Hydrodynamic quantum analogs

- orbital quantization: Larmor levels; double quantization in the SHO
- spin states, Zeeman splitting, spin lattices, spin flips
- statistical projection (`mirage') effects in confined geometries
- Friedel oscillations, corrals, interaction-free measurement
- tunneling, superradiant tunneling and emission; Anderson localization
- single-particle diffraction and interference, Aharanov-Bohm effect
- Uncertainty relations and Exclusion Principles
- boost factors, HOM effect, surreal trajectories, bomb testers
- optical effects: Talbot effect, Bragg scattering, optical ratcheting
- distant, two-particle and multi-particle correlations

Pilot-wave hydrodynamics demonstrates how classical hereditary mechanics may give rise to features taken as evidence of non-locality in QM.

- have motivated the development of generalized pilot-wave theories

A generalized pilot-wave framework

- retain key features of walker system

  (memory, resonance, quasi-monochromatic wave field)

- explore beyond the range of the hydrodynamic system

- discover new quantum-like features; *e.g. stable spin states*

- extend from 2D to 3D

- connect to and inform quantum pilot-wave theories

(*Bush, ARFM, 2015*)

Pilot-wave dynamics: a parametric generalization

$$\kappa_0(1-\Gamma)\ddot{\mathbf{x}}_p + \dot{\mathbf{x}}_p = \frac{2}{(1-\Gamma)^2}\int_{-\infty}^{t} \frac{J_1(|\mathbf{x}_p(t)-\mathbf{x}_p(s)|)}{|\mathbf{x}_p(t)-\mathbf{x}_p(s)|}(\mathbf{x}_p(t)-\mathbf{x}_p(s))e^{-(t-s)}\,ds$$

INERTIA **DRAG** **WAVE FORCING**

where $$\Gamma = \frac{\gamma-\gamma_W}{\gamma_F-\gamma_W}\,, \qquad \kappa_0 = (m/D)^{3/2}k_F\sqrt{gA/2T_F}$$

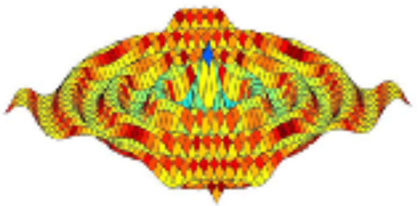
PROXIMITY TO THRESHOLD

$0<\Gamma<1$

CONTAINS ALL FLUID PARAMETERS: BOUNDED IN HYDRODYNAMIC SYSTEM

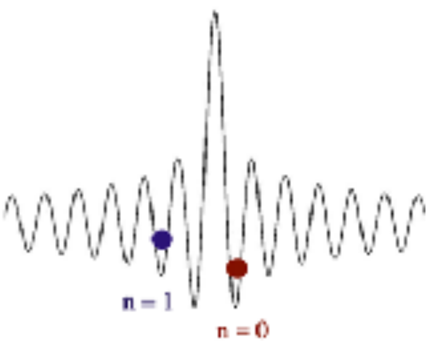
$0.8<\kappa_0<1.6$ in lab

Question: For what values of (κ_0, Γ) does the system look most like QM?



Eg.1 When are hydrodynamic spin states stable?

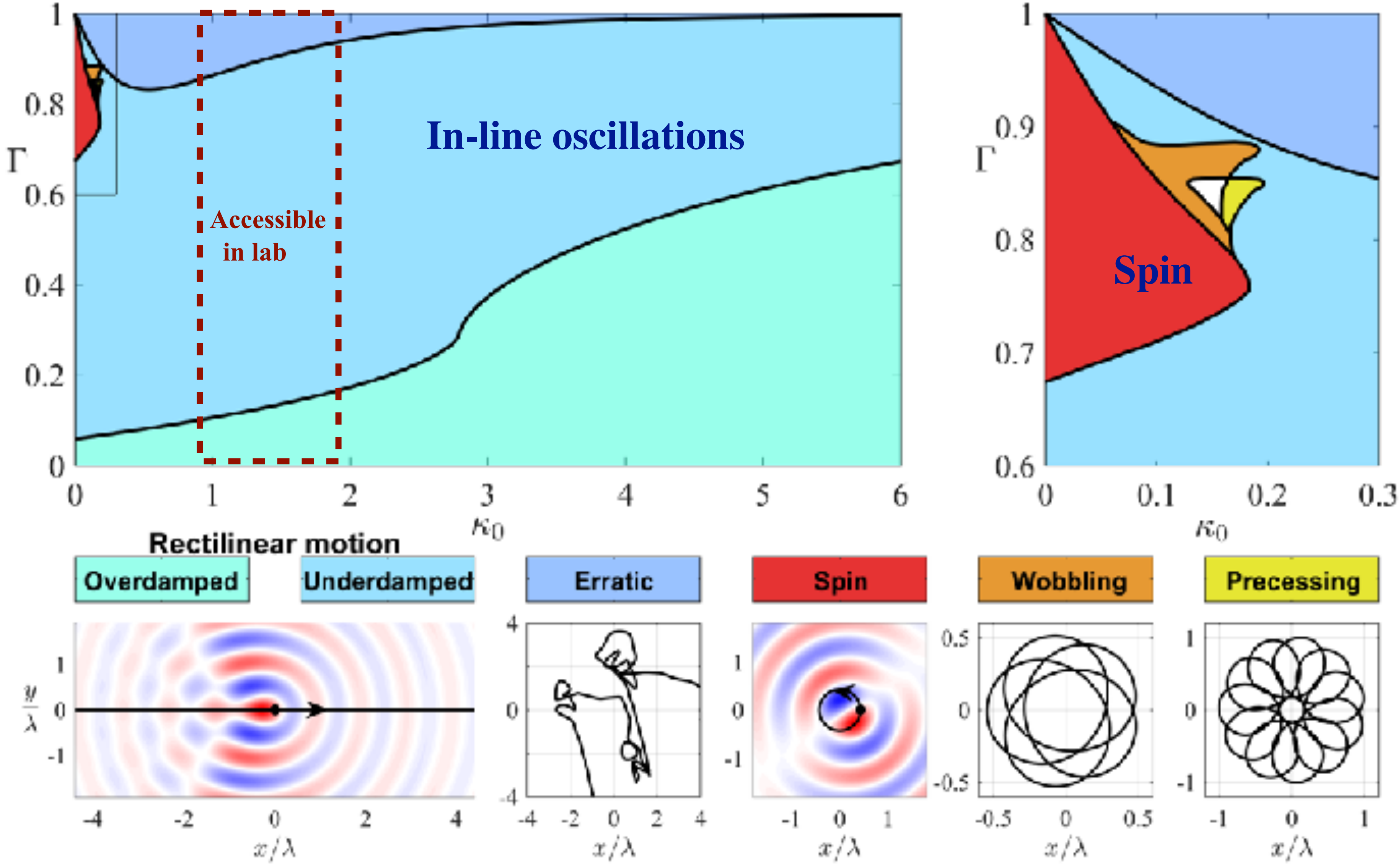
Eg.2 When is orbital quantization the sharpest?



Answer: in the large memory, small-particle-inertia limit

Generalized pilot-wave theory: the free particle in 2D

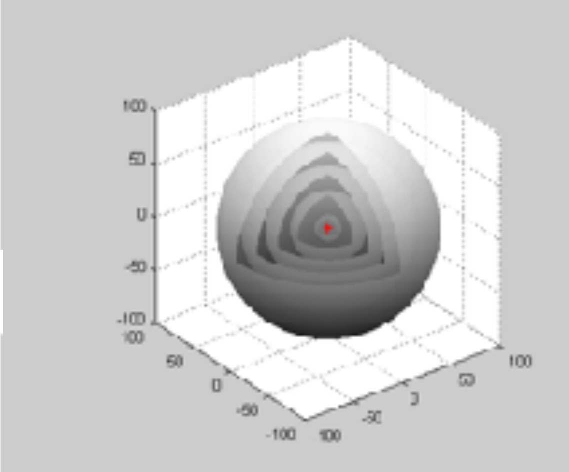
(*Durey & Bush, 2020*)



- stable, wobbling and precessing spin states may obtain
- walking state may be unstable to in-line oscillations with wavelength λ_F
- aperiodic `*jittering*' gives rise to random walk with diffusivity $D \sim U\lambda_F$

Classical pilot-wave dynamics in 3D

Kay, Durey & JB, *PRSA (2025)*

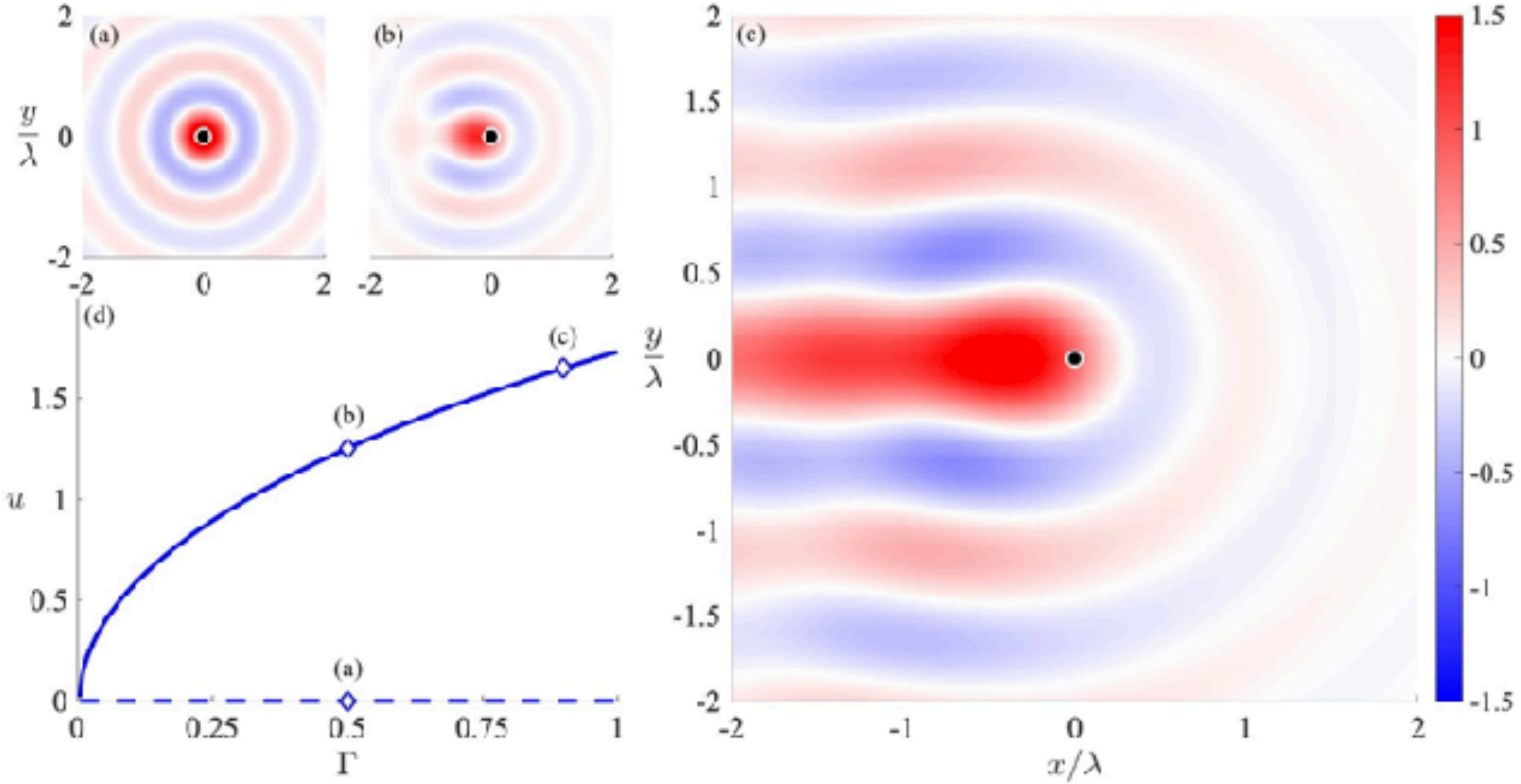


Trajectory: $\kappa_0 \ddot{\mathbf{x}}_p + \dot{\mathbf{x}}_p = -3\nabla h(\mathbf{x}_p, t)$

WAVE $h(\mathbf{x},t) = \int_{-\infty}^{t} \mathcal{H}(k|\mathbf{x}-\mathbf{x}_p(s)|)e^{-\mu(t-s)}ds$ $\quad \mathcal{H} = j_0(x)$

