Lecture 5

Biocapillarity

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18.384 SYLLABUS, Spring 2026

Weekday	Date	Deadline	Topic
M	Feb.2		1. Lecture 1. Introduction
W	Feb.4		2. Lecture 2. Scaling arguments
M	Feb.9		3. Lecture 3. The physics of the beautiful game
W	Feb.11		4. Lecture 4. Interfacial fluid dynamics
T	Feb.17		5. Lecture 5. Biocapillarity
W	Feb.18		6. Lecture 6. Writing and speaking: with Susan Ruff
M	Feb.23		7. Lecture 7. Hydrodynamic quantum analogs
W	Feb.25		8. Student pitches: Lightning round
M	Mar.2		9. Lecture 8. Respiratory disease transmission
W	Mar.4		10. Students talks, Round 1
M	Mar.9		11. Student talks, Round 1
W	Mar.11	Paper 1 due	12. Student talks, Round 1
M	Mart.16		13. Feedback on Round 1
W	Mar.18		14. Students talks, Round 2
	Mar.21-28		**Spring Break**: No class
M	Mar.30		15. Student talks, Round 2
W	Apr.1	Paper 2 due	16. Student talks, Round 2
M	Apr.6		17. Student talks, Round 3
W	Apr.8		18. Student talks, Round 3
M	Apr.13	Paper 3 due	19. Student talks, Round 3
W	Apr.15		20. Feedback on Rounds 2 and 3
M	Apr.20		**Patriot's Day**: No class
W	Apr.22		22. Final Student talks
M	Apr.27		23. Final Student talks
W	Apr.29		24. Final Student talks
M	May.4		25. Final Student talks
W	May.6		26. Final Student talks
M	May.11	Final paper due	27. Final Student talks

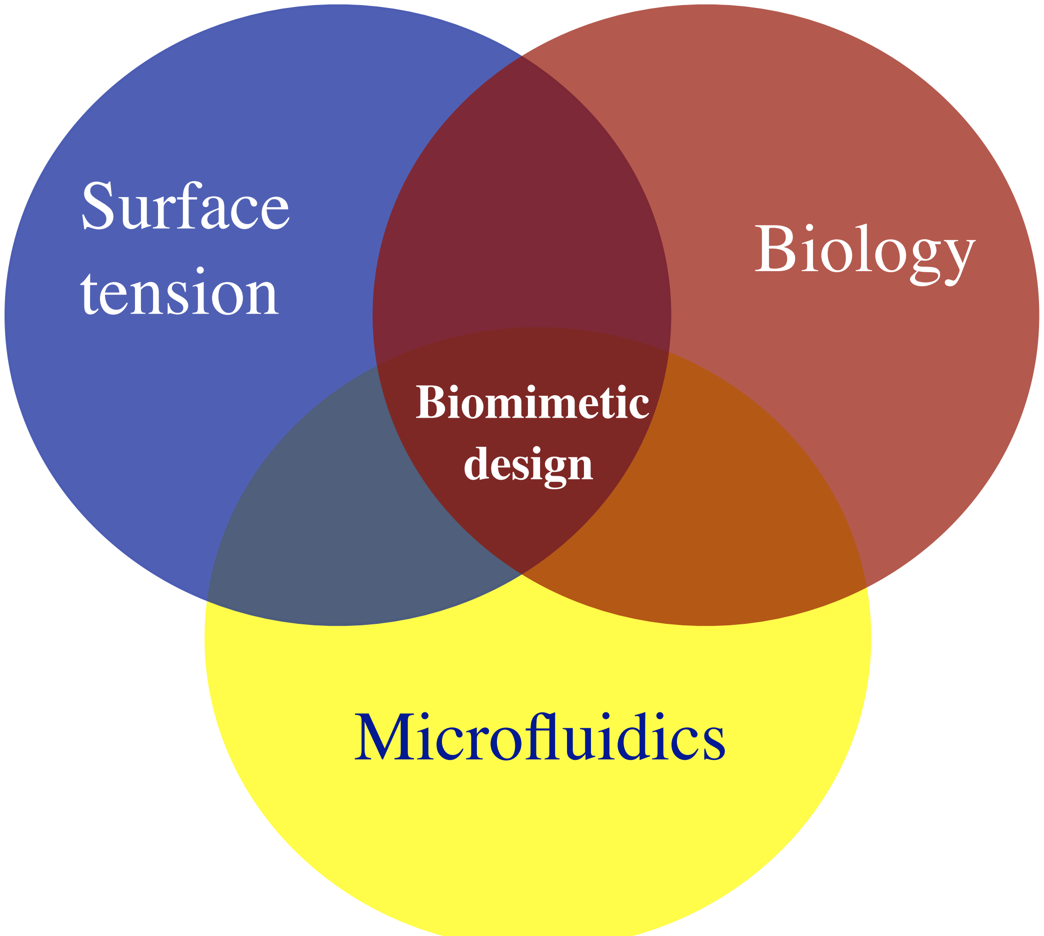
"The imagination is made keener and more correct by continually studying nature and wrestling with it."

- Vincent Van Gogh


"Choose only one master, Nature."
- Rembrandt

MOTIVATION

- to rationalize Nature's designs

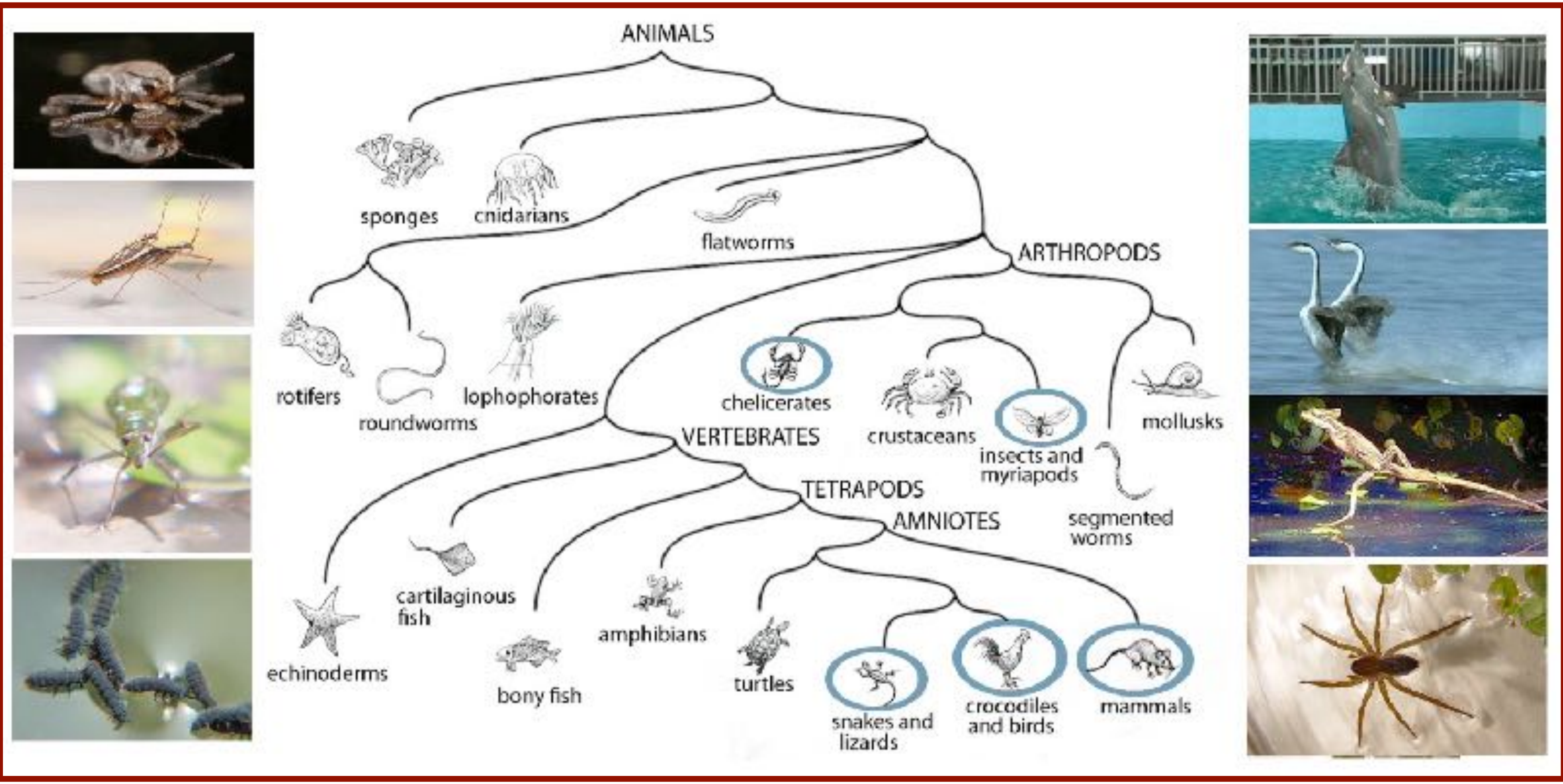


Bonus: to inspire and inform biomimetic design

Walking on water

with David Hu (now at Georgia Tech)

Water-walkers in the tree of life (over 1200 species)



Motivation: foraging on water surface, avoidance of predators

Biolocomotion

- propulsion rationalized in terms of force, energy or momentum transfer
- creature applies a force to its environment: the reaction force propels it

Terrestrials

- frictional forces applied at point of contact with ground
- energy cycled between muscular strain energy, GPE and KE of body

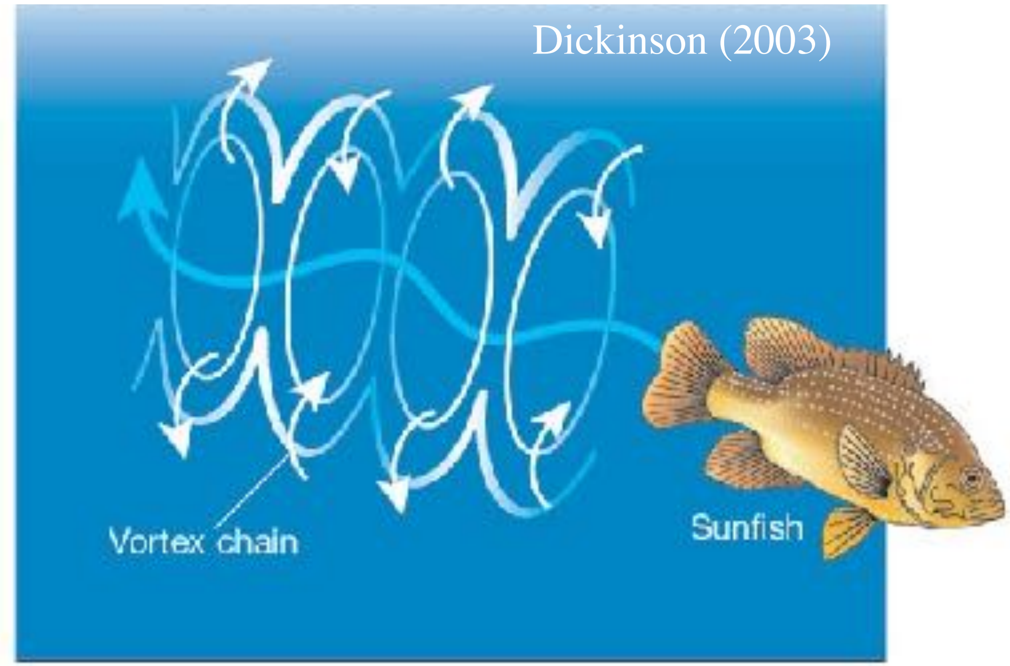
Swimmers and fliers

- hydrodynamic forces applied over body
- strain energy, GPE and KE of body, KE of fluid

Water-walkers

- combination of hydrodynamic and surface forces applied over body
- strain energy, GPE and KE of body, KE of fluid, surface energy

Flying and swimming at high Re



Momentum created by stroke: $$\mathbf{P} = \mathrm{m}\Delta\mathbf{v} = \int_S \mathbf{F}_\mathbf{h}\, dt$$

where $$\mathbf{F}_\mathbf{h} = \int_S \mathbf{n}\cdot\mathbf{T}\,\mathrm{dS} \;,\quad \mathbf{T} = -\,\mathrm{p}\mathbf{I} + \mu\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^\mathbf{T}\right)$$

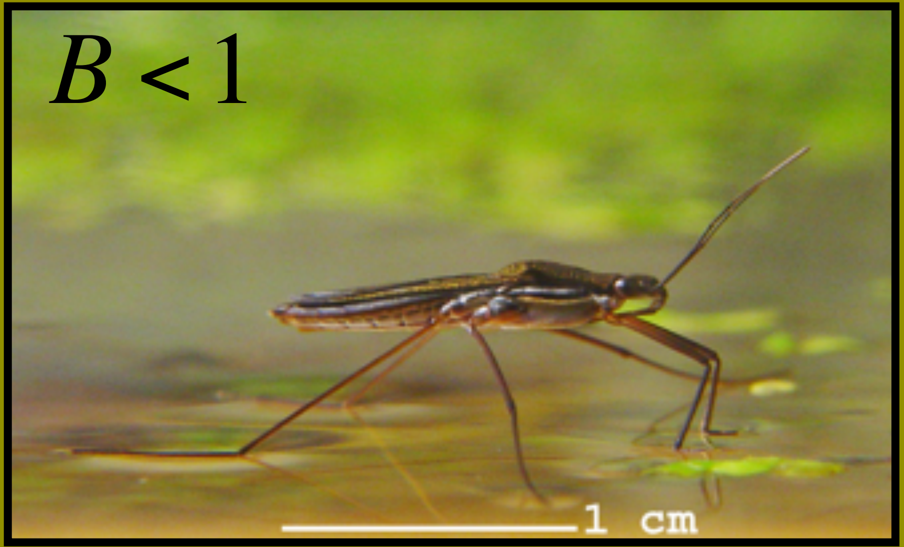
Momentum transferred in wake: $$\mathbf{P}_f = \iiint_V \rho\,\mathbf{u}(\mathbf{x},\mathrm{t})\,\mathrm{dV}$$

Conservation of momentum: $$\mathbf{P} = \mathbf{P}_f$$

Propulsion rationalized by elucidating $\mathbf{F}_\mathrm{h}$ or $\mathbf{P}_f$.

Two modes of weight-support:

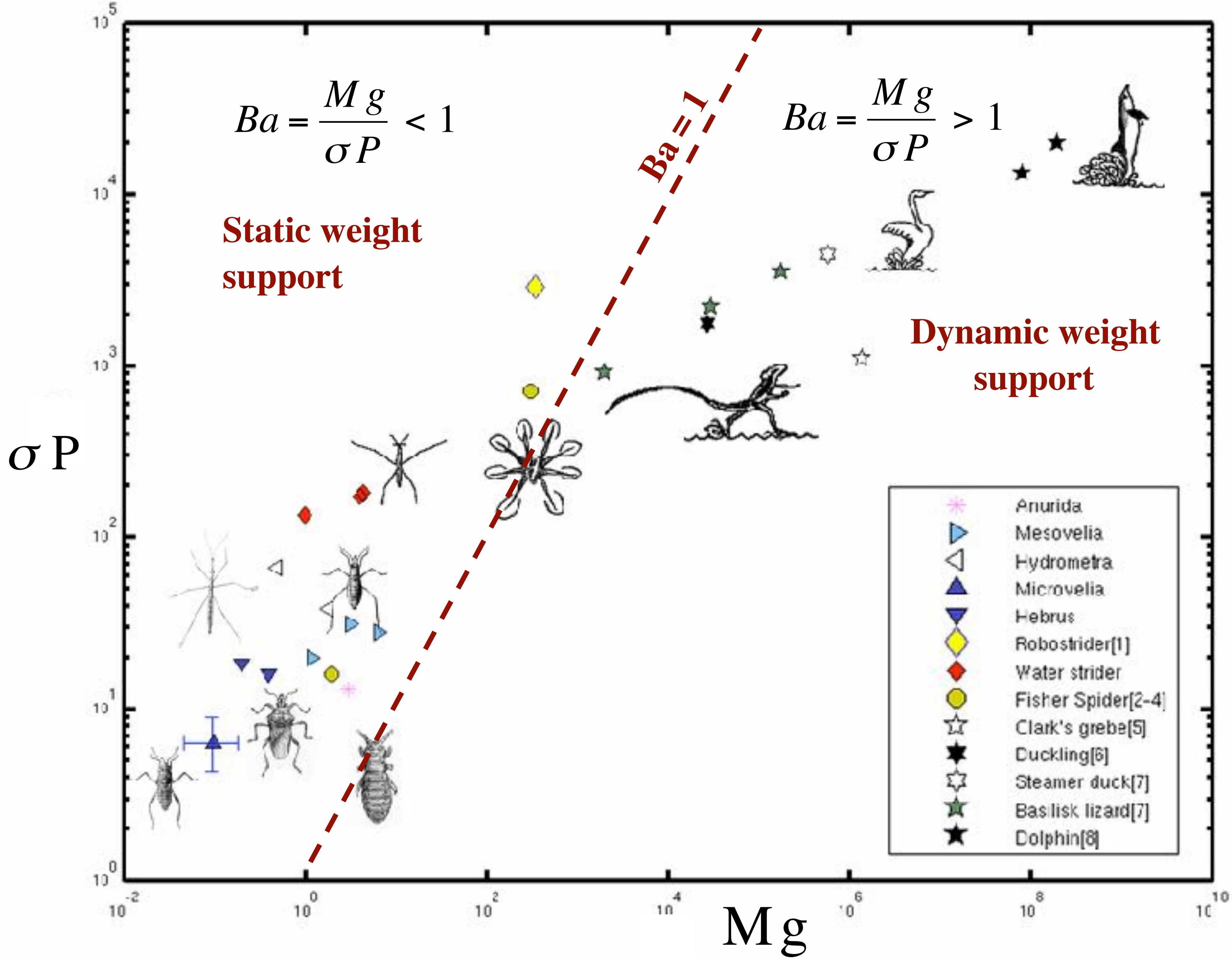
$$B = \frac{Mg}{\sigma P} = \frac{\text{weight}}{\text{surface tension force}}$$



- static weight support by σ

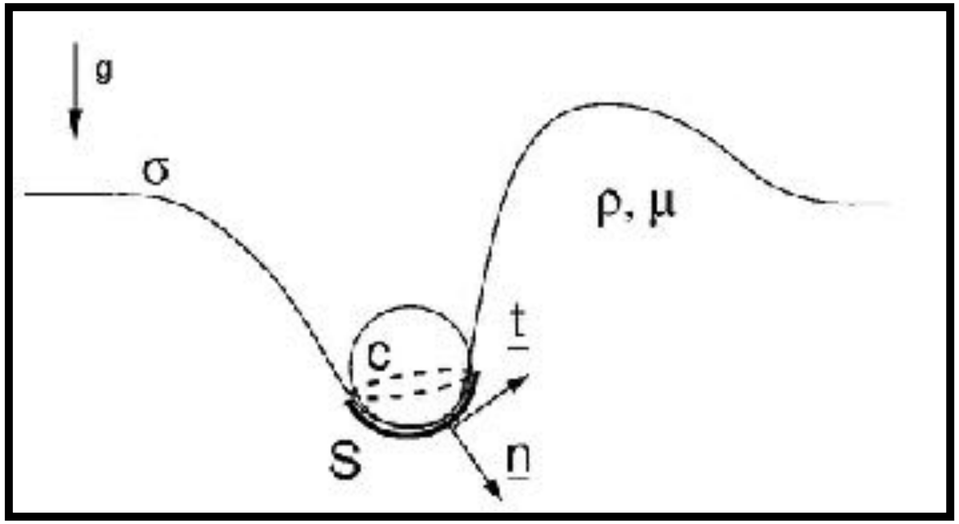
$F \sim 2\sigma L$

- dynamic weight support
- vertical hydrodynamic forces generated by slapping



$Ba = \frac{M g}{\sigma P} < 1$
Ba = 1
$Ba = \frac{M g}{\sigma P} > 1$
Static weight support
Dynamic weight support
σ P
Mg
Anurida
Mesovelia
Hydrometra
Microvelia
Hebrus
Robostrider[1]
Water strider
Fisher Spider[2-4]
Clark's grebe[5]
Duckling[6]
Steamer duck[7]
Basilisk lizard[7]
Dolphin[8]

Lateral propulsion at the interface



$$\underline{F}_H = \int_S \underline{\underline{T}} \cdot \underline{n} \, dS + \int_C \sigma \, \underline{t} \, dl$$

Stress tensor:

$$\underline{\underline{T}} = -p \, \underline{\underline{I}} + \mu \left(\nabla u + (\nabla u)^T \right)$$

Surface tension force:

$$\int_C \sigma \, \underline{t} \, dl = \int_S \sigma \left(\underline{\nabla} \cdot \underline{n} \right) \underline{n} - \underline{\nabla} \sigma \;\; dS$$

curvature pressure

Marangoni stress

For high Re motion:

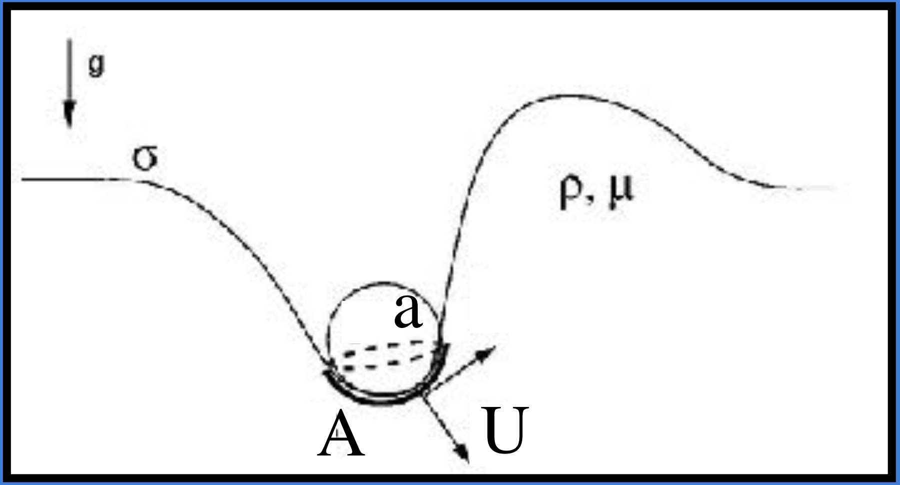
$$\underline{\underline{T}} = -p \, \underline{\underline{I}}$$

where

$$\frac{\partial \phi}{\partial t} + \frac{1}{2} u^2 + \frac{p}{\rho} - \underline{g} \cdot \underline{x} = const$$

time-dependent Bernoulli

Dynamic classification of water-walkers



$$F_H \sim \rho g V_s + \rho U^2 A + \rho V \frac{dU}{dt} + \rho \nu U a + \sigma(\underline{\nabla} \cdot \underline{n}) A - \underline{\nabla} \sigma A$$

buoyancy	form drag	acceleration reaction	viscous drag	curvature	Marangoni

Mathematician: which terms can I discard to get a tractable equation?

Physicist: which forces are used by which creatures?

→ **every force is used by some creature**

Biological classification

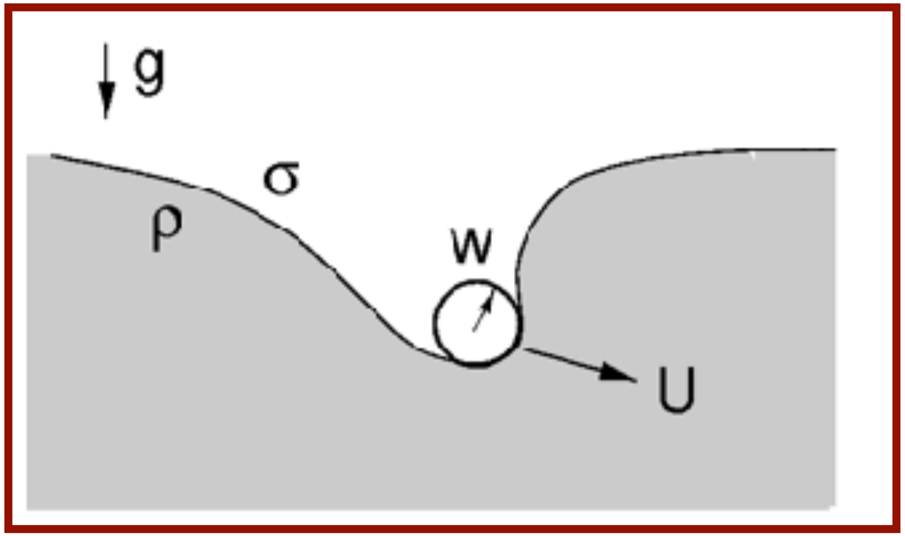
- made along evolutionary grounds

Dynamic classification

- group according to propulsion mechanism
- evaluate relative magnitudes of hydrodynamic forces

	$\rho g z A$	$\rho V dU/dt$	$\rho U^2 A$	$\sigma \nabla \cdot \underline{n} A$	$\underline{\nabla} \sigma A$
Surface slapping					
Rowing & walking					
Surface distortion					
Marangoni propulsion					

Dynamic constraints on large water-walkers $\left(\frac{Mg}{\sigma P} > 1\right)$



- hydrodynamic force must bear body weight:

Crudely, $$\rho\overline{U^2 w^2} > Mg$$

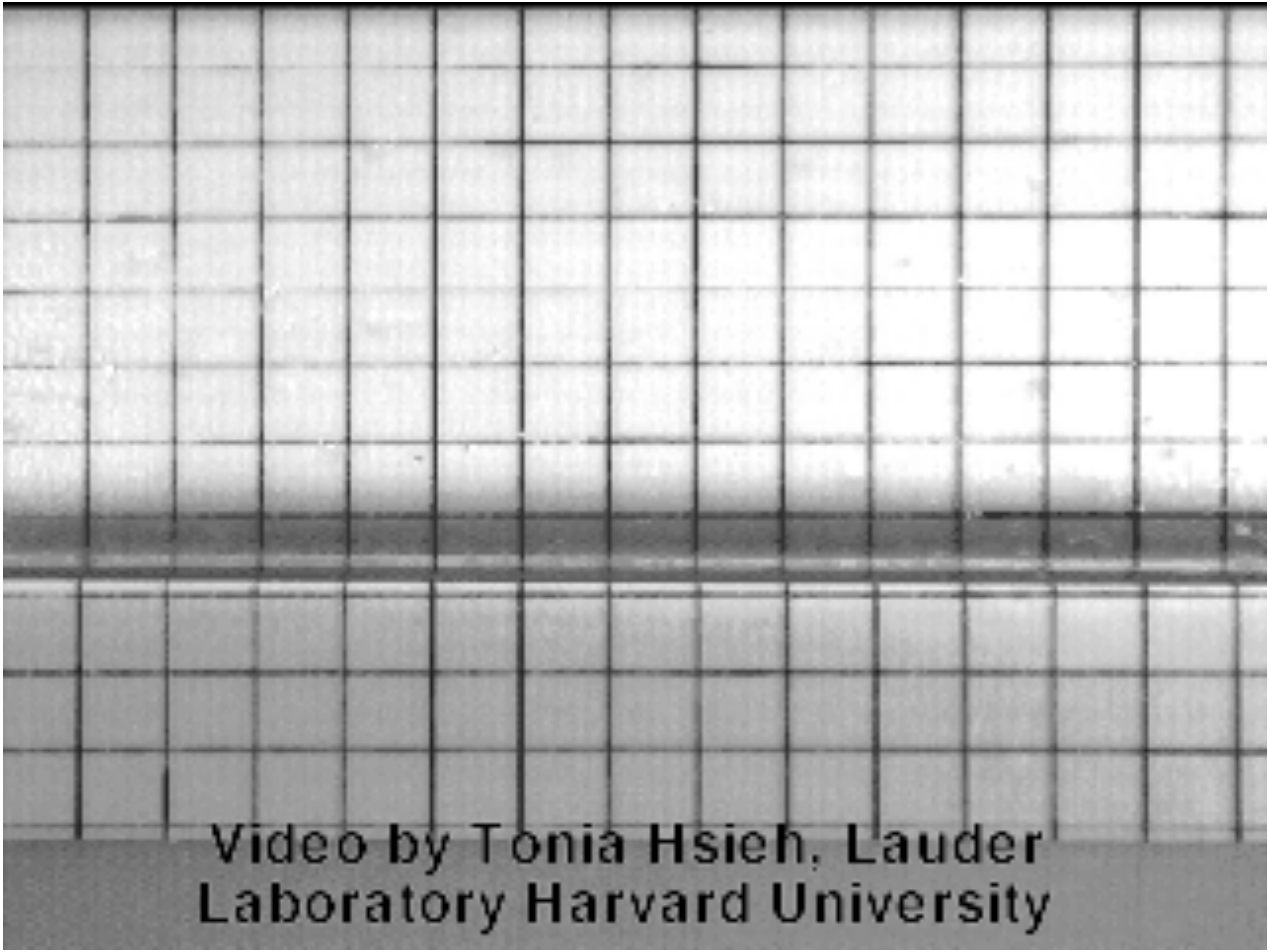
Tougher constraints: **1) power requirements**

2) dynamic stability.

