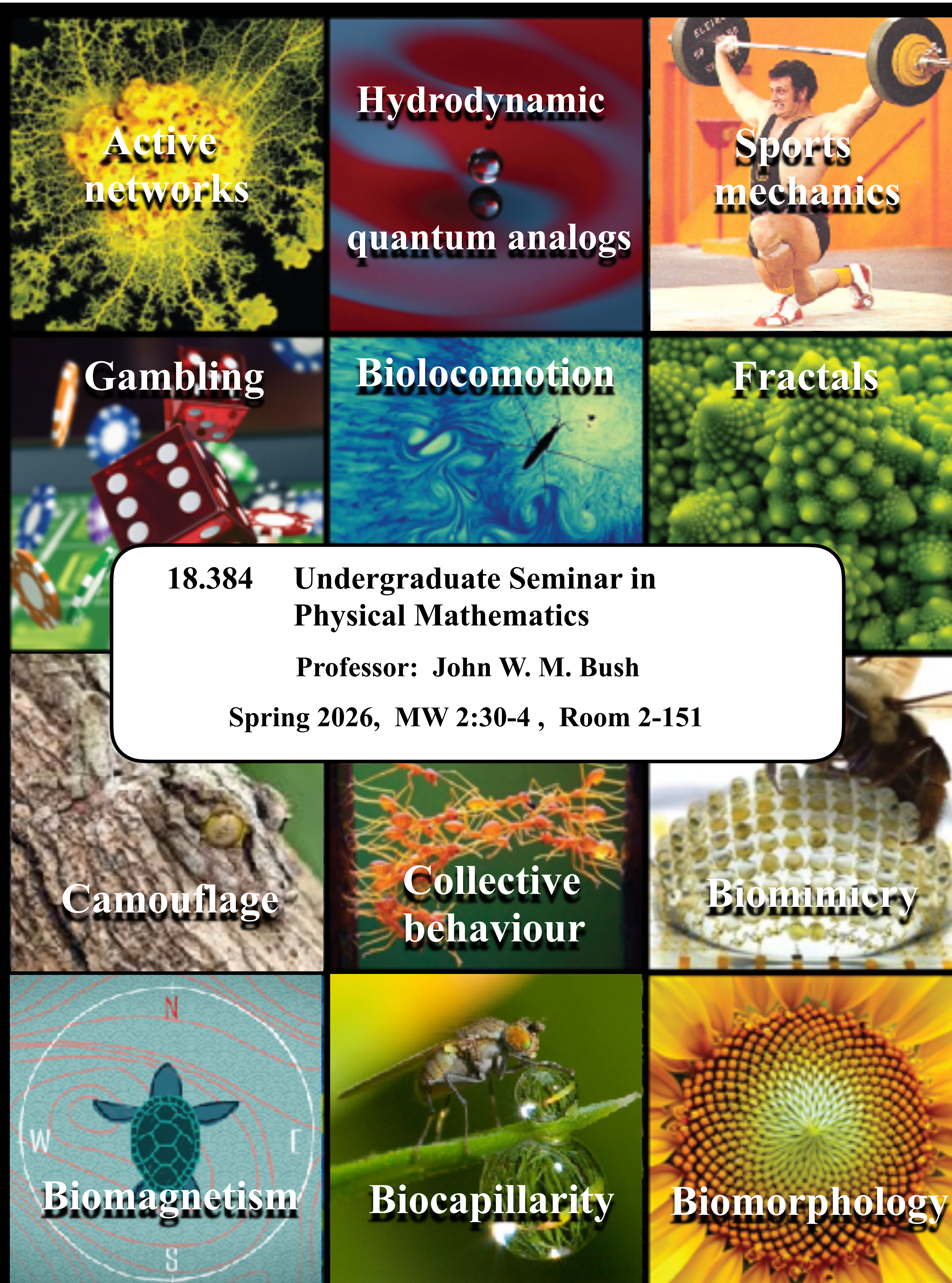
# Designing a Math Talk

18.384 Spring 2026

Susan Ruff
ruff@math.mit.edu

office hour signup:
tinyurl.com/RuffOH-Sp26

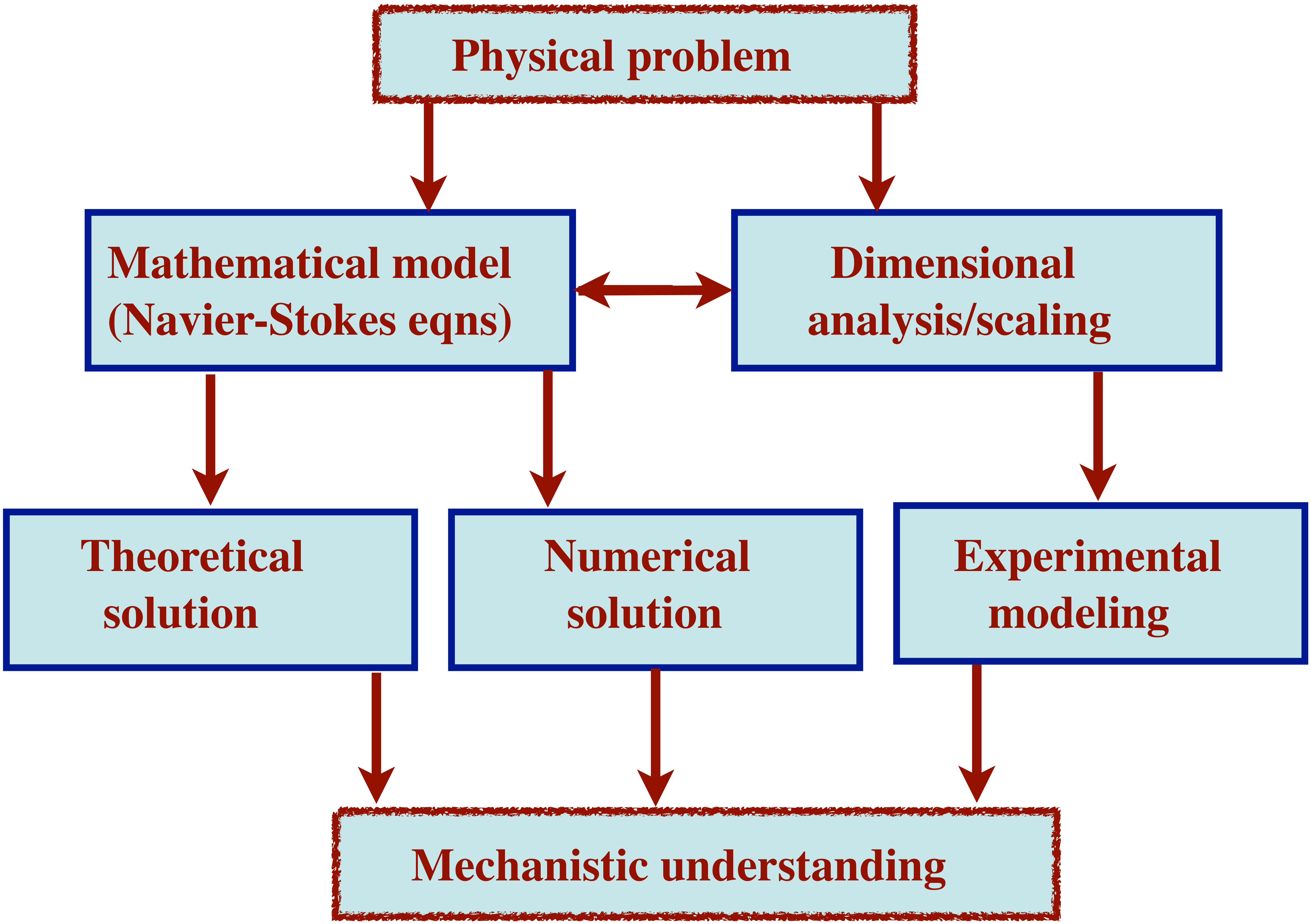



**18.384 Undergraduate Seminar in