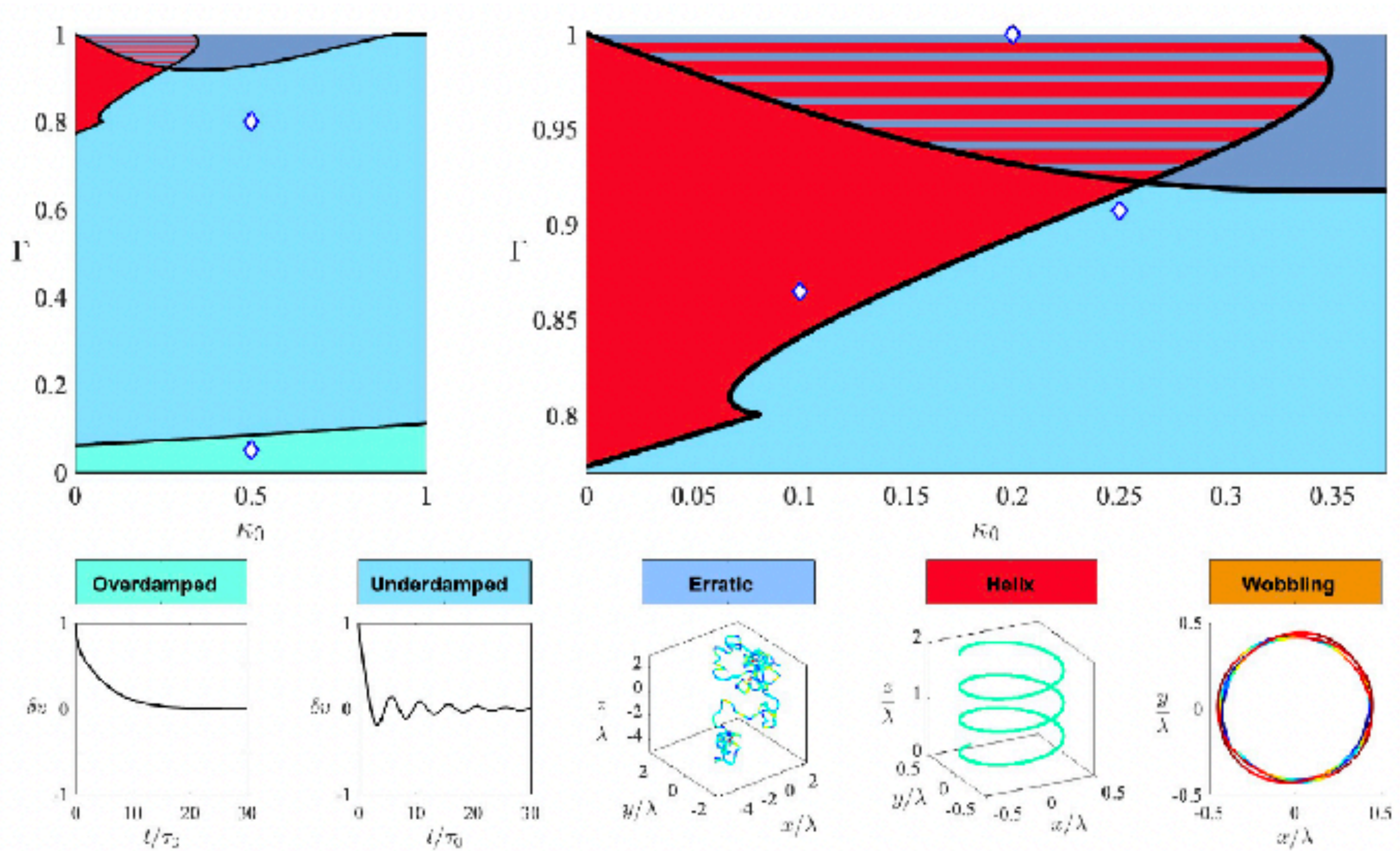
$\mu = 1/\Gamma$

- transitions to self-propelling state at critical memory



Generalized pilot-wave theory: the free particle in 3D

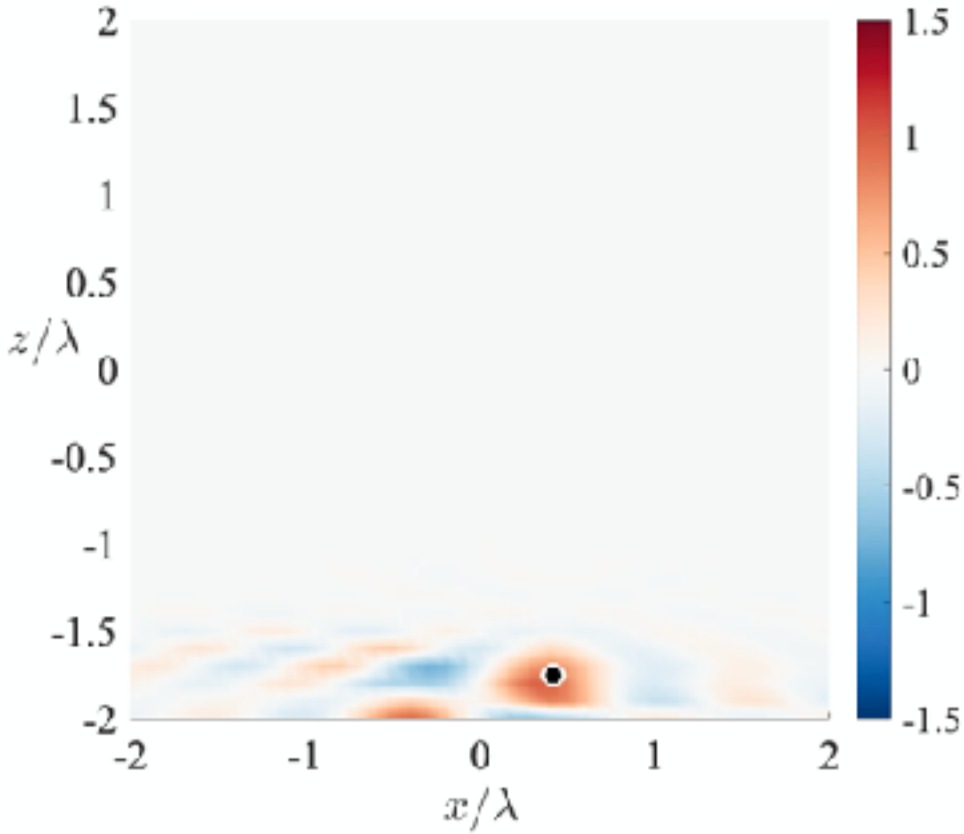
(Kay, *Durey & JB, 2025*)

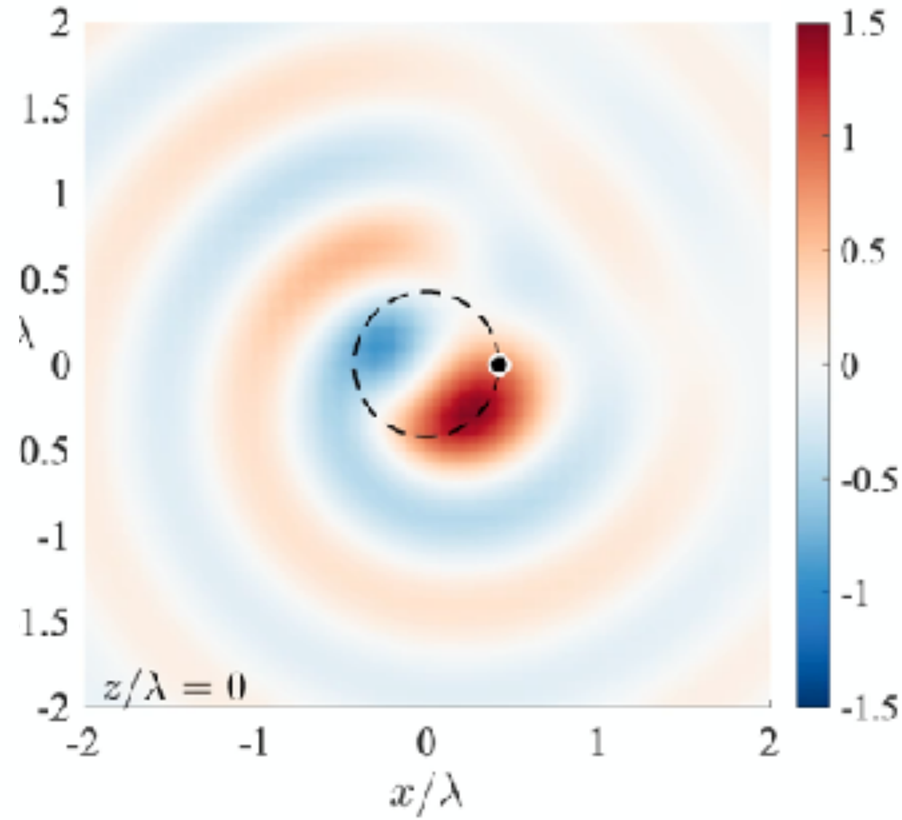


- walking state may be unstable to in-line oscillations with wavelength λ_F
- erratic motion gives rise to random walk with diffusivity $D \sim U\lambda_F$
- stable and wobbling helical spin states may obtain

Helical spin states

- arises in a parameter regime where rectilinear walking is unstable





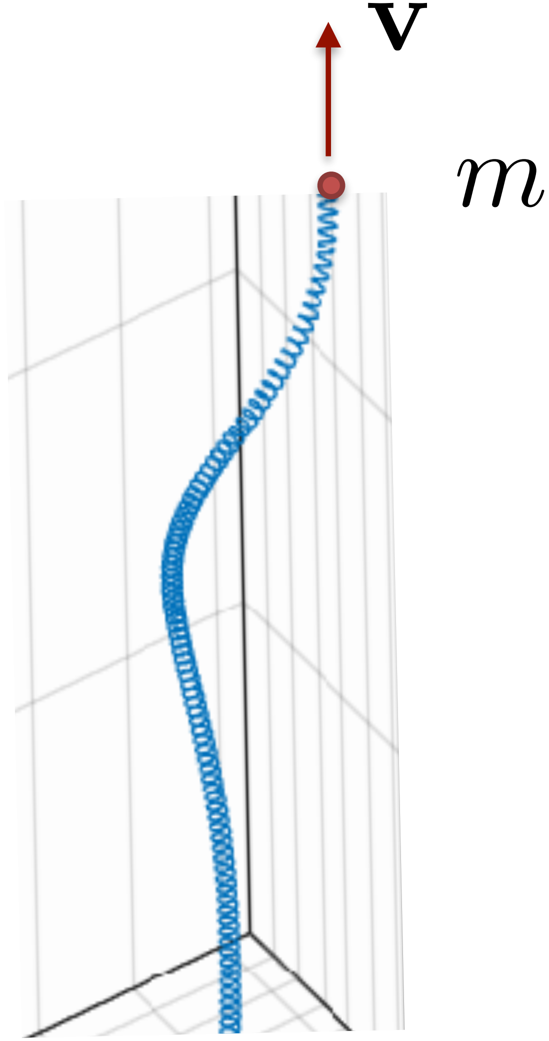
- self-potential constrains particle to follow a helical path
- zooming out indicates motion of particle with intrinsic angular momentum
- reminiscent of classical models of the electron which rely on Compton-scale dynamics (e.g. Hestenes 1990)
- \`instrinsic' spin may be rationalized in terms of classical pilot-wave dynamics
- enables new classical analogs of Stern-Gerlach, spin-orbit coupling

Helical spin state's response to forces

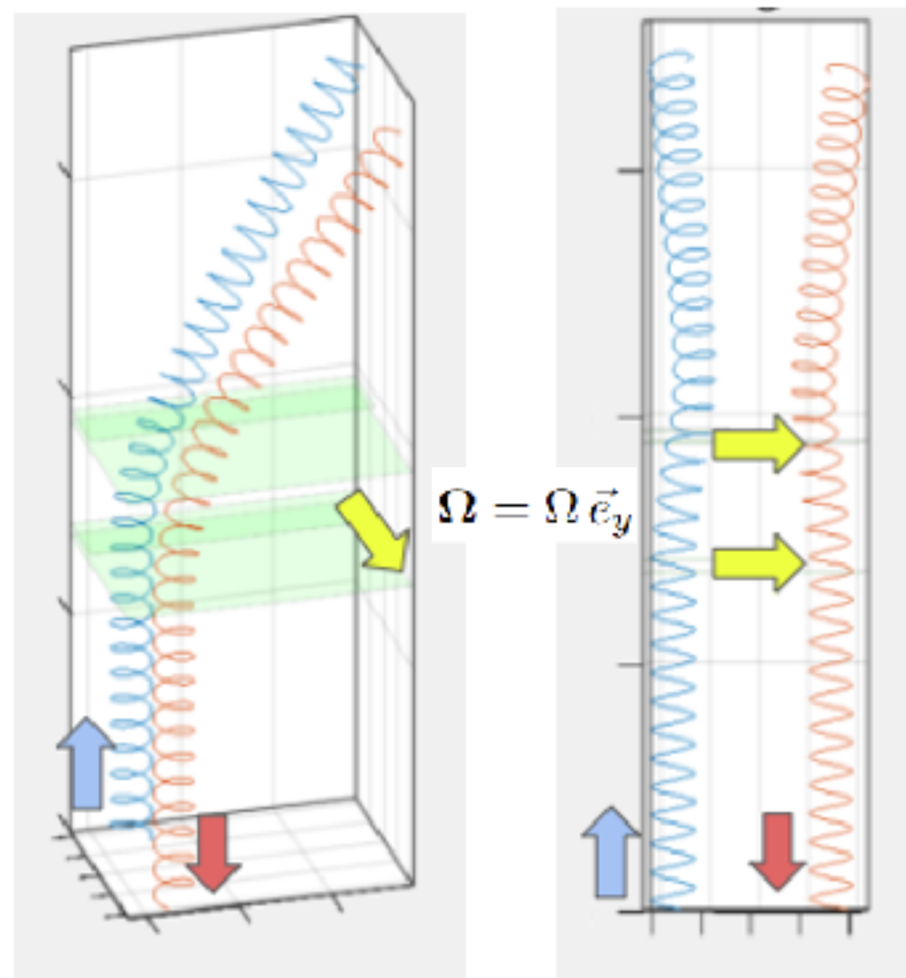
— ongoing work with Diego Chavez and Matt Durey