The skittering frog

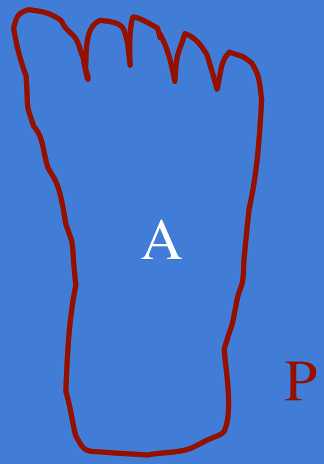
Courtesy of BBC's Natural World

Clark's Grebe: clip courtesy of "Winged Migration"


Video by Tonia Hsieh, Lauder
Laboratory Harvard University

What is the largest creature that can walk on water?

Can man walk on water?



Imagine a man of weight M = 70 kg who can run at U = 10 m/s.

How big must his feet be to walk on water?

Option 1: use surface tension $\sigma = 70$ dynes/cm

Vertical force balance: $$\mathrm{M}\,g = \sigma \mathrm{P}$$

Requires feet with perimeter: $$\mathrm{P} = \frac{\mathrm{M}\,g}{\sigma} \approx 10 \text{ km}$$

Option 2: run via slapping mode

Vertical force balance: $$M\,g = \rho U^2 A$$

Requires feet with area: $$A \approx 1\,m^2$$

Power requirements: unaided, a man would need to run 30 m/s and generate 15 times as much muscle power (Glasheen & McMahon 1996)

Flotation devices required….

Mizugo Ninja, 12th century

Leonardo da Vinci

	$\rho g z A$	$\rho V dU/dt$	$\rho U^2 A$	$\sigma \nabla \cdot \underline{n} A$	$\underline{\nabla} \sigma A$
Surface slapping					
Rowing & walking					
Surface distortion					
Marangoni propulsion					

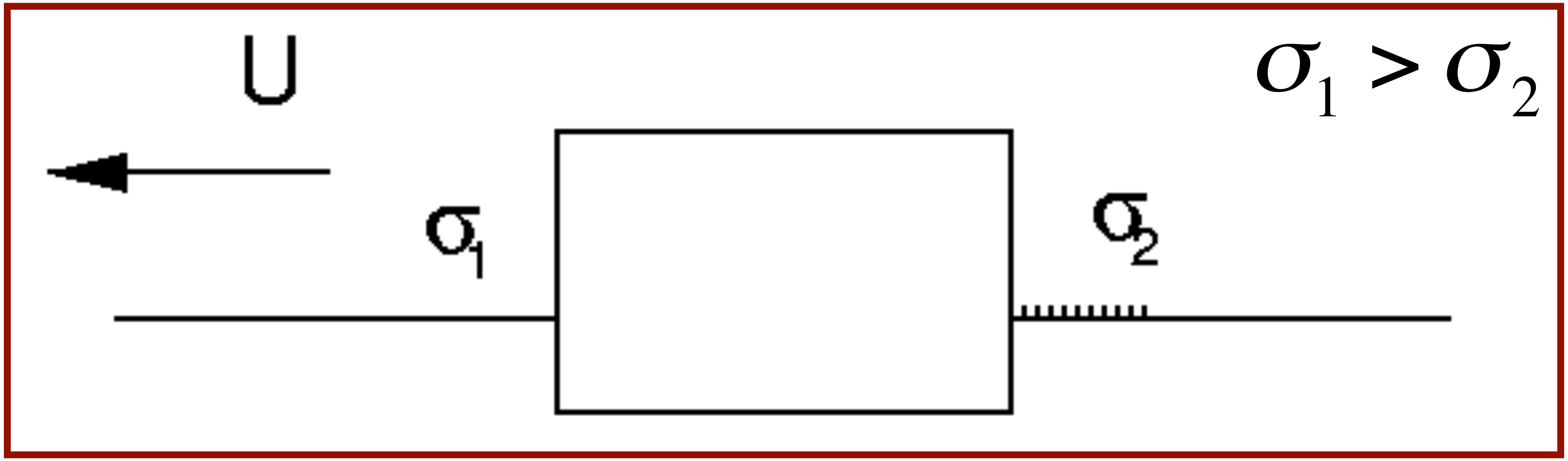
quasi – static propulsion

Quasi-static propulsion

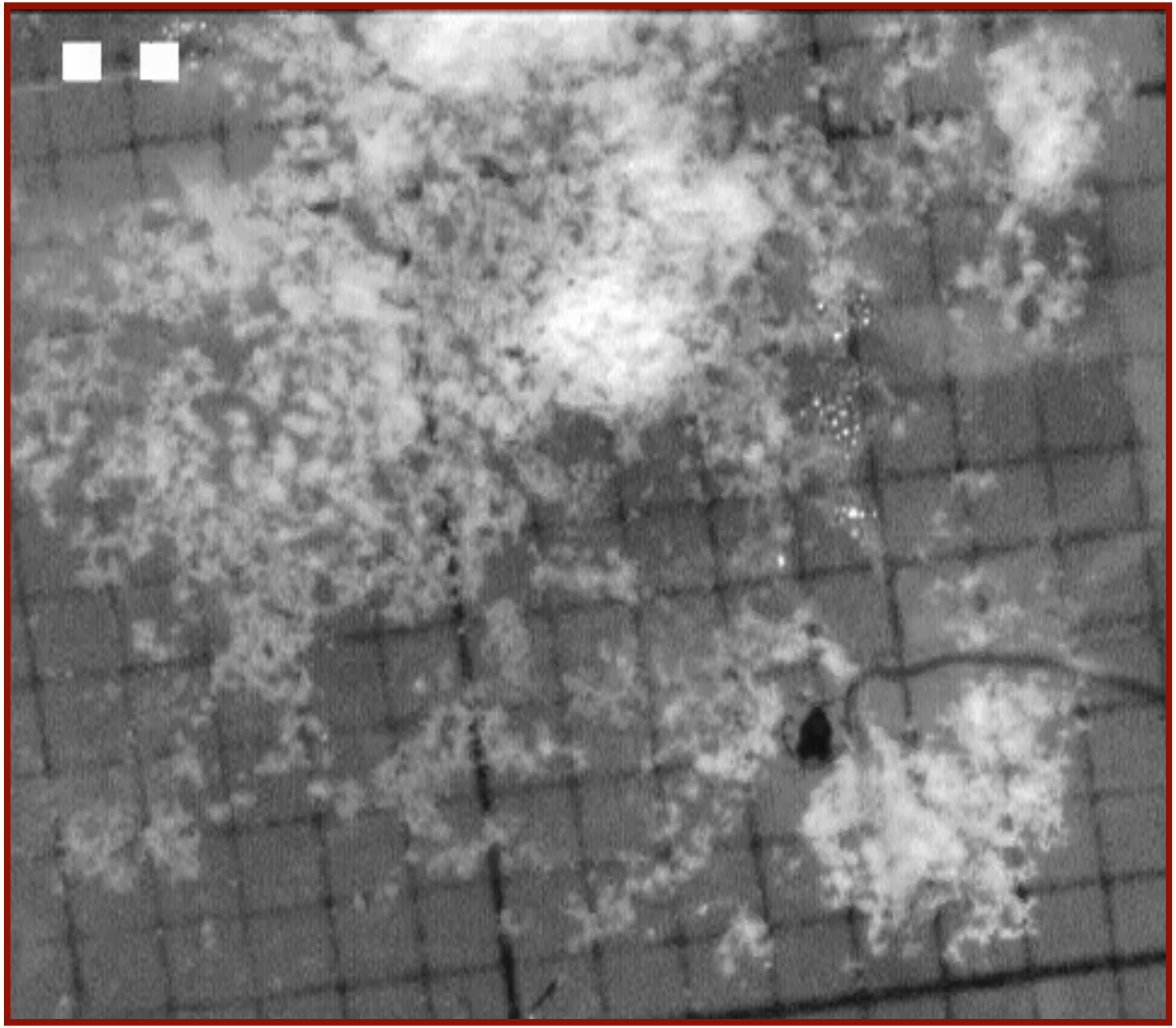
1. Marangoni propulsion
2. Meniscus-climbing

Marangoni propulsion

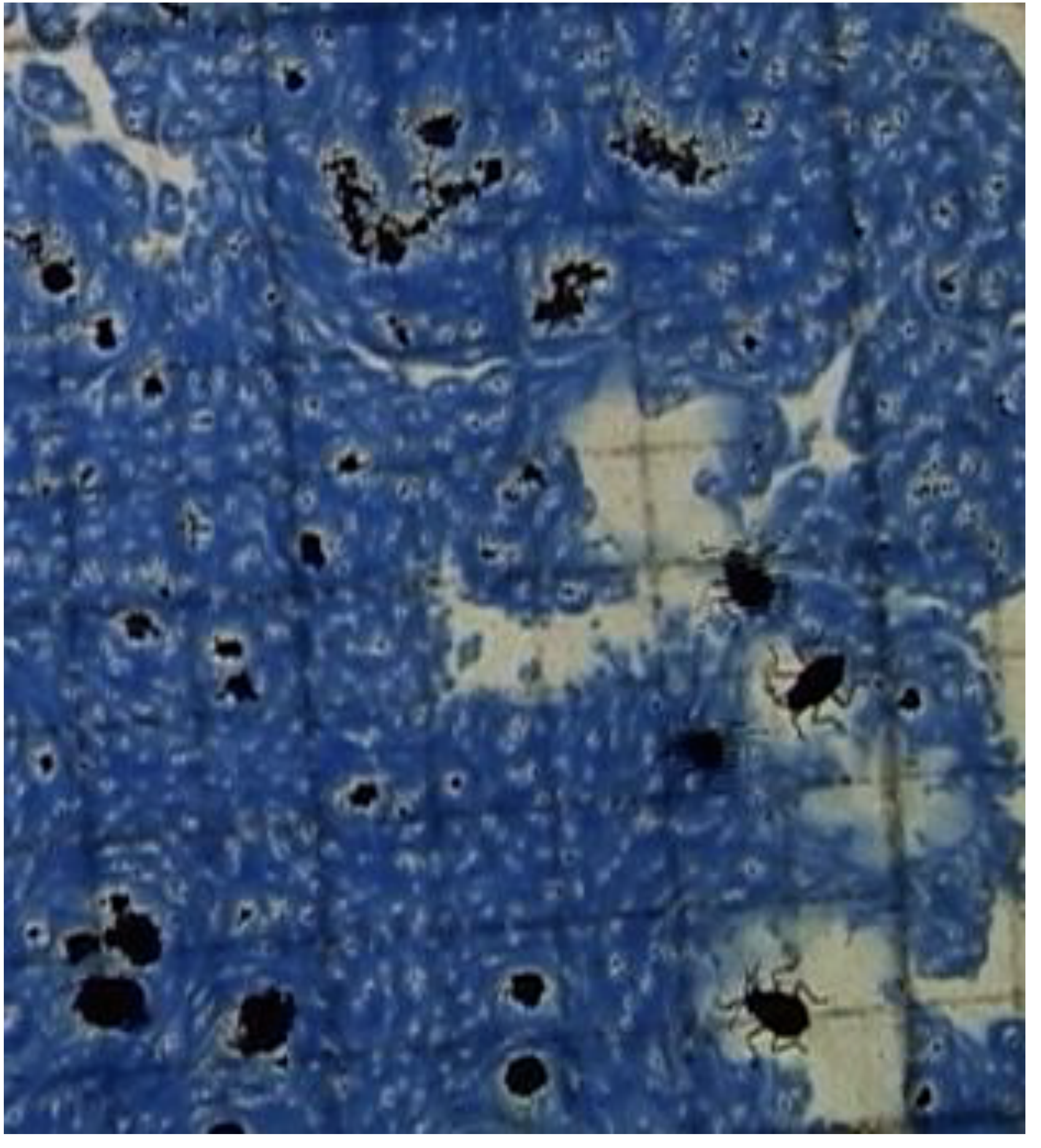
- lateral force generated by surface tension gradient
- quasi-static propulsion
- transfer of chemical potential energy to kinetic energy



Tangential stress, $\nabla\sigma$, may drive lateral motion.



Marangoni propulsion: insect uses lipid as fuel.

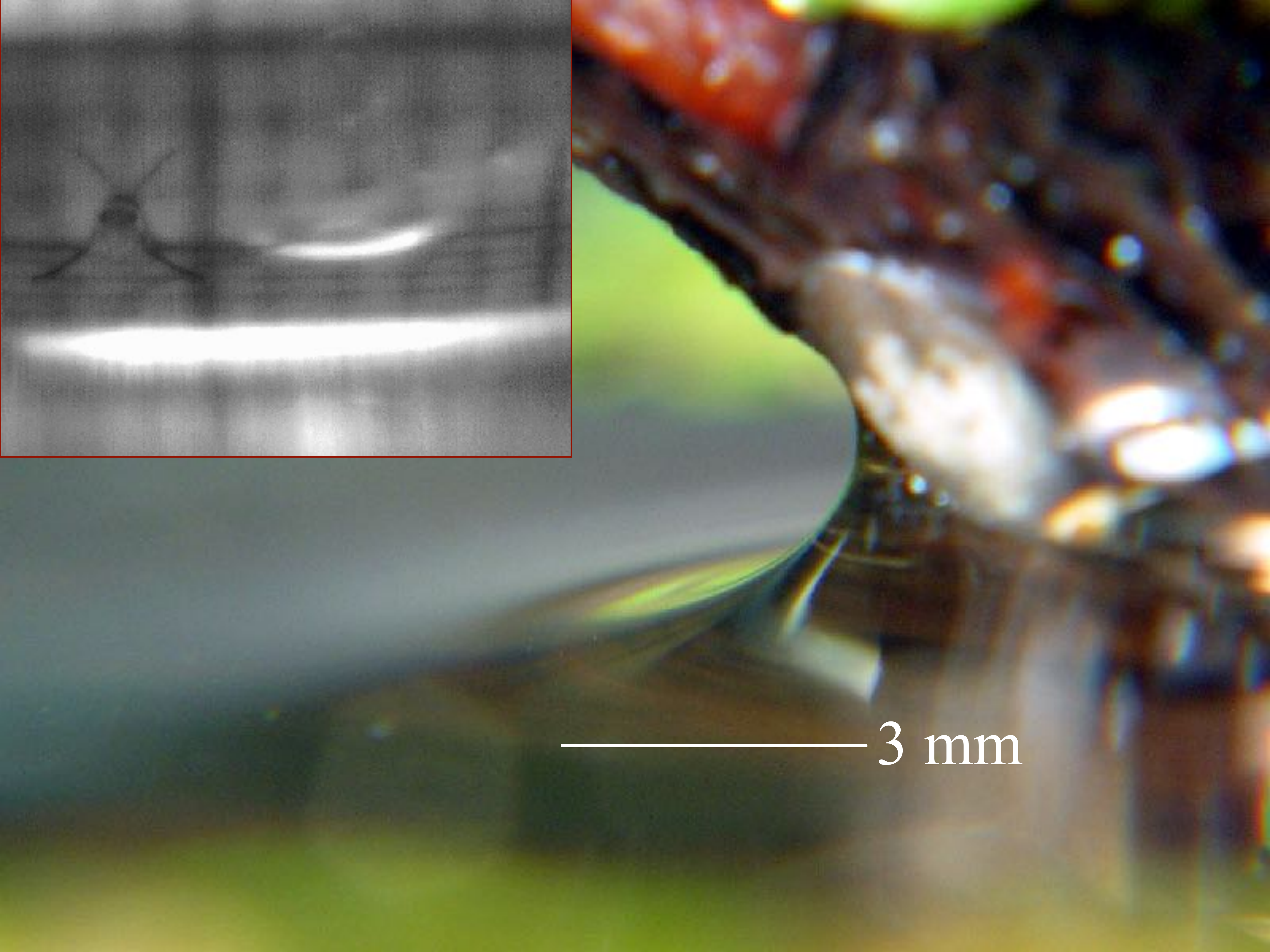


Microvelia clear dye from surface using Marangoni stresses.

	$\rho g z A$	$\rho V dU/dt$	$\rho U^2 A$	$\sigma \nabla \cdot \underline{n} A$	$\underline{\nabla} \sigma A$
Surface slapping					
Rowing & walking					
Surface distortion					
Marangoni propulsion					

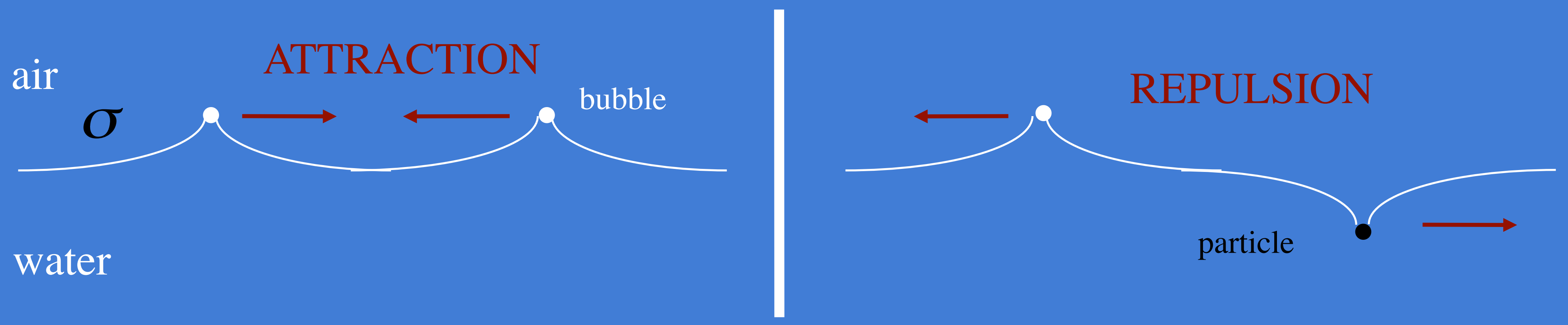
quasi – static propulsion

Meniscus-climbing


3 mm

Capillary forces: The Cheerios effect

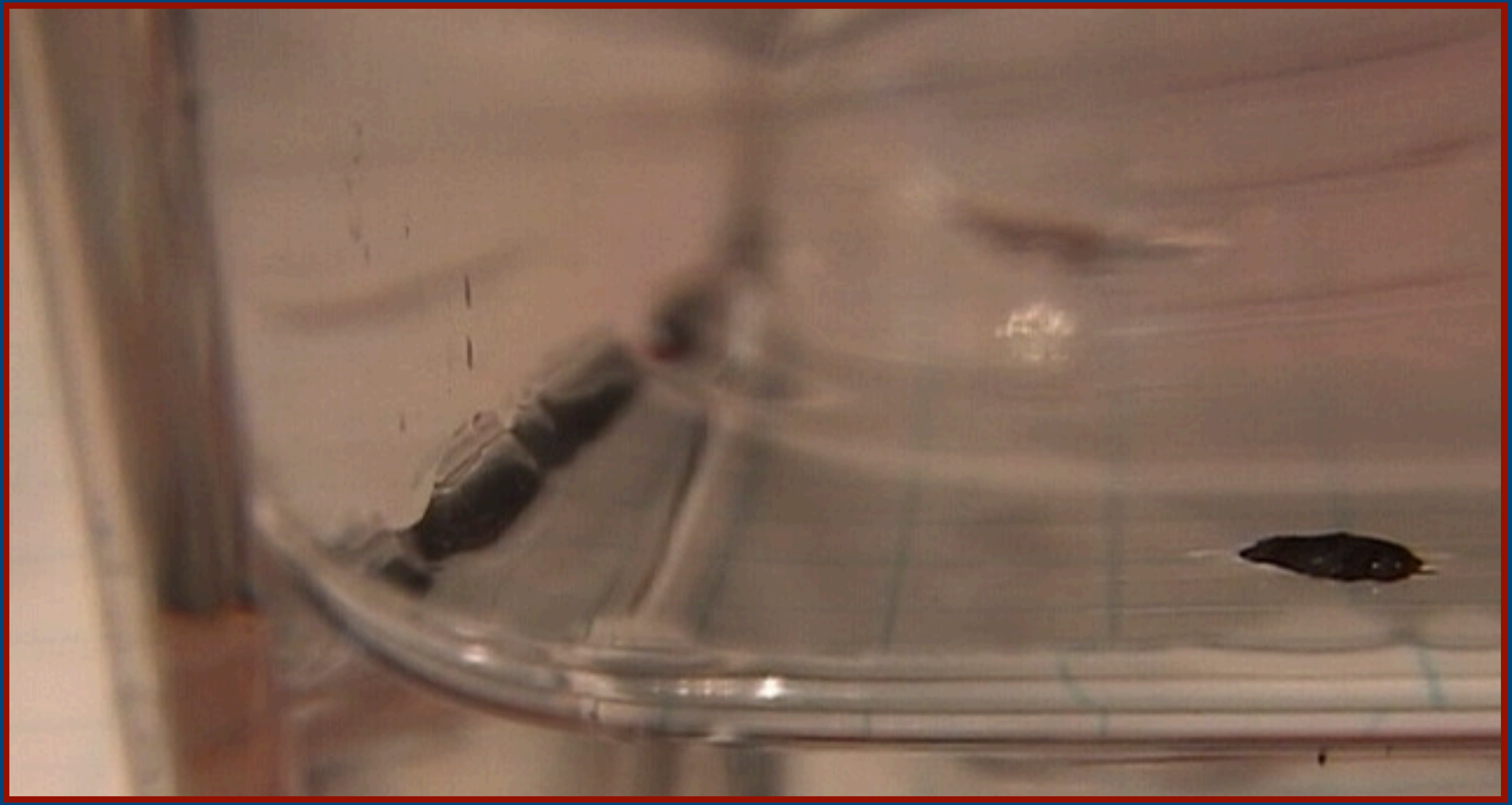
- exist between objects floating at a free surface
- attractive/repulsive for meniscii of the same/opposite sense



- explains the formation of bubble rafts in champagne
- explains the attraction of Cheerios in a bowl of milk
- used by small insects to move themselves along the free surface

