

Lecture 4

Interfacial fluid mechanics

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Motivation

- who cares about surface tension?

As we shall soon see, the surface tension of water dominates gravity on a scale less than the capillary length, ~ 2 mm.

A tour through the world of capillarity

I. Fundamentals

- drops, bubbles, films, sheets, impacts, waves

II. Geophysics

- interfacial effects in climate and the sustenance of life

III. Engineering

- microgravity, microfluidics, biomimicry, biomedical mitigation

IV. Biocapillarity (Lec.5)

- surface tension dominates the world of insects

V. Capillarity in analogizing quantum system (Lec.6)

- modeling quantum systems on the lab scale

Motivation: who cares about surface tension?

Geophysics and environmental science

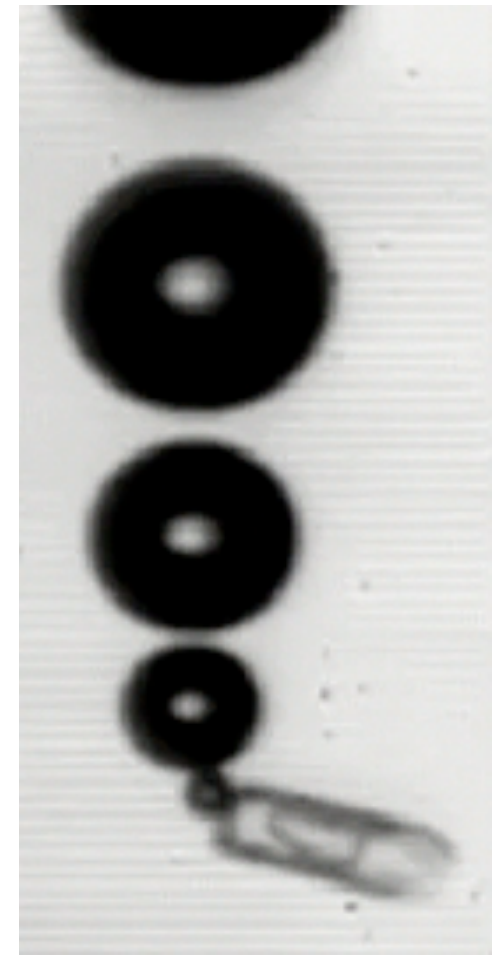
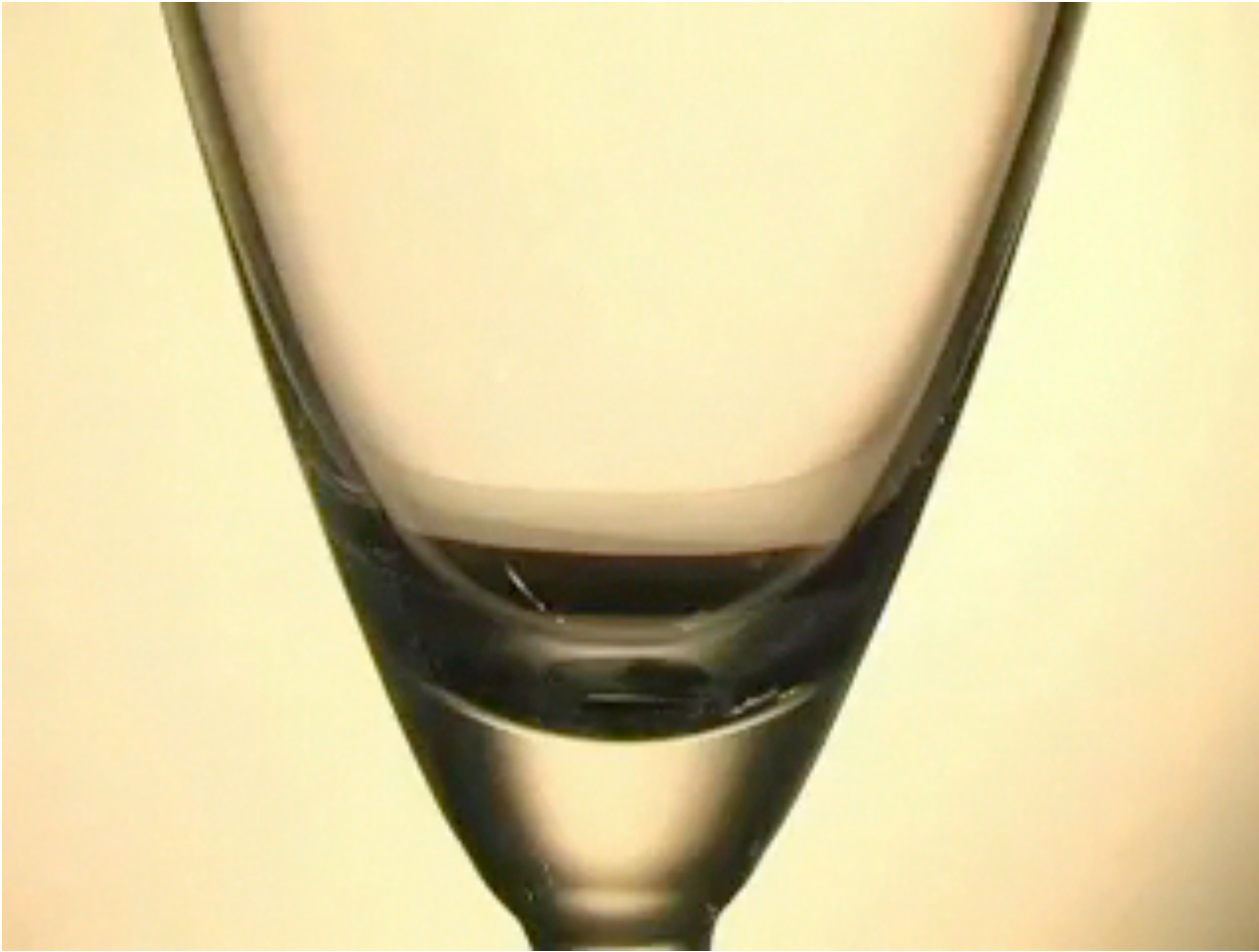
- the dynamics of **raindrops** and their role in the biosphere
- most biomaterial is surface active, sticks to surface of drops/bubbles
- chemical, thermal and biological transport in the **surf zone**
- early life: early vesicle formation, confinement to an interface
- oil recovery, carbon sequestration, groundwater flows
- design of **insecticides** intended to coat insects, leave plant unharmed
- chemical leaching and the water-repellency of soils: *desertification*
- **oil spill** dynamics and mitigation (*e.g.* use of dispersants in BP spill)
- **disease** transmission via droplet exhalation (*e.g.* COVID-19, Lec.7)
- dynamics of magma chambers and volcanoes
- the exploding lakes of Cameroon

The surf zone: bubble and drop creation via mixing

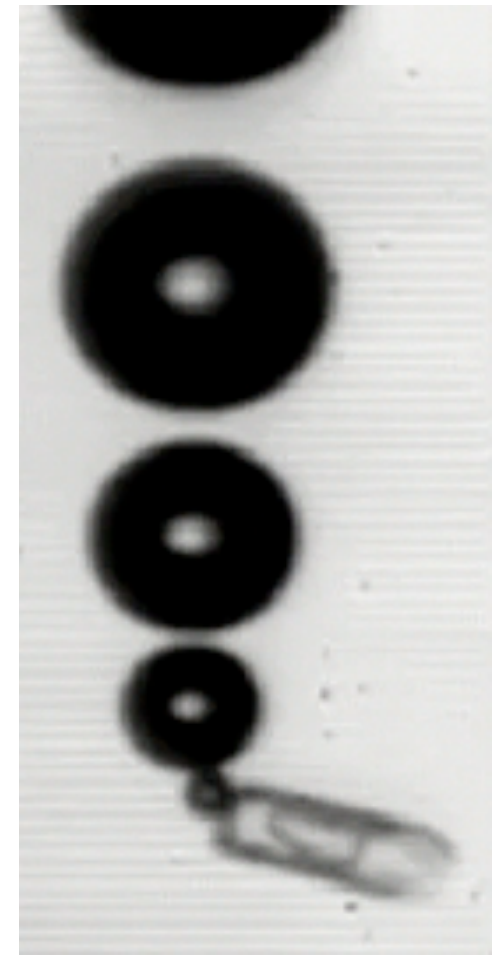
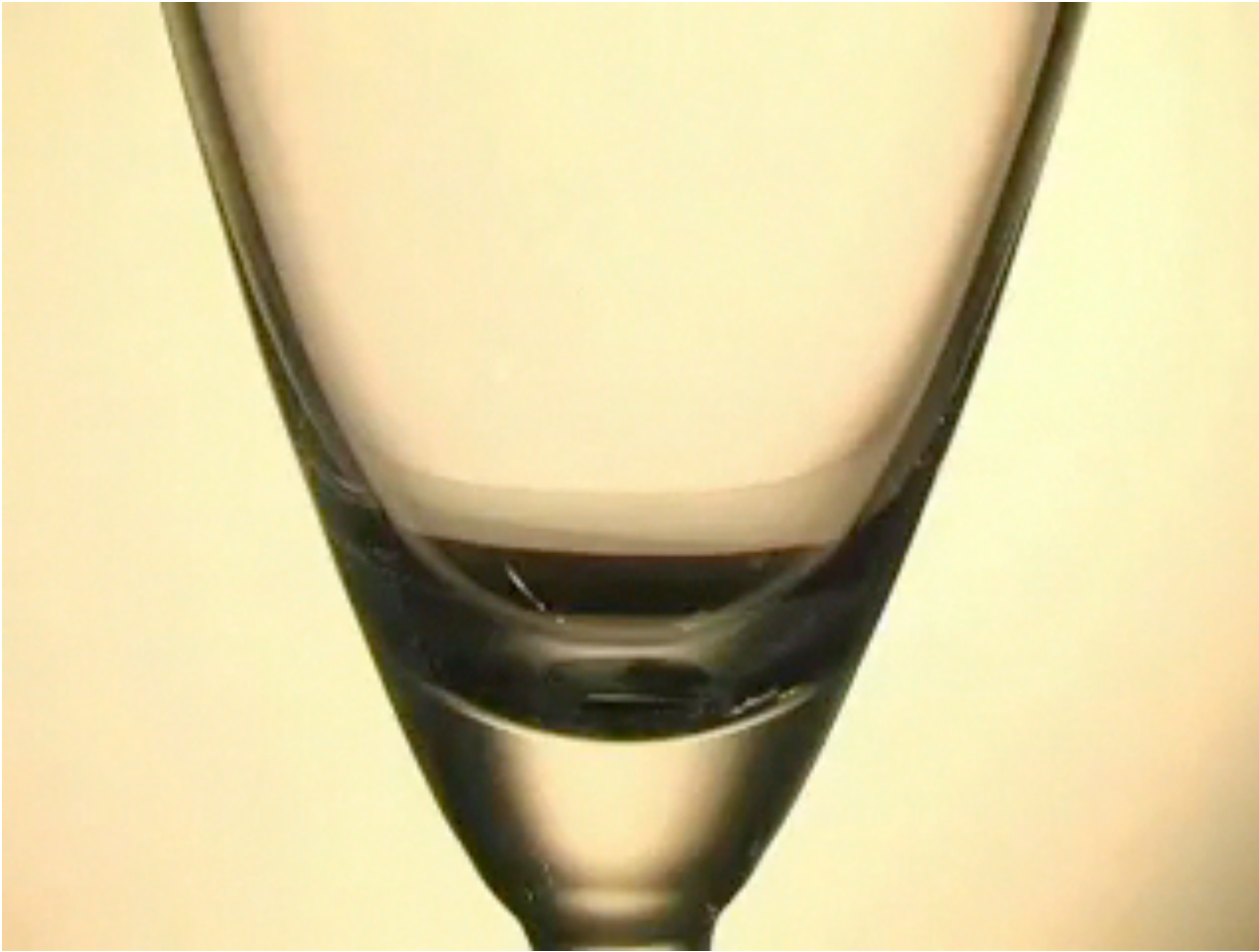


A key site of chemical and biological transport between atmosphere and ocean.

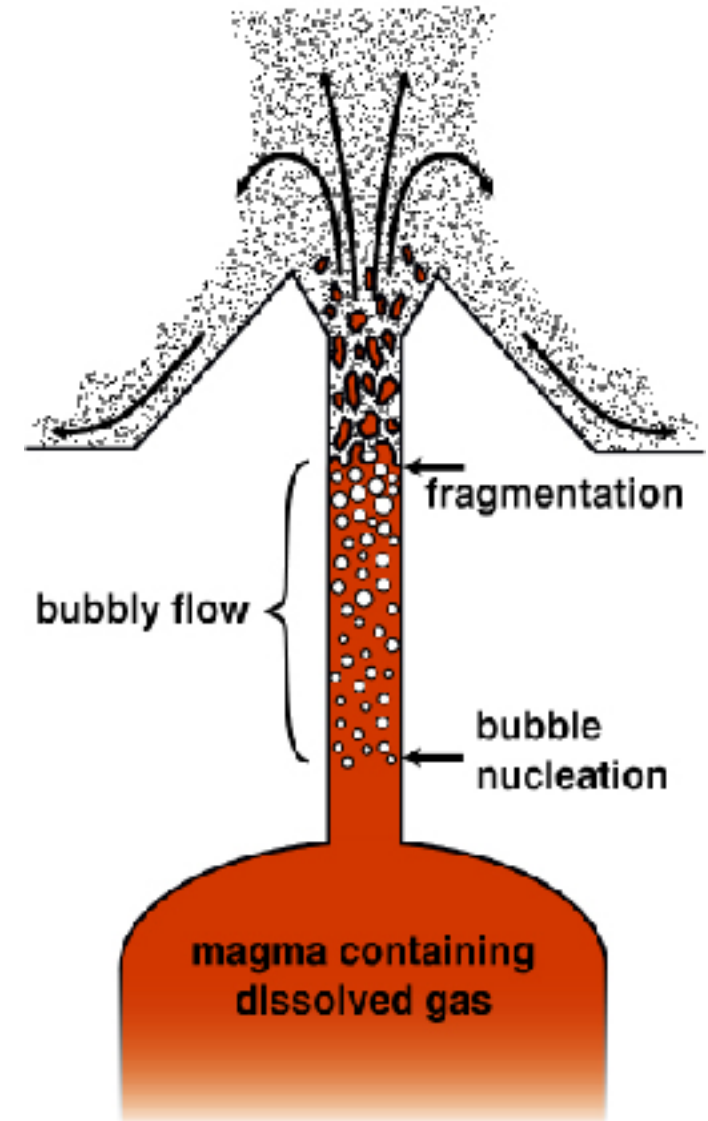
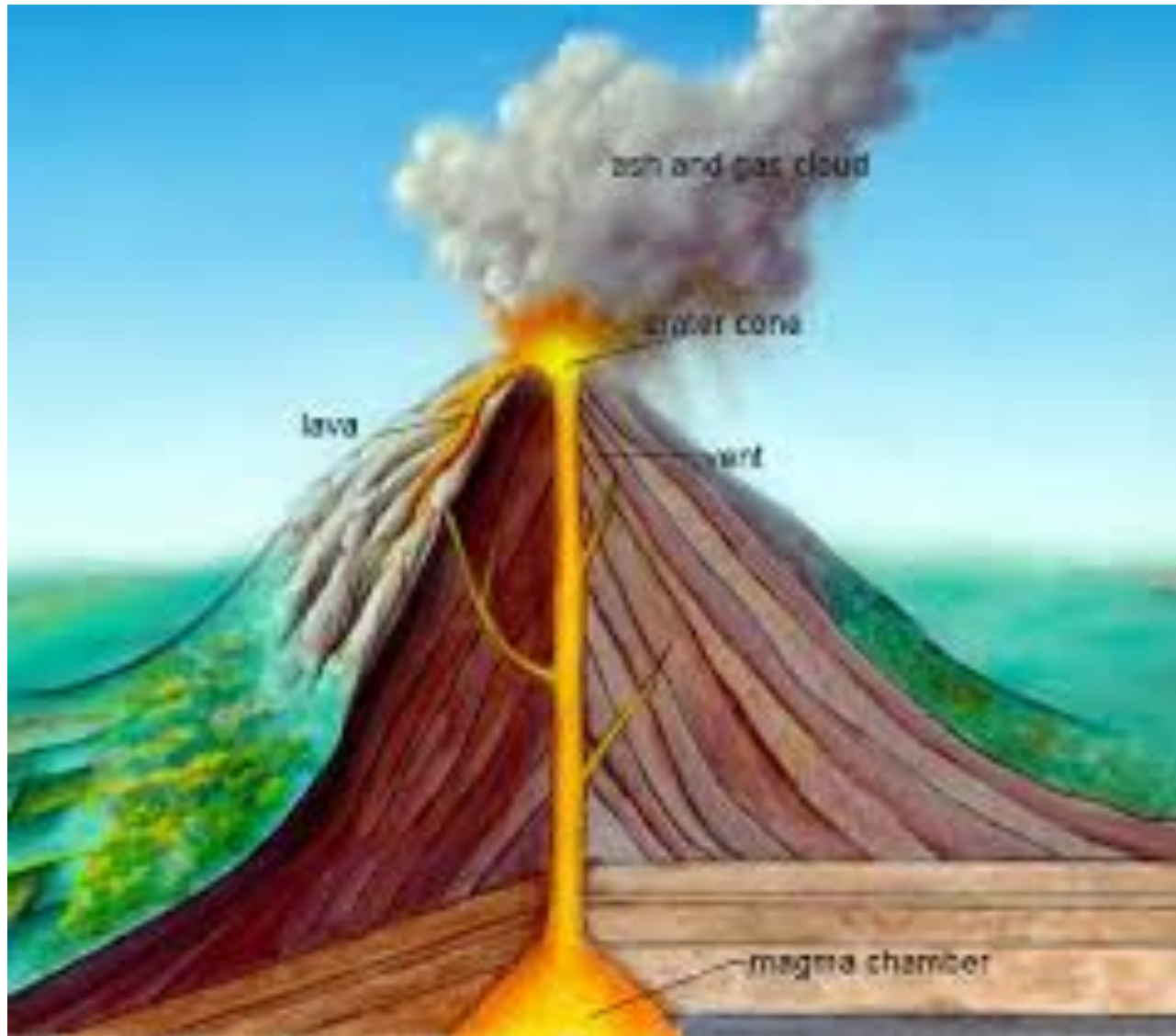
Bubble creation via exsolution of dissolved gas



Bubble creation via exsolution of dissolved gas



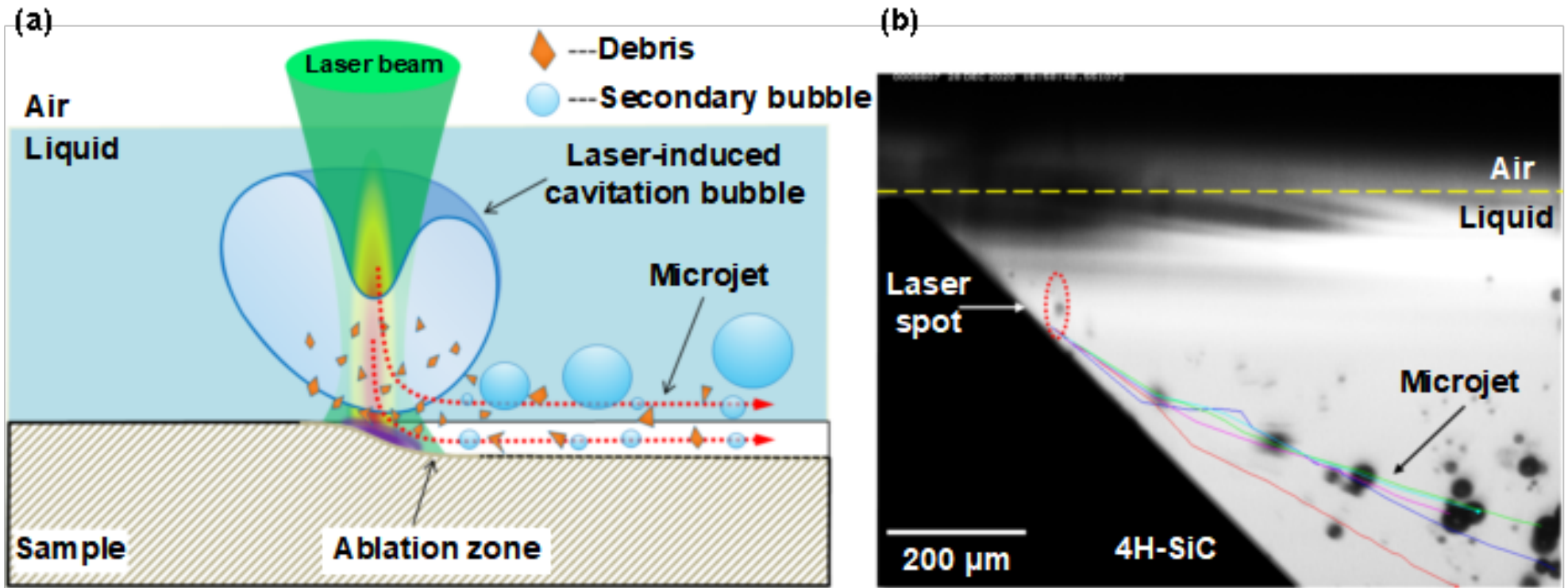
Bubble creation via exsolution of dissolved gas



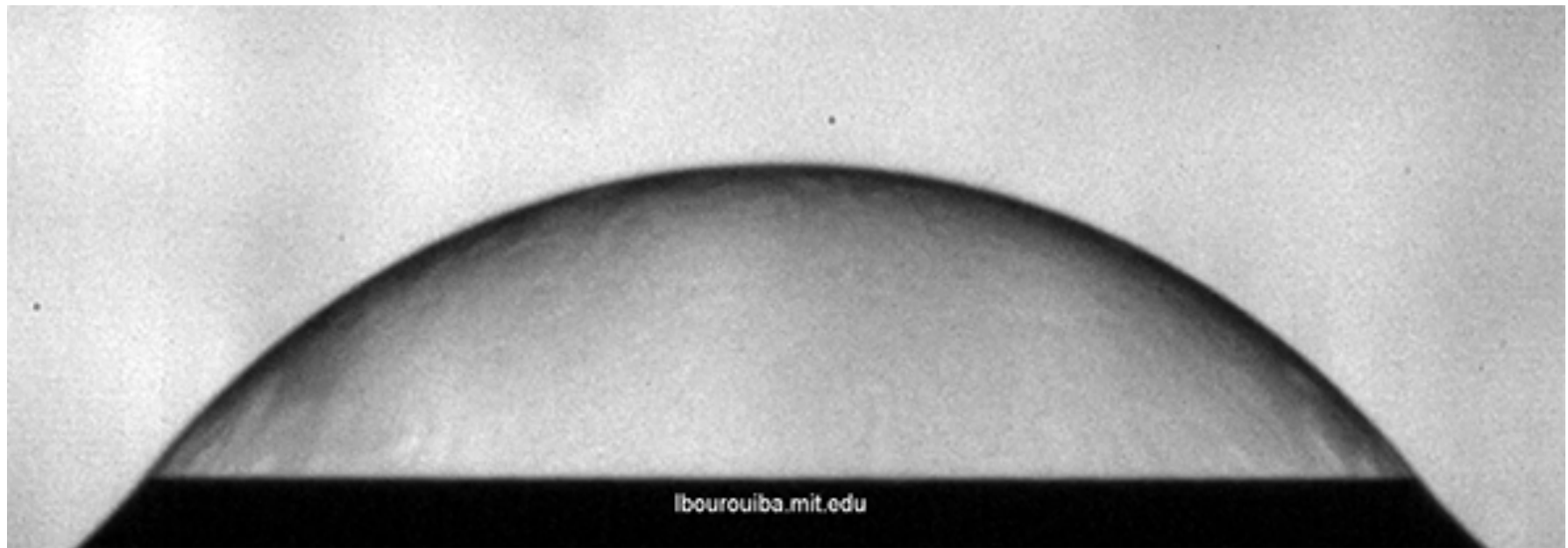
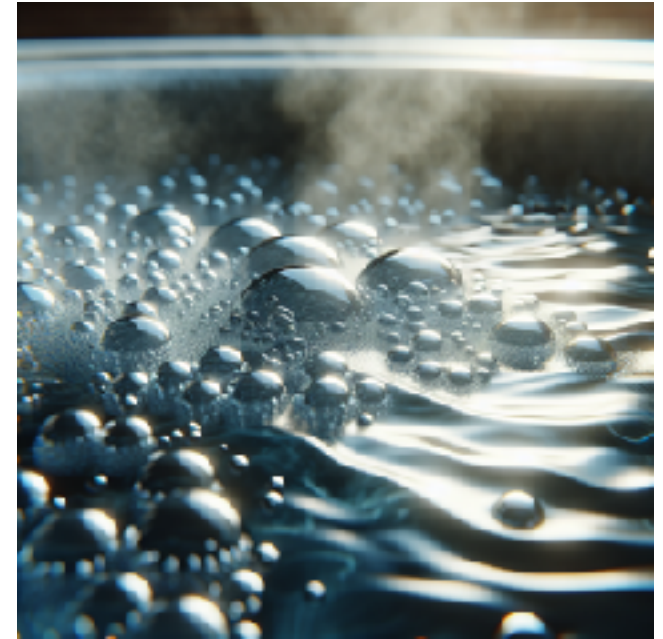
The exploding lakes of Camaroon



Bubble ablation in cancer treatment

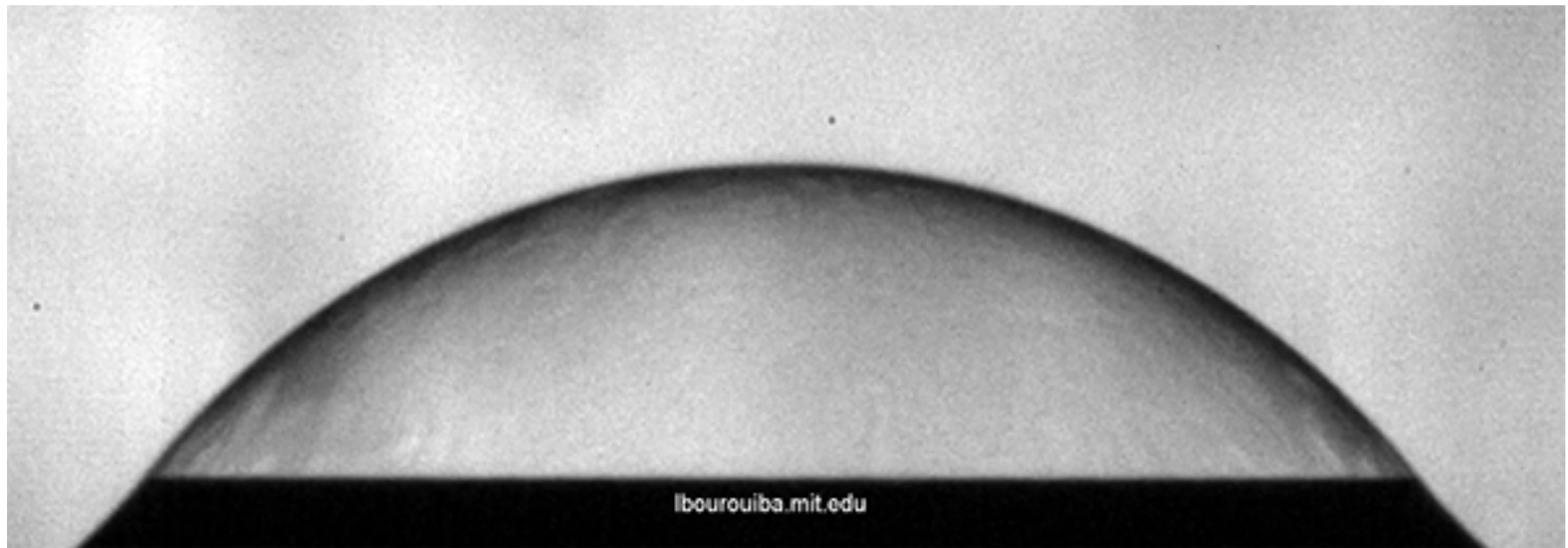
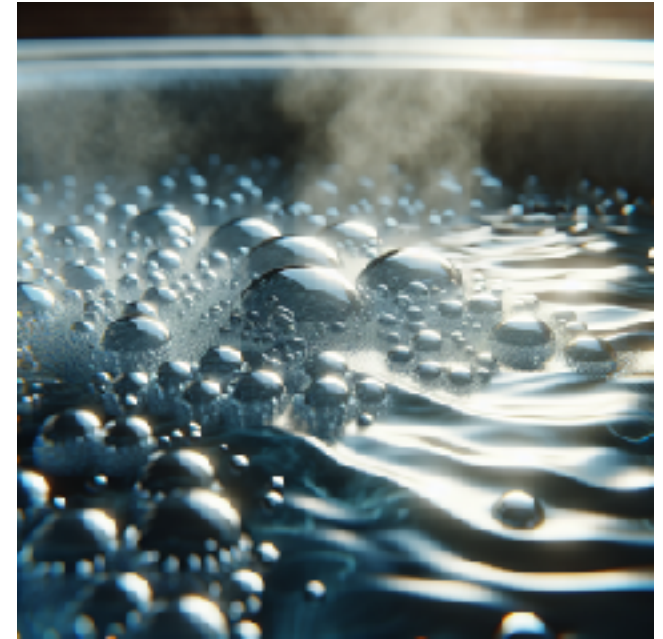


Disease transmission in hot tubs



Bursting bubbles release aerosols.

Disease transmission in hot tubs



Bursting bubbles release aerosols.

Motivation: who cares about surface tension?

Technology

- capillary effects dominant in **microgravity** settings: NASA
- cavitation-induced damage on propellers and submarines
- design of **superhydrophobic surfaces**
e.g. self-cleaning windows, drag-reduction, erosion-resistant surfaces
- inkjet printing
- **microfluidics**: continuous and discrete fluid transport and mixing
- **lab-on-a-chip technology**: medical diagnostics, drug delivery





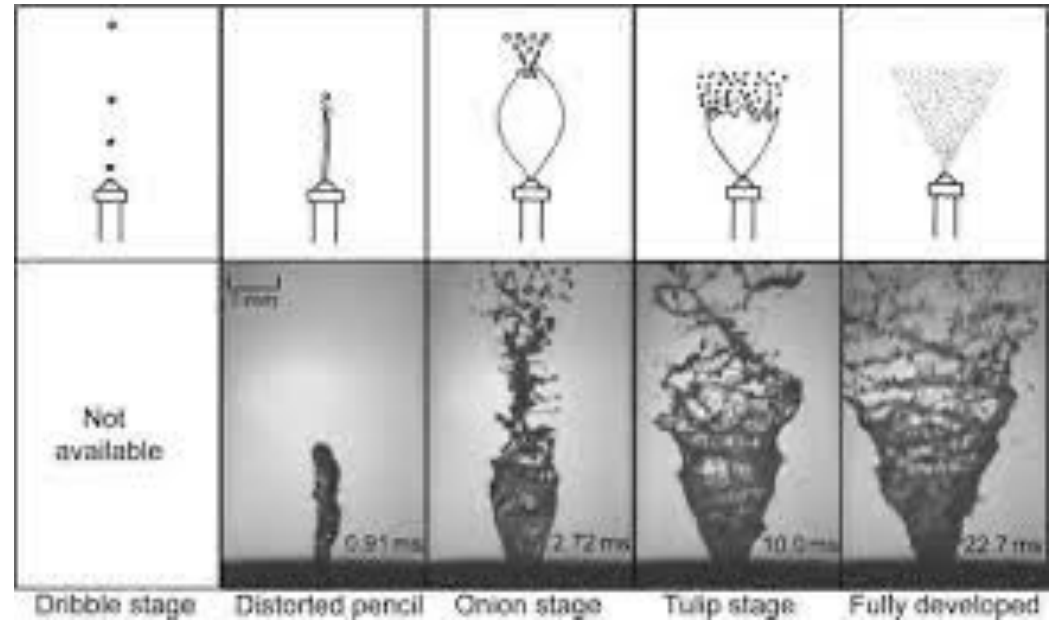
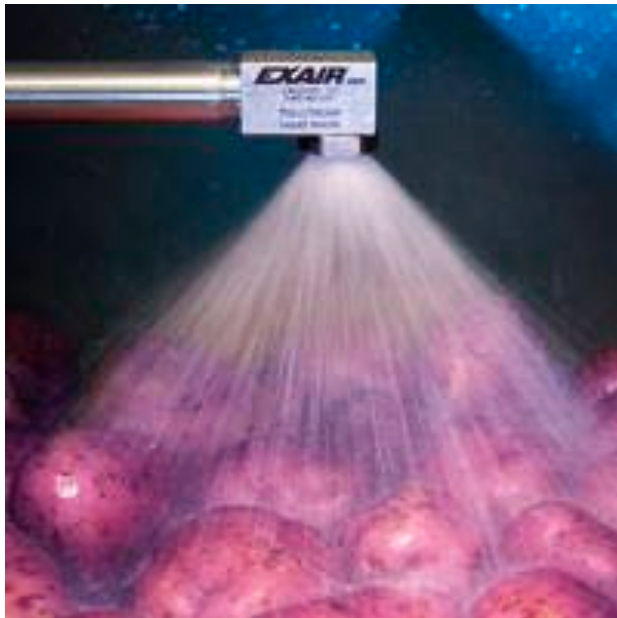
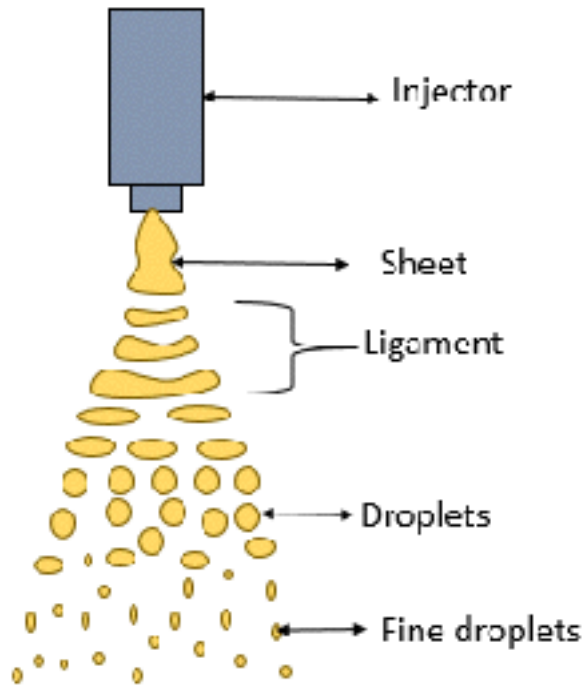
Drinking in space



Drinking in space

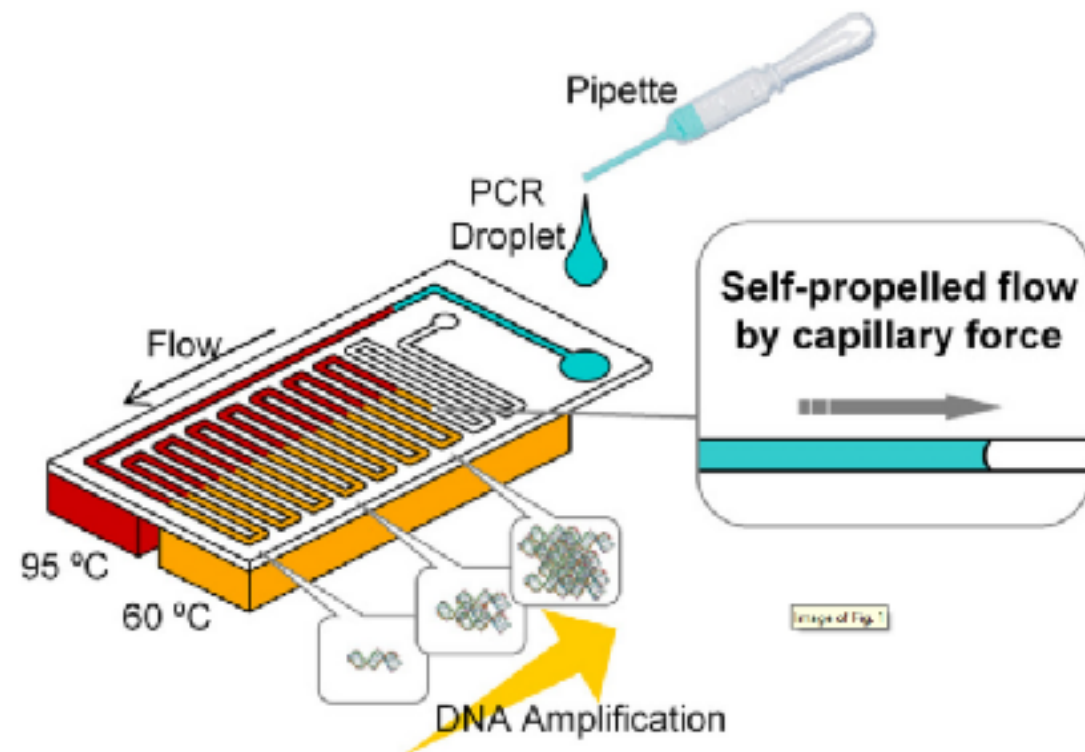
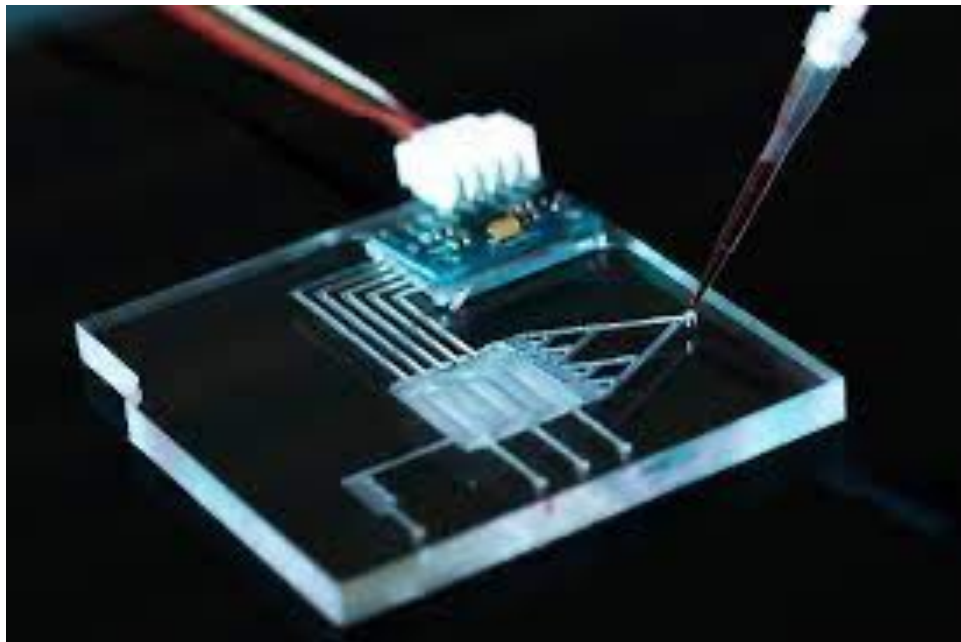


Industrial application: spray atomization



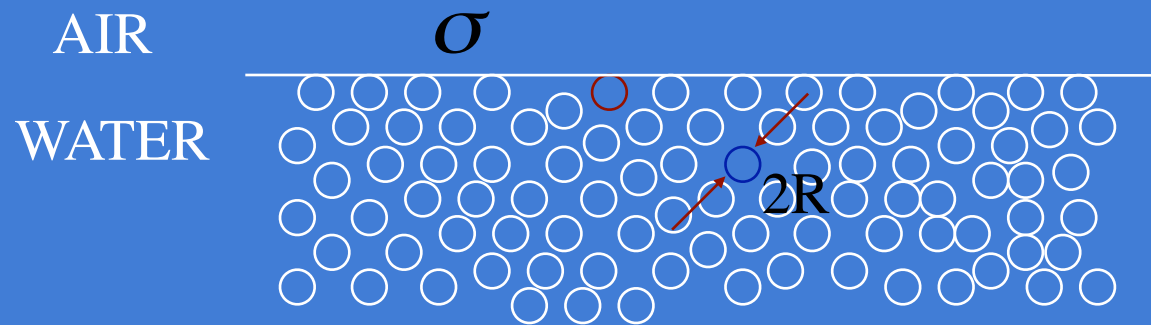
Microfluidics: *'lab on a chip'*

- anticipated in Feynman's 1959 Lecture: *'Plenty of Room at the Bottom'*
- manipulation of fluid in channels of typical scale 10-100 microns
- applications: capillary electrophoresis, DNA analysis, cell sorting
- diagnostics: cancer and pathogen detection
- digital microfluidics: drops and bubbles play the role of vessels



Surface Tension: molecular origins

- each molecule in a fluid feels a cohesive force with surrounding molecules
- molecules at interface feel half this force; are in an energetically unfavourable state
- the creation of new surface is thus energetically costly



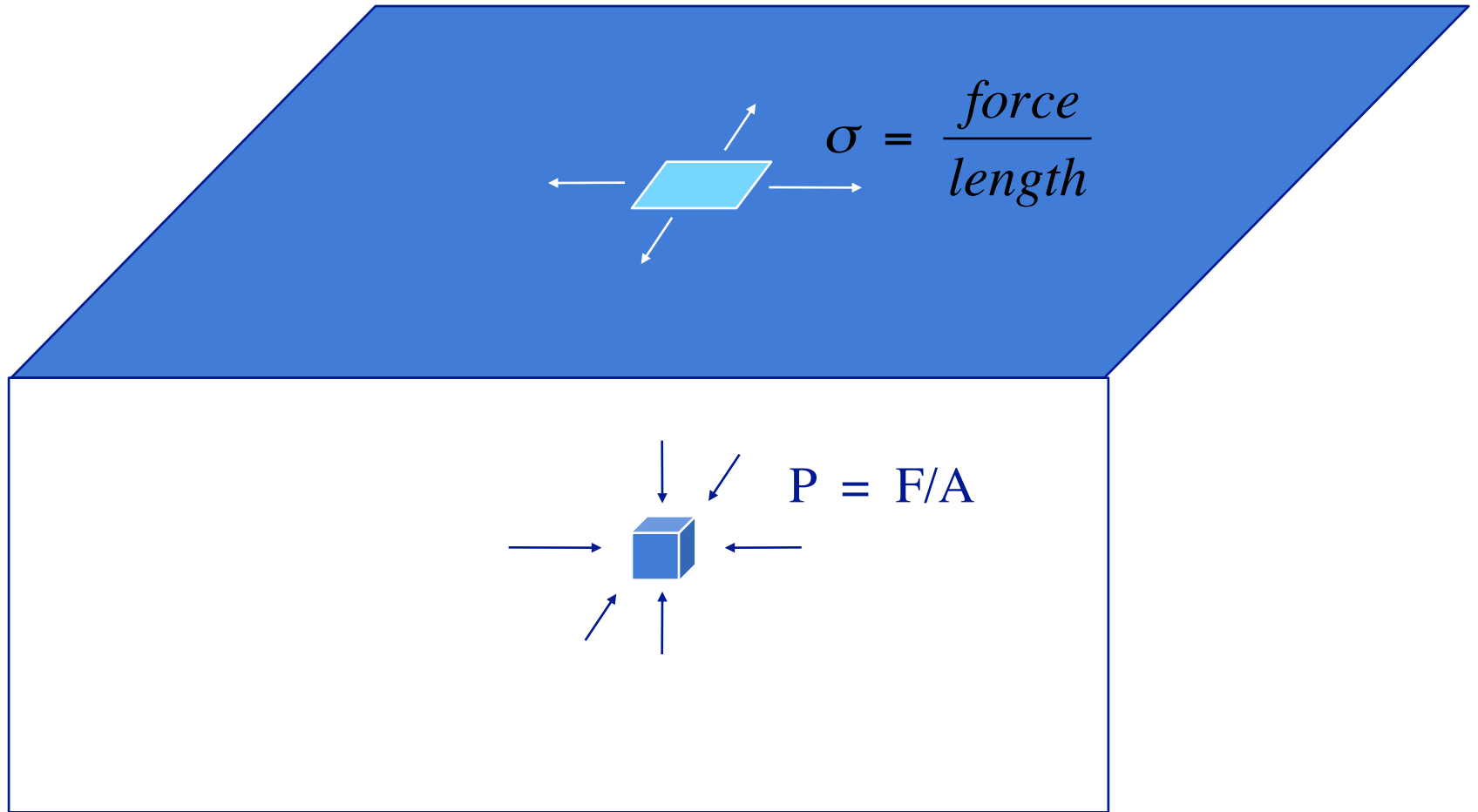
- cohesive energy per molecule of radius R in bulk is U , at surface is $U/2$
- surface tension is this loss of cohesive energy per unit area:

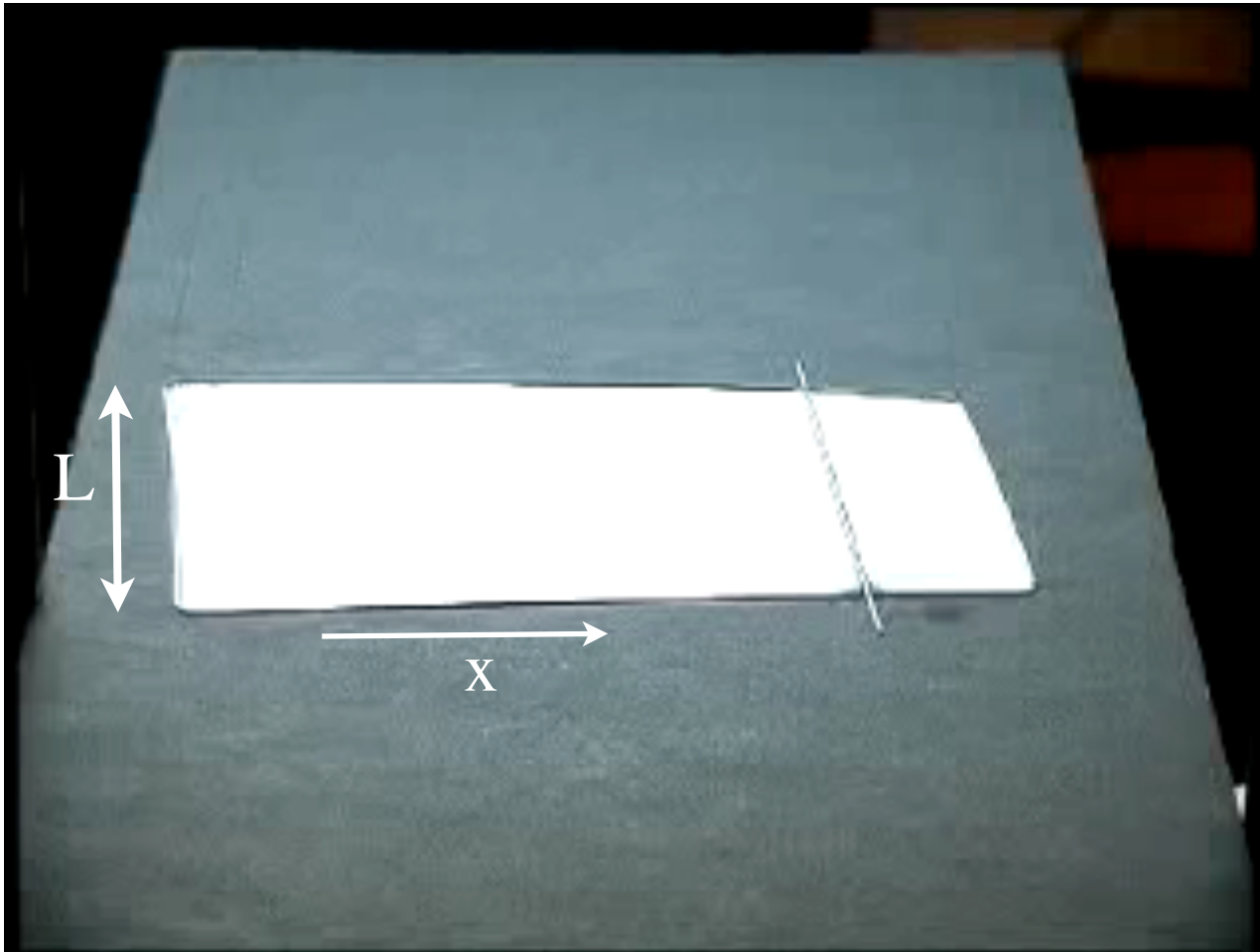
$$\sigma \sim \frac{U}{R^2}$$

Units: $[\sigma] = \frac{\text{ENERGY}}{\text{AREA}} = \frac{\text{FORCE}}{\text{LENGTH}}$

- air-water $\sigma \sim 70$ dyne/cm; oils $\sigma \sim 20$ dyne/cm; liquid metals $\sigma \sim 500$ dyne/cm

Surface tension: analogous to a negative surface pressure

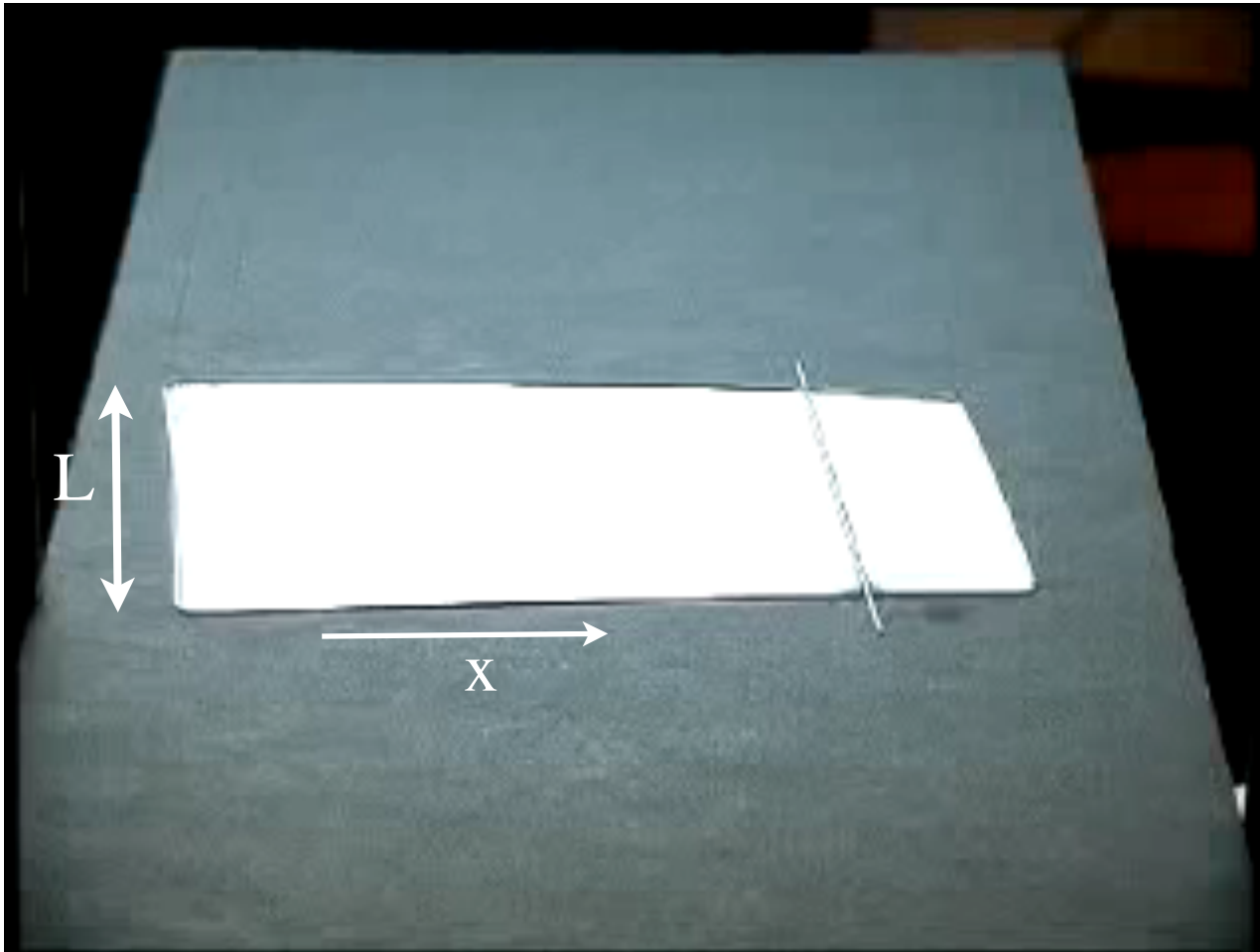




Surface tension: $[\sigma] = \frac{FORCE}{LENGTH} = \frac{ENERGY}{AREA}$

Surface energy: $E_{\sigma} = \int_S \sigma dA = 2 \sigma L x$

Force acting on rod: $F = \frac{dE_{\sigma}}{dx} = 2 \sigma L$

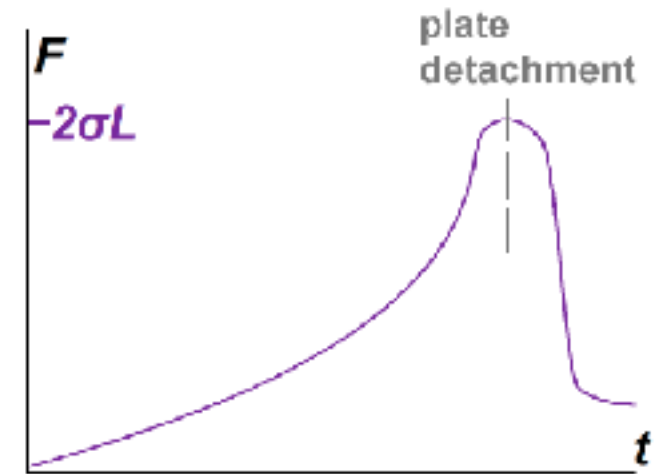
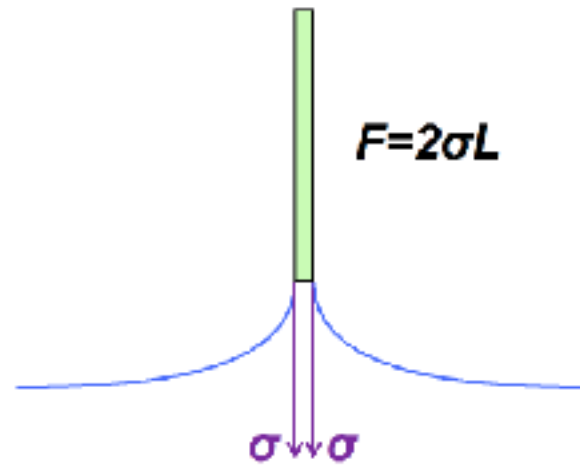
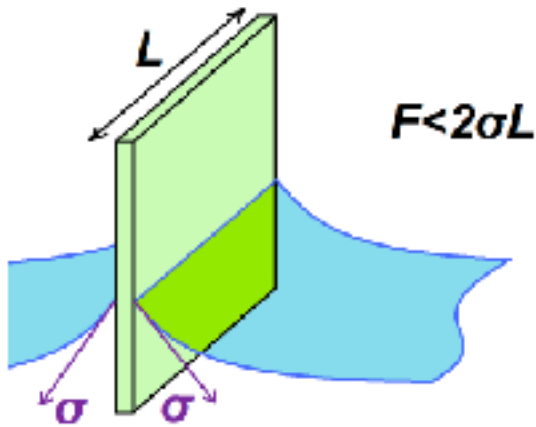


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A simple way to measure surface tension

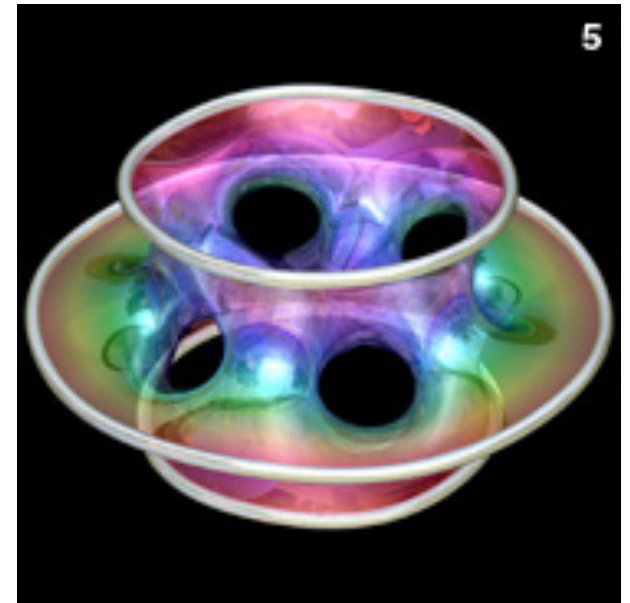
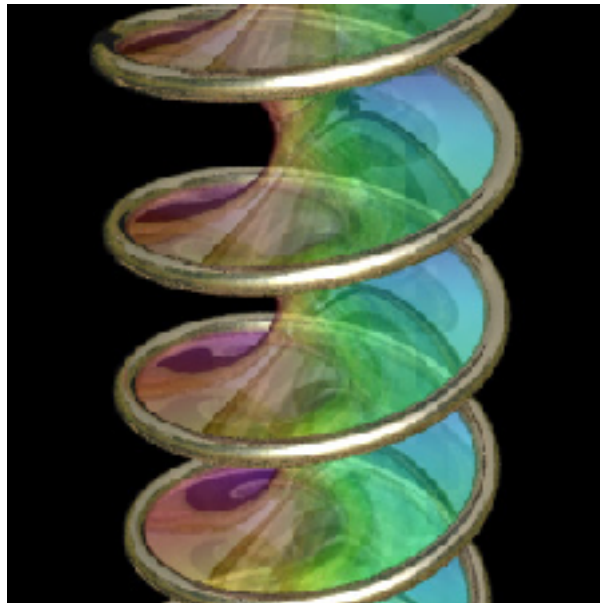
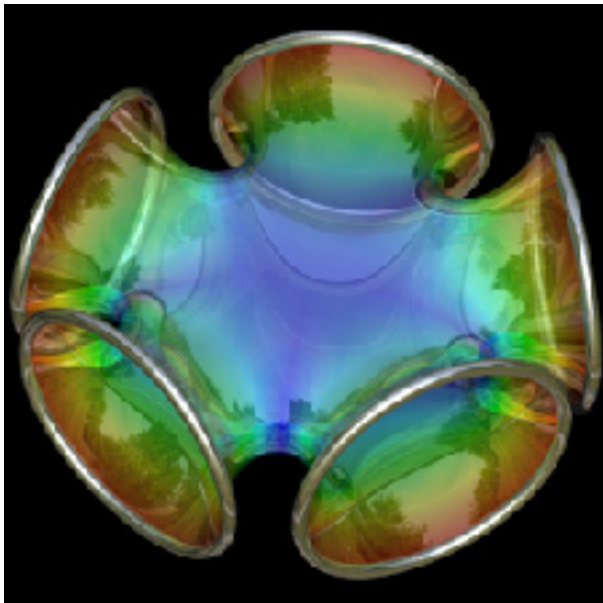
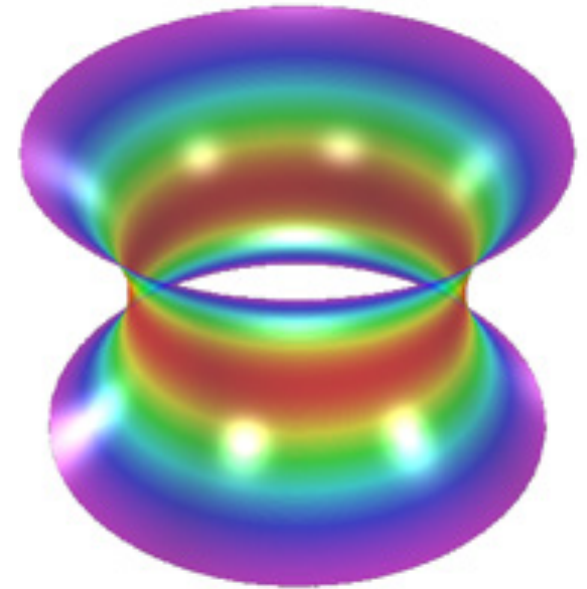


- measure the force required to withdraw a plate from a free surface

The creation of surface is energetically costly

- quasi-static soap films (for which gravity, inertia are negligible) take the form of **minimal surfaces**
- hence their interest to mathematicians:

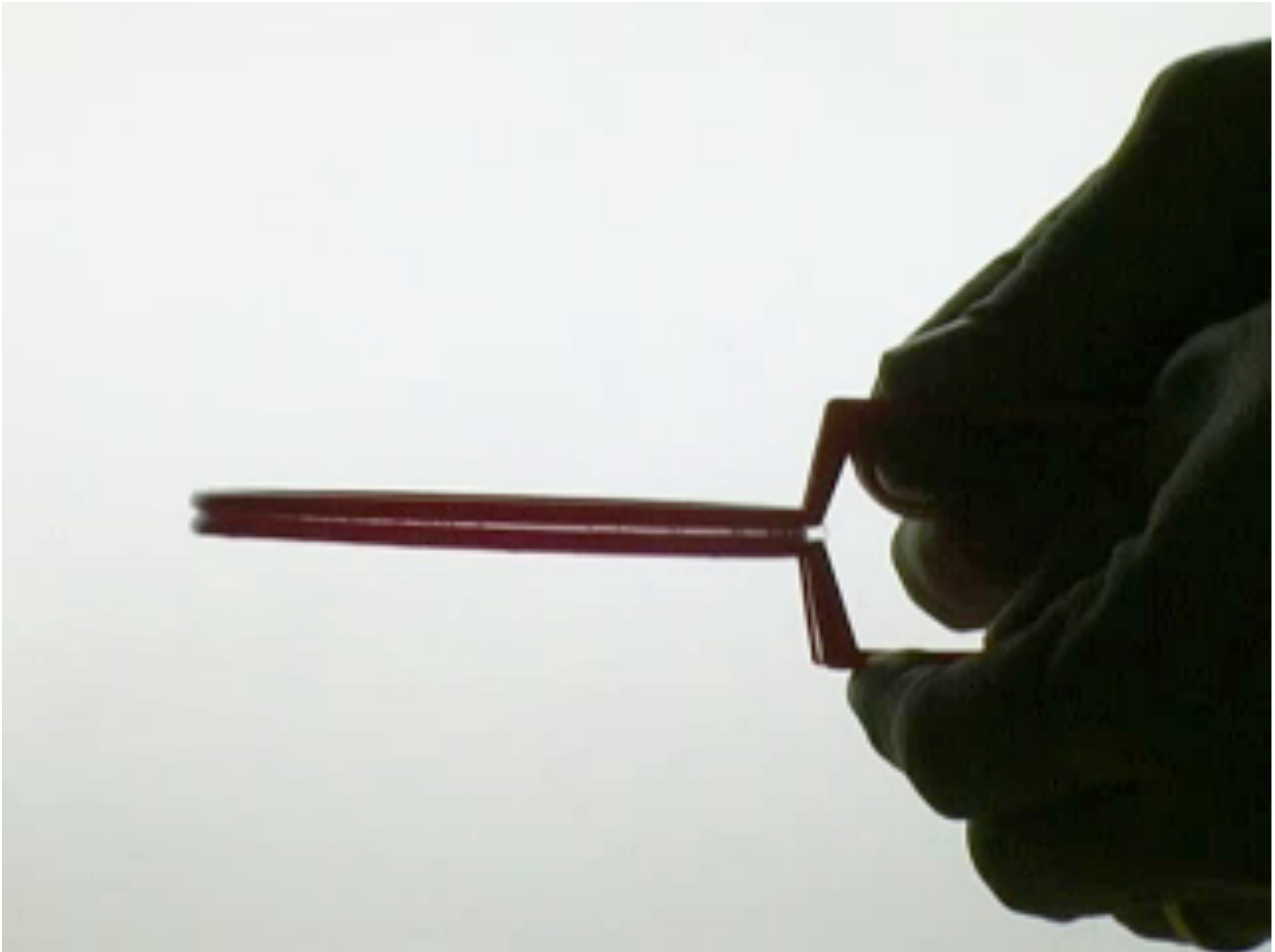
“Find the minimal surface bound by the multiply connected curve C , where C ”



Minimal surfaces: surface energy/area minimized by soap films



Minimal surfaces: surface energy/area minimized by soap films

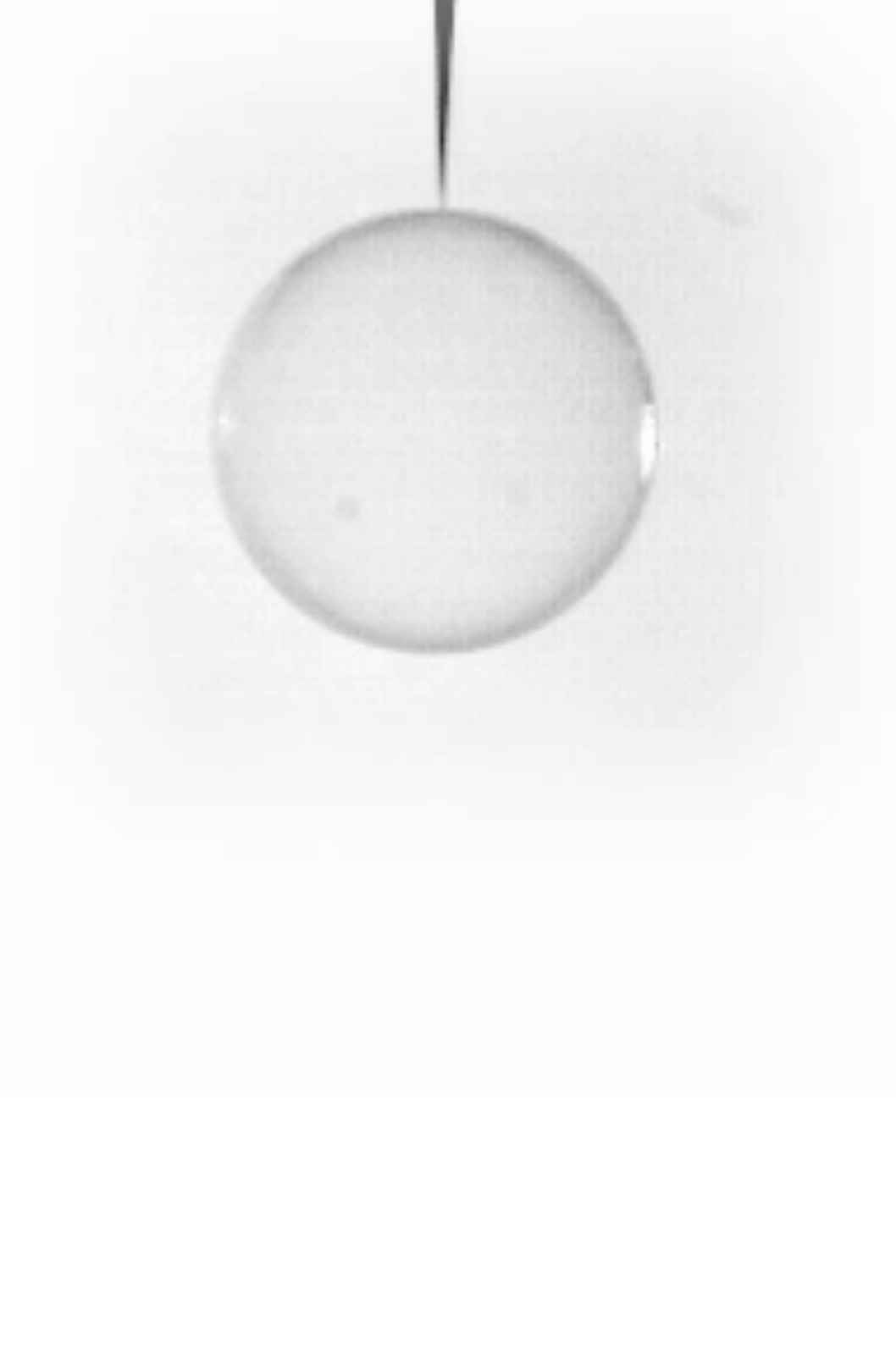


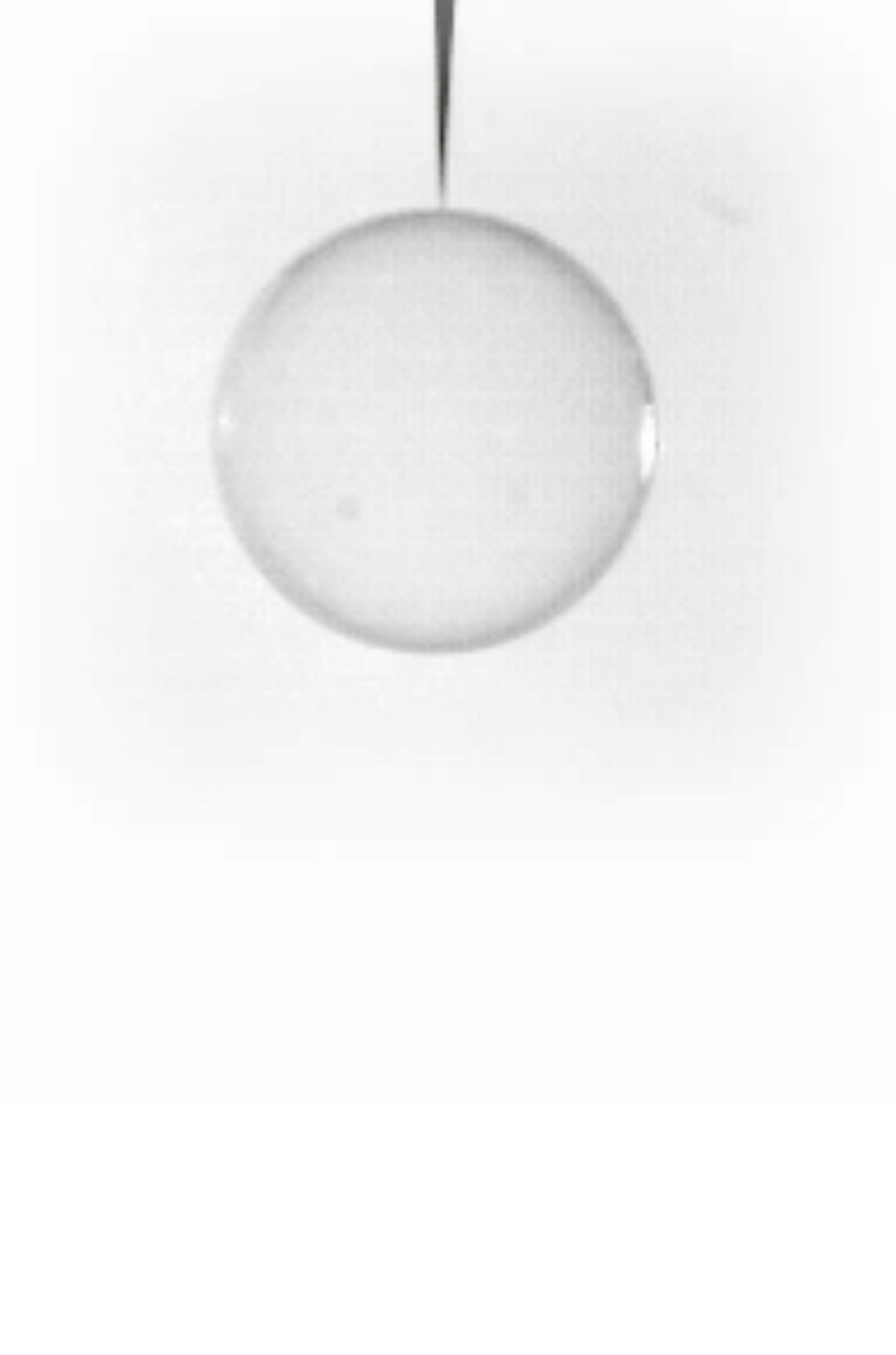
The creation of surface is energetically costly

Thus...

- small drops are nearly spherical
- fluid jets pinch off into droplets
- fluid atomization results in spherical drops
- bubbles and films are fragile
- wet hair sticks together: the “wet look”



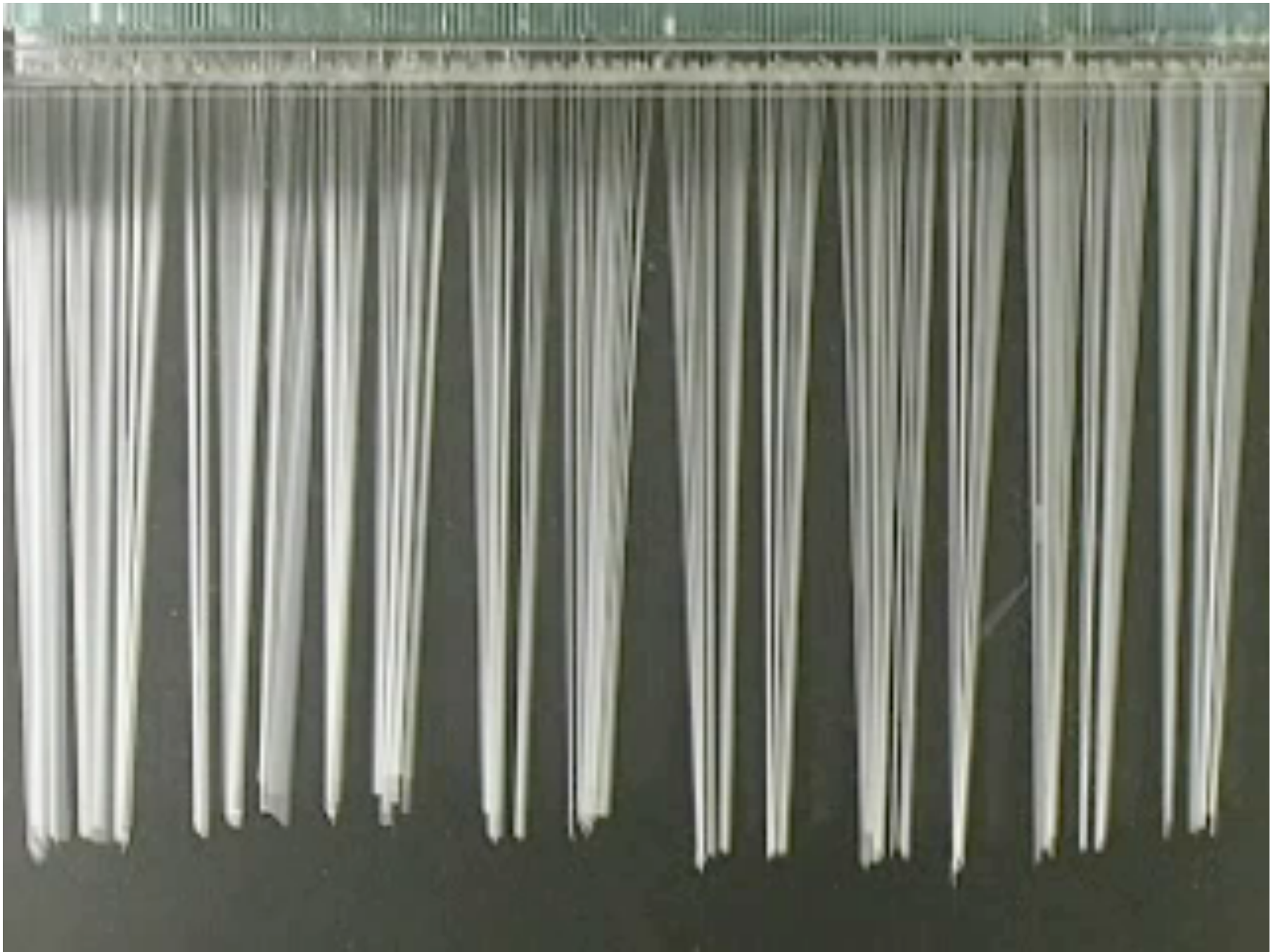




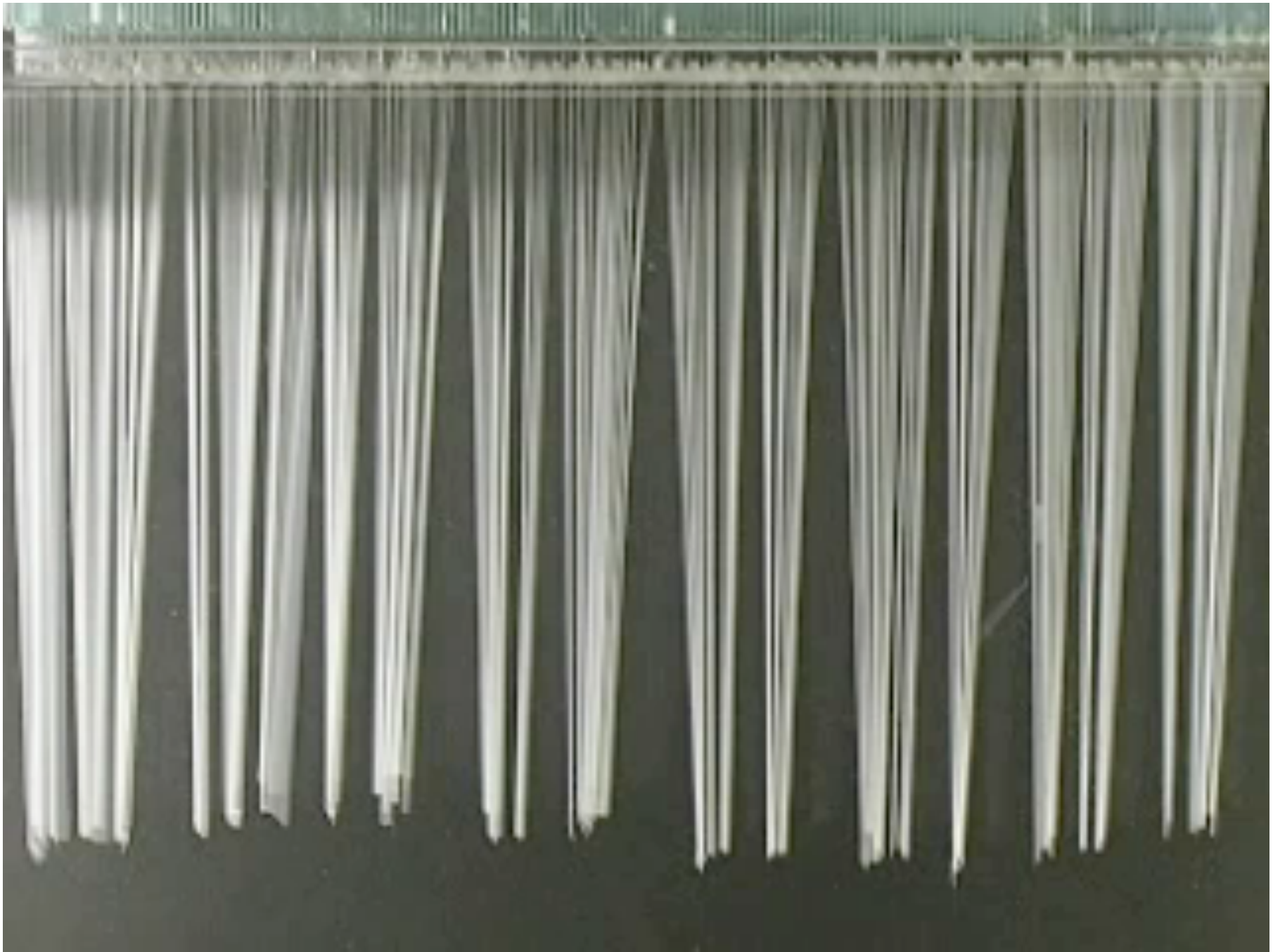
Hair: dry versus wet?



Why is dry hair more voluminous?

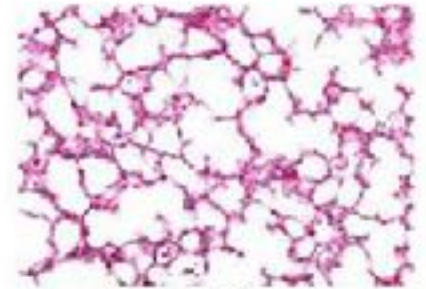
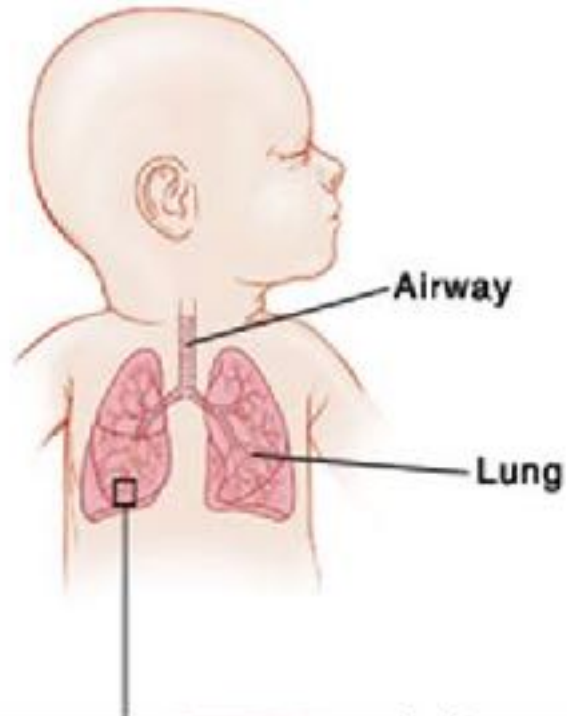


The wet hair instability: threads clump to minimize surface energy

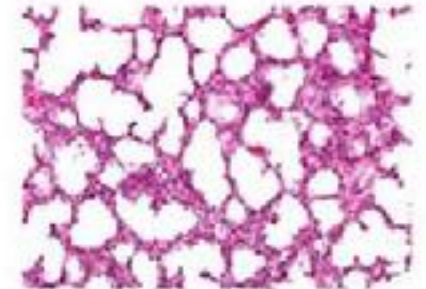


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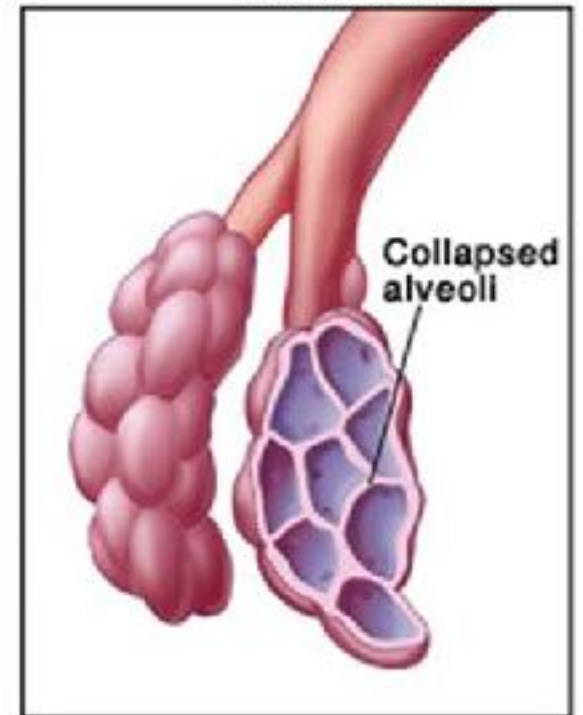
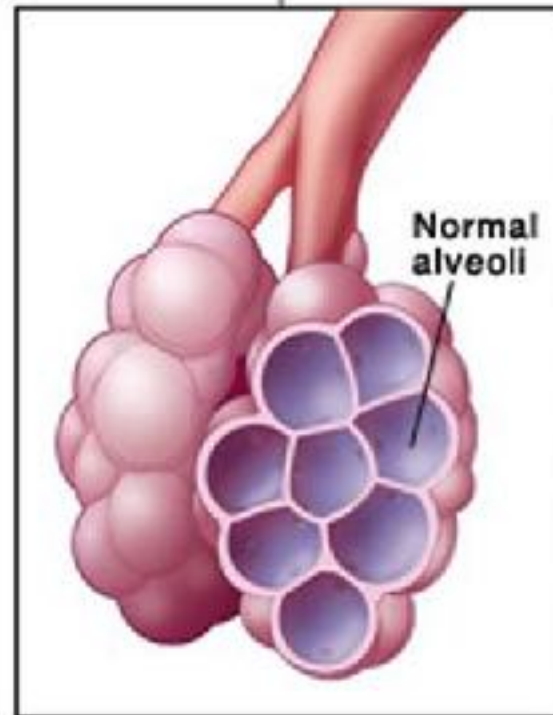
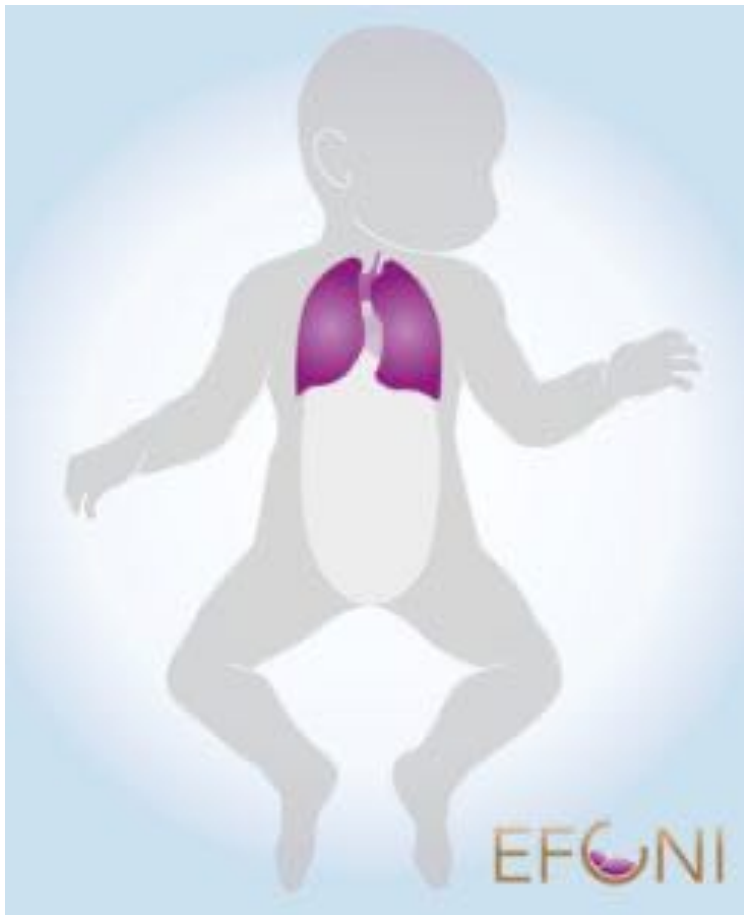
Reducing surface tension saves premature babies



Normal alveoli

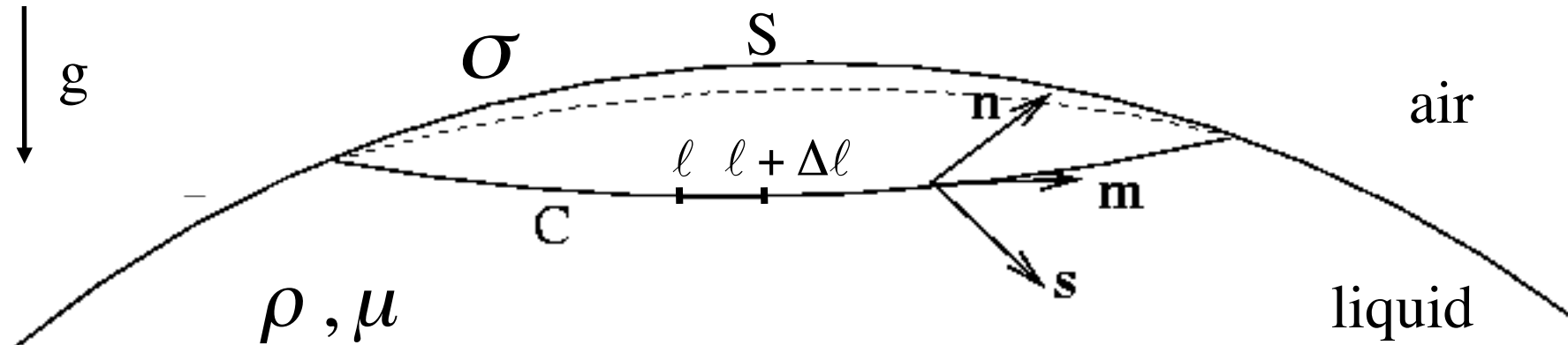


Collapsed alveoli



Surface tension: Geometry

Along any contour C bounding a surface S there is a tensile force per unit length σ acting in the \mathbf{s} direction



Net force on S :

$$\oint_C \sigma \mathbf{s} \, dl = \iint_S \sigma (\nabla \cdot \mathbf{n}) \mathbf{n} \, dS + \iint_S \nabla \sigma \, dS$$

curvature
pressure

Marangoni
stress

1) normal curvature pressure $\sigma \nabla \cdot \mathbf{n}$ resists surface deformation

2) tangential Marangoni stresses may arise from $\nabla \sigma$

Interfacial Fluid Dynamics: Governing Equations

Navier-Stokes:

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{u} \quad , \quad \nabla \cdot \mathbf{u} = 0$$

Boundary Conditions

Normal stress:

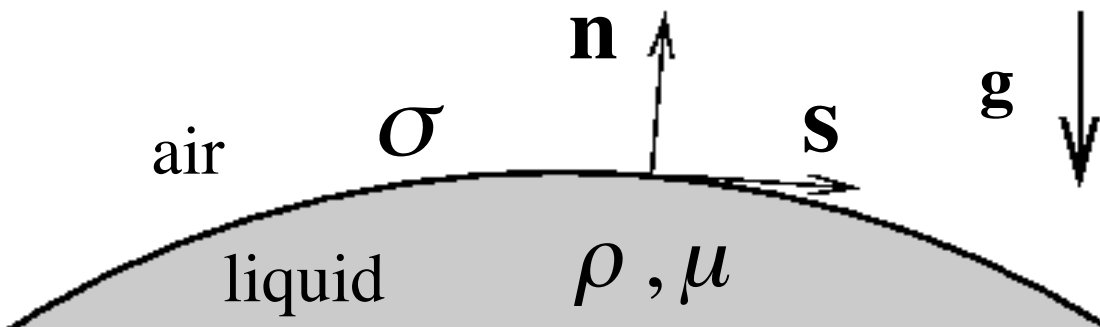
$$\Delta \mathbf{n} \cdot \mathbf{T} \cdot \mathbf{n} = \sigma \nabla \cdot \mathbf{n}$$

Tangential stress:

$$\Delta \mathbf{n} \cdot \mathbf{T} \cdot \mathbf{s} = \nabla_s \sigma$$

Stress tensor

$$\mathbf{T} = -p\mathbf{I} + \mu \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right)$$



Curvature pressure

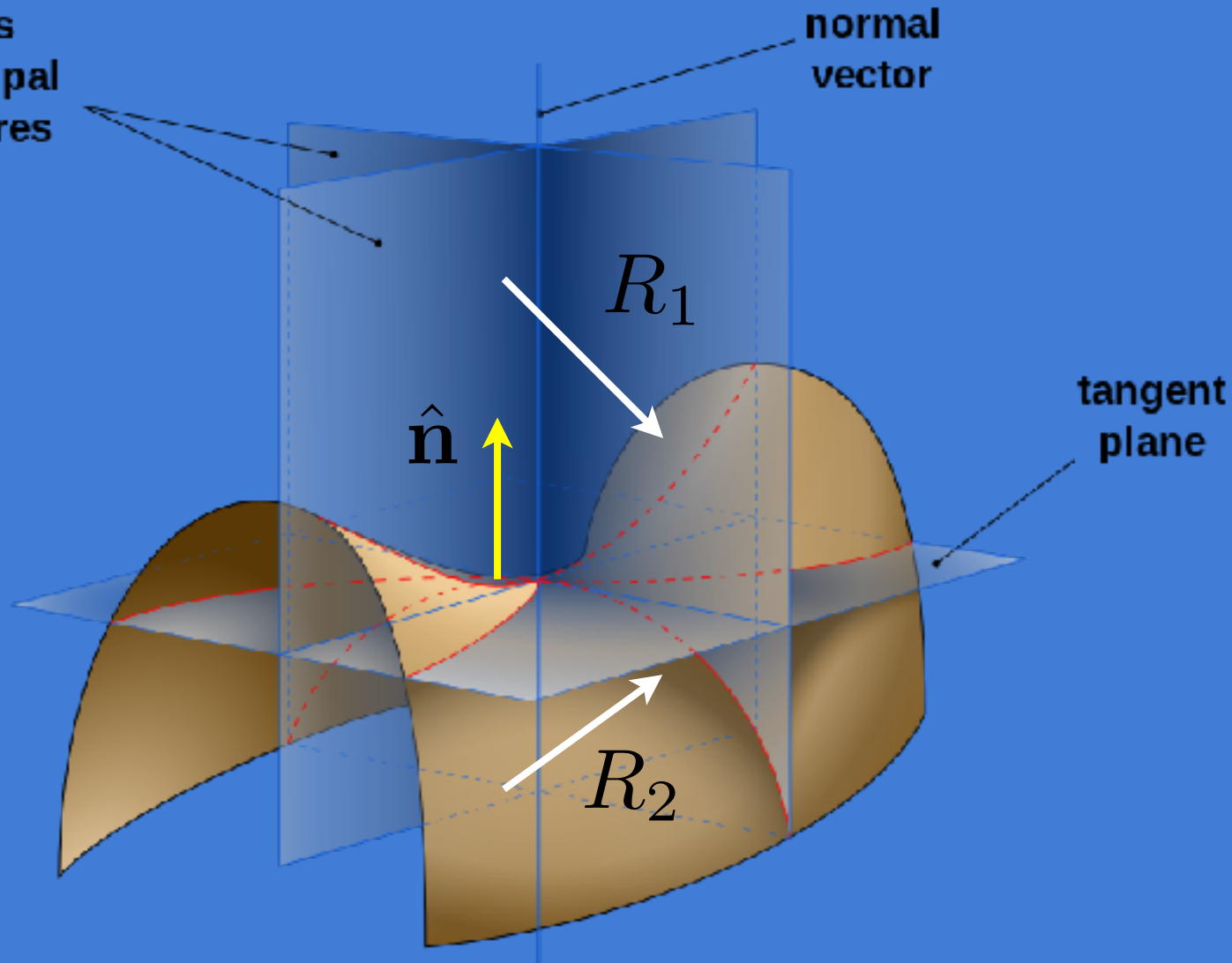
planes
of principal
curvatures

normal
vector

Laplace pressure

$$\Delta P = \sigma \nabla_s \cdot \hat{\mathbf{n}}$$

- pressure jump across a curved interface



Curvature:

$$\nabla \cdot \hat{\mathbf{n}} = \frac{1}{R_1} + \frac{1}{R_2}$$

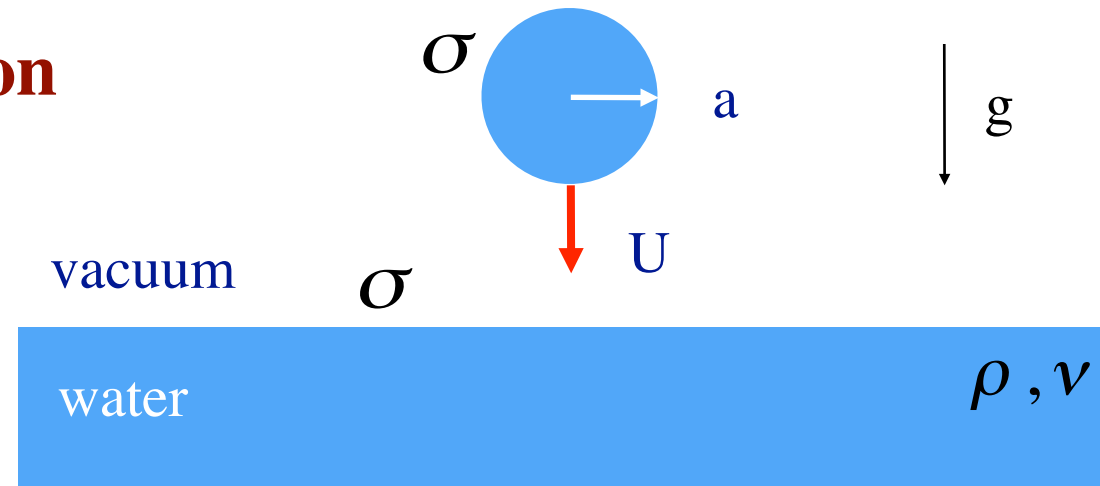
where R_1 , R_2 are the principal radii of curvature

Curvature pressures, $\sigma \nabla \cdot \mathbf{n}$, make the surface behave as a trampoline.



Image: Bo Zhu

The scaling of surface tension



$$W_e = \frac{\rho U^2 a}{\sigma} = \frac{\text{INERTIA}}{\text{CURVATURE}} = \text{Weber number}$$

$$C_a = \frac{\rho \nu U}{\sigma} = \frac{\text{VISCOSITY}}{\text{CURVATURE}} = \text{Capillary number}$$

$$B_o = \frac{\rho g a^2}{\sigma} = \frac{\text{GRAVITY}}{\text{CURVATURE}} = \text{Bond number}$$

Note: σ is dominant relative to gravity when $B_o < 1$

$$\text{i.e. } a < \left(\frac{\sigma}{\rho g} \right)^{1/2} = \ell_c = \text{capillary length} \sim 2\text{mm for air-water}$$

When is surface tension important relative to gravity?

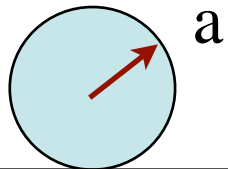
- when curvature pressures are large relative to hydrostatic:

Bond number:
$$B_o = \frac{\rho g a}{\sigma/a} = \frac{\rho g a^2}{\sigma} < 1$$

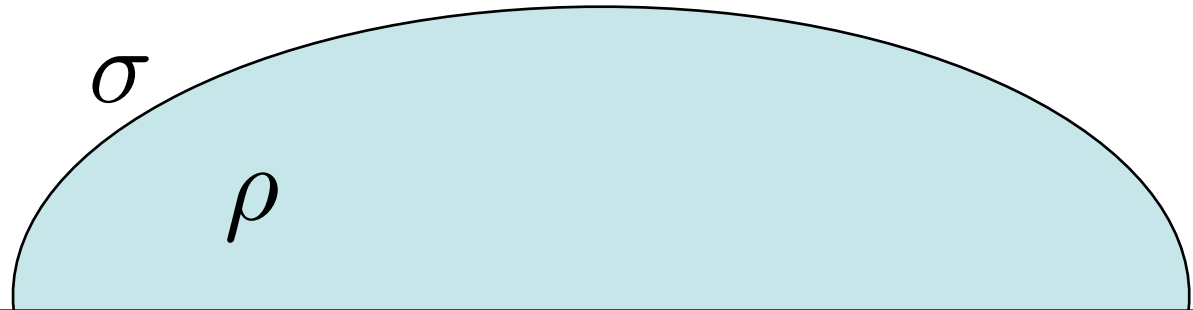
i.e. for drops small relative to the capillary length:

$$a < l_c = \left(\frac{\sigma}{\rho g} \right)^{1/2} \sim 2 \text{ mm for air-water} \quad (\sigma = 70 \text{ dynes/cm})$$

$B_o \ll 1$

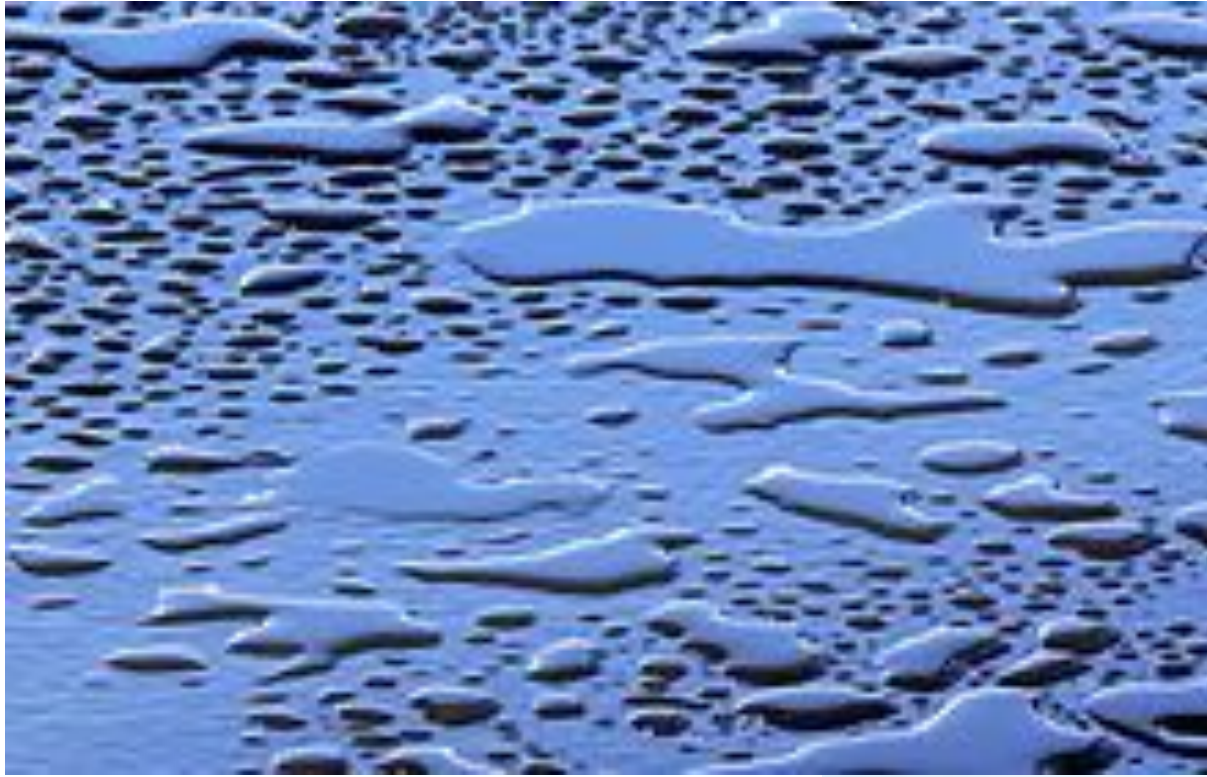


$B_o > 1$



Surface tension dominates the world of insects - and of microfluidics.

Puddles on flat surfaces



Falling rain drops

Force balance:

$$\rho_a U^2 a^2 \sim M g = \frac{4}{3} \pi a^3 \rho g$$

Fall speed: $U \sim \sqrt{\frac{\rho g a}{\rho_a}}$

Drop integrity requires:

$$\rho_a U^2 \sim \rho g a < \sigma / a$$

Small drops

If a drop is small relative to the capillary length

$$a < \ell_c = \sqrt{\sigma / \rho g} \approx 2\text{mm}$$

σ maintains it against the destabilizing influence of aerodynamic stresses.

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Large drops

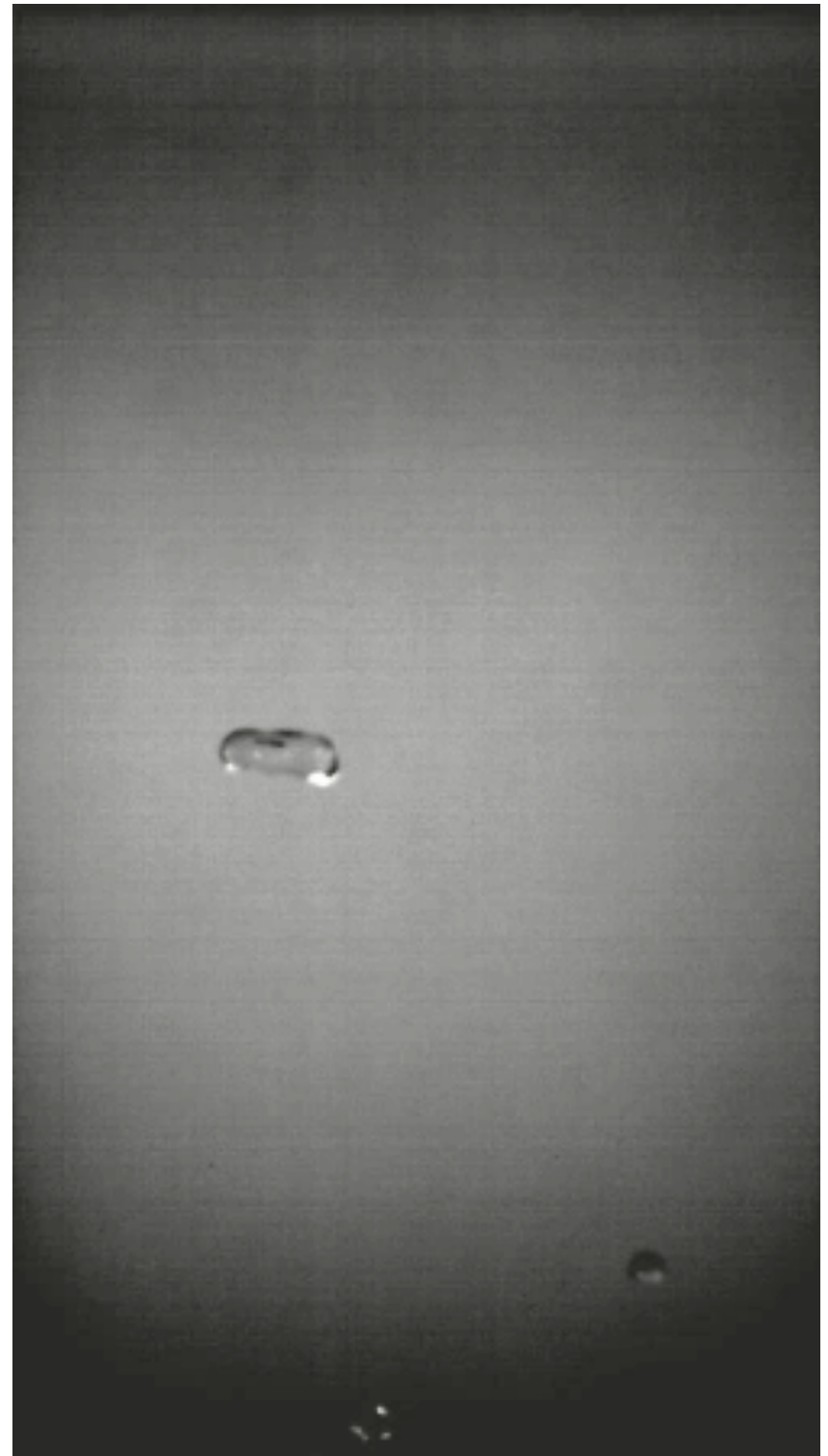
Drops larger than the capillary length

$$a > \ell_c \approx 2\text{mm}$$

break up under the influence of aerodynamic stresses.

The break-up yields drops with size of order:

$$\ell_c \approx 2\text{mm}$$



Large drops

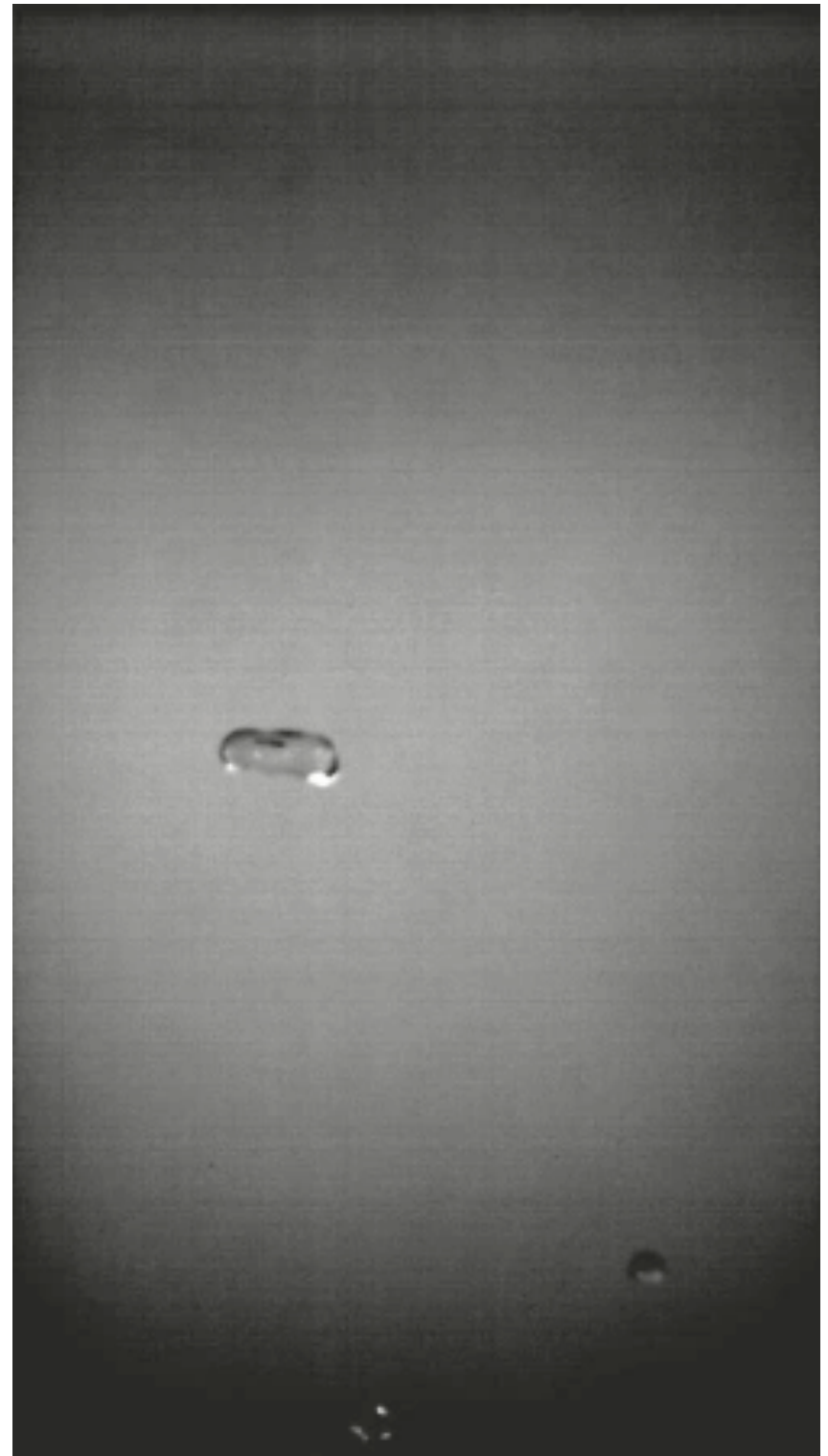
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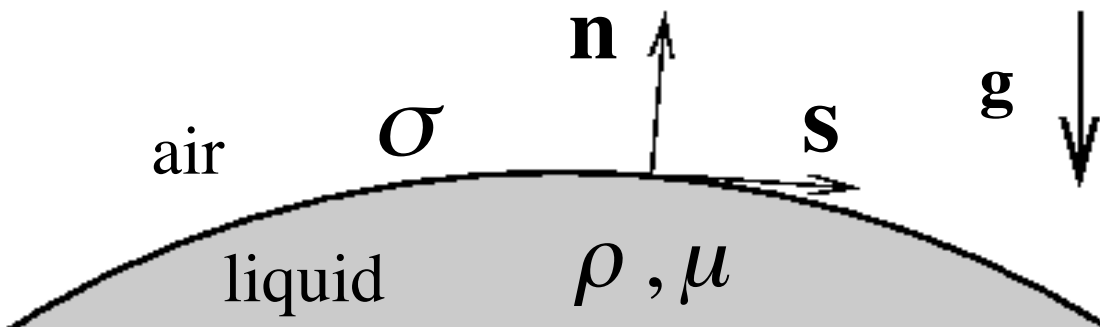
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Tangential stress:

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Stress tensor

$$\mathbf{T} = -p\mathbf{I} + \mu \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right)$$



Fluid Statics

$$\mathbf{T} = -p\mathbf{I} \quad , \quad \hat{\mathbf{T}} = -\hat{p}\mathbf{I}$$

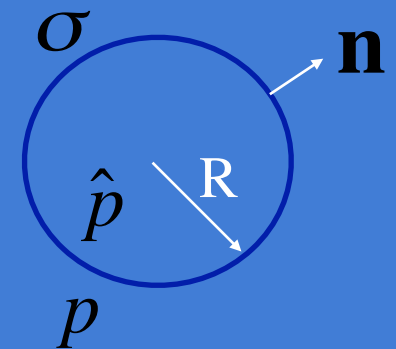
Normal stress balance: $\Delta \mathbf{n} \cdot \mathbf{T} \cdot \mathbf{n} = \sigma (\nabla \cdot \mathbf{n}) \longrightarrow \hat{p} - p = \sigma \nabla \cdot \mathbf{n}$

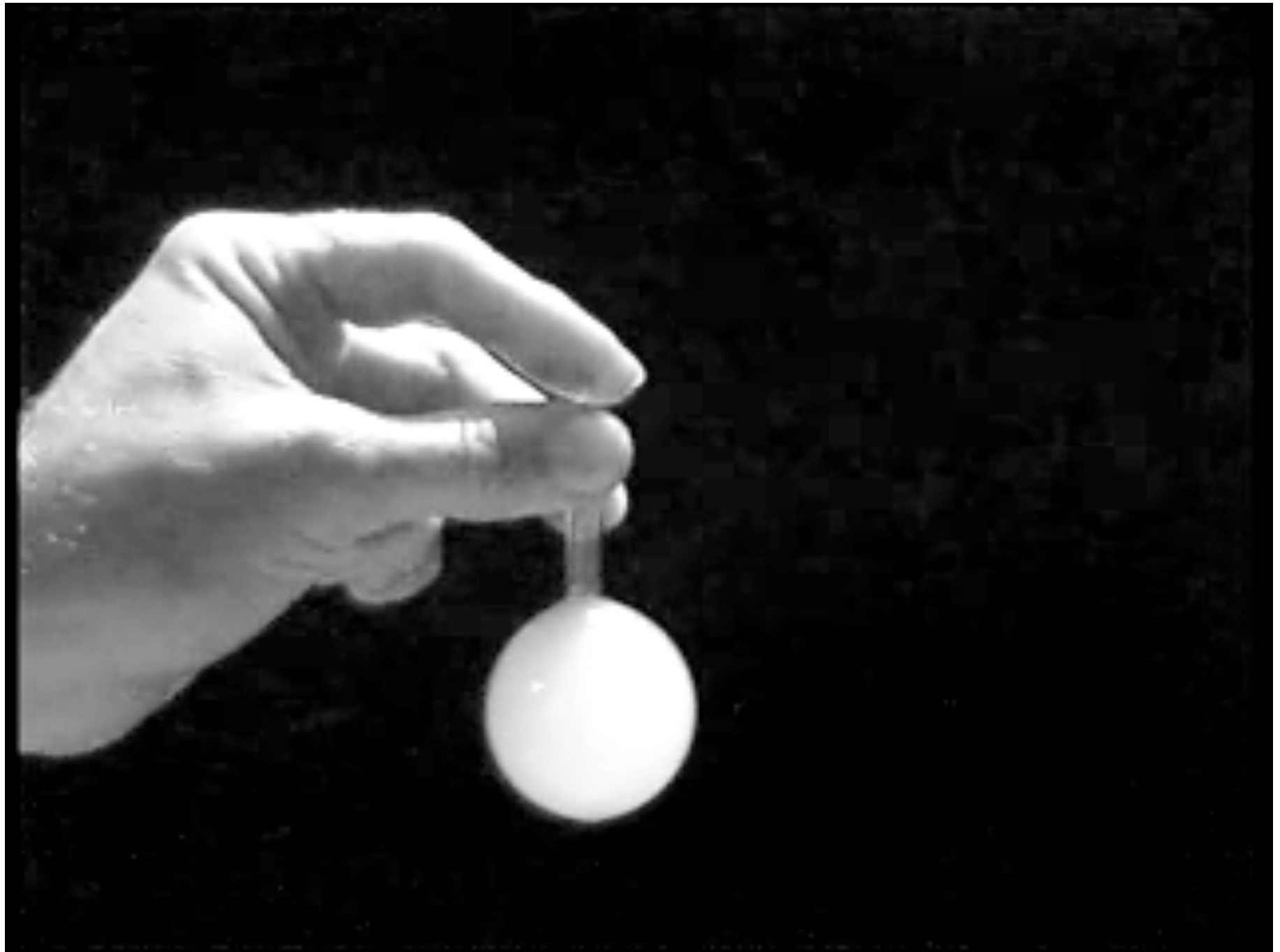
Tangential stress balance: $\Delta \mathbf{n} \cdot \mathbf{T} \cdot \mathbf{t} = \nabla \sigma \longrightarrow 0 = \nabla \sigma$

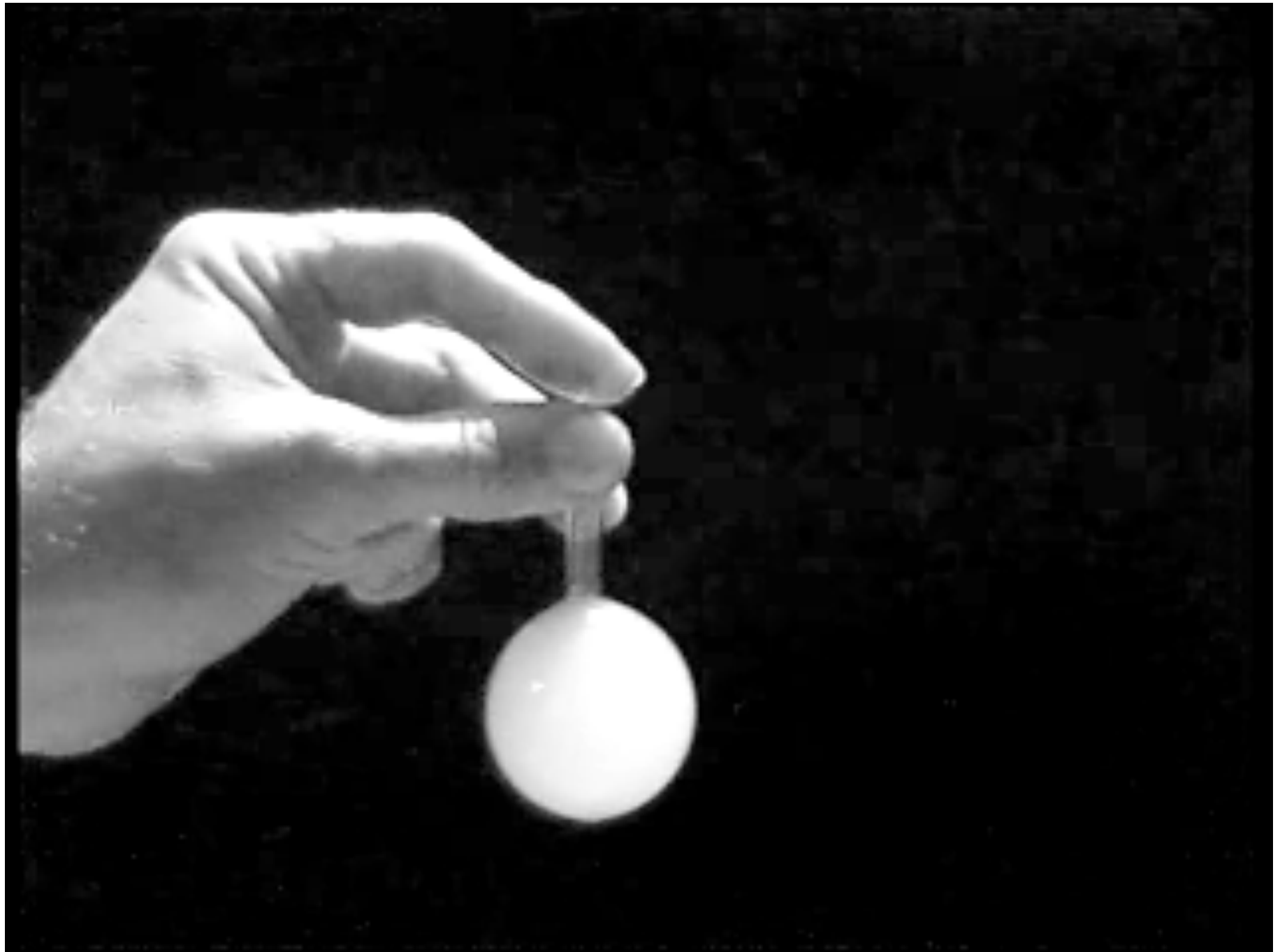
Stationary bubble: what is the pressure drop across a bubble surface?

$$\hat{p} - p = \sigma \nabla \cdot \mathbf{n} = \frac{2\sigma}{R}$$

- smaller bubbles burst more loudly than large ones
- champagne is louder than beer





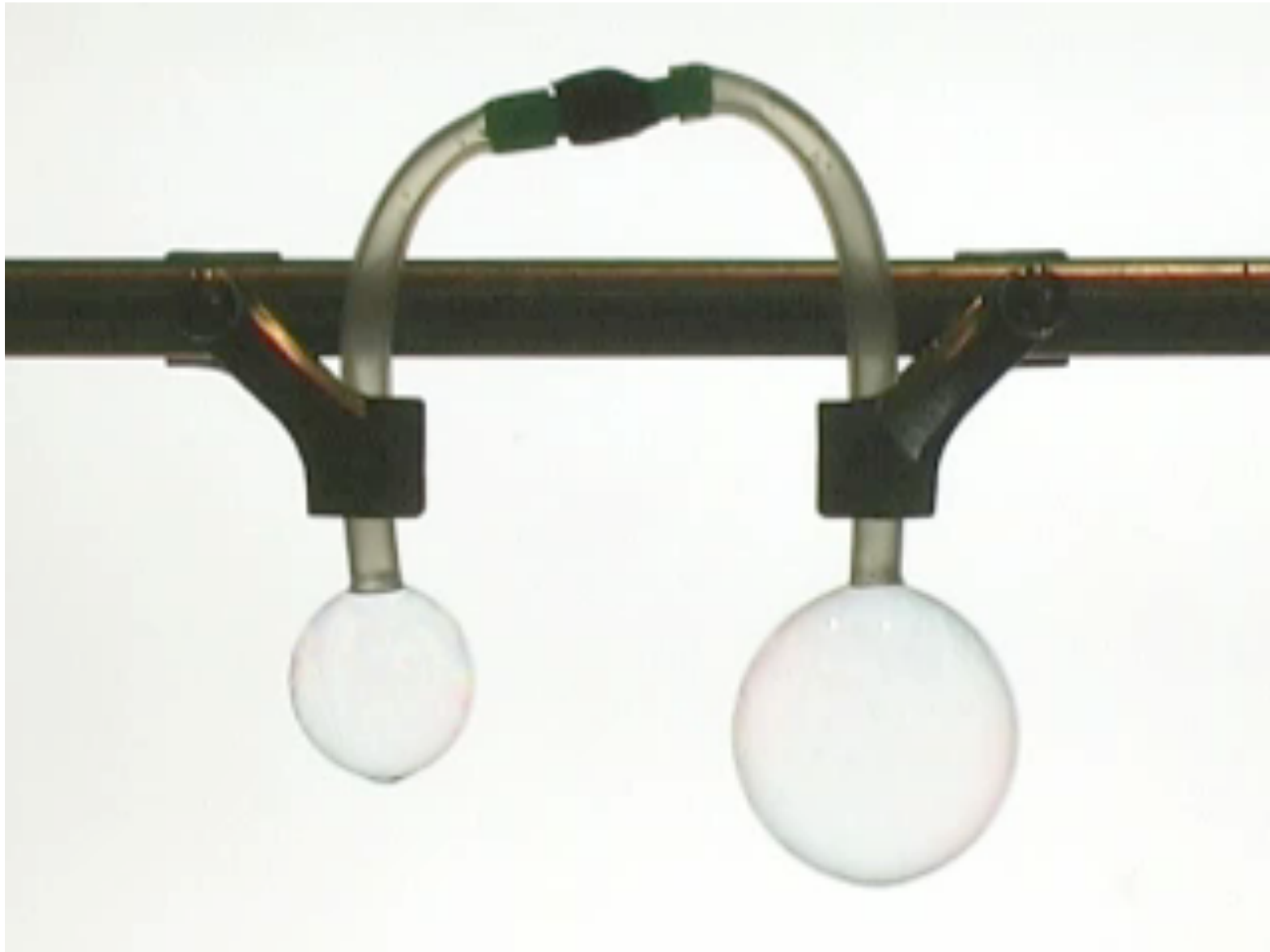


Capillary pressure



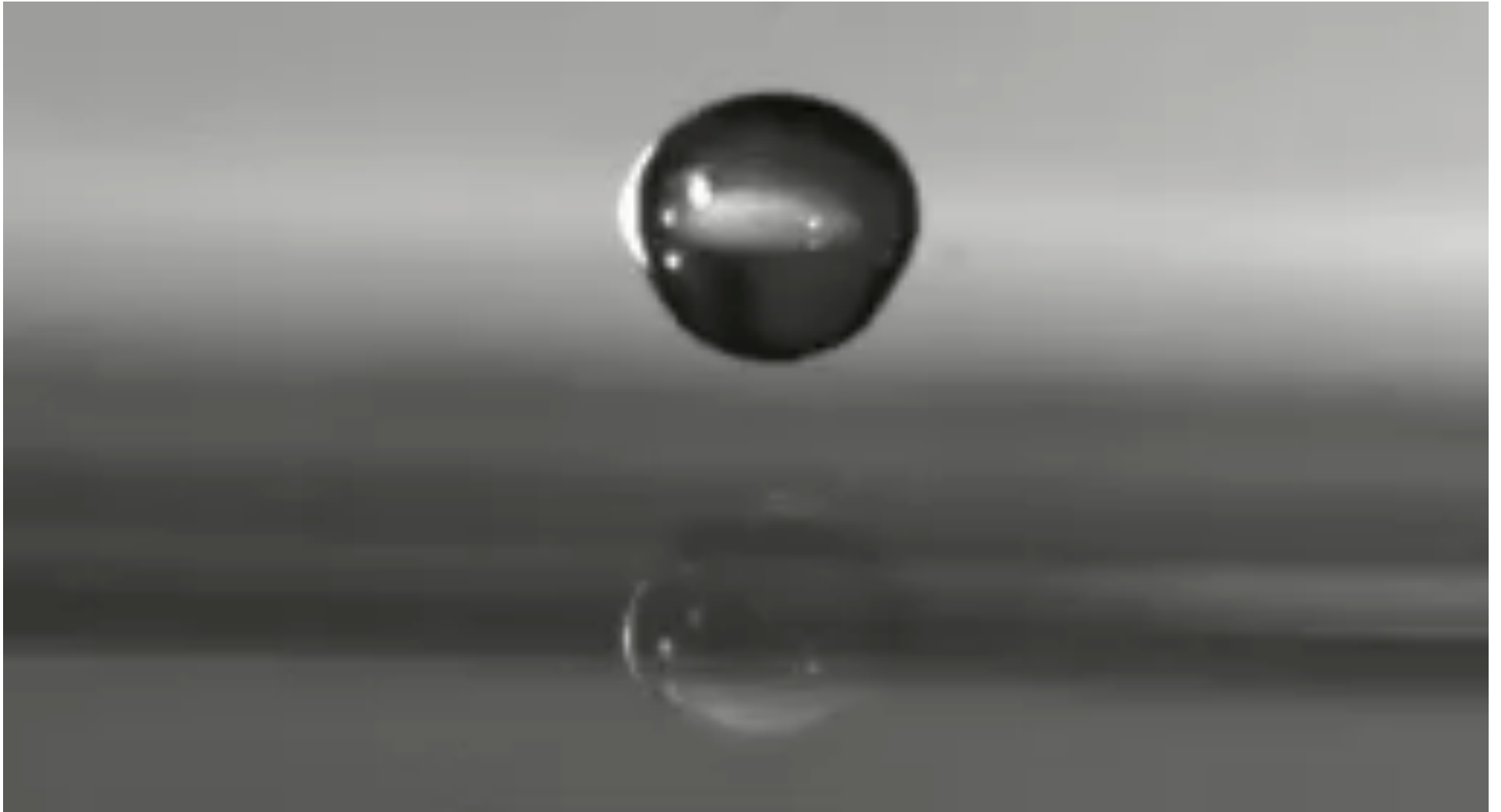
Which way does the air go?