Goal: deduce reduced dynamical description of particle with spin

- exploit separation of timescales between axial and angular motion

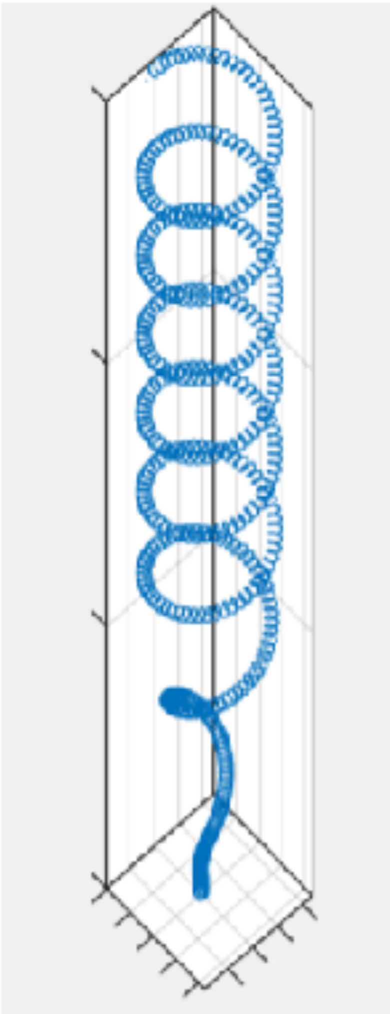


`Stern-Gerlach' splitting



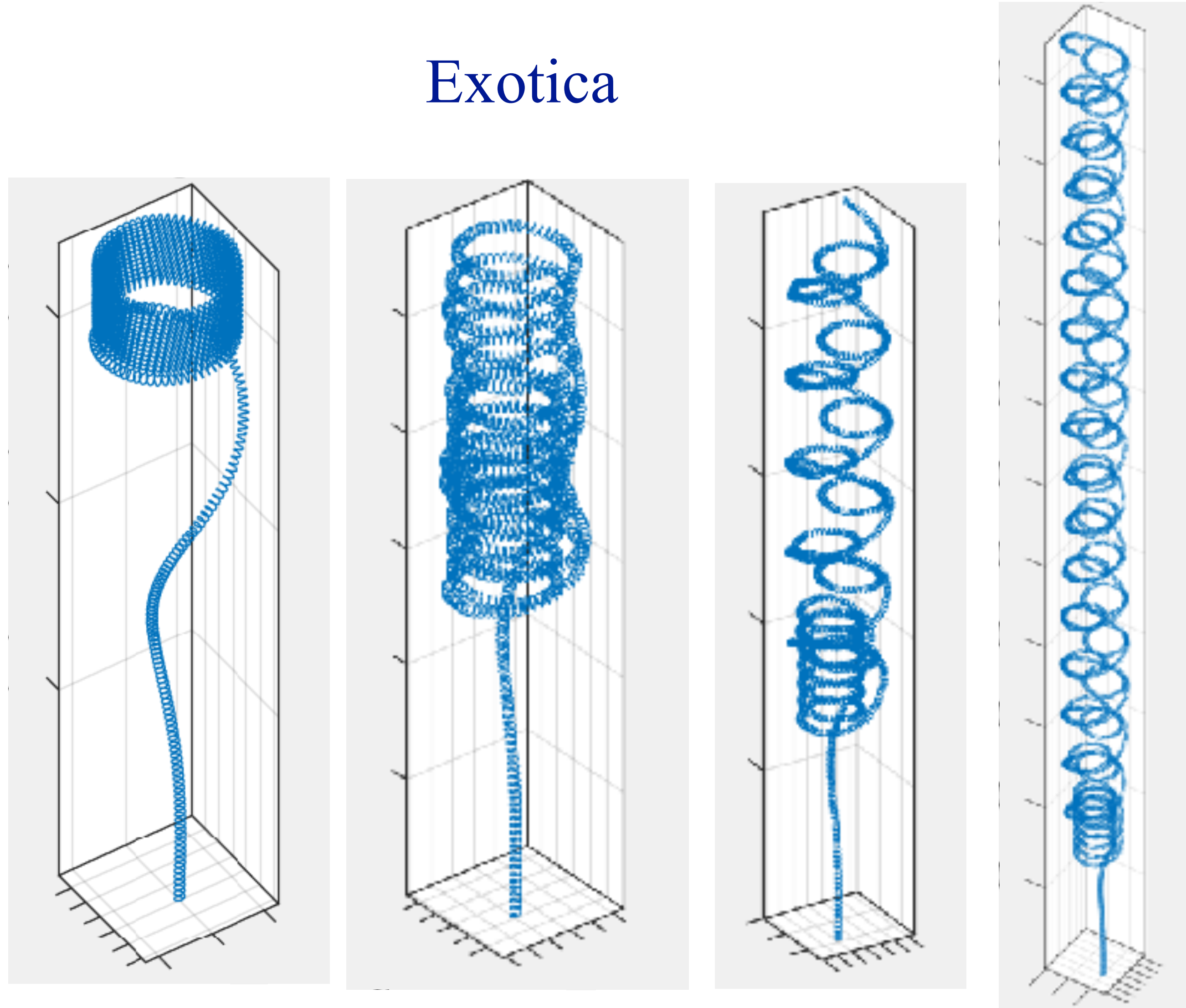
Helical spin states in a Coriolis force

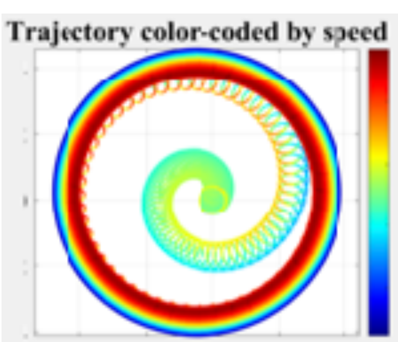
$$\mathbf{F_C} = m\dot{\mathbf{x}}_\mathbf{p} \wedge \boldsymbol{\Omega}$$



$\boldsymbol{\Omega}$

Exotica



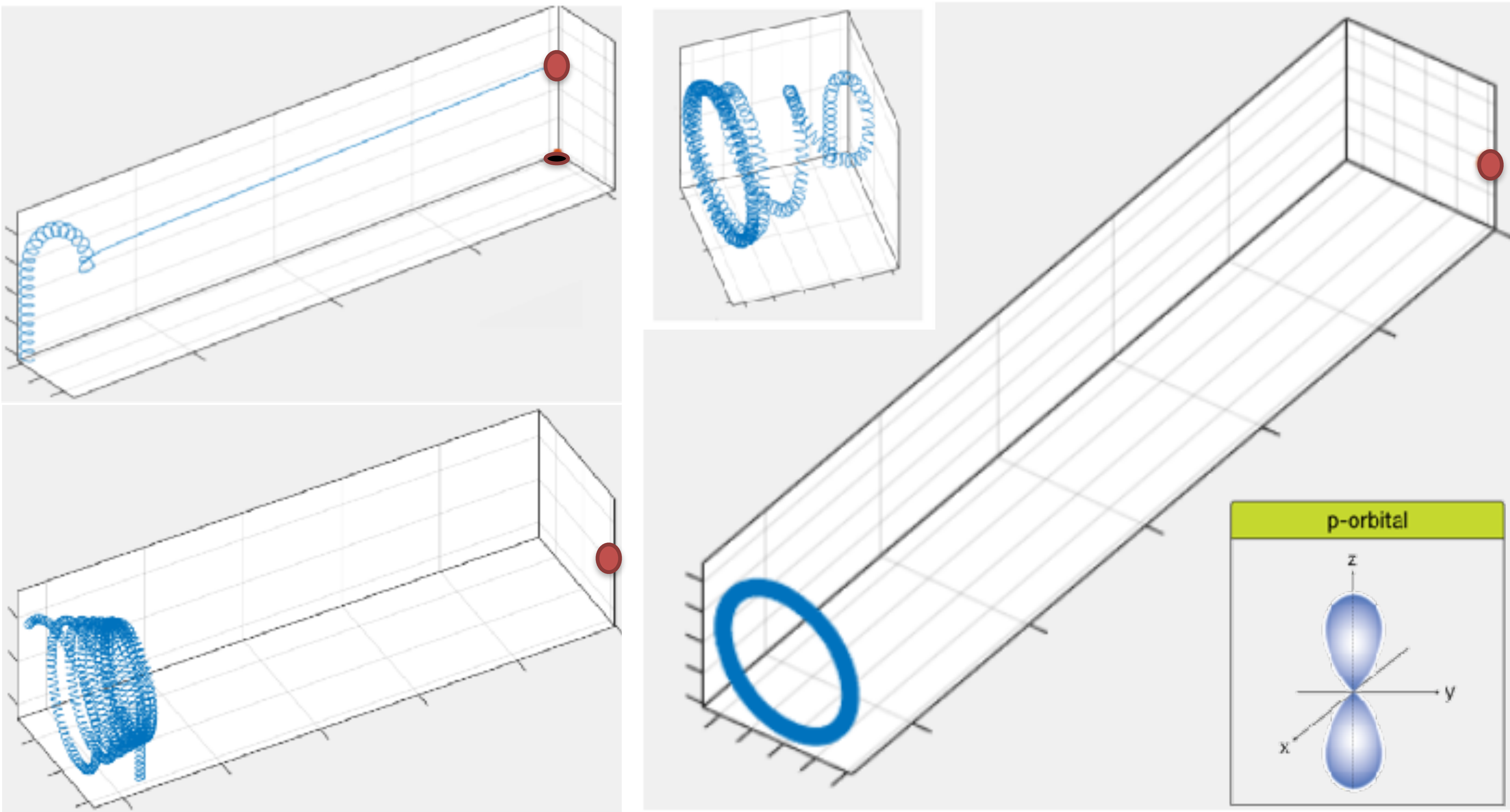


- double-helix: helix spirals along axis aligned with $\boldsymbol{\Omega}$ just as electrons orbit **B** in a cyclotron

Helical spin states in a central force

$$\mathbf{F_G} = -km\frac{\hat{\mathbf{r}}}{r^2}$$

- helix may either escape to infinity, or be drawn into center of force and break



- within a finite range of ICs, the helix forms a torus some distance from the CoF

The Old Hydrodynamic Interpretation of Quantum Mechanics

Schrodinger: $$i\hbar\ \Psi_t\ =\ -\frac{\hbar^2}{2m}\nabla^2\Psi\ +\ V\ \Psi$$

Madelung transformation (1928): $$\boxed{\Psi = \sqrt{\rho}\ e^{iS/\hbar}}$$

Continuity: $$\frac{D\rho}{Dt} + \rho\,\nabla\cdot\mathbf{u} = 0$$

Quantum Hamilton-Jacobi: $$\frac{\partial S}{\partial t} + \frac{1}{2}\,\mathbf{u}^2 - \boxed{\frac{\hbar^2}{2m^2}\,\frac{1}{\sqrt{\rho}}\,\nabla^2\sqrt{\rho}} + \frac{V}{m} = 0$$

QUANTUM POTENTIAL Q

where $\rho = |\Psi|^2$ is the probability density, S is the action,

$\mathbf{u} = \nabla S/m$ is the quantum velocity of probability,

$\mathbf{j} = \rho\,\mathbf{u}$ is the quantum probability flux.

