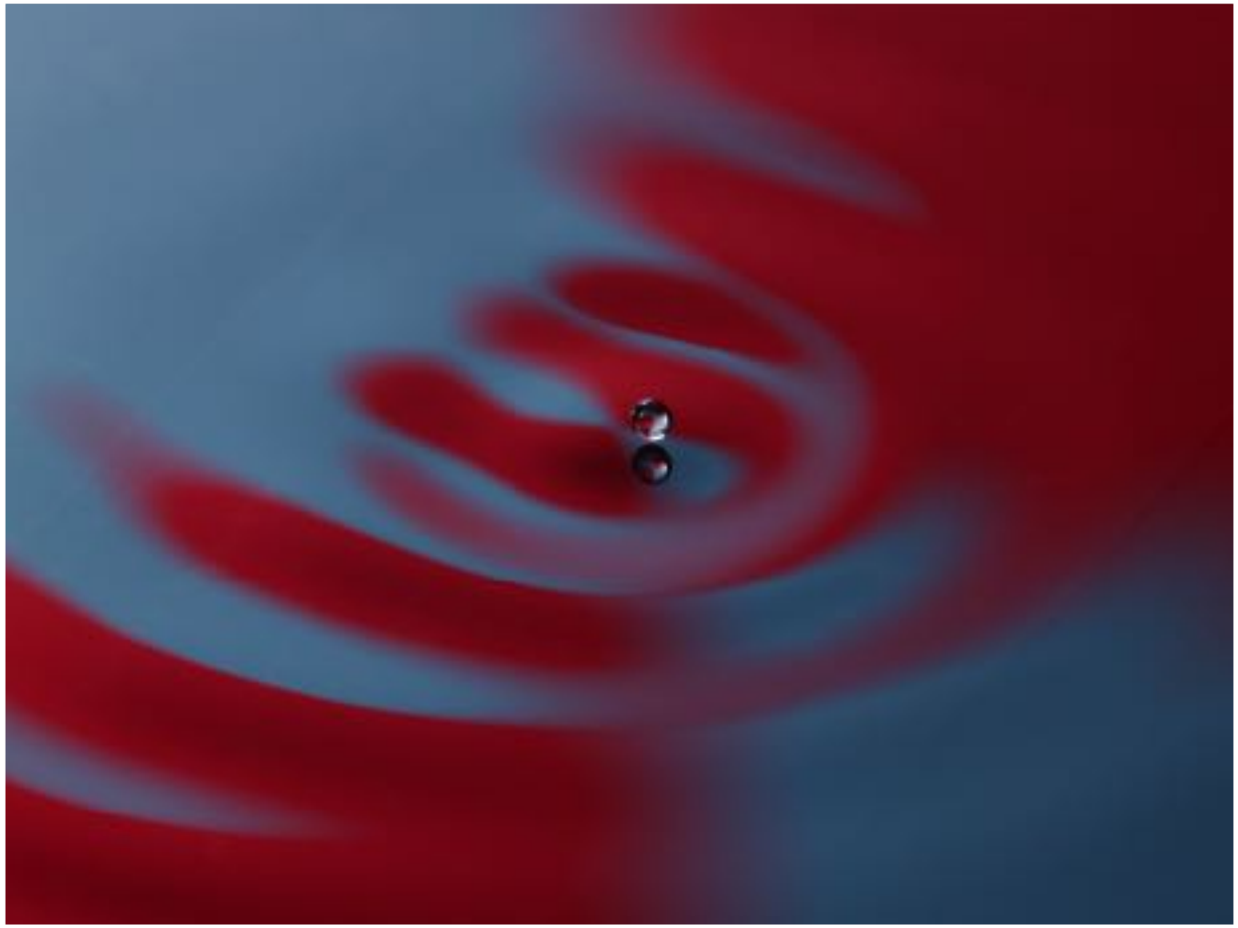
Hydrodynamic quantum analogs



John W. M. Bush

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Newton

`A man may imagine things that are false,
but he can only understand things that are true…'

Feynman

`No one understands quantum mechanics.'

John Bell

`I suspect that what will happen is that physicists will continue to buzz like flies against a closed window, then someone will open a door in the back of the room.'

CLASSICAL PHYSICS

Laplacian determinism

The macroscopic world is deterministic.

Initial conditions uniquely determine outcome.

Chaos theory (Poincaré 1900s, Lorenz 1960s)

There are practical limits to our predictive powers.

Complex systems are sensitive to initial conditions.

Hereditary mechanics (Volterra 1920, Brillouin 1926)

Systems with memory, whose evolution is influenced by their past.

Predictive power requires knowledge of both ICs and history.

Quantum mechanics

- a wave theory that describes the statistics of microscopic particles
- fails to describe particle trajectories — indeed, some flatly deny that they exist

The free particle:

$$E = \hbar\,\omega \qquad\qquad \mathbf{p} = \hbar\,\mathbf{k}$$

- an association of a particle with a wave

But where is the particle, and why does it move?

- an insistence on the completeness of a trajectory-free quantum mechanics has lead to longstanding difficulties

— the proliferation of quantum interpretations

— an abundance of paradoxes and troubling language

— the inference of ***quantum nonlocality*** (superluminal signaling)

Schrodinger's cat: a lampoon of quantum philosophy developed in correspondence between Schrodinger and Einstein

- prior to measurement, the cat is in a superposition of states, alive and dead

The instrumentalist view

- QM is the theory that describes matter on the microscopic scale
- predicts the probabilities of outcome of experimental measurements
- sufficient for the pragmatist interested only in applying the theory

But what is going on under the hood?

Quantum interpretations

- attempts to explain how QM informs our understanding of the physical world
- until the 1980s, one interpretation was preeminent…

The Copenhagen Interpretation

The Copenhagen Interpretation (c.a. 1920s)

`An intellectual deceit borne of despair.' — Schrödinger

Nature is fundamentally probabilistic. Microscopic particles do not move on trajectories. Their dynamics are described entirely by a probability wave that evolves according to Schrodinger's equation. Until an observation is made, the particles exist only in a state of metaphysical limbo. Observation forces the collapse of the wave function, the emergence of the particle into reality.

Represents a drastic change of world view, an abandonment of determinism, realism and locality.

Determinism : the evolution of a physical system can be determined from its initial conditions.

Realism : there is a reality that exists independent of human observation.

Locality : nothing travels faster than the speed of light.

Bohr vs Einstein: the debate over the Nature of Reality (*PRL*, 1935)

The Copenhagen Interpretation

The statistical predictions of QM provide a complete description of reality.

The Realist Stance

There is a hitherto unresolved dynamics underlying quantum statistics.

"I am, in fact, rather firmly convinced that the essentially statistical nature of contemporary quantum theory is solely to be ascribed to the fact that it operates with an incomplete description of physical systems."

"In a complete physical description, the statistical quantum theory would take an approximately analogous position to statistical mechanics within the framework of classical mechanics."

- Albert Einstein

The Copenhagen Interpretation

PRO: **Bohr, Heisenberg**

"*It is clear that* the double slit experiment can in no way be reconciled with the idea that electrons move on paths. In quantum mechanics, there is no such concept as the path of a particle."

- Landau and Lifshitz, 1977

" The idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees, independently of whether or not we observe them ... *is impossible* ."

- Heisenberg, 1958

CON: **Einstein, Schrodinger, Bell**

"There is something rotten in the state of Denmark".

- John Bell

"Bohr brainwashed a whole generation of physicists into believing that the problem had been solved".

- Murray Gell-Mann

An epic take-down

``In the biological sciences, one takes it for granted that in addition to the condition, there must be some other causative factor, not yet identified. One searches for it, tracking down the assumed cause by a process of elimination of possibilities that is sometimes extremely tedious. But persistence pays off; over and over again... Most enzymes, vitamins, viruses, and other biologically active substances owe their discovery to this reasoning process.

What is done in quantum theory today is just the opposite; when no cause is apparent one simply postulates that no cause exists---ergo, the laws of physics are indeterministic and can be expressed only in probability form.

Biologists have a mechanistic picture of the world because, being trained to believe in causes, they continue to use the full power of their brains to search for them---and so they find them. Quantum physicists have only probability laws because for two generations we have been indoctrinated not to believe in causes---and so we have stopped looking for them. Indeed, any attempt to search for the causes of microphenomena is met with scorn and a charge of professional incompetence and 'obsolete mechanistic materialism'."

E.T. Jaynes
Probability Theory: The Logic of Science
Cambridge University Press, 2003

The proliferation of Quantum Interpretations

Interpretation	Year published	Author(s)	Deterministic?	Ontic wavefunction?	Unique history?	Hidden variables?	Collapsing wavefunctions?	Observer role?	Local dynamics?	Counterfactually definite?	Extant universal wavefunction?
Ensemble interpretation	1926	Max Born	Agnostic	No	Yes	Agnostic	No	No	No	No	No
Copenhagen interpretation	1927	Niels Bohr, Werner Heisenberg	No	Some[60]	Yes	No	Some[61]	No[62][63]	Yes	No	No
De Broglie–Bohm theory	1927–1952	Louis de Broglie, David Bohm	Yes	Yes[a]	Yes[b]	Yes	Phenomenological	No	No	Yes	Yes
Quantum logic	1936	Garrett Birkhoff	Agnostic	Agnostic	Yes[c]	No	No	Interpretational[d]	Agnostic	No	No
Time-symmetric theories	1955	Satosi Watanabe	Yes	No	Yes	Yes	No	No	No[64]	No	Yes
Many-worlds interpretation	1957	Hugh Everett	Yes	Yes	No	No	No	No	Yes	Ill-posed	Yes
Consciousness causes collapse	1961–1993	Eugene Wigner, Henry Stapp	No	Yes	Yes	No	Yes	Causal	No	No	Yes
Many-minds interpretation	1970	H. Dieter Zeh	Yes	Yes	No	No	No	Interpretational[e]	Yes	Ill-posed	Yes
Consistent histories	1984	Robert B. Griffiths	No	No	No	No	No[f]	No[g]	Yes	No	Yes
Transactional interpretation	1986	John G. Cramer	No	Yes	Yes	No	Yes[h]	No	No[i]	Yes	No
Objective-collapse theories	1986–1989	Giancarlo Ghirardi, Alberto Rimini, Tullio Weber, Roger Penrose	No	Yes	Yes	No	Yes	No	No	No	No
Relational interpretation	1994	Carlo Rovelli	No[65]	No	Agnostic[j]	No	Yes[k]	Intrinsic[l]	Possibly[m]	No	No
QBism	2010	Christopher Fuchs, Rüdiger Schack	No	No[n]	Agnostic[c]	No	Yes[o]	Intrinsic[q]	Yes	No	No

- encouraged by the philosophical extravagance of the Copenhagen Interpretation

`Hidden variable theories' in quantum mechanics

- seek a rational dynamics that underlies the theory of quantum statistics:
  seek to describe ***particle trajectories***
- generally involve a pilot-wave dynamics in which a particle is guided by a wave

A brief history

- de Broglie (1926) proposed a **double-wave** pilot-wave theory of quantum dynamics: quantum particles vibrate, move in resonance with a monochromatic guiding wave
- von Neuman (1932) alleged to `*prove*' that there could be no hidden variable theory
- Bohm (1952) presented a **single-wave** pilot-wave theory
- Nelson (1966) proposed Stochastic Dynamics, that QM may be understood in terms of a diffusive, random-walk-like process
- Bell violations (Aspect 1982) are taken as proof that any hidden variable theory must be *nonlocal*
- de la Peña & Cetto (1996, 2015) have developed Stochastic Electrodynamics, in which the pilot-wave is sought in the electromagnetic quantum vacuum

Hydrodynamic quantum analogs

- in 2005, Couder and Fort discovered a hydrodynamic pilot-wave system in which a particle moves in resonance with a guiding wave
- the first macroscopic realization of the double-solution pilot-wave dynamics proposed by Louis de Broglie in the 1920s
- exhibits many features of quantum systems previously thought to be exclusive to the microscopic, quantum realm

THE QUESTIONS RAISED

What are the key dynamical features responsible for the quantum-like behavior? ✓

What are the potential and limitations of this hydrodynamic system as a quantum analog?

Can it guide us towards a rational theory for quantum dynamics?

The standard line of thinking

- classical systems cannot mimic quantum systems because QM is nonlocal
- quantum nonlocality is invoked as a rationale for myriad quantum features
  e.g. wave function collapse, slit diffraction, entanglement
- Bell violations are thought to provide a proof of quantum nonlocality

The dissenting view offered by pilot-wave hydrodynamics (PWH)

- PWH is local, but provides a platform for analogizing quantum systems
- PWH exhibits features taken as evidence of nonlocality in QM that may be rationalized in terms of chaotic, hereditary mechanics
- quantum nonlocality is a misinference, rooted in the incompleteness of quantum theory

Two standards of proof in litigation

Civil case: "*more likely than not*"

Criminal case: "*beyond a shadow of a doubt*"

Scientific standards of proof: stricter still

- Wegener (1912) proposed that continental plates drift, but the theory of plate tectonics was not accepted until 1962

Proposal

- let us prosecute the completeness of quantum theory in a *civil case* by demonstrating:

  1) classical pilot-wave dynamics enables hydrodynamic quantum analogs,
  2) this system is a macroscopic realization of de Broglie's pilot-wave theory,
  3) theoretical extensions allow for a broader class of quantum analogs, more quantum features

- let us leave the *criminal prosecution* for remote posterity…

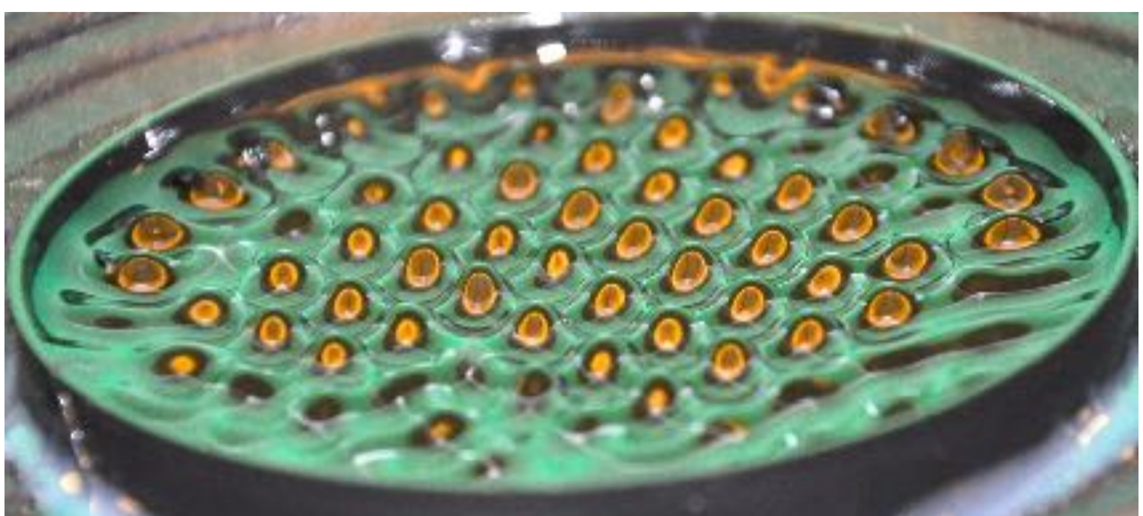
Faraday waves

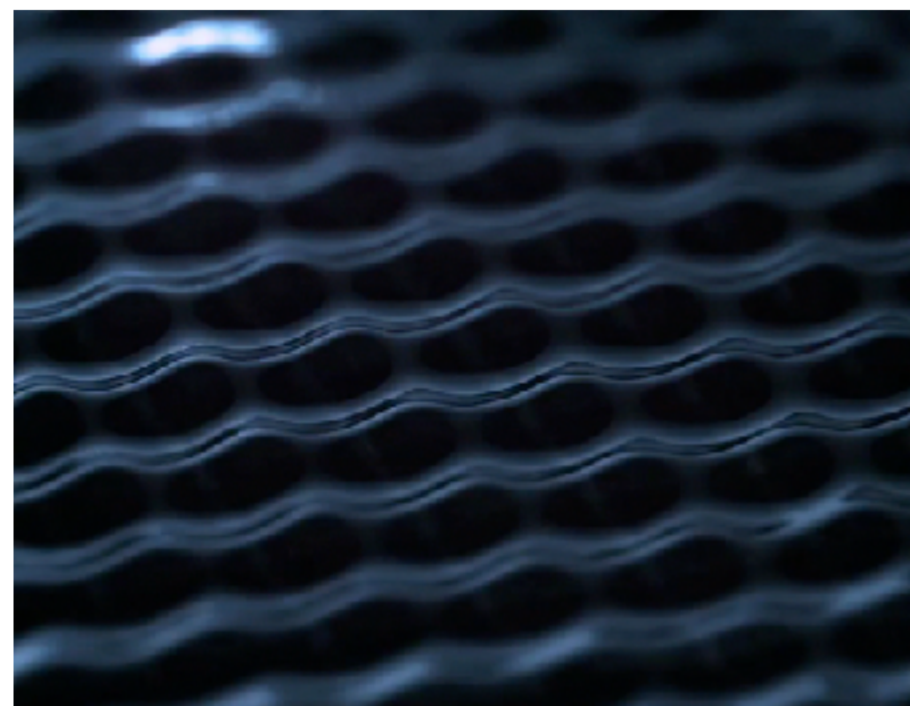
Faraday (1831)

