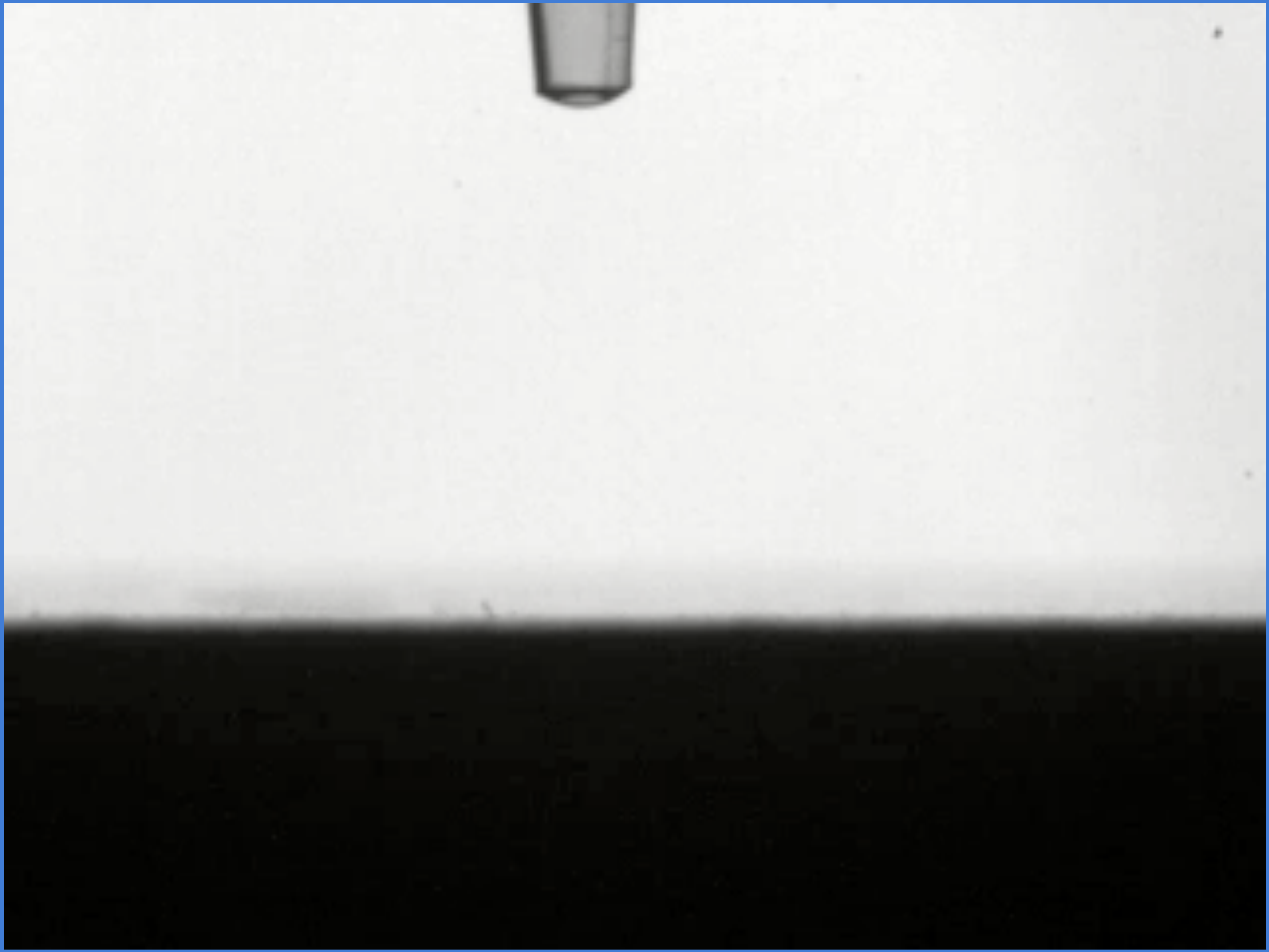
Wetting

Who cares about wetting?