Bohmian Mechanics (1952)

David Bohm

- equate quantum velocity of probability $\mathbf{u}$ and particle velocity $\dot{\mathbf{x}}_p$
- solve Schrodinger's equation for Ψ , from which Q is computed
- solve trajectory equation

$$\boxed{m\,\ddot{\mathbf{x}}_p = -\nabla Q - \nabla V}$$

NONLOCAL

Successes

- restores realism, the notion of particle trajectories, to quantum theory

FIGURE 1: An ensemble of trajectories for the two-slit experiment, uniform in the slits. (adapted by Gernot Bauer from Philippidis, Dewdney, & Hiley 1979: 23, fig. 3)

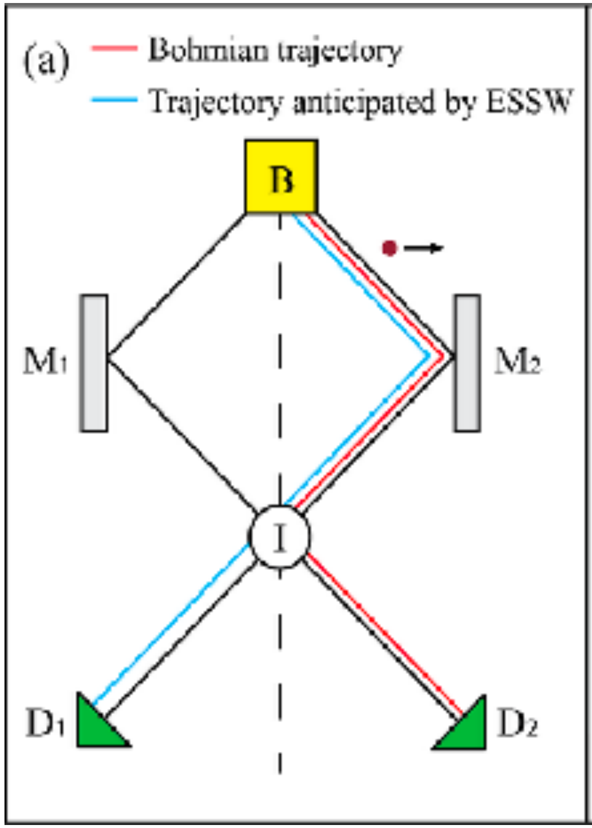
- a counterexample of the Impossibility Proofs that held sway at the time

Surreal trajectories

- Englert, Sully, Süssman and Walther (ESSW). 1992

- proposed an interference experiment intended to expose the shortcomings of Bohmian mechanics

`Bohmian trajectories are at odds with common sense: they are not real, they are surreal.'



- their reasoning was criticized by Aharanov & Vaidman (1996), who concluded:

` ESSW does not show that Bohmian mechanics is inconsistent, only that Bohmian trajectories behave differently from what one would expect classically.'

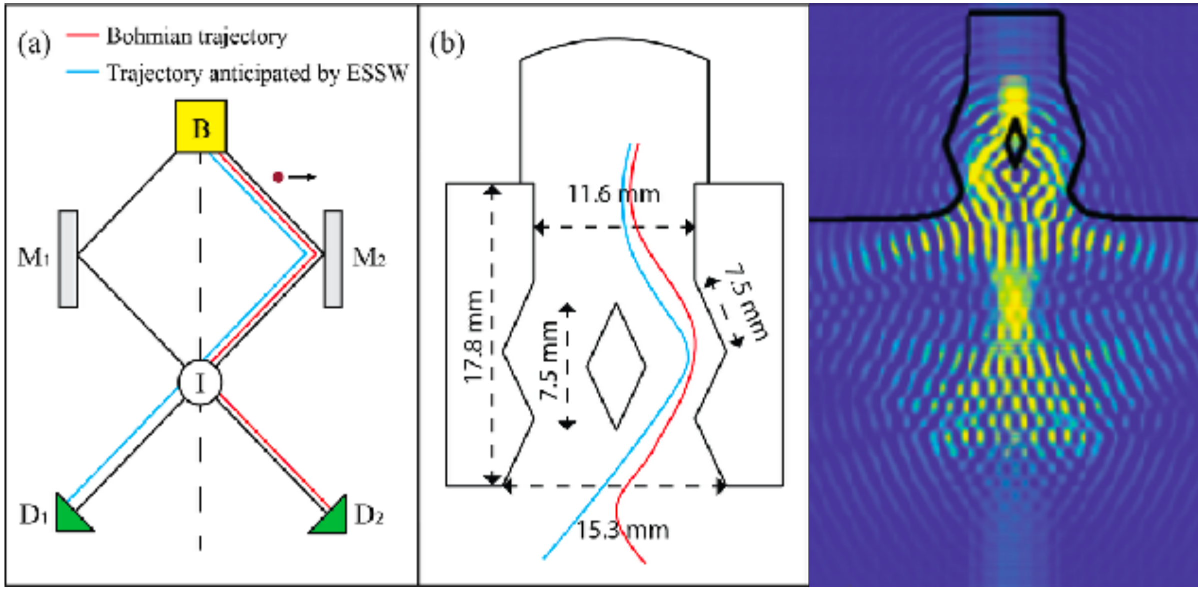
- experimental investigations using `weak measurement' found mean trajectories consistent with the surreal trajectories *(Mahler et al., 2016)*

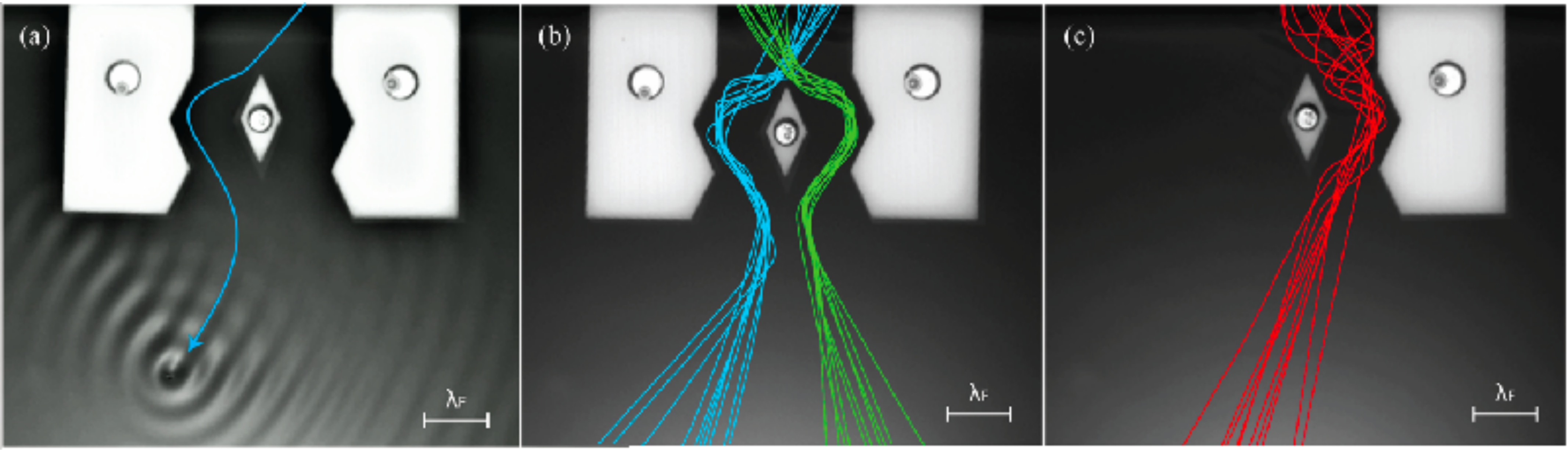
`We demonstrate that the trajectories seem surreal only if one ignores their manifest nonlocality.'

Real surreal trajectories

- *Frumkin, Struyve, Darrow, JB (PRA, 2022)*

- arise in the walker system at high memory





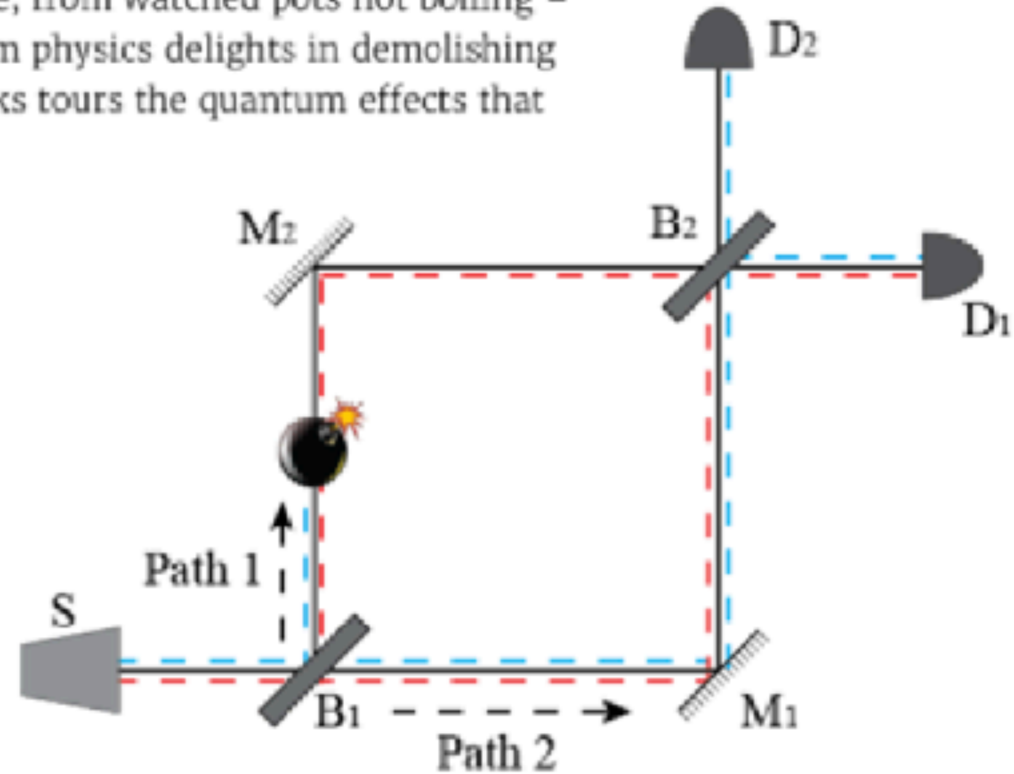
- \`surreal' trajectories are not at odds with classical intuition informed by a familiarity with pilot-wave hydrodynamics

- may be readily understood as a manifestation of non-Markovian pilot-wave dynamics, with no need to invoke \`nonlocality'

NewScientist

Seven wonders of the quantum world

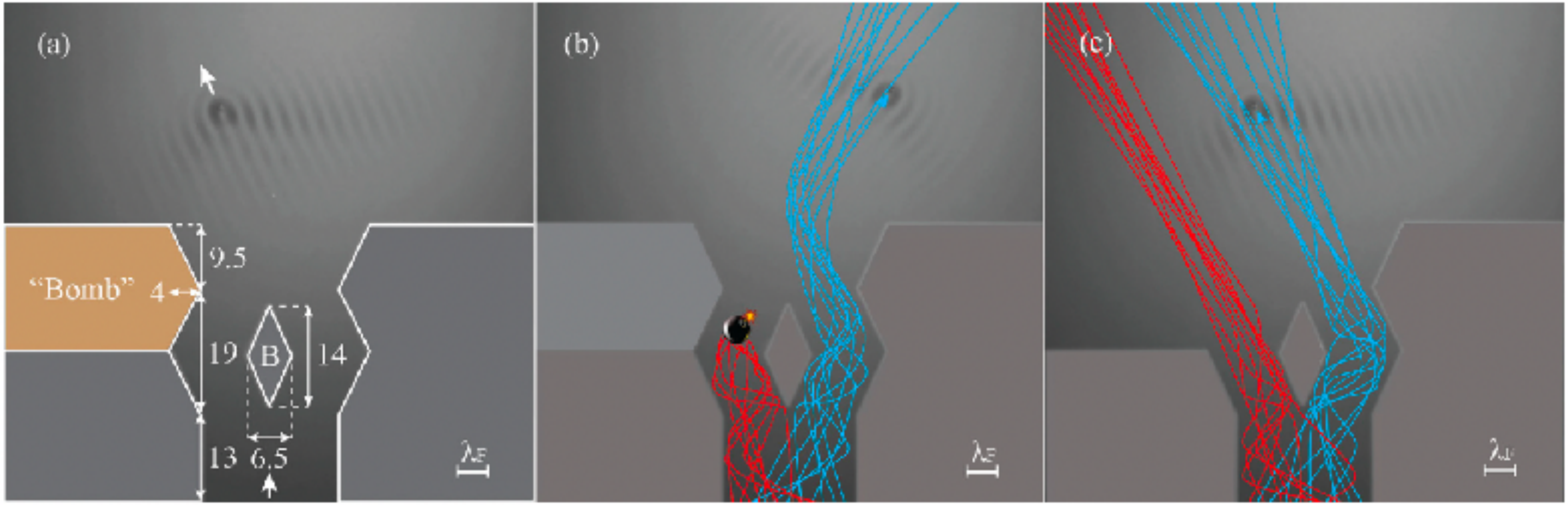
From undead cats to particles popping up out of nowhere, from watched pots not boiling – sometimes – to ghostly influences at a distance, quantum physics delights in demolishing our intuitions about how the world works. Michael Brooks tours the quantum effects that are guaranteed to boggle our minds.