- surface undulations with twice the forcing period, a parametric instability
- arise above a threshold γ_F that depends on fluid depth, viscosity, surface tension

$$\gamma > \gamma_F \approx 4g$$

Vibrational acceleration: $\gamma = A(2\pi f)^2$

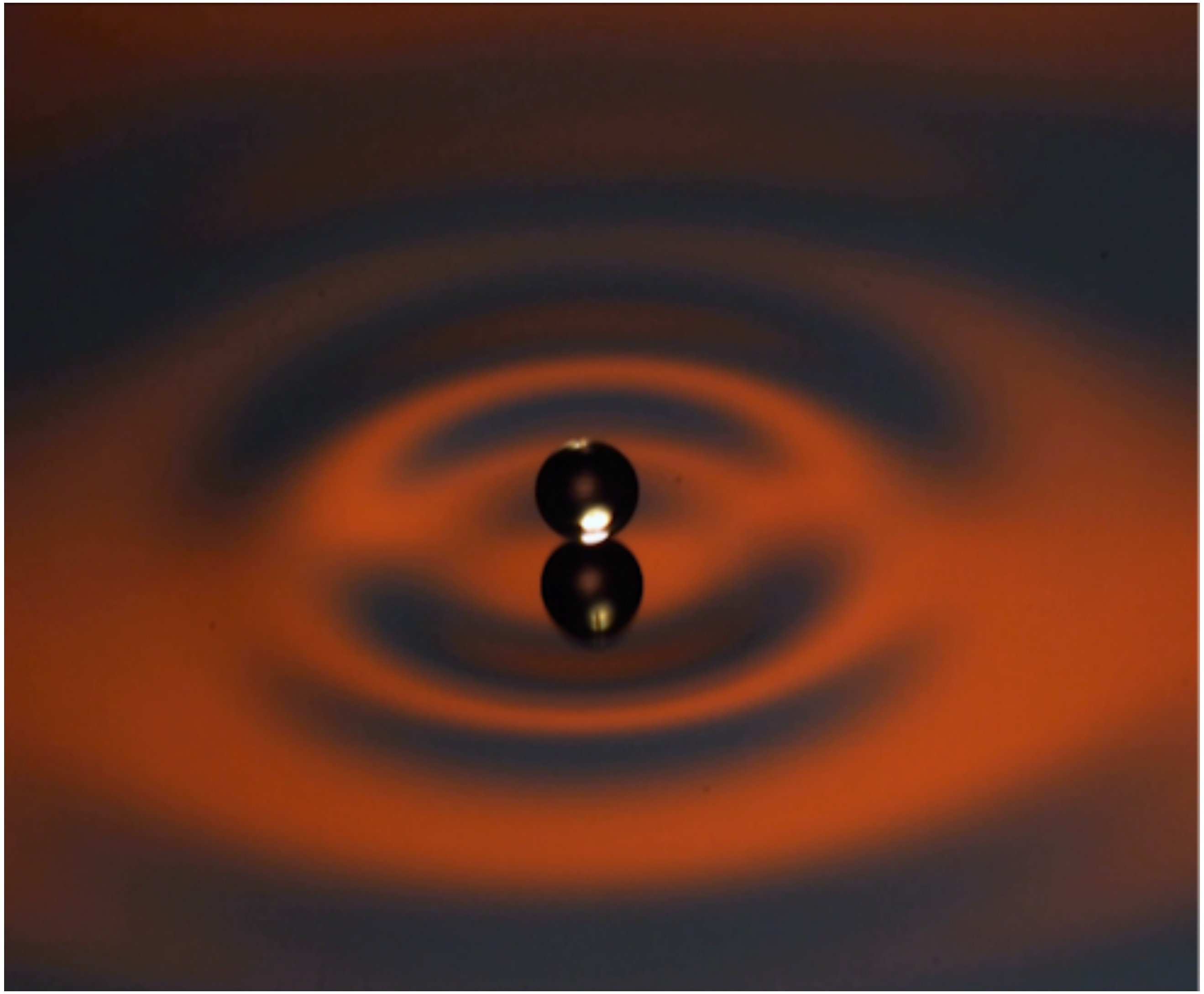




Noncoalescence on a vibrated fluid bath

Jearl Walker (1978), Protière et al. (2005)

- coalescence avoided provided impact time is less than time required for air layer between drop and bath to drain to ~100 nm



f ~ 50 Hz

50cS
Si oil

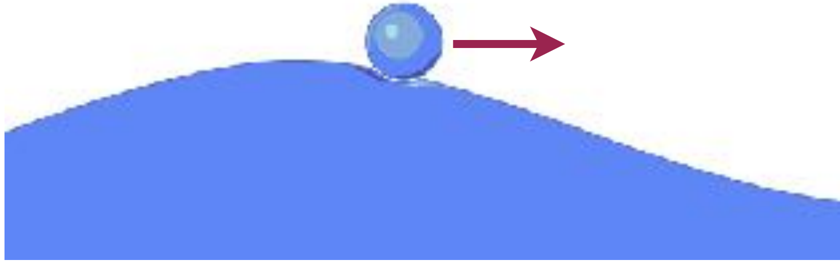
The Couder walker

Couder et al. (2005)

- ***resonance condition***: millimetric drop bounces at Faraday frequency
- resonant bouncing droplets may be destabilized by their wave field, *walk*
- spatially extended *walkers* consist of both droplet and guiding/pilot wave
- drop dynamics is ***non-Markovian, hereditary***: wave force depends on its history
- proximity to Faraday threshold prescribes longevity of waves, `**path memory**'

Eddi et al. (2011)

Strobed pilot-wave dynamics



- strobe the system once per bounce cycle
- conceals the vertical dynamics responsible for the guiding wave
- drop appears to surf on the interface, dressed by a quasi-monochromatic pilot-wave field that is stationary in the drop's frame of reference

Wave-particle duality at the macroscopic scale

"Both matter and radiation possess a remarkable duality of character, as they sometimes exhibit the properties of waves, at other times those of particles.

Now *it is obvious that* a thing cannot be a form of wave motion and composed of particles at the same time - the two concepts are too different."

- Heisenberg, *On Quantum Mechanics (1930)*

Can the wave-particle duality manifest in this classical system yield insight into the longstanding conceptual difficulties of quantum mechanics?

The stroboscopic model

Molacek & JB (2013)
Oza, Rosales & JB (2013)

$$m\ddot{\mathbf{x}}_p + D\dot{\mathbf{x}}_p = -mg\nabla h(\mathbf{x}_p, t)$$

WAVE FORCE

Average dynamics over one bounce:

$$\nabla h(\mathbf{x}, t) = -Ak_F \int_{-\infty}^{t} \frac{J_1(k_F|\mathbf{x} - \mathbf{x}_p(s)|)}{|\mathbf{x} - \mathbf{x}_p(s)|}(\mathbf{x} - \mathbf{x}_p(s))e^{-(t-s)/(T_F M_e)}\, ds$$

MEMORY $M_e = \frac{T_d}{T_F(1 - \gamma/\gamma_F)}$

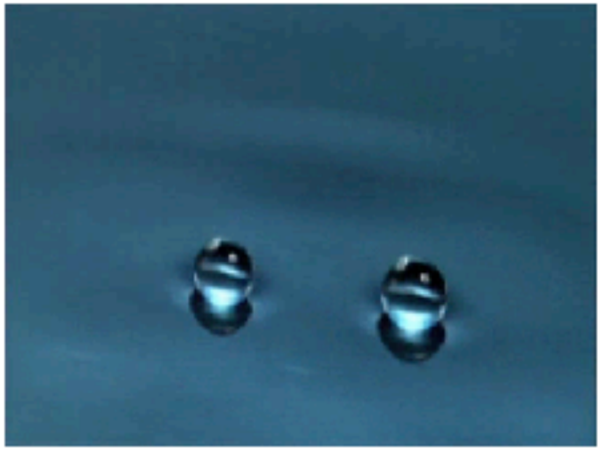
$F = mgAk_F$

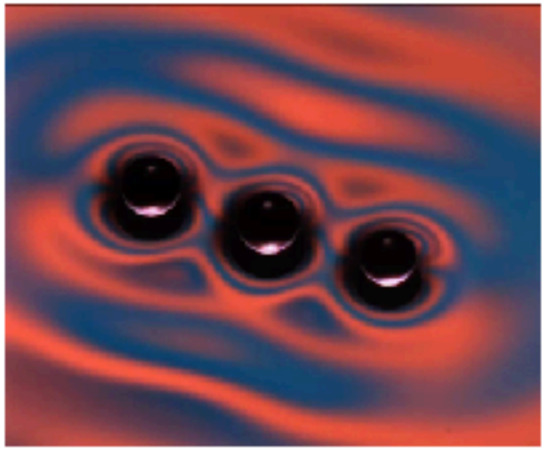
$$m\ddot{\mathbf{x}}_p + D\dot{\mathbf{x}}_p = \frac{F}{T_F}\int_{-\infty}^{t} \frac{J_1(k_F|\mathbf{x}_p(t) - \mathbf{x}_p(s)|)}{|\mathbf{x}_p(t) - \mathbf{x}_p(s)|}(\mathbf{x}_p(t) - \mathbf{x}_p(s))\, e^{-(t-s)/(T_F M_e)}\, ds$$

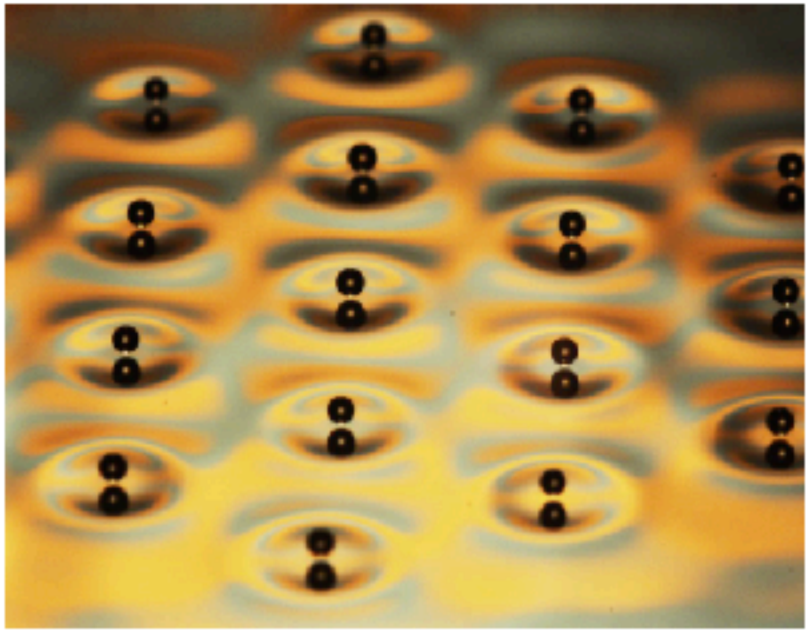
- droplet explores its self potential, its quasi-monochromatic pilot-wave field
- quantum features emerge in the high `*memory*' limit, where its pilot-wave field is most persistent

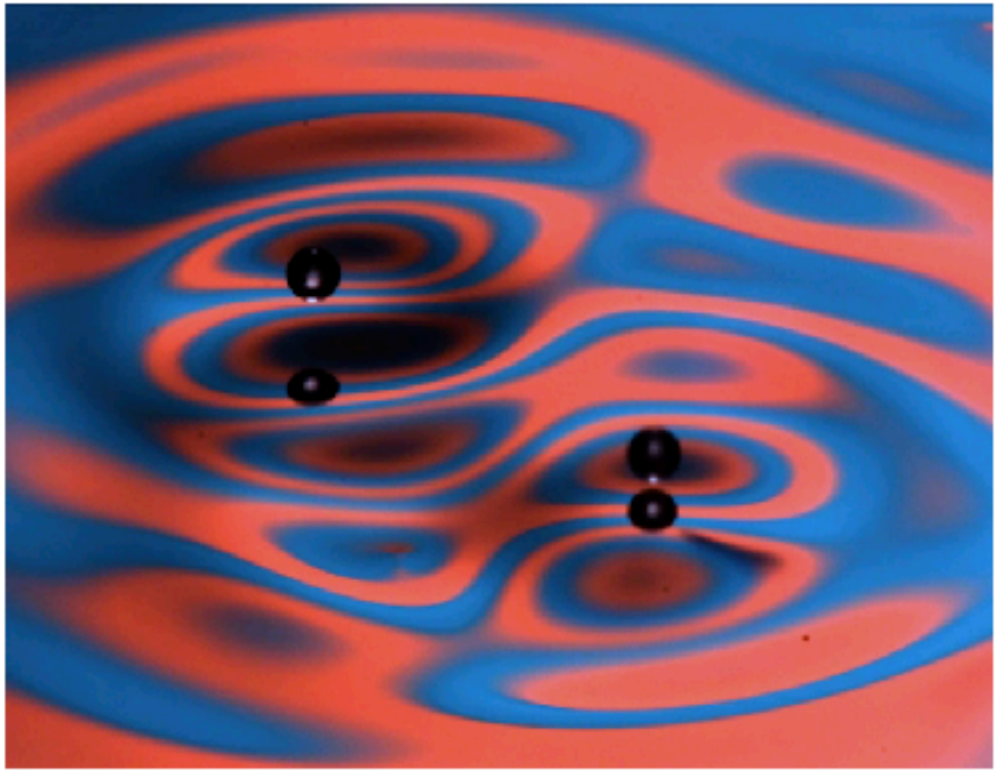
Static and dynamic bound states

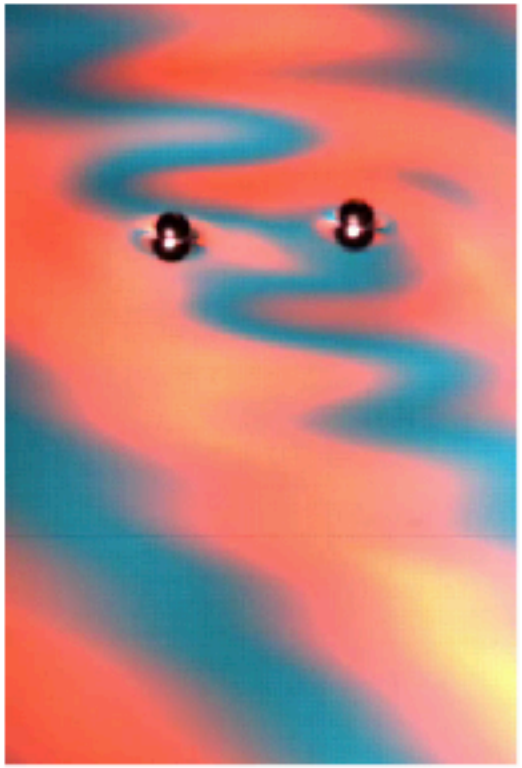
- quantized by quasi-monochromatic pilot wave