Capillary pressure

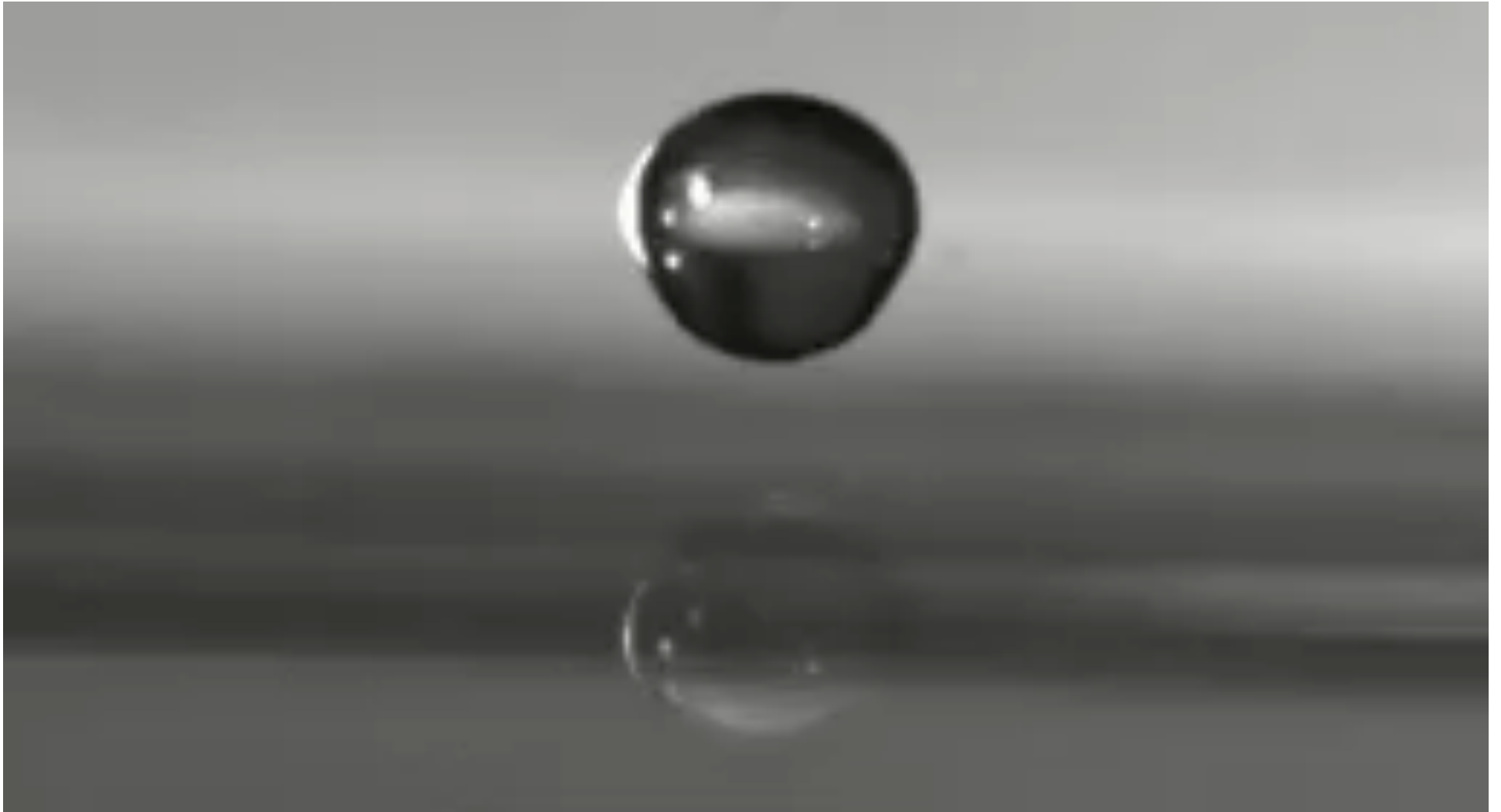


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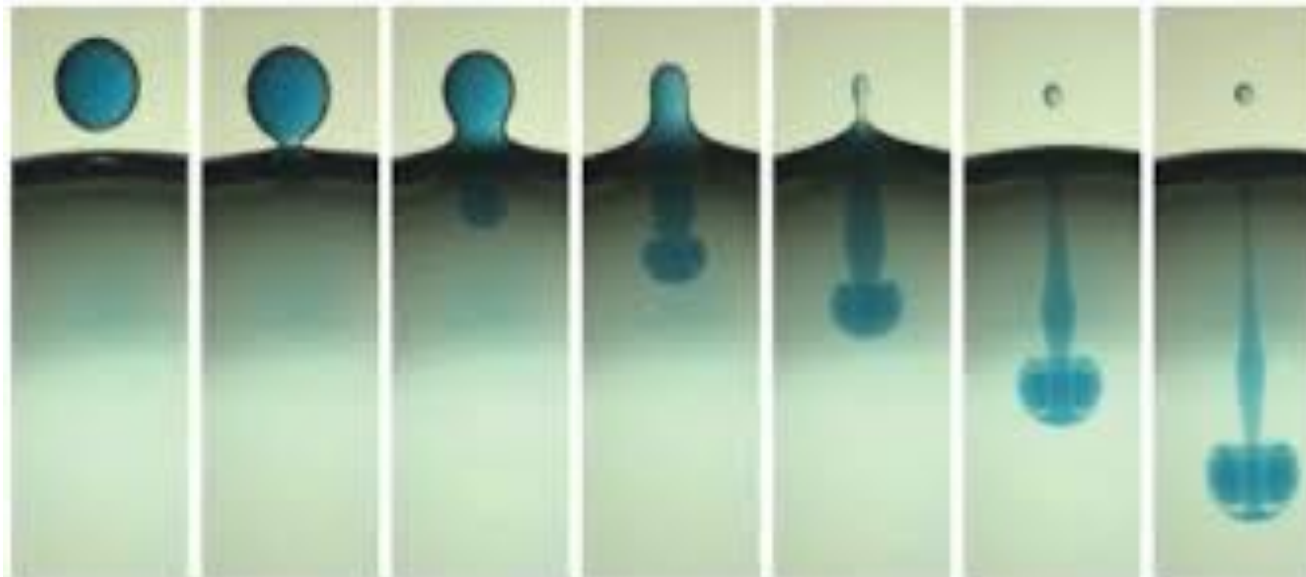
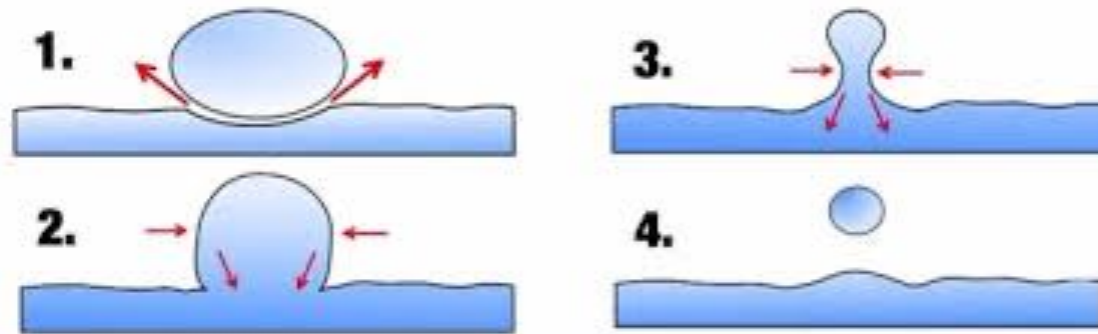
The coalescence cascade



The coalescence cascade



Vortex generation following drop coalescence



Coalescence Cascade: Scaling

- coalescence forced by combination of gravity, capillary pressure. Which?

$$\frac{\text{gravity}}{\text{curvature}} \sim \frac{\rho g R_1}{\sigma / R_1} = B_o \ll 1$$

- coalescence resisted by combination of inertia and viscosity. Which?

$$\frac{\text{inertia}}{\text{viscosity}} \sim \frac{U R_1}{\nu} = \text{Re} \gg 1 \Rightarrow$$

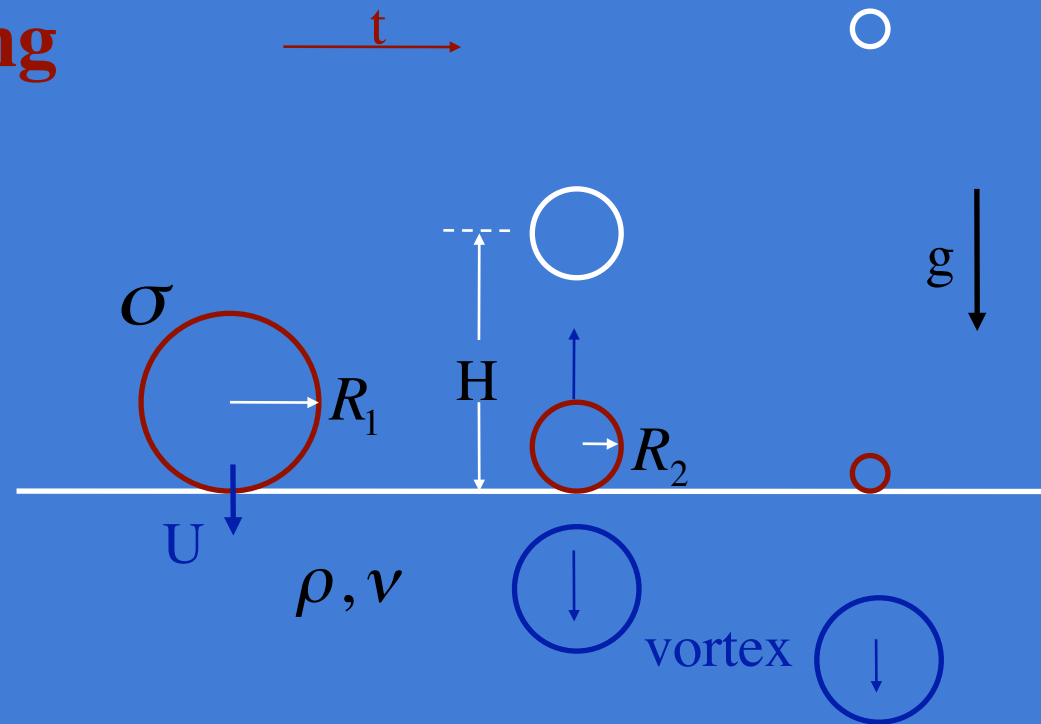
coalescence forced by capillary pressure, resisted by inertia

Vortex speed: $\rho U^2 \sim \sigma / R_1 \Rightarrow U \sim \sqrt{\sigma / \rho R_1}$

Volume change: $\Delta V = V_1 - V_2 \sim \pi R_1^2 U \tau \sim \pi R_1^3$

$$\Rightarrow \Delta V = \frac{4\pi}{3} (R_1^3 - R_2^3) \approx \pi R_1^3 \Rightarrow R_2 \approx 0.6 R_1$$

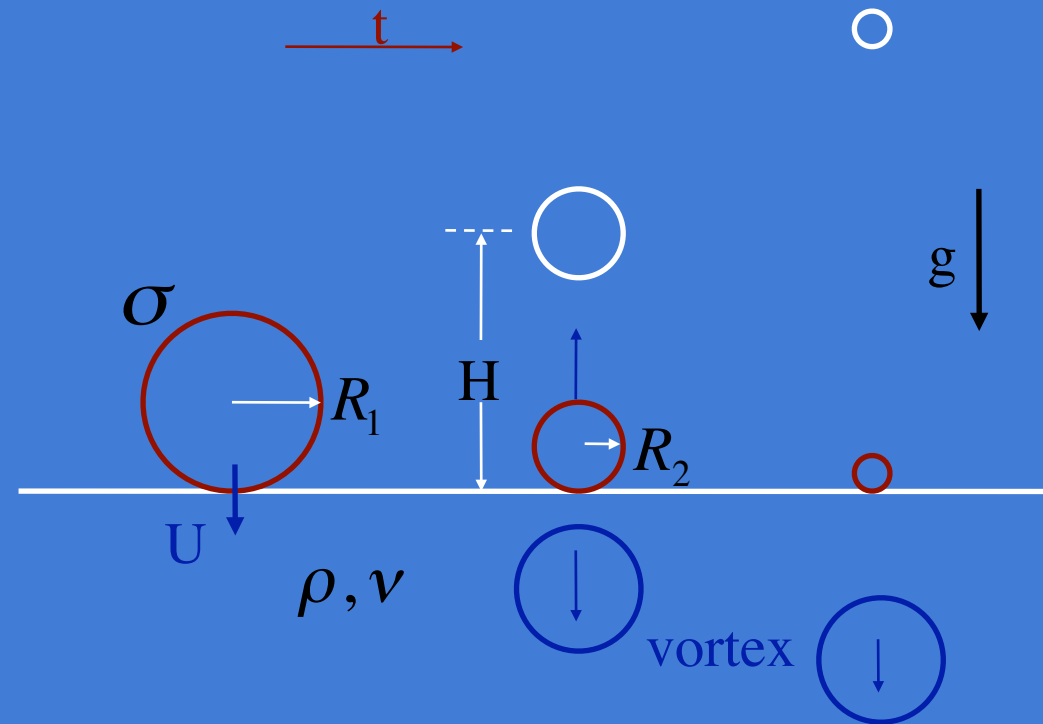
Consistent with experimental observations: $0.5 R_1 < R_2 < 0.7 R_1$



Coalescence Cascade

What is the leap height H ?

Assume viscous dissipation negligible during coalescence.



Energetics

$$\cancel{M_1 g R_1} + \sigma A_1 = M_2 g H + \sigma A_2 + \cancel{\frac{1}{2}(M_1 - M_2)U^2}$$

$$\Rightarrow H = \frac{3\sigma R_1^2 - R_2^2}{\rho g R_2^3} \approx \frac{9\sigma}{\rho g} \frac{1}{R_1} \quad \text{since } R_2 \approx 0.6 R_1$$

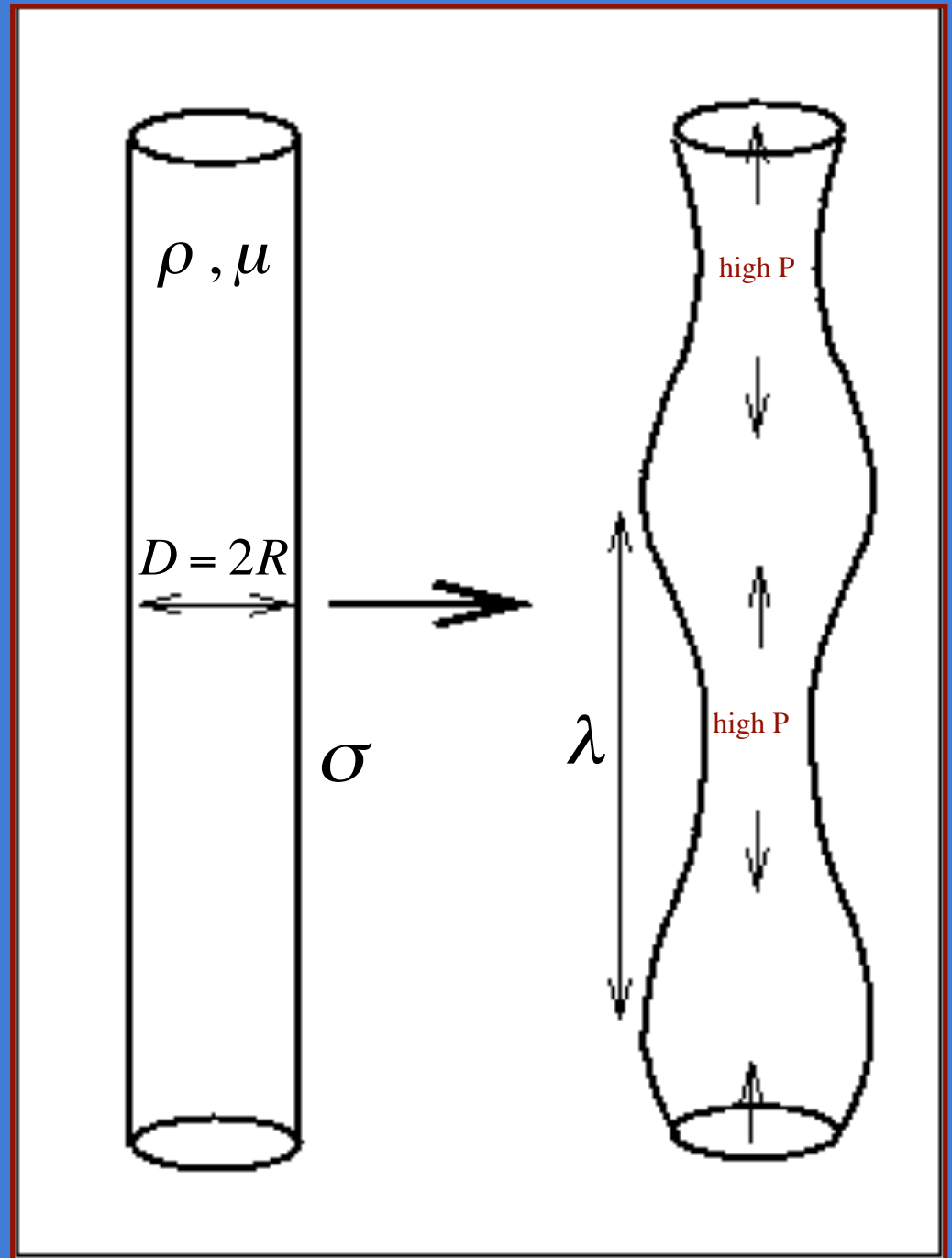
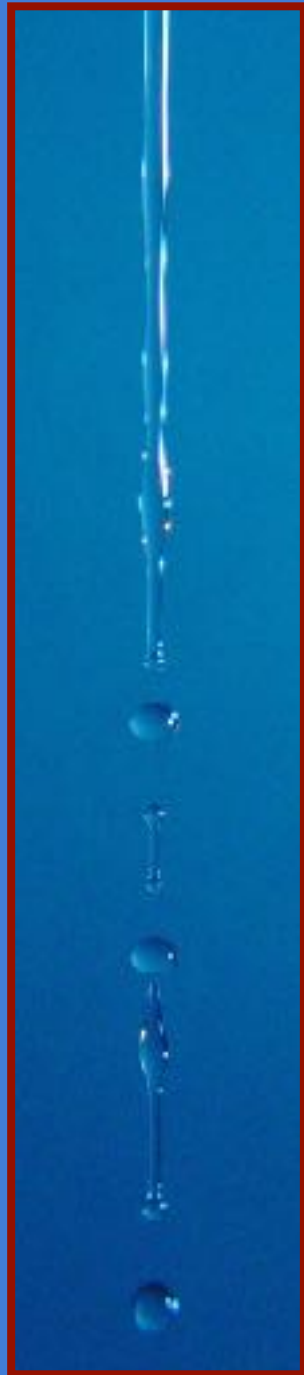
Smaller drops leap higher:

$$\frac{H}{R_1} \sim \frac{1}{R_1^2}$$

consistent with
experimental
observations

Rayleigh-Plateau Instability (Rayleigh 1900)

Capillary
pinch-off
of a fluid
thread



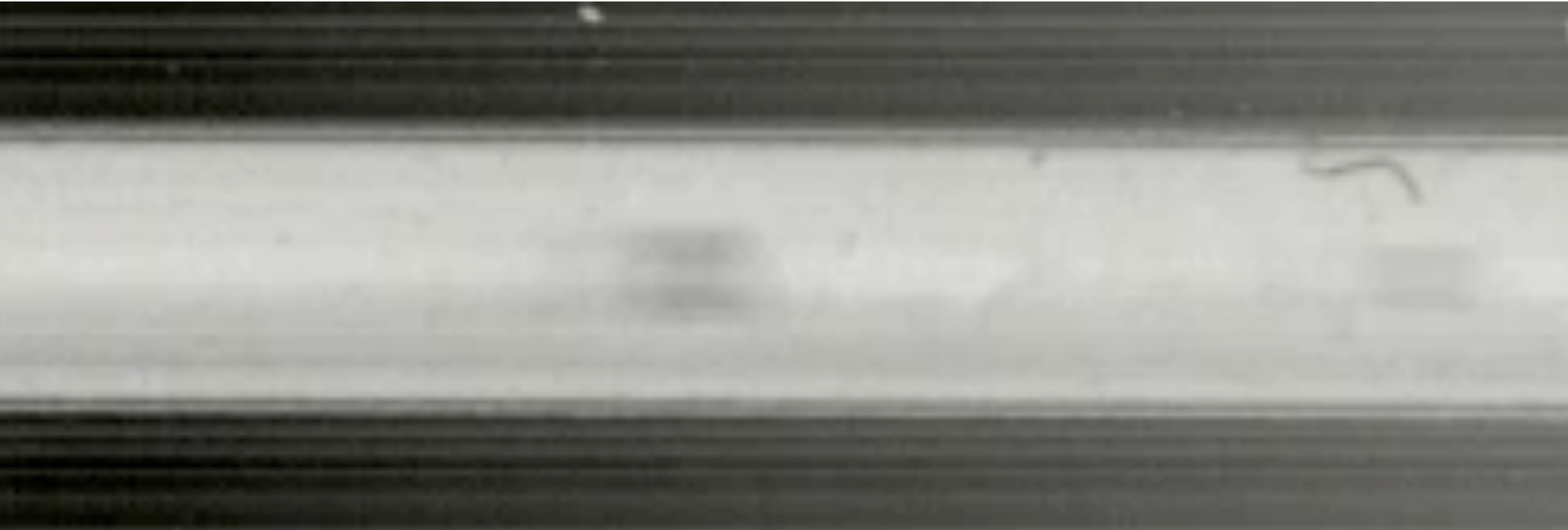
Coating a wire



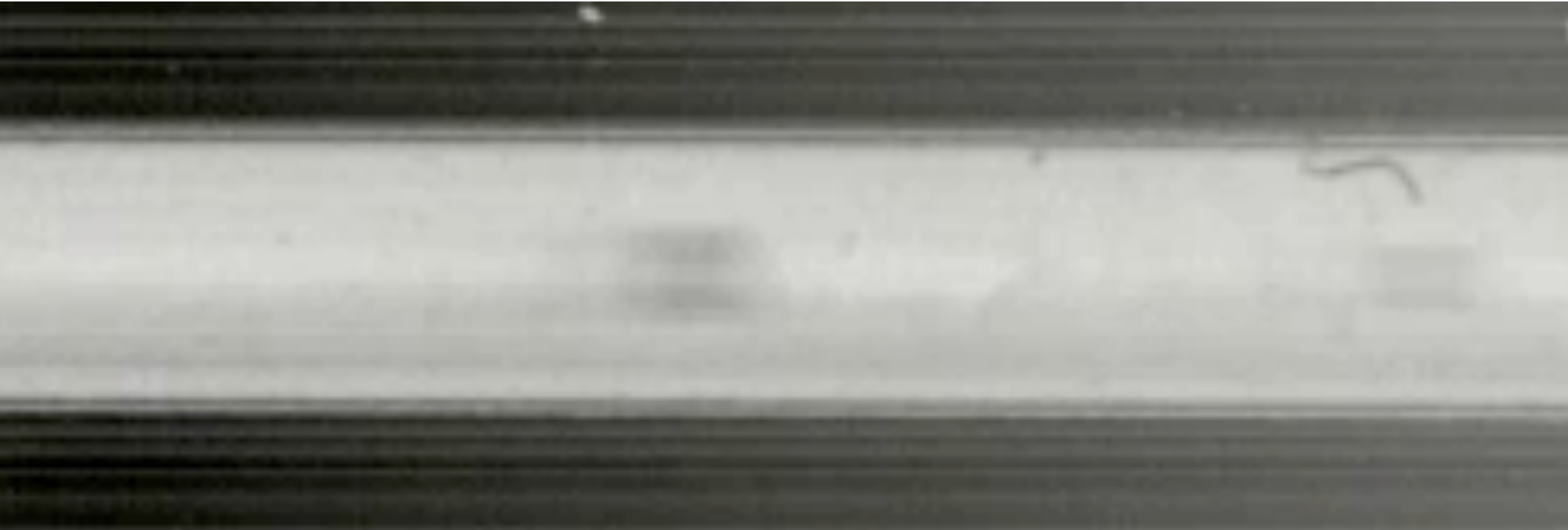
Coating a wire



Bubble formation within a hose



Bubble formation within a hose



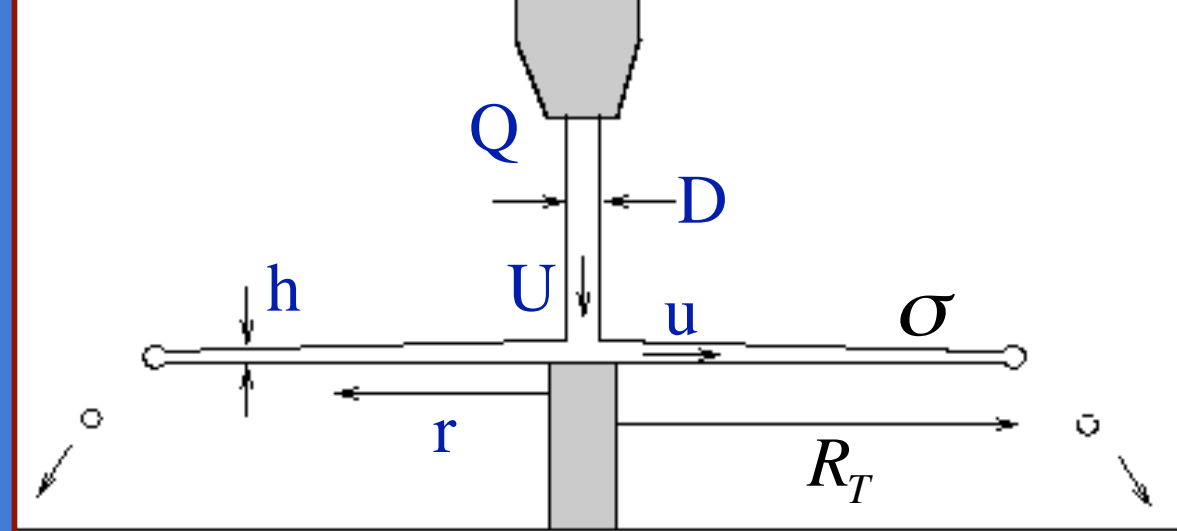
Fluid sheets



Stable sheet

(Savart 1833, Taylor 1959)

$$We = \frac{\rho U^2 D}{\sigma} < 1000$$



- sheet radius: balance of surface tension and inertia

$$\rho u^2 h \sim 2\sigma$$

- sheet thickness : $h = \frac{Q}{2\pi r u} \sim \frac{1}{r}$

- Taylor radius:

$$R_T = \frac{\rho Q u}{4\pi\sigma}$$

- toroidal sheet rim releases drops through Rayleigh instability

The water bell



Water bells (Savart 1833, Taylor 1959)

- form prescribed by balance of inertia, gravity and capillarity

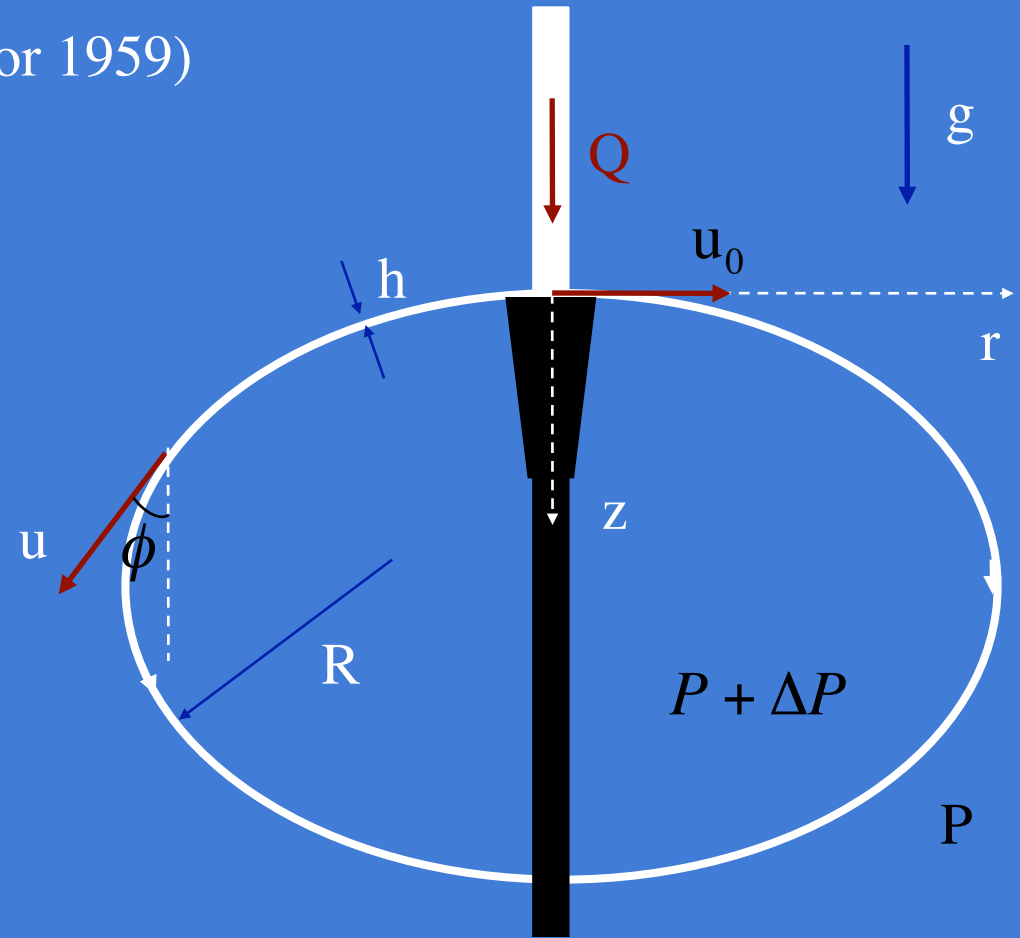
Continuity: $Q = 2\pi h r u$

Energy conservation:

$$u^2 = 2gz + u_0^2$$

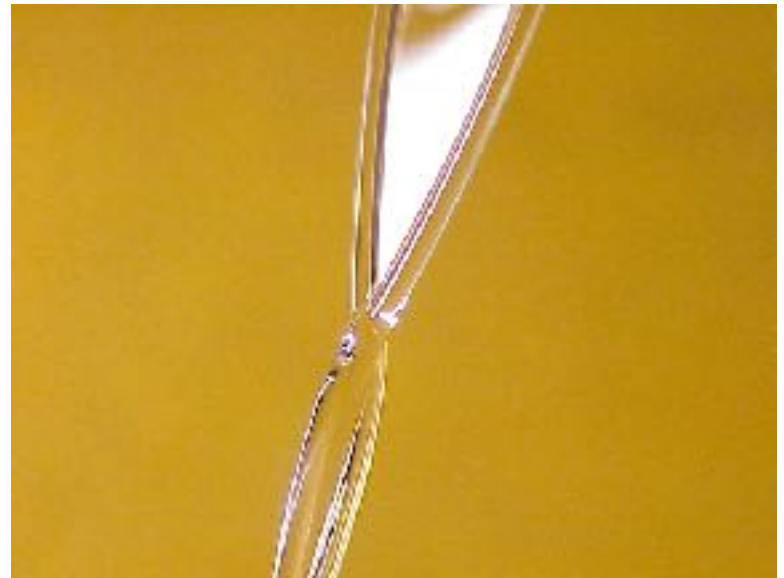
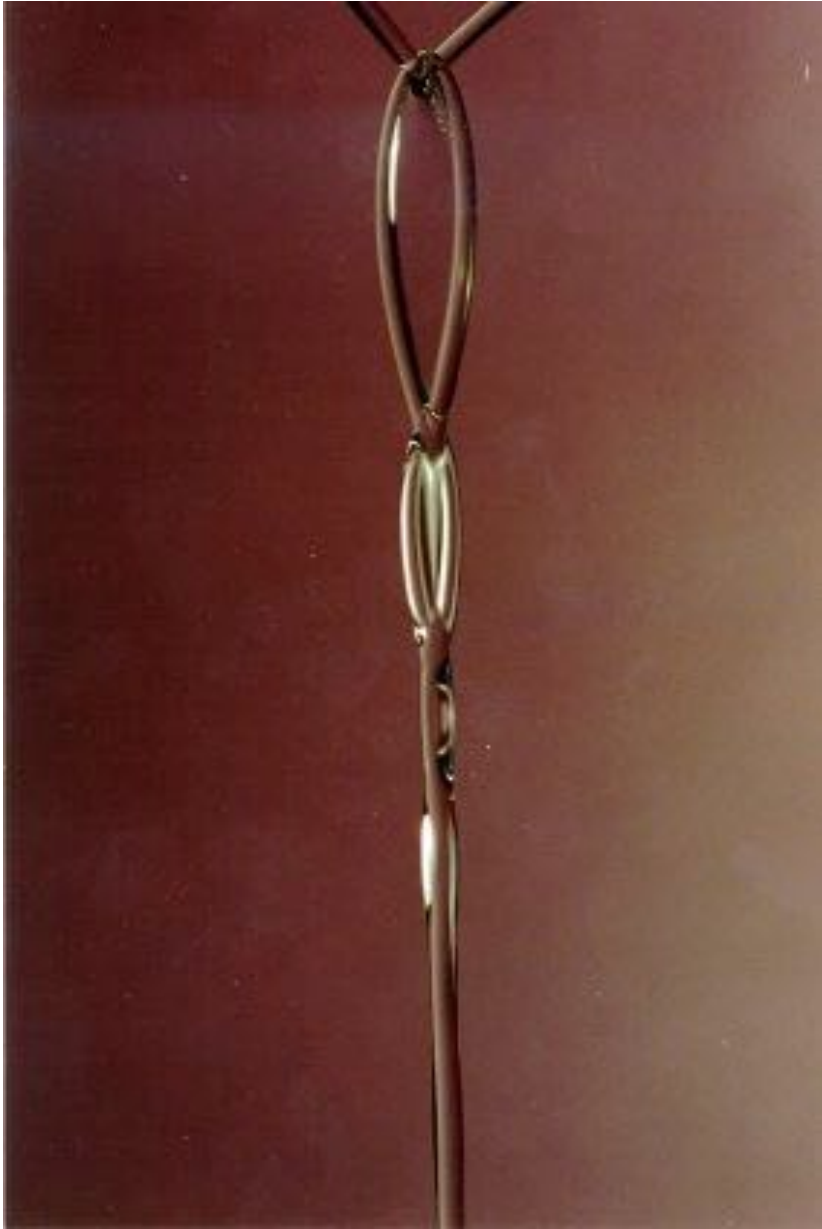
Normal force balance:

$$\underbrace{\frac{2\sigma}{R} + \frac{2\sigma \cos \phi}{r}}_{\text{curvature}} + \underbrace{\rho g h \sin \phi}_{\text{gravity}} = \Delta P + \underbrace{\frac{\rho h u^2}{R}}_{\text{inertia}}$$



- bell closes owing to influence of out-of-plane curvature

Colliding jets



Fluid fishbones

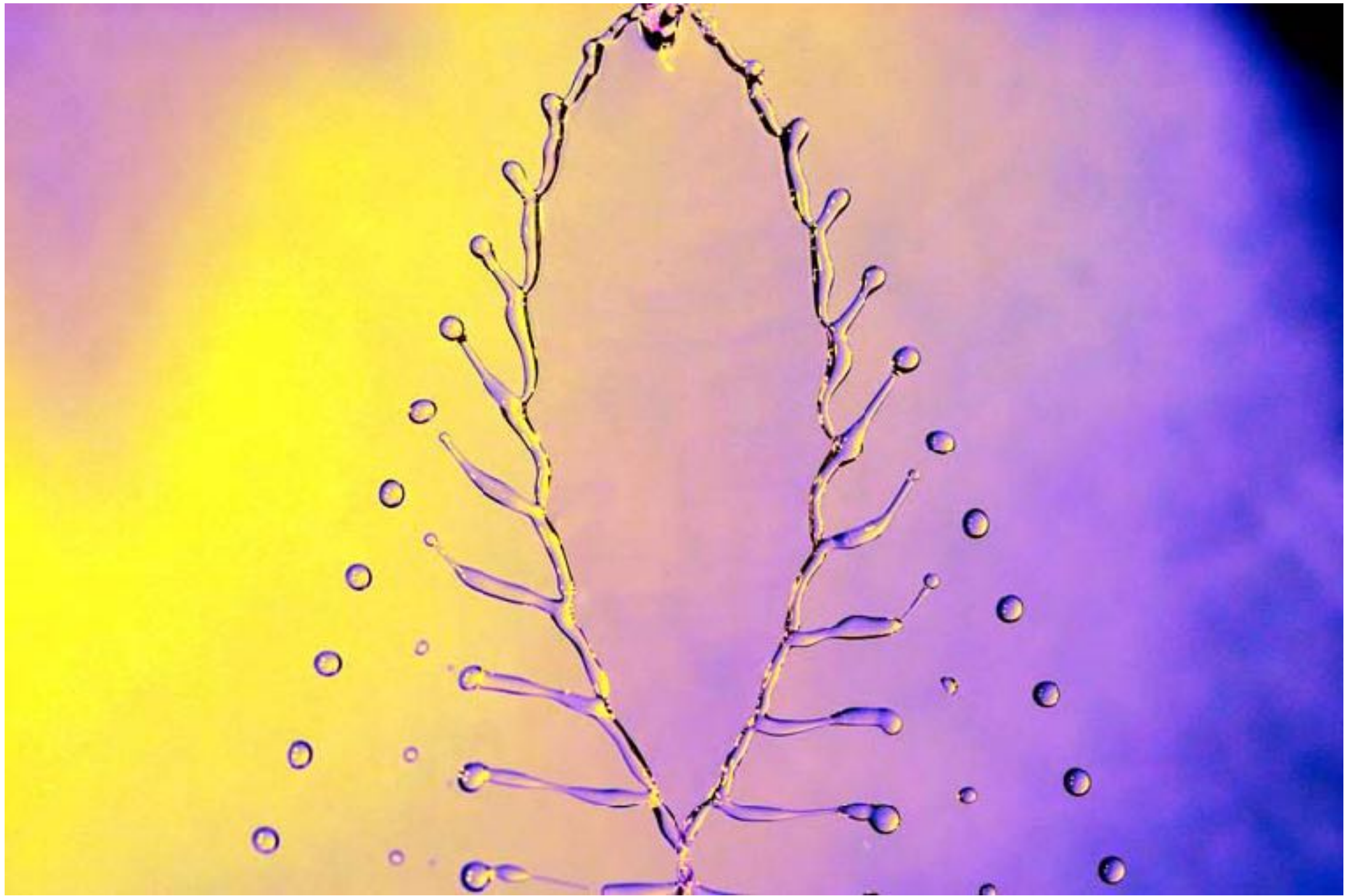


- sheet rim destabilizes through Rayleigh-Plateau instability

Fluid fishbones



- sheet rim destabilizes through Rayleigh-Plateau instability



Fluid-fluid impact



How do we rationalize the resulting forms?



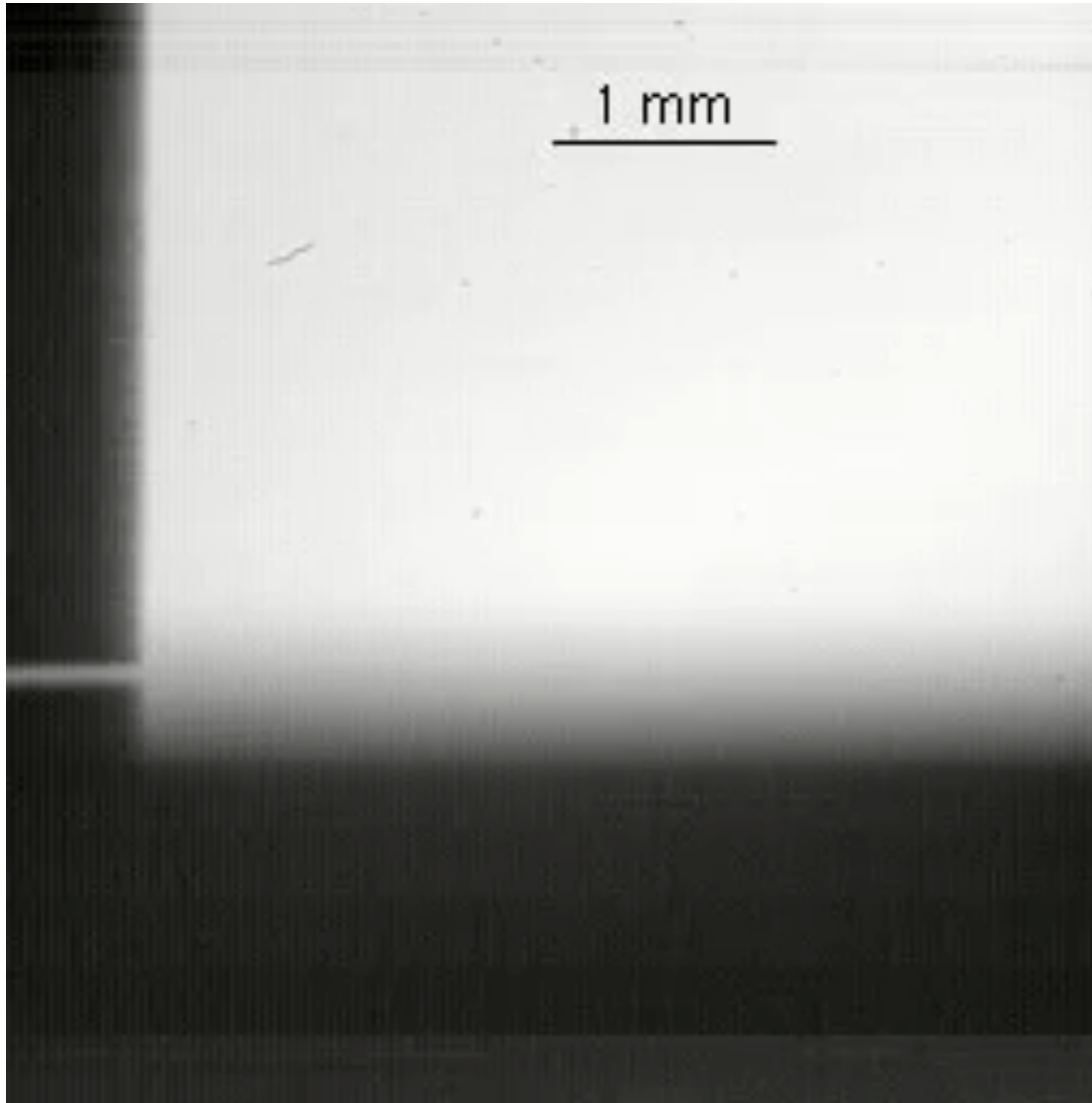
“I never before realized so strongly the splendour and beauty of the mere physical forms of Nature.

A wonderful thing is the curious repetition of the same forms, of the same design almost, in the shape of the falling water.

It gave me a sense of how completely what seems to us the wildest liberty of Nature is restrained by governing laws.”

- **Oscar Wilde**, on viewing Niagara Falls (the Canadian side)

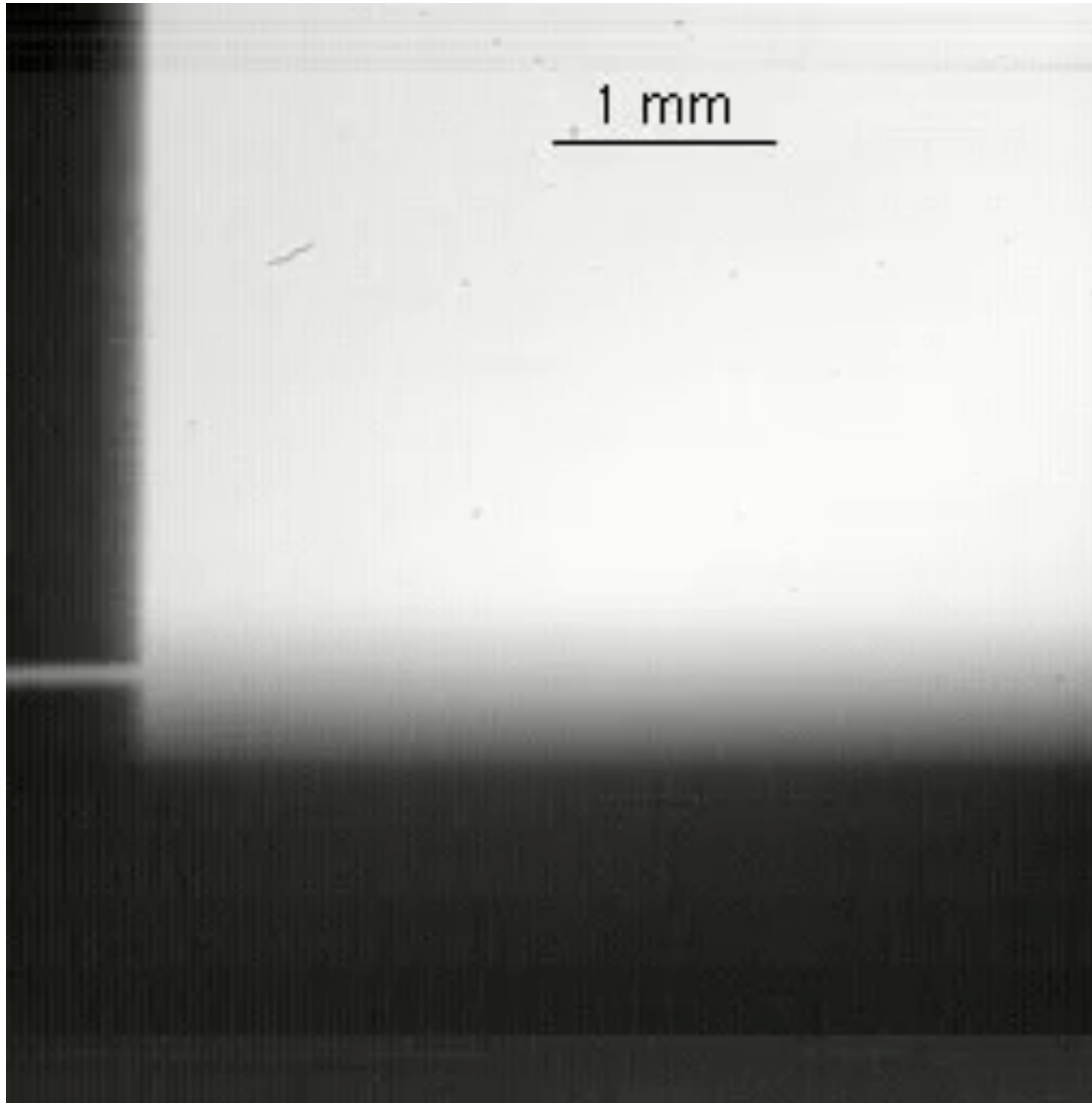
Capillary rise: the planar meniscus



Interface shape:

$$h(x) = \ell_c \cot\theta_e e^{-x/\ell_c}$$

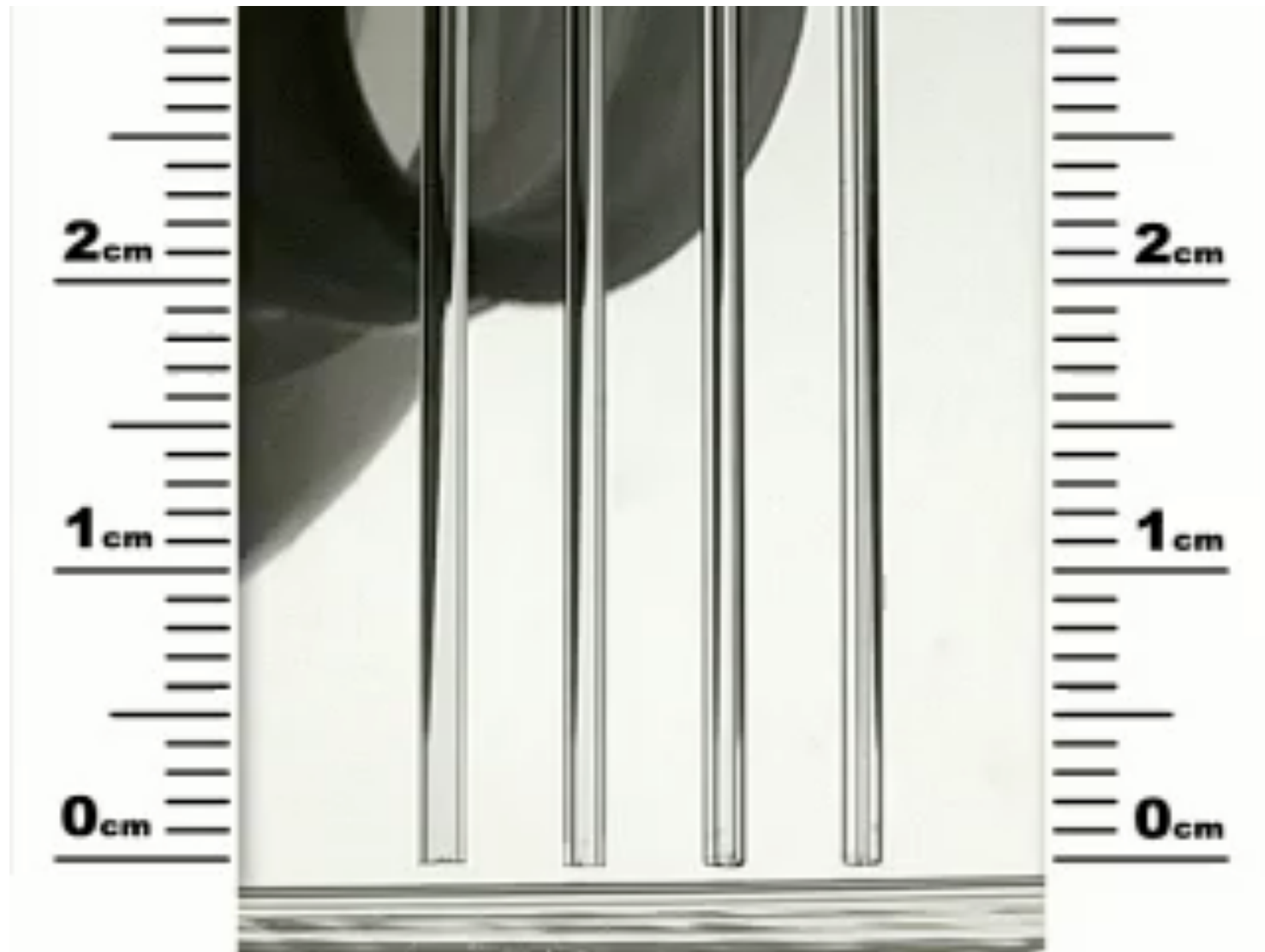
Capillary rise: the planar meniscus



Interface shape:

$$h(x) = \ell_c \cot\theta_e e^{-x/\ell_c}$$

Capillary rise in tubes



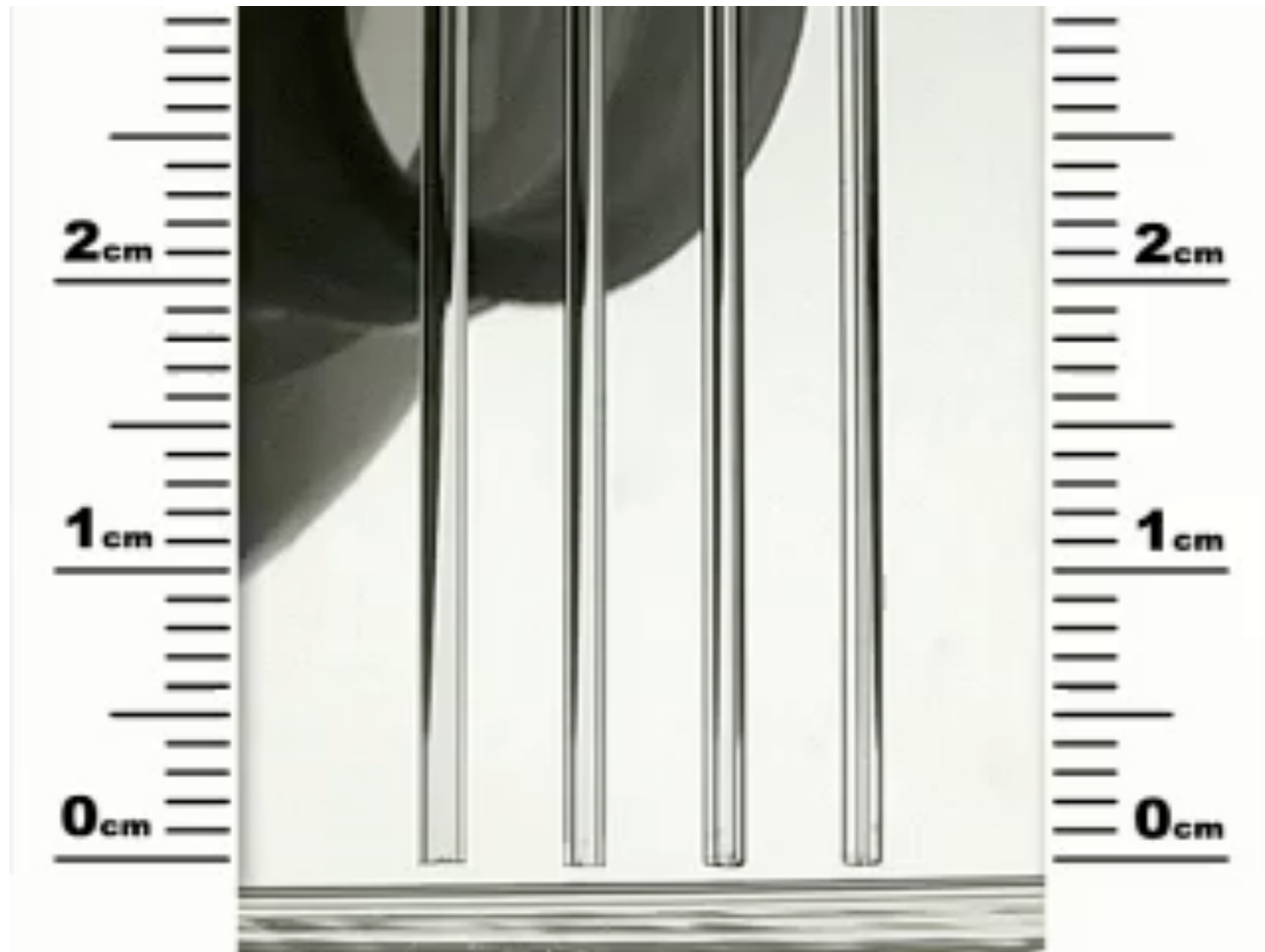
Scaling for rise height

$$\rho g H \sim \sigma / R$$



$$H \sim \frac{\sigma}{\rho g R}$$

Capillary rise in tubes



Scaling for rise height

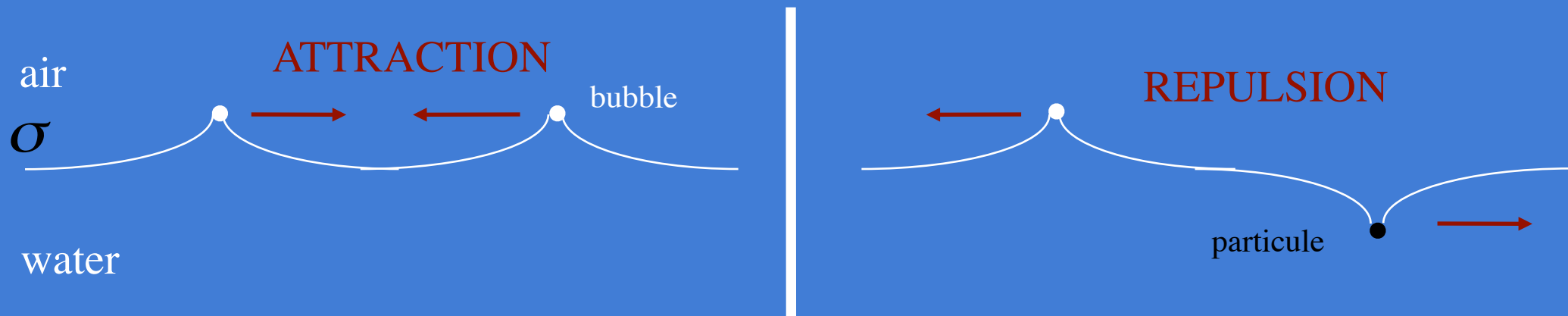
$$\rho g H \sim \sigma / R$$



$$H \sim \frac{\sigma}{\rho g R}$$

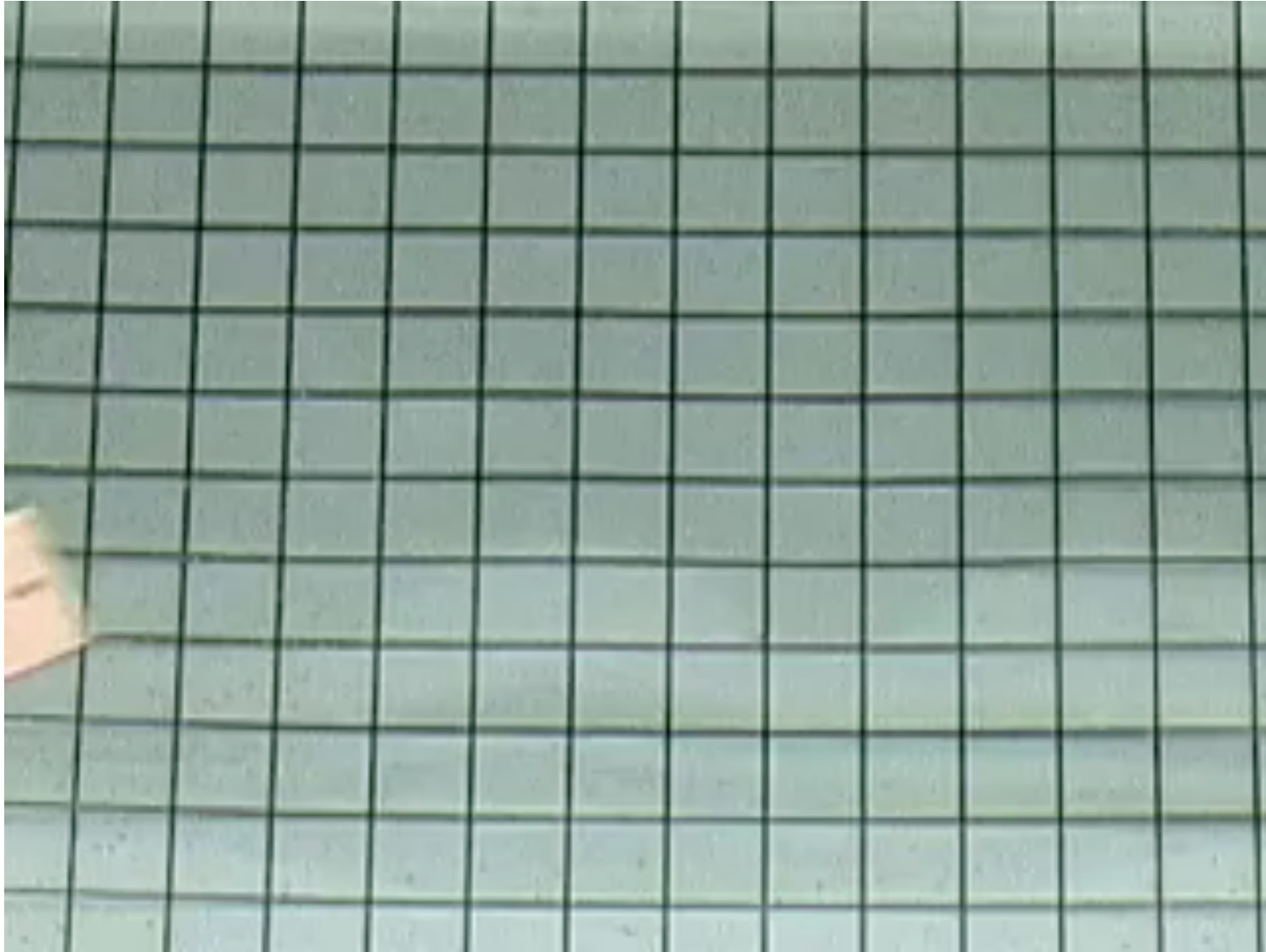
Capillary forces

- exist by virtue of the interaction of the menisci of floating bodies
- attractive/repulsive if the menisci are of the same/opposite sense

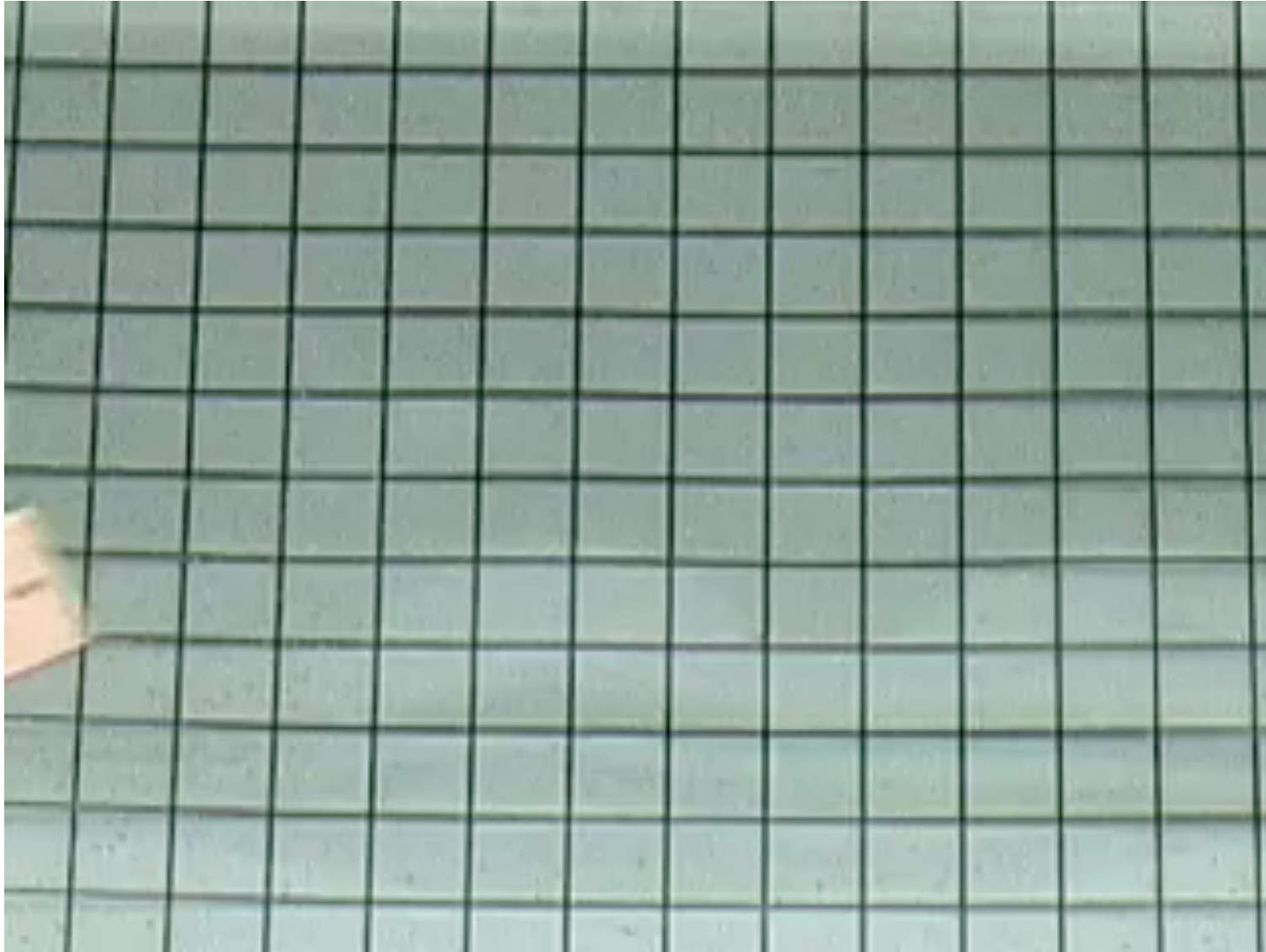


- explains the formation of bubble rafts on champagne
- explains the mutual attraction of Cheerios, and their attraction to the walls
- utilized for self-assembly on the microscale

Floating copper

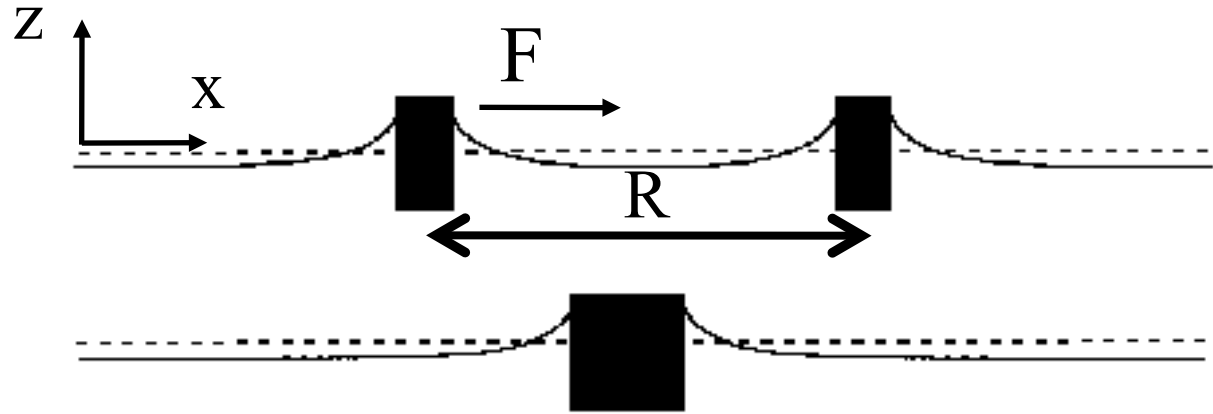


Floating copper



Capillary attraction

$$E_{Tot} = E_S + GPE$$

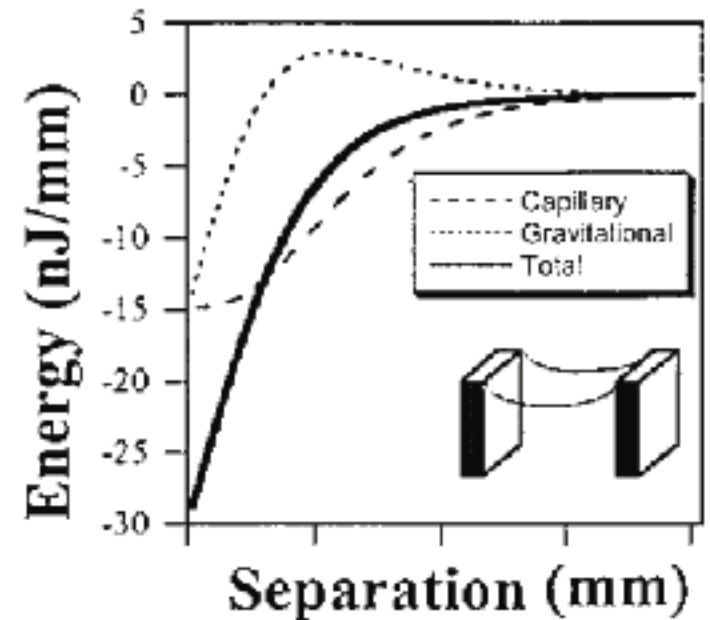


$$E_S = \sigma A(R)$$

$$GPE(R) = \int_{x=-\infty}^{x=\infty} \int_{z=0}^{h(x)} \rho g dz dx$$

$$\Rightarrow F(R) = -\frac{dE_{Tot}(R)}{dR}$$

Gryzbowski *et al* 2001



Interfacial Fluid Dynamics: Governing Equations

Navier-Stokes:

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{u} \quad , \quad \nabla \cdot \mathbf{u} = 0$$

Boundary Conditions

Normal stress:

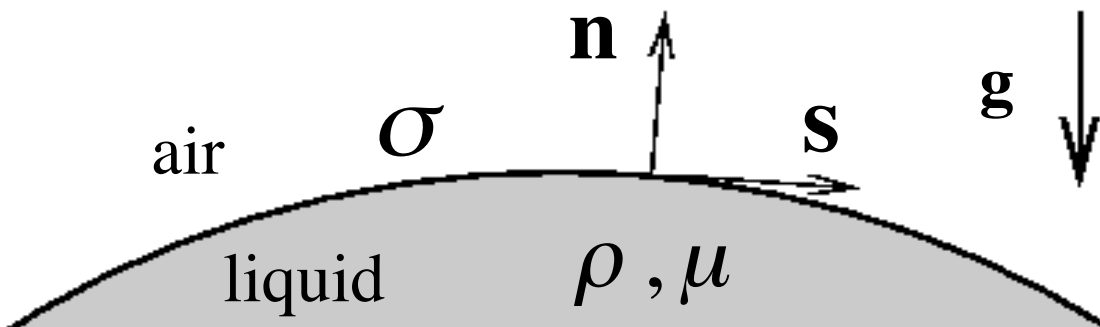
$$\Delta \mathbf{n} \cdot \mathbf{T} \cdot \mathbf{n} = \sigma \nabla \cdot \mathbf{n}$$

Tangential stress:

$$\Delta \mathbf{n} \cdot \mathbf{T} \cdot \mathbf{s} = \nabla_s \sigma$$

Stress tensor

$$\mathbf{T} = -p\mathbf{I} + \mu \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right)$$

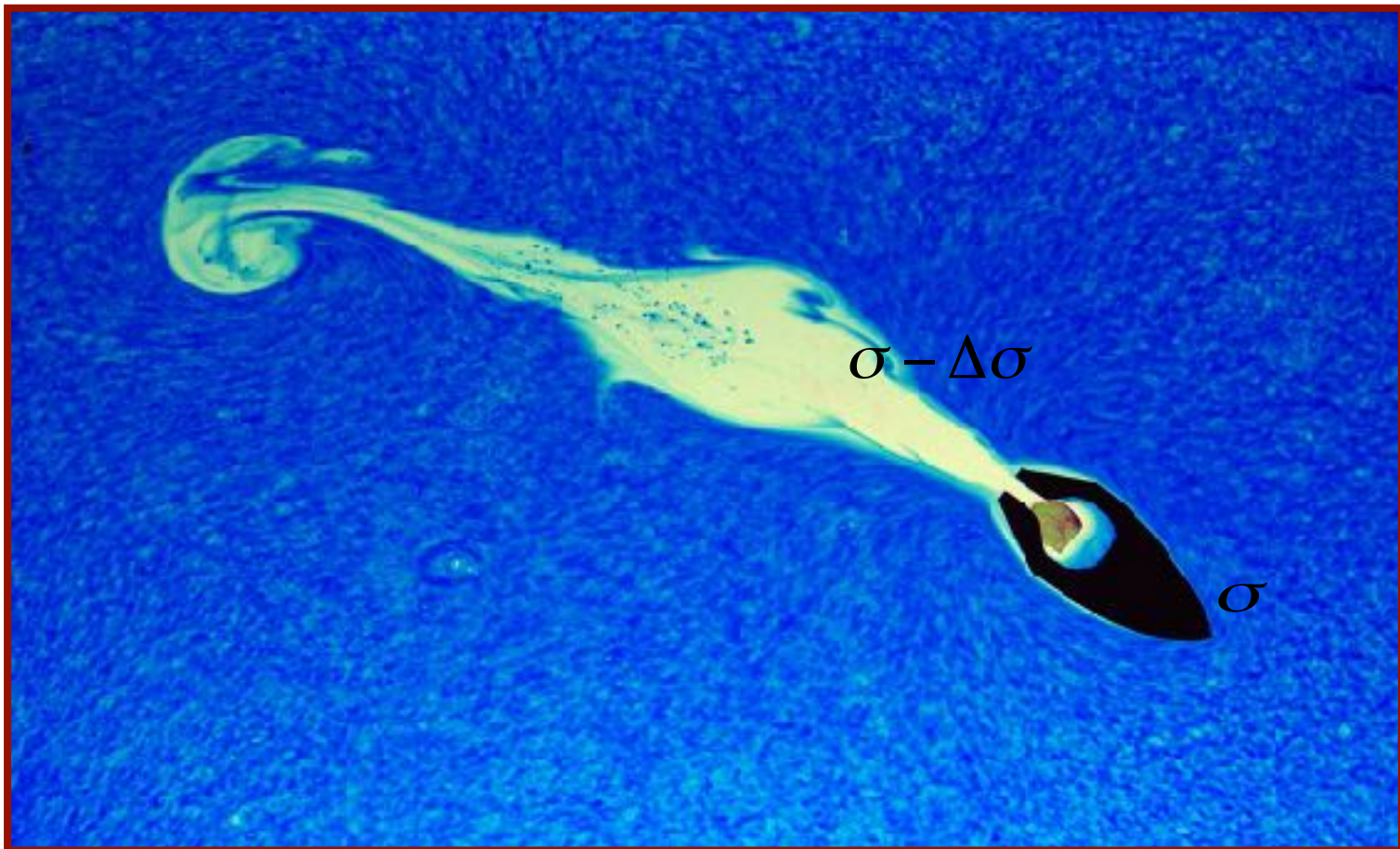


Marangoni Flows

- flows dominated by the influence of surface tension gradients

Recall tangential stress BC: $\Delta \mathbf{n} \cdot \mathbf{T} \cdot \mathbf{s} = \nabla \sigma$

- $\nabla \sigma$ may arise due to dependence of $\sigma(T, c, \Gamma)$

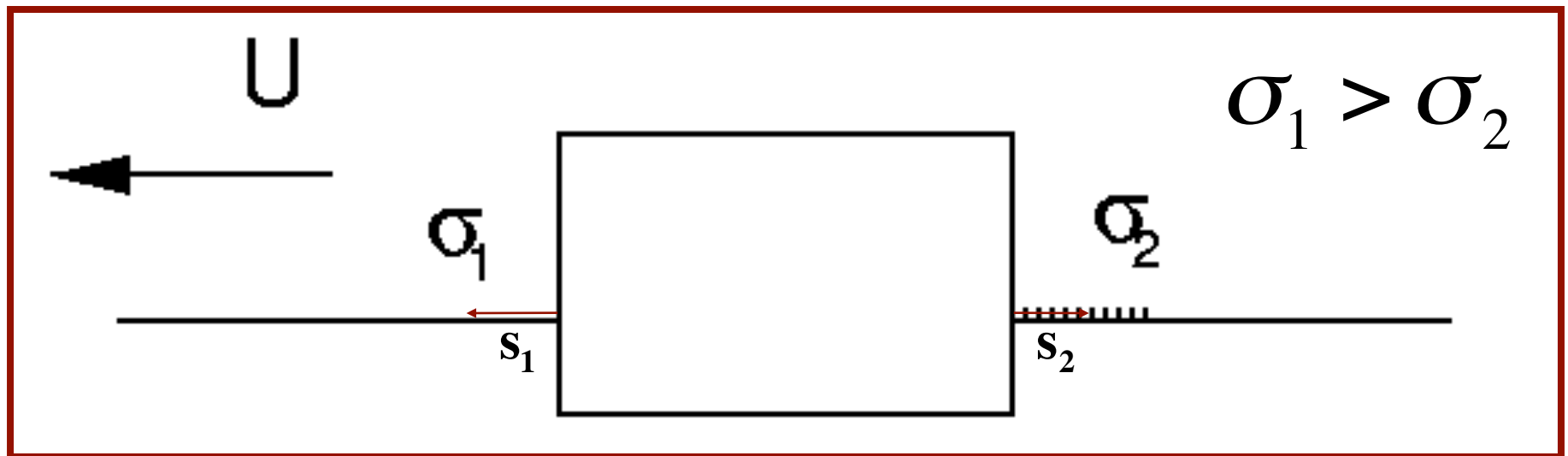


Marangoni propulsion

- lateral force may be generated by surface tension gradient

$$F = \int_C \sigma \mathbf{s} d\ell$$

integrate around
contact line



e.g. water-walking insects, dispersal of pine needles

- motion driven by soap cannot be sustained in a closed container
- motion may be sustained if driven by a volatile component (e.g. camphor)

Camphor boats



Nakata (2005)

Camphor boats



Nakata (2005)

Camphor boat races



Nakata (2006)

Camphor boat races



Nakata (2006)

The cocktail boat: fueled by alcohol



Video: Lisa Burton

The cocktail boat: fueled by alcohol



Video: Lisa Burton

COMMUNICATION

Biomimicry and the culinary arts

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Abstract

We present the results of a recent collaboration between scientists, engineers and chefs. Two particular devices are developed, both inspired by natural phenomena reliant on surface tension. The cocktail boat is a drink accessory, a self-propelled edible boat powered by alcohol-induced surface tension gradients, whose propulsion mechanism is analogous to that employed by a class of water-walking insects. The floral pipette is a novel means of serving small volumes of fluid in an elegant fashion, an example of capillary origami modeled after a class of floating flowers. The biological inspiration and mechanics of these two devices are detailed, along with the process that led to their development and deployment.

(Some figures may appear in colour only in the online journal)

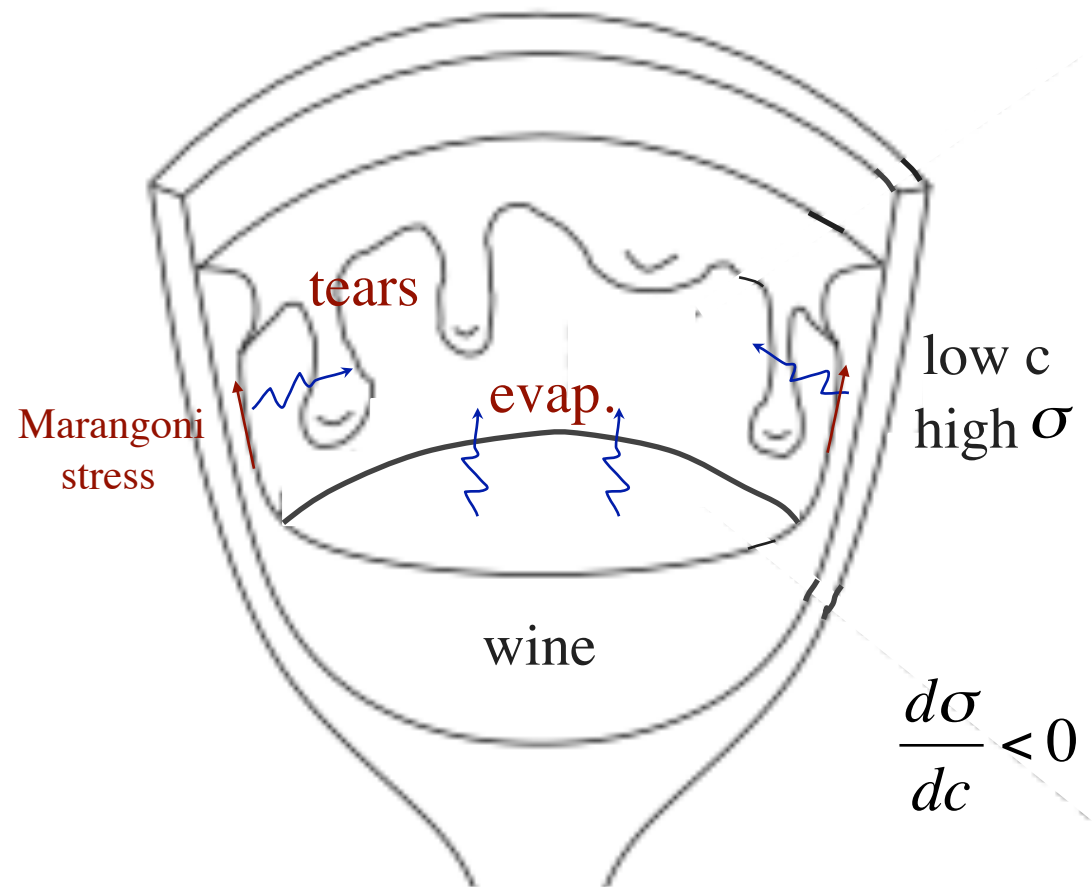


1. Introduction

“El gran llibre, sempre obert i que cal esforçar se a llegir, és el de la Naturalesa.”⁵

– Antoni Gaudi

dominate those of gravity for fluid systems small relative to the capillary length $l_c = \sqrt{\sigma/\rho g}$, where ρ represents the fluid density and g gravity. For air–water systems, the capillary length corresponds roughly to the size of a raindrop. Surface tension is thus an important player in the lives of



The tears of wine

“Who hath sorrow? Who hath woe? They that tarry long at the wine.
Look not though upon the strong red wine *that moveth itself aright*.
At the last it biteth like a serpent and stingeth like an adder.”

- Proverbs 23: 29-32 (c.a. 950 BC)

King Solomon, “the wisest man that ever lived”.

The tears of wine



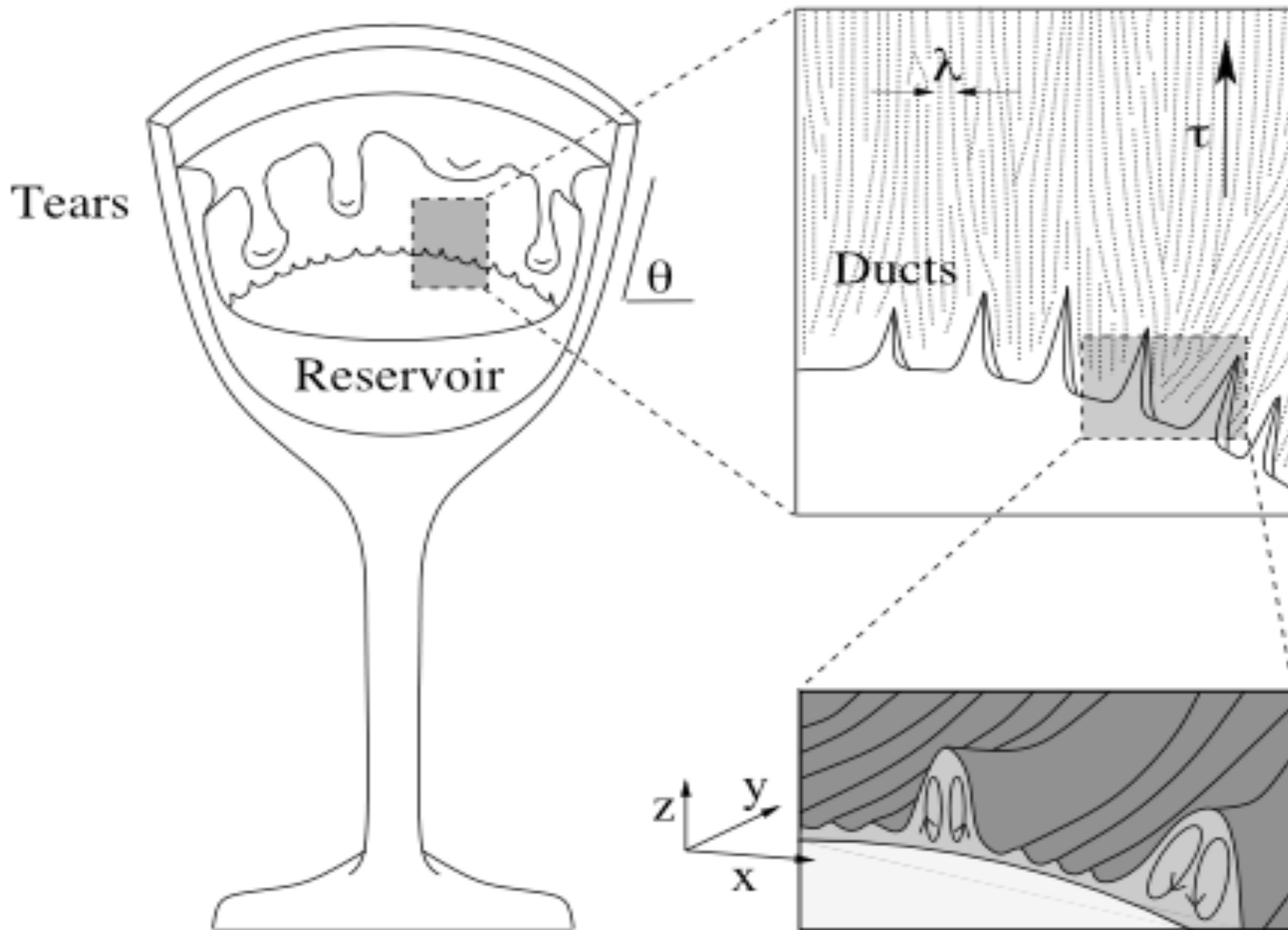
The first `Marangoni flow' studied scientifically (Thomson 1855).

The tears of wine



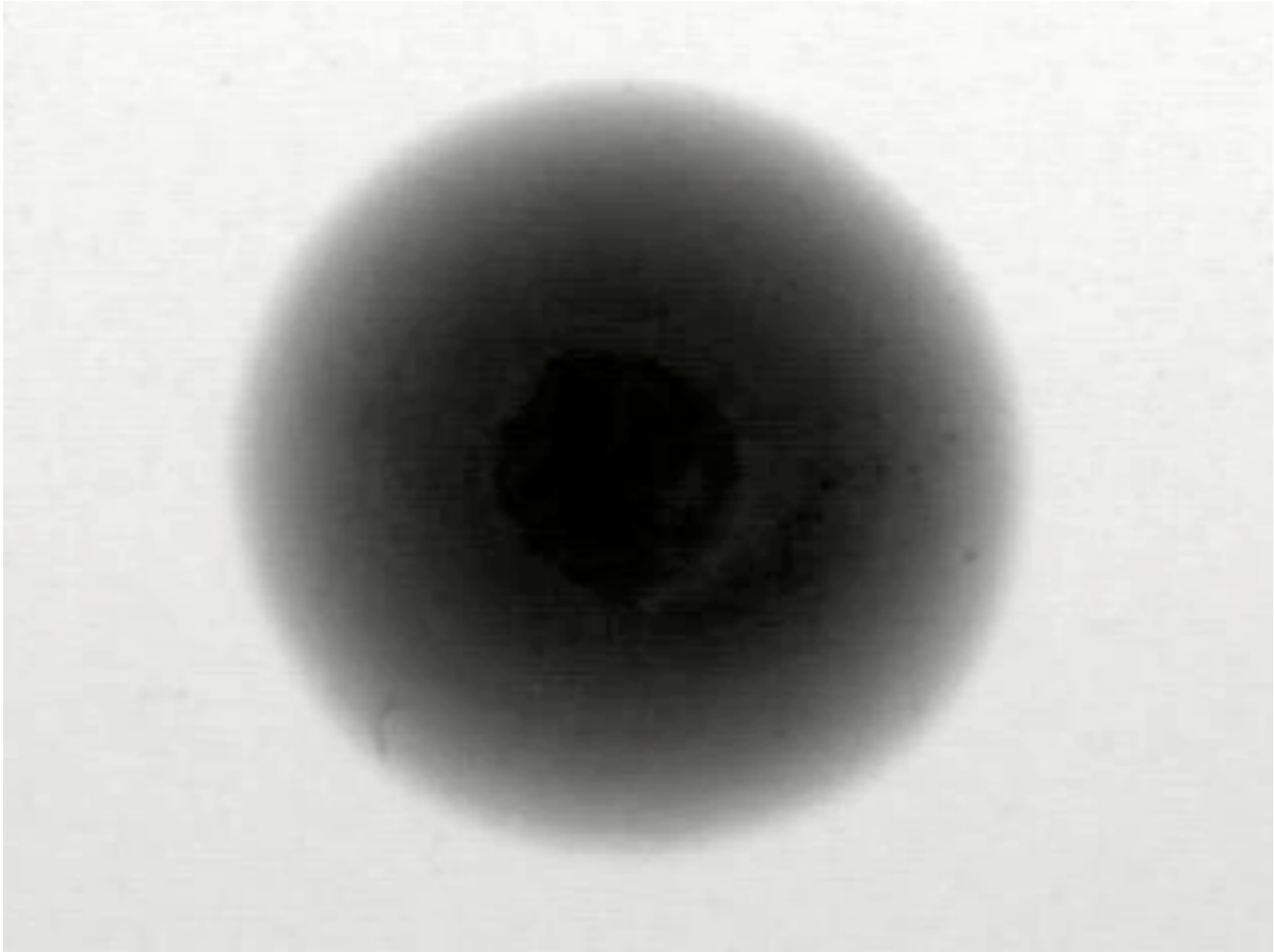
The first `Marangoni flow' studied scientifically (Thomson 1855).

The tear ducts of strong wine (Hosoi & Bush 2001)



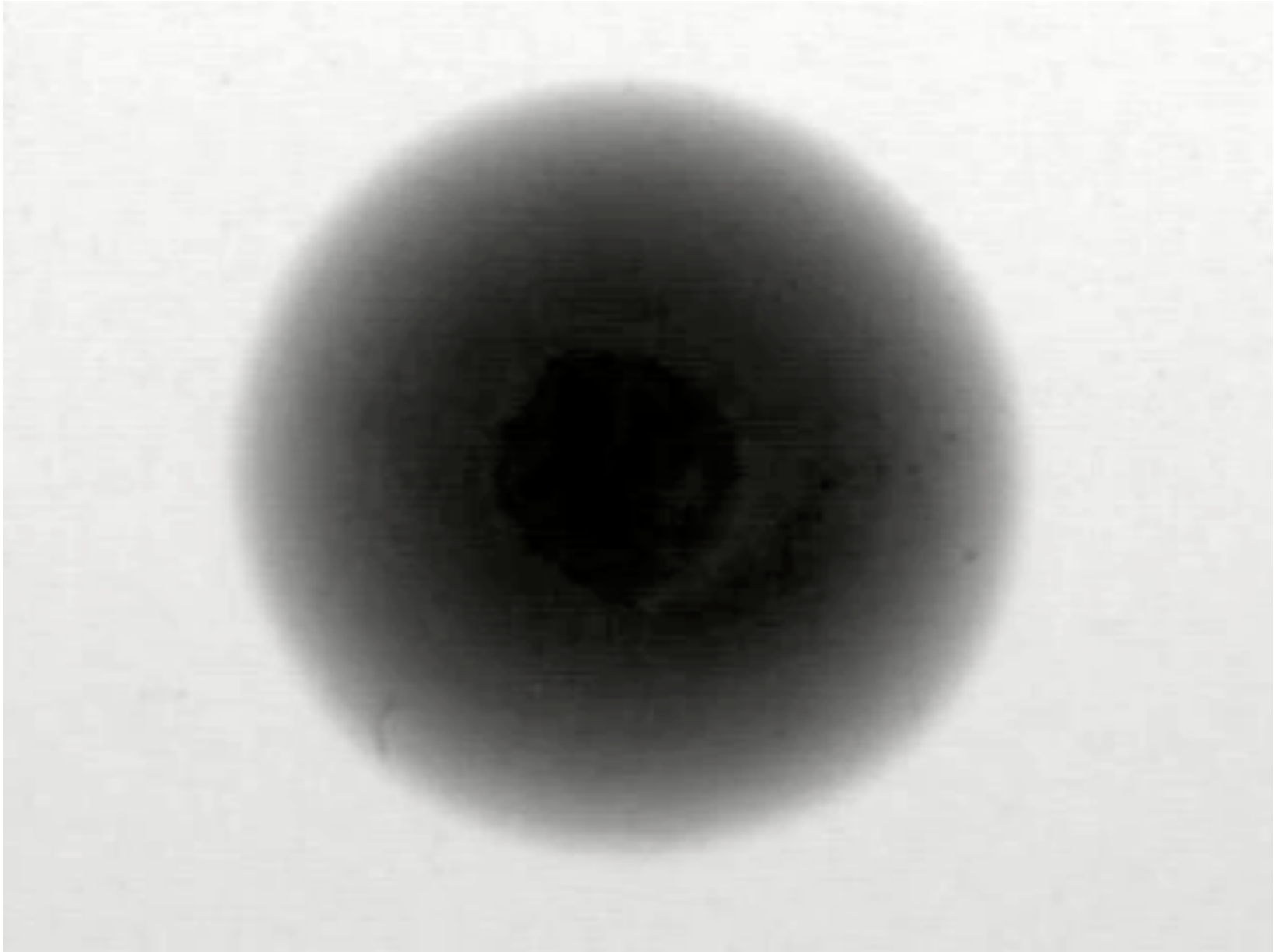
- layer marked by streamwise vortices, Marangoni convection rolls

The coffee ring problem



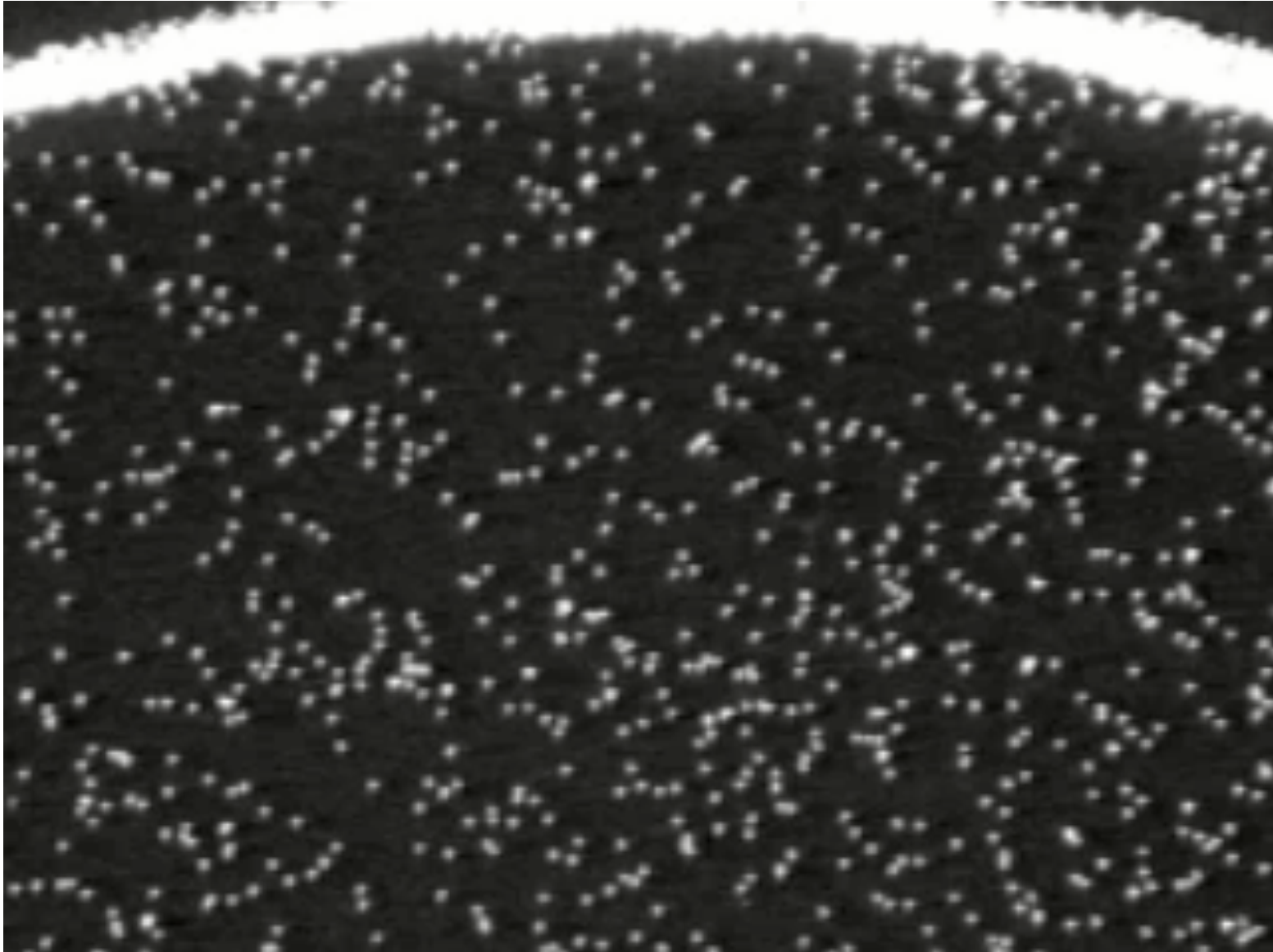
Why does an evaporating coffee drop leave a ring?