Physical Mathematics**

**Professor: John W. M. Bush**

**Spring 2026, MW 2:30-4 , Room 2-151**

# Problem solving in fluid mechanics

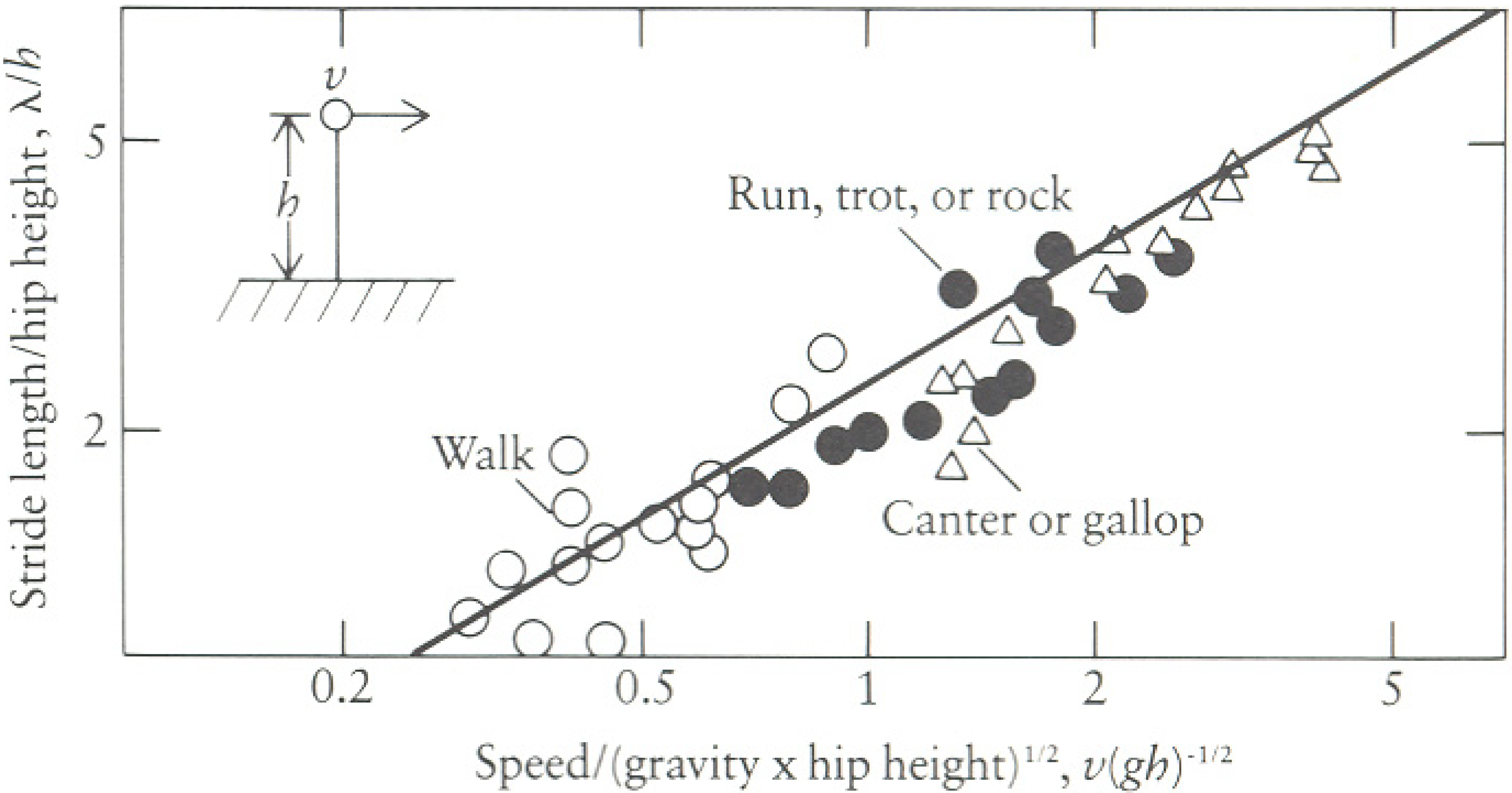


# Reynolds numbers of swimmers and flyers

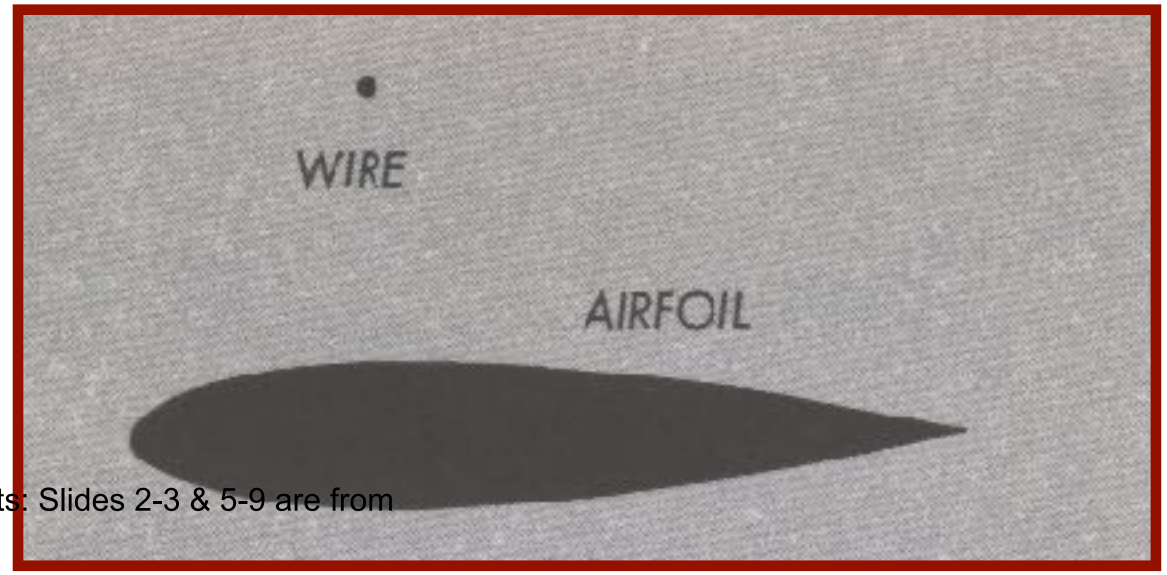