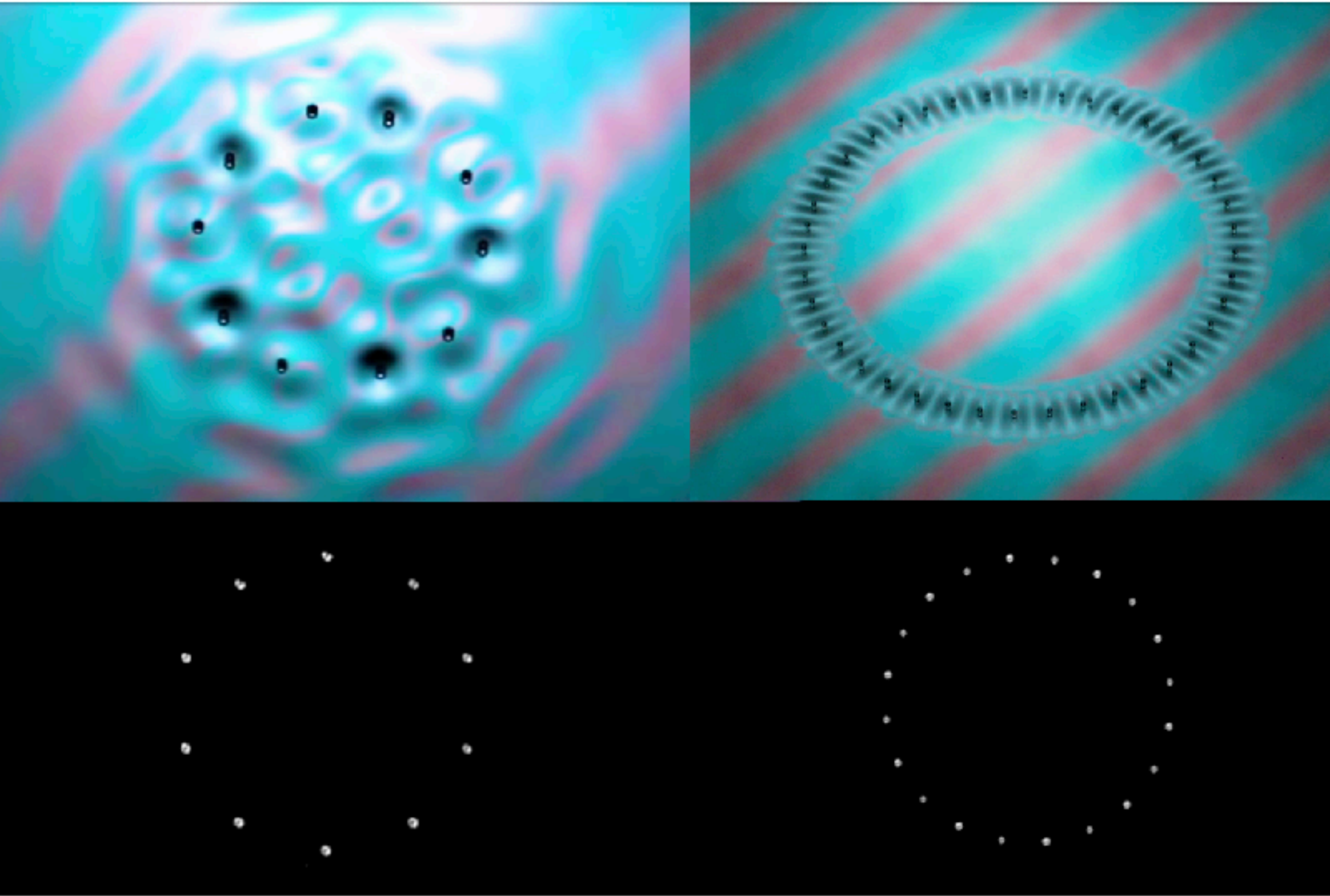






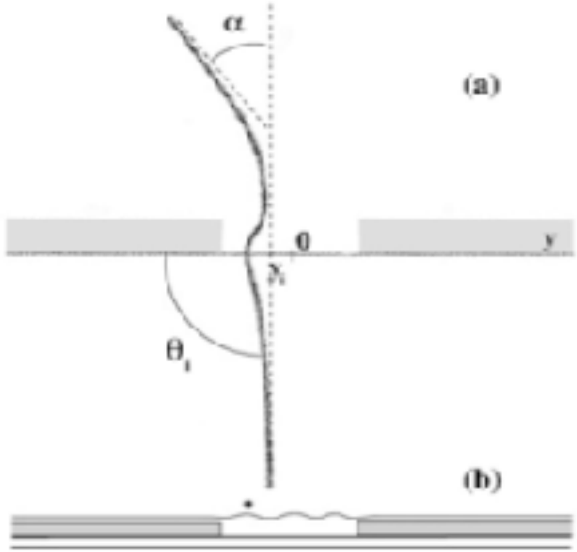
Rings of bouncing droplets

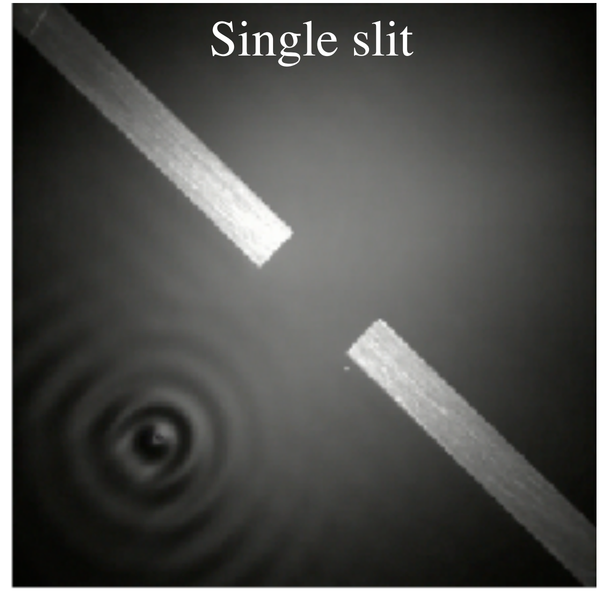
Couchman & Bush (2020)
Thomson, Couchman & Bush (2020)

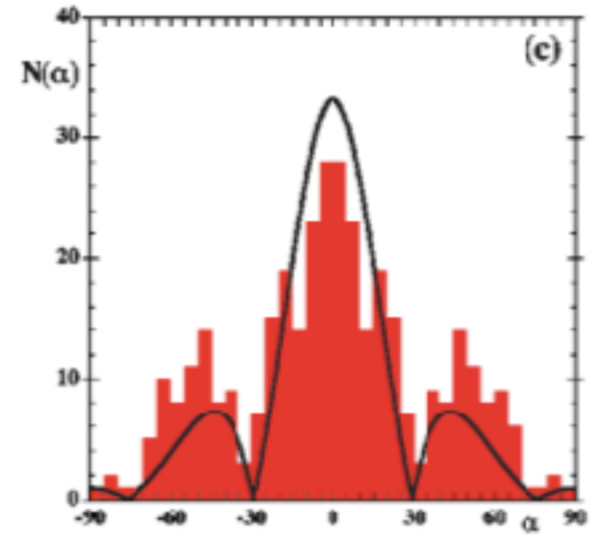


Single-walker diffraction and interference

Couder & Fort (2005)

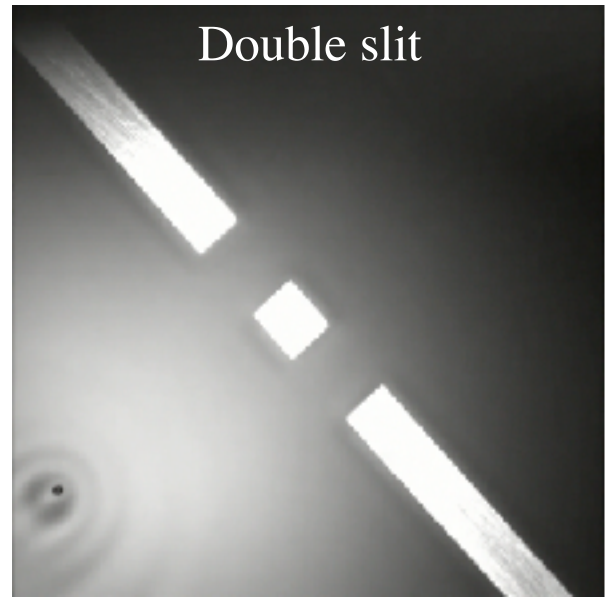


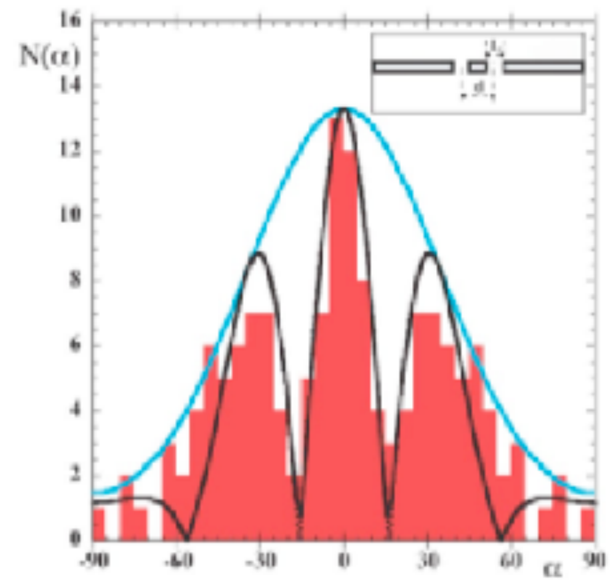




“ A phenomenon which is *impossible, absolutely impossible*, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the only mystery.”

- Richard Feynman

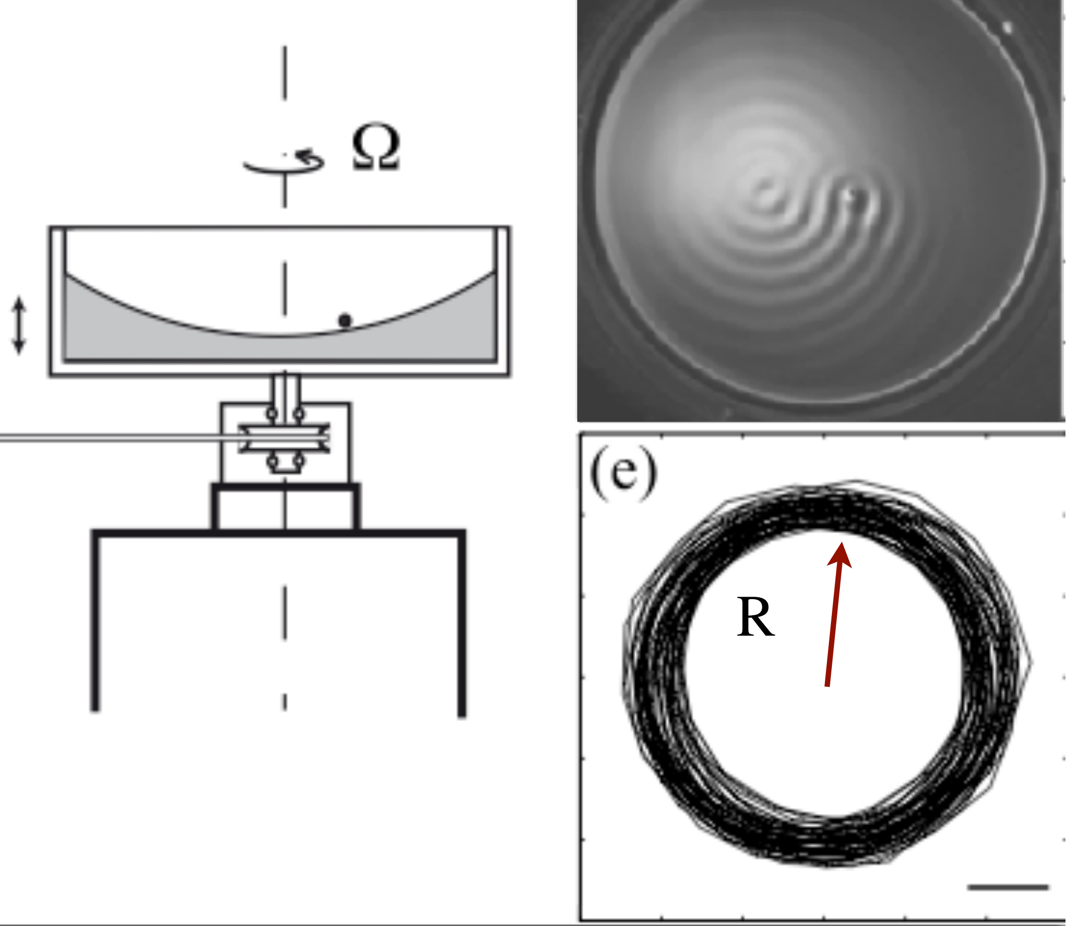




- **coherent, wave-like statistics emerge from chaotic pilot-wave dynamics**
- revisited several times by several groups: two key features are robust

Walkers in a rotating frame

Fort et al. (2010)



- execute circular orbits on which inertial, Coriolis forces balance:

$$\rho V^2/R = 2\rho\Omega V$$

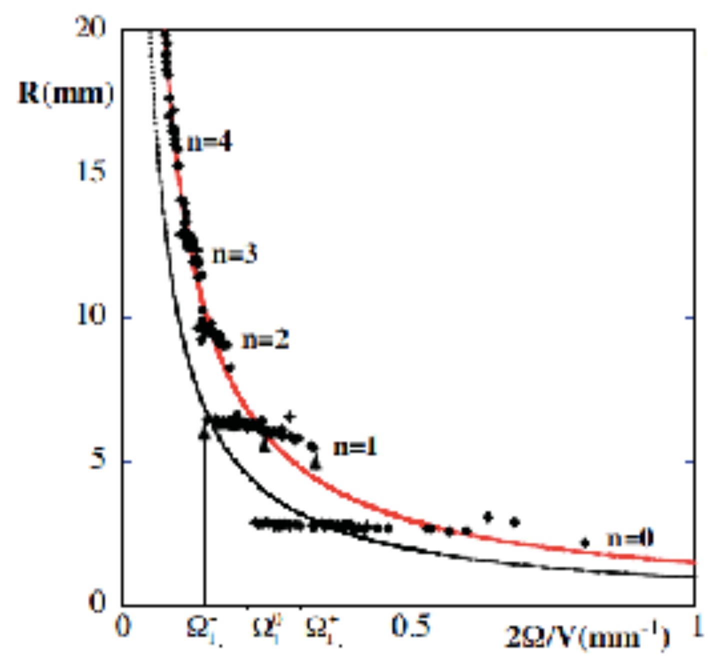
- one expects an orbital radius:

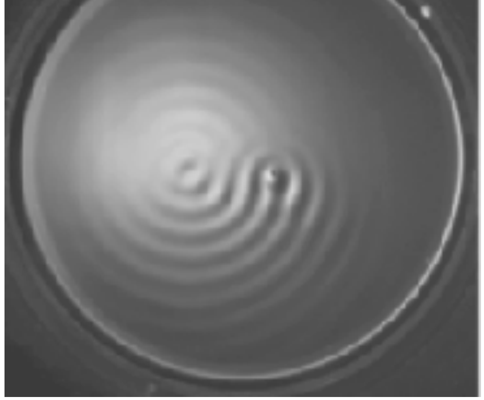
$$R = V/(2\Omega)$$

- in the long-memory limit, a 3rd force, the wave force, induces *orbital quantization:*

$$R_n \sim \frac{1}{2}(n+1/2)\lambda_F$$

- walker confined to move in the troughs of its associated wave field



Landau orbits		Inertial orbits
• charge q of mass m orbits in a magnetic field $\mathbf{B}$		• walker of mass m orbits in a vortex $2\mathbf{\Omega}$
$\mathbf{F_B} = q(\mathbf{v} \wedge \mathbf{B})$	Force	$\mathbf{F_C} = -m(\mathbf{v} \wedge 2\mathbf{\Omega})$
$R_L = mv/(qB)$	Radius	$R_C = v/(2\Omega)$
$\tau_L = m/(qB)$	Period	$\tau_C = 1/(2\Omega)$
$R_n = \frac{1}{\pi}(n+1/2)\lambda_{dB}$	Orbit levels	$R_n \sim \frac{1}{2}(n+1/2)\lambda_F$
Larmor levels		*Couder levels*
λ_{dB}	Quantum step	λ_F
de Broglie wavelength		Faraday wavelength

Pilot-wave dynamics in a rotating frame

Oza, Harris, Rosales & Bush (2013)

$$m\ddot{\mathbf{x}} + D\dot{\mathbf{x}} = \frac{F}{T_F}\int_{-\infty}^{t} \frac{J_1(k_F|\mathbf{x}(t)-\mathbf{x}(s)|)}{|\mathbf{x}(t)-\mathbf{x}(s)|}(\mathbf{x}(t)-\mathbf{x}(s))\, e^{-(t-s)/(T_F M_e)}\, ds - 2m\boldsymbol{\Omega}\times\dot{\mathbf{x}}$$

Coriolis force

Seek orbital solutions: $r_p(t) = r_0\ ,\ \theta_p(t) = \omega t$

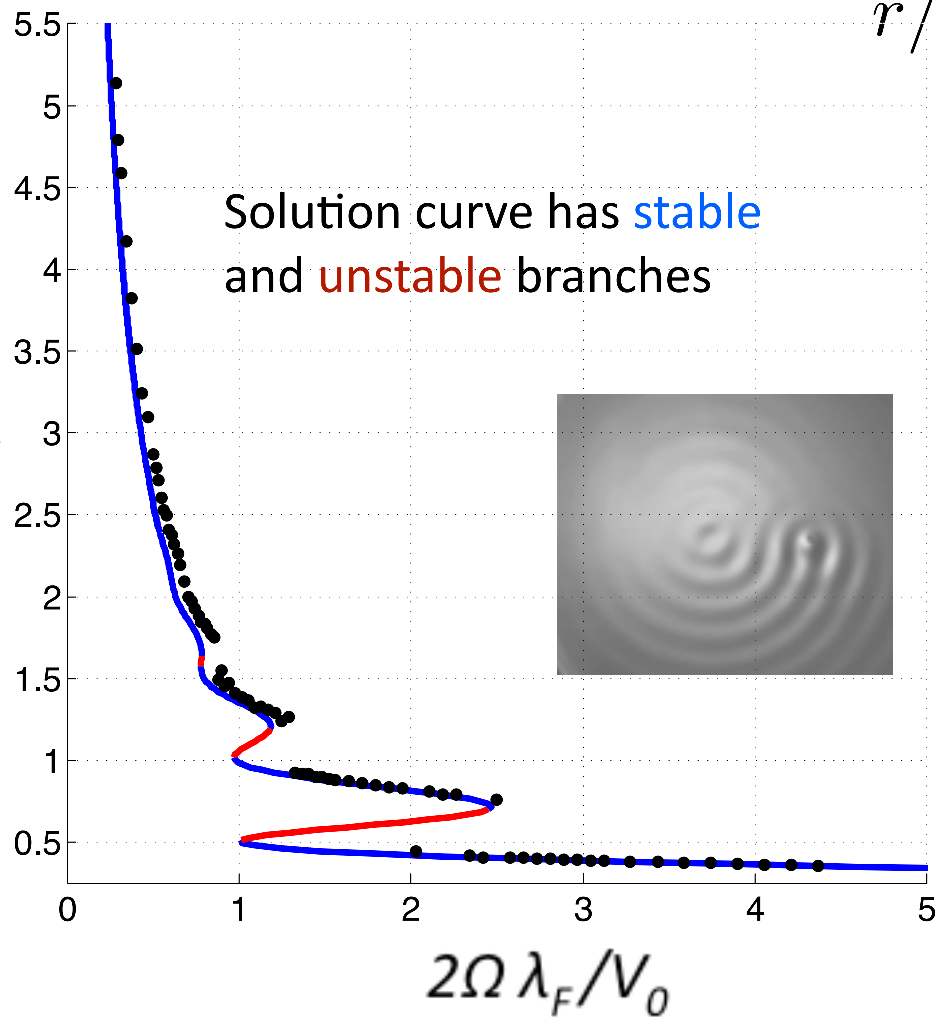
$$-mr_0\omega^2 = \frac{F}{T_F}\int_0^\infty J_1\left(2k_F r_0 \sin\frac{\omega z}{2}\right)\sin\frac{\omega z}{2} e^{-z/(M_e T_F)}\, dz + 2mr_0\Omega\omega$$

$$Dr_0\omega = \frac{F}{T_F}\int_0^\infty J_1\left(2k_F r_0 \sin\frac{\omega z}{2}\right)\cos\frac{\omega z}{2} e^{-z/(M_e T_F)}\, dz$$