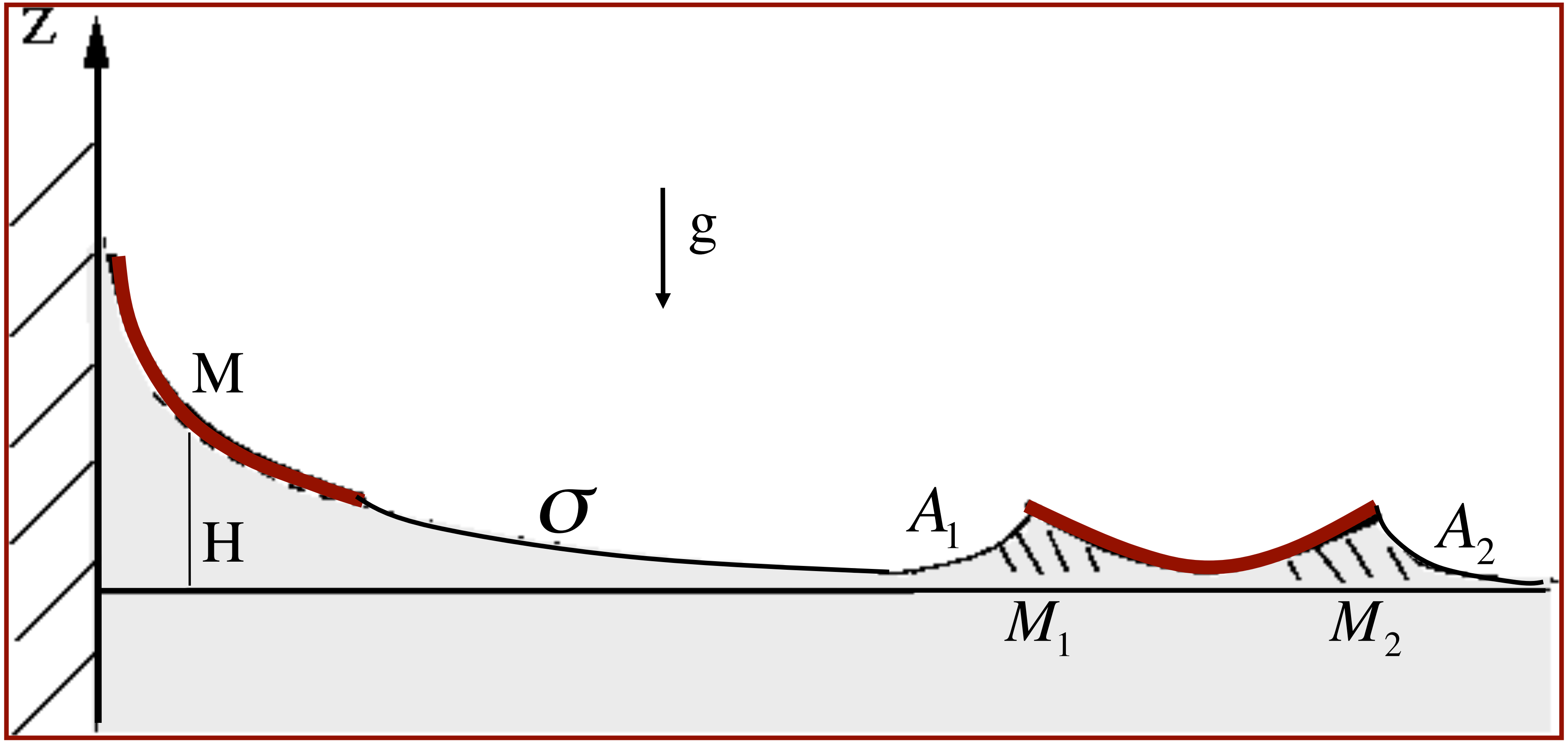
Meniscus climbing

Hu & Bush (2005)



- Anurida arches its back to match curvature of meniscus
- anomalous surface energy exceeds GPE associated with climb

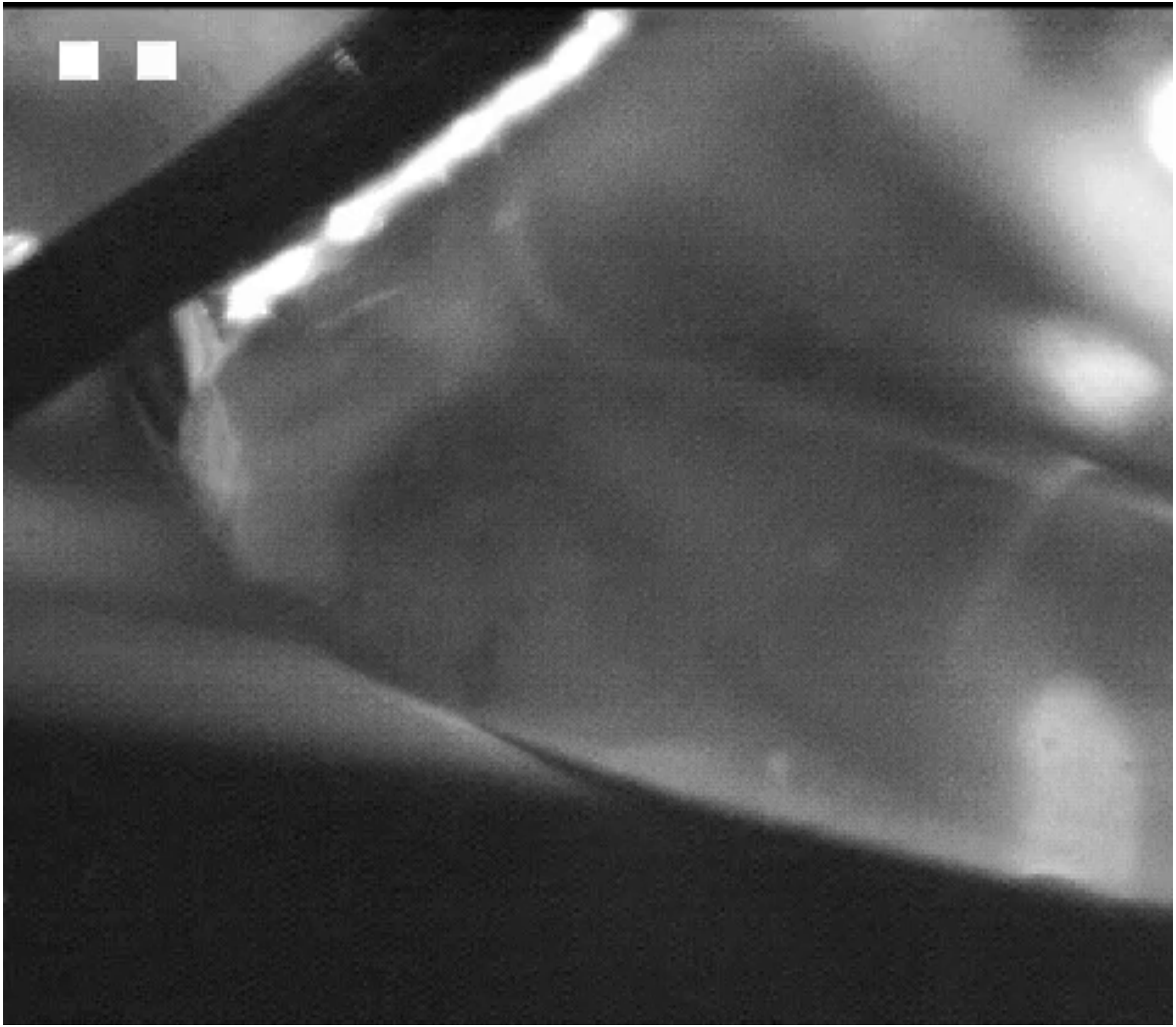
Meniscus-climbing: Energetics



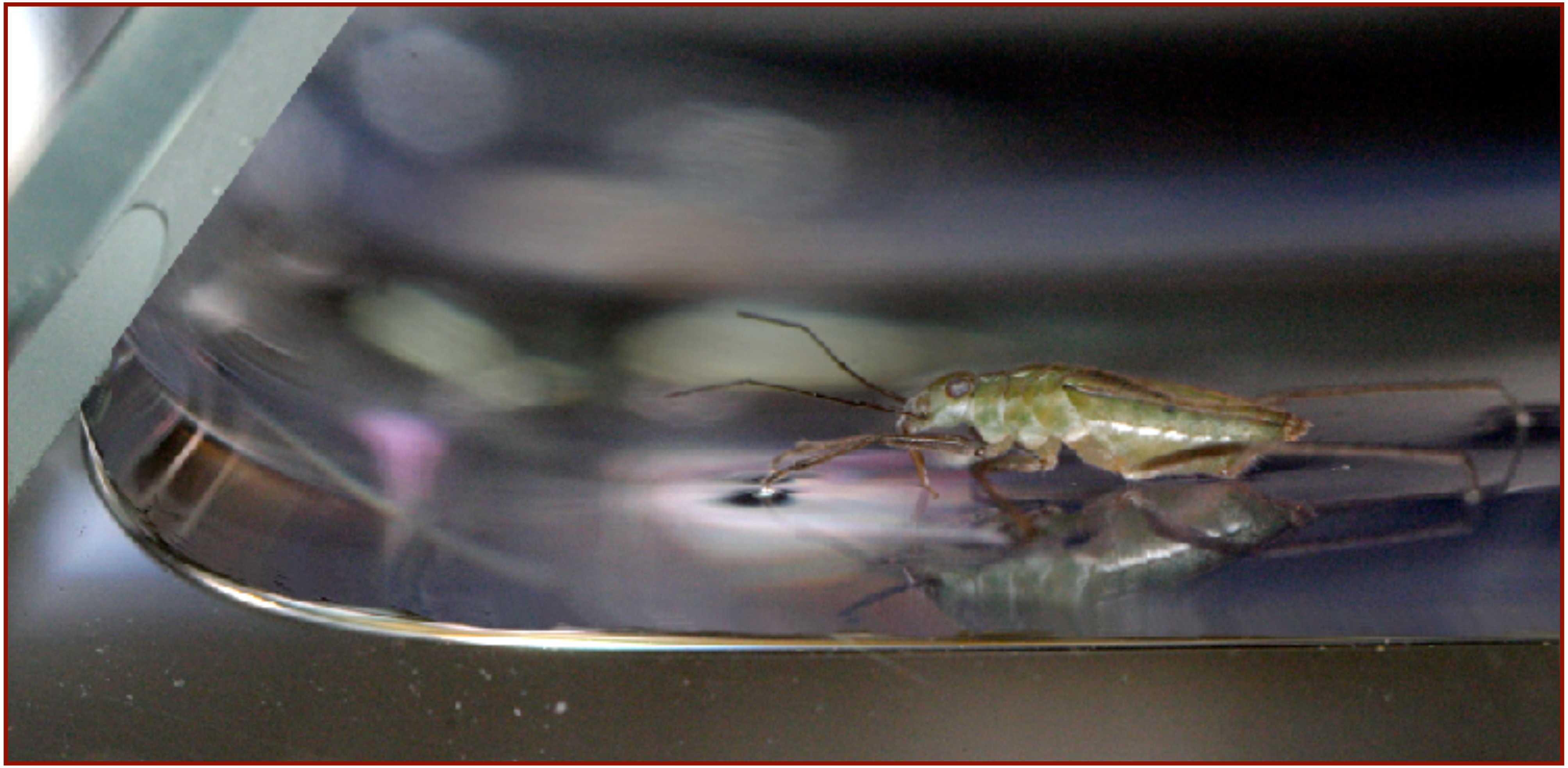
Body climbs provided total energy minimized:

$$\sigma(A_1 + A_2) + M_1 g h_1 + M_2 g h_2 > MgH$$

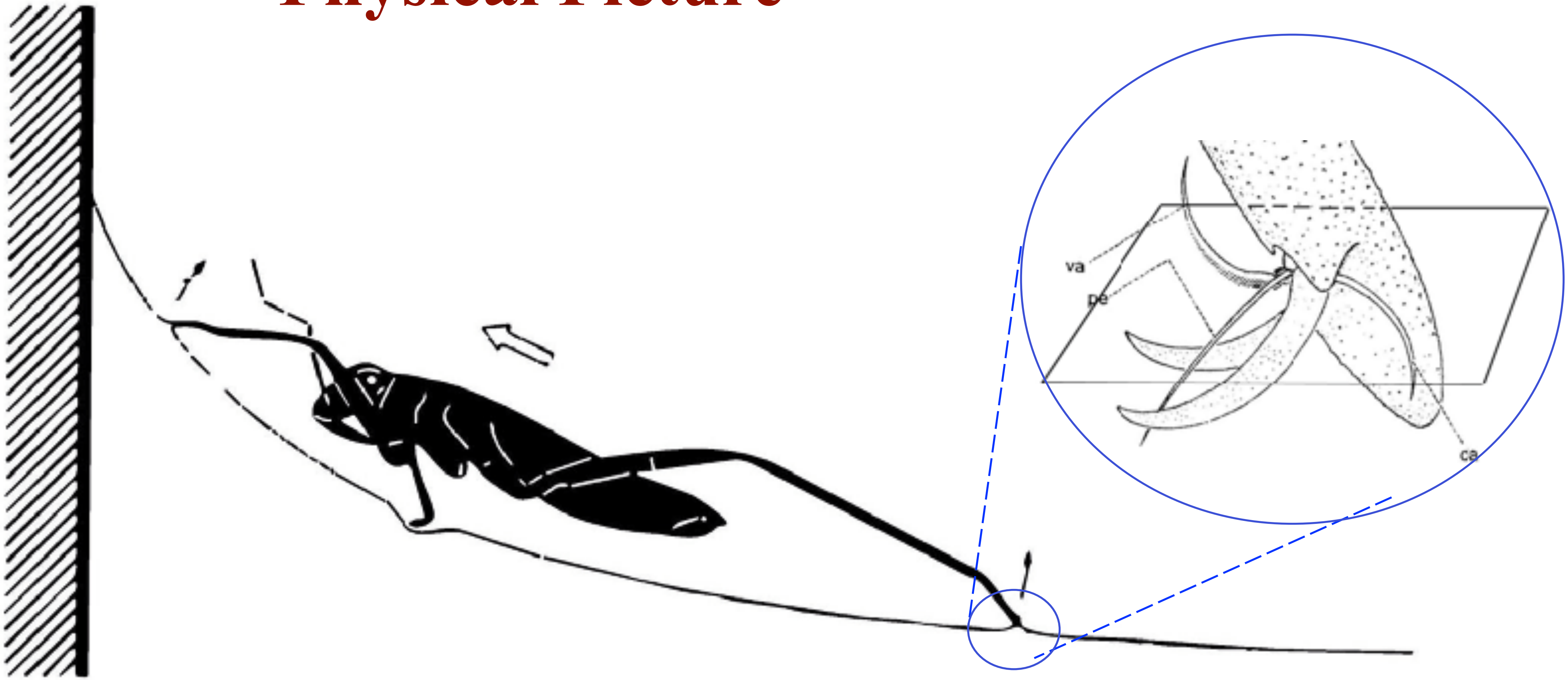
Meniscus-climbing for non-wetting insects



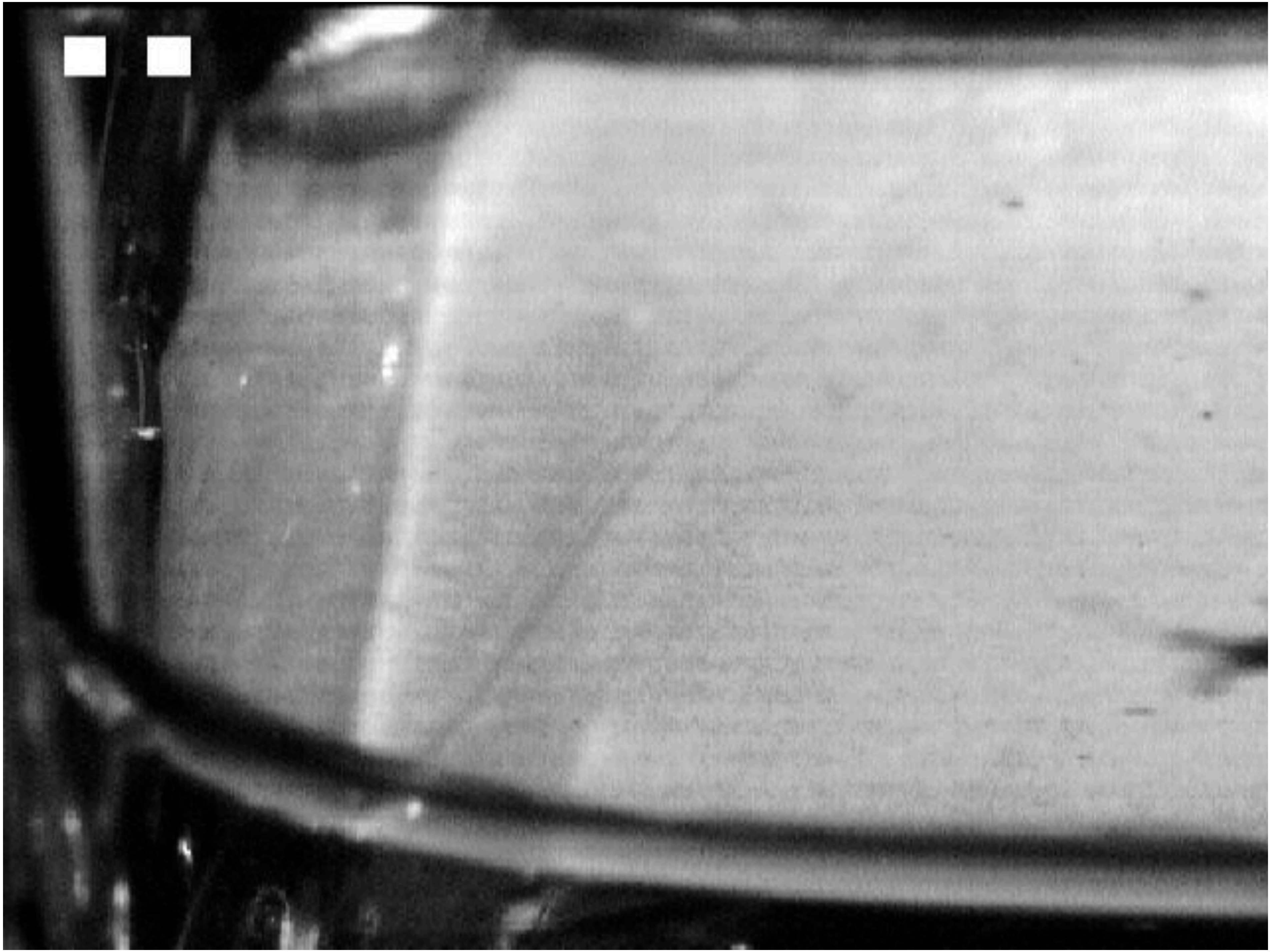
Microvelia



Physical Picture



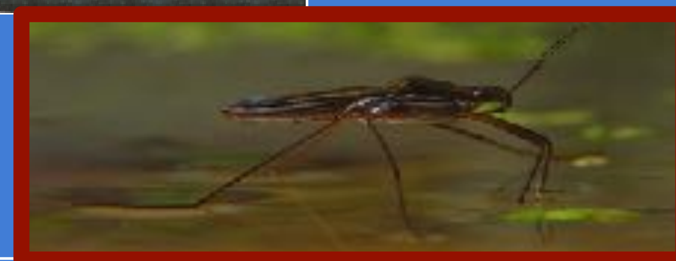
- exploit attraction between like-signed menisci
- pull up with front legs to generate lateral force
- pull up with rear legs to balance torque
- push down with middle legs to support weight




29 September 2005 | www.nature.com/nature | $10
THE INTERNATIONAL WEEKLY JOURNAL OF SCIENCE
nature
A SLIPPERY SLOPE
Meniscus-climbing insects scale the heights
INSIDE Surfaces and interfaces
nature insight
DARK MATTER
Lost and found in space
SELF REPLICATION
Robots get it right
PLANT GROWTH
Elusive gibberellin receptor
OCEAN ACIDIFICATION
Calcifying organisms hit by 2100
NATUREJOBS
Berlin's pulling power

Other uses for capillary attraction

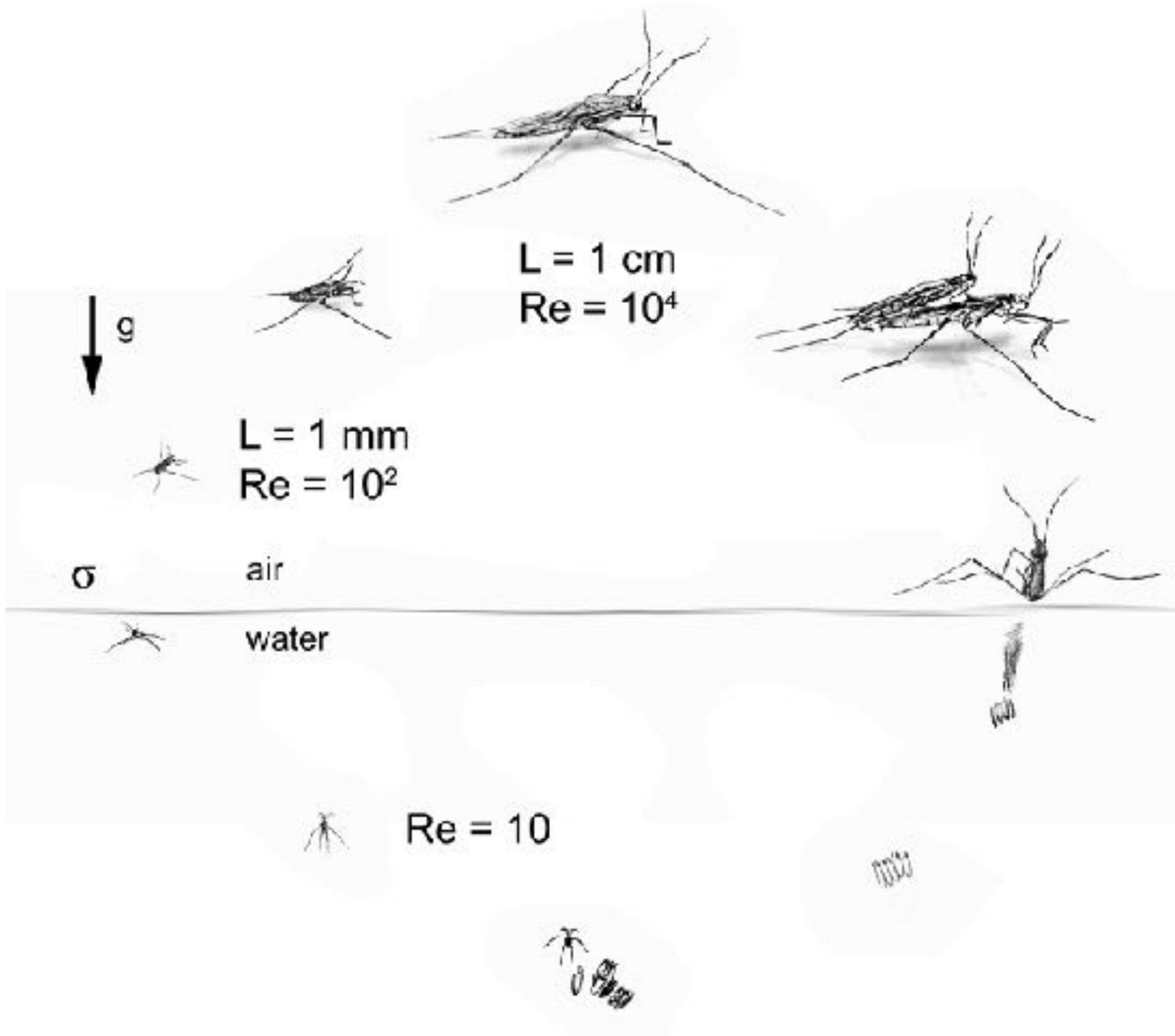
Anurida colony

	$\rho g z A$	$\rho V dU/dt$	$\rho U^2 A$	$\sigma \nabla \cdot \underline{n} A$	$\underline{\nabla} \sigma A$
Surface slapping					
Rowing & walking					
Surface distortion					
Marangoni propulsion					

The Water Strider (*Gerridae*)

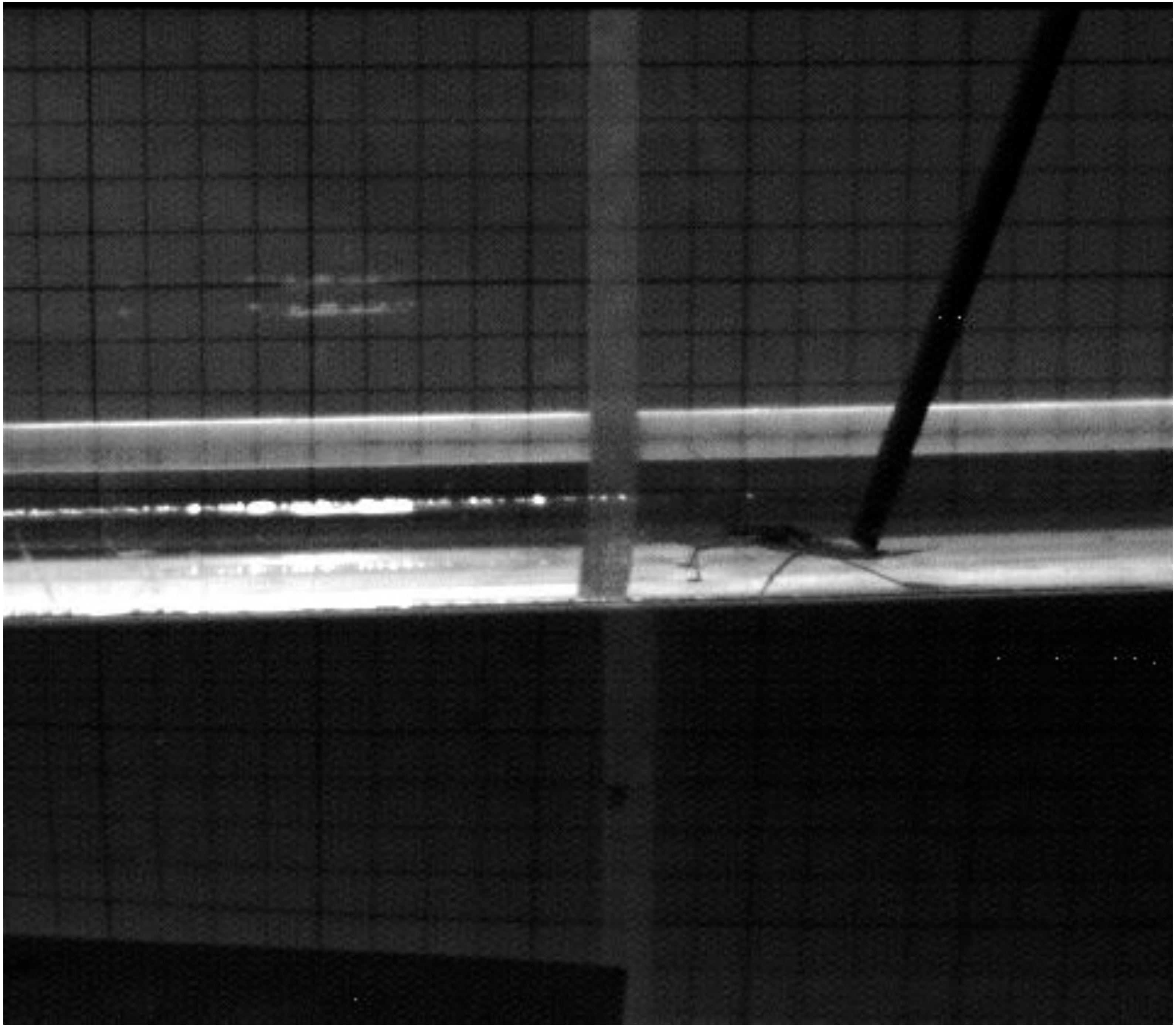
- weight: 1-10 dynes
- contact leg length ~ 1 cm; perimeter ~ 5 cm
- hairy non-wetting legs