$$\frac{\text{Pressure drag}}{\text{Viscous drag}} = \frac{Ua}{\nu} \equiv \text{Re}$$

| | |
|---|---|
| Whale | 300,000,000 |
| Tuna | 30,000,000 |
| Duck in flight | 300,000 |
| Dragonfly | 30,000 |
| Copepod | 300 |
| Smallest insect in flight | 30 |
| Swimming larvae | 0.3 |
| Sea urchin sperm | 0.03 |
| Bacteria | 0.00001 |

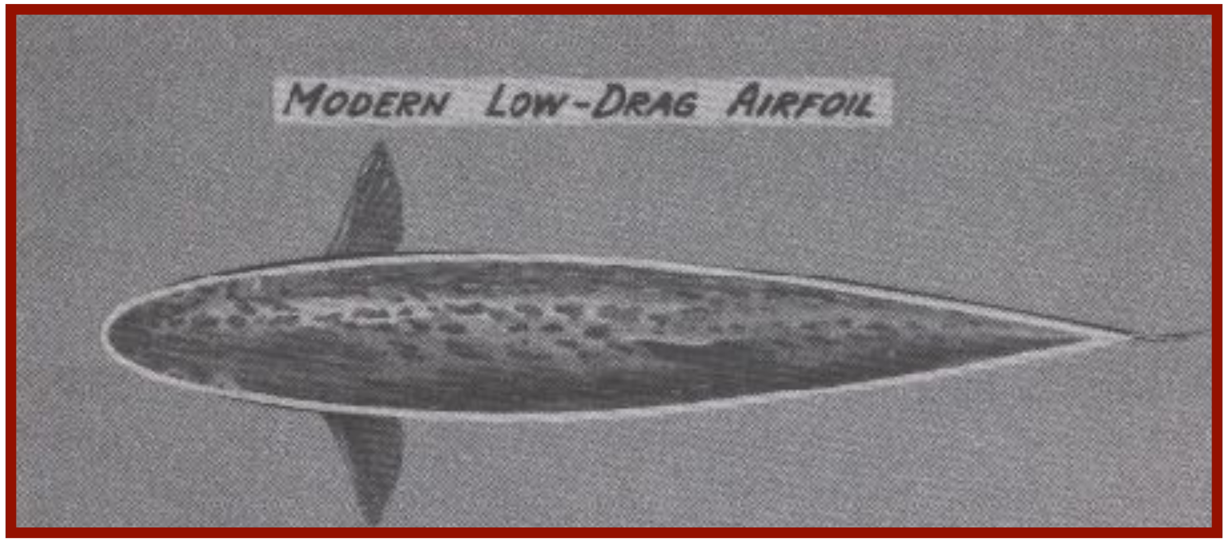
**Observation:** gaits change when $Fr = \frac{U^2}{gh} \sim 1$ for limbed terrestrials.