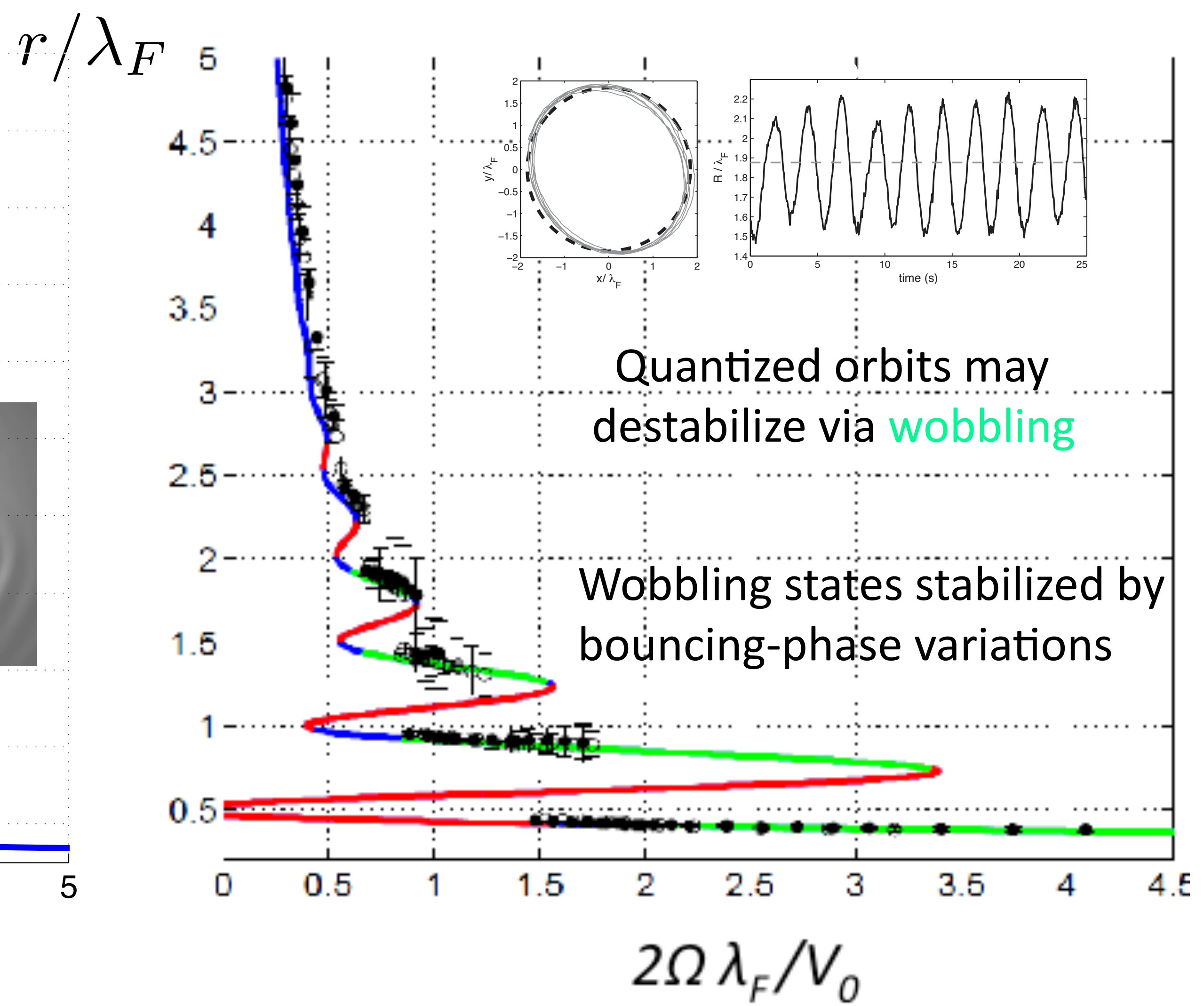
nonlinear system of equations in *(r_0, ω)*

Quantized orbits in a rotating frame

Mid memory



High memory

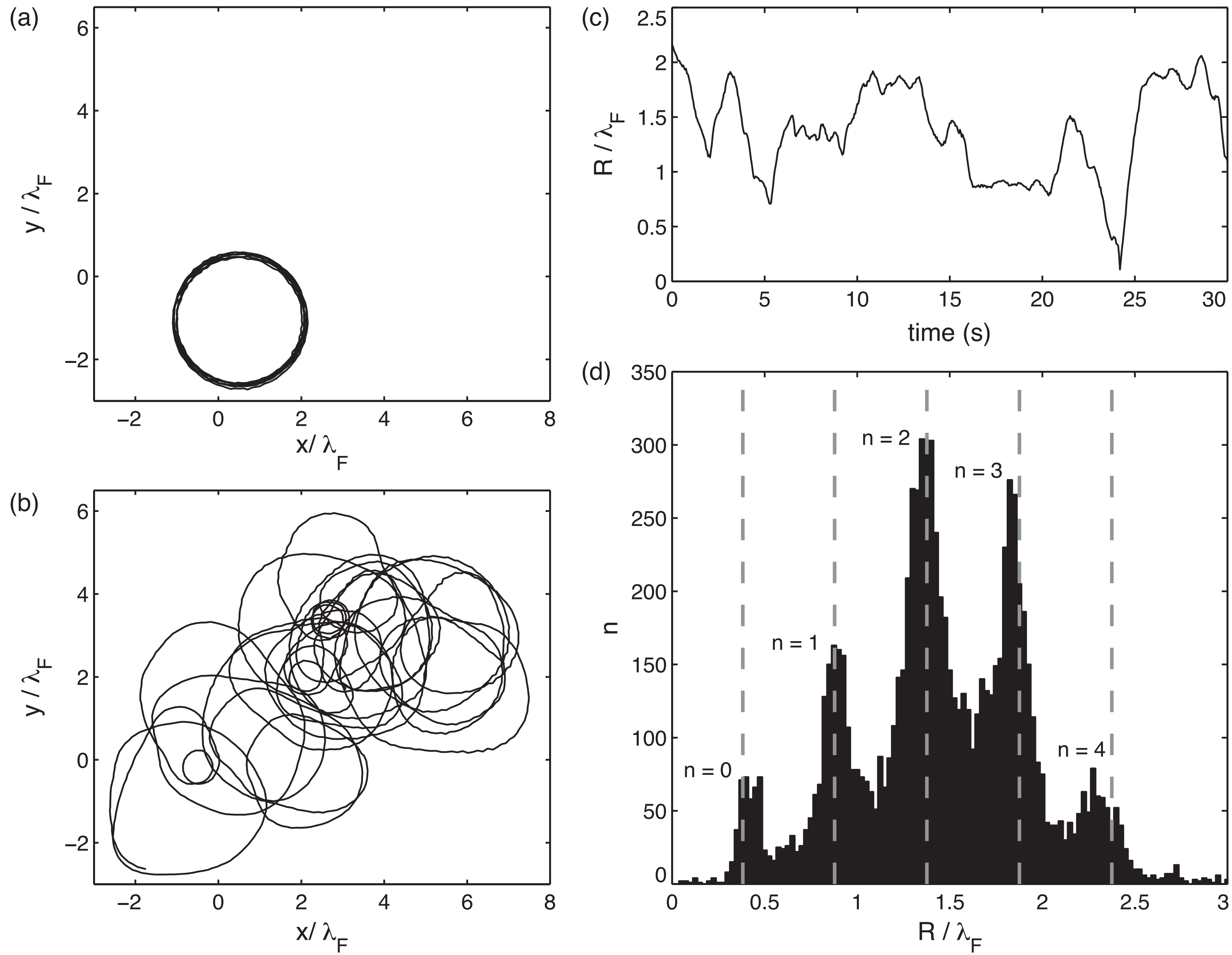


- **stable quantized orbits**

- **stable and unstable orbits**

Chaotic, orbital pilot-wave dynamics

Harris & Bush (2015)



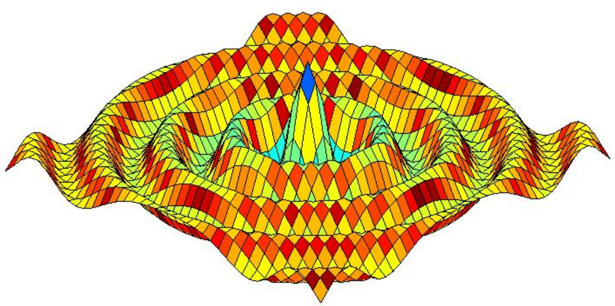
- intermittent switching between weakly unstable periodic orbits
- `*superposition of states*' reflects chaotic dynamics over a self-potential

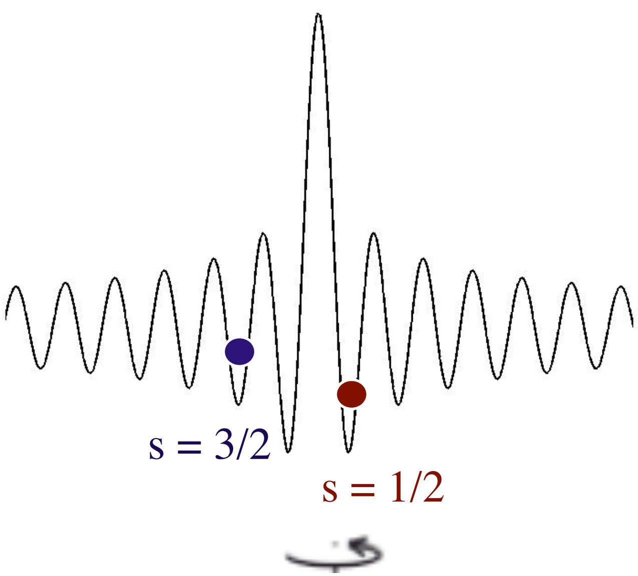
Hydrodynamic spin states at ultra-high memory?

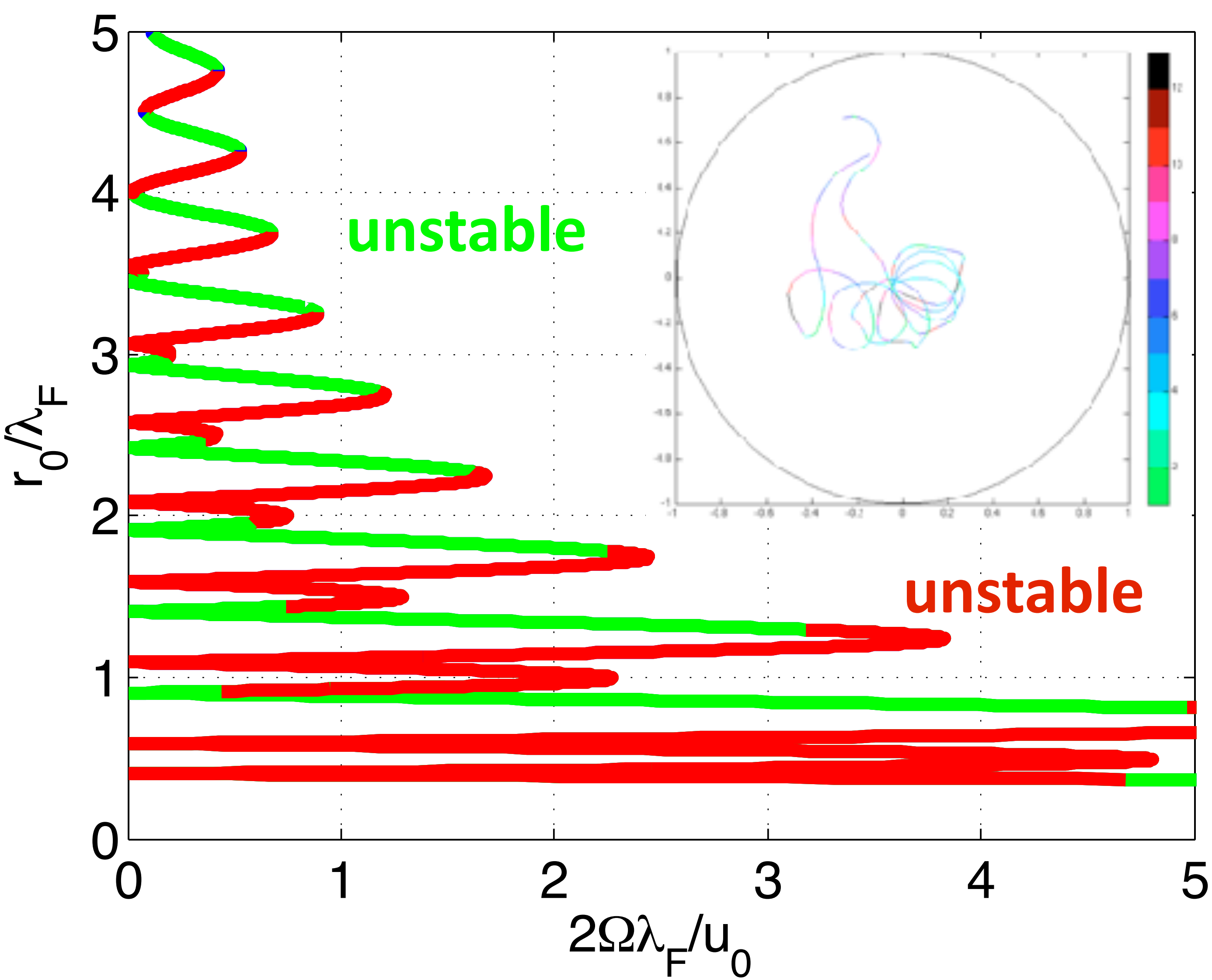
UNSTABLE!

Balance between inertial and wave force.

Orbital radii split by applied rotation.





