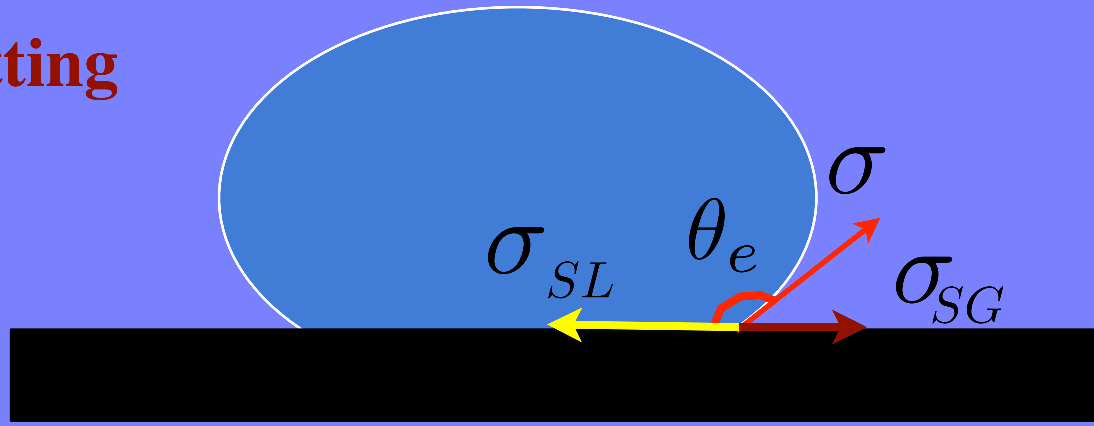
Lecture 4

Development of Interfacial BCs

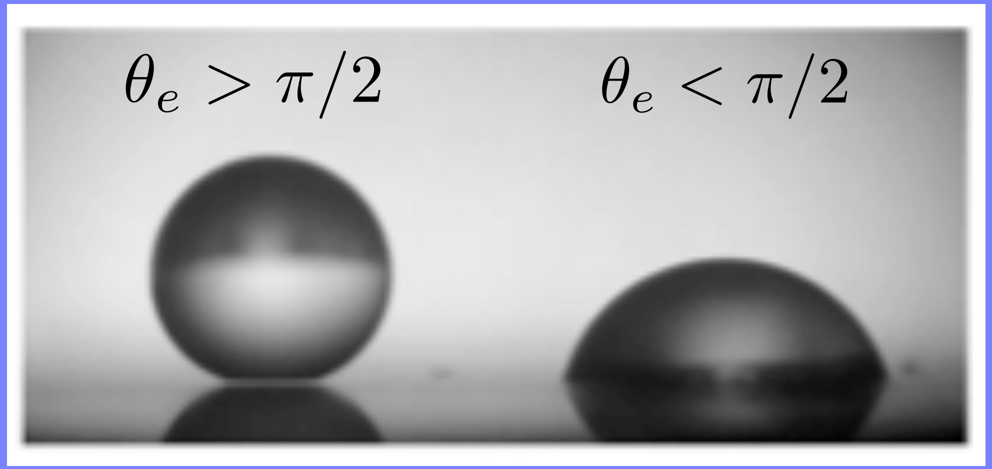
Fluid-Solid Contact: Partial wetting



Equilibrium contact angle θ_e

Energy differential: $dW = dx\,(\sigma_{SG} - \sigma_{SL}) - dx\,\sigma\,\cos\theta_e$