# How does the speed of a fish depend on its size?




Credits: Slides 2-3 & 5-9 are from




Propulsive force: $F \sim L^2$

Hydrodynamic resistance: skin friction $D \sim \mu \dfrac{U}{\delta} L^2$

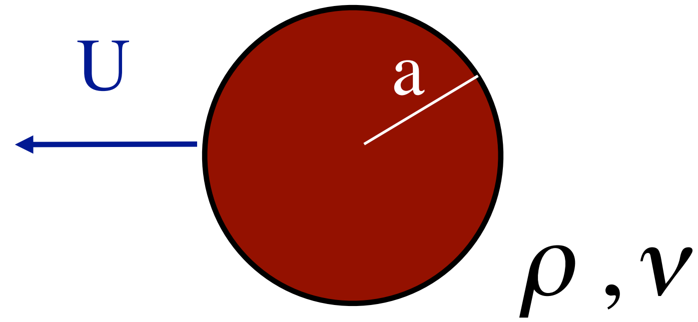
where $\delta \sim L\ Re^{-1/2}$ is the boundary layer thickness

$Re = \dfrac{UL}{\nu}$ is the Reynolds number

Force balance (F=D) indicates $U \sim L^{1/3}$

**Scaling:** the determination of the interdependency of variables in a physical system

**Method:** consider dominant force balance




**Dimensional analysis gave:** $D = C_D(\mathrm{Re})\,\rho\,U^2\,a^2$

- form of $C_D(Re)$ yields dependence at low and high Re

**Scaling approach:**

Re << 1: viscous drag dominant

$$D \sim \frac{\mu U}{a} \cdot a^2 \sim \rho \nu U a$$

VISCOUS STRESS

SURFACE AREA

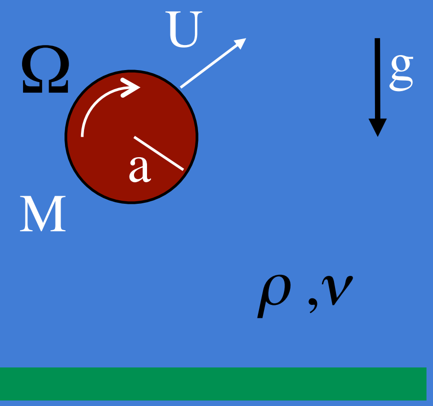
Re >> 1: pressure drag dominant

$$D \sim \rho\, U^2 \cdot a^2$$

FORE-AFT PRESSURE DROP

EXPOSED AREA

# Aerodynamic length




- deduce length, time over which ball changes direction via scaling

$$m\frac{d\mathbf{U}}{dt} = m\mathbf{g} - C_d\,\pi a^2 \rho\, U^2\,\hat{\mathbf{s}} + C_L\,\pi a^3 \rho\,\mathbf{\Omega}\times\mathbf{U}$$

$$\rho_B V \frac{U^2}{L_a} \sim \rho V U \Omega \quad \longrightarrow \quad L_a \sim \frac{U}{\Omega}\frac{\rho_B}{\rho}$$

AERODYNAMIC LENGTH

# Turning time

$$\tau_T \sim \frac{L_a}{U} \sim \frac{1}{\Omega}\frac{\rho_B}{\rho}$$

# Audience can’t read & listen at same time:

Give time to read

## First-order sentences

- H10 is about truth of positive existential sentences

$$(\exists x_1 \exists x_2 \cdots \exists x_n)\ p(x_1, \ldots, x_n) = 0.$$

- Harder problem: Find an algorithm to decide the truth of arbitrary first-order sentences, in which any number of bound quantifiers $\exists$ and $\forall$ are permitted, e.g.,

$$(\exists x)(\forall y)(\exists z)(\exists w) \quad (x \cdot z + 3 = y^2) \ \vee \ \neg(z = x + w).$$

If variables range over integers, this is undecidable (since it is harder than the original H10).

But what if variables range over rational numbers?

# Audience can't read & listen at same time:

## Give time to read; or make slides readable "at a glance"

### First-order sentences

Undecidability in number theory

Bjorn Poonen

- H10 is about truth of positive existential sentences

$$(\exists x_1 \exists x_2 \cdots \exists x_n)\ p(x_1, \ldots, x_n) = 0.$$

- Harder problem: Find an algorithm to decide the truth c arbitrary first-order sentences, in which any number of bound quantifiers $\exists$ and $\forall$ are permitted, e.g.,

$$(\exists x)(\forall y)(\exists z)(\exists w) \quad (x \cdot z + 3 = y^2)\ \vee\ \neg(z = x + w)$$

If variables range over integers, this is undecidable (since it is harder than the original H10).

