#### 18.355 FLUID DYNAMICS

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#### **GRADING SCHEME**

- 70% : 7 problem sets (group discussion encouraged)
- 30% : course project on subject of your choosing

There is **no required text** for the course, which will be based on the lecture notes; however, the following are recommended supporting material.

## SUGGESTED TEXTS

#### Batchelor, G.K. An Introduction to Fluid Dynamics. Cambridge University Press

#### Acheson, D. J. Elementary Fluid Dynamics

A pair of comprehensive texts that present much of classical fluid dynamics in great depth.

Tritton, D.J. Physical Fluid Dynamics. Oxford University Press

A very readable and well-illustrated account of many fluid phenomena described from a physical perspective.

## RELATED GOOD READS

Simpson, J.E. Gravity Currents, Cambridge University Press

A clear concise description of gravity currents in their many guises with an outstanding collection of photos. A must read for those interested in geophysical and environmental fluid dynamics.

McMahon, T.A. and Bonner, J.T. On Size and Life, Scientific American Books

An inspirational book describing the influence of geometrical scaling in biology. Includes an excellent introduction to dimensional analysis and some beautiful scaling arguments. A must read for those interested in biological fluid dynamics.

Van Dyke, M. An album of Fluid Motion

Samimy, Breuer, Leal and Steen, A Gallery of Fluid Motion

Collections of photographs of a variety of fluid motions.

# **DETAILED COURSE OUTLINE**

# Lecture 1

• motivation and course philosophy

# Lectures 2-4: Dimensional Analysis and Scaling

- dimensional analysis and Buckingham's Theorem
- examples: frequency of a pendulum; Taylor's result for the atom bomb blast cloud; pressure inside a bubble; drag on a sphere
- dynamic similarity and its utility in experimental modeling
- scaling: physically informed dimensional analysis
- examples: the scaling of bouncing drops, birds in flight, rowboats
- demo: video of drop coalescence cascade

# Lecture 5-8: Derivation of Navier Stokes Equations

- basic concepts: continuum hypothesis, Lagrangian/Eulerian descriptions
- Reynold's Transport Theorem
- conservation of mass and the continuity equation
- Cauchy's momentum equation
- the state of stress in a fluid: pressure and rate of strain tensor
- the stress tensor: definition and interpretation
- constitutive equations: Newtonian and non-Newtonian fluids
- demos: ferrofluids; custard powder; rod climbing

# Lecture 9: Thermodynamics

- conservation of energy
- mechanical and thermal energy equations
- the heat equation

# Lecture 10: Boundary Conditions

- no-slip and kinematic conditions
- derivation of the stress balance equation at an interface
- normal and tangential stress balance equtions
- demos: the water strider; soap boat, tears of wine, the beating heart

#### Lecture 11: Nondimensionalization of N-S equations and BCs

- dimensionless groups are those deduced from dimensional analysis
- important groups: Reynolds, Bond, Froude, Strouhal numbers

### Lecture 12: Kinematics

- decompose velocity gradient into rate of strain and vorticity tensors
- circulation and Kelvin's Theorem
- vortex dynamics: examples of vortex stretching

#### Lecture 13-15: Unidirectional Flows - some exact solutions

- steady flows: Couette and Poiseuille flows
- unsteady flows: Rayleigh's first and second problems (motivate similarity solutions via dimensional analysis)
- start up flow in a pipe (solve via Sturm-Liouville theory)

## Lecture 16: Invisicid Flow

- Bernoulli's equation and its applications: flow through a weir; the Egyptian water clock; shape of a falling fluid jet
- the momentum integral: e.g. the expanding pipe junction
- demos: ping pong balls on a hair dryer; canoe in the wind

#### Lecture 17

- the use of potential flow for describing irrotational flows
- irrotational flow past a sphere (generate soln from harmonic functions)
- drag calculation and D'Alembert's Paradox