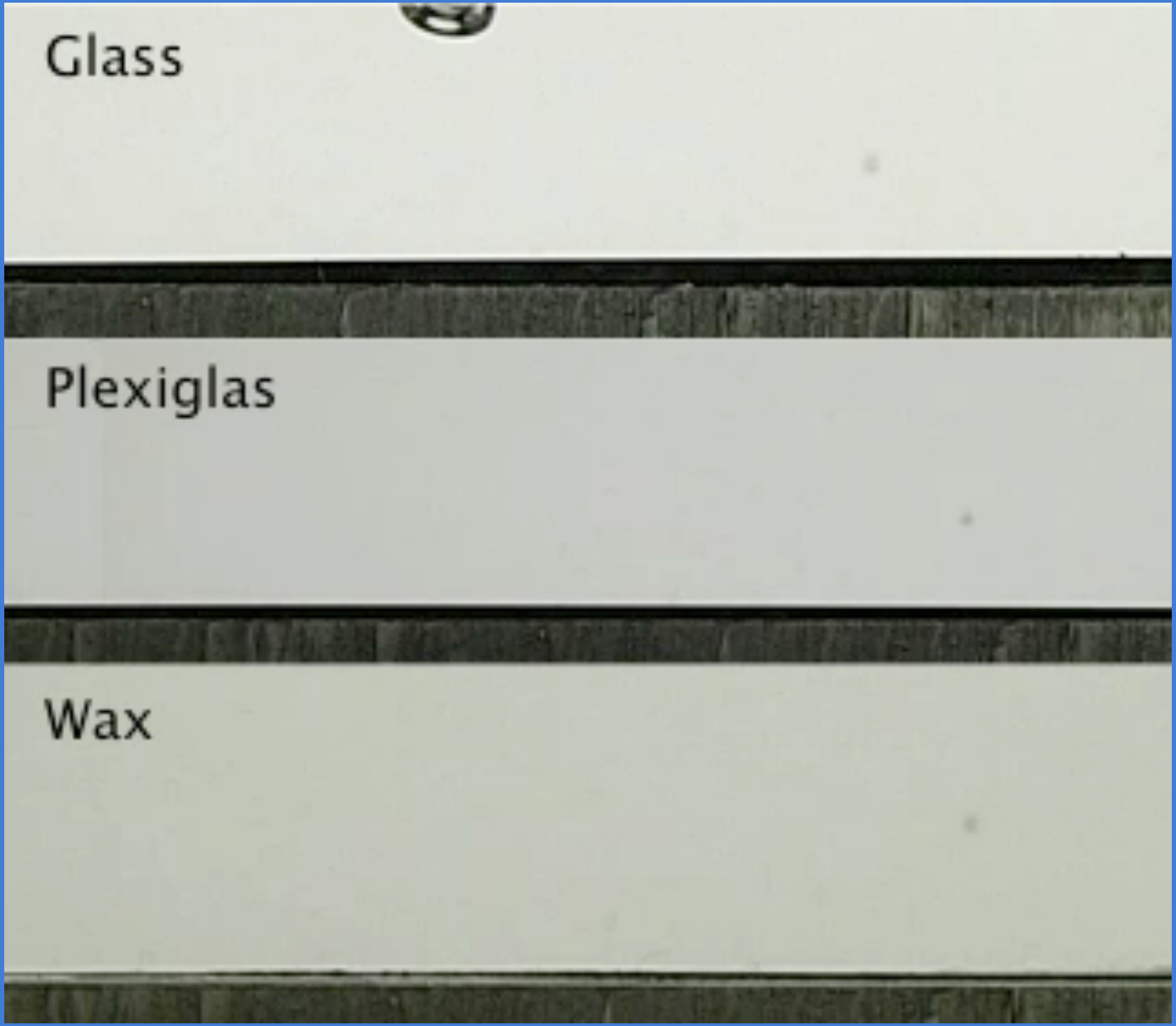
Young's relation: $\boxed{\sigma\ \cos\theta_e = \sigma_{SL} - \sigma_{SG}}$