- in absence of bomb, interference always causes photon to arrive at D1
- with bomb, particle either detonates bomb (Path 1) or arrives at D2 or D1 with equal probability
- if bomb is present 50% of the time, then you can detect it 25% of the time via a particle that took Path 2, so never interacted with it

A hydrodynamic analog of the quantum bomb tester

(and 'interaction-free measurement') - Frumkin & JB *(PRA, 2023)*



- submerged topography (orange) plays the role of the `bomb'
- in the absence of the bomb, all trajectories go to the left
- in the presence of the bomb, surreal trajectories arise:
  - — the pilot wave interacts with the boundaries, altering the drop's trajectory
- if a bomb is present 50% of the time, then 25% of the time, the droplet will detect a bomb along a path it didn't take

Bohmian Mechanics (1952)

David Bohm

- equate quantum velocity of probability $\mathbf{u}$ and particle velocity $\dot{\mathbf{x}}_p$
- solve Schrodinger's equation for Ψ , from which Q is computed
- solve trajectory equation

$$\boxed{m\,\ddot{\mathbf{x}}_p = -\nabla Q - \nabla V}$$

NONLOCAL

Shortcomings

- Einstein's objection: it is `*nonlocal*' by virtue of the quantum potential Q
- there is no mechanism for creating the guiding wave, which is imposed by fiat

Extensions (Bohm & Vigier 1954)

- invoke a stochastic forcing $\nabla\Phi_S$ from a `sub quantum realm':

$$\boxed{m\,\ddot{\mathbf{x}}_p = -\nabla Q - \nabla V + \nabla\Phi_S}$$

- particles jostle about $\mathbf{u}$ like Brownian motion of gas molecules about streamlines

	Bohmian mechanics	Walkers
WAVELENGTH	λ_B	λ_F
GUIDANCE	$m\,\ddot{\mathbf{x}}_p = -\nabla Q - \nabla V + \nabla\Phi_S$ NONLOCAL … AD HOC	$m\,\ddot{\mathbf{x}}_\mathbf{p} = -D\,\dot{\mathbf{x}}_\mathbf{p} + \nabla\eta(\mathbf{x},t) - \nabla V$ LOCAL
NON-LOCAL WAVE POTENTIAL	$\mathbf{Q} = -\frac{\hbar^2}{m^2}\frac{1}{\sqrt{\rho}}\nabla^2\sqrt{\rho}$ QUANTUM POTENTIAL	$\bar{\eta}(\mathbf{x}) = \eta_B * \mu(\mathbf{x})$ MEAN WAVE FIELD $\Phi \sim -[\nabla\bar{\eta}(\mathbf{x})]^2$ PONDEROMOTIVE POTENTIAL
STOCHASTIC FORCING	$\nabla\Phi_S$ ARBITRARY, *ad hoc*	$-\nabla\eta^*(\mathbf{x},t)$ PERTURBATION WAVE FIELD
WAVE ORIGIN	NONE	PARTICLE VIBRATION

Particle vibration on the Compton scale

- Frank Wilczek (*The Lightness of Being*, 2008): **`a poem in two lines'…**

RELATIVITY	**QM**
$E = m\ c^2$	$E = \hbar\ \omega$

Einstein-de Broglie relation: $$mc^2 = \hbar\omega$$

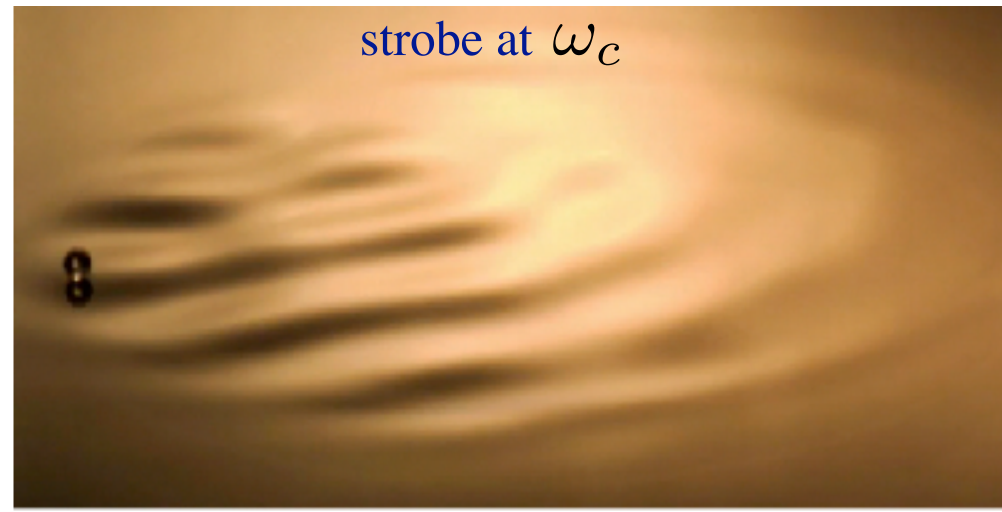
Natural frequency: $$\boxed{\omega_c = \frac{mc^2}{\hbar}}$$ **Compton frequency**

- de Broglie (1926) suggested microscopic particles have an internal clock at ω_c that generates a wave that moves in concert with the particle

→ **particles move in resonance with a guiding or `pilot' wave field**

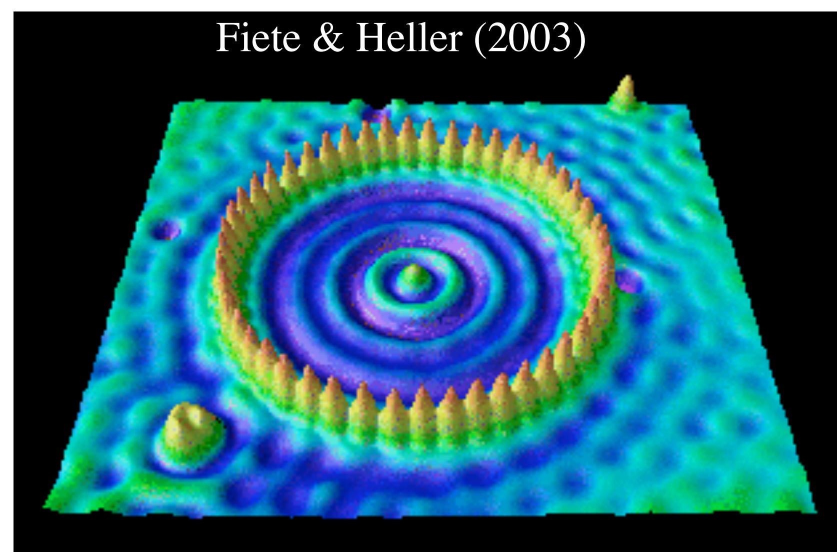
de Broglie's pilot-wave theory

- **fast** dynamics: particle oscillations at $\omega_c = \frac{m_0 c^2}{\hbar}$ create pilot-wave field centered on particle



- **intermediate** pilot-wave dynamics: particle rides its guiding wave field such that

$$\mathbf{p} = \hbar\, \mathbf{k}$$



- **long-term statistical** behaviour described by standard quantum theory

	de Broglie	Walkers
WAVE TRIGGER	**ZITTERBEWEGUNG**	**Bouncing**
VIBRATION FREQUENCY	$\omega_c = \frac{m_o c^2}{\hbar}$	$\omega_d = \sqrt{\frac{\sigma}{m}}$
WAVES	**Matter waves**	**Capillary Faraday**
WAVE-PARTICLE RESONANCE	**Harmony of phases**	$\omega_d = \omega_F$
WAVE ENERGETICS	$mc^2 \longleftrightarrow \hbar\omega$	$mgH \longleftrightarrow$ Surface Energy
KEY PARAMETER	$\hbar$	σ
STATISTICAL WAVELENGTH	λ_B	λ_F
VIBRATION LENGTH	$\lambda_c = h/mc$	λ_F

JB, *ARFM* *(2015)*

Schrodinger's equation: Origins

- de Broglie's pilot-wave theory was criticized on the grounds that he did not have a theoretical description of his pilot wave

- Schrodinger was, like Einstein, a supporter of de Broglie's theory, and so derived an equation to describe the pilot wave

Ingredients

PLANCK RELATION

$$\boxed{E = \hbar\,\omega}$$

DE BROGLIE RELATION

$$\boxed{p = \hbar\,k}$$

$$E = \hbar\omega = \frac{1}{2}mv^2 = \frac{1}{2m}\hbar^2 k^2 \longrightarrow \omega = \frac{1}{2m}\,\hbar k^2$$

What equation has this dispersion relation?

Schrodinger's Equation:

$$\boxed{i\hbar\,\frac{d\Psi}{dt} = -\frac{\hbar^2}{2m}\,\nabla^2\Psi}$$

- despite his objections, Schrodinger's equation was adopted as a description of the statistics of quantum systems

The Klein-Gordon Equation

$$\boxed{\frac{1}{c^2}\Psi_{tt} \;-\; \nabla^2\Psi \;+\; \frac{m^2c^2}{\hbar^2}\,\Psi \;=\; 0}$$

Dispersion relation: $\omega = \omega_c(1+\beta^2)^{1/2}$

where $\underbrace{\omega_c = \frac{mc^2}{\hbar}}_{\text{COMPTON}}$, $\beta = \frac{v}{c} = \frac{\hbar k}{mc} = \frac{k}{k_c}$, $k_c = \frac{mc}{\hbar}$

Seek solution: $\Psi(\mathbf{x},t) \;=\; \underbrace{e^{-i\frac{mc^2}{\hbar}t}}_{\text{FAST}}\;\underbrace{\Psi^s(\mathbf{x},t)}_{\text{SLOW}} \longrightarrow$ `Strobed' solution

$$\cancel{\frac{\hbar^2}{2mc^2}\,\Psi^s_{tt}} + \boxed{i\hbar\;\Psi^s_t \;=\; -\,\frac{\hbar^2}{2m}\,\nabla^2\Psi^s}$$ **LSE**

if $\Psi^s(\mathbf{x},t)$ is slowly varying, with a frequency ω s.t. $\omega/\omega_c \ll 1$

- → strobing at ω_c would reveal a wave envelope that is a solution to the LSE.
- → such strobing conceals the origins of the wave, particle vibration at ω_c
- → by starting with LSE, Bohm neglects the fast time responsible for waves

So, what is the matter wave field in QM?

"What is it that waves in wave mechanics? We have no idea…"

— J.S. Bell (1993)

- de Broglie suggested that the field satisfies the Klein-Gordon equation, as governs the Higgs field

- workers in Stochastic Electrodynamics (SED) suggest an EM pilot wave

  (*de la Pena*, *Cetto*, *Valdes-Hernandes 2015*)

- several have suggested gravitational waves: matter waves arise in the fabric of spaced-time (*Feoli & Scarpetta 1998, D'Errico 2023*)

What might de Broglie have tried … had he had MATLAB …?

Relativistic, classical pilot-wave theory

Darrow & Bush (2024)

- combine particle and field Lagrangians at the level of actions

$$\mathcal{S} = \mathcal{S}_{\text{field}} + \mathcal{S}_{\text{particle}} + \mathcal{S}_{\text{interaction}}$$

$$\downarrow$$

$$\mathcal{S}_{\text{field}} = \frac{1}{2}\int_{\Omega} d^4 q \; \left(\partial^\mu \phi \partial_\mu \phi - m^2 \phi^2\right)$$

$$\mathcal{S}_{\text{particle}} = -\int_0^{t'} dt \; m c^2 \gamma^{-1}$$

$$\mathcal{S}_{\text{interaction}} = \int_0^{t'} dt \; \gamma^{-1} \left(a\phi(q_p) - b\tau\gamma \dot{q}_p^\mu \partial_\mu \phi(q_p)\right)$$

- one is free to choose the manner of the wave-particle coupling via the interaction action

Coupled wave and guidance equations

$$\boxed{\begin{aligned} (\partial_\mu \partial^\mu + m^2)\phi &= \gamma^{-1}(a+b)\delta^3(q - q_p) \\ d_t\left((m - a\phi(q_p))\gamma \dot{q}_p\right) &= \gamma^{-1}(a+b)\nabla \phi(q_p) \end{aligned}}$$

Coupling constants a, b

$$\longrightarrow$$

Current model

$$\boxed{\begin{aligned} (\partial_\mu \partial^\mu + m^2)\phi &= \gamma^{-1} b \delta^3(q - q_p) \\ d_t\left(m\gamma \dot{q}_p\right) &= \gamma^{-1} b \nabla \phi(q_p) \end{aligned}}$$

