Surface tension:

History, motivation and physical origins

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I. History

- surface tension in antiquity

II. Motivation

- who cares about surface tension?

III. Physical origins

- a heuristic discussion
I. The history of surface tension
Surface tension in antiquity

Hero of Alexandria (~0 BC)

• greek mathematician and engineer, ``the greatest experimentalist of antiquity”

• exploited capillarity in a number of inventions described in his book *Pneumatics*, including the water clock

Pliny the Elder (~0 BC)

• author, natural philosopher, army and naval commander of the early Roman Empire

• described the glassy wakes of ships

“True glory comes in doing what deserves to be written; in writing what deserves to be read; and in so living as to make the world happier.”

“Truth comes out in wine.”
The Tears of Wine

“Who hath sorrow? Who hath woe? They that tarry long at the wine. Look not though upon the strong red wine that moveth itself aright. At the last it biteth like a serpent and stingeth like an adder.”

- Proverbs 23: 29-32 (c.a. 950 BC)

King Solomon, “the wisest man that ever lived”. 
The tears of wine

The first `Marangoni flow’ studied scientifically (Thomson 1855).
Surface tension in history

Leonardo da Vinci (1452-1519)
• reported capillary rise in his notebooks
• hypothesized that mountain streams are fed by capillary networks

Francis Hauksbee (1666-1713)
• conducted systematic investigation of capillary rise
• his work was described in Newton’s Opticks, but no mention was made of Hauksbee

Benjamin Franklin (1706-1790)
• polymath: scientist, inventor, politician
• examined the ability of oil to suppress waves
Surface tension in history

Pierre-Simon Laplace (1749-1827)
- French mathematician and astronomer
- Elucidated the concept and theoretical description of the meniscus: hence, Laplace pressure

Thomas Young (1773-1829)
- Polymath, solid mechanician, scientist, linguist
- Demonstrated wave nature of light with ripple tank expts
- Described wetting of a solid by a fluid

Joseph Plateau (1801-1883)
- Belgian physicist, continued his expts after losing his sight
- Extensive study of capillary phenomena, soap films, minimal surfaces, drops and bubbles
II. Motivation

• who cares about surface tension?
Motivation: who cares about surface tension?

As we shall soon see, surface tension dominates gravity on a scale less than the capillary length, ~2 mm.

Biology

- all small creatures live in a world dominated by surface tension
- surface tension important for insects for many basic functions
- weight support and propulsion at the water surface
- adhesion and deadhesion via surface tension
- the pistol shrimp: hunting with bubbles (VIDEO)
- the archer fish: hunting with drops (VIDEO)
- underwater breathing and diving via surface tension
- natural strategies for water-repellency in plants and animals
- the hydraulics of trees
- the dynamics of lungs and the role of surfactants and impurities
Hunting with bubbles

The Pistol Shrimp
Motivation: who cares about surface tension?

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The Archer Fish
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- the hydraulics of trees
- the dynamics of lungs and the role of surfactants for premature infants
Motivation: who cares about surface tension?

Geophysics and environmental science

- the dynamics of raindrops and their role in the biosphere
- most biomaterial is surface active, sticks to surface of drops/bubbles
- chemical, thermal and biological transport in the surf zone
- early life: early vesicle formation, confinement to an interface
- oil recovery, carbon sequestration, groundwater flows
- design of insecticides intended to coat insects, leave plant unharmed
- chemical leaching and the water-repellency of soils: desertification
- oil spill dynamics and mitigation (e.g. use of dispersants in BP spill)
- disease transmission via droplet exhalation (e.g. COVID-19)
- dynamics of magma chambers and volcanoes
- the exploding lakes of Cameroon

Ref. Drops and bubbles in the environment, Bourouiba & Bush (2012)
Motivation: who cares about surface tension?

Technology

- capillary effects dominant in microgravity settings: NASA (Video)
- cavitation-induced damage on propellers and submarines
- design of superhydrophobic surfaces
e.g. self-cleaning windows, drag-reduction, erosion-resistant surfaces
- lab-on-a-chip technology: medical diagnostics, drug delivery
- microfluidics: continuous and discrete fluid transport and mixing (Video)
- tracking submarines with their surface signature
- inkjet printing
- the bubble computer (Video)
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Bouncing drops and digital microfluidics

- A means of sorting droplets according to size, mixing
Emulsification via bouncing

Oil

Water

T. Gilet
Motivation: who cares about surface tension?

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• the bubble computer (MFM)
III. The physical origins of surface tension
Surface Tension: molecular origins

- each molecule in a fluid feels a cohesive force with surrounding molecules
- molecules at interface feel half this force; are in an energetically unfavourable state
- the creation of new surface is thus energetically costly

![Diagram of surface tension](image)

- cohesive energy per molecule of radius \( R \) in bulk is \( U \), at surface is \( U/2 \)
- surface tension is this loss of cohesive energy per unit area:
  \[
  \sigma \sim \frac{U}{R^2}
  \]
  