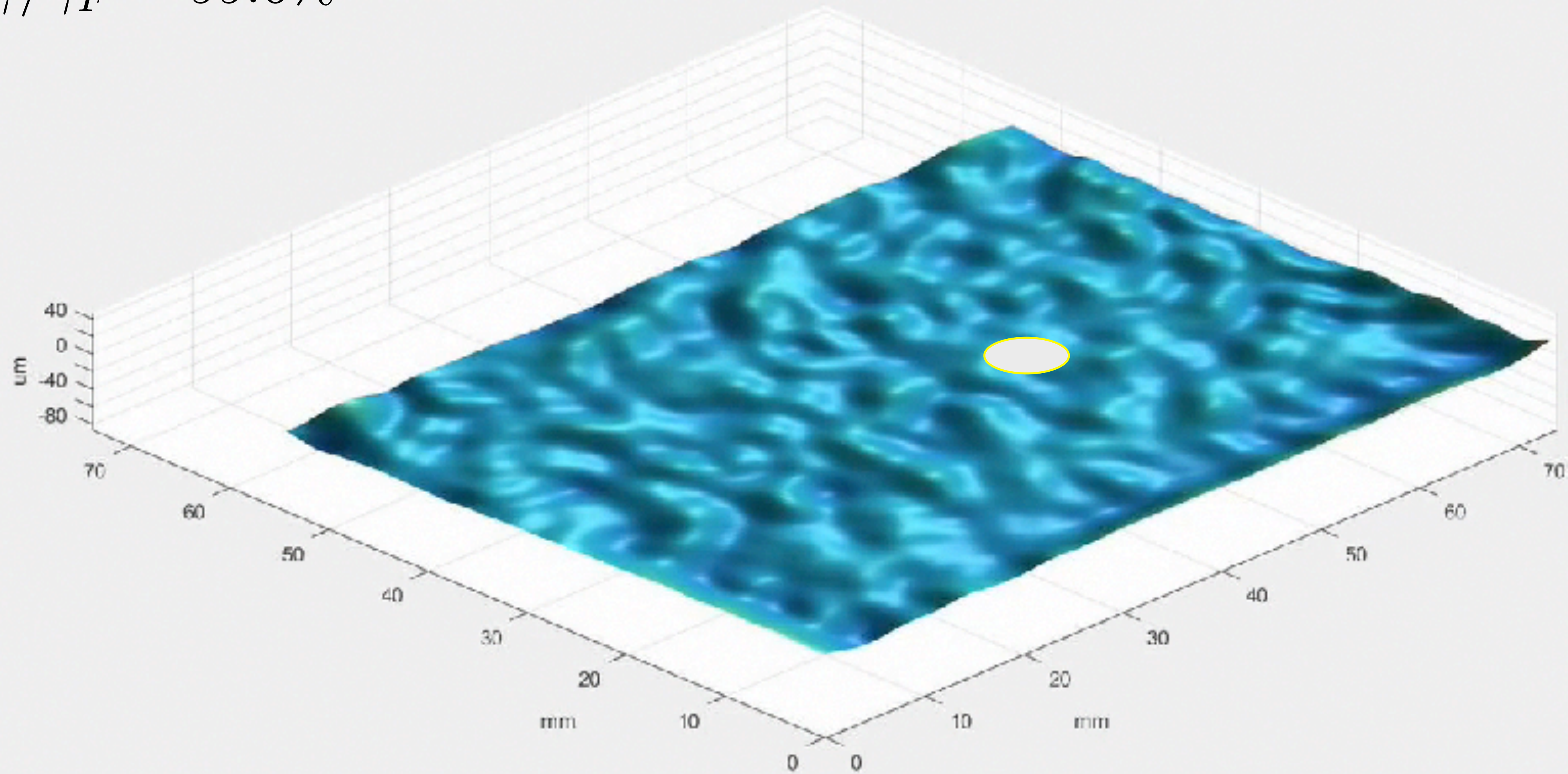


Oza, Rosales & Bush (2018)

Walker scattering from a submerged pillar

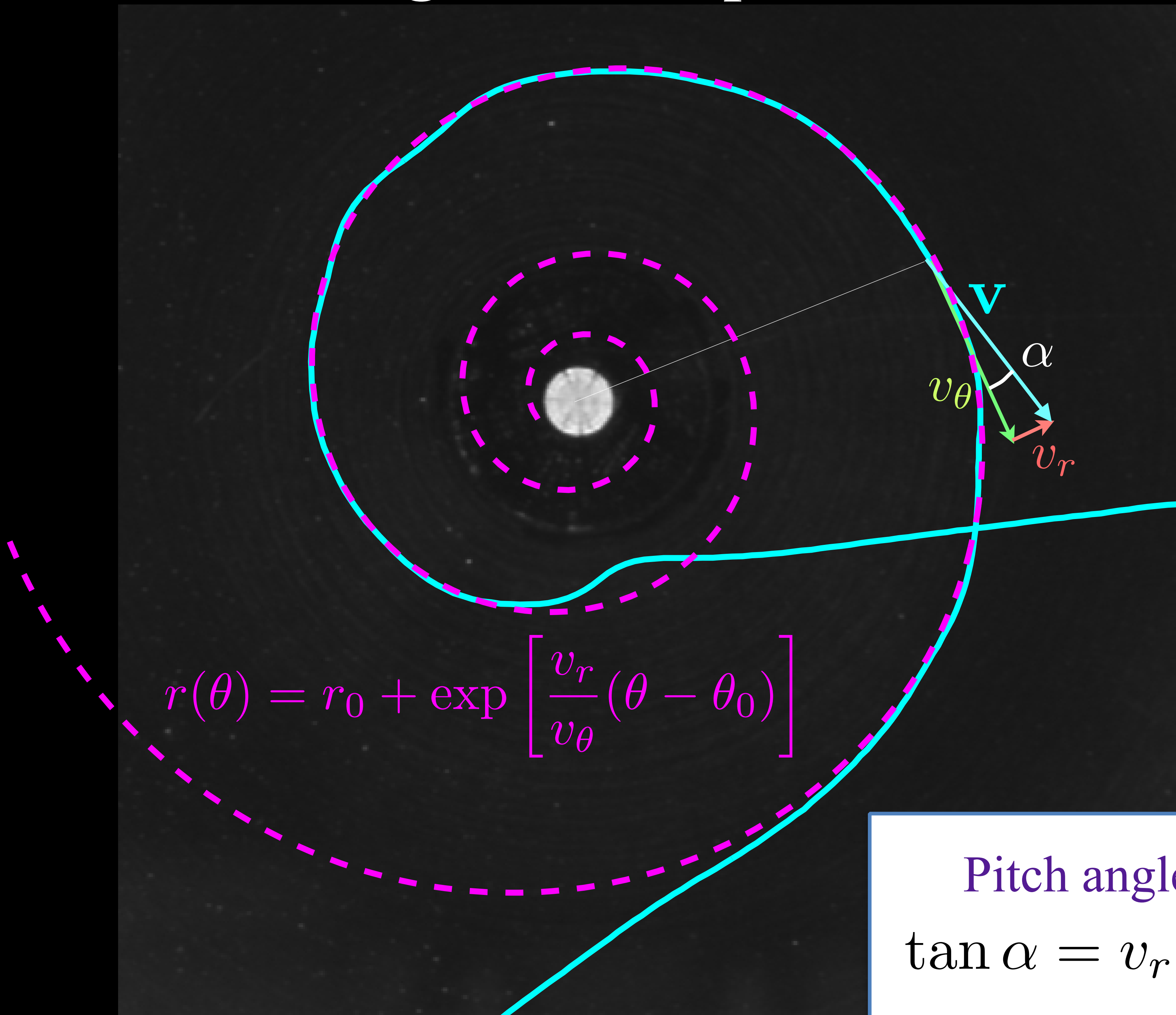
Damiano *et al*. 2016, Harris *et al*. 2018

$\gamma/\gamma_F = 99.6\%$



- pillar acts to locally suppress the walker-induced wave field

Logarithmic spiral



$$r(\theta) = r_0 + \exp\left[\frac{v_r}{v_\theta}(\theta - \theta_0)\right]$$

Pitch angle

$$\tan\alpha = v_r / v_\theta$$

Infer wave-mediated pillar-induced force from trajectory

$$\frac{d}{dt}\,\mathbf{p_w} \;=\; \mathbf{F_p}$$

Force required for a logarithmic spiral:

$$\mathbf{F}_p = 2\pi\,\gamma_B m\;\mathbf{v}\times\boldsymbol{\Omega}$$

where $\boldsymbol{\Omega} = \dfrac{v_\theta}{2\pi r}\,\hat{\mathbf{k}}$ is the walker's instantaneous angular velocity

- identical forms of Coriolis force acting on a mass $\mathbf{F_C} = 2m(\mathbf{v}\wedge\boldsymbol{\Omega})$ and the Lorentz force acting on a charge $\mathbf{F_B} = q(\mathbf{v}\wedge\mathbf{B})$ was the basis for the analogy between inertial orbits and Larmor levels *(Fort et al. 2010)*

- here, it indicates that the walker is similar to a charge moving in the magnetic field associated with its own motion

→ **hydrodynamic self-induction**

- a wave-mediated force gives rise to apparent `***spooky action at a distance***'

Gravitoelectromagnetism

- in limit of weak spacetime curvature (weak gravitational fields)

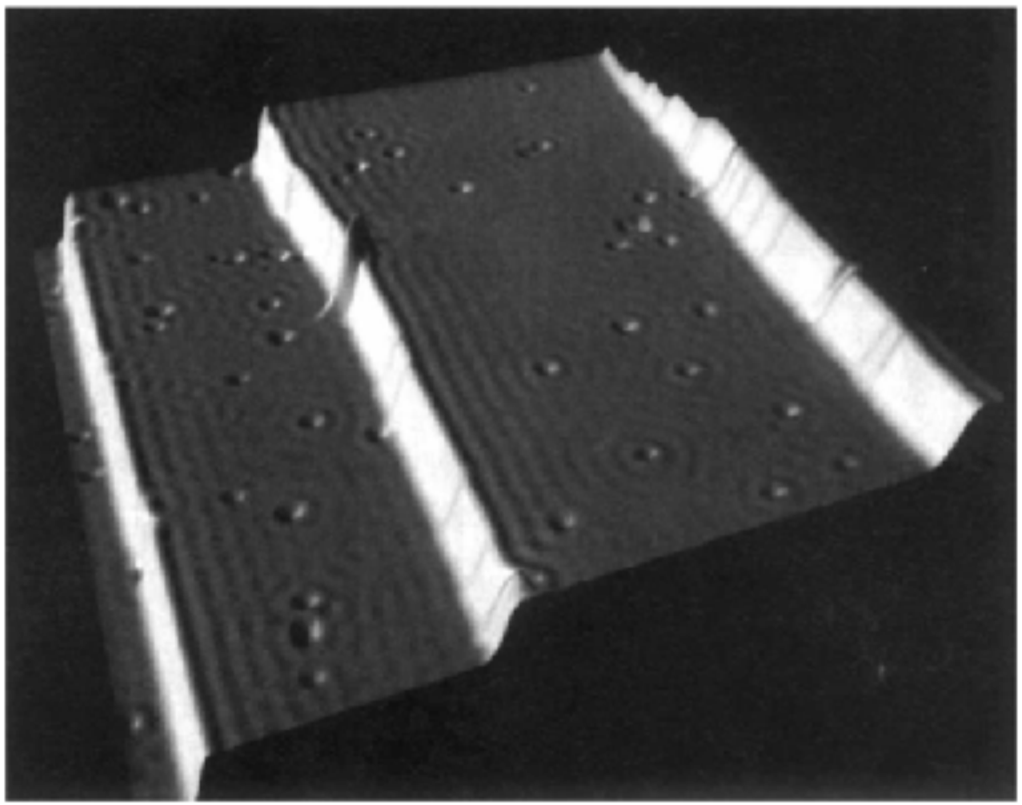
GEM equations	Maxwell's equations
$\nabla \cdot \mathbf{E}_\text{g} = -4\pi G \rho_\text{g}$	$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$
$\nabla \cdot \mathbf{B}_\text{g} = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E}_\text{g} = -\frac{\partial \mathbf{B}_\text{g}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{B}_\text{g} = -\frac{4\pi G}{c^2}\mathbf{J}_\text{g} + \frac{1}{c^2}\frac{\partial \mathbf{E}_\text{g}}{\partial t}$	$\nabla \times \mathbf{B} = \frac{1}{\epsilon_0 c^2}\mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$

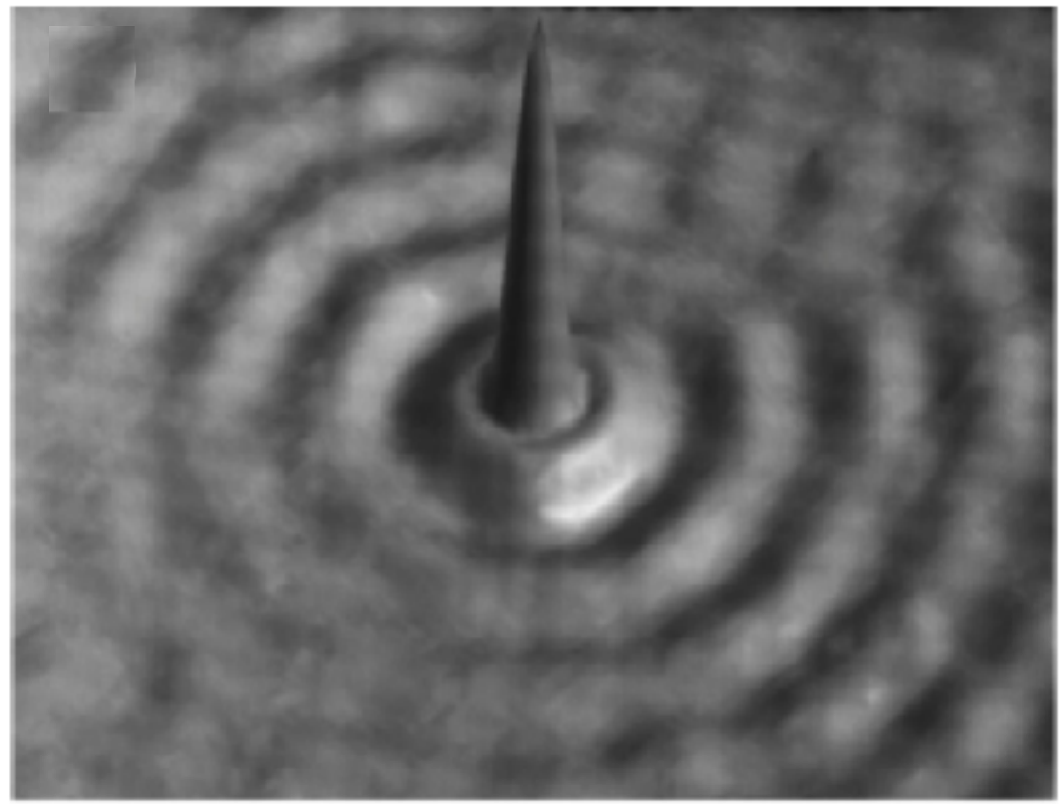
Lorentz force

GEM equation	EM equation
$\mathbf{F}_\mathbf{g} = m\left(\mathbf{E}_\text{g} + \mathbf{v} \times 4\mathbf{B}_\text{g}\right)$	$\mathbf{F}_\mathbf{e} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$

Friedel oscillations

- modulations of the probability density of the electron-sea on a substrate due to the presence of a scattering impurity
- taken as evidence of the finite size of an electron



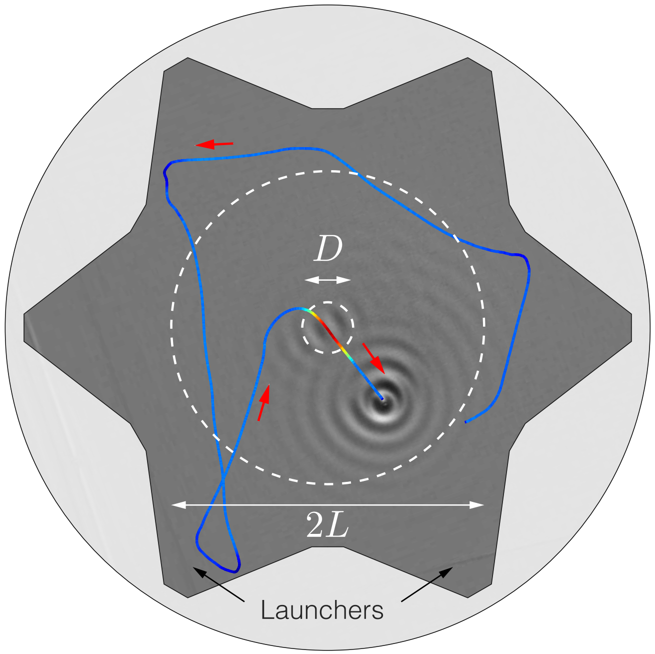


Unknown interaction mechanism

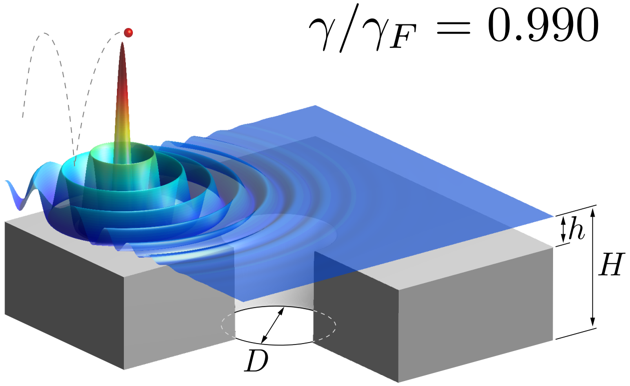
Modeled as localized scattering potentials