Single coupling constant b

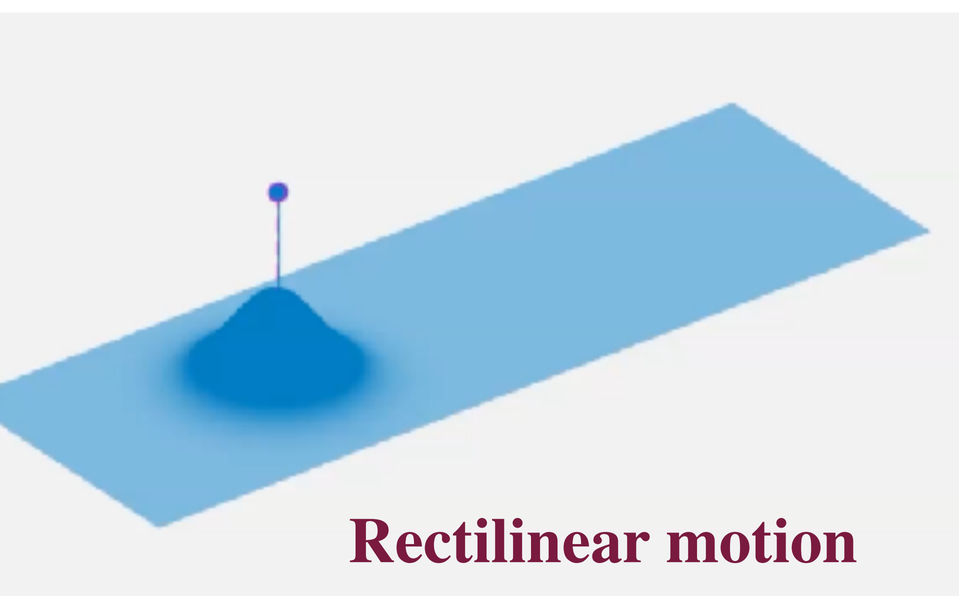
Relativistic, classical pilot-wave theory

Darrow & Bush (2024)

$$\boxed{\begin{gathered}(\partial_\mu \partial^\mu + m^2)\phi = \gamma^{-1} b \delta^3 (q - q_p) \\ d_t\,(m\gamma\dot{q}_p) = \gamma^{-1} b \nabla\phi(q_p)\end{gathered}}$$

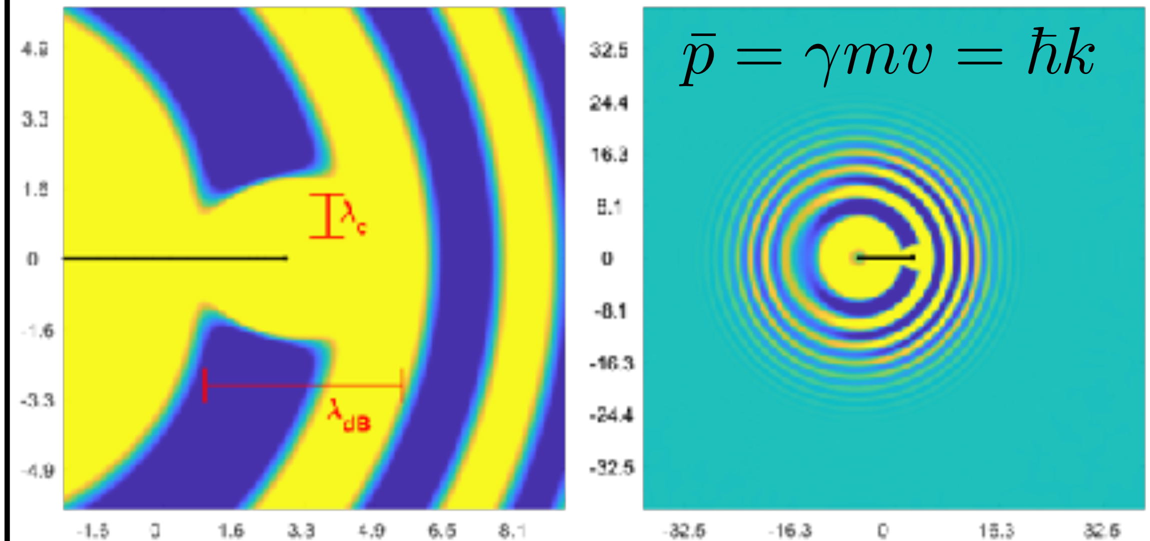
Key features

- Lorentz invariant
- correct classical limit
- emergent oscillations at $\omega_c = \dfrac{mc^2}{\hbar}$



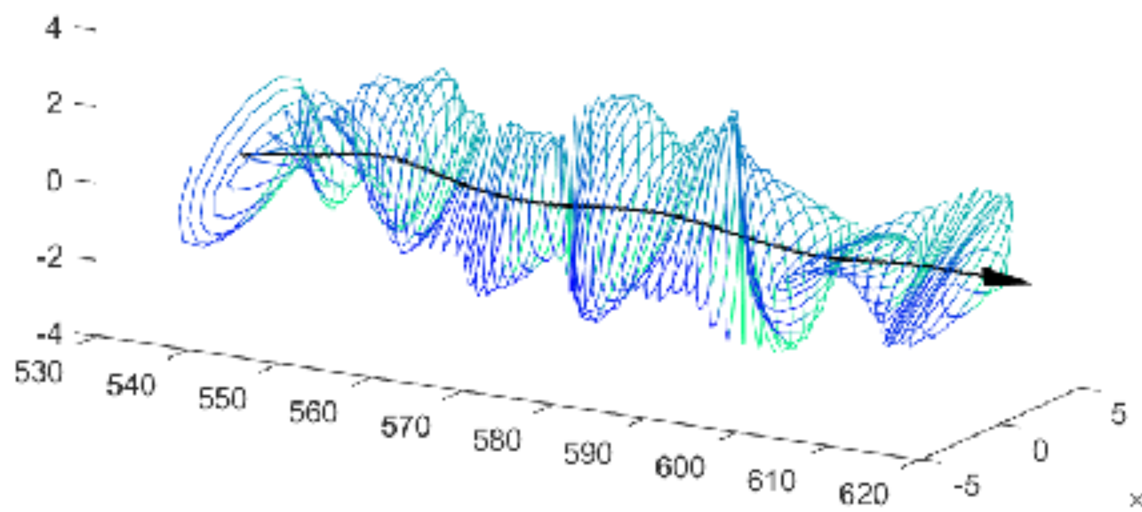
Free particle

- self-propels in a rectilinear fashion



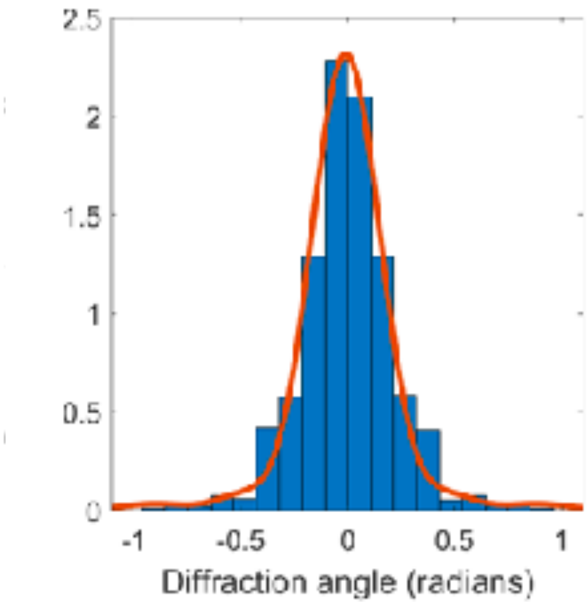
Near boundaries

- Zitter consistent with Heisenberg Uncertainty

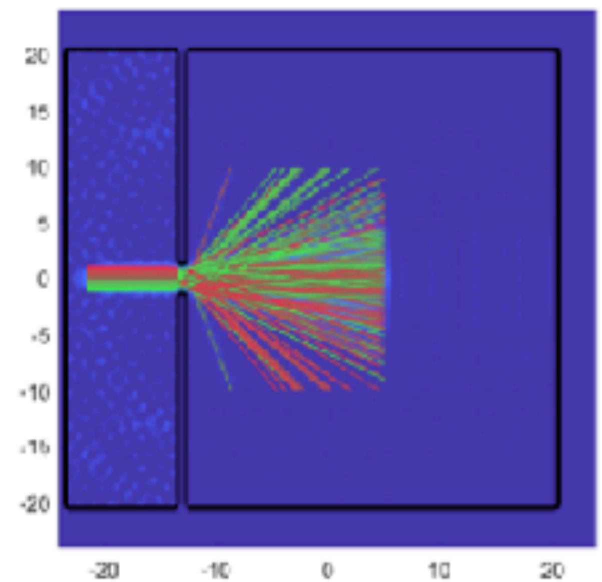


Fraunhofer diffraction with our classical pilot-wave theory

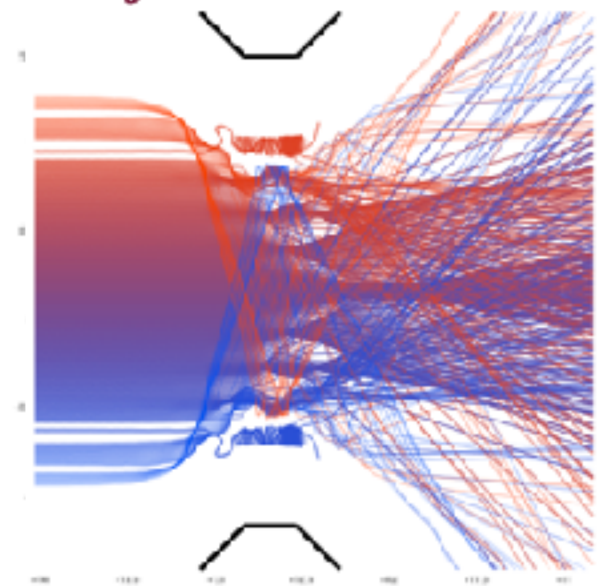
Single slit diffraction pattern



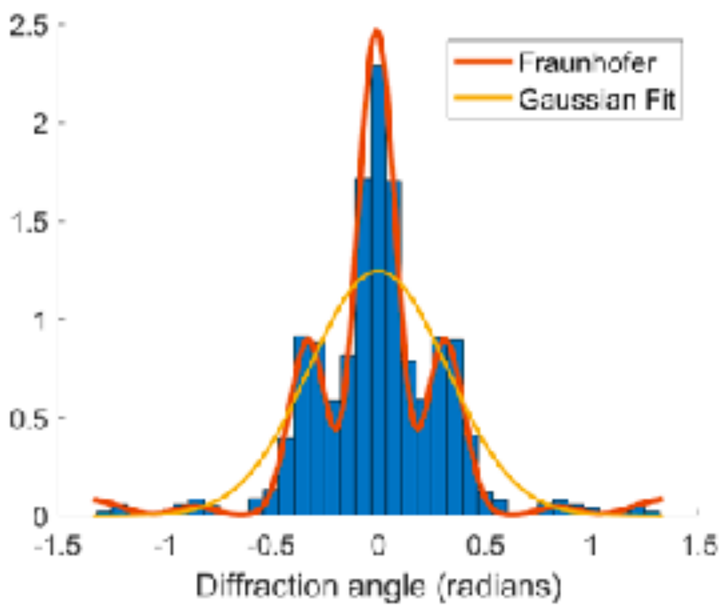
Trajectories



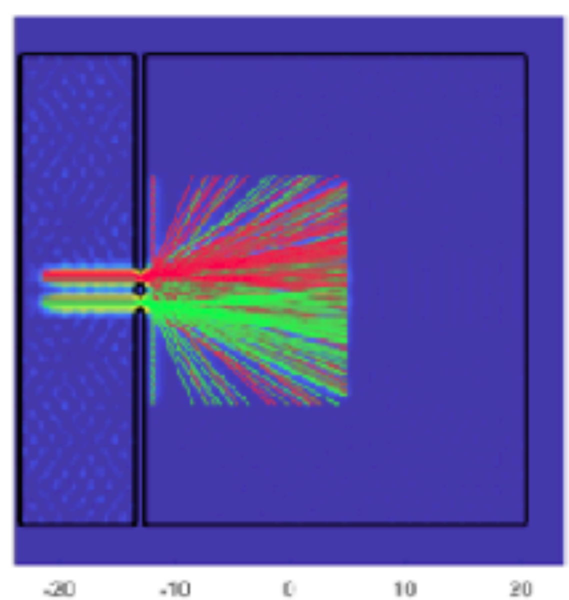
Trajectories inside slit



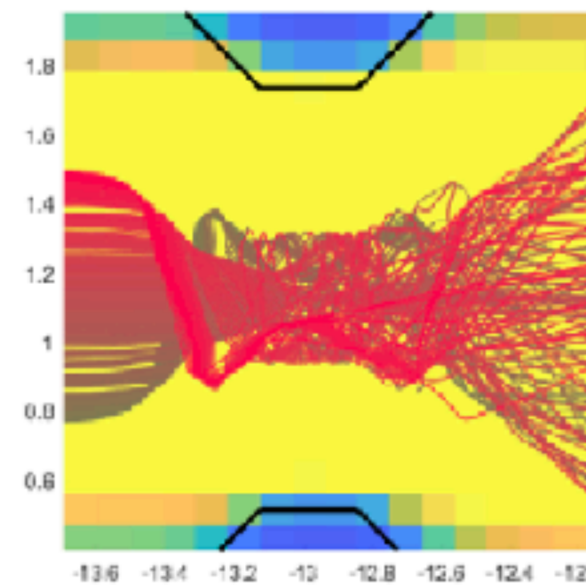
Double slit pattern



Trajectories



Trajectories inside slit



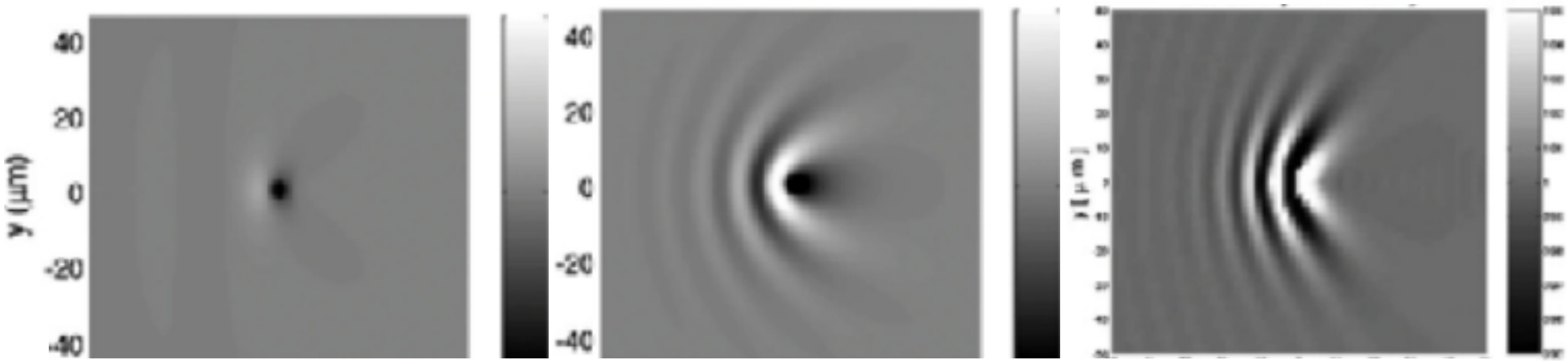
Darrow & Bush (PRR, 2025)