Hydrophobic surface

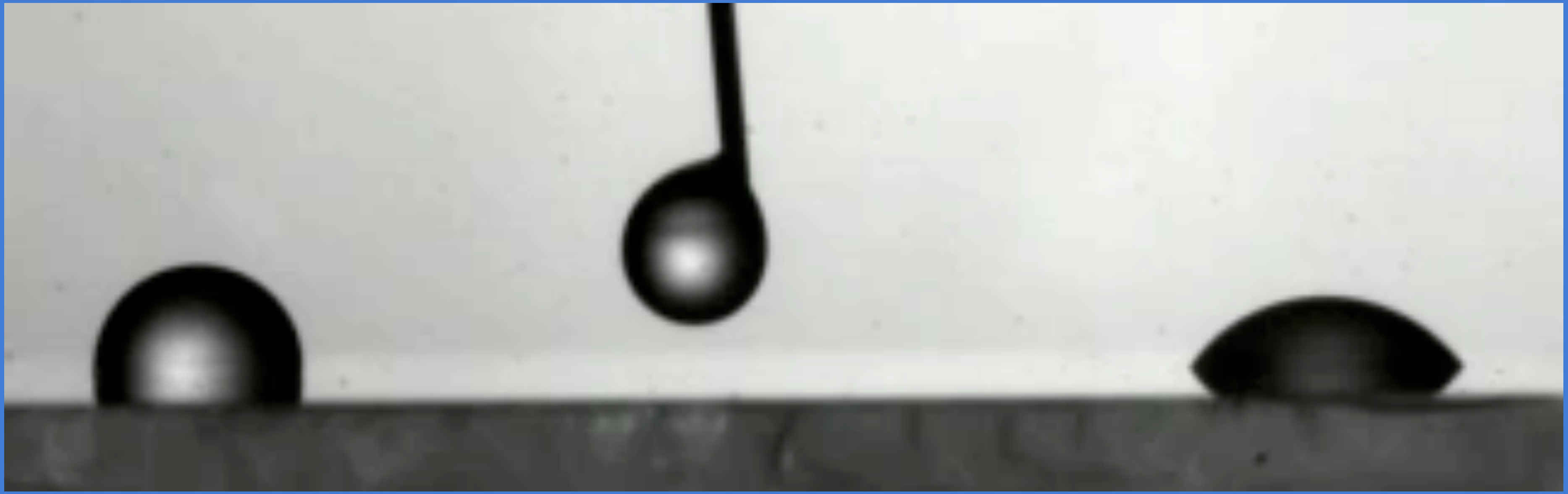
Hydrophilic surface

Puddles

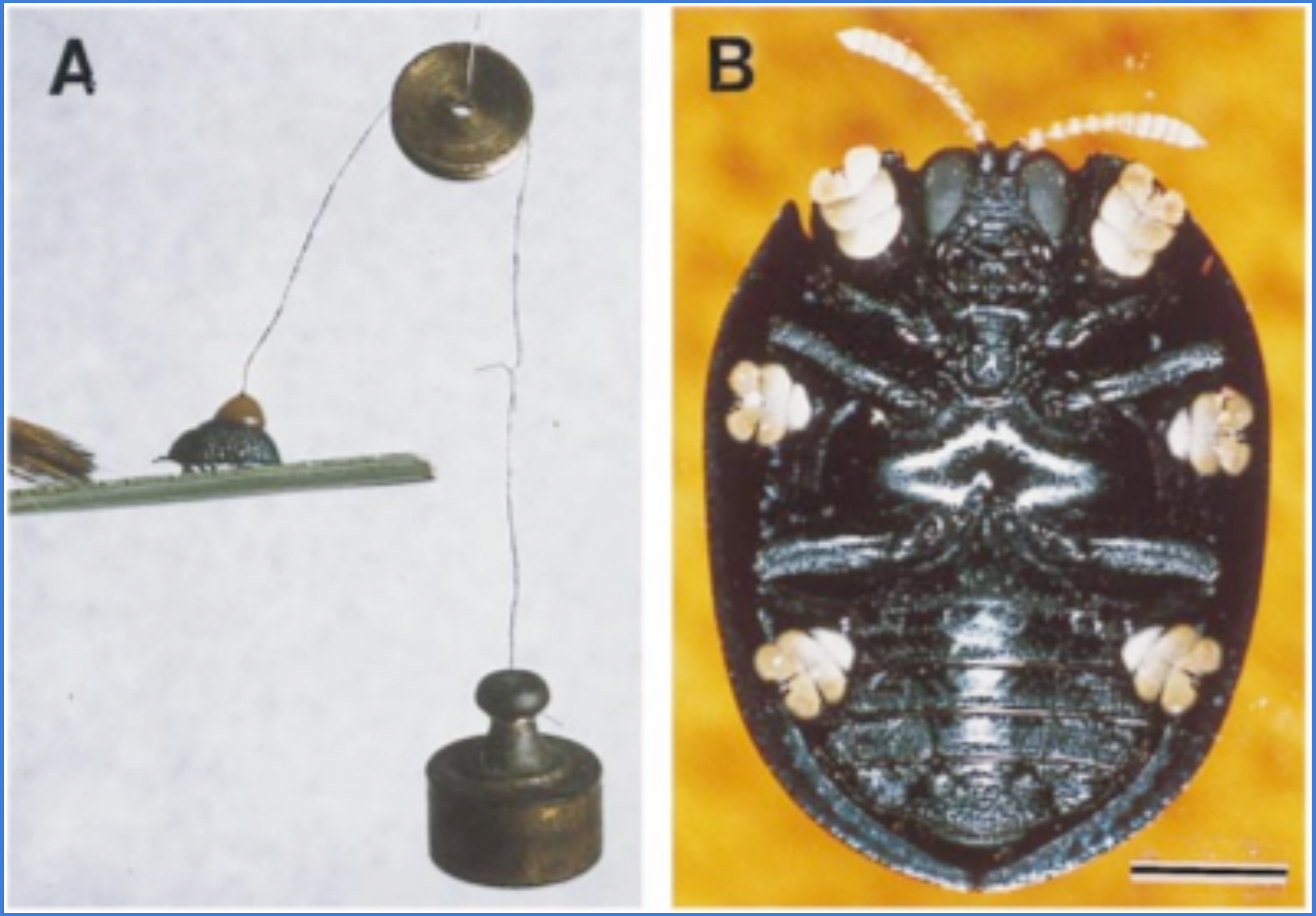


Partial wetting

Spontaneous motion in response to a wettability gradient



Capillary adhesion in nature



18.357: Lecture 5

I. Interfacial boundary conditions

II. Fluid statics: menisci, floating bodies

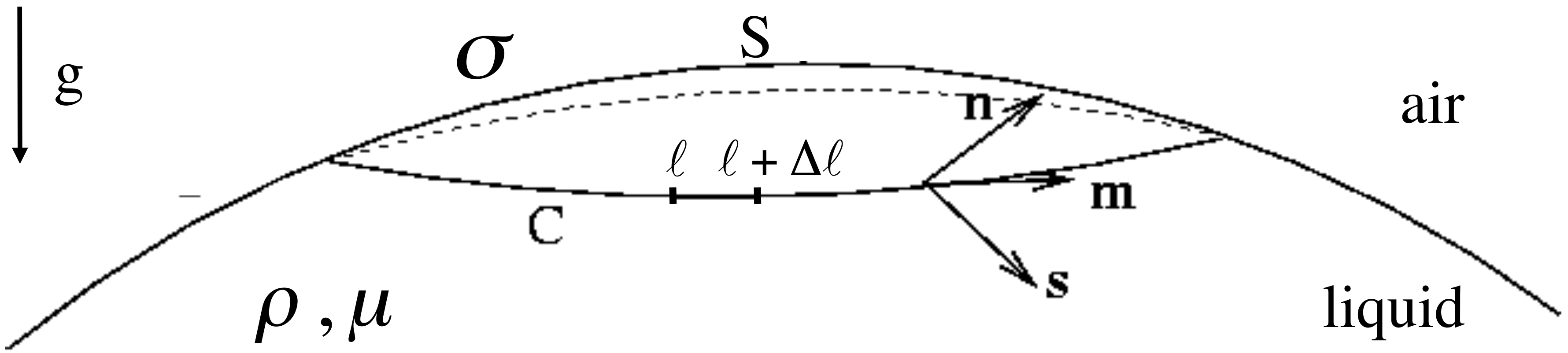
John W. M. Bush

Department of Mathematics
MIT

The governing equations

Surface tension: Geometry