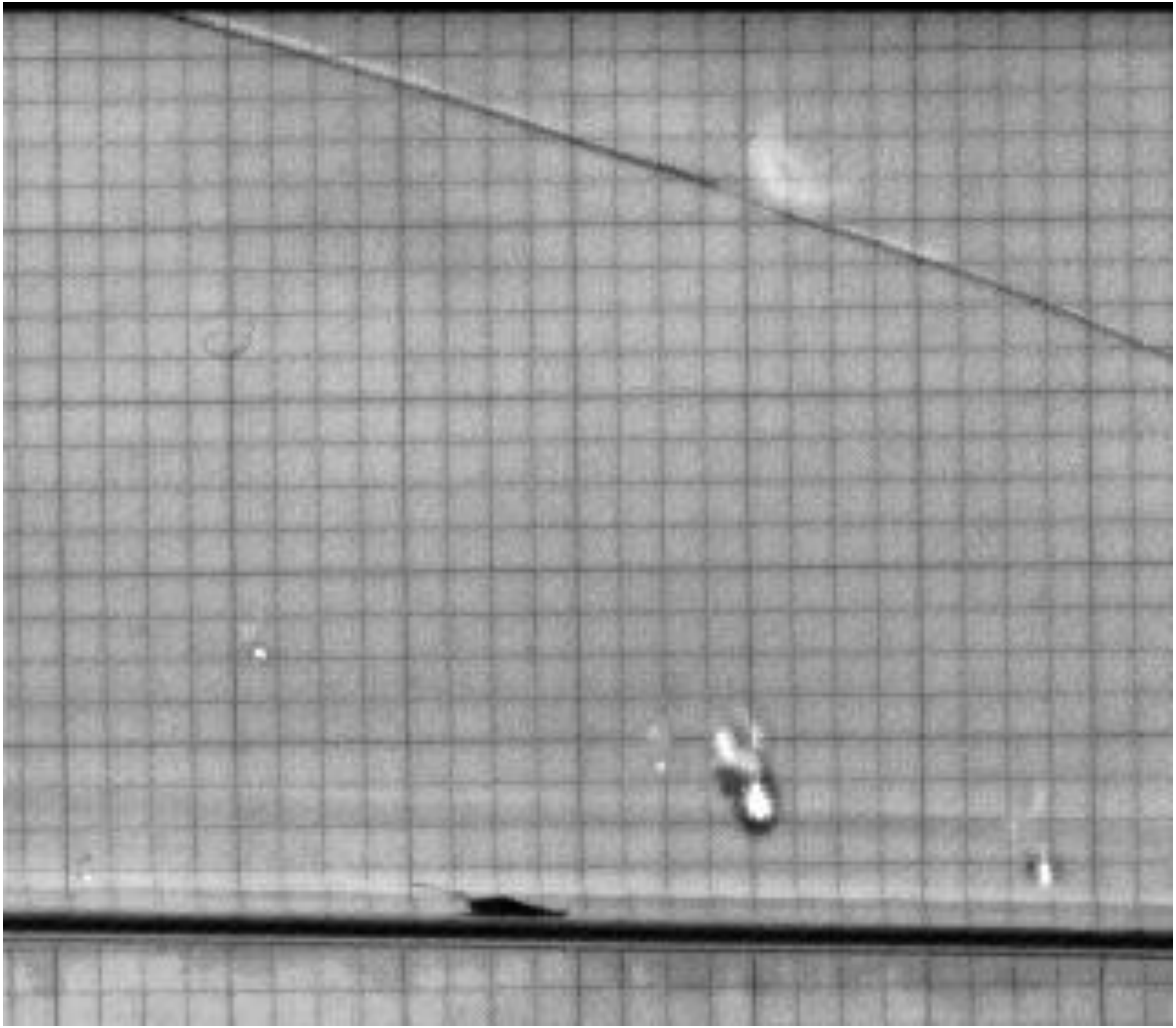
The life cycle

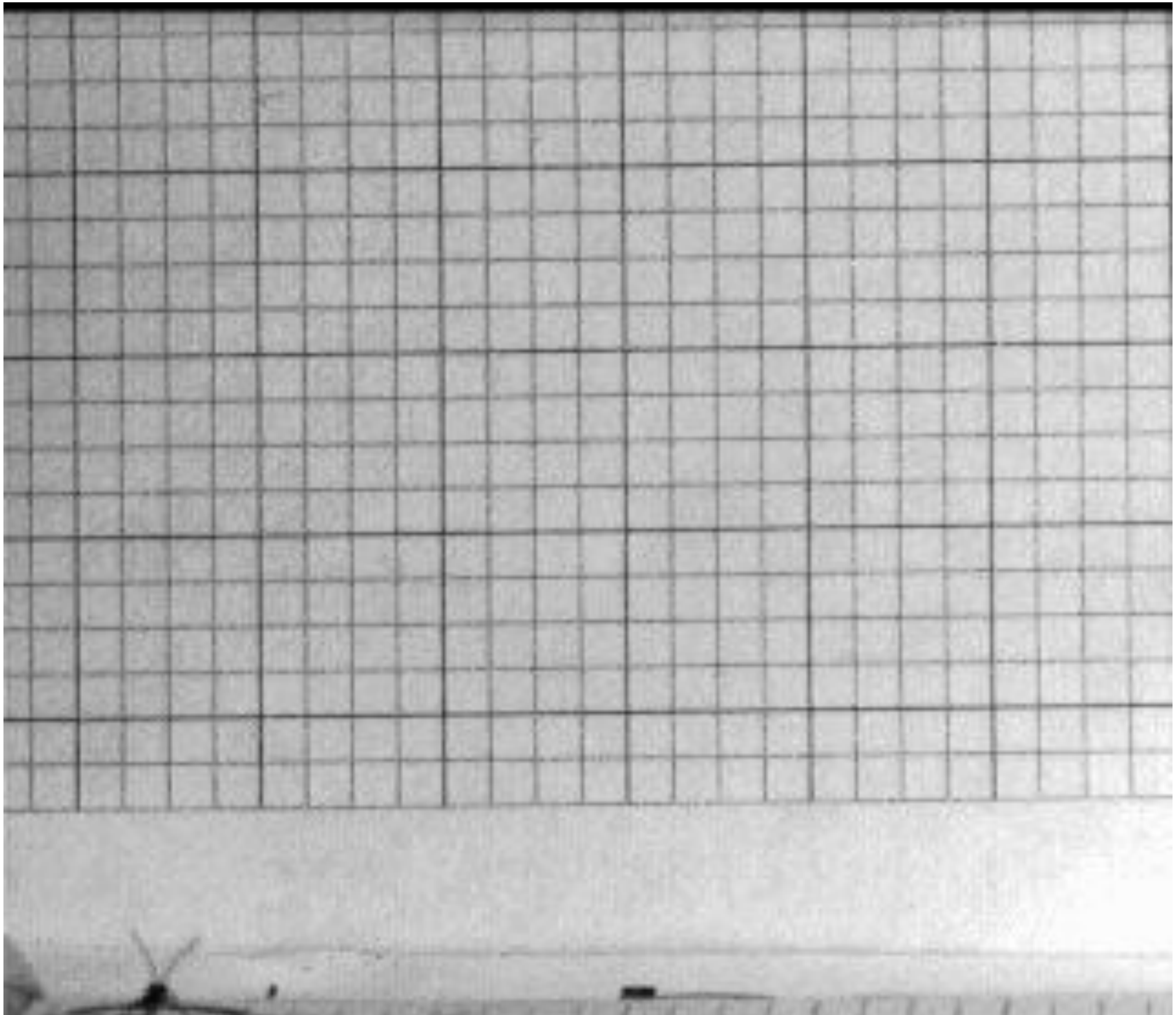


Crossing the interface

[redacted]

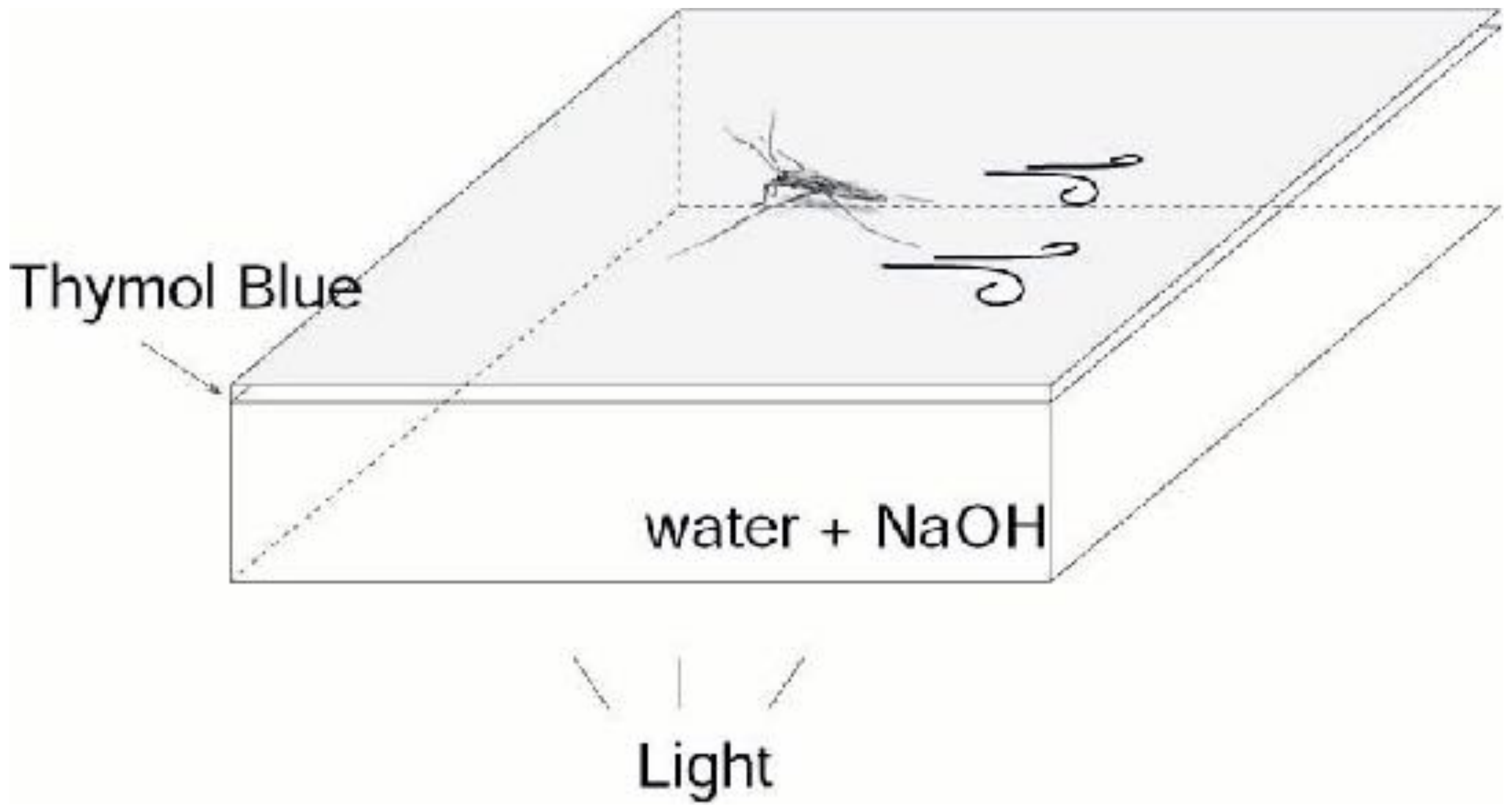






Experimental Technique

A. Insects, spiders maintained in lab
B. High-speed cinematography
C. Particle tracking (Kalliroscope, pliolite)
D. Dye studies (food colouring, thymol blue)



Observations

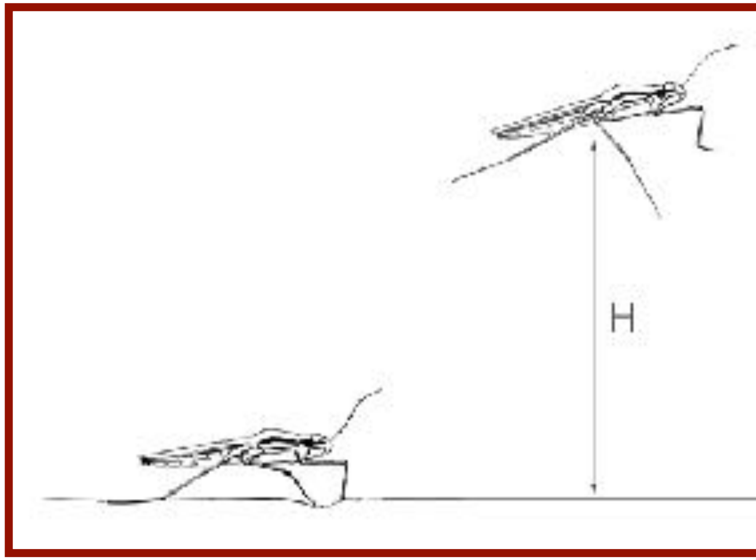
- peak speed ~ 150 cm/s;   momentum ~ 1 g cm/s
- stroke Reynolds number: $\mathrm{Re} = VL_2/\nu \sim 1000$

Propulsive force: F ~ 50 dynes

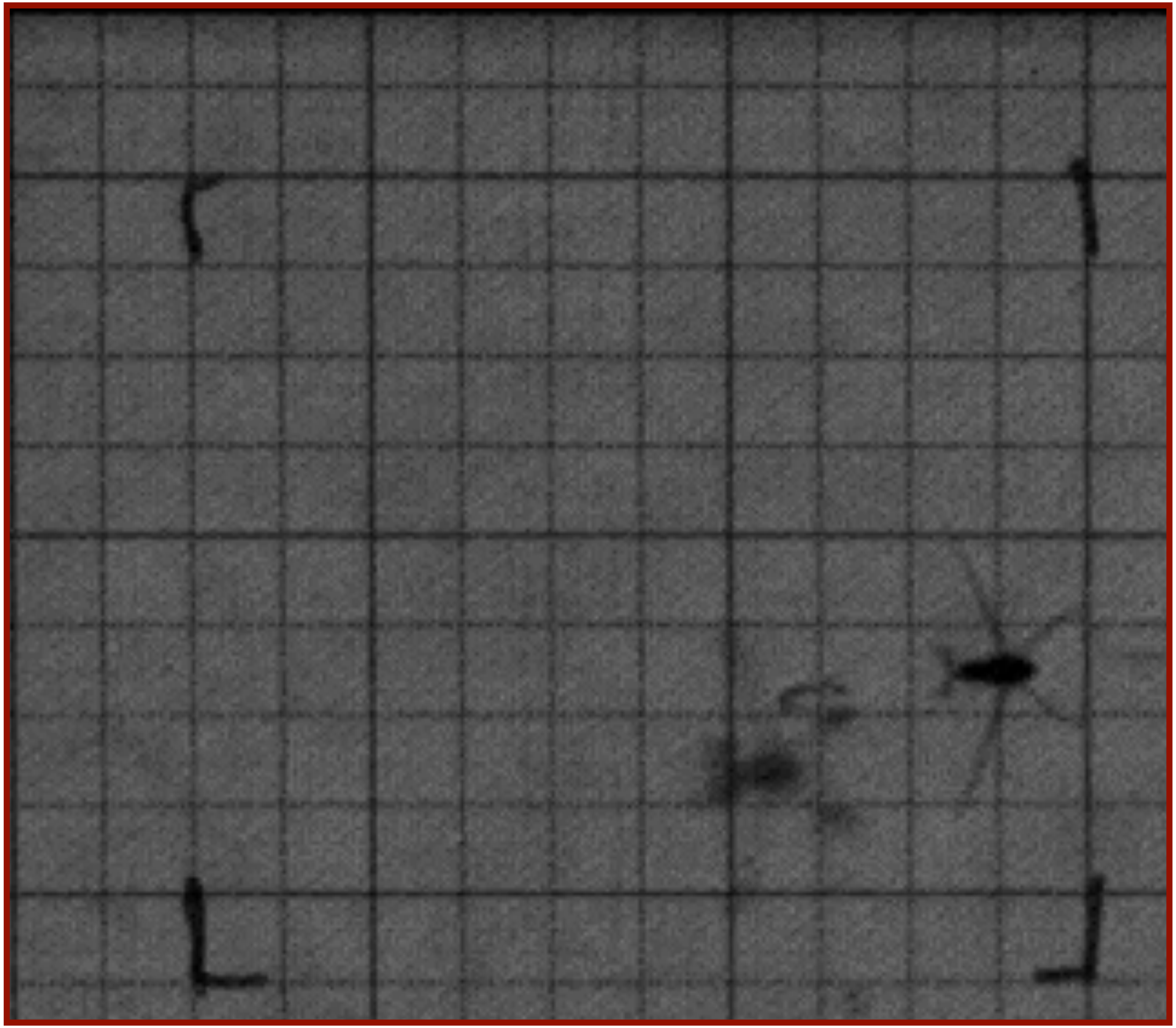
Lines of evidence:

Acceleration: F = M dv/dt

Leap height: F = MgH/L



⟹ strider ideally tuned to life at the interface



Denny's Paradox (1993)

- infant water striders cannot swim:

> " Exactly how they manage to propel themselves across the water surface remains a mystery."

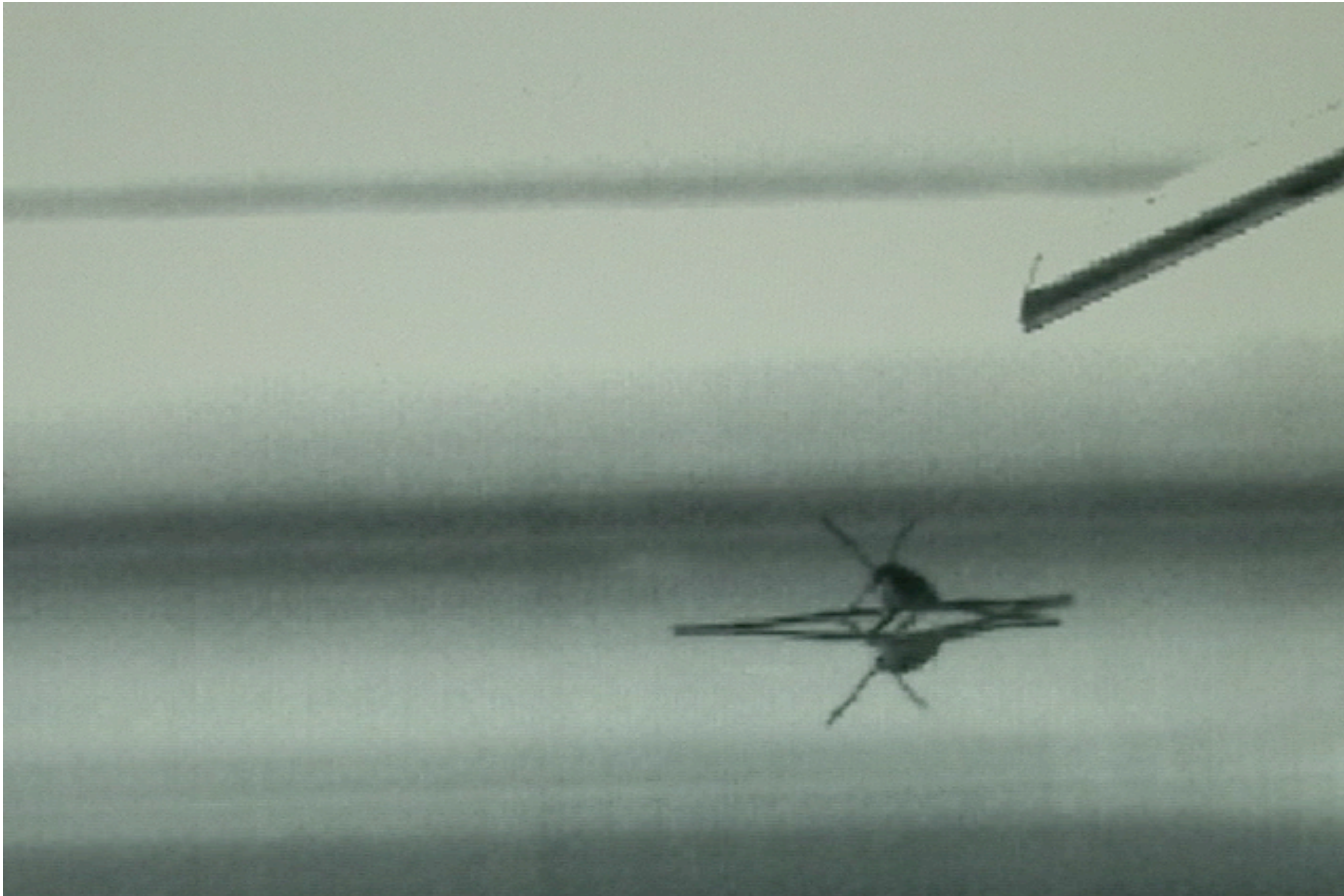
Reasoning

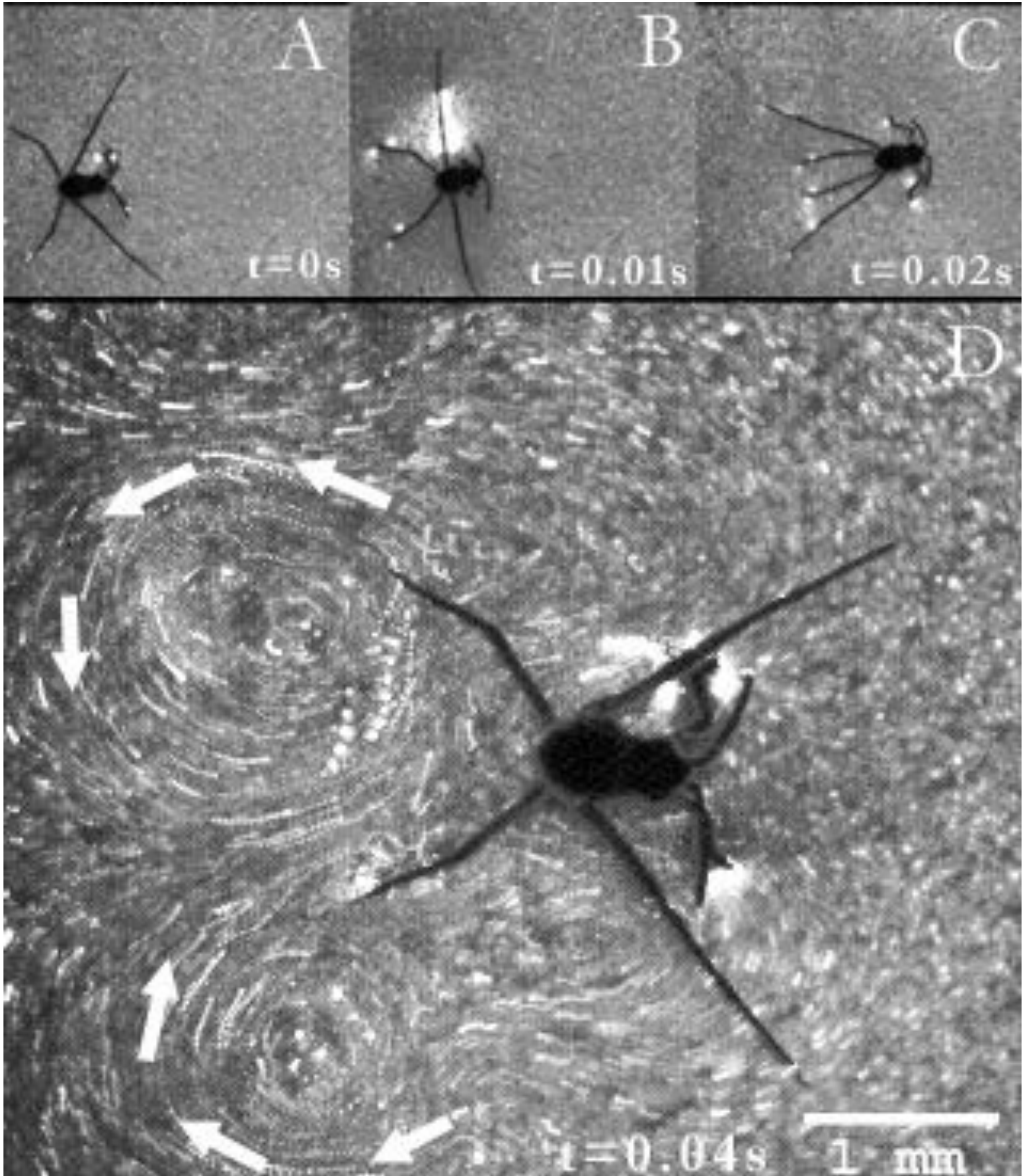
- assumed momentum transfer exclusively in waves