Pilot-wave hydrodynamics supports the conjecture of de Broglie (and its modern extension, SED) that quantum dynamics may be understood in terms of a vibrating particle moving in resonance with its pilot-wave field.

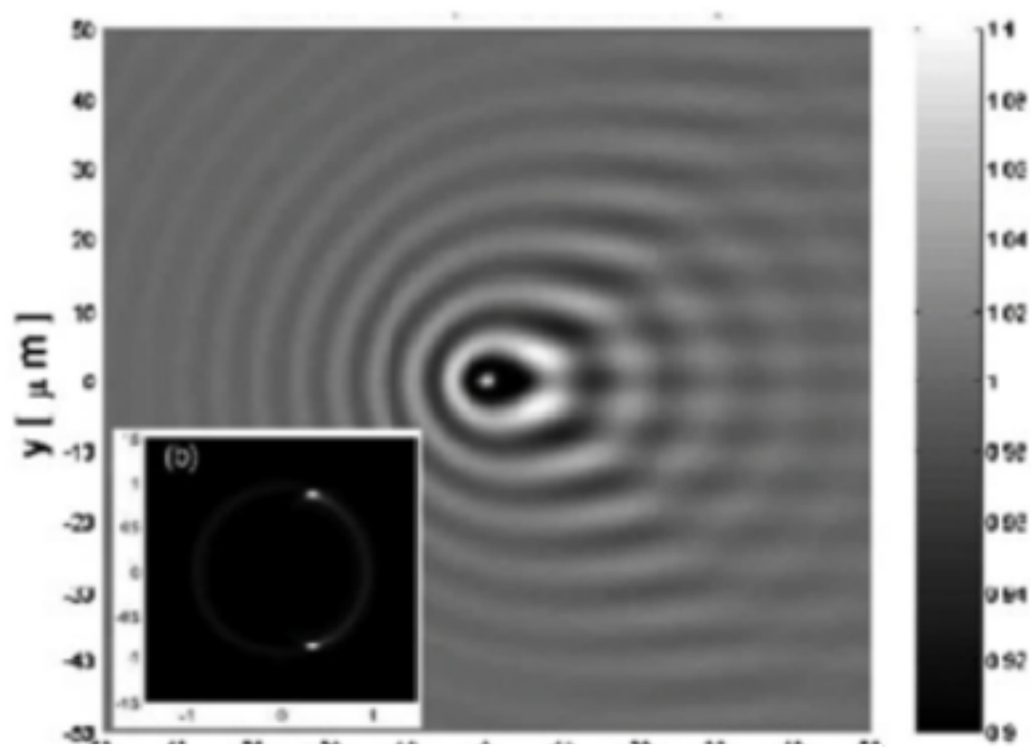
How would such a pilot-wave field look on the quantum scale?

We already know…

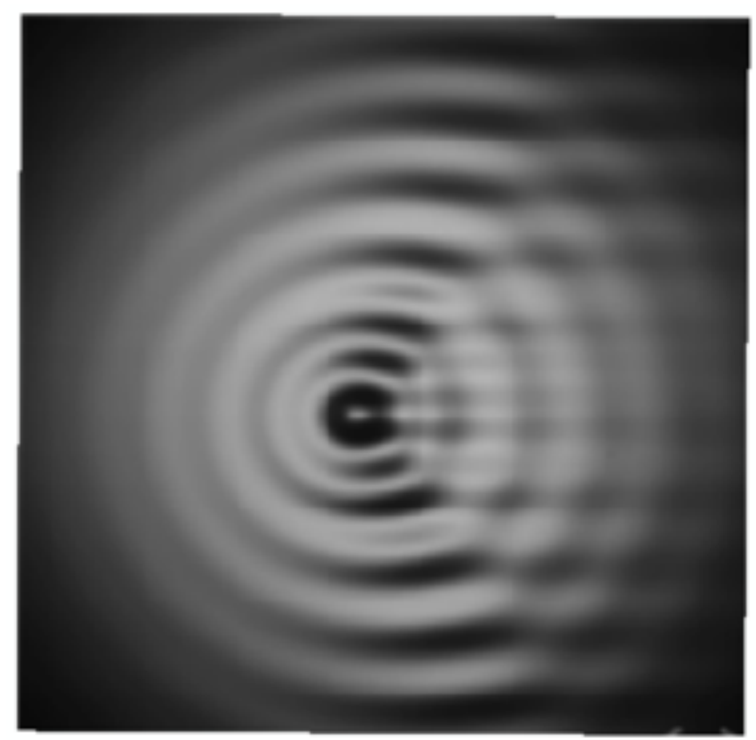
Quantum fluids of light: polariton flow over a vibrating defect



Carusotto & Ciuti (Rev. Mod. Phys. 2013)



Exciton-polariton



Walker

The New Hydrodynamic Interpretation of QM

- an amalgam of de Broglie's pilot-wave theory and the walker system
- the particle vibration is the source of the particle's guiding wave

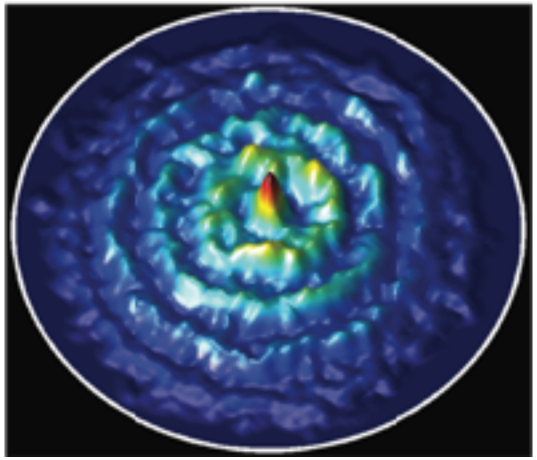
``Bush (2015) has further explored this sort of possibility for the emergence of a Bohmian version of quantum mechanics from something like classical fluid dynamics. A serious obstacle to the success of such a program is the quantum entanglement and nonlocality characteristic of many-particle quantum systems."

— *S. Goldstein, Bohmian Mechanics (Stanford Encyclopedia of Philosophy, 2021)*

- nonlocality is obviously *not* a feature of the walker system, need not be invoked for any of the myriad established HQAs

Nonnonlocality: misinferences of non-locality from local hereditary pilot-wave dynamics

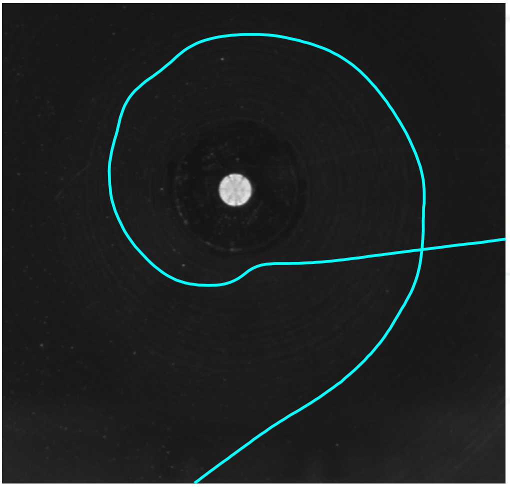
Harris et al. (2013)



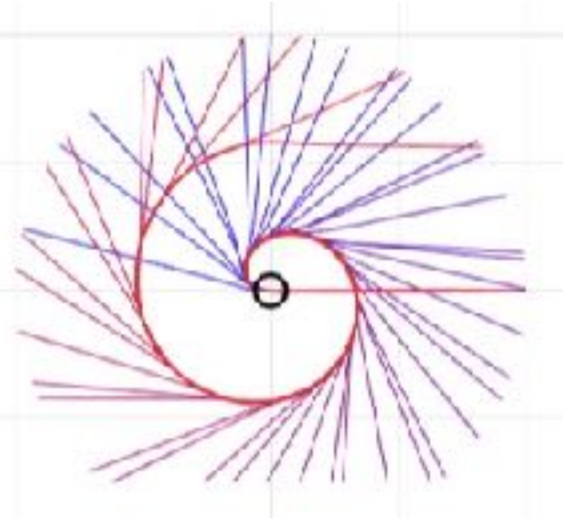
1. Wave function collapse

- act of observation causes instantaneous collapse of statistical wave form

2. Spooky action at a distance



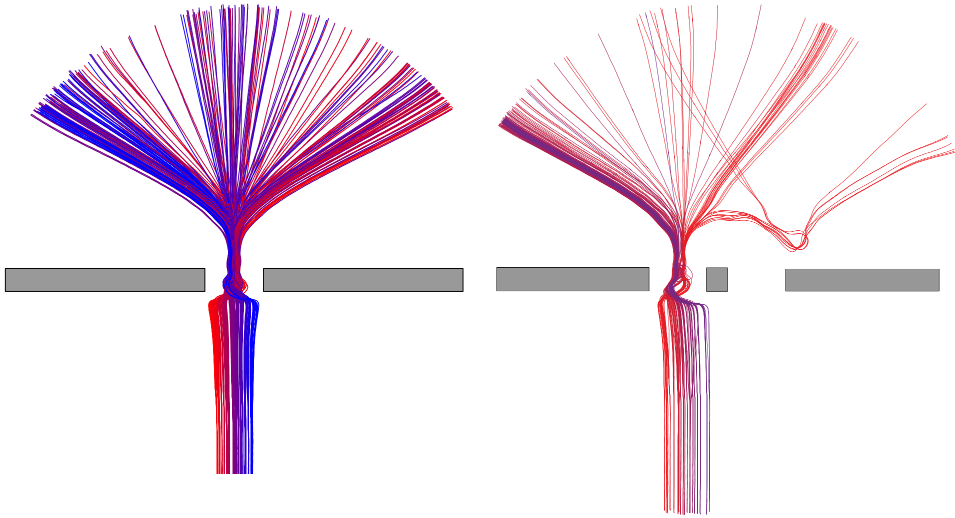
Harris et al. (2018)



Saenz et al. (2018)

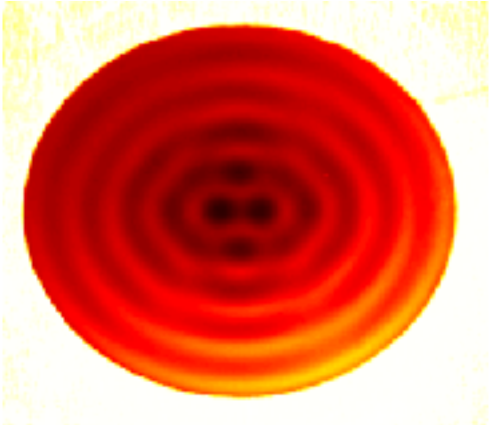
- interaction with pillars and wells: wave-mediated local forces give rise to apparently non-local lift forces

Couder & Fort (2006), Pucci et al. (2017)



- double-slit experiment: walker feels both slits by virtue of its spatial delocalization