The coffee ring problem

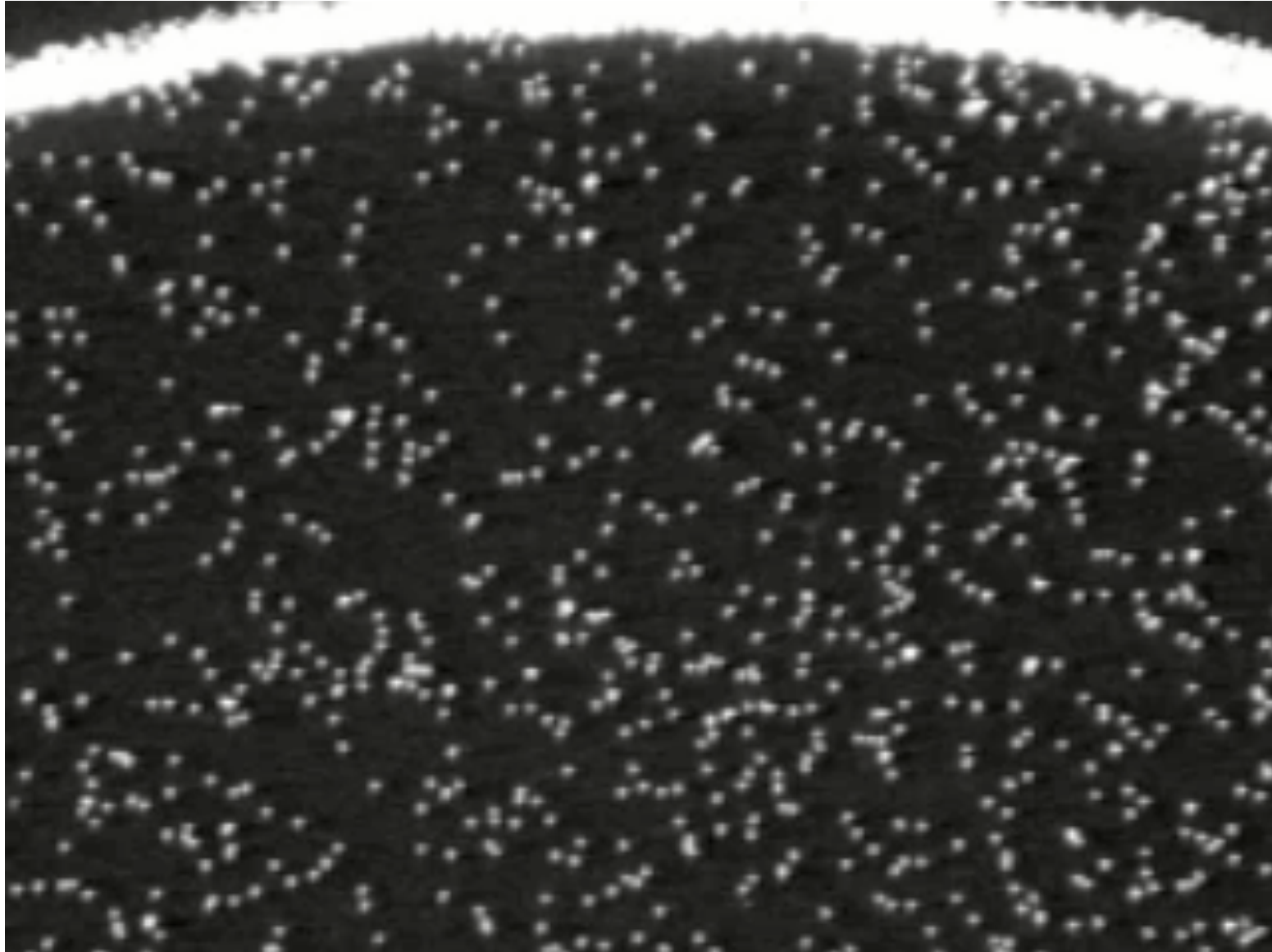


Why does an evaporating coffee drop leave a ring?

The flow in an evaporating coffee drop



The flow in an evaporating coffee drop



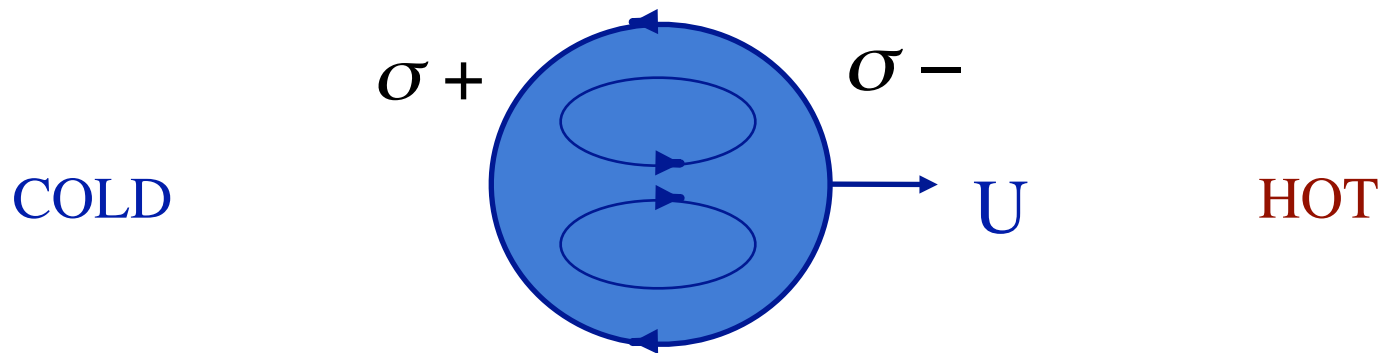
Thermocapillary Flows

- Marangoni flows induced by temperature gradients $\sigma(T)$
- Navier-Stokes coupled to heat equation through BCs

$$\frac{\partial T}{\partial t} + u \cdot \nabla T = \kappa \nabla^2 T$$

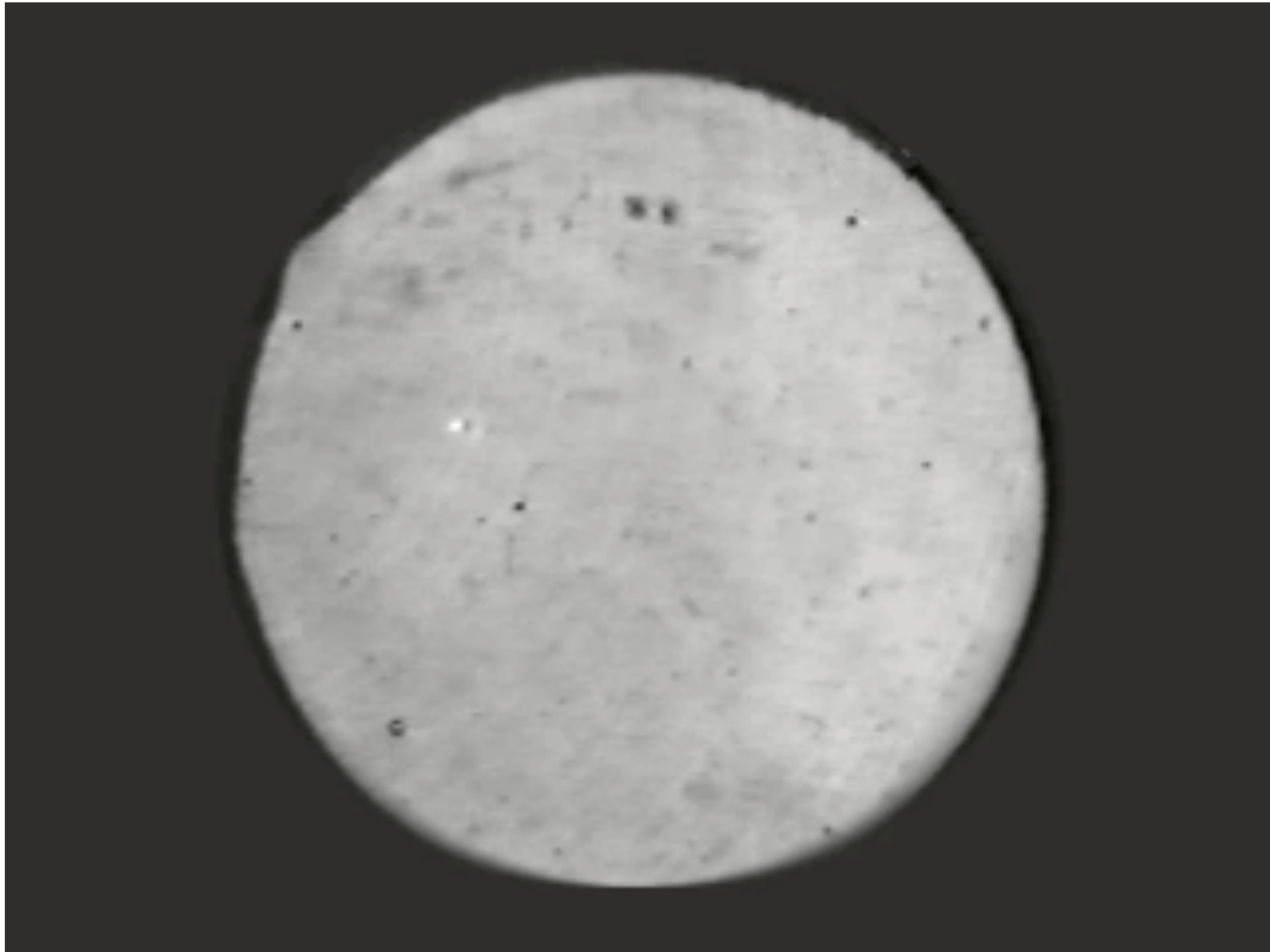
e.g. thermocapillary drop motion: drops migrate toward heat source

Young, Goldstein & Block (1962)

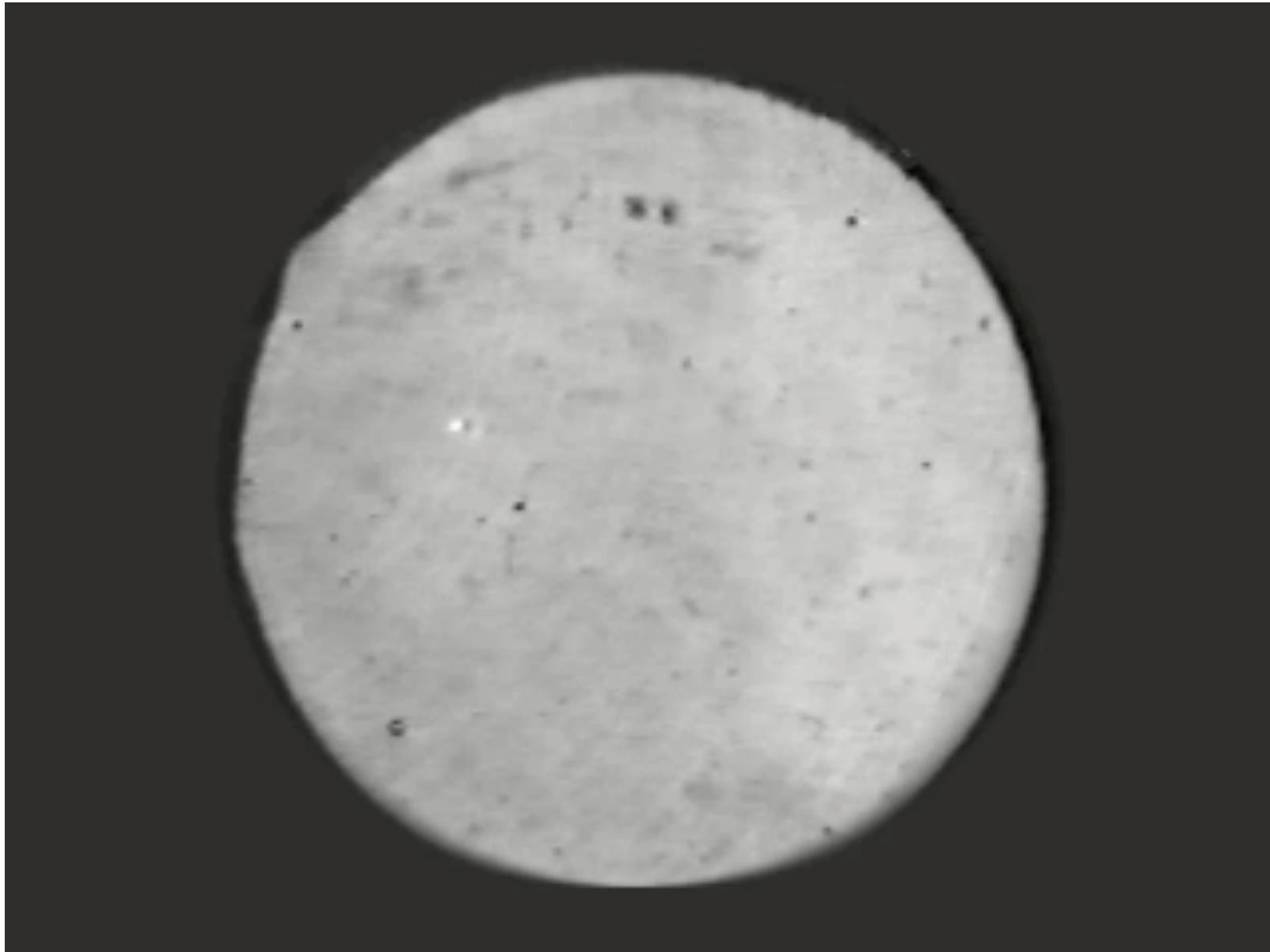


- used for fluid manipulation in zero gravity

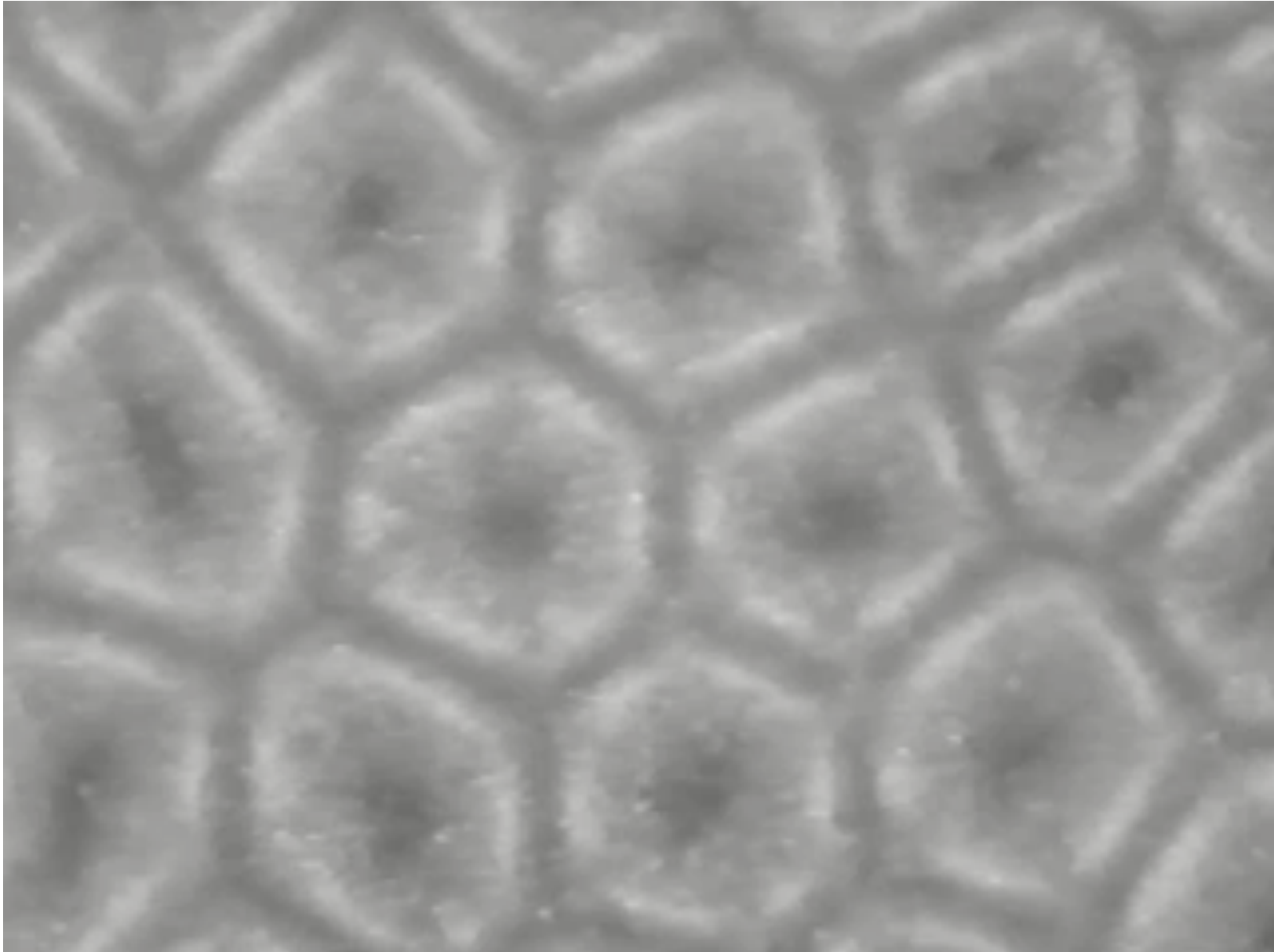
Thermal convection in a plane layer



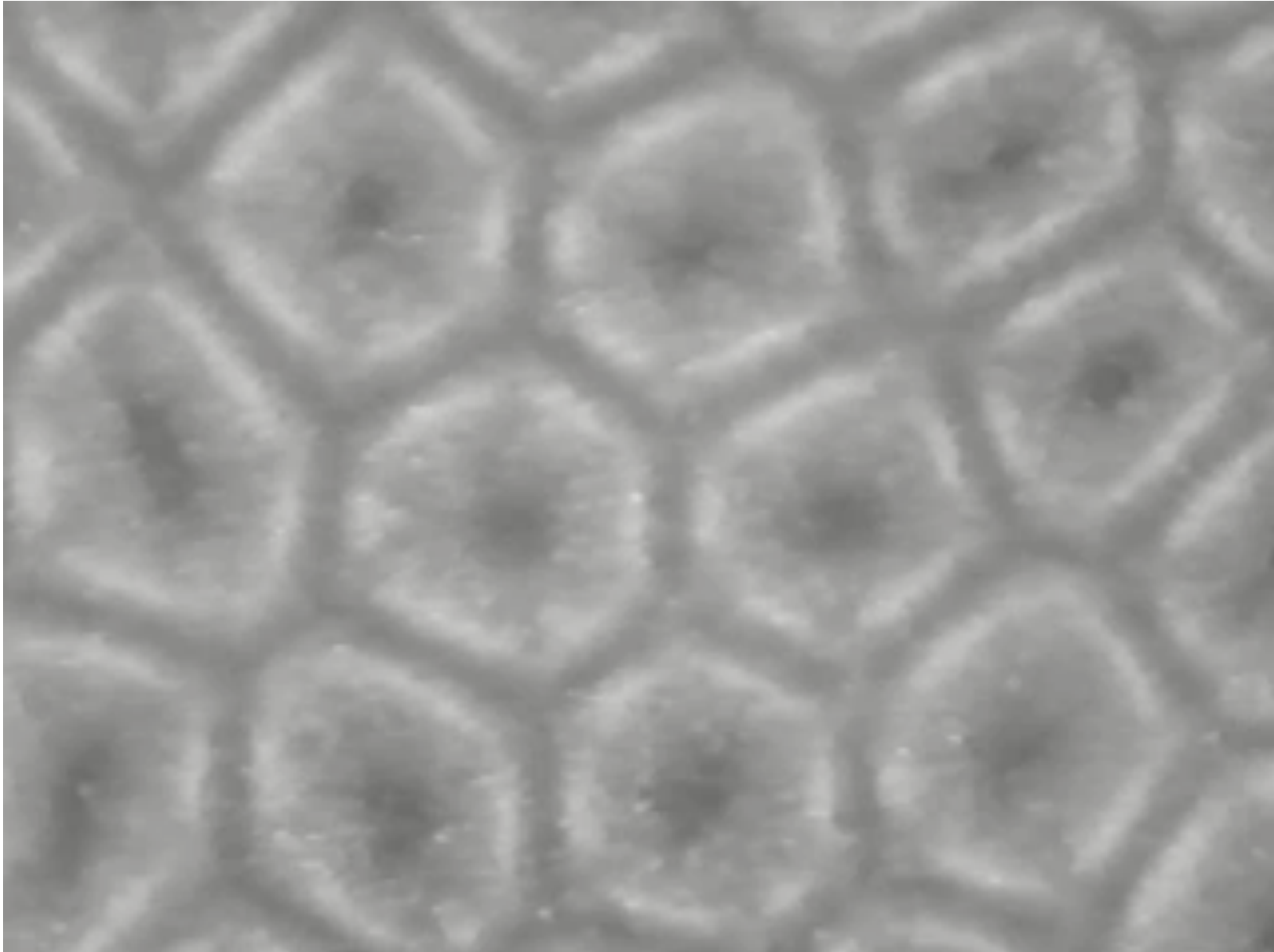
Thermal convection in a plane layer



Thermal convection in a plane layer

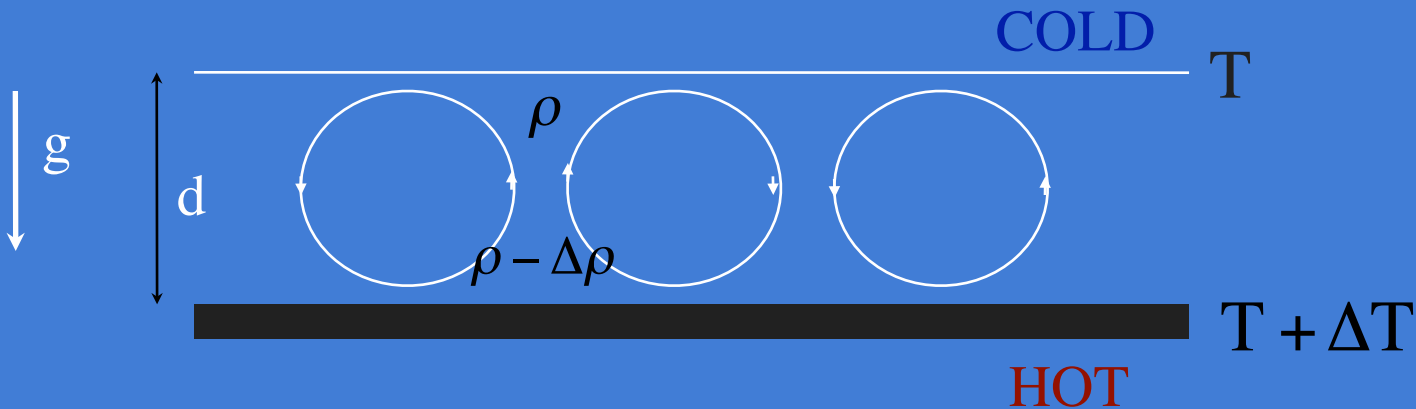


Thermal convection in a plane layer



THERMAL CONVECTION

Rayleigh-Benard $\rho(T) = \rho_0[1 + \alpha(T - T_0)]$

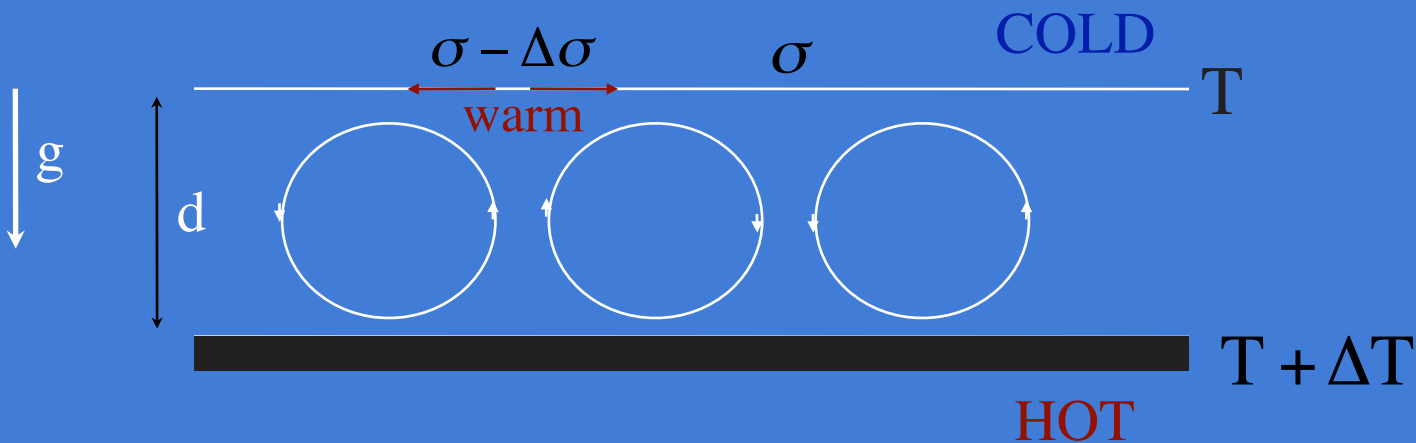


Stability prescribed by:

$$Ra = \frac{g\alpha\Delta T d^3}{\kappa\nu}$$

Rayleigh number

Marangoni-Benard $\sigma(T) = \sigma_0 - \Gamma(T - T_0)$



$$Ma = \frac{\Gamma\Delta T d}{\kappa\mu}$$

Marangoni number

Note: Marangoni convection dominates for thin films

Michela Geri problem

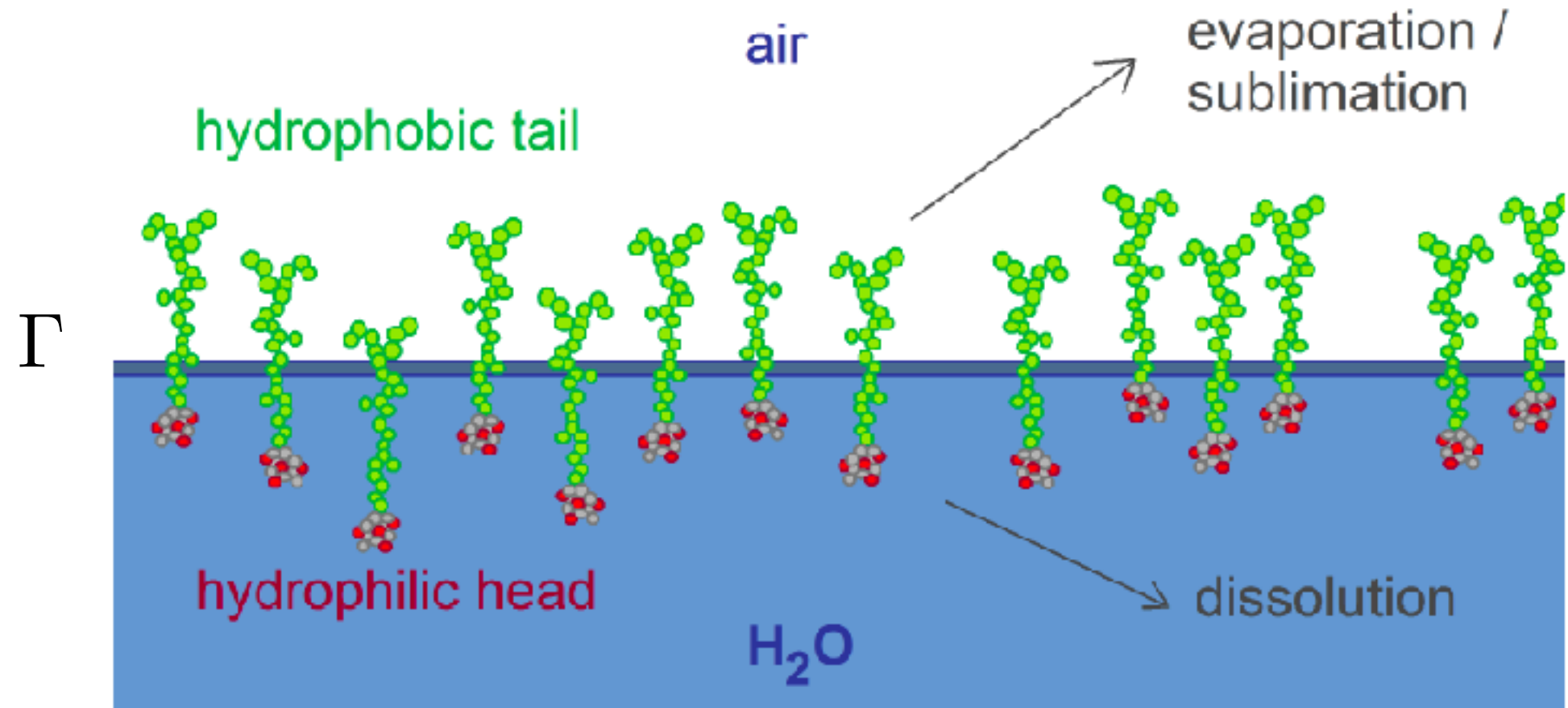


Michela Geri problem



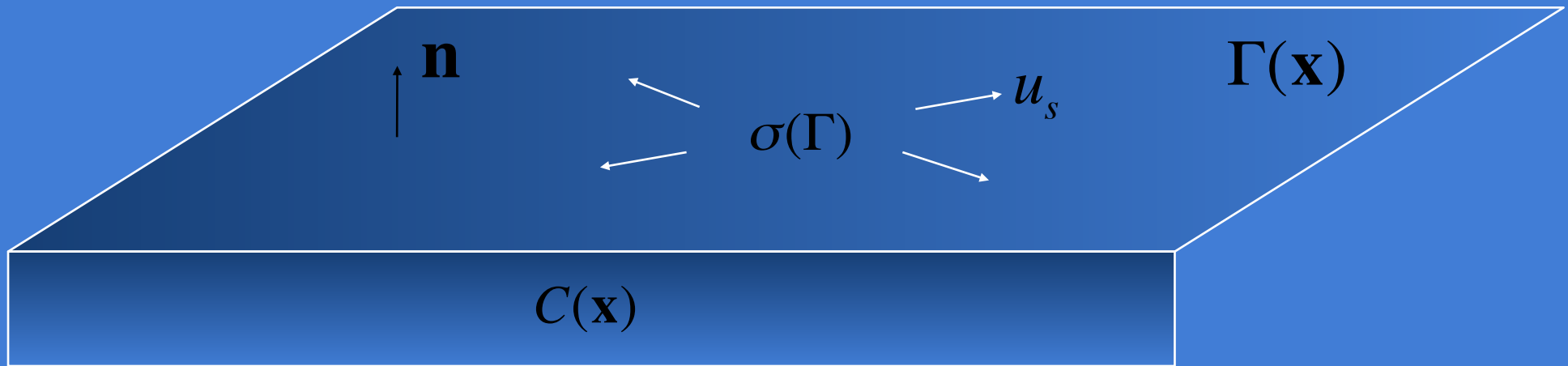
Surfactants: surface-active reagents

- molecules that find it energetically favourable to reside at an interface
e.g. commercial detergents



- generally act to reduce σ locally, $\frac{d\sigma}{d\Gamma} < 0$: may induce Marangoni flows

The evolution of a surfactant-laden interface



Since $\sigma(\Gamma)$, N-S equations and BCs must be augmented by

Surfactant evolution equation:

$$\frac{\partial \Gamma}{\partial t} + \nabla_s \cdot (\Gamma u_s) + \Gamma (\nabla_s \cdot n)(u \cdot n) = J(\Gamma, C) + D_s \nabla_s^2 \Gamma$$

advection

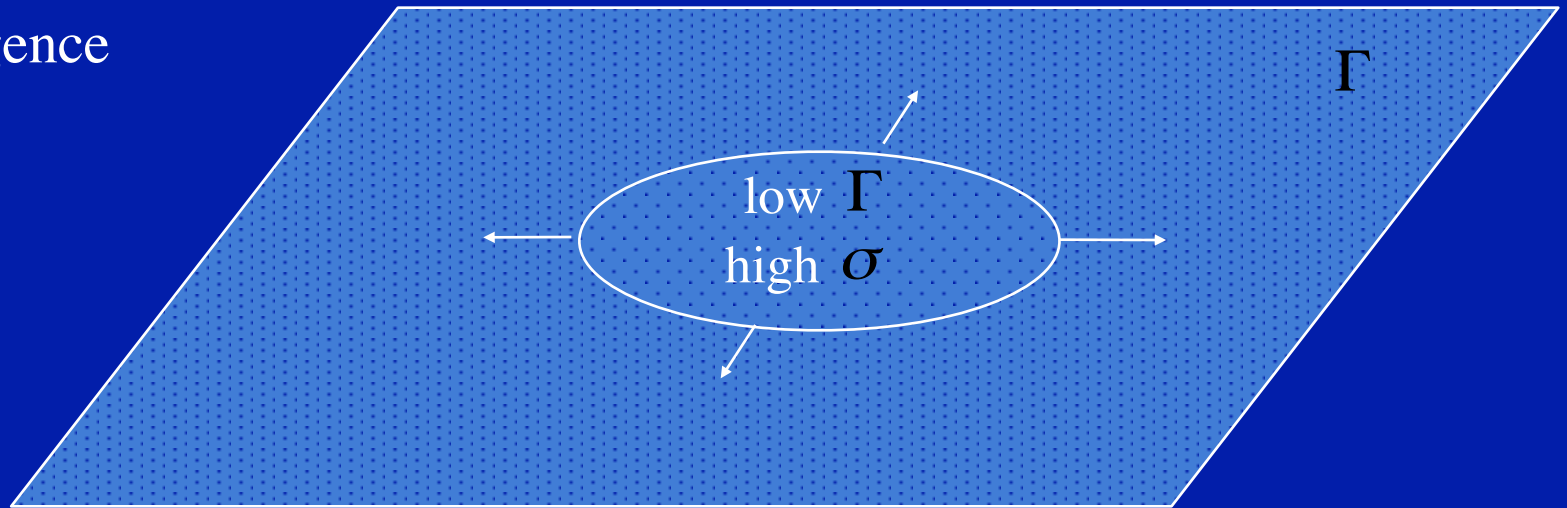
surface
expansion

exchange
with bulk

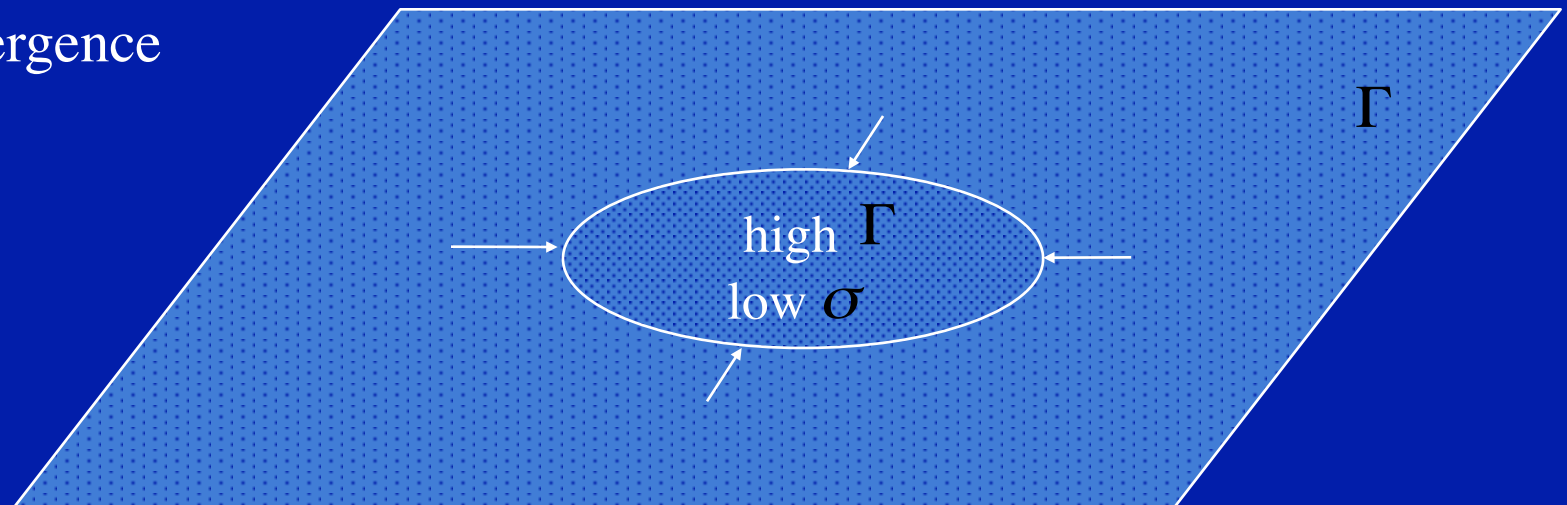
surface
diffusion

Surfactants: impart effective elasticity to contaminated interfaces through resisting flows with non-zero surface divergence

Surface divergence

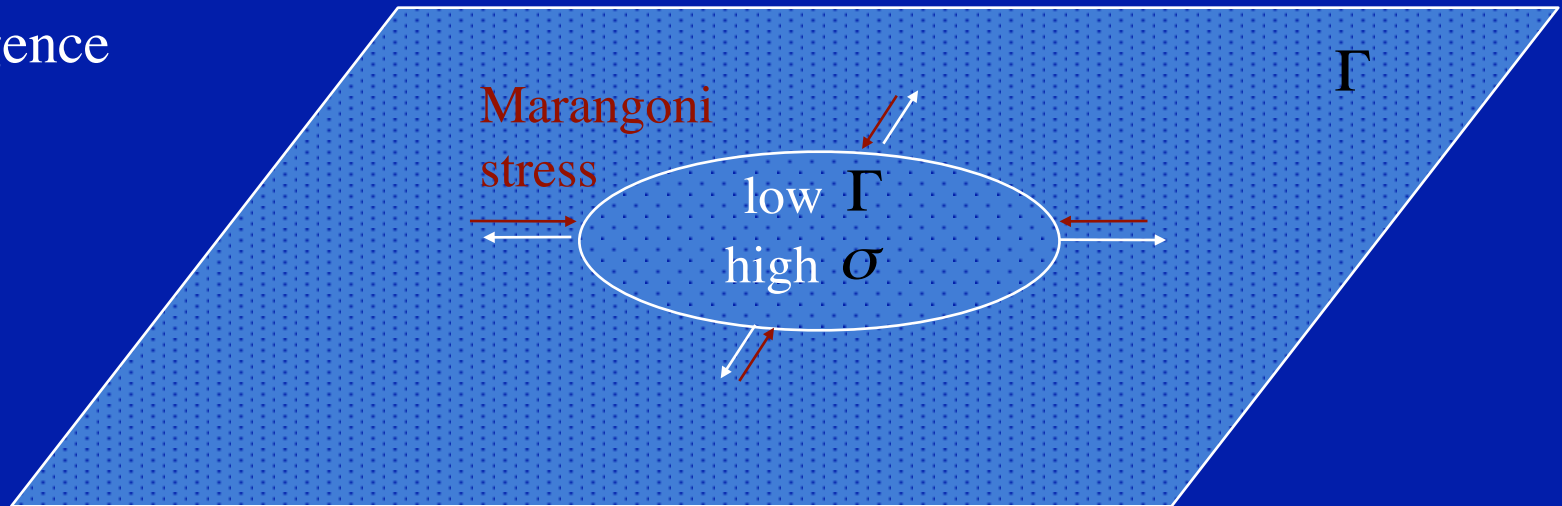


Surface convergence

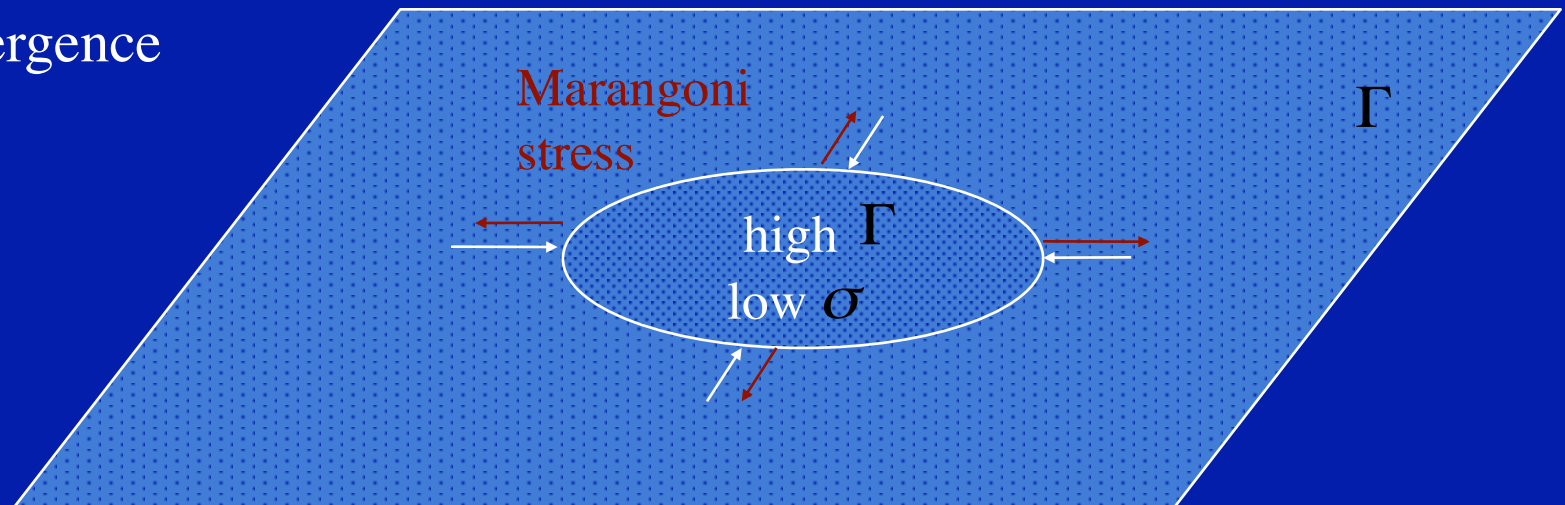


Surfactants: impart effective elasticity to contaminated interfaces through resisting flows with non-zero surface divergence

Surface divergence

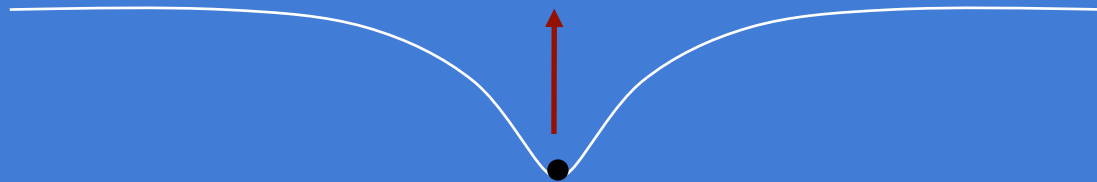


Surface convergence



Clean interface = 'slippery trampoline'

- resists deformation through generation of normal curvature pressures
- cannot generate traction on the interface