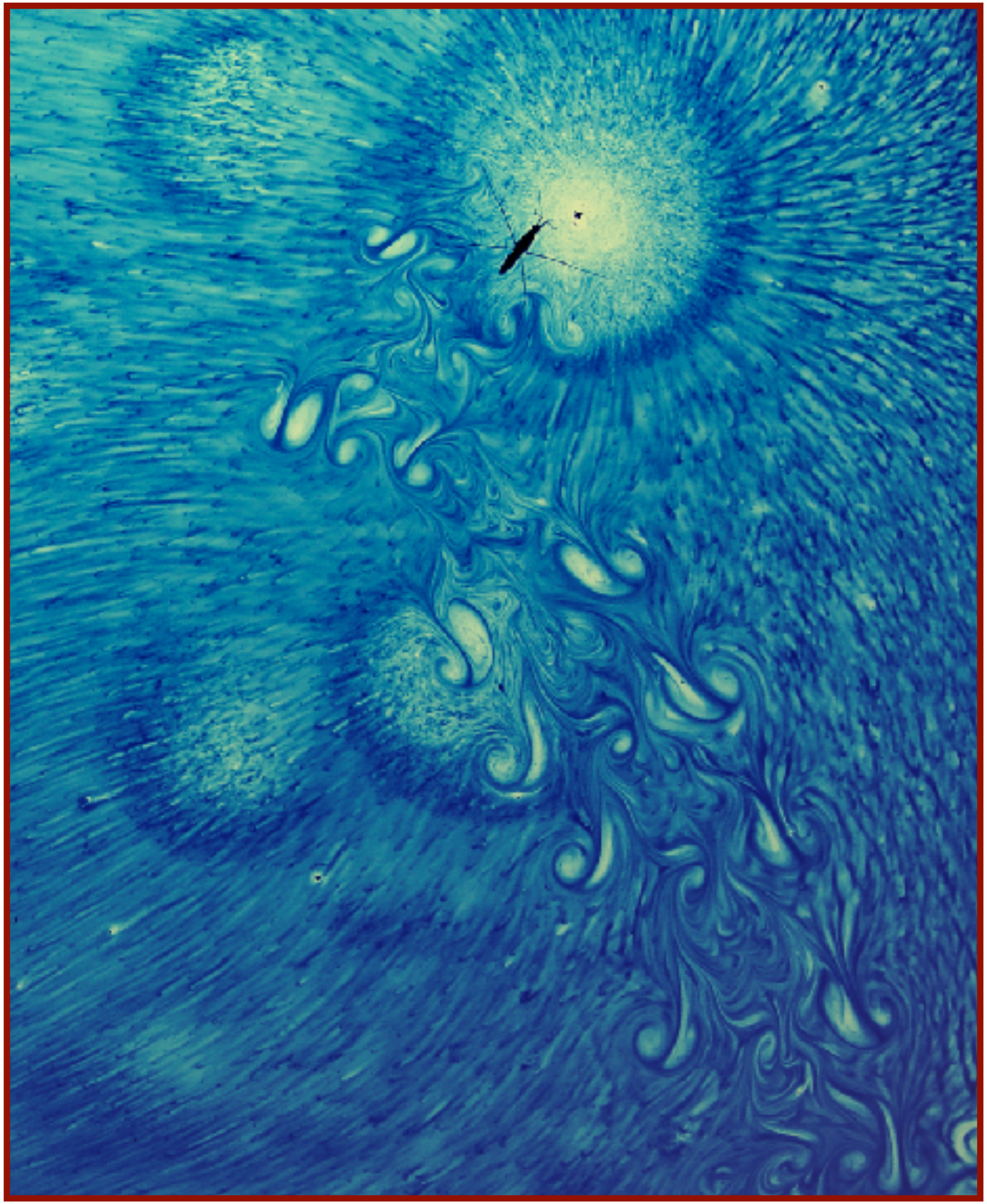
- to generate waves, legs must exceed

$$c_m = (4g\sigma/\rho)^{1/4} = 23 \text{ cm/s}$$

- infant leg speed $< c_m$




A
t=0s
B
t=0.01s
C
t=0.02s
D
t=0.04s
1 mm




7 August 2003

International weekly journal of science

nature

30.4
No Nature
no impact

www.nature.com/nature

Walking on water

The physics of water strider motion

Accelerated vaccine
Targeting Ebola virus

Early Solar System
Comets change their story

Particle physics
Whatever happened to antimatter?

naturejobs Chicago for energy

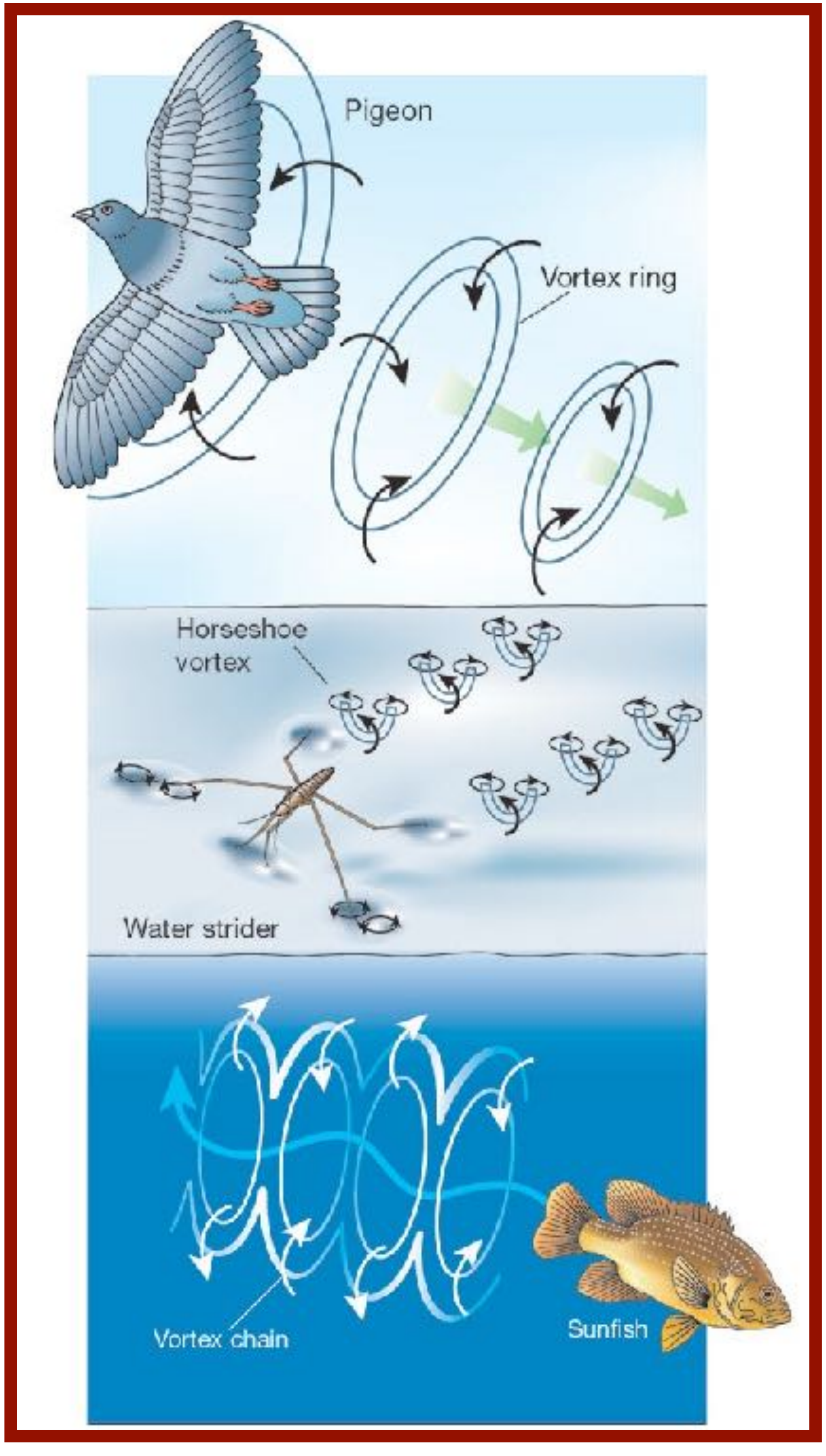
Conclusions

- Denny's Paradox resolved
- momentum transferred principally in vortices, not waves
- striders row, using their legs as oars, menisci as blades

Flying

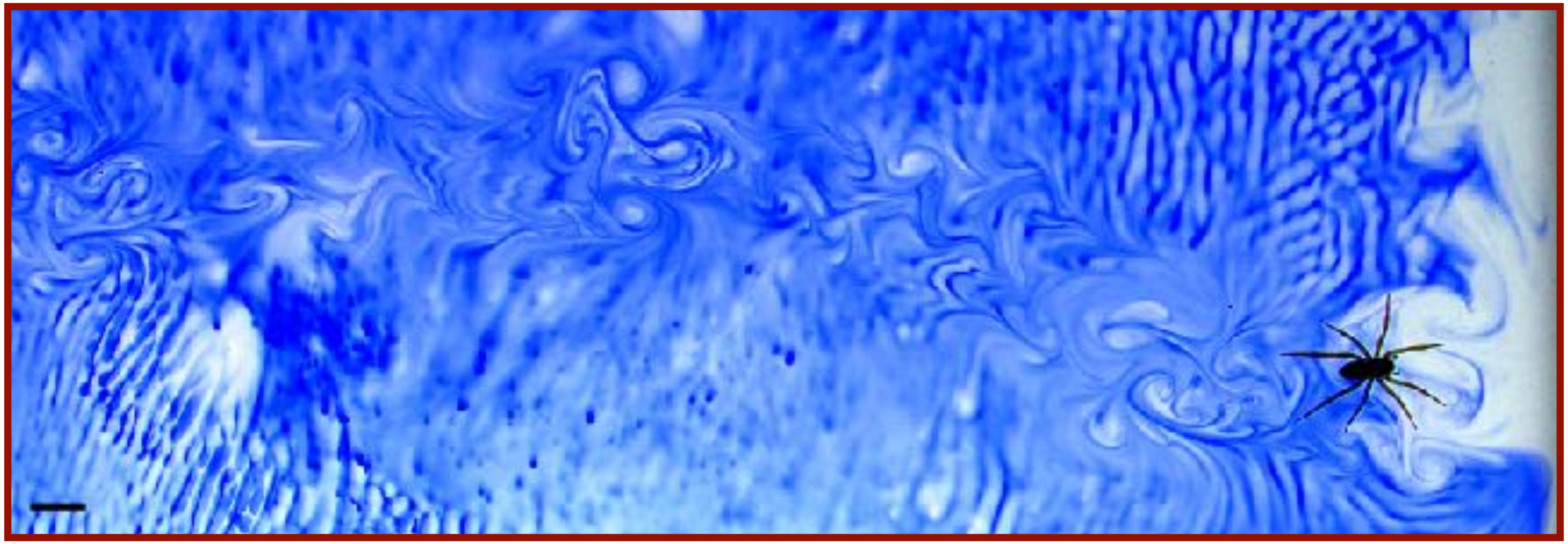
Rowing

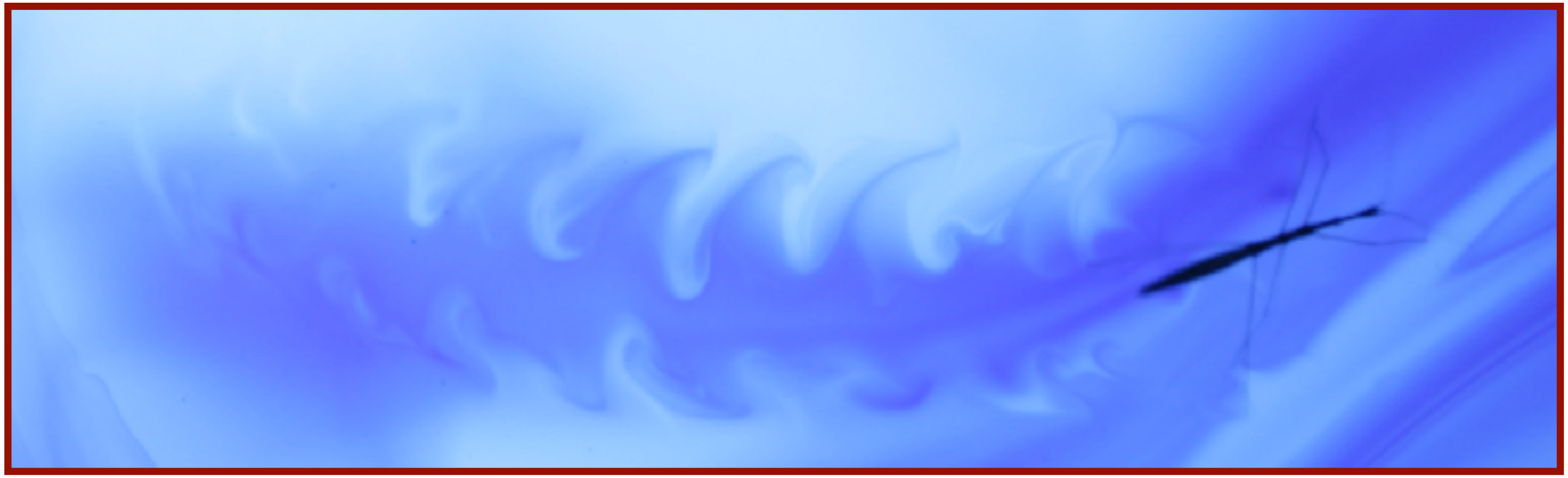
Swimming



Generic Physical Picture

- water-walking creatures propel themselves forward through momentum transfer via subsurface vortices



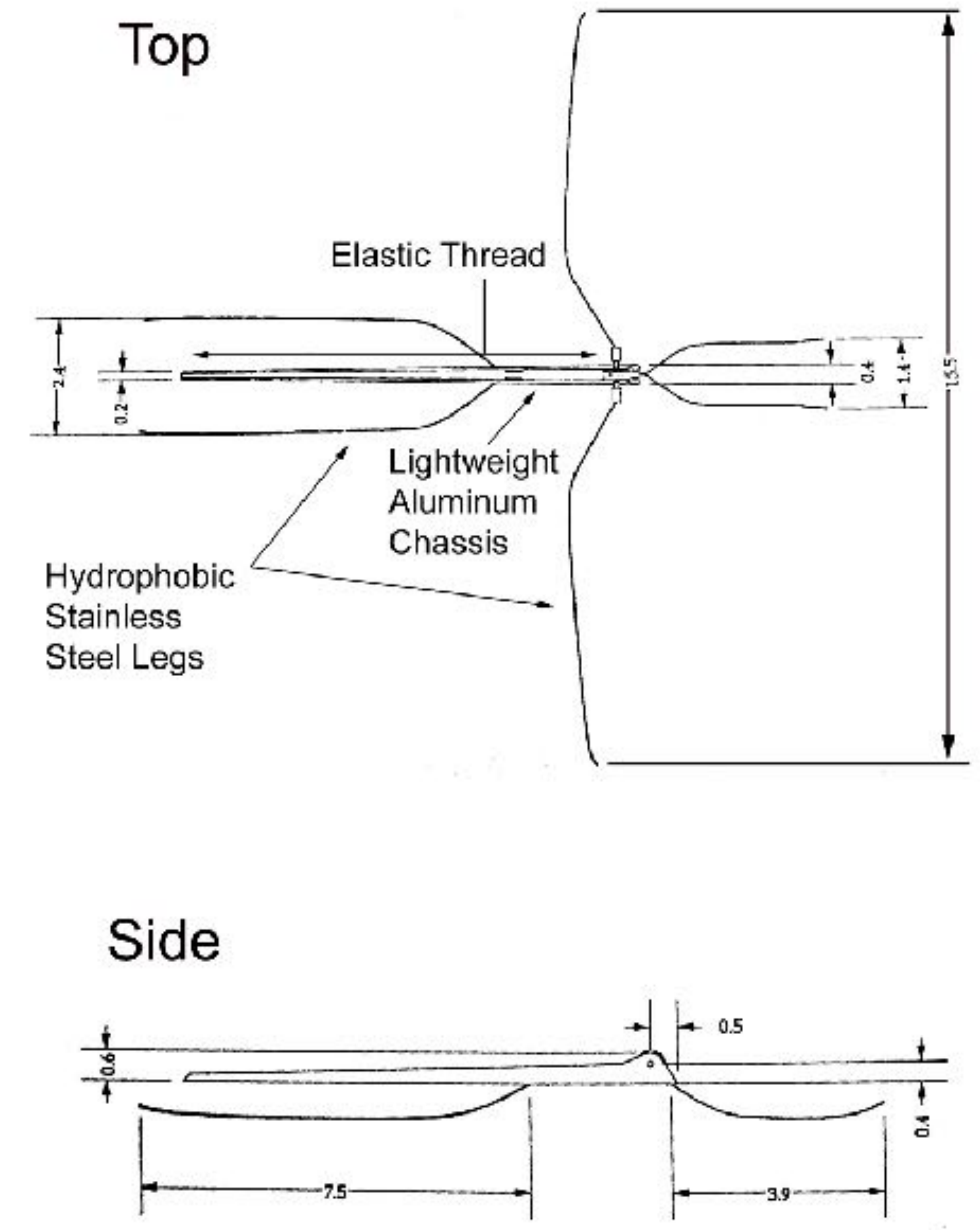


ROBOSTRIDER

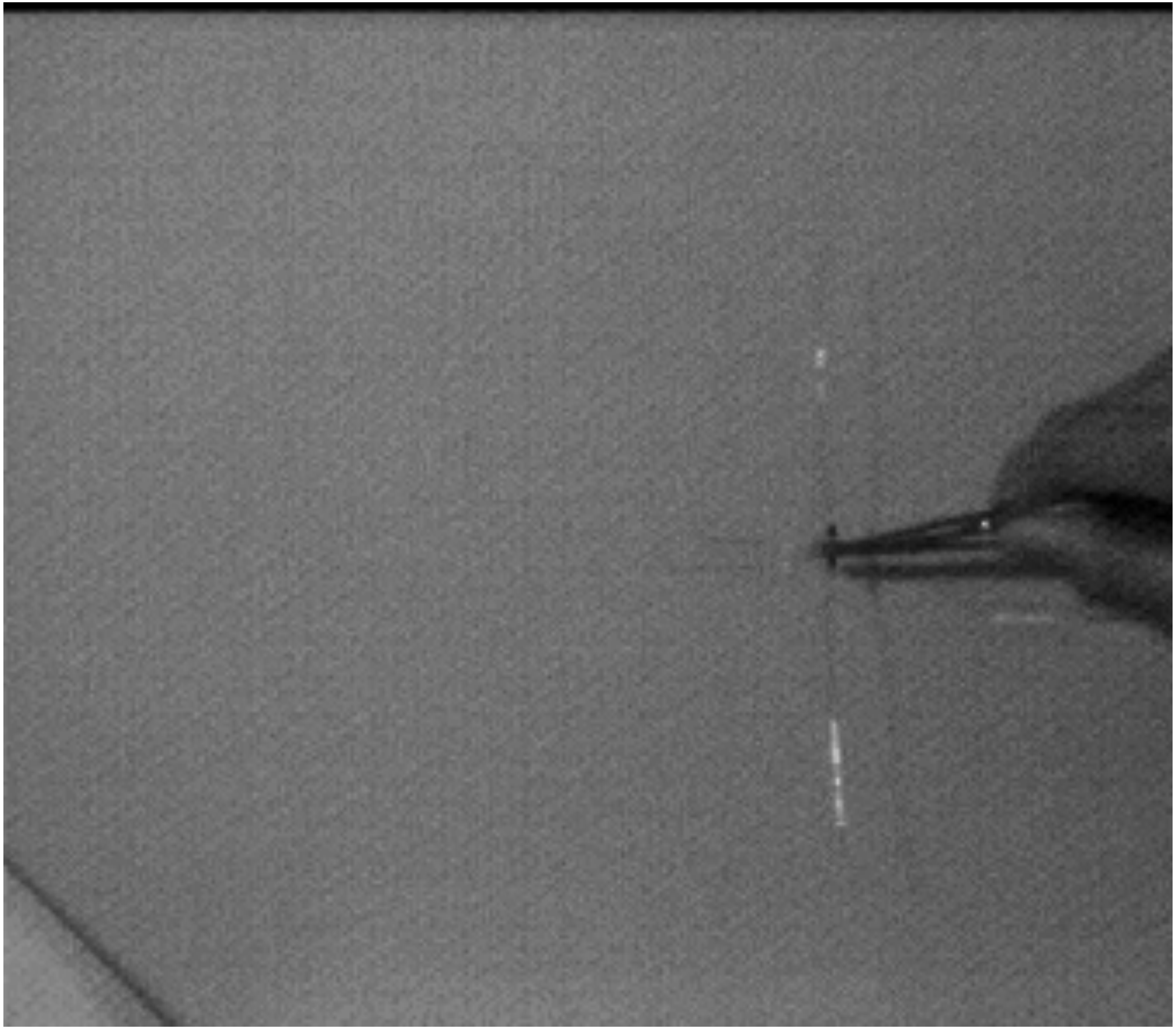
Goal: to design and construct the first mechanical water walker

Design Criteria

1. Non-wetting legs
2. $M_c < 1$: weight supported by σ
3. Force/length on driving legs $< 2\sigma$
4. Form consistent with natural counterpart



Top
Elastic Thread
2.4
0.2
0.4
1.4
15.5
Lightweight
Aluminum
Chassis
Hydrophobic
Stainless
Steel Legs
Side
0.5
0.6
0.4
7.5
3.9



NanoRobotics Lab @ Carnegie Mellon

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Water Walker
A miniature water strider robot

Goal: To develop a microrobot that can manouver on water with power efficiency and agility.

Approach: To understand the physics of water striders to model their characteristics of floating on the surface of water. We are using micro-actuators to simulate water striders movements. We are also investigating different materials to improve the robot's ability to float on water.

Benefits: Water strider robots will be small and relatively efficient. Because it is on the surface of water and light, the robot will be highly agile and can reach inaccessible areas for many different applications.

Videos:
Video 1: Water strider robot moving across the surface

Members: Steve Suhr, Sang Jun Lee, Yun Seong Song, Metin Sitti

Publicity:
Associated Press Article featured in: Forbes.com, Yahoo News, and Wired.

In Turkish - MSNBC, e-kolay, sabah.com, Milliyet.com.

[Moving Water strider robot prototype]

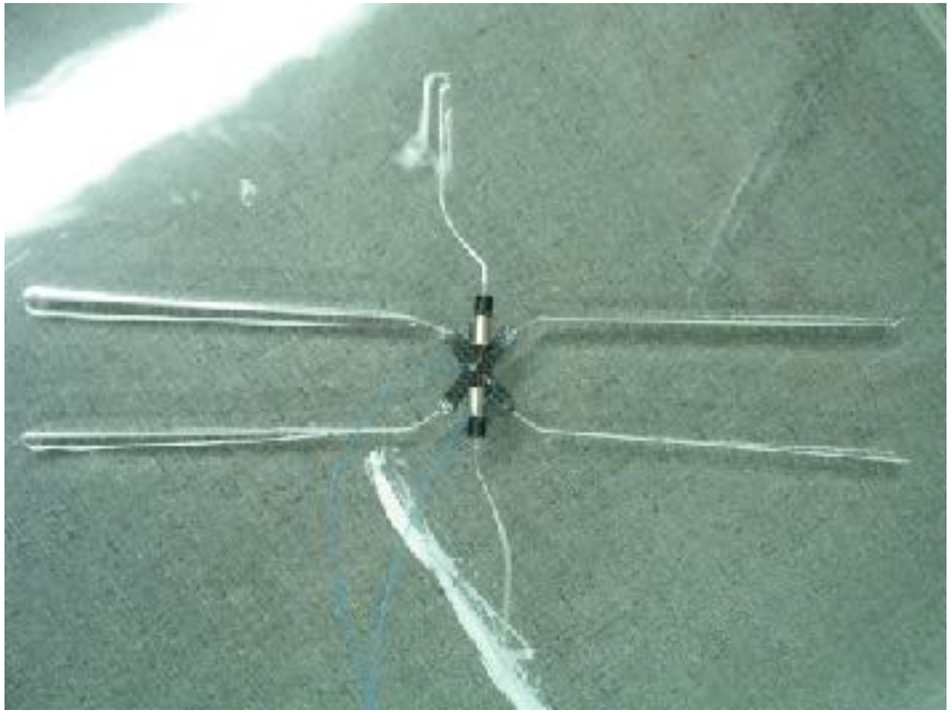
[Moving Water strider robot prototype]

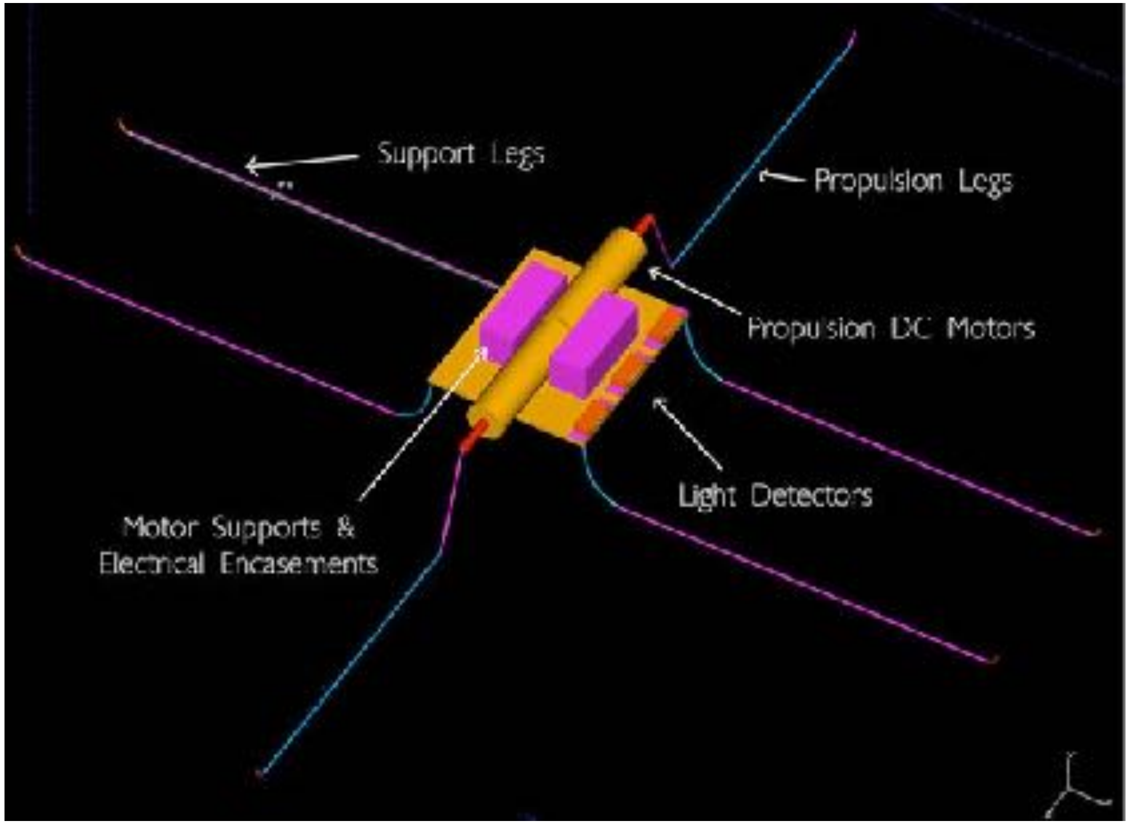
[Conceptual 3-D CAD model of the water strider robot. A, B, C and D are the supporting legs; E and F are the actuating legs; G is the body with sensors, power sources and a wireless communication module; H is the middle actuator; and I and J are the right/left actuators.]