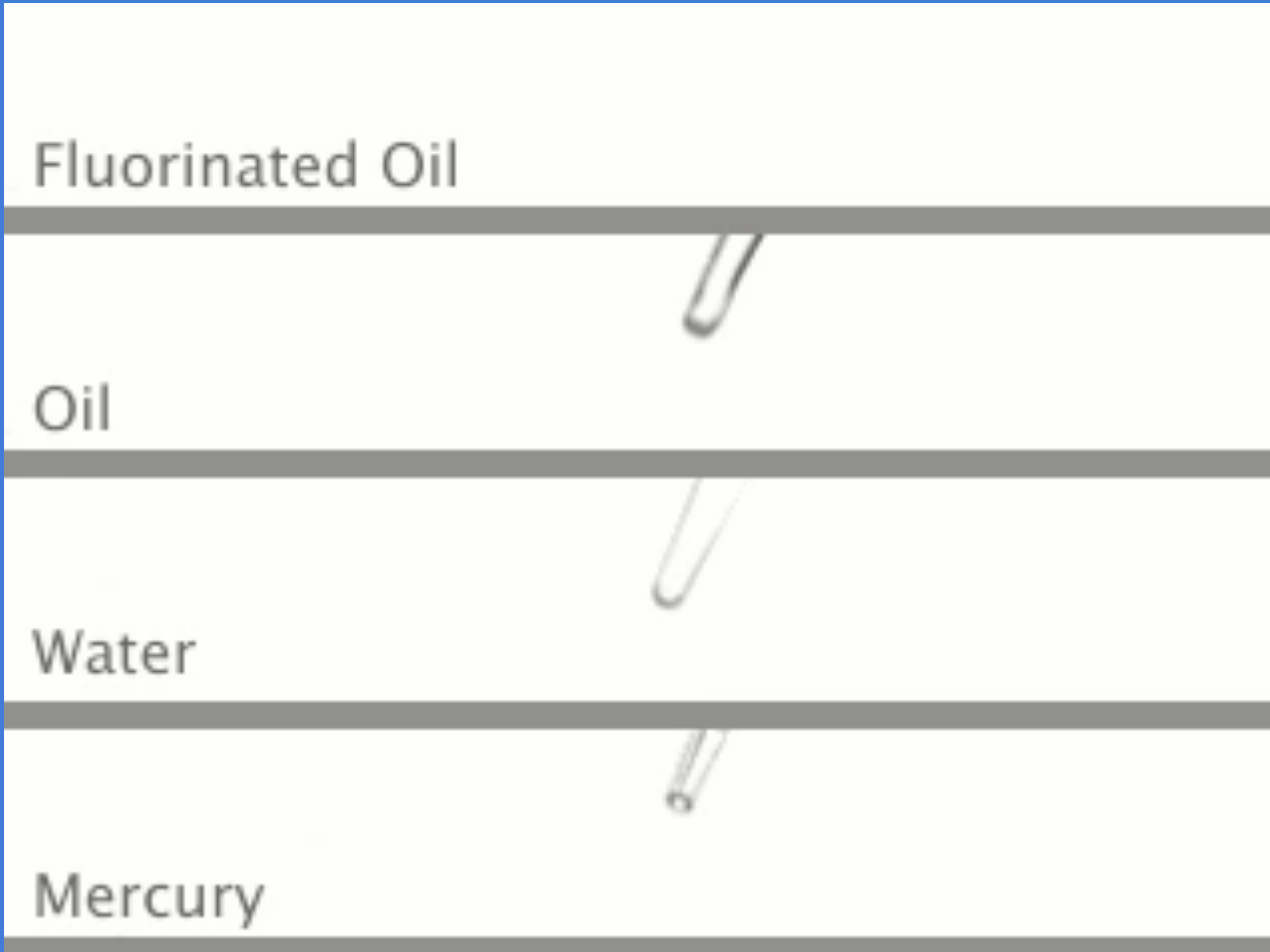
The world’s smallest lizard: the Brazilian Pygmy Gecko