Surfactant-laden interface = trampoline

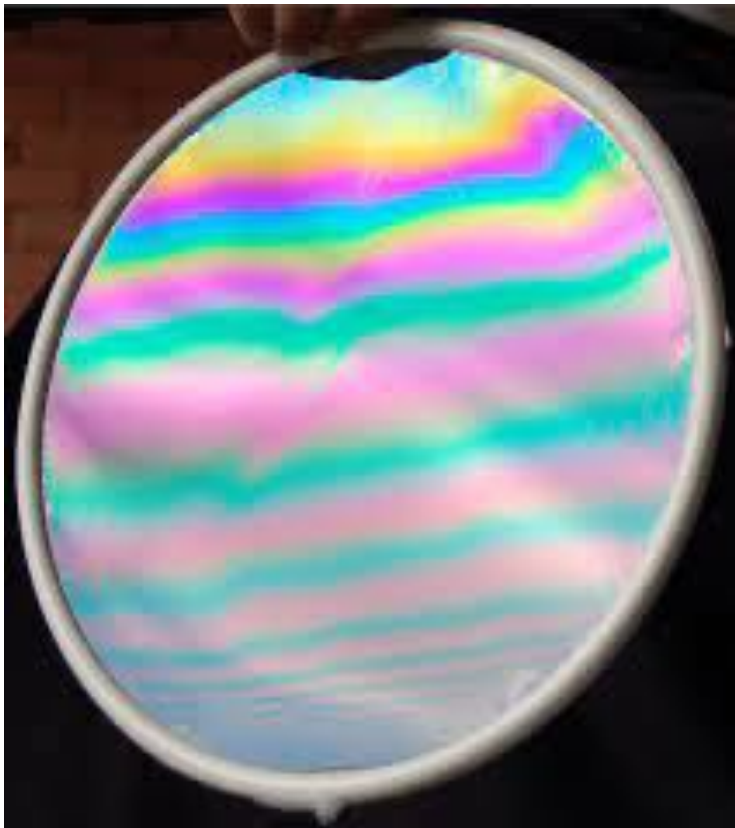
- resists surface deformation as does a clean interface
- can support tangential stresses via Marangoni elasticity

Soap films

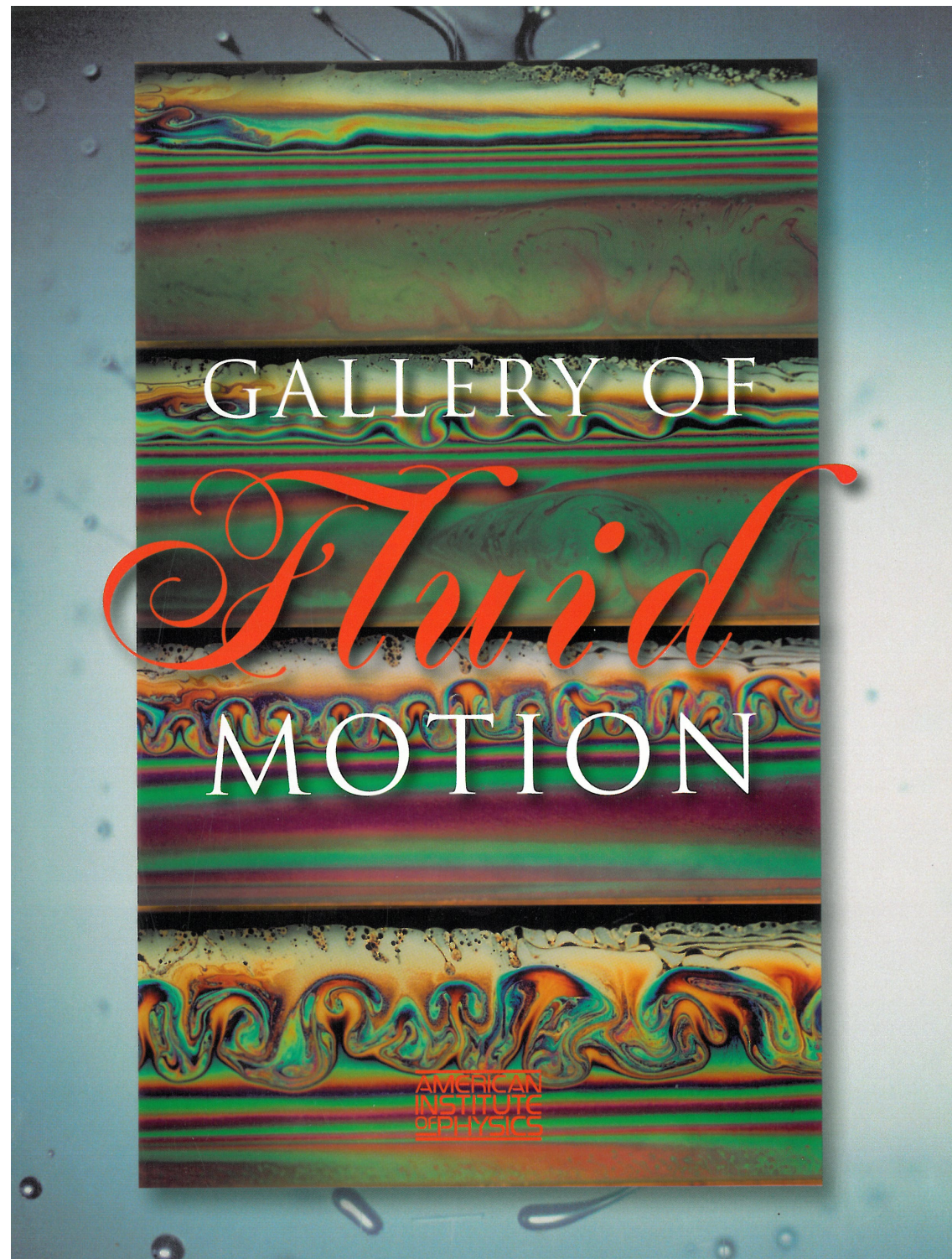


- prevalent in art owing to their aesthetic appeal

Soap films



- color due to interference of light reflected from both sides of film
- film thickness comparable to wavelength of light

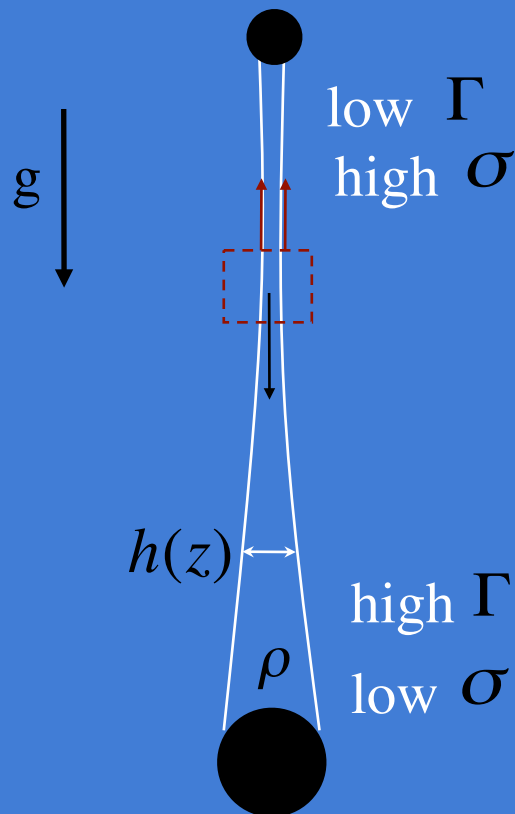


Soap films

- stabilized against rupture by presence of surfactants

Draining soap film

- weight of film supported by Marangoni stress



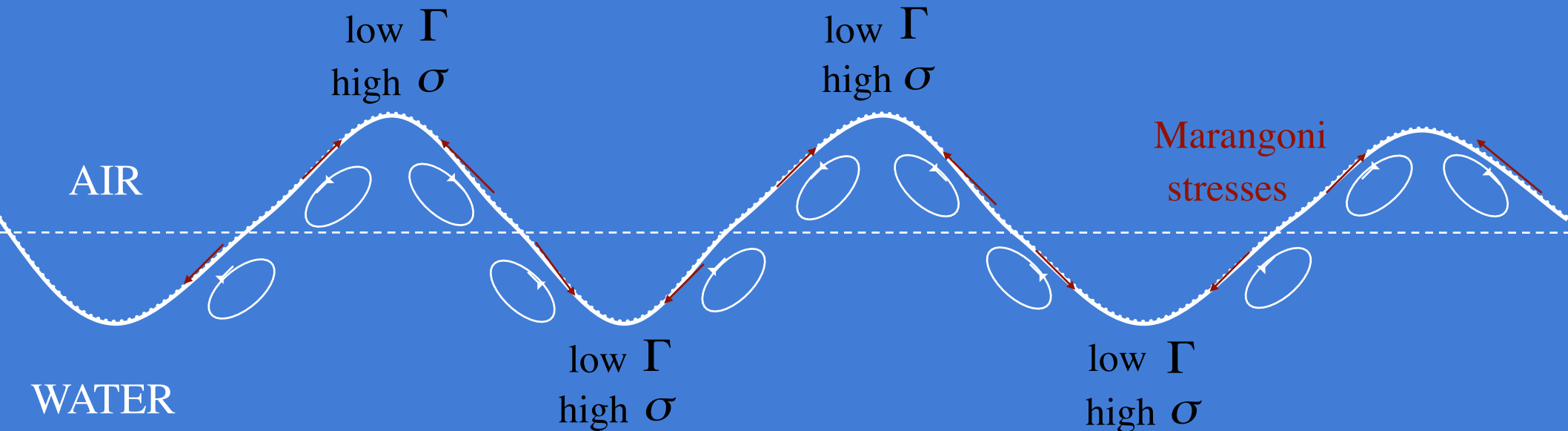
$$\rho g h(z) = 2 \frac{d\sigma}{dz} = 2 \frac{d\sigma}{d\Gamma} \frac{d\Gamma}{dz}$$

The suppression of capillary waves by surfactant

- wave motion generates regions of surface divergence

- concomitant surfactant gradients generate Marangoni stresses $\frac{d\sigma}{d\Gamma} < 0$

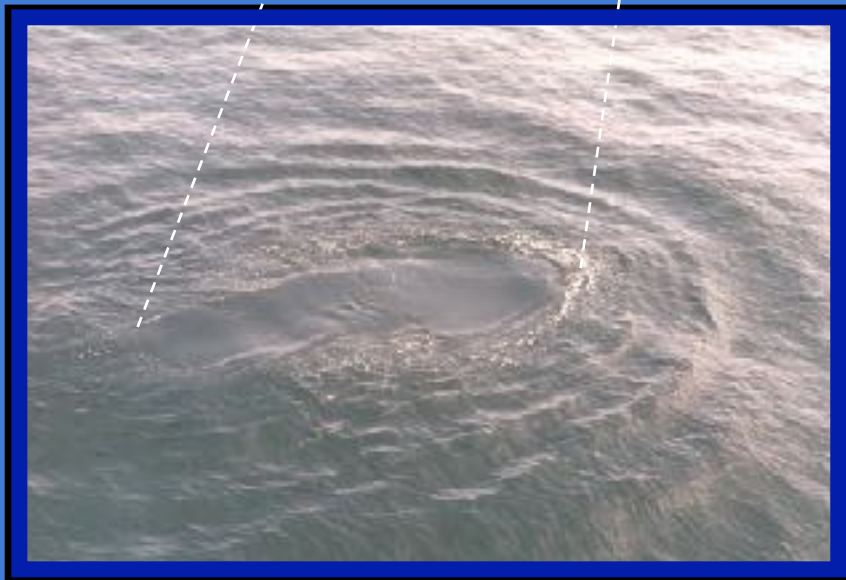
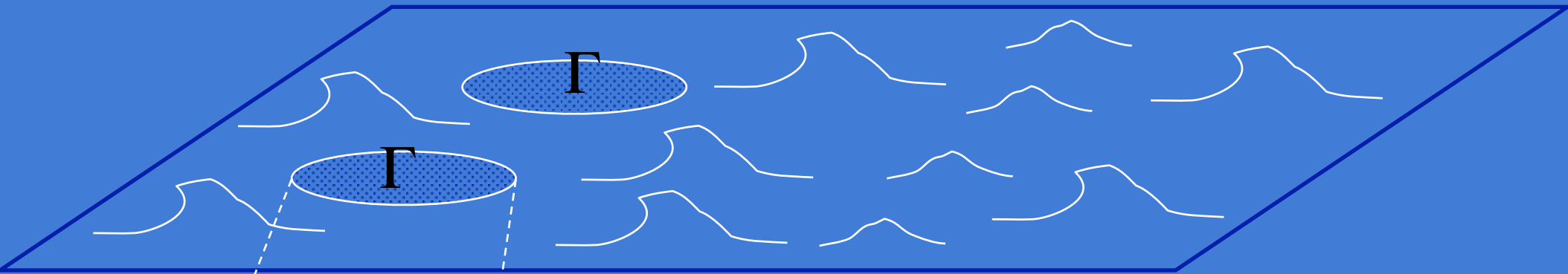
- resulting small scale flows extremely dissipative



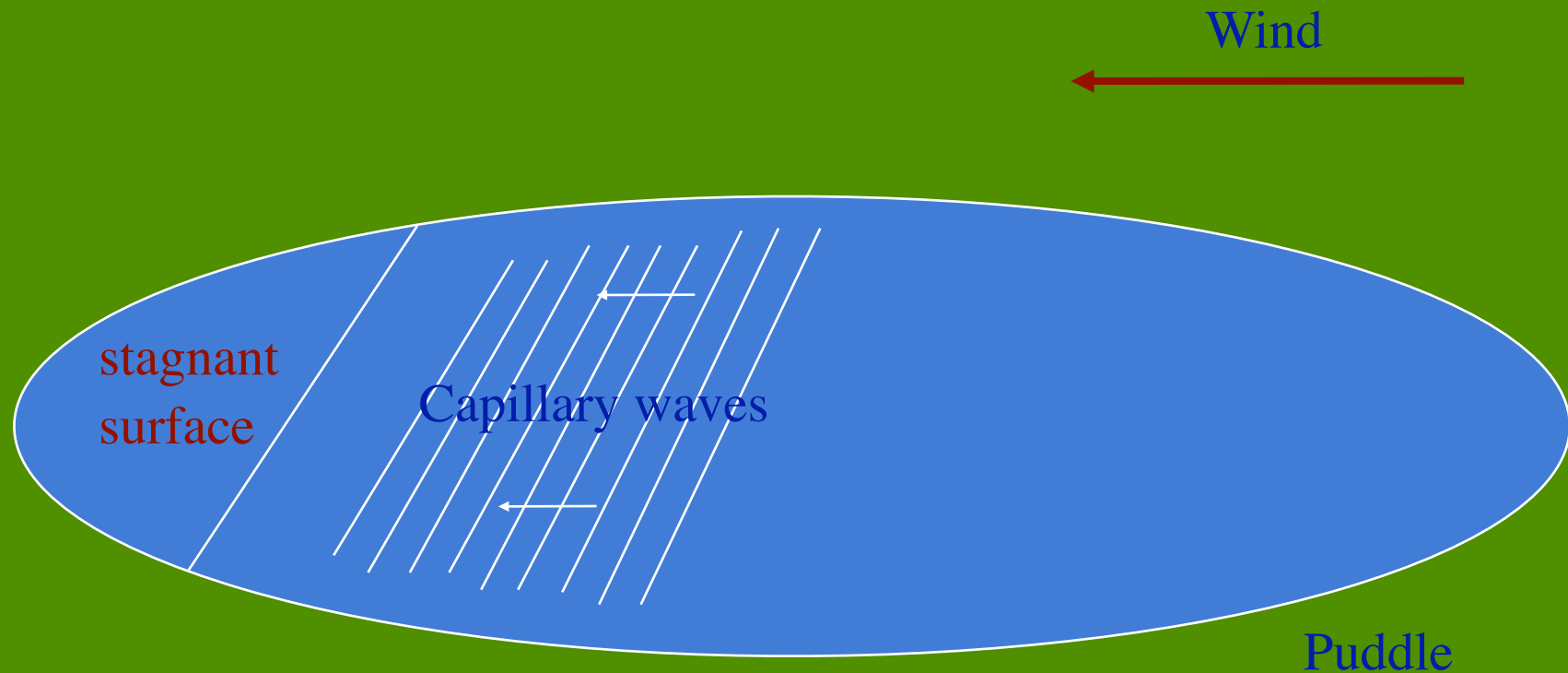
- flat ship wakes first remarked upon by Pliny the Elder
- examined by Benjamin Franklin, motivated by Bermudan spear fishermen
- now used to track submarines: flat wakes visible on satellite images

The footprints of whales

- surfactants (biomaterial in water column) swept to surface by diving whales
- suppress capillary waves and cascade to larger scale waves
- similar rationale for the glassy wake of ships

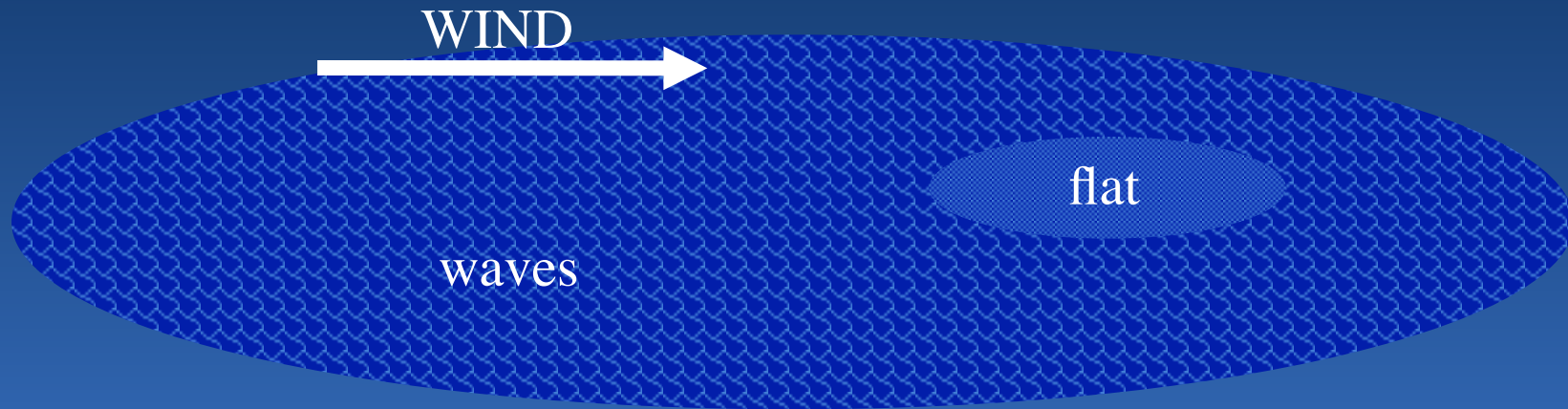


The dynamics of puddles



- surfactants swept to lee of puddle by wind stress
- Marangoni stress balances wind stress \implies stagnant surface
- capillary waves suppressed by surfactant in lee of puddle

Surfactants and a murder mystery

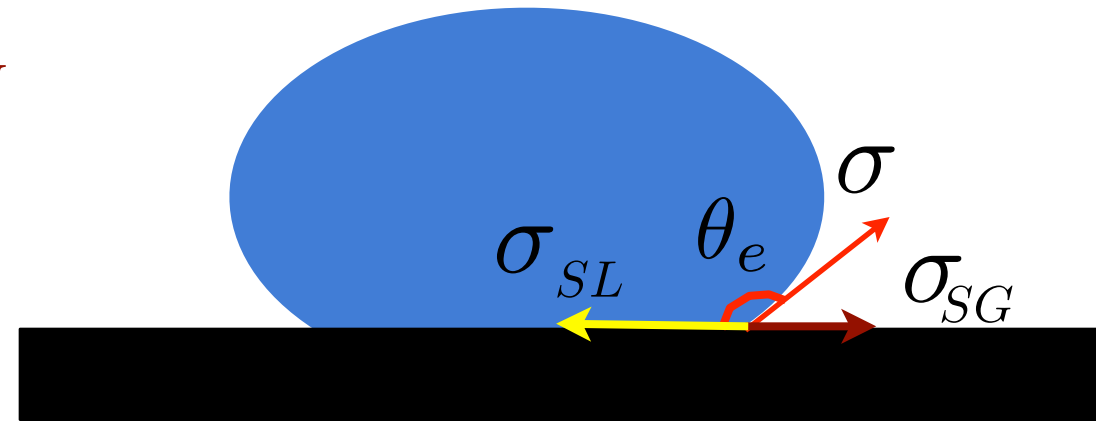


Who dunnit?

“In the autumn of 1878 a man committed a terrible crime in Somerset, which was for some time involved in deep mystery. His wife, a handsome and decent mulatto woman, disappeared suddenly and entirely from sight, after going home from church on Sunday, October 20. Suspicion immediately fell upon the husband, a clever young fellow of about thirty, but no trace of the missing woman was left behind, and there seemed a strong probability that the crime would remain undetected. On Sunday, however, October 27, a week after the woman had disappeared, some Somerville boatmen looking out towards the sea, as is their custom, were struck by observing in the Long Bay Channel, the surface of which was ruffled by a slight breeze, a long streak of calm such as, to use their own illustration, a cask of oil usually diffuses around it when in the water. The feverish anxiety about the missing woman suggested some strange connection between this singular calm and the mode of her disappearance. Two or three days after – why not sooner I cannot tell you – her brother and three other men went out to the spot where it was observed, and from which it had not disappeared since Sunday, and with a series of fish-hooks ranged along a long line dragged the bottom of the channel, but at first without success. Shifting the position of the boat, they dragged a little further to windward, and presently the line was caught. With water glasses the men discovered that it had caught in a skeleton which was held down by some heavy weight. They pulled on the line; something suddenly gave way, and up came the skeleton of the trunk, pelvis, and legs of a human body, from which almost every vestige of flesh had disappeared, but which, from the minute fragments remaining, and the terrible stench, had evidently not lain long in the water. The husband was a fisherman, and Long Bay Channel was a favourite fishing-ground, and he calculated, truly enough, that the fish would very soon destroy all means of identification; but it never entered into his head that as they did so their ravages, combined with the process of decomposition, would set free the matter which was to write the traces of his crime on the surface of the water. The case seems to be an exceedingly interesting one; the calm is not mentioned in any book on medical jurisprudence that I have, and the doctors seem not to have had experience of such an occurrence. A diver went down and found a stone with a rope attached, by which the body had been held down, and also portions of the scalp and of the skin of the sole of the foot, and of clothing, by means of which the body was identified. The husband was found guilty and executed.”

Fluid-Solid Contact: WETTING

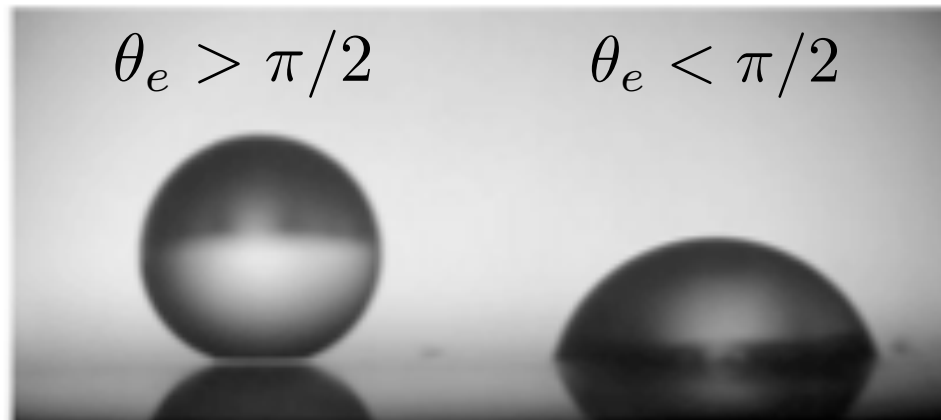
Equilibrium contact angle θ_e



Energy differential: $dW = dx (\sigma_{SG} - \sigma_{SL}) - dx \sigma \cos\theta_e$

Young's relation:

$$\sigma \cos\theta_e = \sigma_{SL} - \sigma_{SG}$$



Hydrophobic
surface

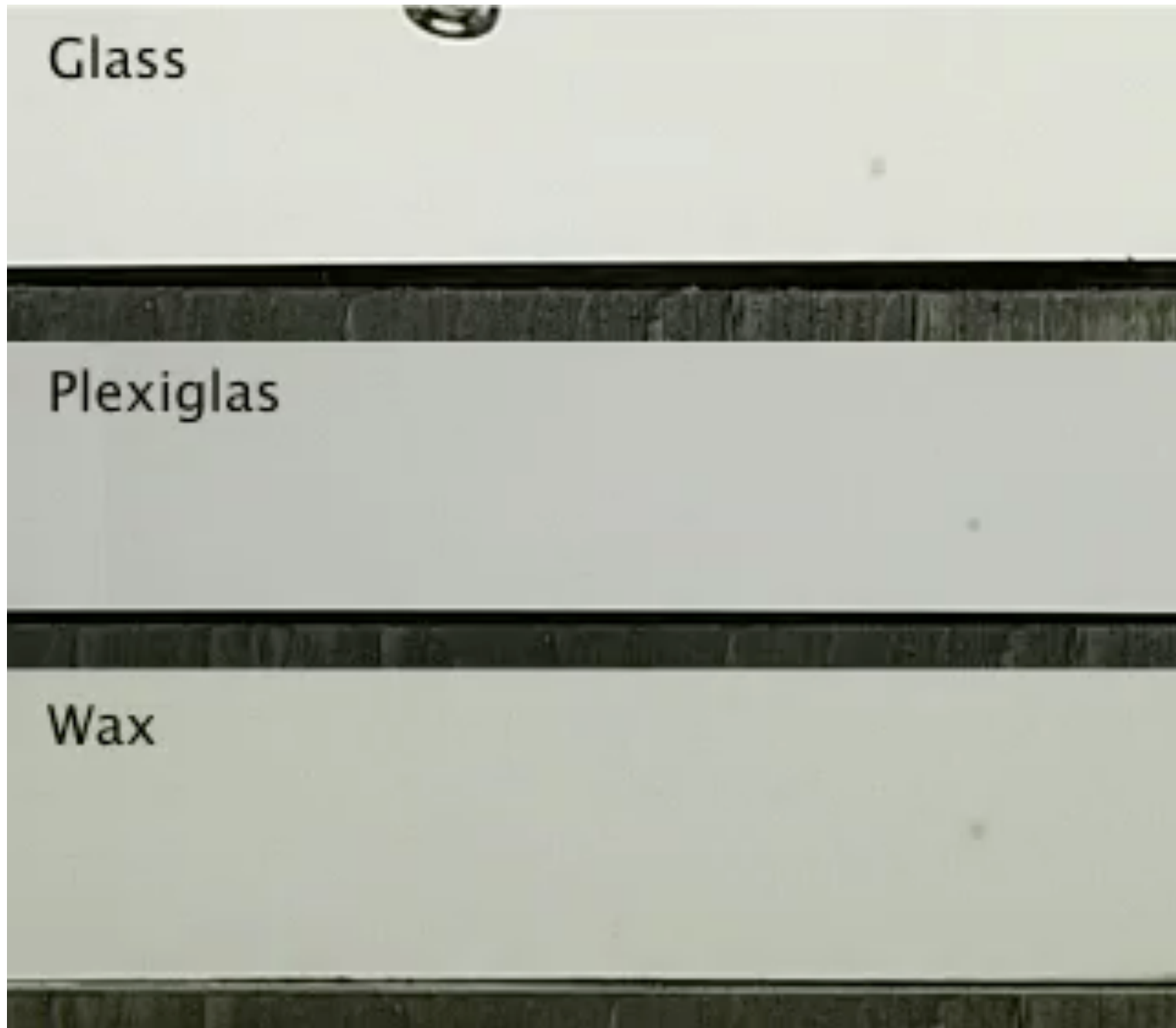
Hydrophilic
surface



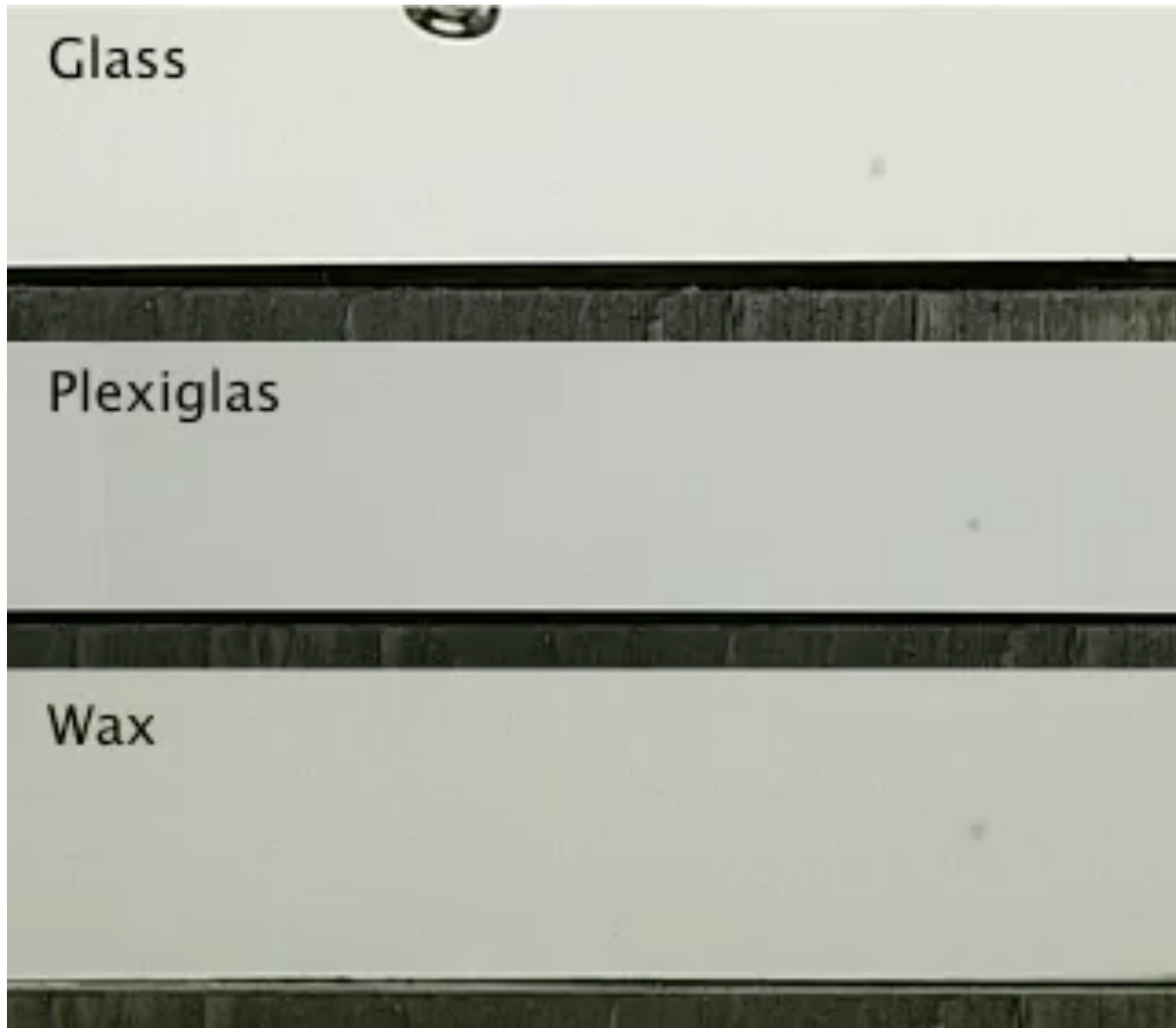
Total wetting on a flat solid



Total wetting on a flat solid



Partial wetting



Partial wetting

Fluorinated Oil



Oil



Water



Mercury

Fluorinated Oil



Oil



Water



Mercury

The raindrop paradox

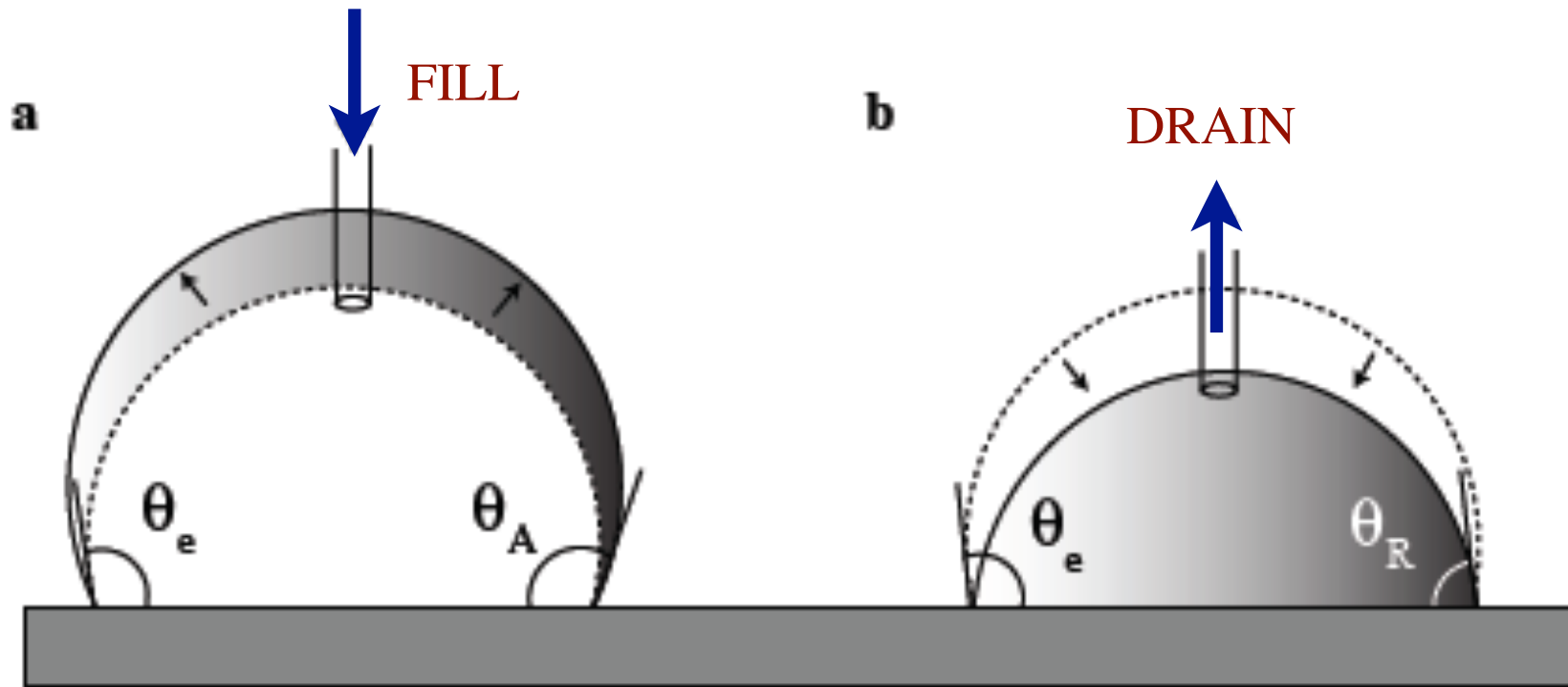


Why do rain drops stick?

Contact angle hysteresis

Static contact angle is not uniquely θ_e

Reality: drop is stable over a range of $\theta_r < \theta < \theta_a$



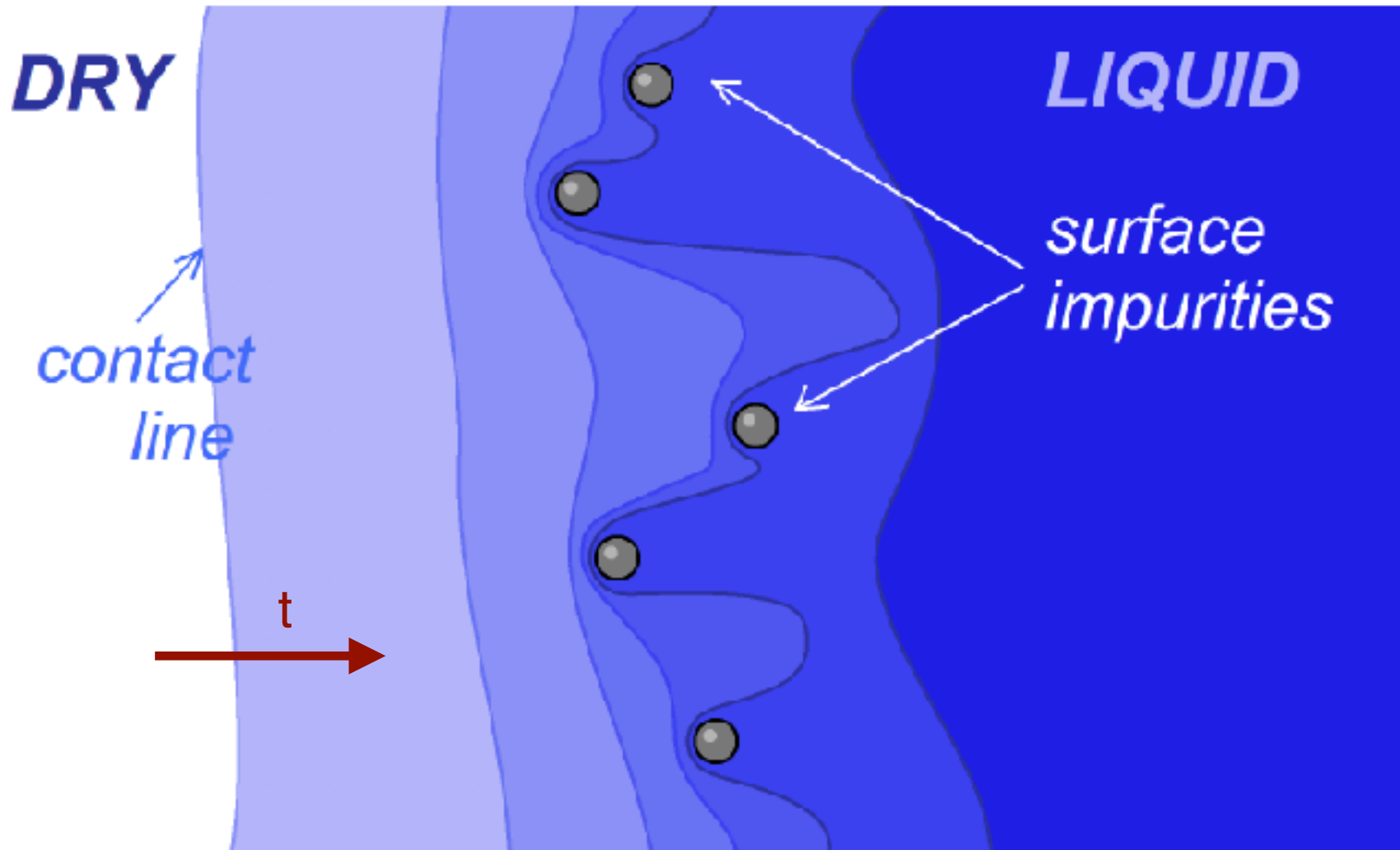
→ FORCE of ADHESION resists drop motion

increases with $\Delta\theta = \theta_a - \theta_r$

Origins: advancing contact lines pinned on surface irregularities

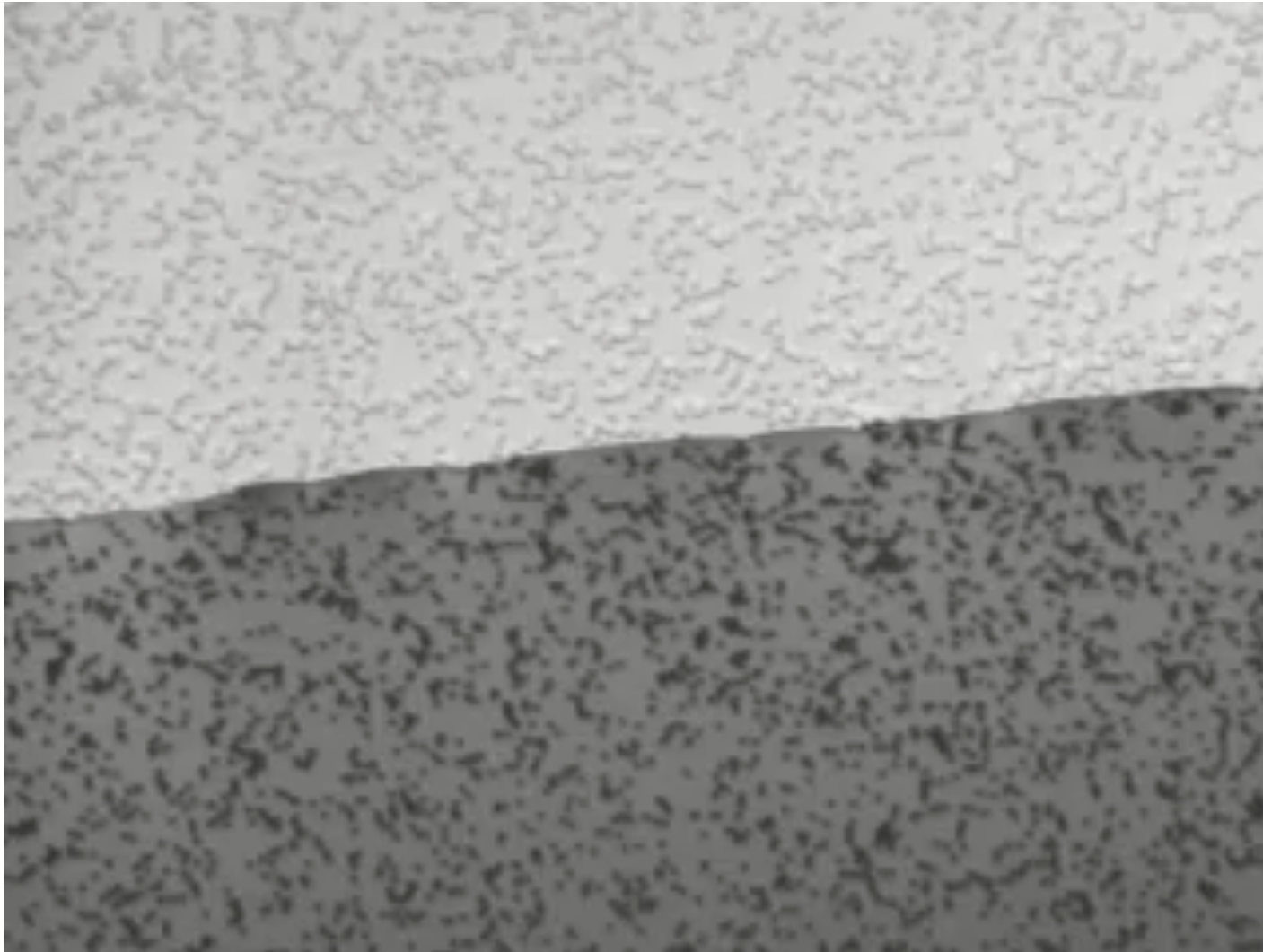
The origins of contact angle hysteresis

- motion of contact line past chemical/textural irregularities is energetically costly



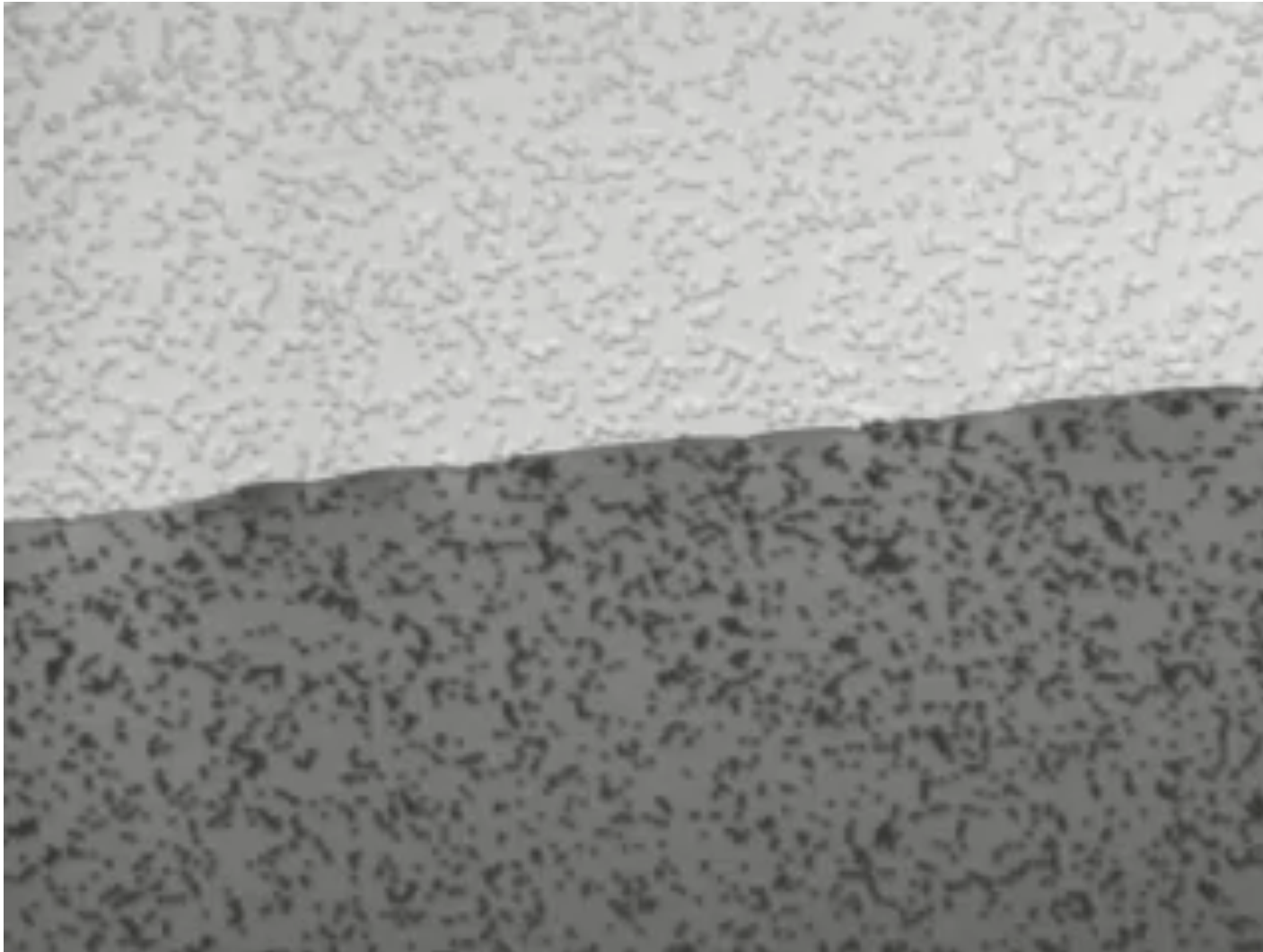
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The origins of contact angle hysteresis