The return of Robostrider

by R. Brasso & collaborators

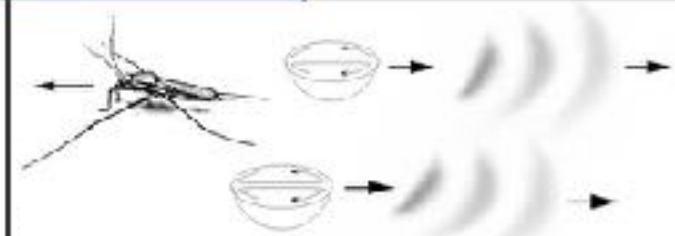
Dept. of Mechanical Engineering
Columbia University





» solar powered?!?

SUMMARY

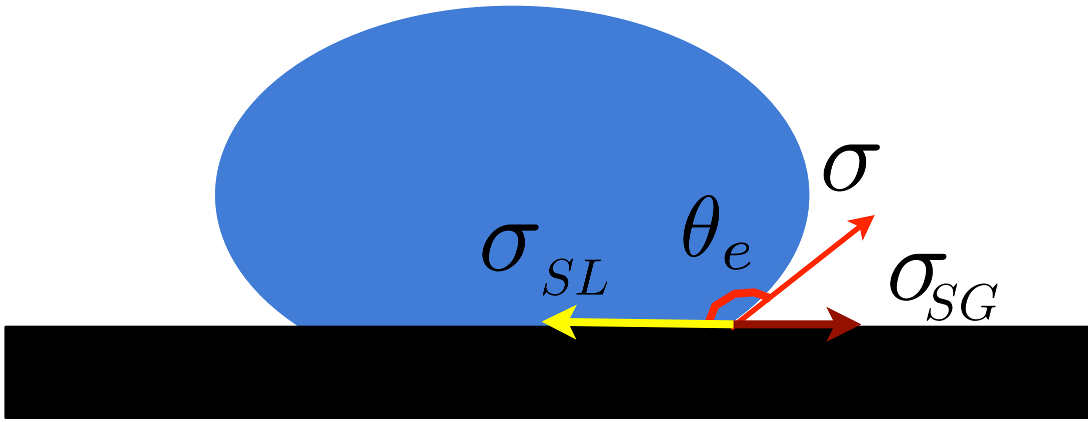
	Buoyancy	Added mass	Inertia	Curvature	Marangoni
Surface slapping	 Hsieh & Lauder (2004)				
Rowing & walking			 Hu, Chan & Bush (2003)		
Meniscus climbing				 Hu & Bush (2005)	
Marangoni propulsion					

But what is happening on the microscale?

Walking on water: a closer look

with Manu Prakash

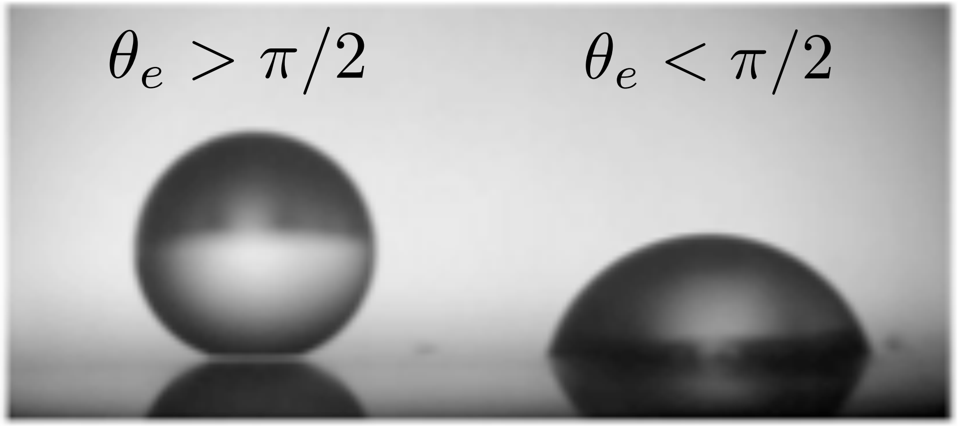
Fluid-Solid Contact: WETTING



Equilibrium contact angle θ_e

Energy differential: $dW = dx\,(\sigma_{SG} - \sigma_{SL}) - dx\,\sigma\,\cos\theta_e$

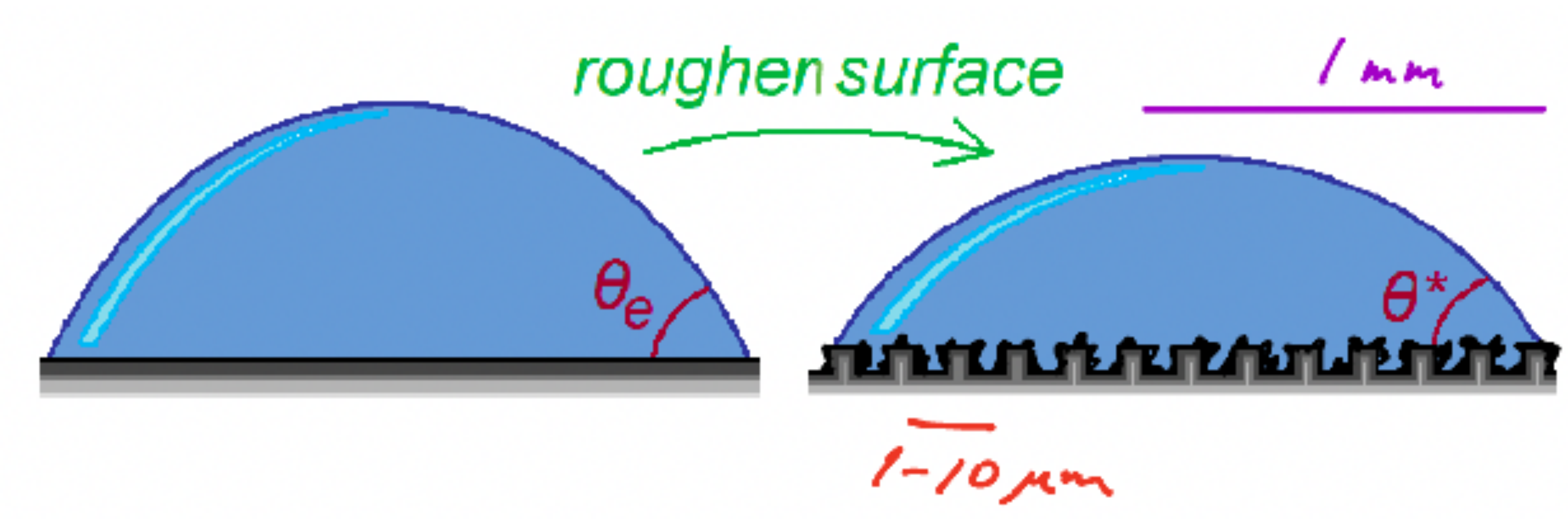
Young's relation: $$\sigma\ \cos\theta_e = \sigma_{SL} - \sigma_{SG}$$



Hydrophobic surface

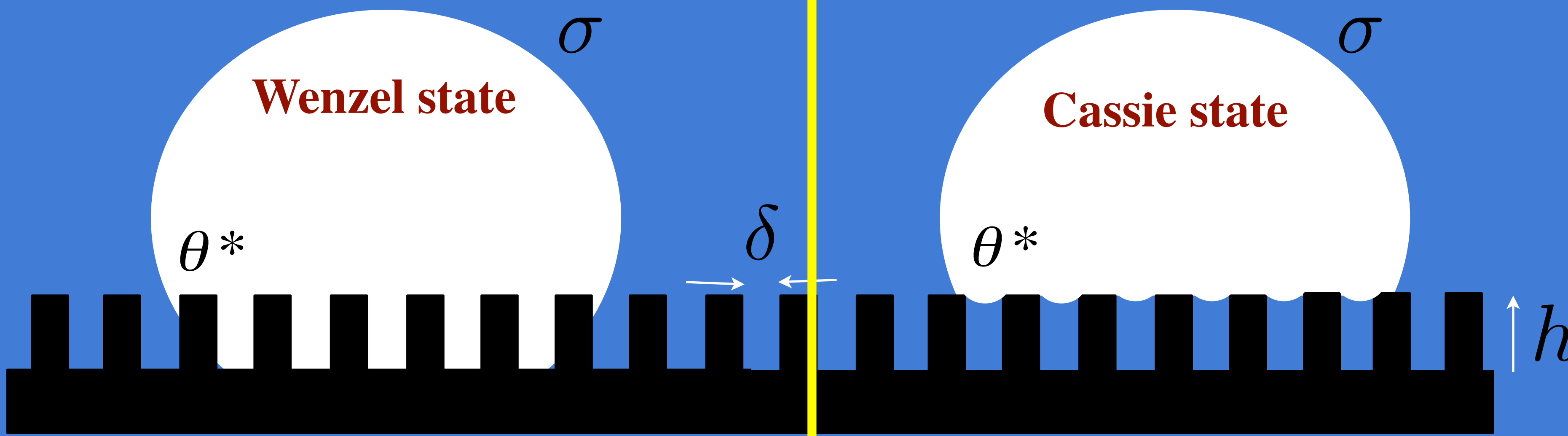
Hydrophilic surface

Effective contact angle on a rough solid



Three possible states

Wetting of a rough hydrophobic surface: Wenzel vs. Cassie



$$dW = r\ dx\ (\sigma_{SG} - \sigma_{SL}) - dx\ \sigma\ cos\theta^*$$

$$\cos\theta^* = r\ \cos\theta$$

where r is total/planar area

θ^* INCREASES, but $\Delta\theta$ INCREASES

$$\cos\theta^* = -1 + f_s + f_s \cos\theta$$

where f_s is exposed/planar area

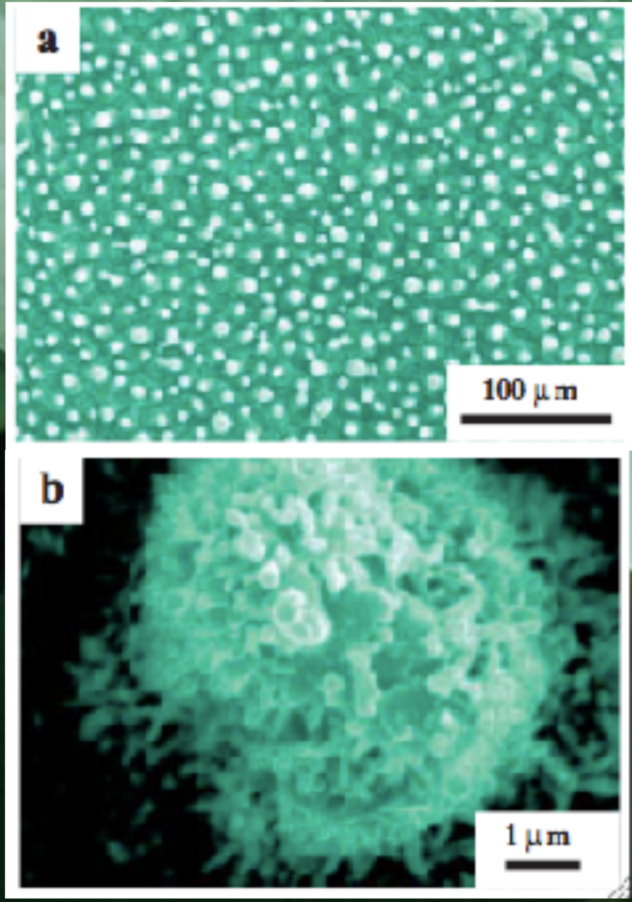
θ^* INCREASES

$\Delta\theta$ DECREASES

Water-repellency: requires the maintenance of a Cassie state

Water repellency in nature

"One who performs his duty without attachment, surrendering the results unto the Supreme Being, is unaffected by sinful action, as the lotus leaf is untouched by water."
Bhagavad Gita 5.10



Feng et al. (2004)

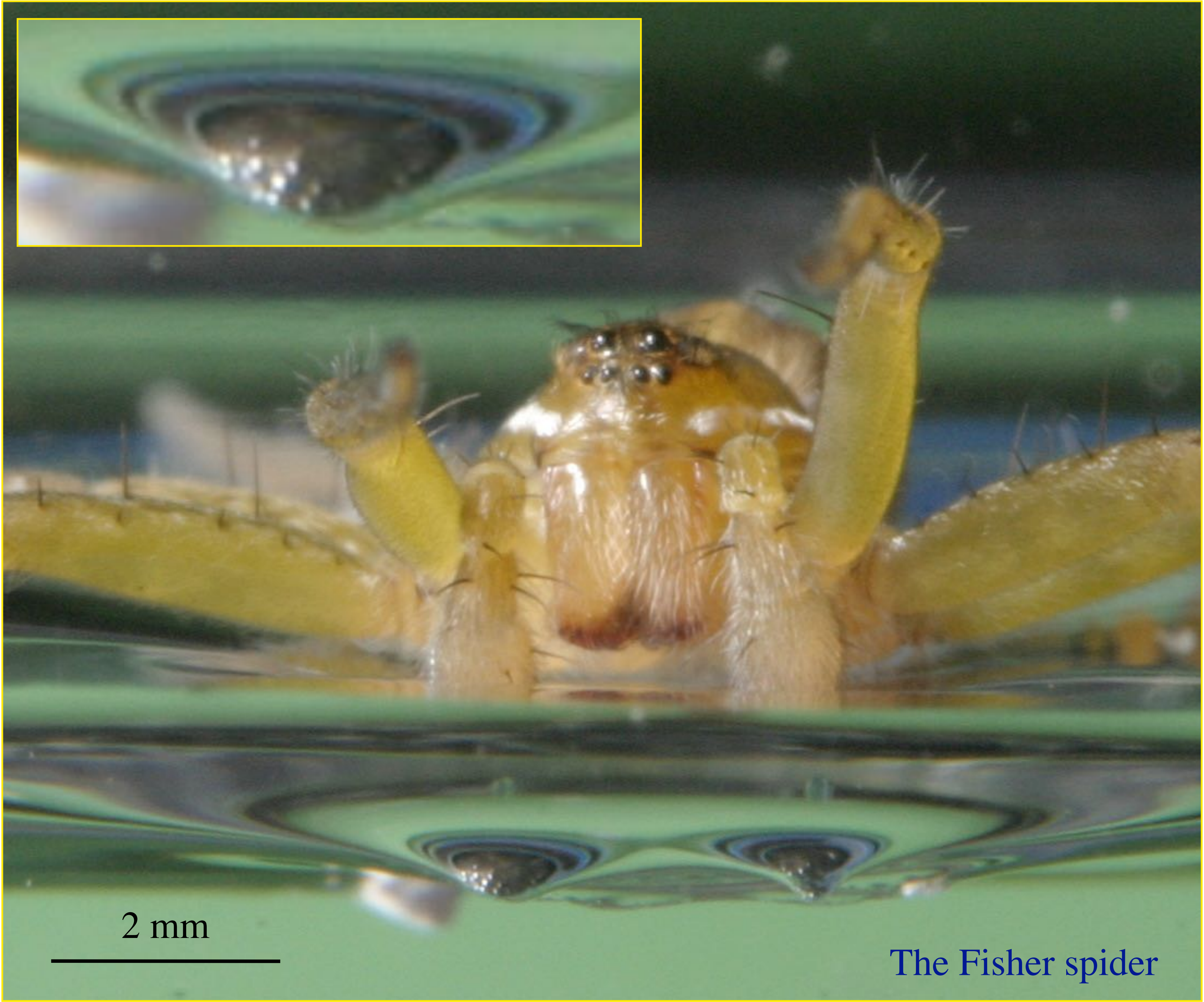
- the lotus leaf is superhydrophobic and self-cleaning by virtue of its waxy surface roughness

The wetting properties of spiders

(Suter et al. 2004)

- water-walking spiders have dense coats of hydrophobic hairs

- many terrestrial spiders have sparse covers of hydrophilic hairs

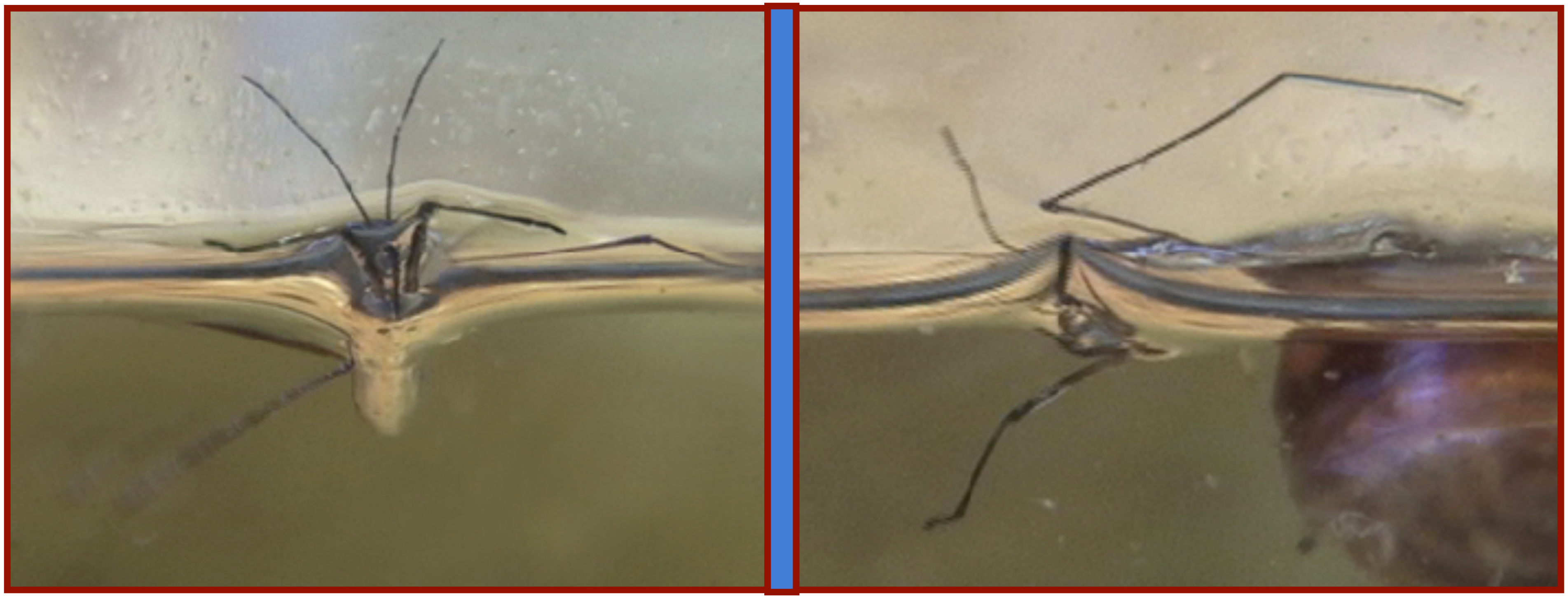

2 mm
The Fisher spider

The integument of water-walking insects and spiders

- body and legs covered in dense mat of fine hairs: “the Lotus Effect”

- hair layer increases surface area and so energetic cost of wetting
- hair mat thus discourages wetting, enhances water-repellency

The struggle of a partially submerged water strider (in soapy water)



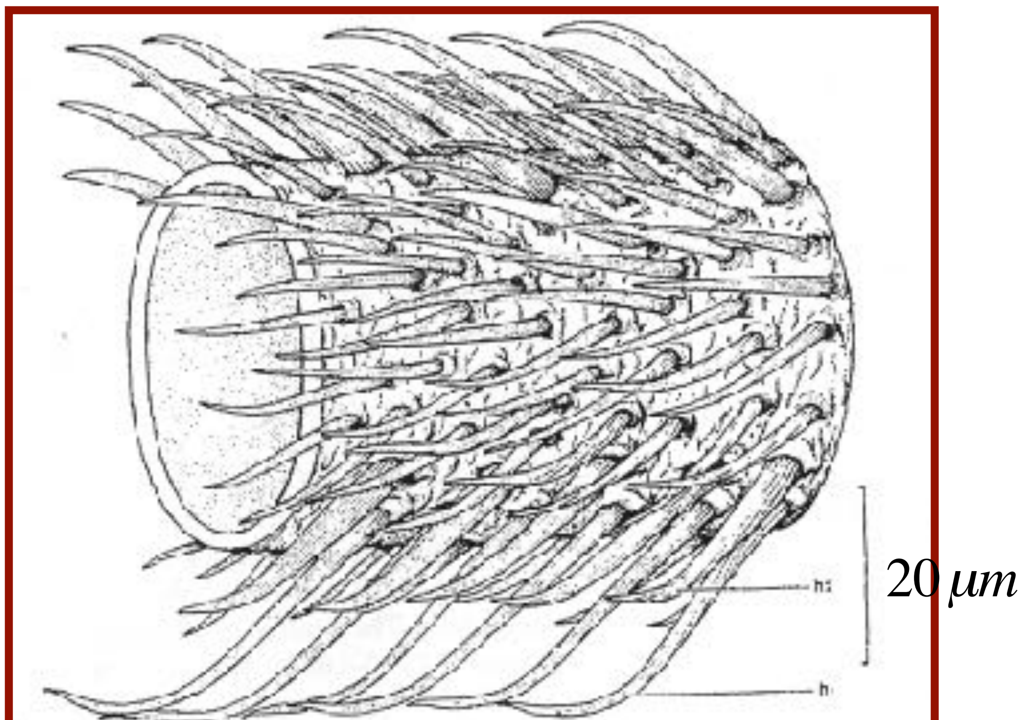
σ = 70 dynes/cm

- body weight: $W = M g \sim 5$ dynes
- total contact perimeter (leg plus body length): $P \sim 7$ cm
- force required to cross the interface: $\sigma P \sim 500 \text{ dynes} \sim 100\, W$

Hair cover on water-walking insects

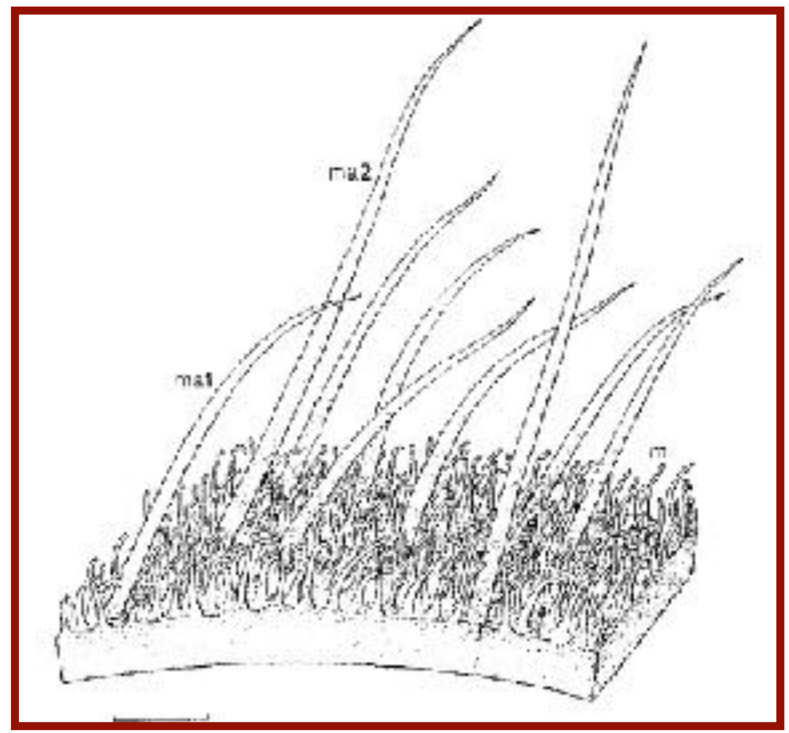
- on the legs, there is a dense layer of tilted hairs of uniform length
- on the body, there are two distinct hair layers: the macro- and microtrichia
    1) the long hairs protrude through the interface
    2) the short hair layers are impenetrable by water, and so maintain an air layer, or plastron, when the insect is submerged

Tarsal segment of gerridae



Andersen (1976)