WALKER-WELL INTERACTION

Experimental Setup



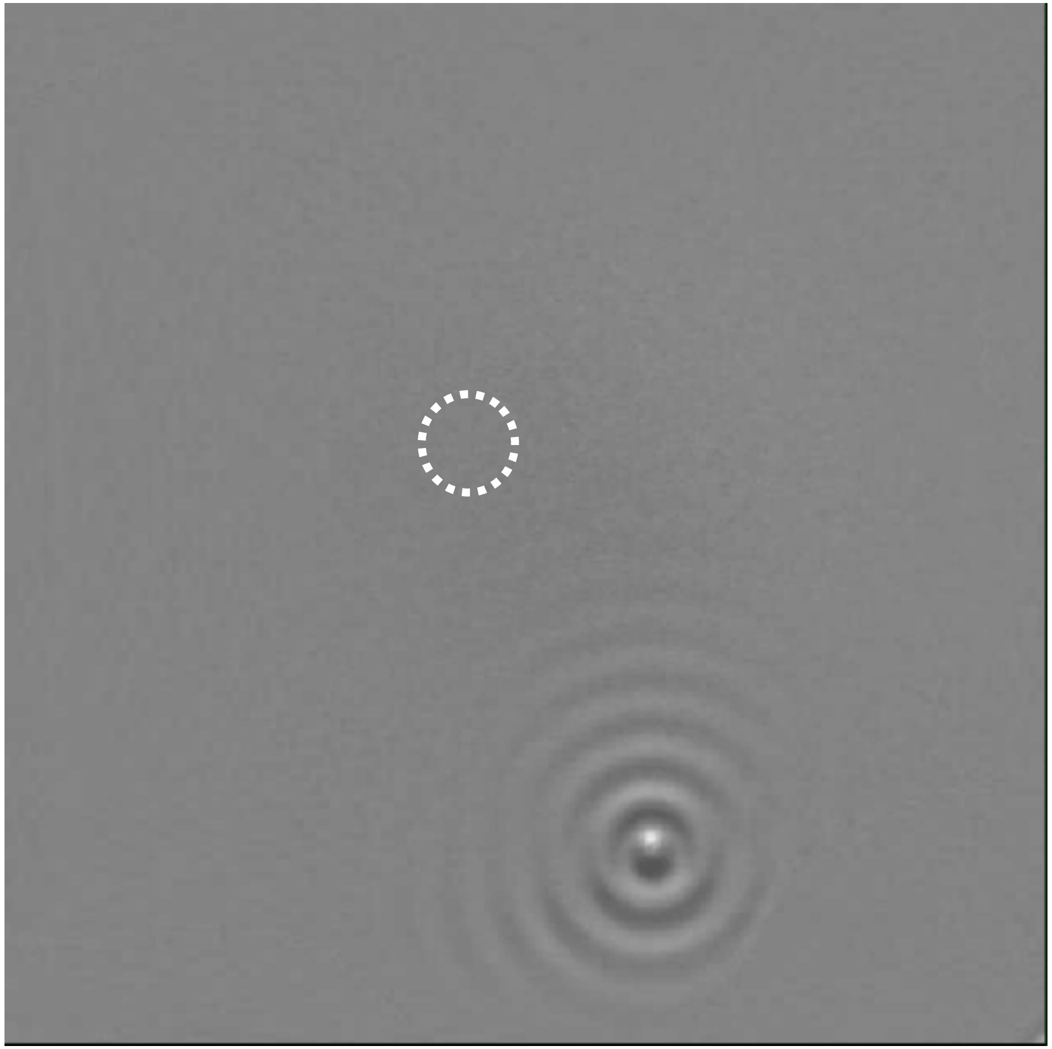
$\gamma/\gamma_F = 0.990$



Well

Region of high excitability

$$\gamma_F^H < \gamma_F < \gamma_F^h$$

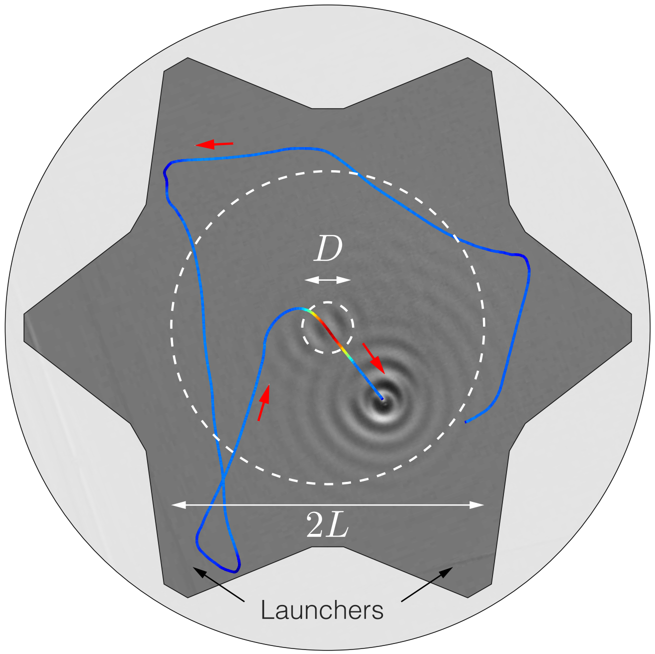


Drop drawn in along an Archimedean spiral

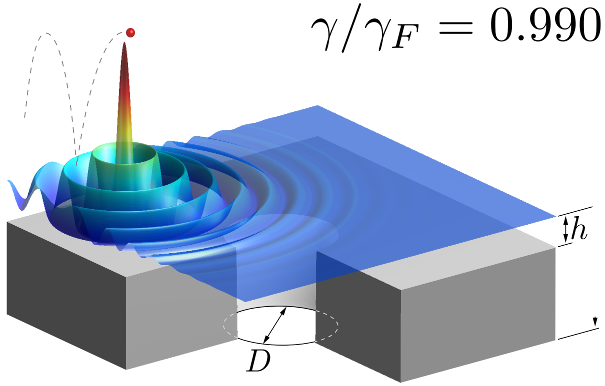
Speed modulations induced by interaction with waves generated above the well

WALKER-WELL INTERACTION

Experimental Setup



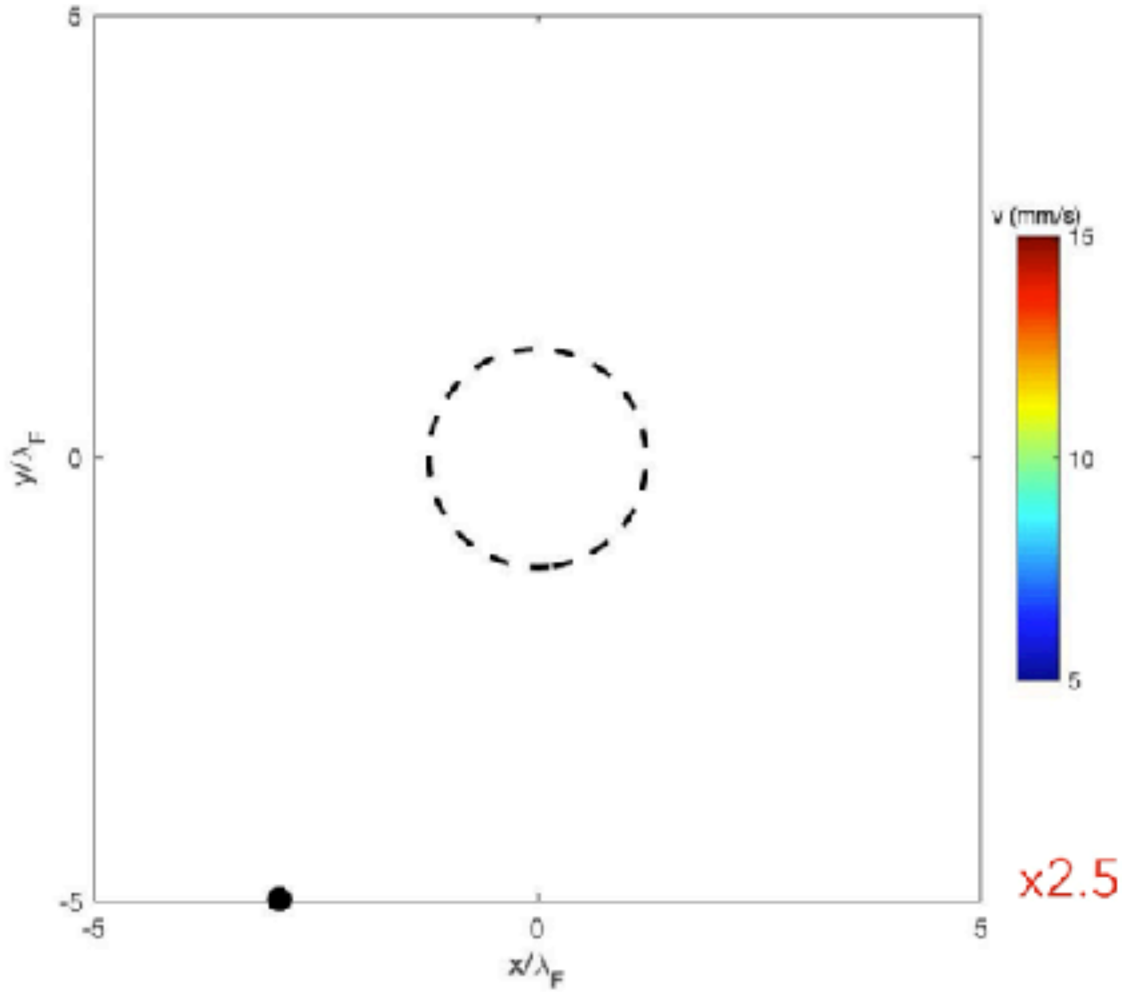
$\gamma/\gamma_F = 0.990$



Well

Region of high excitability

$$\gamma_F^H < \gamma_F < \gamma_F^h$$



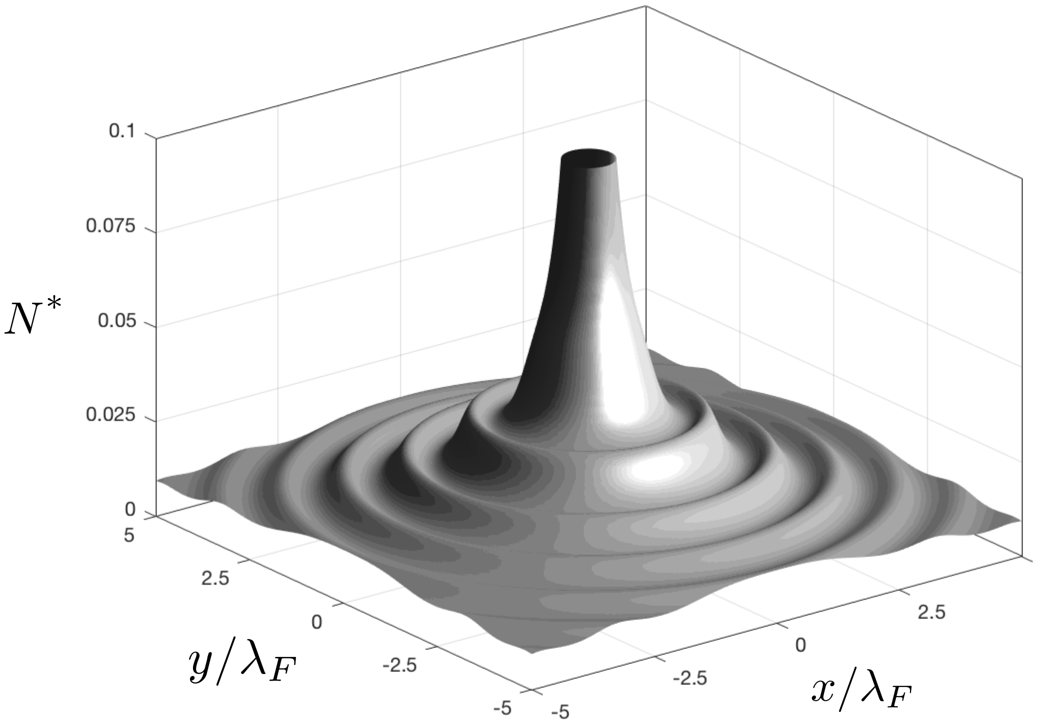
Drop drawn in along an Archimedean spiral

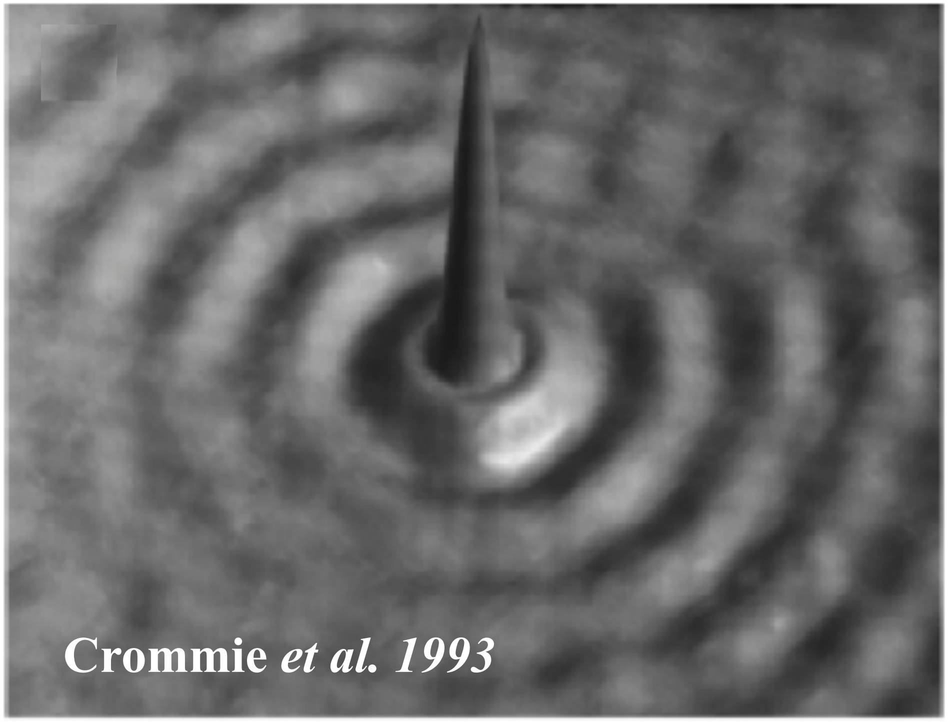
Speed modulations induced by interaction with waves generated above the well

A hydrodynamic analog of Friedel oscillations

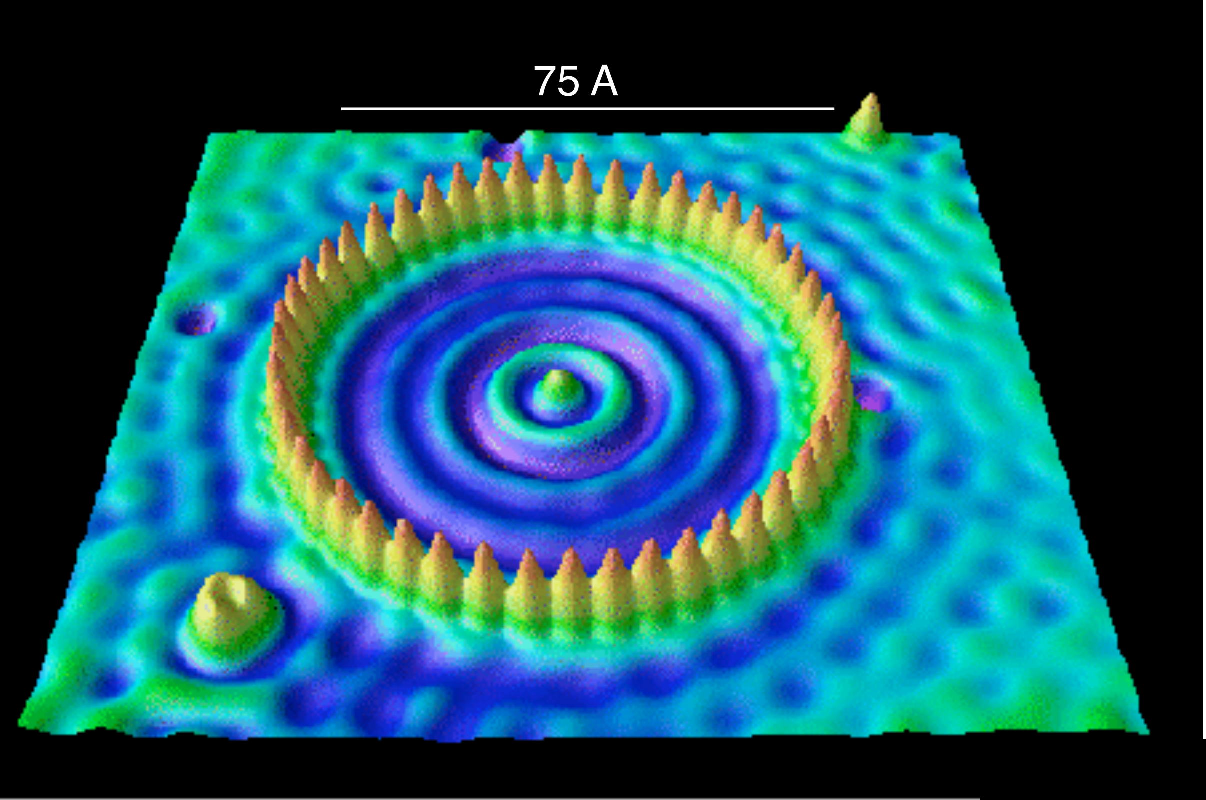
Sáenz, Cristea-Platon & Bush (Sci. Advances, 2020)