Along a contour C bounding a surface S there is a tensile force per unit length σ acting in the $\mathbf{s}$ direction



Net force on S:

$$\boxed{\oint_C \sigma\, \mathbf{s}\; d\ell = \iint_S \sigma\, (\nabla \cdot \mathbf{n})\, \mathbf{n}\; dS \;+\; \iint_S \nabla\sigma\, dS}$$

curvature pressure

Marangoni stress

1) normal curvature pressure $\sigma \nabla \cdot \mathbf{n}$ resists surface deformation

2) tangential Marangoni stresses may arise from $\nabla\sigma$

Governing Equations

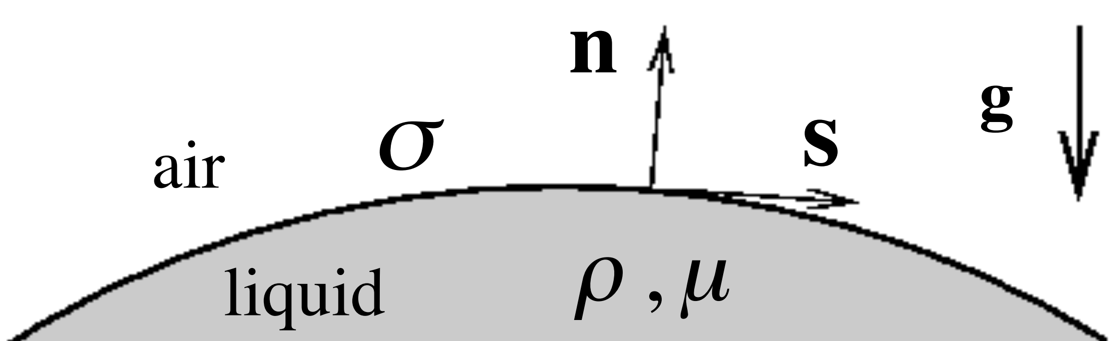
Navier-Stokes equations:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \rho\mathbf{g} + \mu\nabla^2\mathbf{u} \quad , \qquad \nabla\cdot\mathbf{u} = 0$$

Boundary Conditions

Normal stress: $$\mathbf{n}\cdot\mathbf{T}\cdot\mathbf{n}\,| = \sigma\nabla\cdot\mathbf{n}$$

Tangential stress: $$\mathbf{n}\cdot\mathbf{T}\cdot\mathbf{s}\,| = \nabla_s\sigma$$



Stress tensor

$$\mathbf{T} = -p\mathbf{I} + \mu\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right)$$