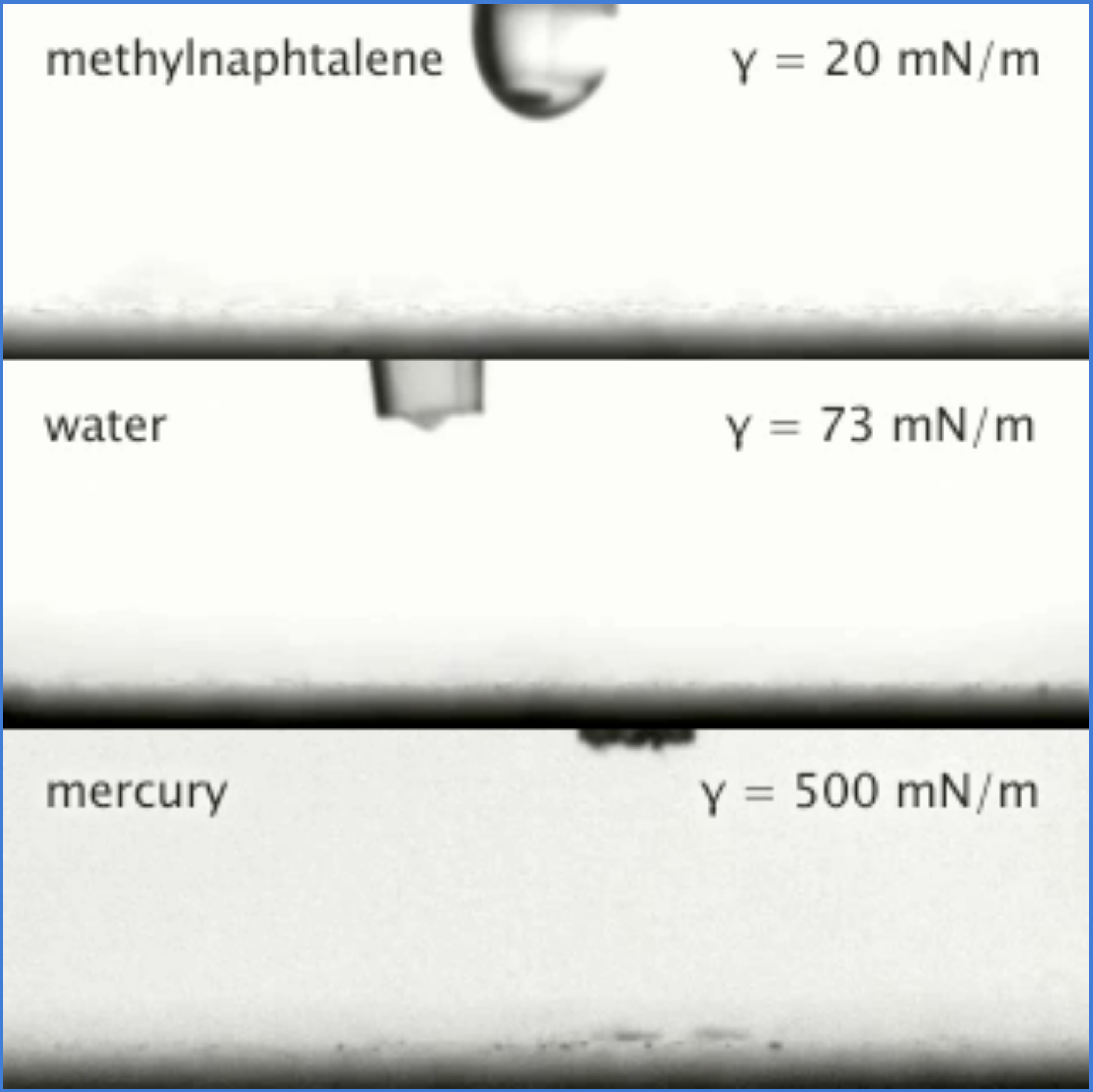


Partial wetting



Total wetting


Fluorinated Oil
Oil
Water
Mercury


methylnaphtalene
γ = 20 mN/m
water
γ = 73 mN/m
mercury
γ = 500 mN/m

Water on glass