- arises due to wave-induced speed modulations in outgoing trajectory



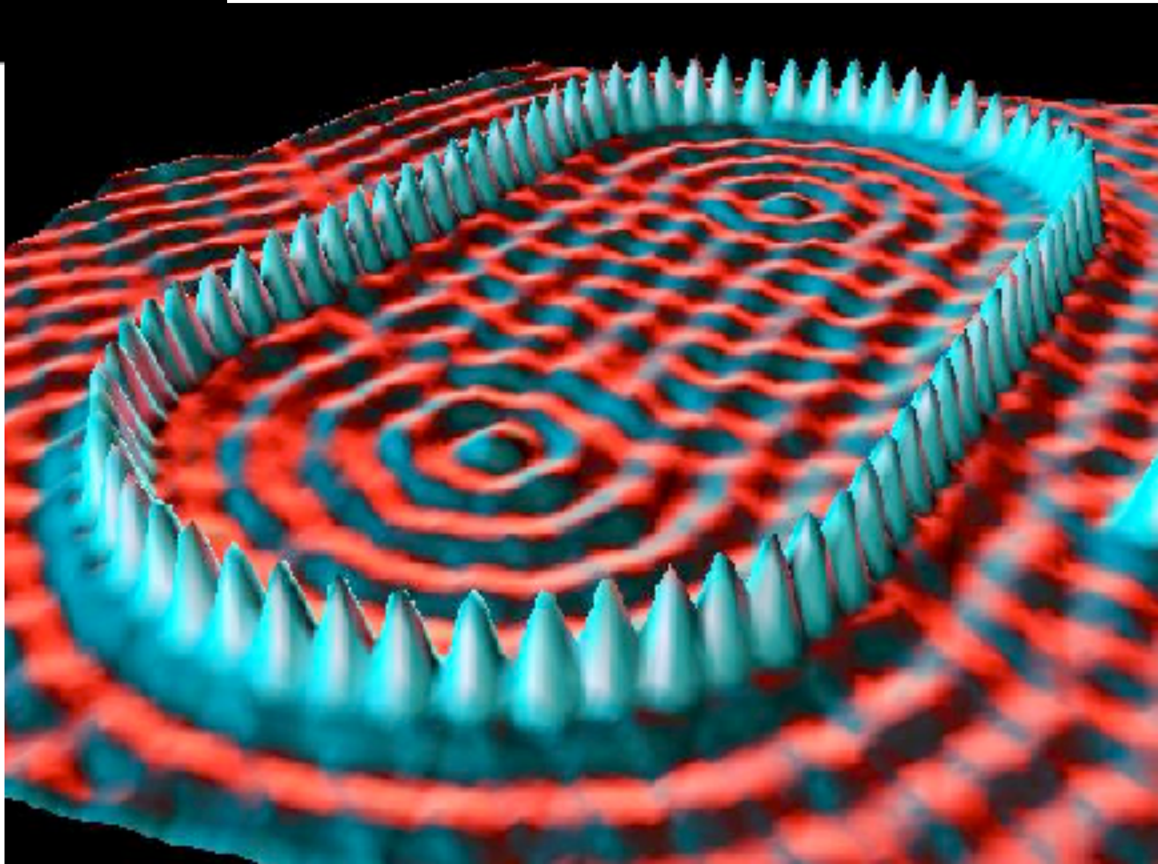


Friedel-like oscillations are *not* inconsistent with the notion of particle trajectories



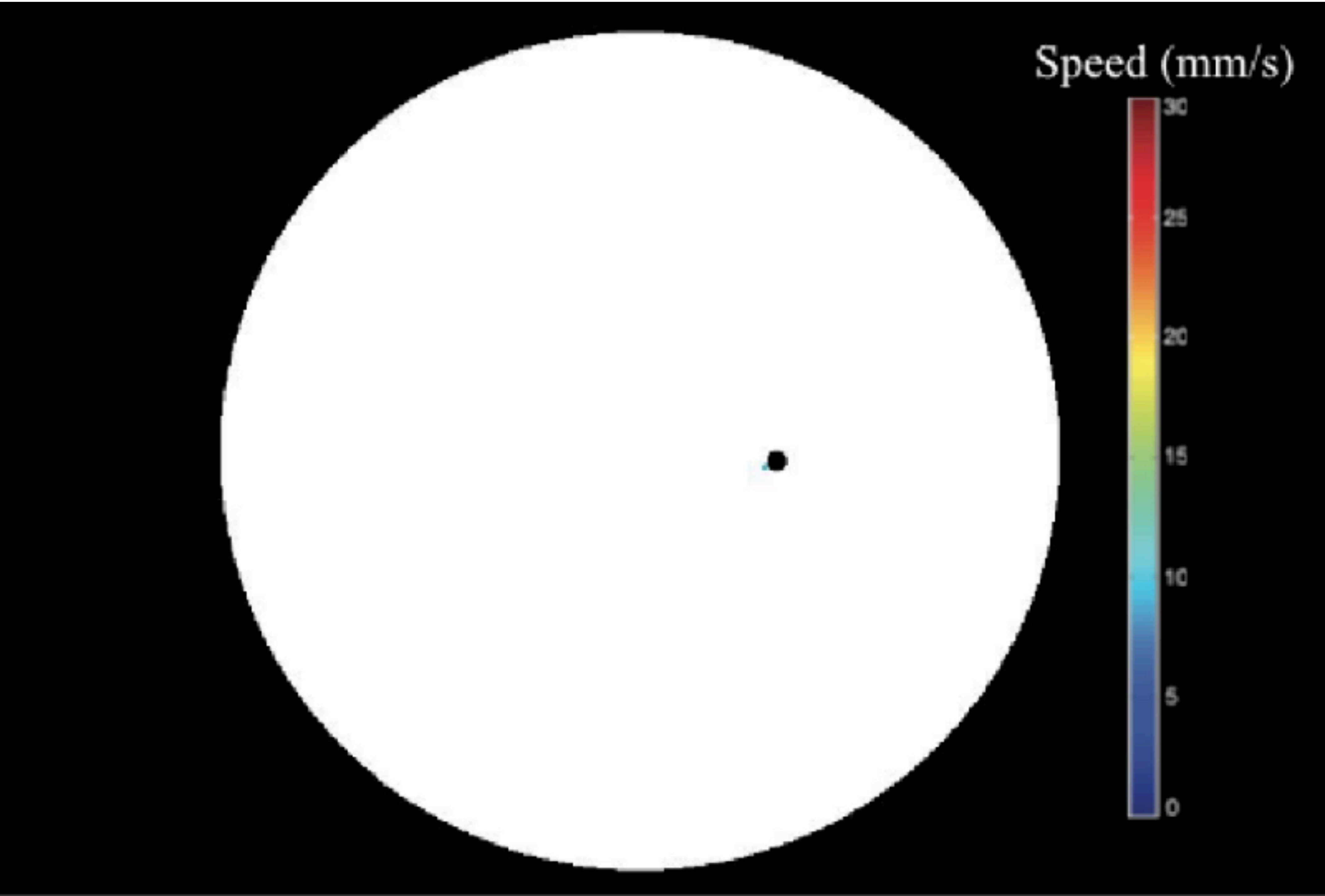
The quantum corral

Crommie, Lutz & Eigler (1993)
Fiete & Heller (2003)

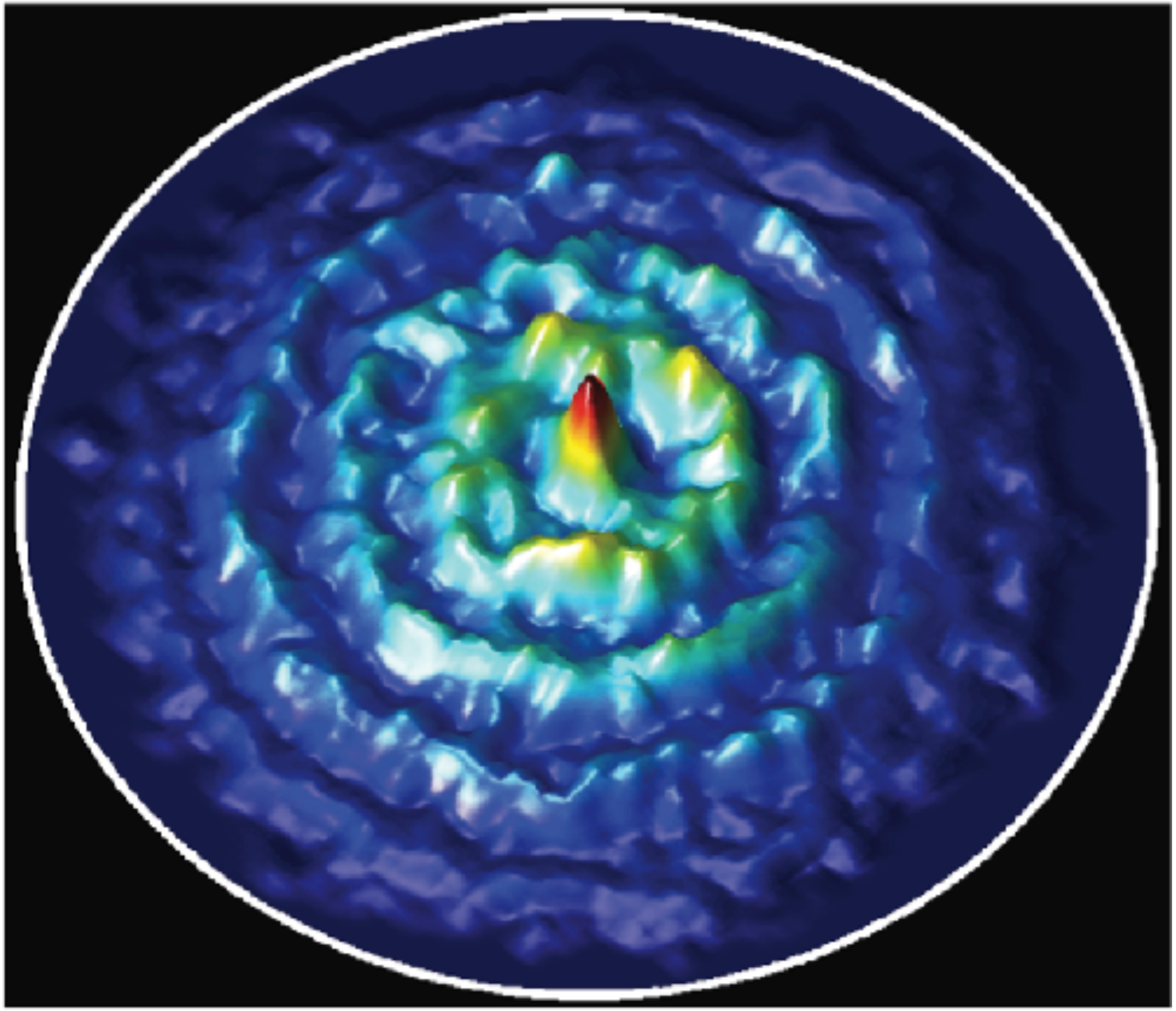


- de Broglie waves evident in the pdf of a sea of electrons trapped on a metal surface, excited by an SEM

Droplet walking in a circular corral



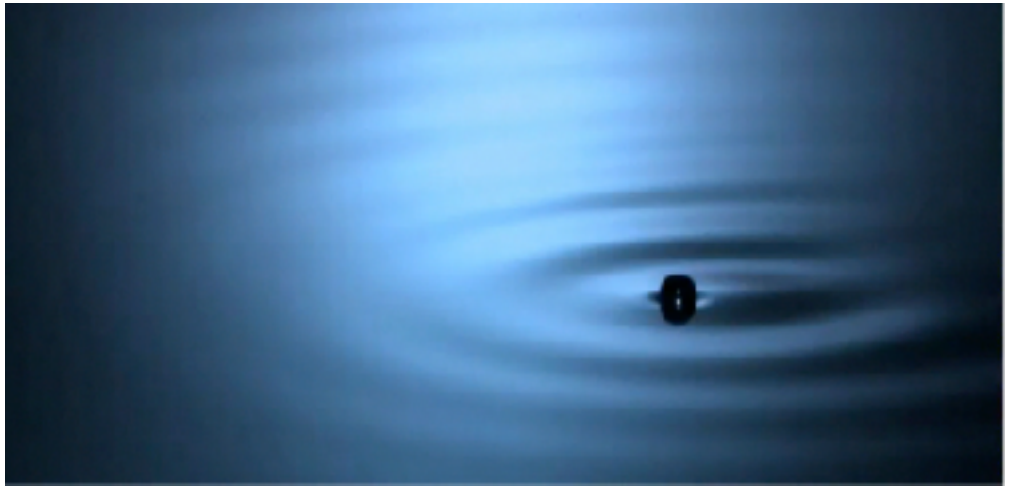
Probability density function



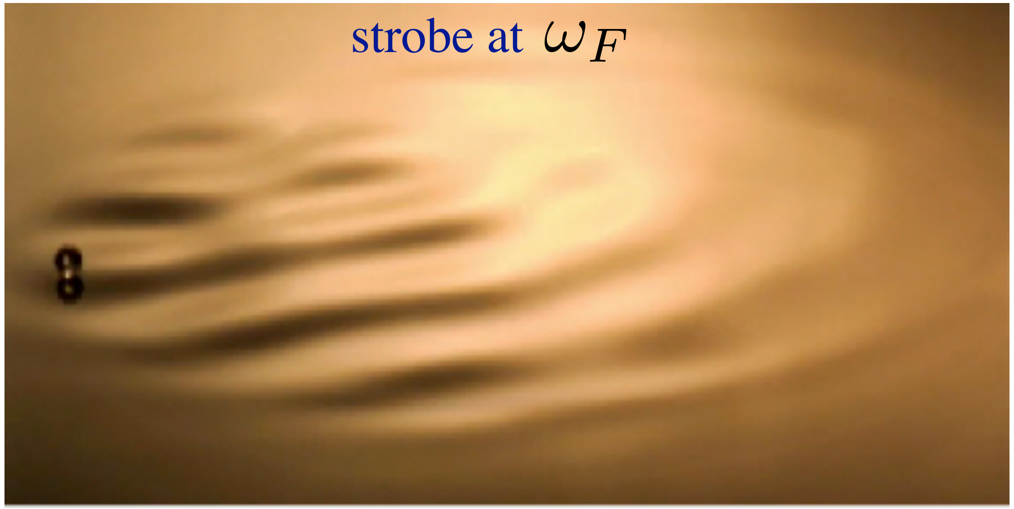
Harris, Moukhtar, Fort, Couder & Bush (PRE, 2013)

- **emergent statistics not inconsistent with the notion of particle trajectories**
- coherent, wave-like statistics emerge from chaotic pilot-wave dynamics
- nonresonant bouncing over pilot wave induces *ponderomotive effects*

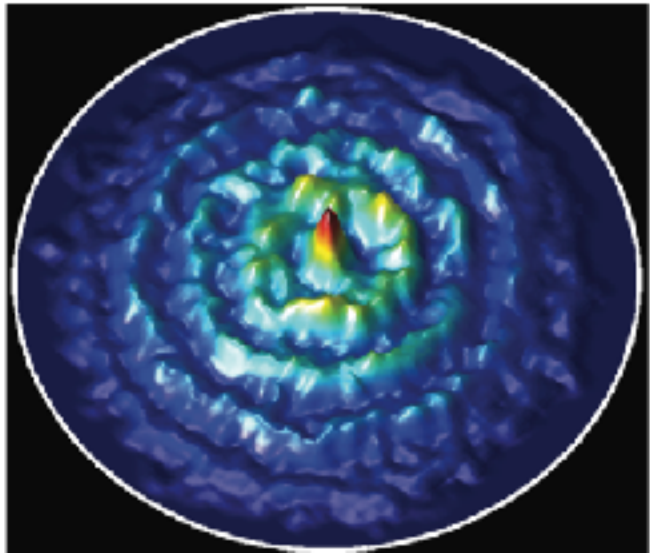
Emerging physical picture: 3 time scales



- **fast dynamics of wave generation**:
  drop bouncing at ω_F generates quasi-monochromatic pilot-wave field