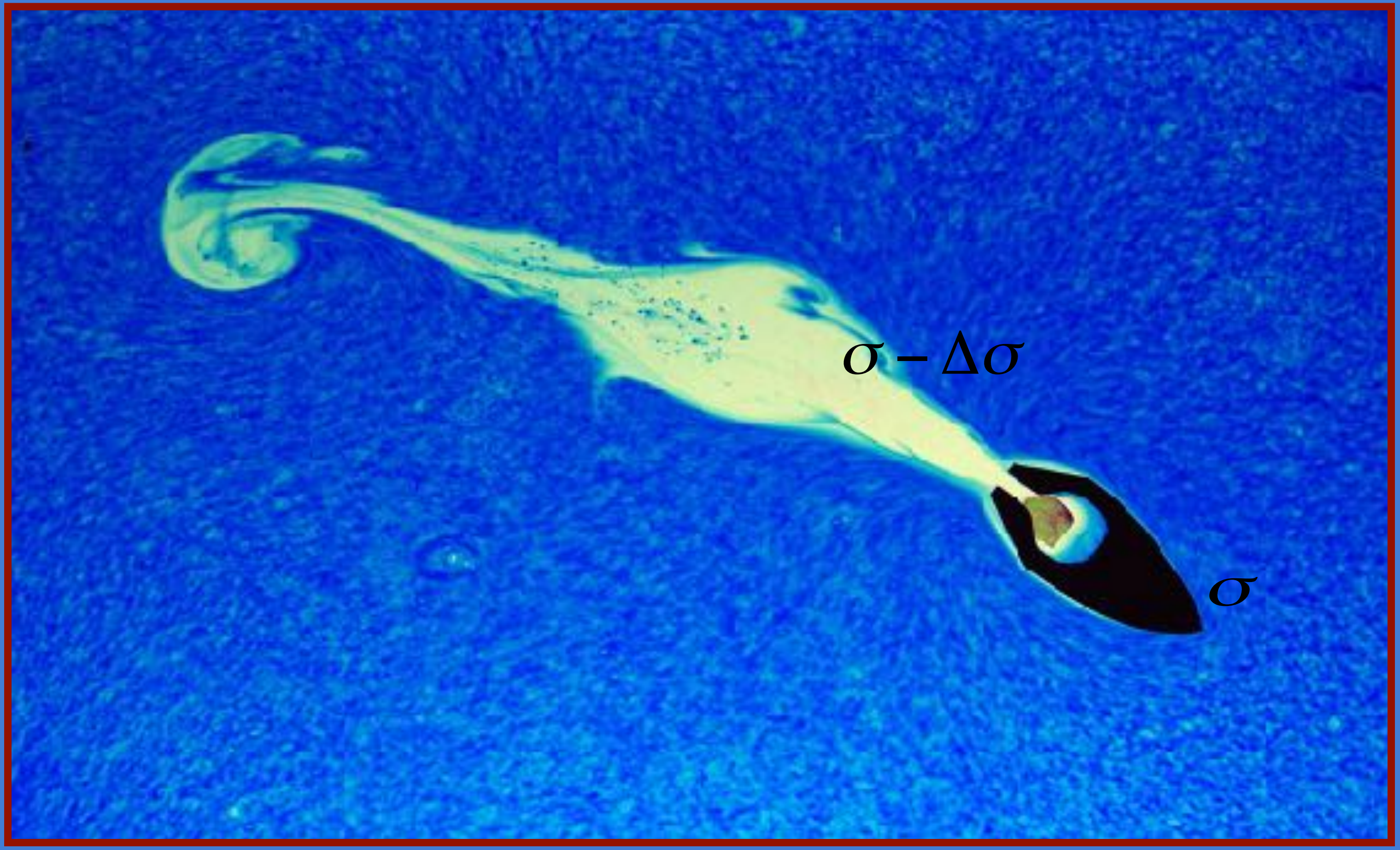
Capillary forces support the weight of water-walking insects.

Marangoni Flows

- flows dominated by the influence of surface tension gradients

Recall tangential stress BC: $\Delta\, \mathbf{n}\cdot\mathbf{T}\cdot\mathbf{s} = \nabla\sigma$