But what if variables range over rational numbers?

### First-order sentences

**H10**: Find algorithm to decide truth of

$$(\exists x_1 \exists x_2 \cdots \exists x_n)\ p(x_1, \ldots, x_n) = 0$$

← positive existential sentence

**Harder**: Find algorithm to decide truth of, e.g.,

arbitrary first-order sentence

$$(\exists x)(\forall y)(\exists z)(\exists w) \quad (x \cdot z + 3 = y^2)\ \vee\ \neg(z = x + w)$$

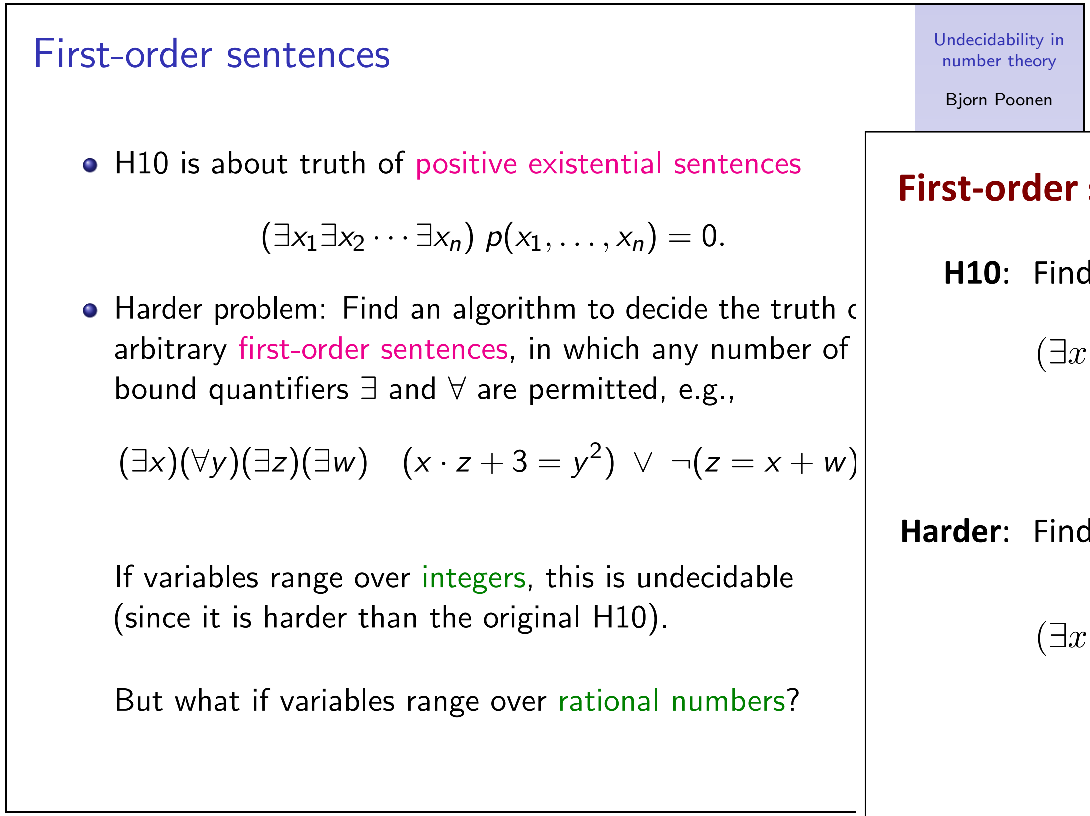
Any number of $\exists, \forall$

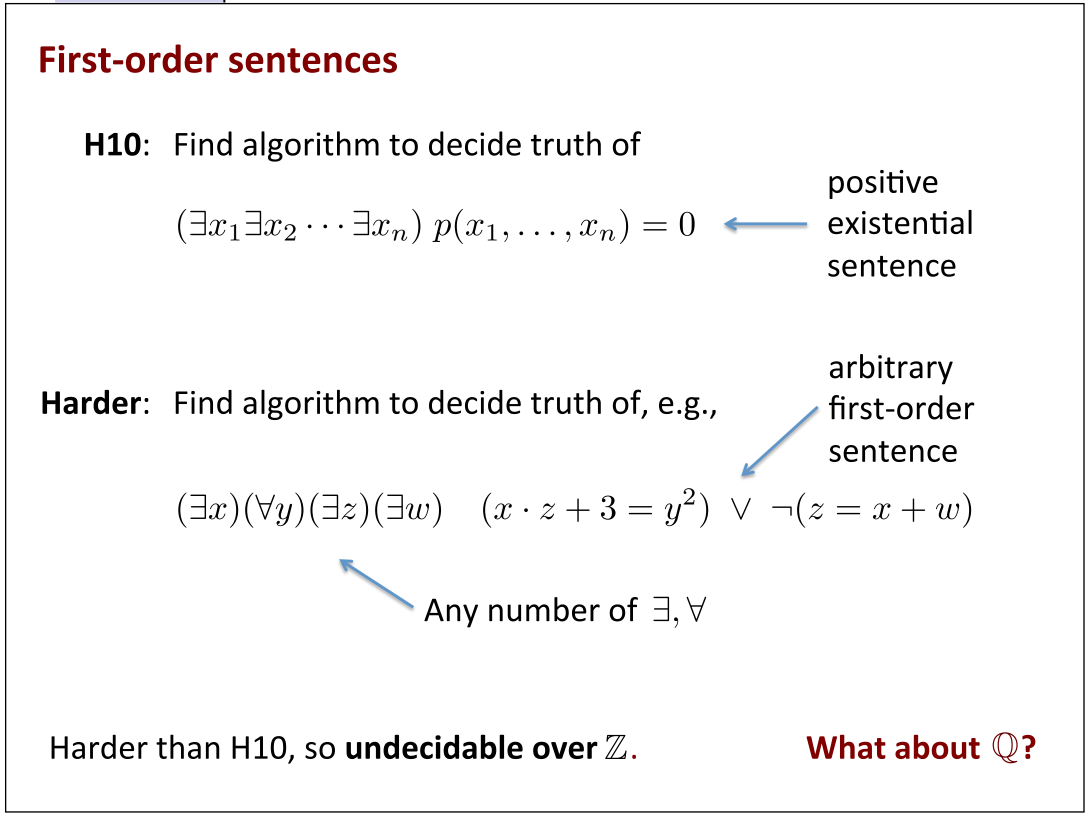
Harder than H10, so **undecidable over $\mathbb{Z}$.**

**What about $\mathbb{Q}$?**

# Audience can't read & listen at same time:

Give time to read; or make slides readable "at a glance"




Use Color
Short text
White space
Alignment
Exact repetition
Meaningful line breaks:

# Audience can't read & listen at same time:

Give time to read; or make slides readable "at a glance"

First-order sentences

Undecidability in number theory

Bjorn Poonen

- H10 is about truth of positive existential sentences

$$(\exists x_1 \exists x_2 \cdots \exists x_n)\ p(x_1, \ldots, x_n) = 0.$$

- Harder problem: Find an algorithm to decide the truth c arbitrary first-order sentences, in which any number of bound quantifiers $\exists$ and $\forall$ are permitted, e.g.,