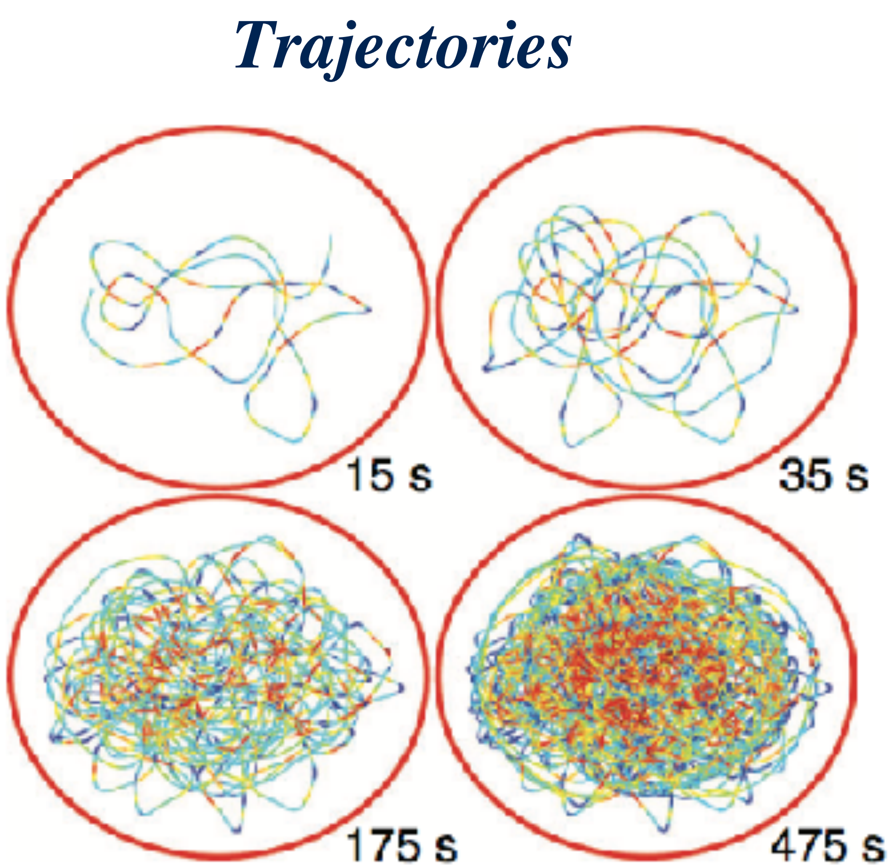


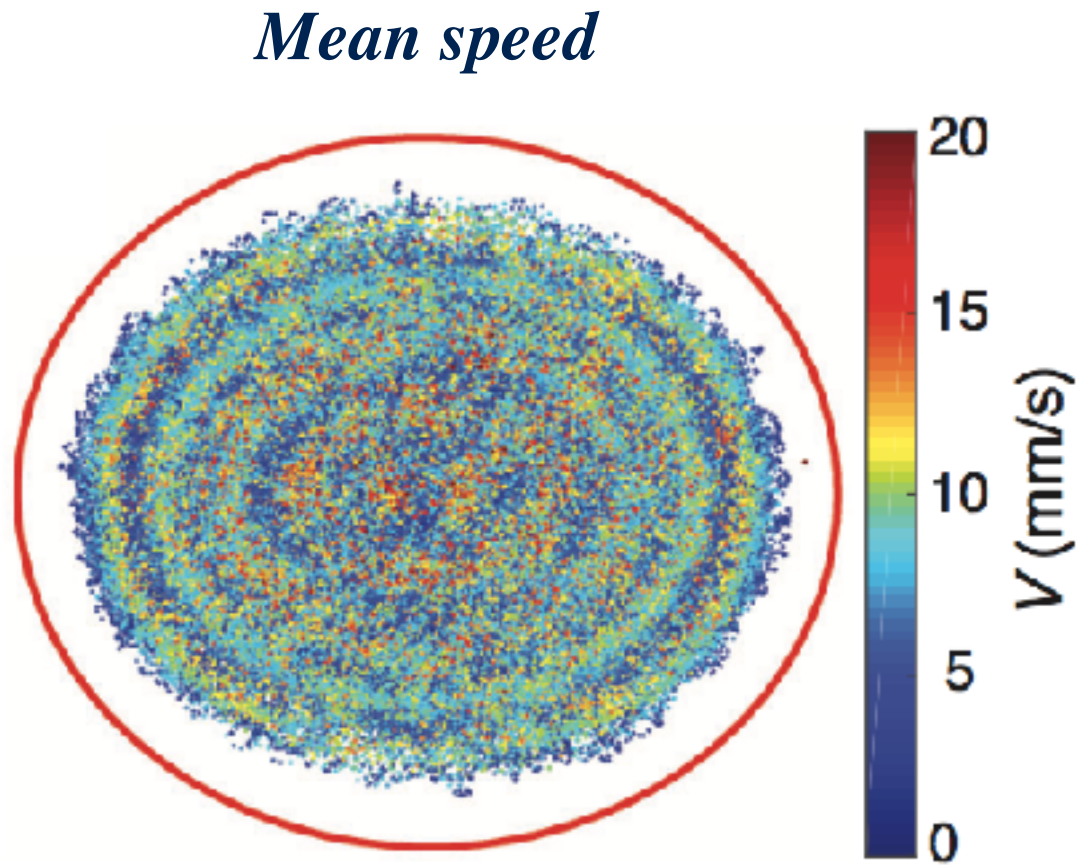
- **intermediate pilot-wave dynamics:**
  particle rides its pilot wave



- **slow timescale of statistical convergence**:
  statistics emerge through influence of the mean pilot wave

The elliptical corral



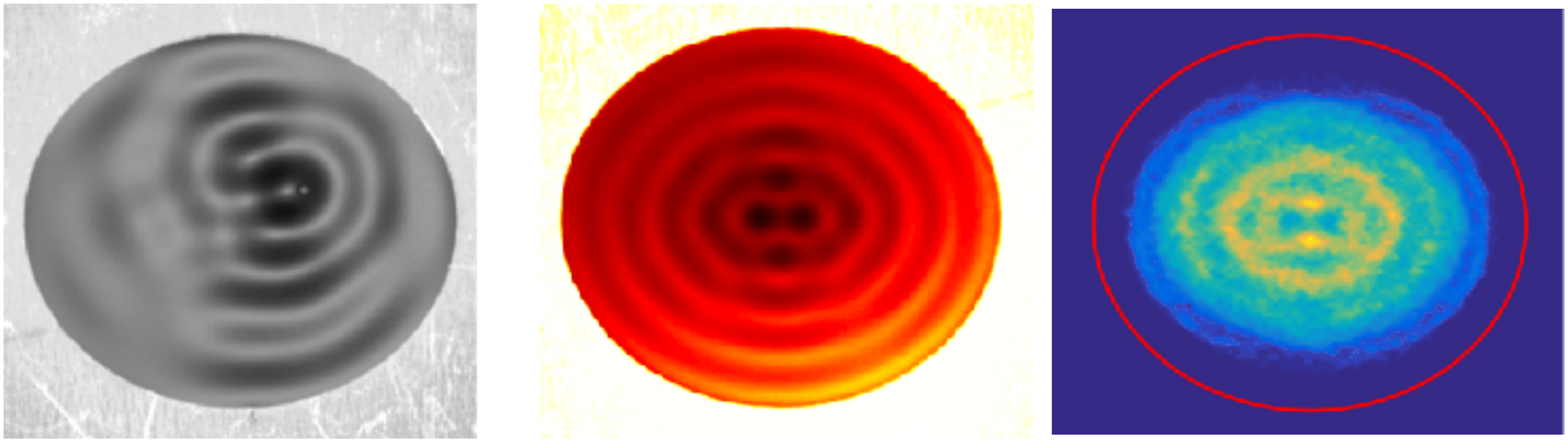


- correlation between position and speed, induced by *ponderomotive effects*
- exhibits *statistical projection effects* analogous to the `quantum mirage'

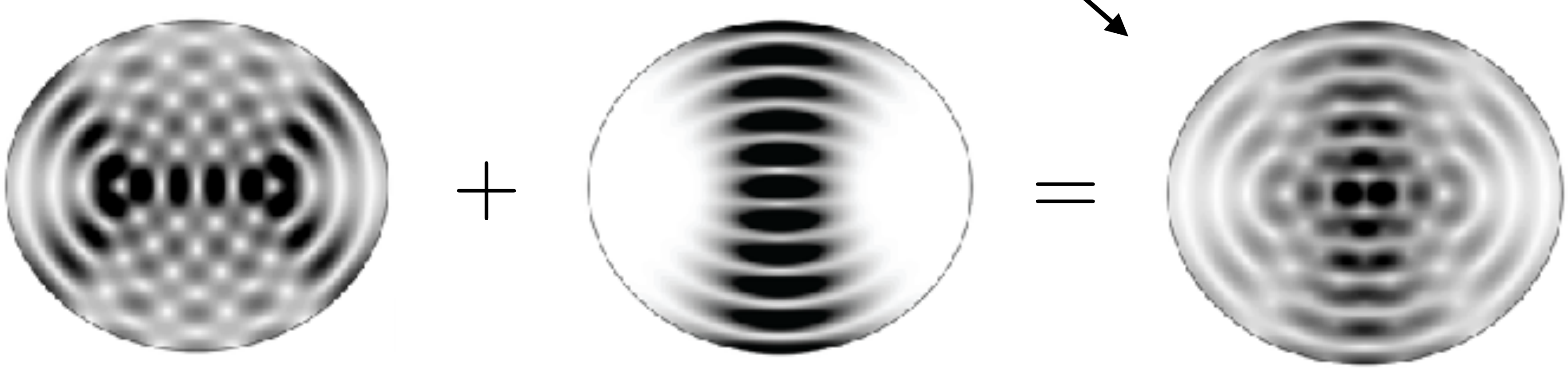
- Sáenz, Cristea-Platon & Bush, Nat. Phys. (2018)

A striking equivalence

Instantaneous wave | **Average slope** | **Particle's histogram**



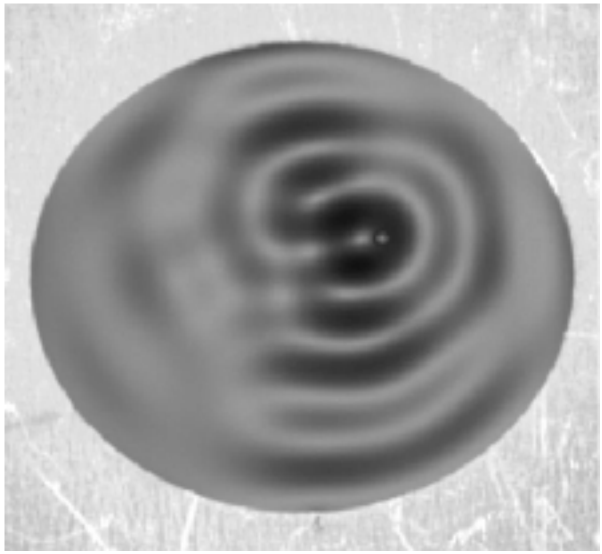
Mode superposition



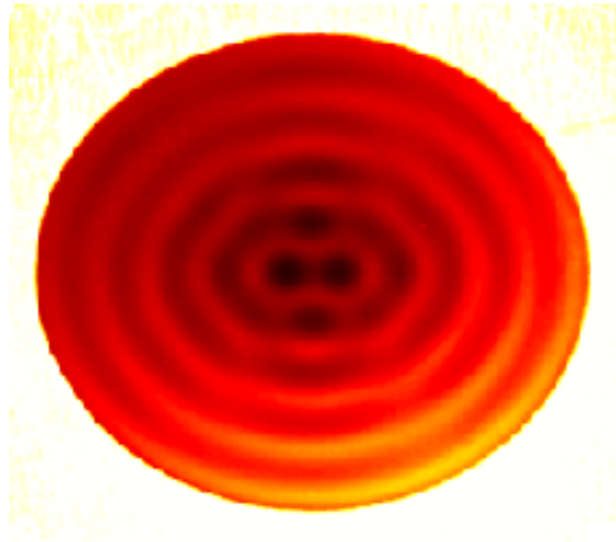
A superposition of statistical states

The mean pilot-wave field

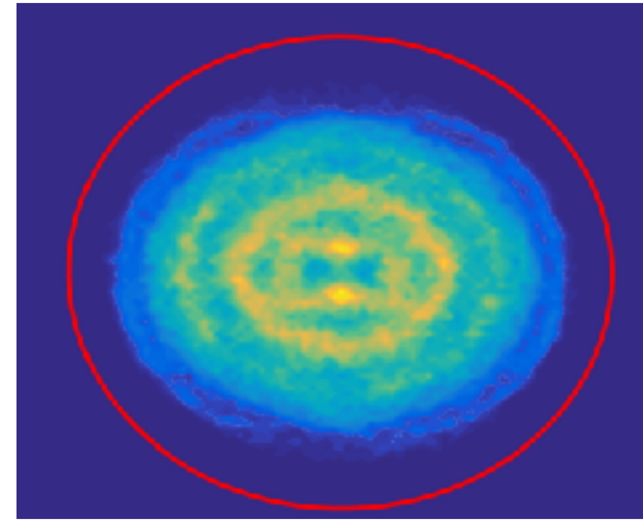
Instantaneous wave



Average slope

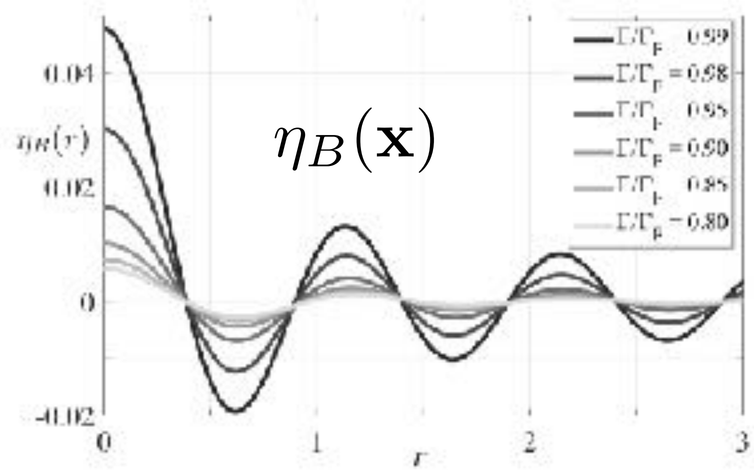


Particle's histogram $\mu(\mathbf{x})$



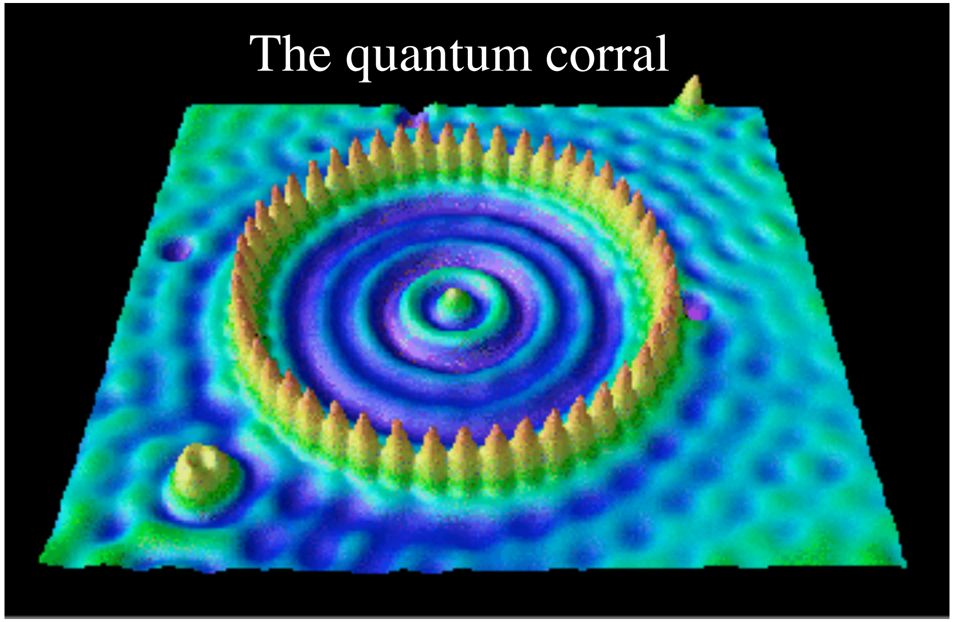
Theorem *(Durey, Milewski & JB, 2018)*

$$\bar{\eta}(\mathbf{x}) = \int_{\mathbf{R^2}} \eta_{\mathbf{B}}(\mathbf{x}-\mathbf{y})\ \mu(\mathbf{y})\ \mathbf{dy} = (\eta_{\mathbf{B}} * \mu)(\mathbf{x})$$

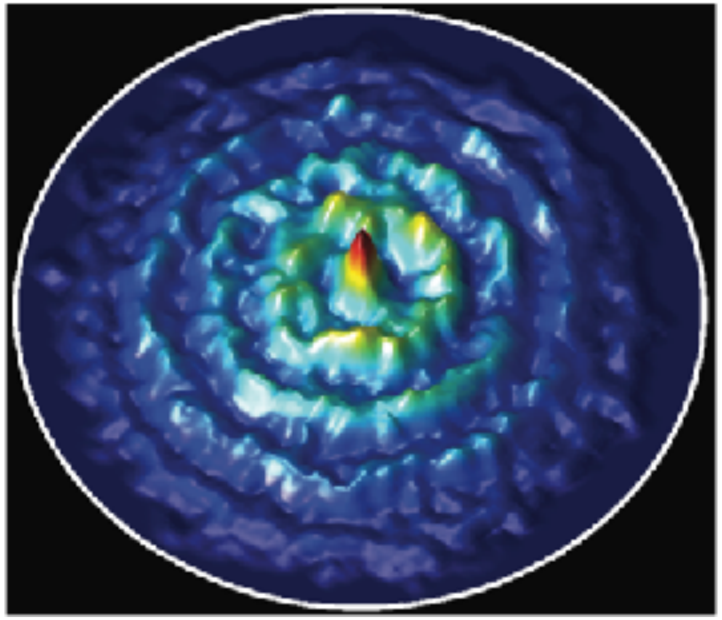


- the average wave field, $\bar{\eta}(\mathbf{x})$, corresponds to the convolution of the *pdf*, $\mu(\mathbf{x})$ and the wave field of a stationary bouncing droplet, $\eta_B(\mathbf{x})$
- in the long-memory limit, this serves as a self-potential, in both corrals and orbital pilot-wave dynamics, and may induce ponderomotive effects

Quantum mechanics	Walking droplets
Statistical description	**Statistical description**



Fiete & Heller (2003)



Harris et al. (2012)

Underlying dynamics	**Underlying dynamics**
Hidden Variable Theory ?	**Exposed variable theory** rationalizes emergent statistics in terms of pilot-wave-mediated ponderomotive potential.