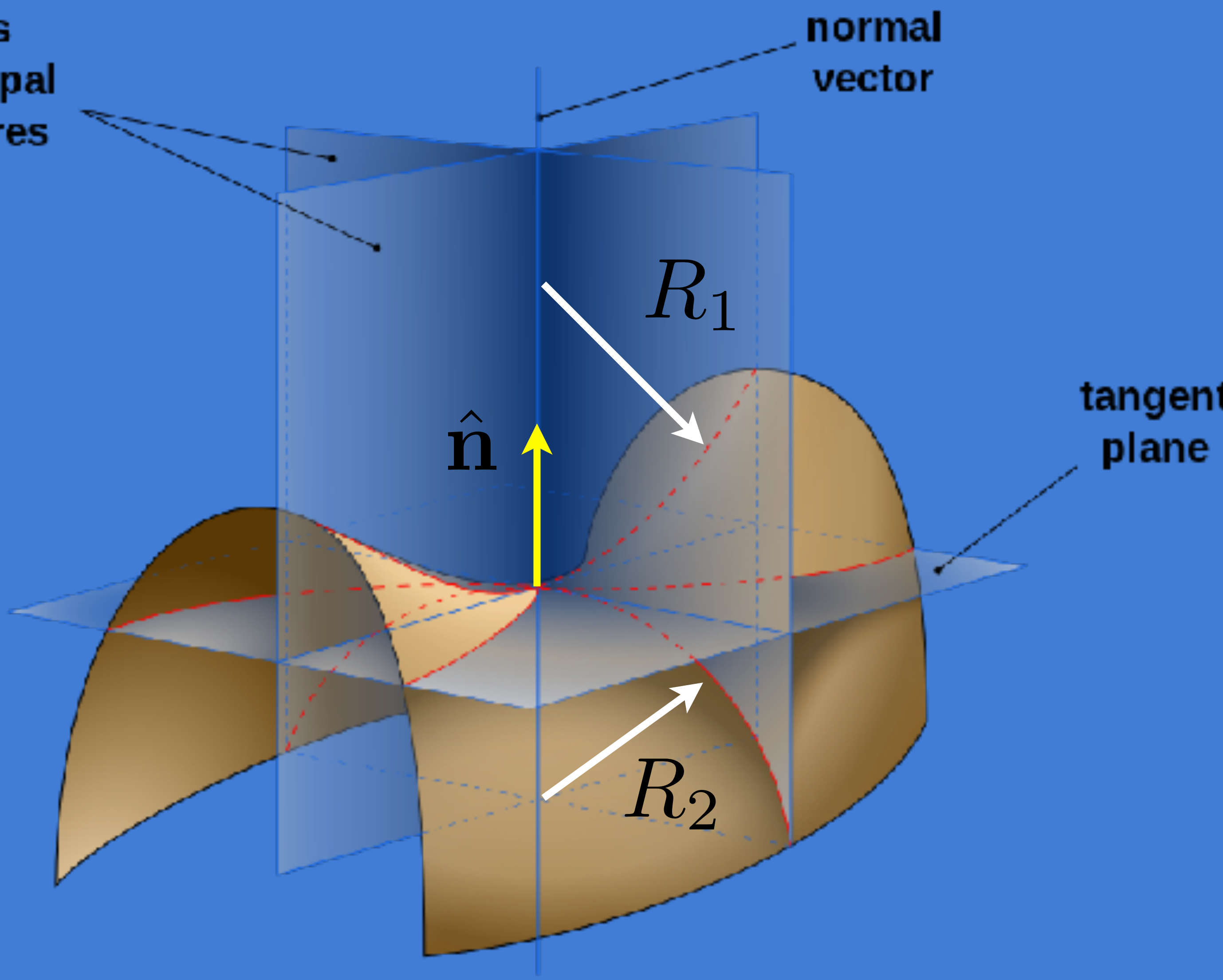
- $\nabla\sigma$ may arise due to dependence of $\sigma(T, c, \Gamma)$



Marangoni Flows

- flows dominated by the influence of surface tension gradients
- $\nabla\sigma$ may arise due to dependence of $\sigma(T, c, \Gamma)$

The cocktail boat: fueled by alcohol



Curvature

$$\nabla \cdot \hat{\mathbf{n}} = \frac{1}{R_1} + \frac{1}{R_2}$$

where R_1 , R_2 are the principal radii of curvature