$$(\exists x)(\forall y)(\exists z)(\exists w) \quad (x \cdot z + 3 = y^2) \ \vee \ \neg(z = x + w)$$

If variables range over integers, this is undecidable (since it is harder than the original H10).

But what if variables range over rational numbers?

**First-order sentences**

**H10**: Find algorithm to decide truth of

$$(\exists x_1 \exists x_2 \cdots \exists x_n)\ p(x_1, \ldots, x_n) = 0$$

← positive existential sentence

**Harder**: Find algorithm to decide truth of, e.g.,

arbitrary first-order sentence

$$(\exists x)(\forall y)(\exists z)(\exists w) \quad (x \cdot z + 3 = y^2) \ \vee \ \neg(z = x + w)$$

Any number of $\exists, \forall$

Harder than H10, so **undecidable over** $\mathbb{Z}$.

**What about $\mathbb{Q}$?**

Use
- Color
- Short text
- White space
- Alignment
- Exact repetition
- Meaningful line breaks:

**Test for yourself which sentence is faster to read.**

**Test for yourself which sentence is faster to read.**

Credits: Slides 2-3 & 5-9 are course slides (by John Bush);
Slide 4 is Vincent van Gogh's Memory of the Garden at Etten (Ladies of Arles)
Slides 10-13 include a slide by Bjorn Poonen (as noted).

*I'm happy to meet to help you with your slide talk.*
*You can email me to arrange a time:* *ruff@math.mit.edu*
*or sign up for a time here:* http://tinyurl.com/RuffOH-Sp24

*--Susan*