## Lecture 18: Irrotational Viscous Flows

- prove that viscous forces vanish in a viscous irrotional flow
- hydrodynamic drag in an irrotational flow
- surface deformation for an irrotational vortex
- demo: hydrodynamic forces on floating bodies in a vortex

# Lecture 19: Unsteady Inviscid Flows

- water waves: derivation of dispersion relation
- special cases: gravity and capillary waves in deep and shallow water
- interpretation of waves generated by body moving at surface

# Lecture 20: Instability of Superposed Sheared Fluids

- solution of linear problem via normal modes
- deduction of dispersion relation and Squires Theorem
- special cases: Rayleigh-Taylor and Kelvin-Helmholtz instabilities

### Lecture 21: Translation of a body in 3D irrotational flow

- hydrodynamic forces on body: buoyancy, steady and acceleration forces
- most general form of D'Alembert's paradox: steady force vanishes
- express acceleration force in terms of added mass tensor
- example: acceleration from rest of a speherical bubble

### Lecture 22: Potential Flow Theory

- the streamfunction and streamfunction-vorticity equation
- complex potentials for 2D irrotational flows
- simple applications: flow at a wall angle; sources and sinks; flow past a half body; flow past a cylinder with circulation
- analogies with electrostatics: method of images, superposition

• discussion of the limits of the potential flow description

# Lecture 23: The Dynamics of Sports Balls

- hydrodynamic drag at high Reynolds numbers
- skin friction in flow past streamlined bodies
- form drag and separation in flow past bluff bodies
- the role of surface roughness on hydrodynamic drag
- the role of altitude and humidity on balls in flight
- the Magnus effect and its applications in sports and ballistics
- the reverse Magnus effect and its prominence in beach ball soccer
- applications: soccer, golf, ping pong, baseball, cricket
- video: Brazilian free kicks

# Lecture 24-27: Lubrication Theory

- generic physical picture: derivation of governing equations, range of validity
- viscous gravity currents (motivate similarity soln from dimensional analysis)
- the squeeze film
- the thrust bearing: calculate normal and tangential stress on slider

# Lecture 28-29: Stokes Flows

- steady and unsteady Stokes equations: range of validity
- general properties of Stokes flows: no inertia, quasi-steady, linearity, reversibility
- consequences of linearity for particle motion
- solving simple problems by application of time reversibility

# Lecture 30-33: Vector Methods for Stokes Flows

- preliminaries: definition of true- and pseudo-vectors, harmonic functions
- the Hinch Method: express velocity and pressure fields in terms of harmonic pseudoand real vector and scalar potentials; deduce soln by inspection

- Stokes flow past a sphere and drag calculation (compare to onerous streamfunction formulation)
- rotating sphere in an infinite fluid: flow and torque
- translation of a spherical drop: Hadamard-Rybczinski formula; discussion of influence of surfactants
- the romocapillary drop motion: calculate drop translation speed

## Lecture 34: Stokes flow past a cylinder

- deduce sol'n via Hinch Method: note impossibility to satisfy BCs
- Stokes Paradox: interpret with a scaling argument
- resolution via the Oseen Method: linearization of inertial term; matched asymptotic expansions

## Lecture 35-36: Flow in Porous Media

- definition of Darcy velocity and deduction of Darcy's Law
- flow past a cylindrical porosity anomaly via vector methods
- the Hele-Shaw cell as an experimental analogue of Darcy flow
- examples: gravity currents and bubbles in a Hele-Shaw cell

### Lecture 37-39: Rotating Flows

- derivation of Navier-Stokes equations in a rotating frame
- vortex dynamics in a rotating frame
- statics in a rotating frame: shapes of bodies of revolution
- nondimensionalization of governing equations: Rossby and Ekman numbers
- geostrophic flow and the Taylor-Proudman theorem
- viscous boundary layers: the Ekman spiral
- axial particle motion in a rotating flow
- inertial, cyclostrophic and geostrophic vortices
- spin-down of inertial and geostrophic vortices
- demos: Taylor column, drop motion in a rotating tank