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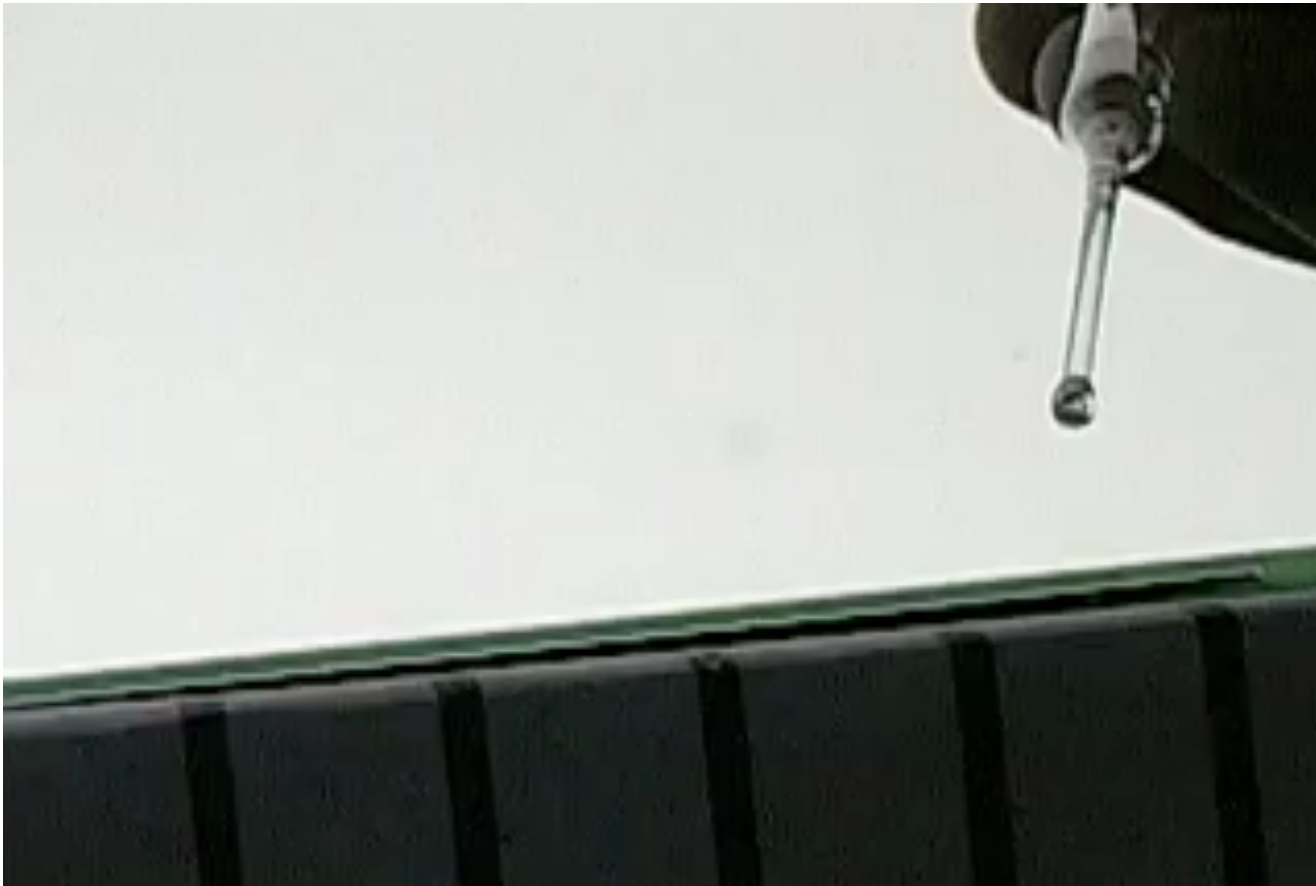
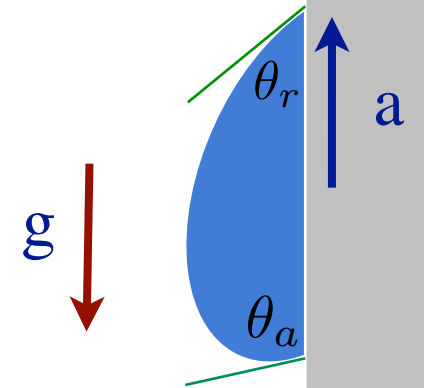
The force of adhesion (Dussan & Chow 1983)

Raindrop stuck on a window

- small drops supported by contact line resistance

$$F_c \sim 2\pi a \sigma (\cos \theta_r - \cos \theta_a)$$

- drops grow by accretion until weight prompts rolling



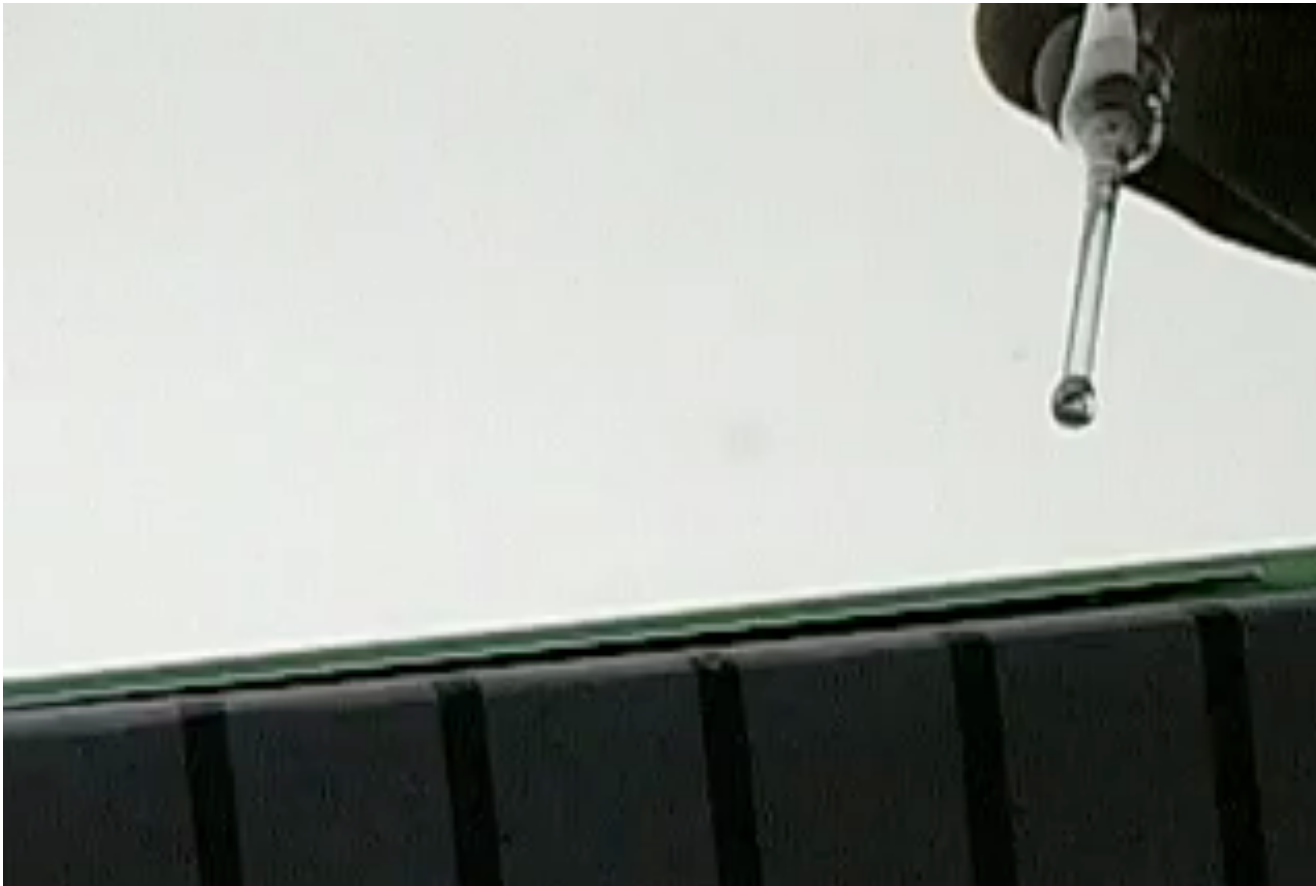
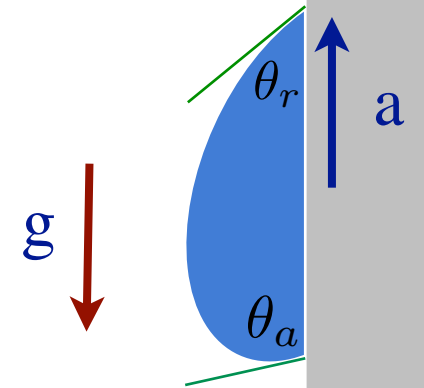
The force of adhesion (Dussan & Chow 1983)

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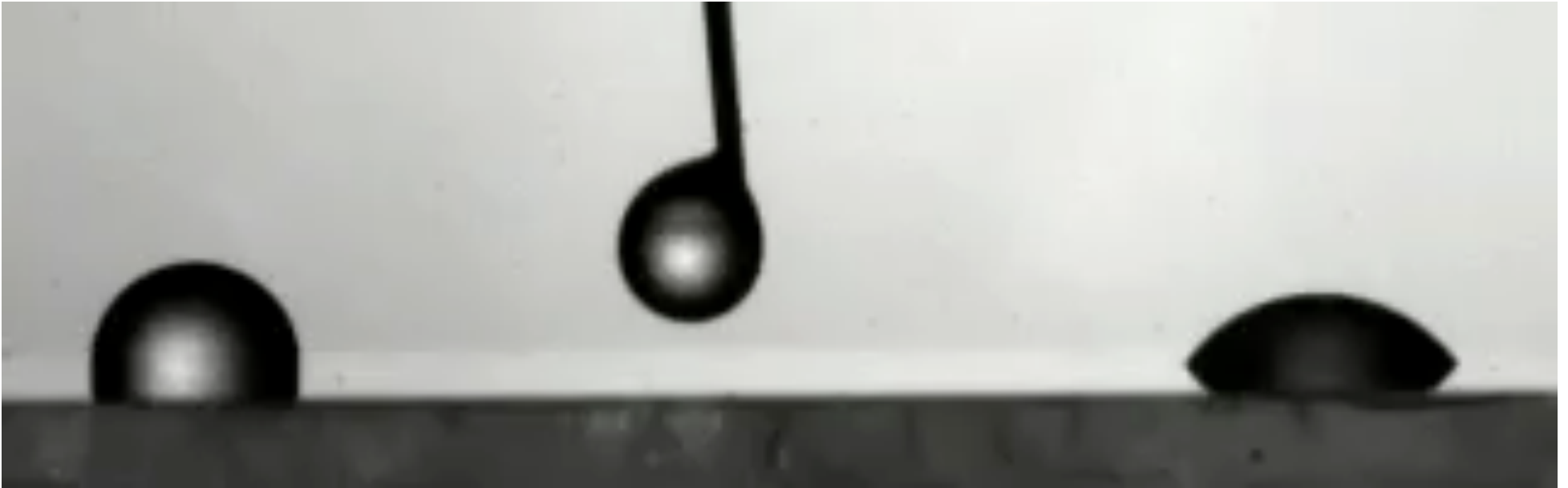
- small drops supported by contact line resistance

$$F_c \sim 2\pi a \sigma (\cos \theta_r - \cos \theta_a)$$

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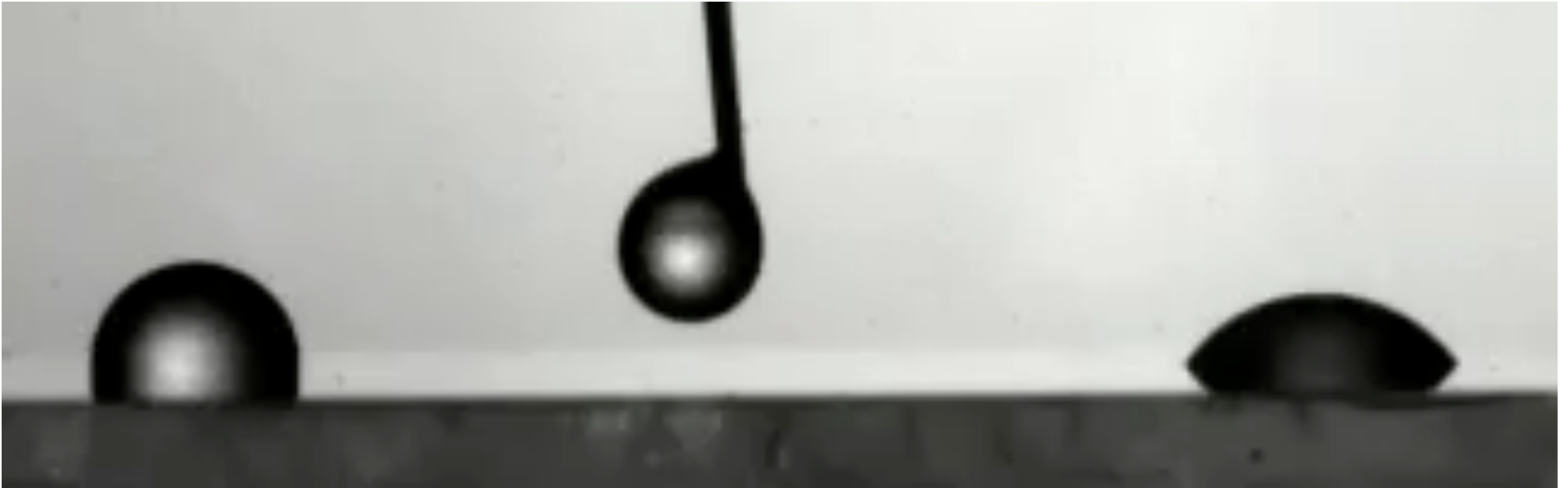


Spontaneous motion in response to a wettability gradient



- lateral chemical force must overcome contact force

Spontaneous motion in response to a wettability gradient



- lateral chemical force must overcome contact force

Spontaneous motion in response to a chemical gradient



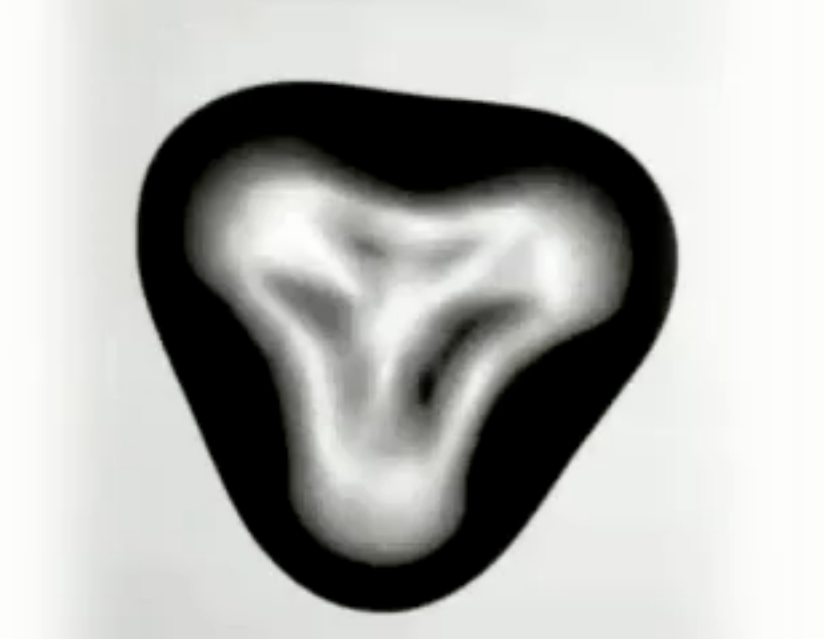
- lateral chemical force must overcome contact force

Spontaneous motion in response to a chemical gradient



- lateral chemical force must overcome contact force

Overcoming contact forces via vibration

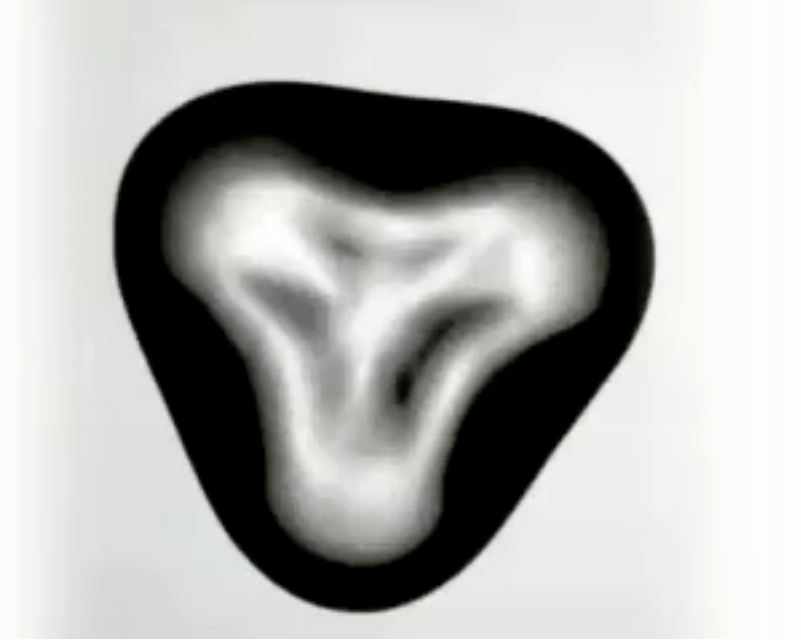


- force at drop's natural frequency

$$\rho U^2 \approx \sigma / R$$

→
$$\omega \sim \left(\frac{\sigma}{\rho R^3} \right)^{1/2}$$

Overcoming contact forces via vibration



- force at drop's natural frequency

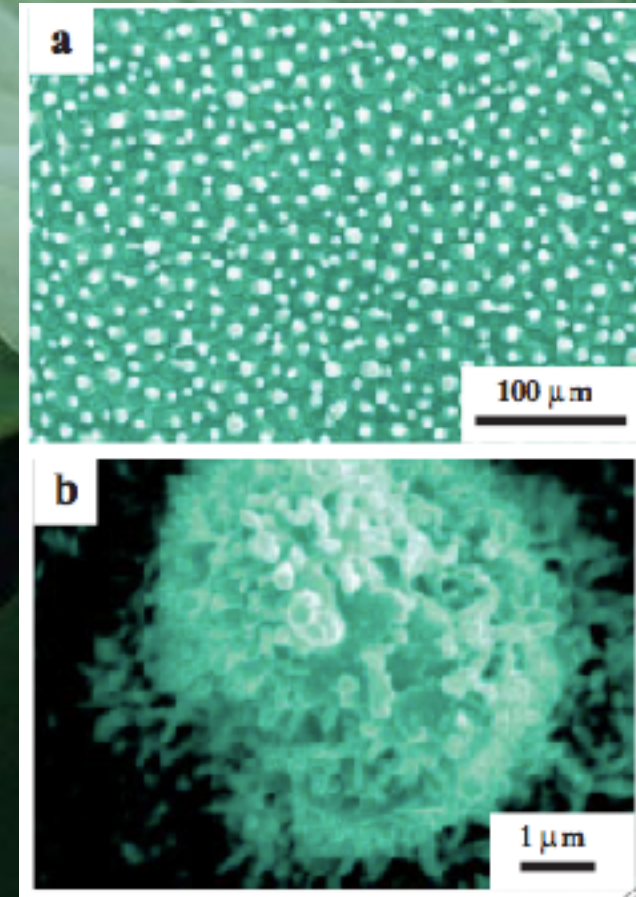
$$\rho U^2 \approx \sigma / R$$

→
$$\omega \sim \left(\frac{\sigma}{\rho R^3} \right)^{1/2}$$

Water repellency in nature

“One who performs his duty without attachment, surrendering the results unto the Supreme Being, is unaffected by sinful action, as the lotus leaf is untouched by water.”

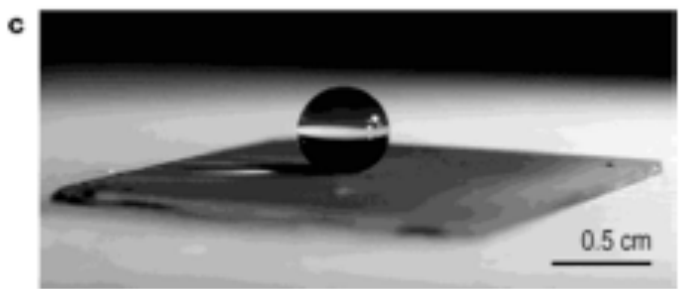
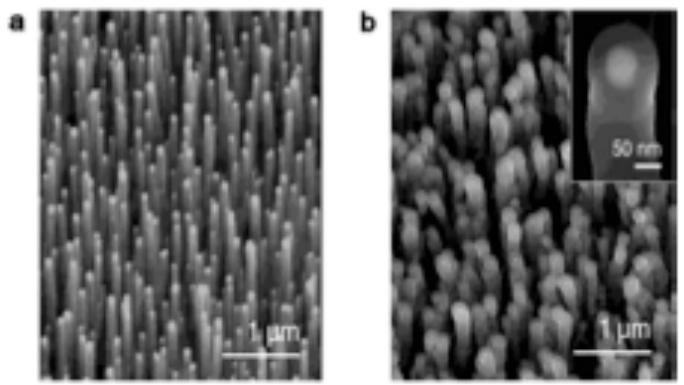
Bhagavad Gita 5.10



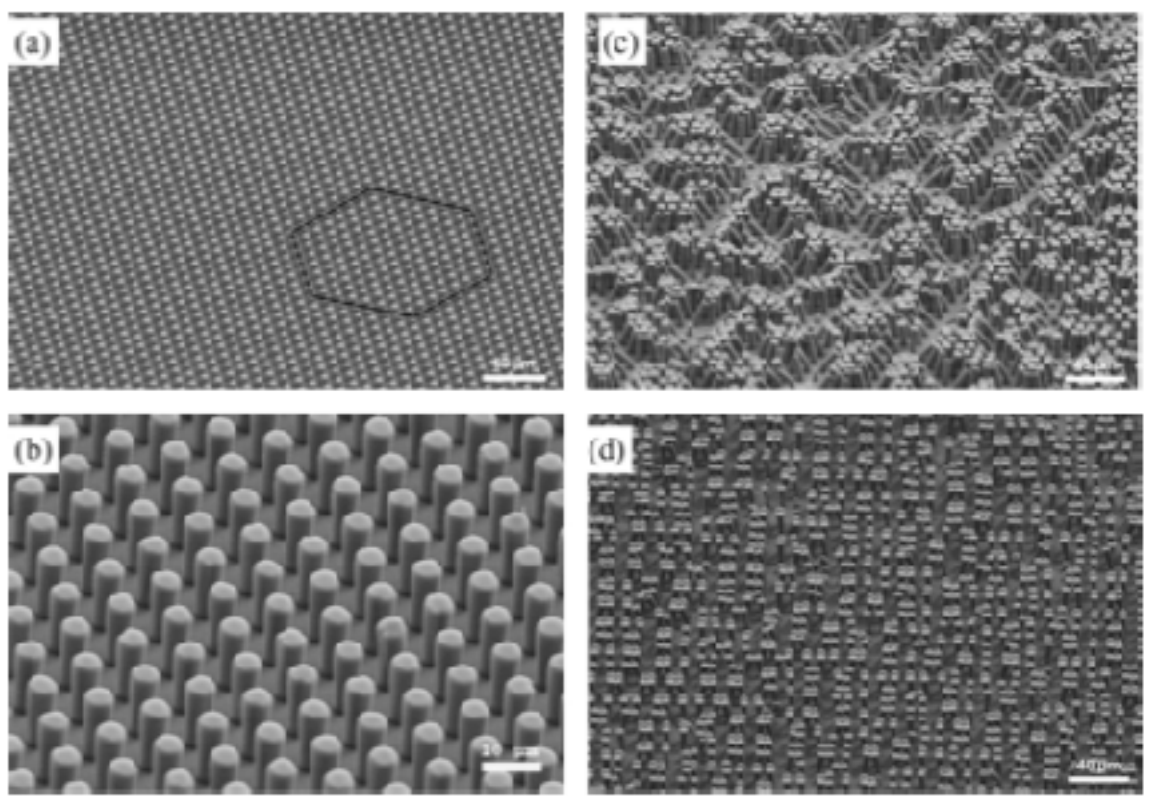
Feng et al. (2004)

- the lotus leaf is superhydrophobic and self-cleaning by virtue of its waxy surface roughness

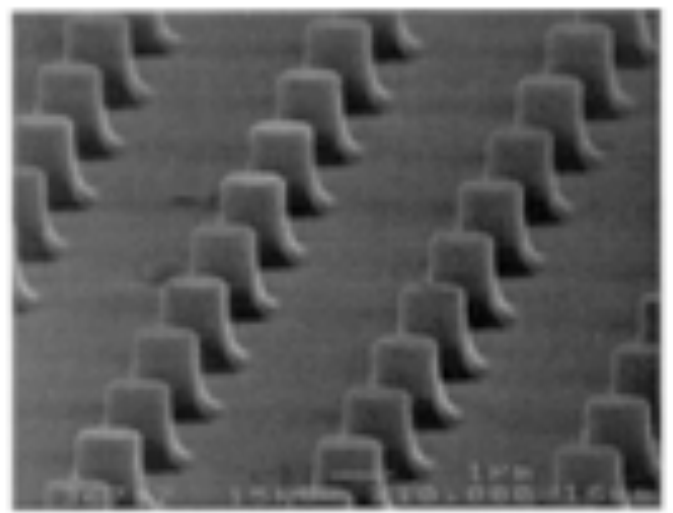
Biomimetic water-repellent surfaces: viable with new microfab techniques



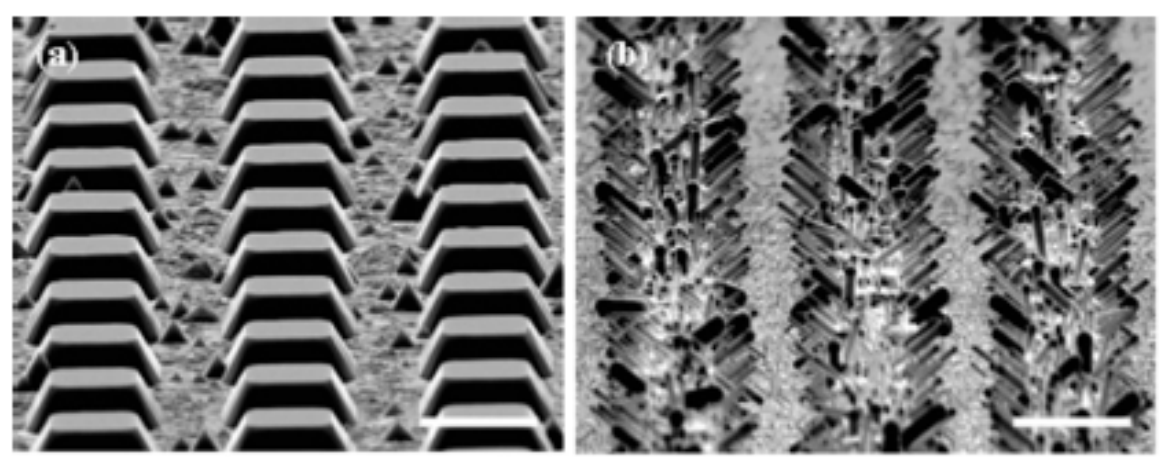
Lau et al. (2003)



Greiner et al. (2007)

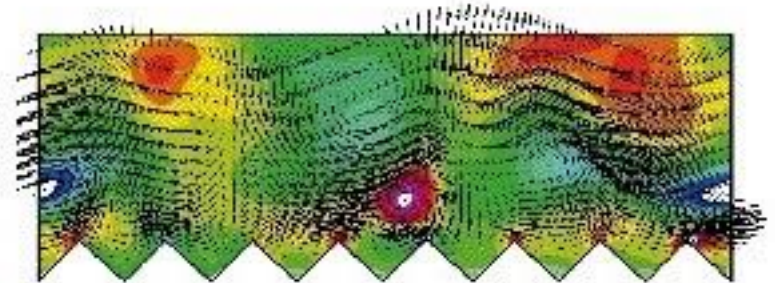
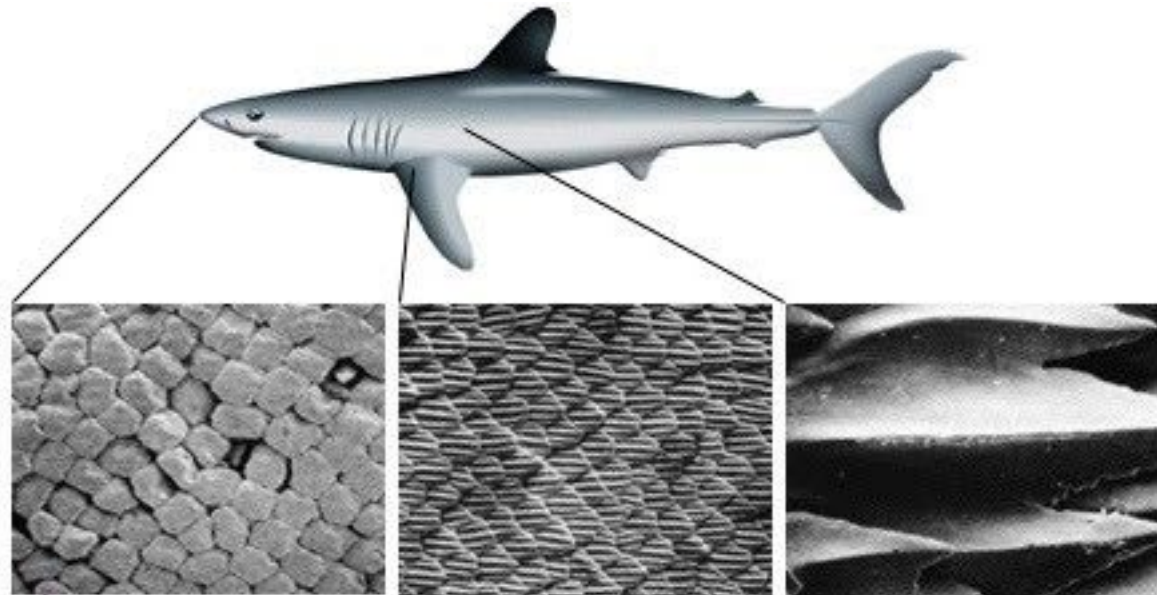
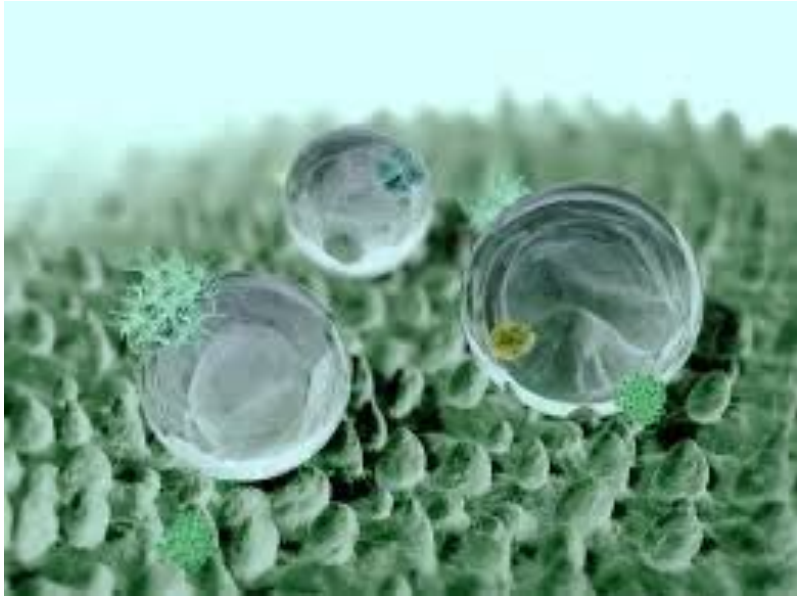


Bico et al. (1999)

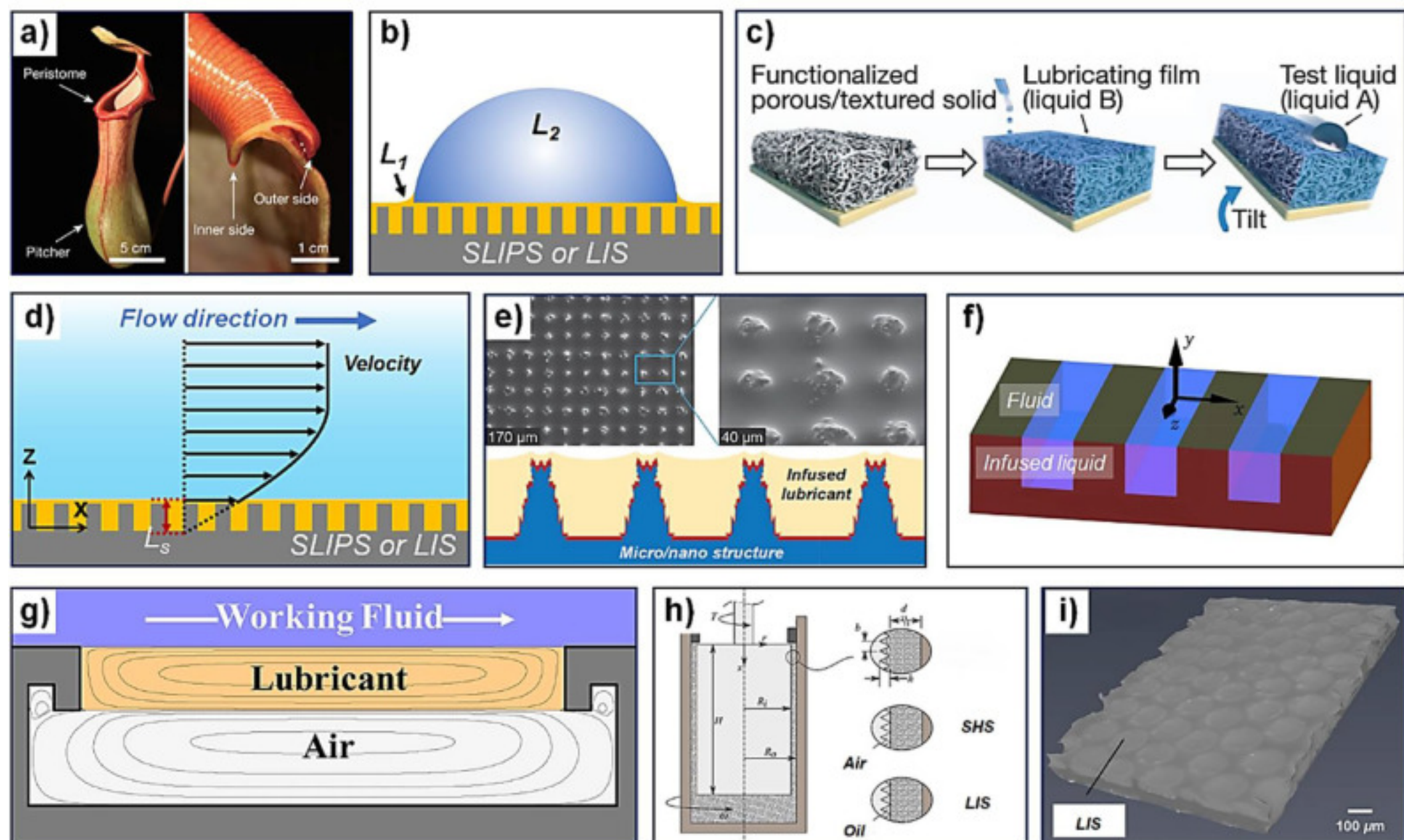


Cao et al. (2007)

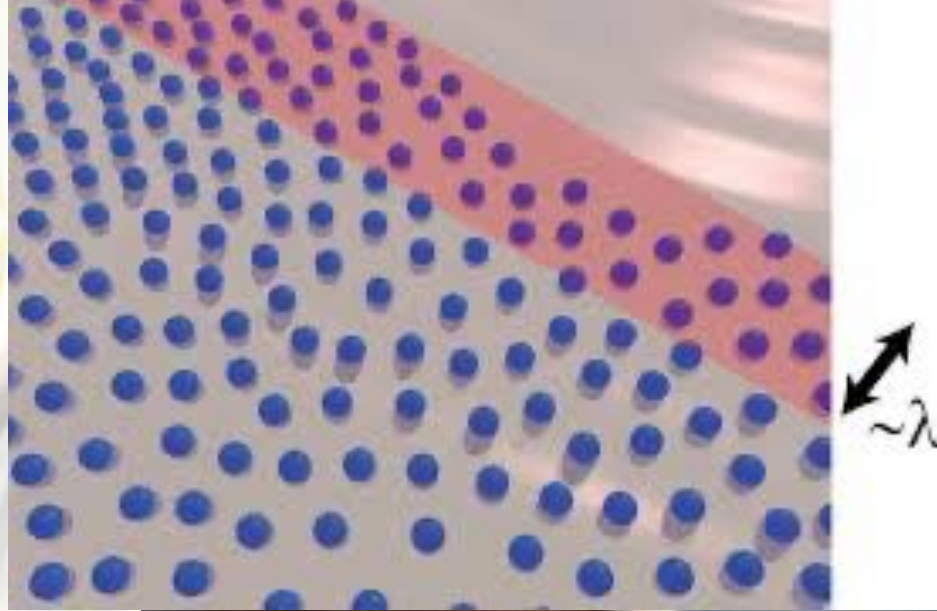
Biomimetic surfaces: for water-repellency, self-cleaning, and drag reduction



Slippery surfaces



Some project ideas



What are the origins of the iridescence on a cup of tea?

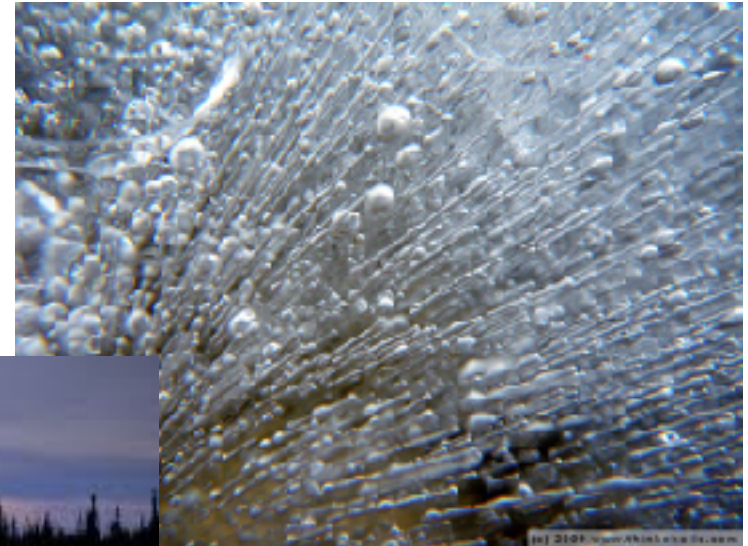
Frost



Interfacial effects in snowflake formation



Bubbles in ice



What sets their shape?

The Thoreau- Reynolds Ridge



The Thoreau-Reynolds Ridge



- arise upstream of blocked flow regions; *e.g.* floating log, canoe paddles
- surfactants adjoining the blocker suppress waves, impede surface flow

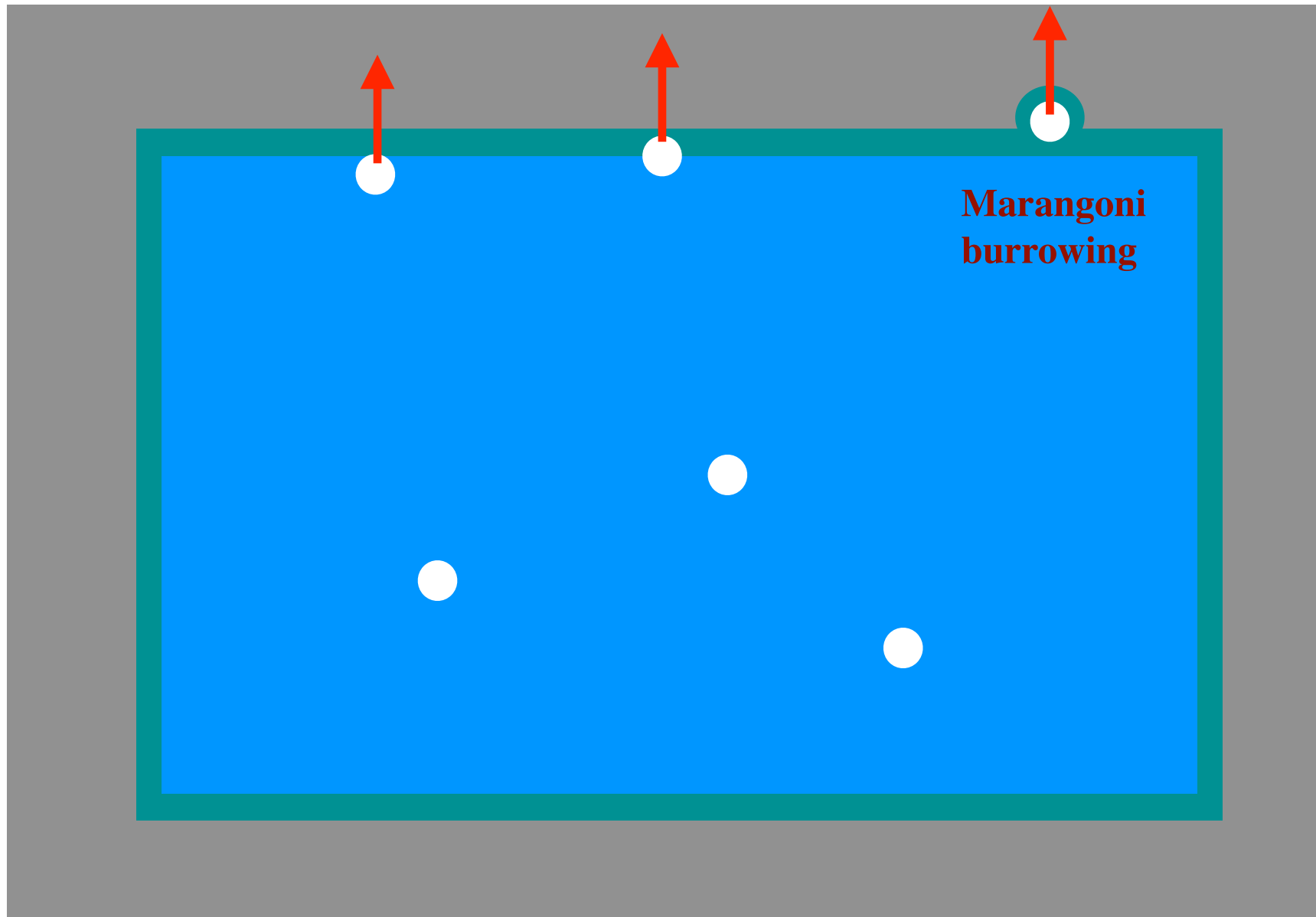
The Thoreau-Reynolds Ridge



- arise upstream of blocked flow regions; *e.g.* floating log, canoe paddles
- surfactants adjoining the blocker suppress waves, impede surface flow

Marangoni Doomsday

The storage of radioactive liquid waste.



Fireworks

What do bubbles have
to do with it?



Sparklers

Inoue et al.

Physics of skiing and skating



How important are interfacial effects at the ice surface?

Physics of curling



The reverse Magnus effect?