Sternum of water strider

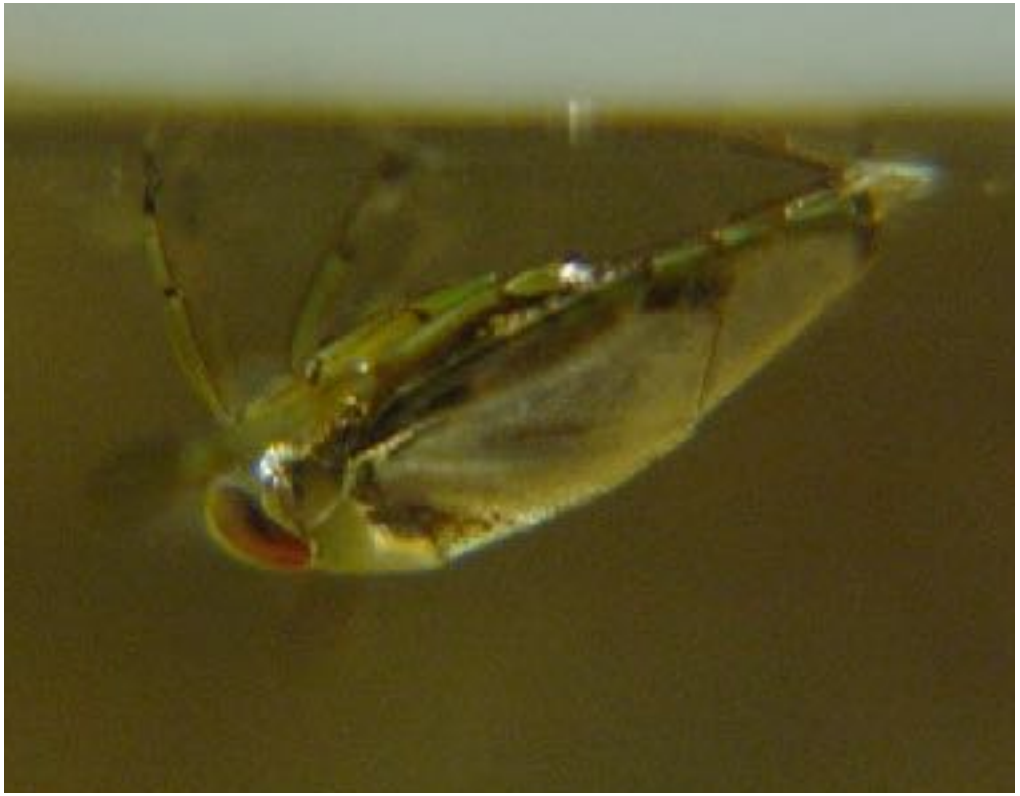


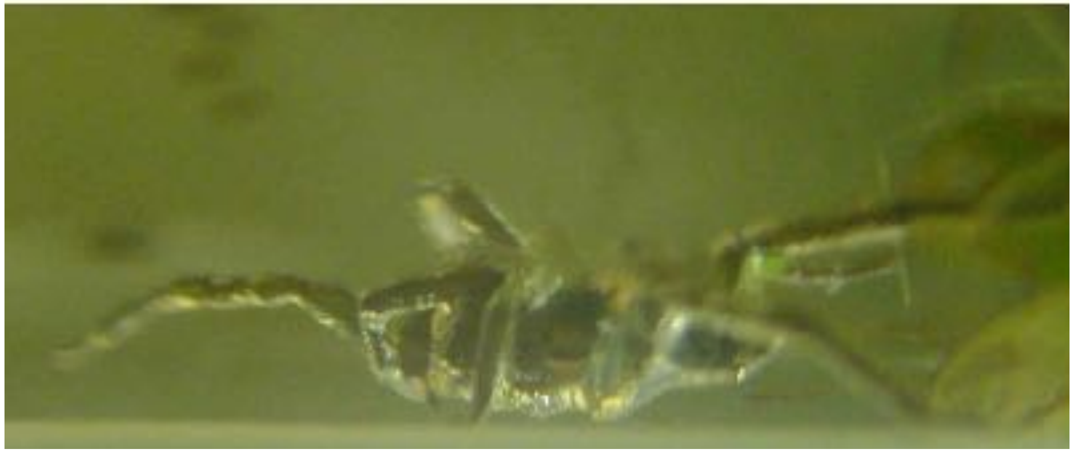
10 μm

Andersen (1977)

Underwater breathing via water-repellency

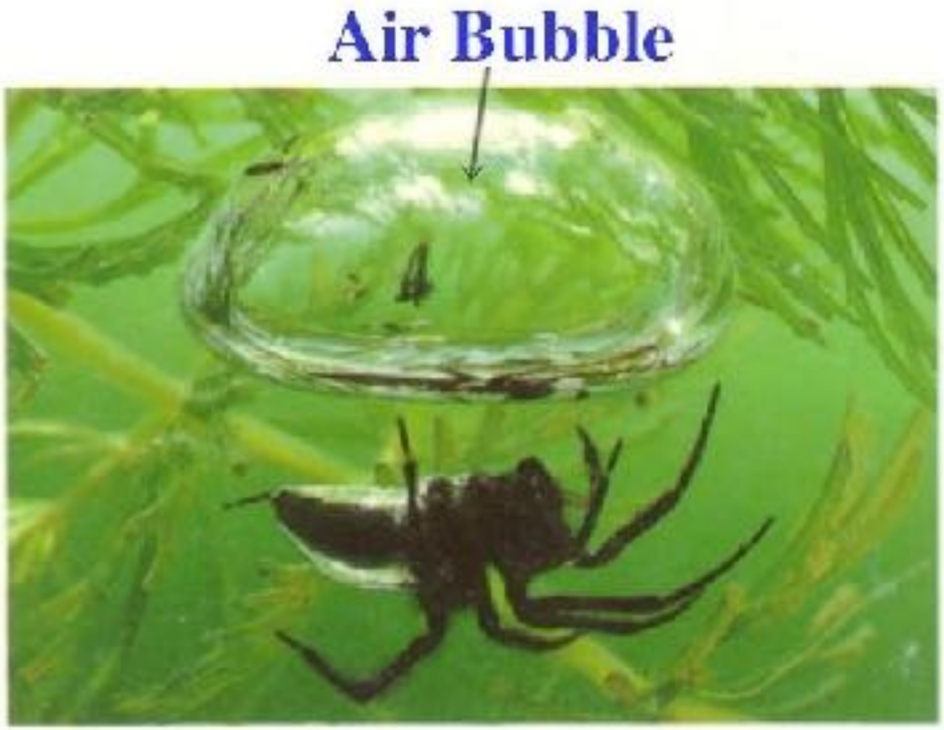
- thin air layer, termed the `plastron', trapped on body surface





- plastron serves as external gill
- oxygen diffuses into plastron, enabling extended dives
- may sustain bug indefinitely

Flynn & Bush (2007)

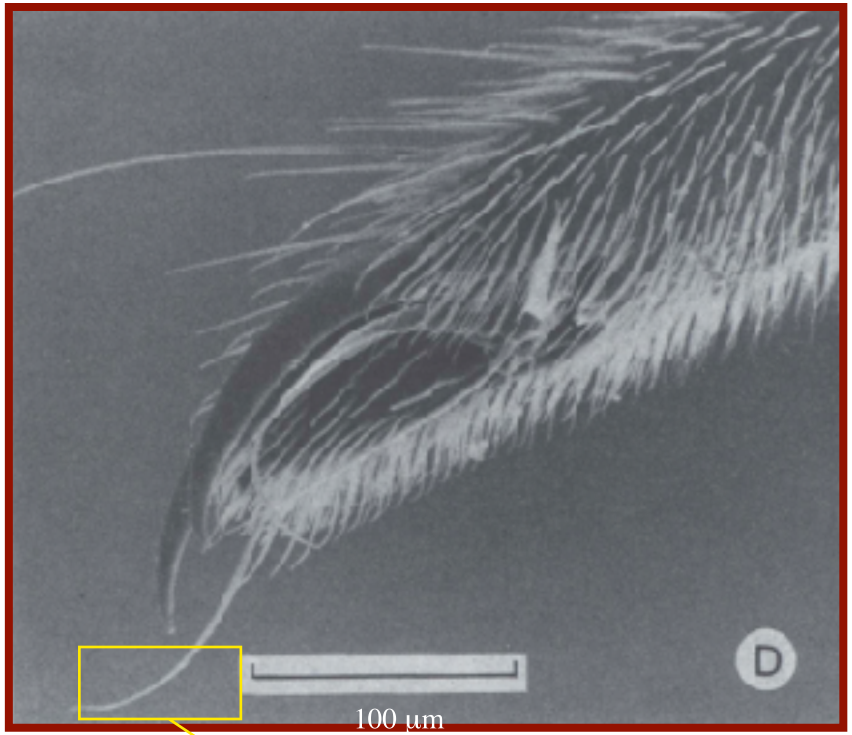

Air Bubble

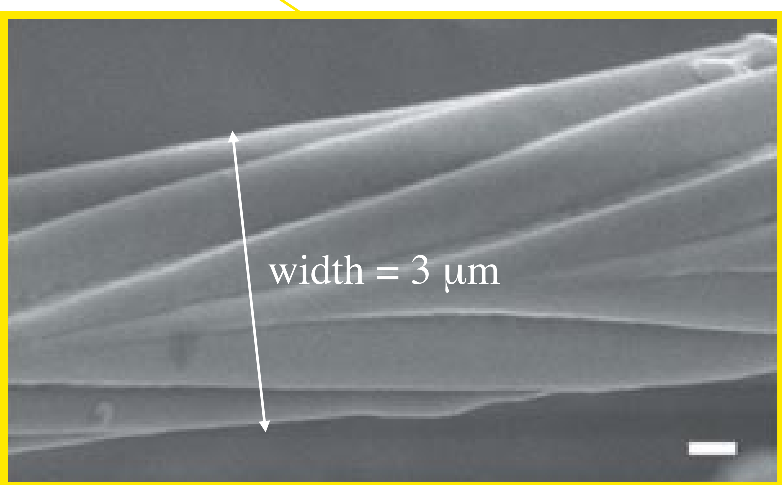

ARKive
© www.osf.uk.com

The microstructure of the insect leg

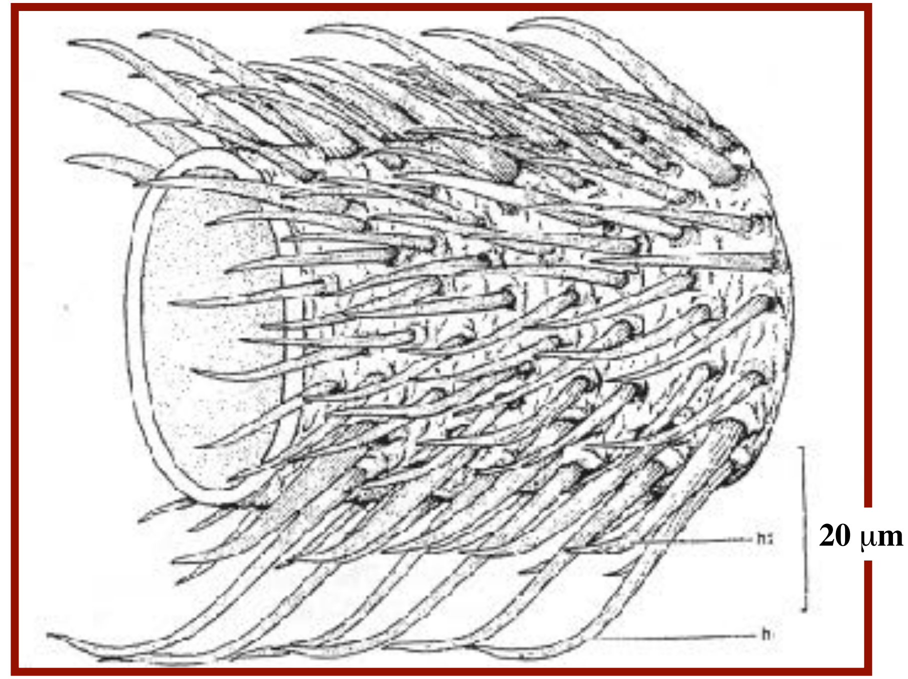
(Andersen 1976, Gao & Jiang 2006)

N = 10,000 hairs/mm²

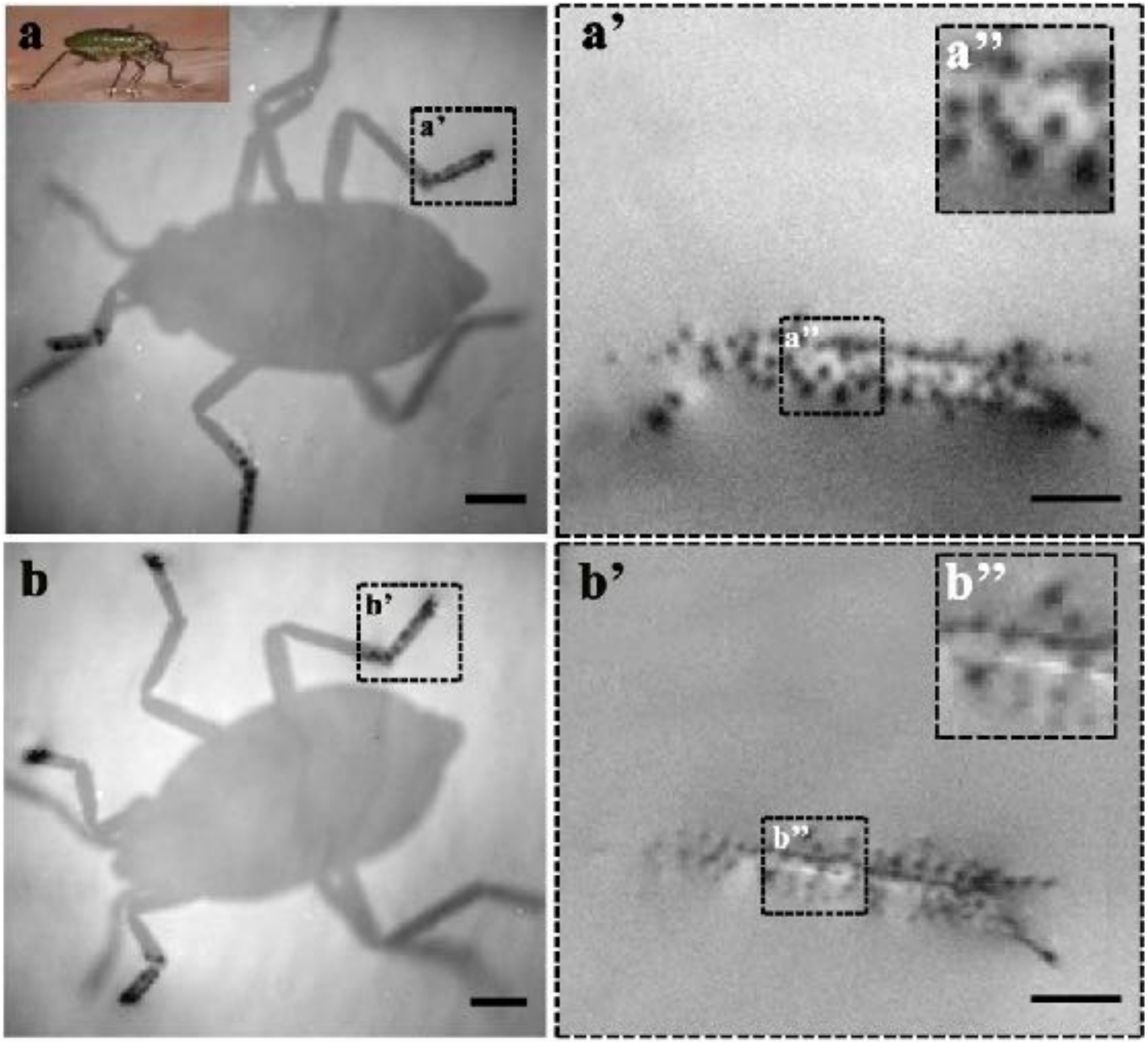




hairs covered with
fluted microgrooves

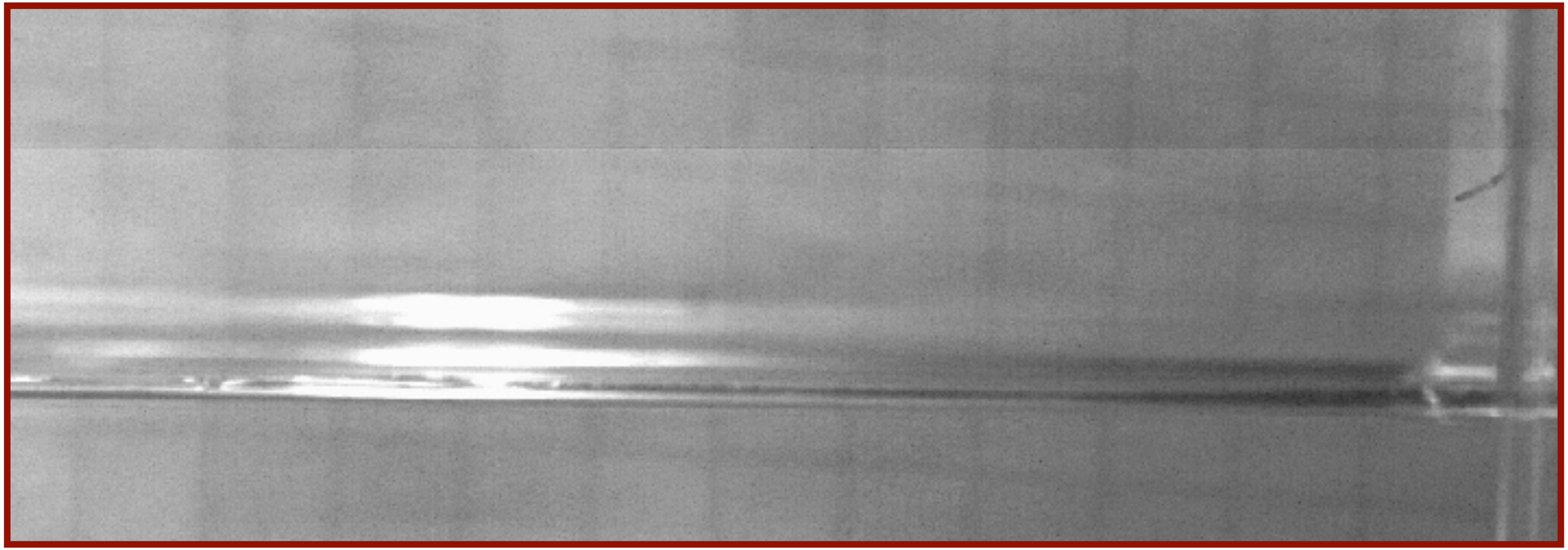


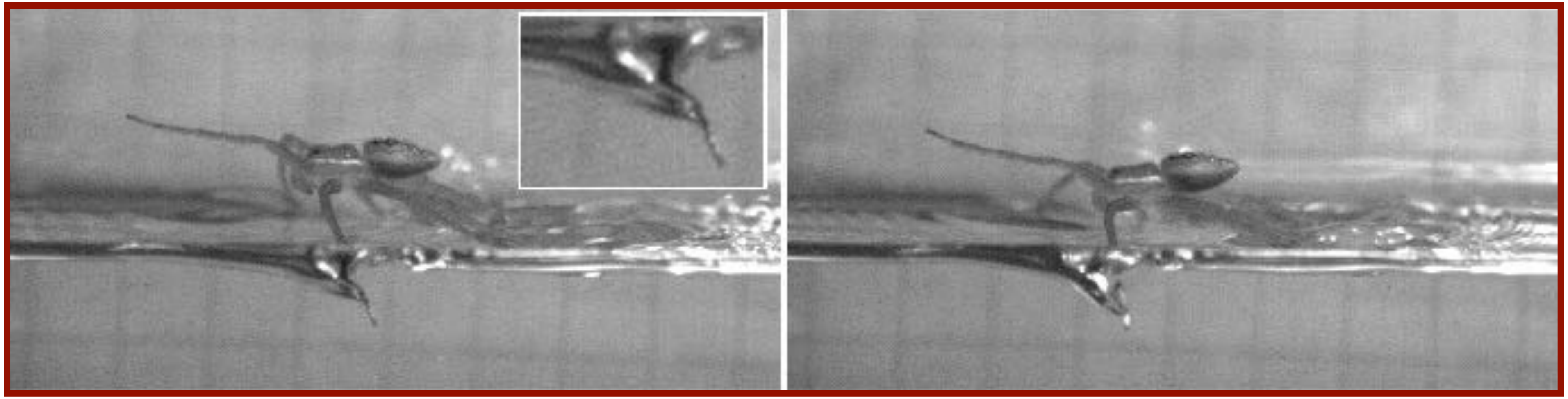
Water-walking arthropods: in a Cassie state



