

## Problem Set 5

18.355, Fall 2014  
Due Oct.29

### 1. *The falling jet*

Consider a laminar stream of water ejected from a nozzle of radius  $a$  falling vertically under the influence of gravity  $g\hat{z}$  its surface tension being  $\sigma$ . Assume that the jet's vertical velocity  $U(z)$  is uniform across the jet, so independent of the distance  $r$  from the axis of symmetry.

a) By applying Bernoulli's Theorem at two points on the jet, one at the inlet, and one downstream, deduce expressions for both the jet speed  $U(z)$  and radius  $r(z)$ . (Do not solve these, as they are messy).

b) In the limit of large Bond number,  $Bo = \rho g a^2 / \sigma \gg 1$ , solve the equations. Interpret your solution.

c) By doing an experiment at home in your kitchen sink, indicate the limitations of the solutions deduced above.

2. Demonstrate that the viscous contribution to the force per unit volume in a fluid may be expressed in terms of the vorticity  $\omega$  as  $\mu \nabla \wedge \omega$ . Thus demonstrate that, if a flow is irrotational, its dynamics may be described by Euler's equations, and the hydrodynamic force on a body in the flow may be deduced by integrating the fluid pressure over the body surface.

### 3. *Irrotational flow past a cylinder*

a) Deduce the form of two-dimensional irrotational flow past a stationary cylinder by superposing the velocity potentials describing uniform flow and flow forced by a uniformly translating cylinder.

b) Demonstrate that the drag on the cylinder vanishes identically.

### 4. *The Magnus Effect*

Consider the flow past a spinning cylinder. In a real fluid, the angular motion would act to impart a net circulation to the flow through the action of fluid viscosity. This circulation (denoted by the constant  $\Gamma$ ) may be incorporated artificially into an irrotational flow model by adding an irrotational vortex potential  $\phi = \theta\Gamma/(2\pi)$  to the velocity potential deduced in Problem 3.

a) Deduce an expression for the resulting velocity field.

b) By examining the location of stagnation points in the flow, deduce the dependence of the form of the flow on the dimensionless spin number  $S = \Gamma/(4\pi aU)$ , and make rough sketches of the flow for  $S < 1$ ,  $S = 1$  and  $S > 1$ .

c) Demonstrate that the drag on the cylinder still vanishes.

d) Deduce an expression for the transverse force (or 'lift') on the cylinder.

*Note:* the generation of lift through the interaction of circulation and translation is the cause of many peculiarities in the dynamics of sports balls, including Roberto Carlos's classic Brazilian free kick (see course poster).

**5.** *The momentum integral*

a) For steady, inviscid flow in the presence of a force field  $\mathbf{f} = -\nabla\Psi$ , derive and interpret the following statement, which is valid for any closed surface  $S$  within the fluid:

$$\int_S [ \rho(\mathbf{u} \cdot \mathbf{n})\mathbf{u} + (p + \Psi)\mathbf{n} ] dS = 0.$$

b) A buoyant bubble rises steadily through water. Consider the surface  $S$  to be a large vertical cylinder centered on the bubble. From a), deduce an expression for the difference in the vertical momentum fluxes through the upper and lower surfaces of the cylinder. You will need to consider the force balance on the rising bubble.

c) Using similar means, deduce the vertical momentum flux across a horizontal plane beneath a hovering bird of mass  $m$ .