

## Problem Set 7

18.355, Fall 2014  
Due Wednesday Nov. 26

1. Verify that the Stokes and continuity equations are satisfied by the representation

$$\mathbf{u}(\mathbf{x}) = \nabla\phi + \mathbf{x} \wedge \nabla\Psi + \nabla(\mathbf{x} \cdot \mathbf{A}) - 2\mathbf{A} \quad , \quad p(\mathbf{x}) = 2\mu \nabla \cdot \mathbf{A} \quad ,$$

where  $\phi, \Psi$  and  $\mathbf{A}$  are harmonic functions.

2. A rigid sphere of radius  $a$  is held stationary in a pure straining flow, so that

$$\mathbf{u}(\mathbf{x}) = 0 \text{ on the particle surface, and } \mathbf{u} \rightarrow \mathbf{E} \cdot \mathbf{x} \text{ as } |\mathbf{x}| \rightarrow \infty \quad .$$

Determine the velocity and pressure fields in the fluid. Note:  $\mathbf{E}$  is the constant traceless and symmetric rate-of-strain tensor of the undisturbed flow in the absence of the particle.

3. A rigid sphere of radius  $a_1$  rotates steadily inside a larger concentric sphere of radius  $a_2$ . The rotational velocity of the inner sphere is  $\boldsymbol{\Omega}$ .

a) Assuming that the Reynolds number is small, determine the velocity and pressure fields in the fluid occupying the annular region between the spheres.

b) Deduce an expression for the torque required to rotate the inner sphere. Evaluate the torque for the case of extremely large  $a_2/a_1$ .

### BONUS (1 point and \$1 million from the Clay Institute)

*The Clay Prize question:* Prove or give a counter-example of the following statement...

*In three space dimensions and time, given an initial velocity field, there exists a vector velocity and a scalar pressure field, which are both smooth and globally defined, that solve the Navier-Stokes equations.*