  Units: \( [\sigma] = \frac{\text{ENERGY}}{\text{AREA}} = \frac{\text{FORCE}}{\text{LENGTH}} \)

- air-water \( \sigma \sim 70 \) dyne/cm; oils \( \sigma \sim 20 \) dyne/cm; liquid metals \( \sigma \sim 500 \) dyne/cm
Surface Tension

- a tensile force per unit length acting at gas-liquid interfaces

Nomenclature

- $\sigma$ denotes surface tension
- $\gamma$ denotes interfacial tension (as arises at any interface: liquid-liquid, solid-liquid, solid-gas)

A note on units: I prefer cgs

1 dyne = 1 g cm/s^2 = 10^{-5} N = cgs unit of force

For comparison,

1 atm = 101 kPa = 10^5 N/m^2 = 10^6 dynes/cm^2

$[\sigma] = \text{dynes/cm} = \text{mN/m}$
Some numbers for

\[ \sigma \sim \frac{U}{R^2} \]

<table>
<thead>
<tr>
<th>Interface</th>
<th>(\sigma) (dynes/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>air - water (20 °C)</td>
<td>72</td>
</tr>
<tr>
<td>air - soapy water</td>
<td>30 - 35</td>
</tr>
<tr>
<td>air - water w/ superwetting agent</td>
<td>20</td>
</tr>
<tr>
<td>air - water (100 °C)</td>
<td>58</td>
</tr>
<tr>
<td>air - ethanol</td>
<td>23</td>
</tr>
<tr>
<td>oil - water</td>
<td>(\gamma \sim 40)</td>
</tr>
<tr>
<td>air - olive oil</td>
<td>30</td>
</tr>
<tr>
<td>air - Si oil</td>
<td>20</td>
</tr>
<tr>
<td>air - He (4 °K)</td>
<td>0.1</td>
</tr>
<tr>
<td>air - molten glass (800 °K)</td>
<td>500</td>
</tr>
<tr>
<td>air - mercury</td>
<td>415</td>
</tr>
<tr>
<td>air - glycerol</td>
<td>63</td>
</tr>
</tbody>
</table>

\(H_2O\): hydrogen bonds
— high \(U\)

Oils: \(U \sim\) van de Waals
\sim kT \sim 1/40 eV at 25 °C

Mercury: strongly cohesive
liquid metal: \(U \sim 1\)eV
Surface tension in flocks, schools and swarms?

Might the cost of being on the edge give rise to analogous behavior?
Might the cost of being on the edge give rise to analogous behavior?
Related (course project style) question: the dynamics of cycling pelotons?
What is an interface?

- an idealized surface between two **immiscible** fluids; e.g. oil-water, air-water, oil-air
- there is no surface tension between miscible fluids, e.g. water-salt water
- in reality, the interface is rough on a molecular scale

Roughness scale $\delta$

- equate anomalous surface energy with thermal agitation energy

$$\sigma \delta^2 \sim kT$$

$$\delta \sim \left(\frac{kT}{\sigma}\right)^{1/2}$$

- treating the interface as sharp is consistent with the continuum hypothesis, wherein one assumes fluids are smooth beyond 10 molecular dimensions
Evaporation across a fluid interface

- thermal agitation overcomes interfacial tension
The creation of surface is energetically costly

- quasi-static soap films (for which gravity, inertia are negligible) take the form of minimal surfaces
- hence their interest to mathematicians:

"Find the minimal surface bound by the multiply connected curve C, where C ...."
The minimal surface between a pair of rings

A catenoid when the rings are close, a pair of circles when they are far apart.
The rupture of a catenoidal soap film

There is a critical distance at which a catenoid breaks.
The creation of surface is energetically costly

Thus:

- small drops are nearly spherical (MFM)
- fluid jets pinch off into droplets (MFM)
- fluid atomization results in spherical drops (MFM)
- wet hair sticks together: the “wet look” (MFM)
- bubbles and films are fragile (MFM)
methylNaphtalene \( \gamma = 20 \text{ mN/m} \)

water \( \gamma = 73 \text{ mN/m} \)

mercury \( \gamma = 500 \text{ mN/m} \)
The creation of surface is energetically costly

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The wet hair instability: threads clump to minimize surface energy
The creation of surface is energetically costly

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- bubbles and films are fragile (MFM)
Surface tension: analogous to a negative surface pressure

- gradients in surface tension necessarily drive surface motion

\[ \sigma = \frac{\text{force}}{\text{length}} \]

\[ P = \frac{F}{A} \]
Surface tension: \[ [\sigma] = \frac{\text{FORCE}}{\text{LENGTH}} = \frac{\text{ENERGY}}{\text{AREA}} \]

Surface energy: \[ E_\sigma = \int_S \sigma \, dA = 2 \sigma L x \]

Force acting on rod: \[ F = \frac{dE_\sigma}{dx} = 2 \sigma L \]
If you use a hair, you can achieve neutral buoyancy, and so have a two-dimensional model of a balloon in the atmosphere.
A simple way to measure surface tension

- measure the force required to withdraw a plate from a free surface
Capillary forces support the weight of water-walking insects.