Mesovelia

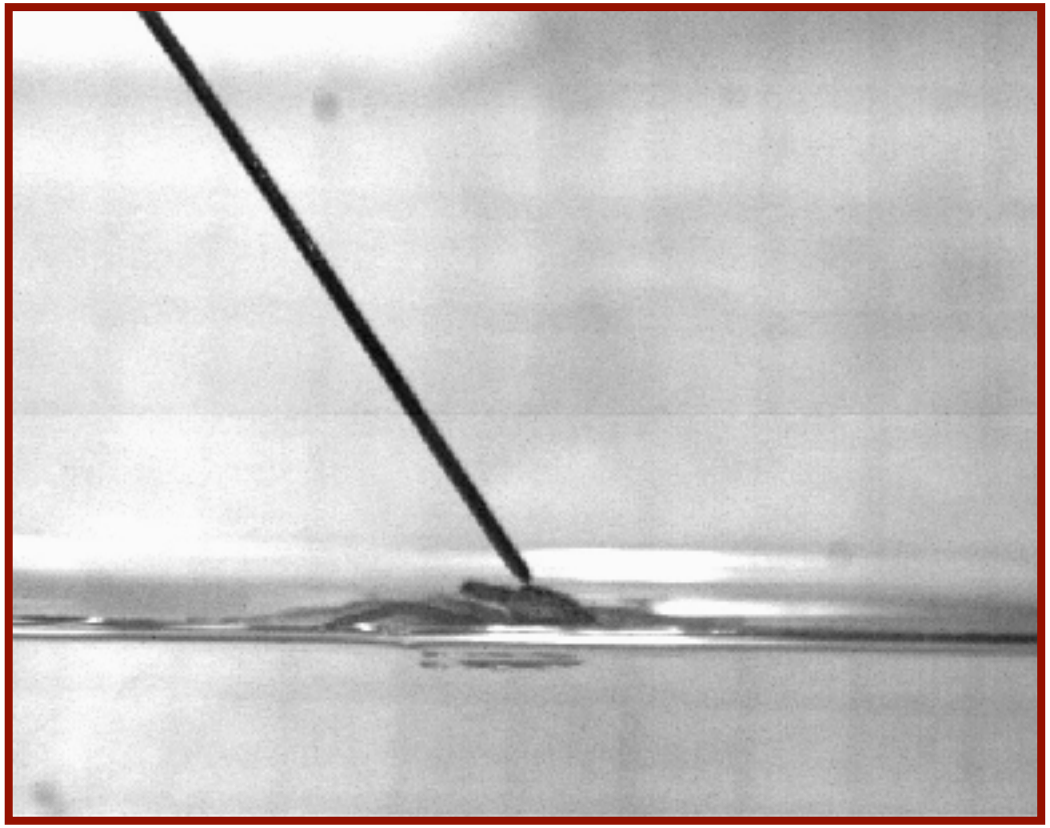
The galloping fisher spider

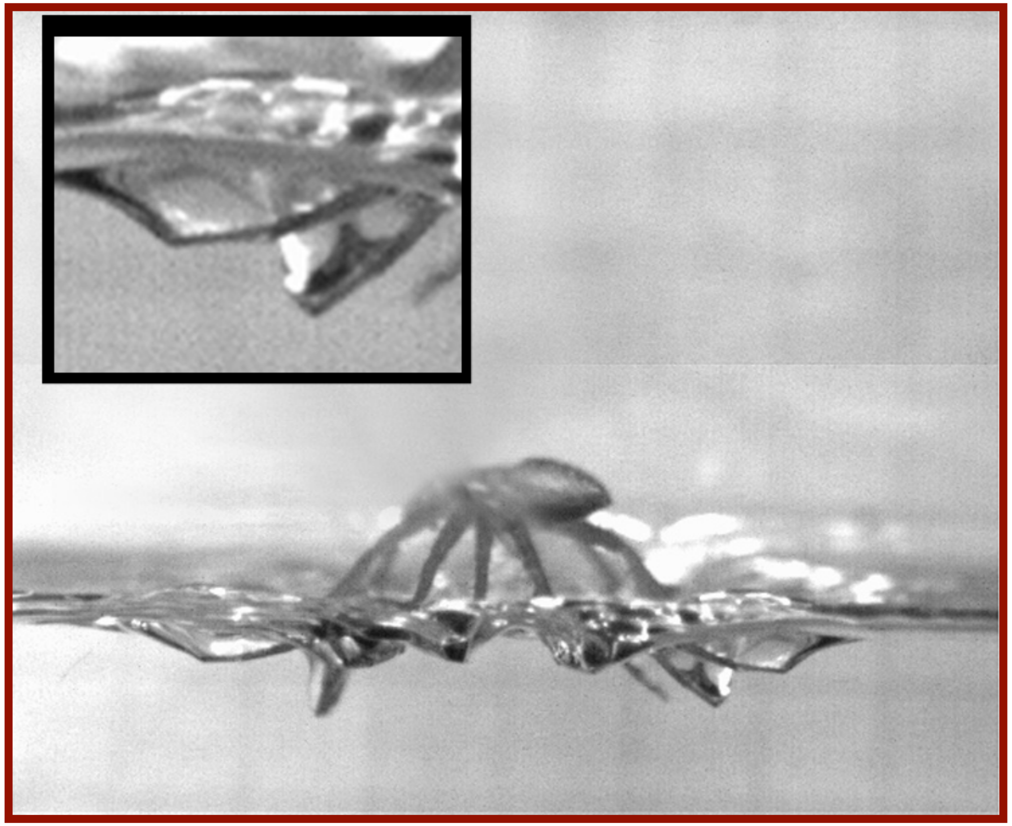




- leg penetrates free surface but avoids wetting by maintaining a thin air layer

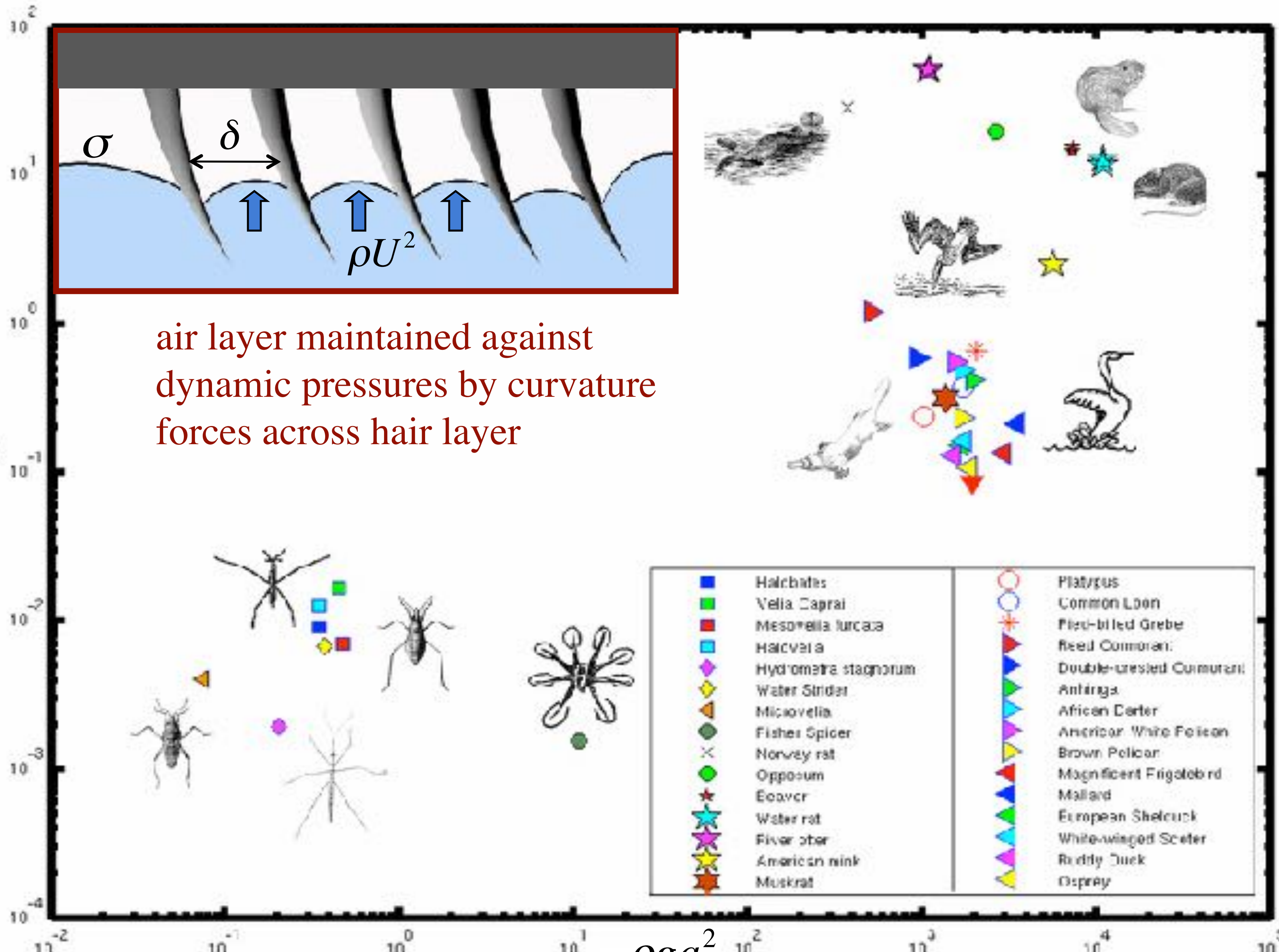
The leap of the fisher spider





- leg penetrates free surface but avoids wetting by maintaining a thin air layer

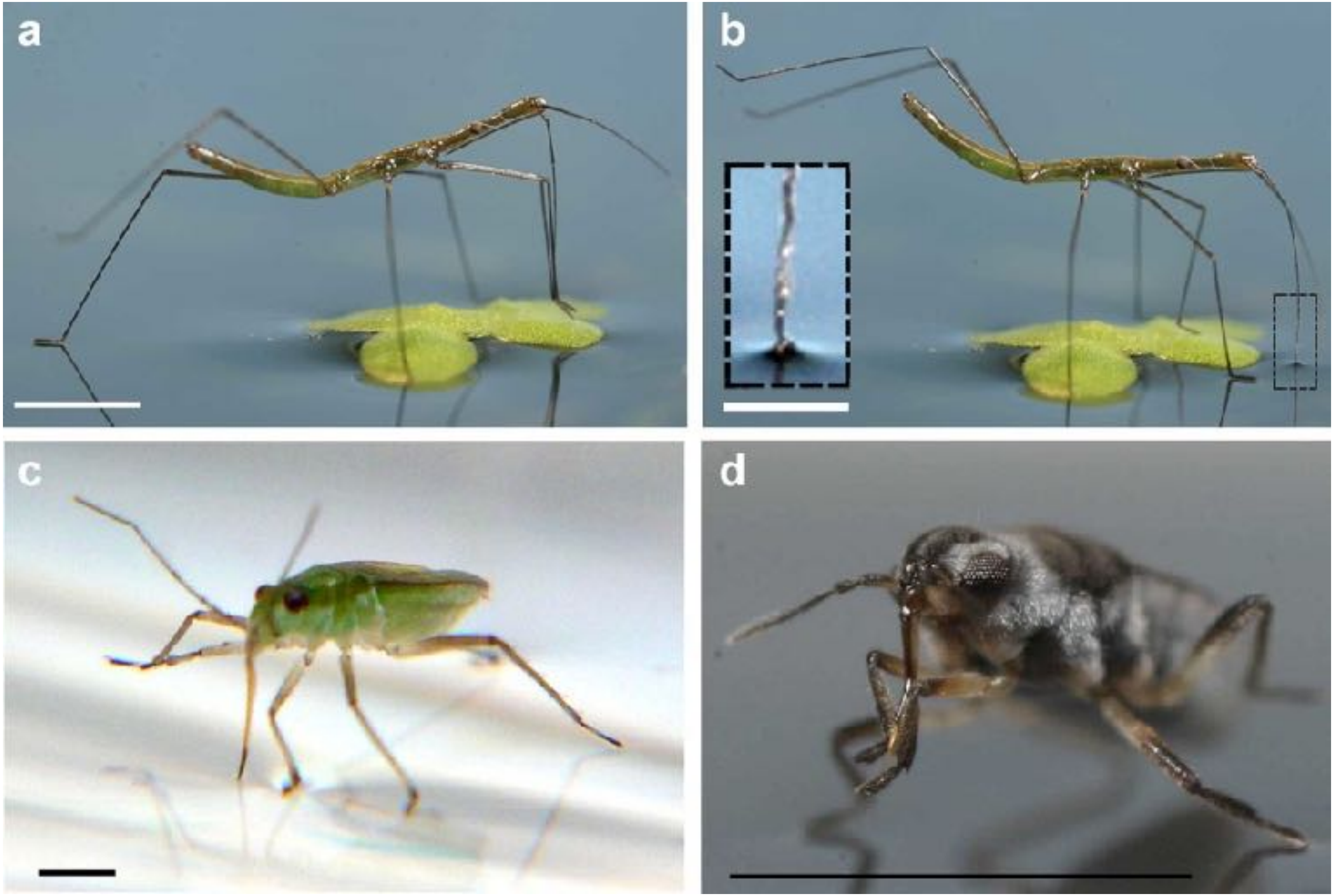
Can water-walking arthropods maintain a Cassie state?



$$We_s = \frac{\rho U^2 \delta}{\sigma}$$

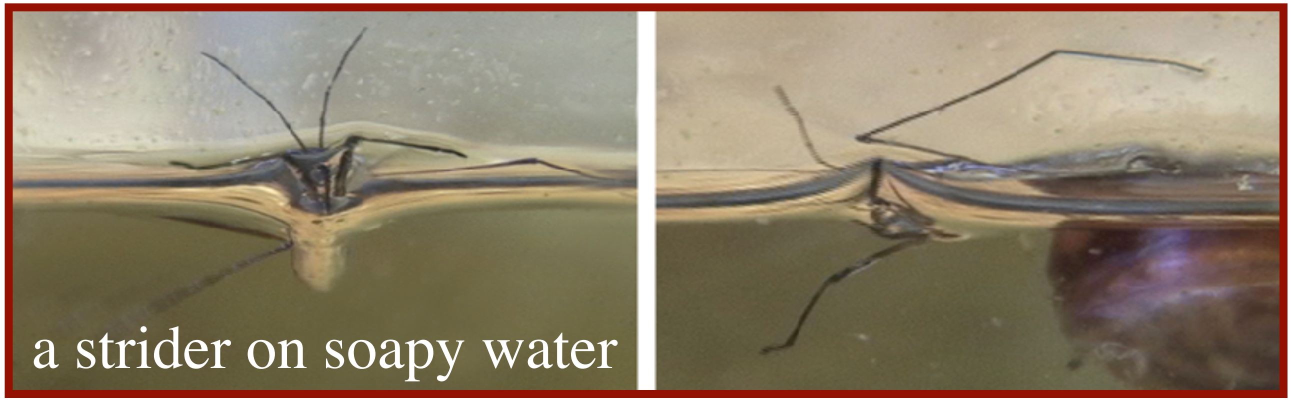
$$B_o = \frac{\rho g a^2}{\sigma}$$

Maintenance of their Cassie state prompts frequent grooming sessions.



Conundrum

- in order to avoid falling through the interface, water-walking insects must be water-repellent



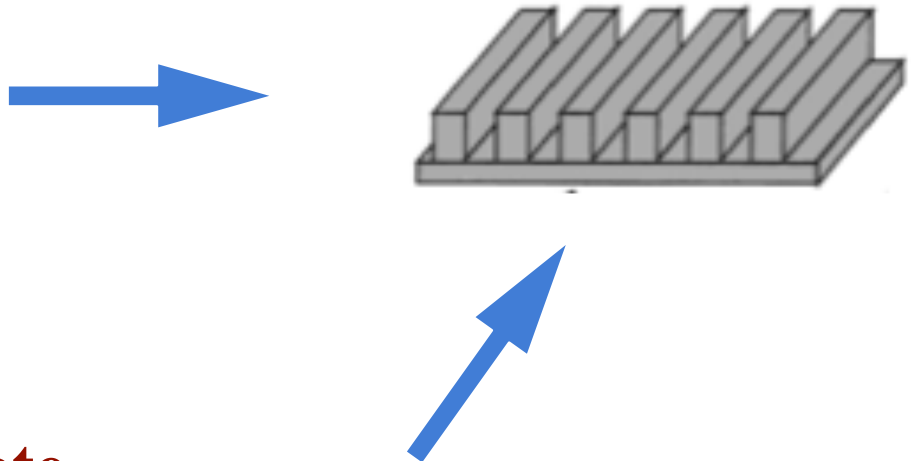
- water-repellent surfaces experience minimal traction on the free surface

- water-walking insects propel themselves by striking the surface

HOW?

Drag reduction and superhydrophobicity

Choi et al. (2006)
Min & Kim (2006)
Joseph et al. (2006)



Wenzel state

- drag reduced substantially on flow along nanogrooves

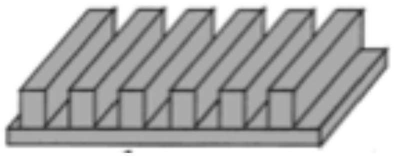
→ basis for `riblet' technology

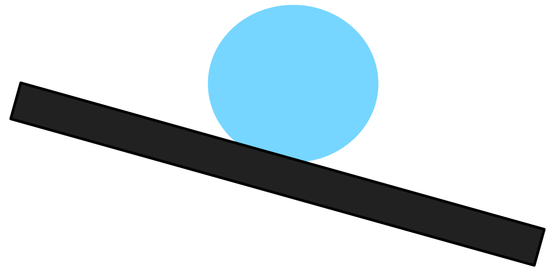
Cassie-Baxter state

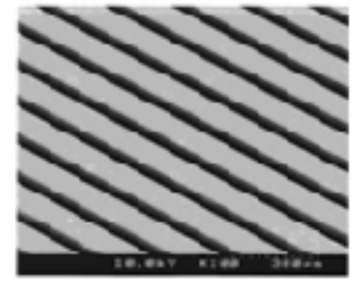
- drag reduced for flow along nanogrooves
- drag increased for flow across nanogrooves

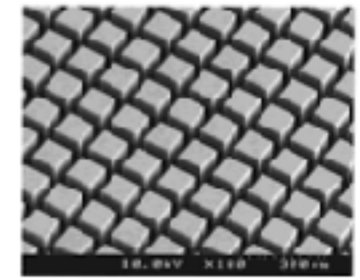
Surface texturing and directional adhesion

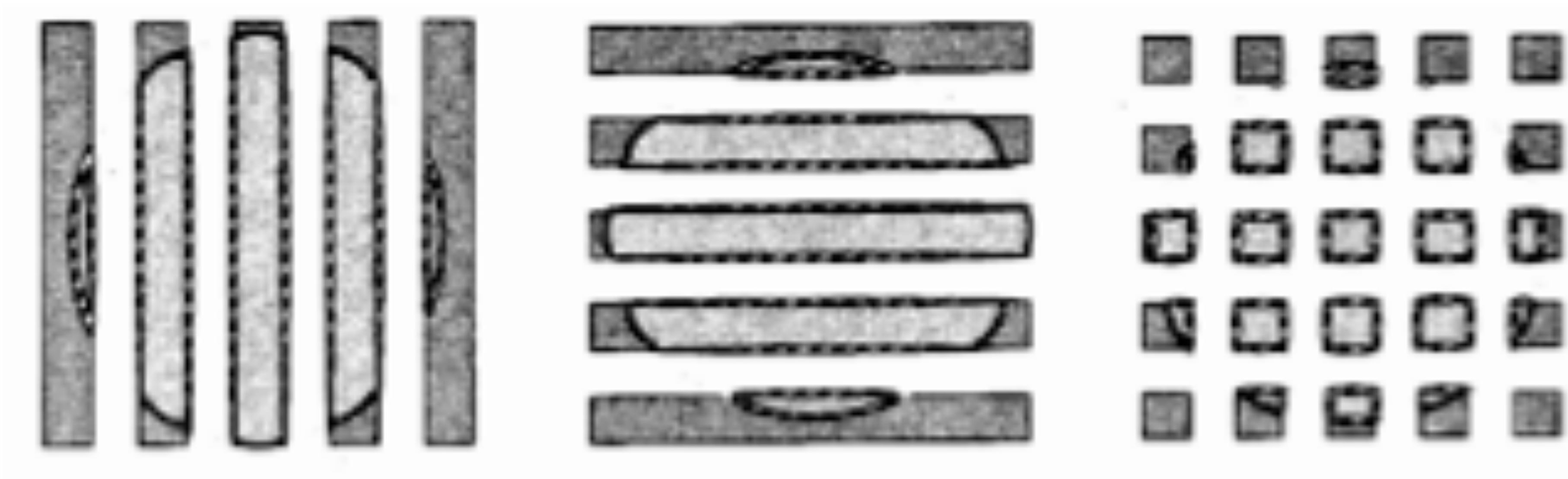
Yoshimitsu et al. (2002)









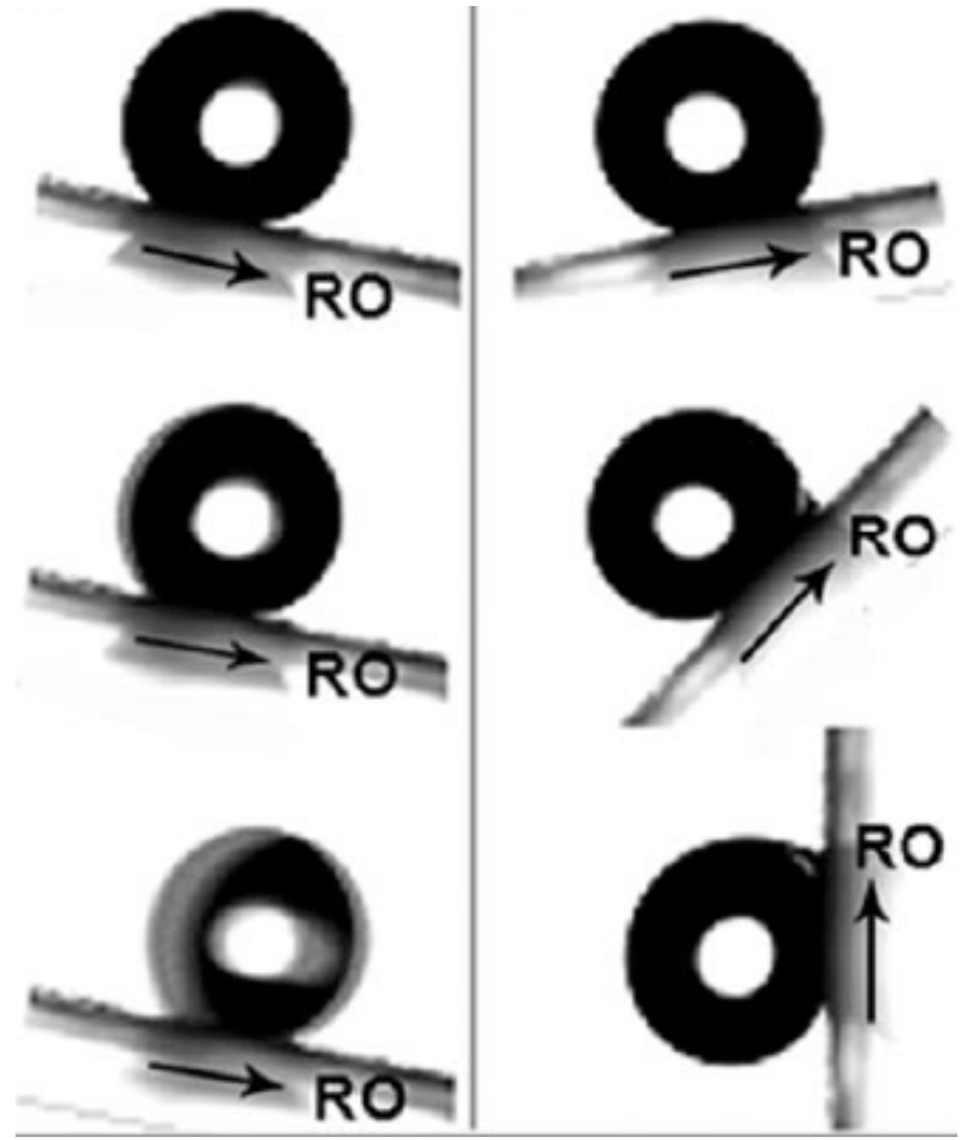


- drops move most easily along nanogrooves
- greatest resistance to motion perpendicular to grooves
- texturing introduces anisotropy in contact line resistance

Unidirectional adhesion

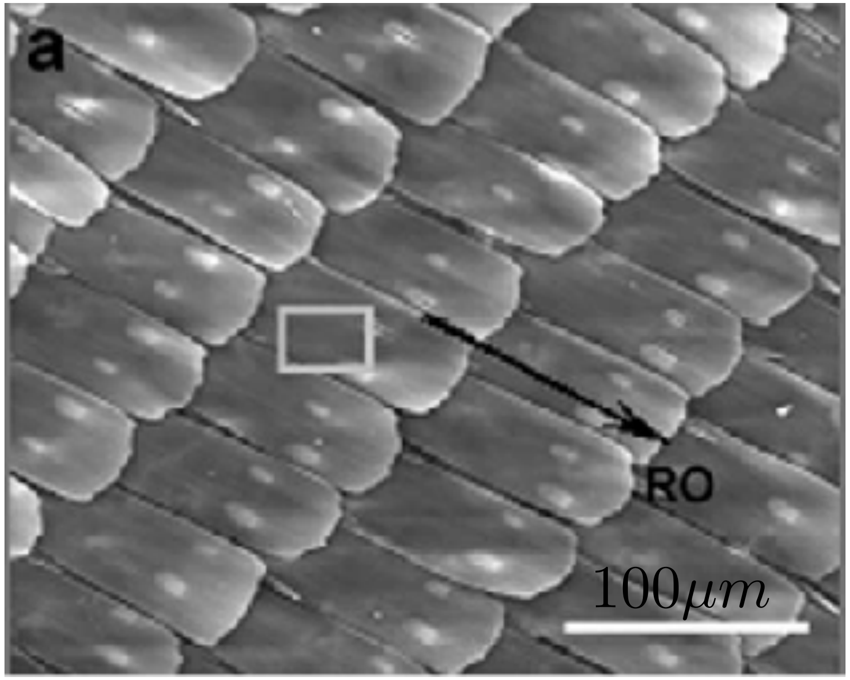
on the butterfly wing

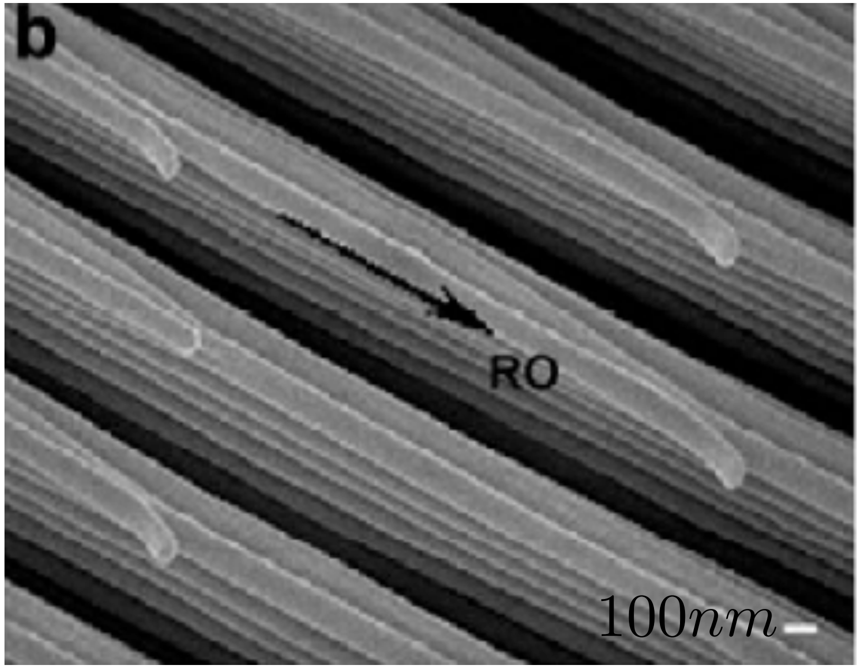
Zheng et al. (2007)

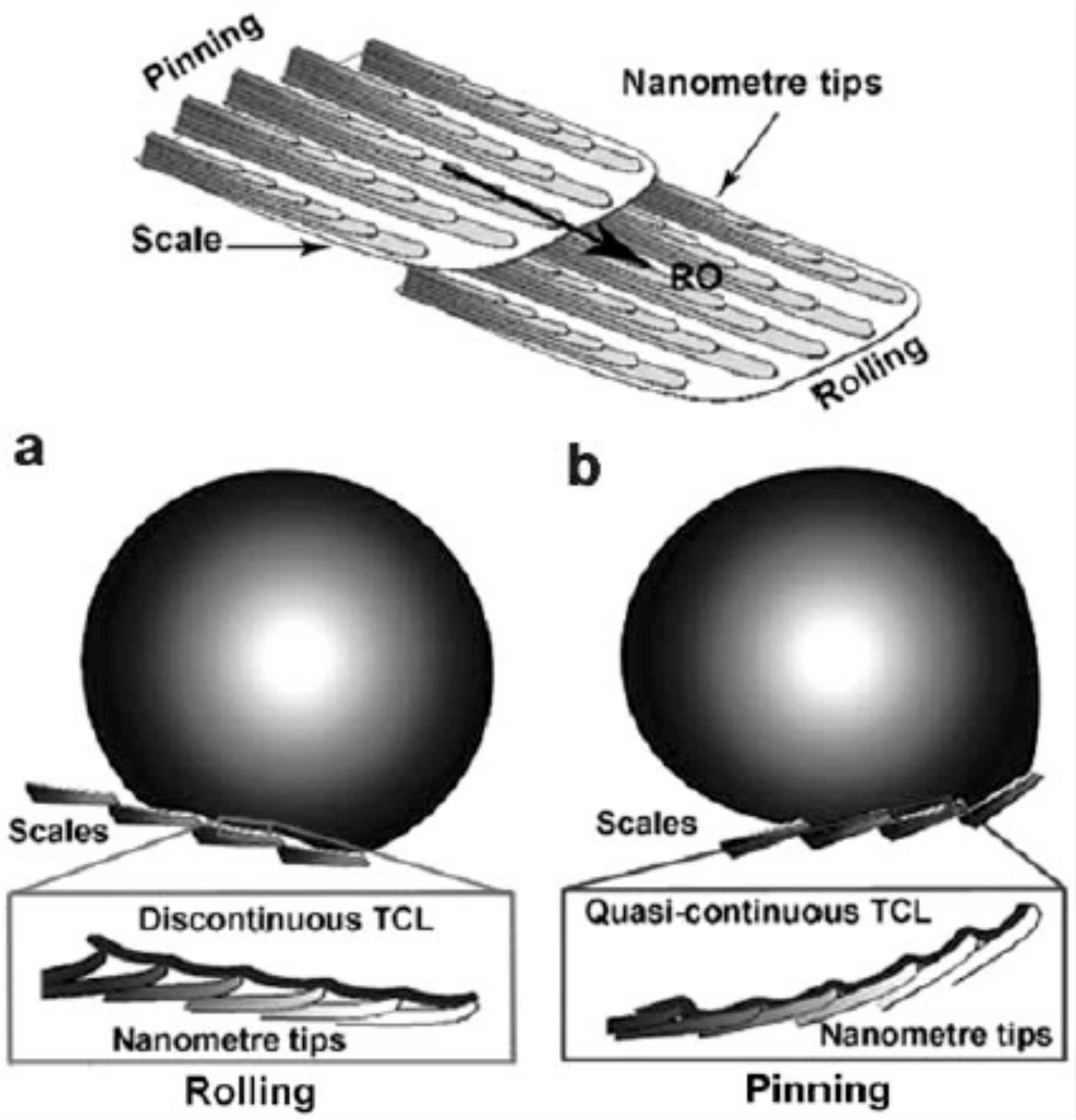


Unidirectional adhesion

Zheng et al. (2007)







How do flying insects cope with rain?