Nonnonlocality: misinferences of non-locality

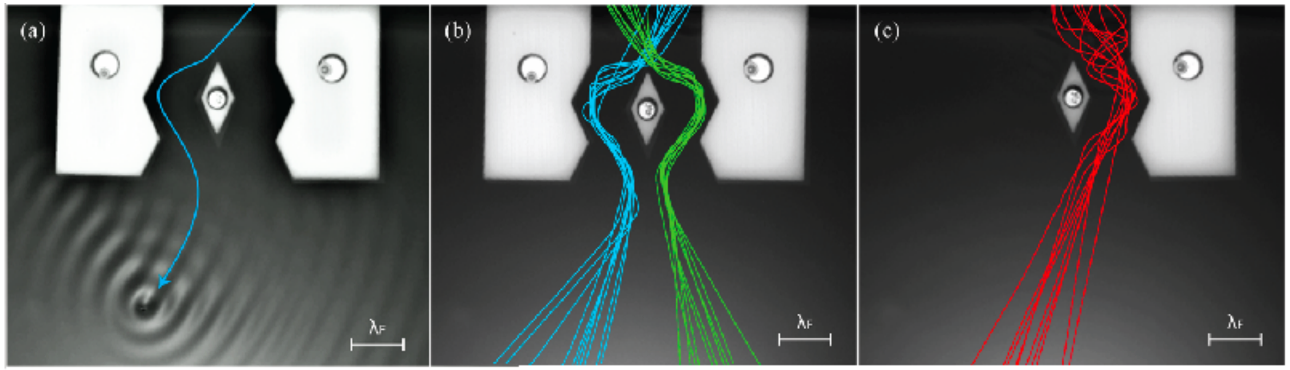


3. Mean pilot-wave potential

- plays role comparable to nonlocal quantum potential in Bohmian mechanics *(Durey et al. 2018)*

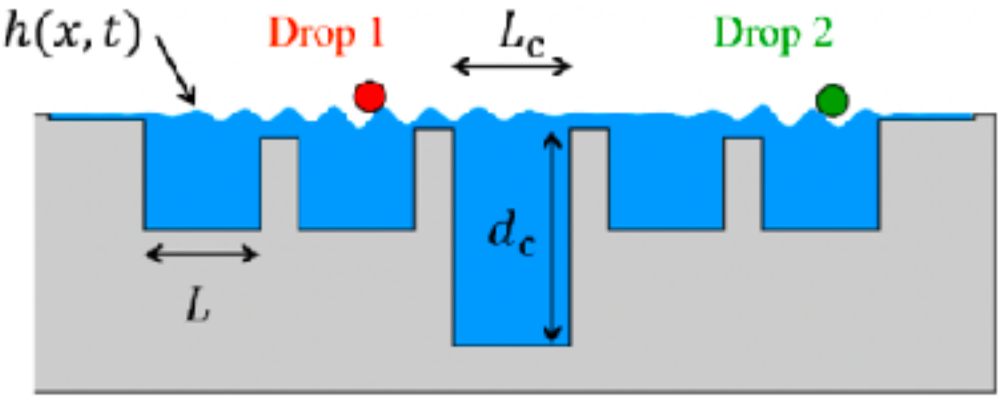
4. Surreal trajectories and interaction-free measurement

(Frumkin & JB, 2024)



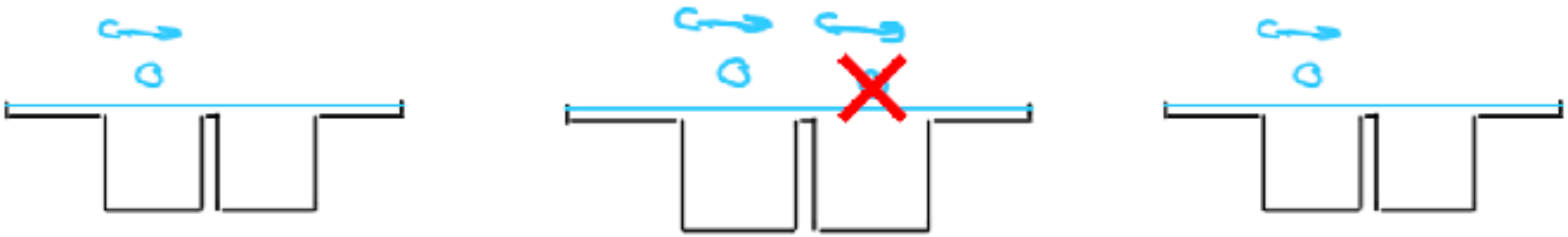
5. Distant correlations between drop pairs

- drops may become synchronized via local, wave-mediated forces
- Bell violations achieved in a static test

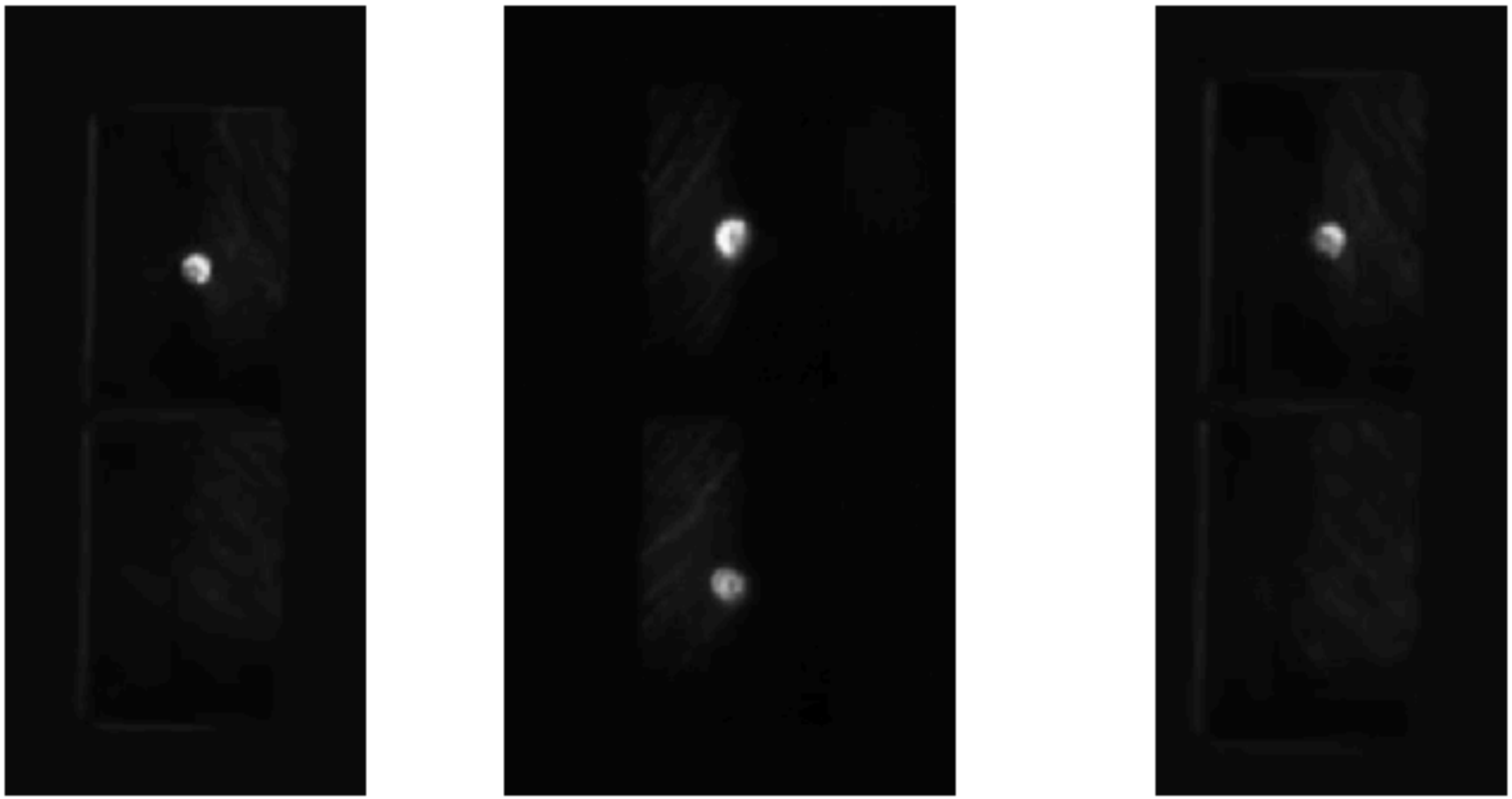


(Papatryfonos, Vervoort, Nachbin, Labousse, JB, 2024)

Towards a dynamic Bell test



- surviving drop retains memory of its lost partner



Might memory account for entanglement?

The Quantum Problem (Holland 1993)

Individual events in QM are unpredictable, but a coherent statistics emerges when one considers a large number of such events.

Central puzzling phenomena

- ***self-interference*** of single particles; e.g. single- and double-slit
- ***tunneling***: a quantum particle can pass through a barrier forbidden to a classical particle
- ***stability of matter***: atoms and molecules exist only in certain discrete energy states, and do not collapse (as would their classical counterparts). During transition between states, a discrete quantum of energy is exchanged with the EM field.
- ***spin***: a novel type of nonclassical internal angular momentum, as is apparent in, for example, the Stern-Gerlach experiment
- ***nonlocal correlations***: the properties of one particle can depend on those of an arbitrarily distant system with which it has interacted in the past

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The appeal of walking droplets

- timescales of bouncing, lateral motion, statistical convergence all accessible

- a macroscopic example of a particle moving in response to its own wave field, the theoretical description of which is notoriously difficult on the microscale

  — *e.g.* the Lorentz-Abraham-Dirac equation of EM

- connects to a number of hidden-variable quantum theories:

  — de Broglie's double-wave solution, Bohmian mechanics, Stochastic Mechanics (*Nelson 1964*), Stochastic Electrodynamics (*de la Peña, Cetto & Hernandez 2018*)

- provides a vehicle for revisiting QM with a fresh perspective, a progressive approach, a physical picture that might allow us to understand it

- suggests hydrodynamics may provide the platform for marrying QM with GR

  — relation between gravitational waves and matter waves?

Conclusion

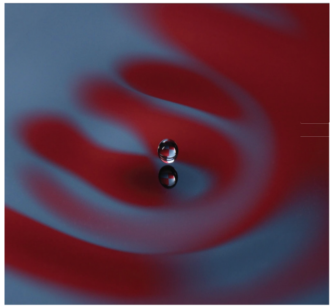
The hydrodynamic pilot-wave system

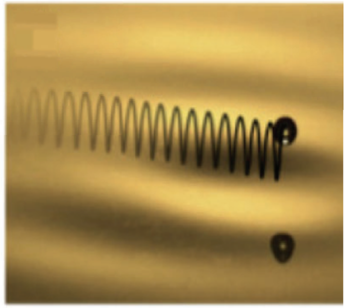
- provides a vehicle to redefine the boundaries between classical and QM
- extends the range of classical systems to include features previously thought to be exclusive to the microscopic, quantum realm
- provides a conceptual framework, a progressive approach, for understanding QM

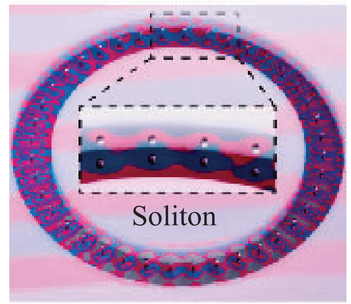
Pilot-wave hydrodynamics demonstrates how classical hereditary mechanics may give rise to features taken as evidence of non-locality in QM.

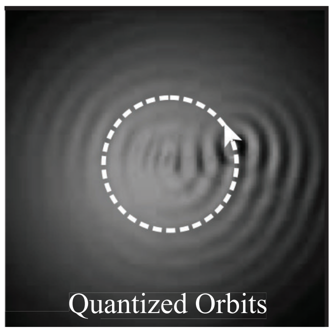
- is reminiscent of the de Broglie's pilot-wave theory of quantum dynamics
- suggests the shortcomings of quantum pilot-wave theories of de Broglie and Bohm
- has motivated a new class of theoretical models of classical pilot-wave dynamics

Hydrodynamic quantum analogs









Spring 2024

18.S996: Hydrodynamic Quantum Analogs

Prof. John W. M. Bush

MW 2:30-4pm, Room 2-143

Prerequisites: A course in waves (e.g. 8.03) or fluid mechanics (e.g. 18.354, 18.355, 2.25) or permission of Instructor

Millimetric droplets self-propelling along the surface of a vibrating liquid bath represent a macroscopic realization of wave-particle duality, and of a dynamics of the form proposed for microscopic quantum particles by Louis de Broglie in the 1920s. Since their discovery, experimental studies have shown that these walking droplets exhibit many features previously thought to be exclusive to the microscopic, quantum realm. The system thus represents a new platform for exploring and redefining the boundary between classical and quantum behavior. A hierarchy of theoretical models have been developed with a view to rationalizing the emergent quantum behavior and connecting to extant models of quantum dynamics. In this course, we will focus on reviewing the experimental studies and theoretical models of the hydrodynamic system, as well as its successes and shortcomings as a quantum analog. We will also review related realist models of quantum dynamics, including Bohmian mechanics and de Broglie's original theory of matter waves.

For further information, please contact Prof. Bush at bush@math.mit.edu