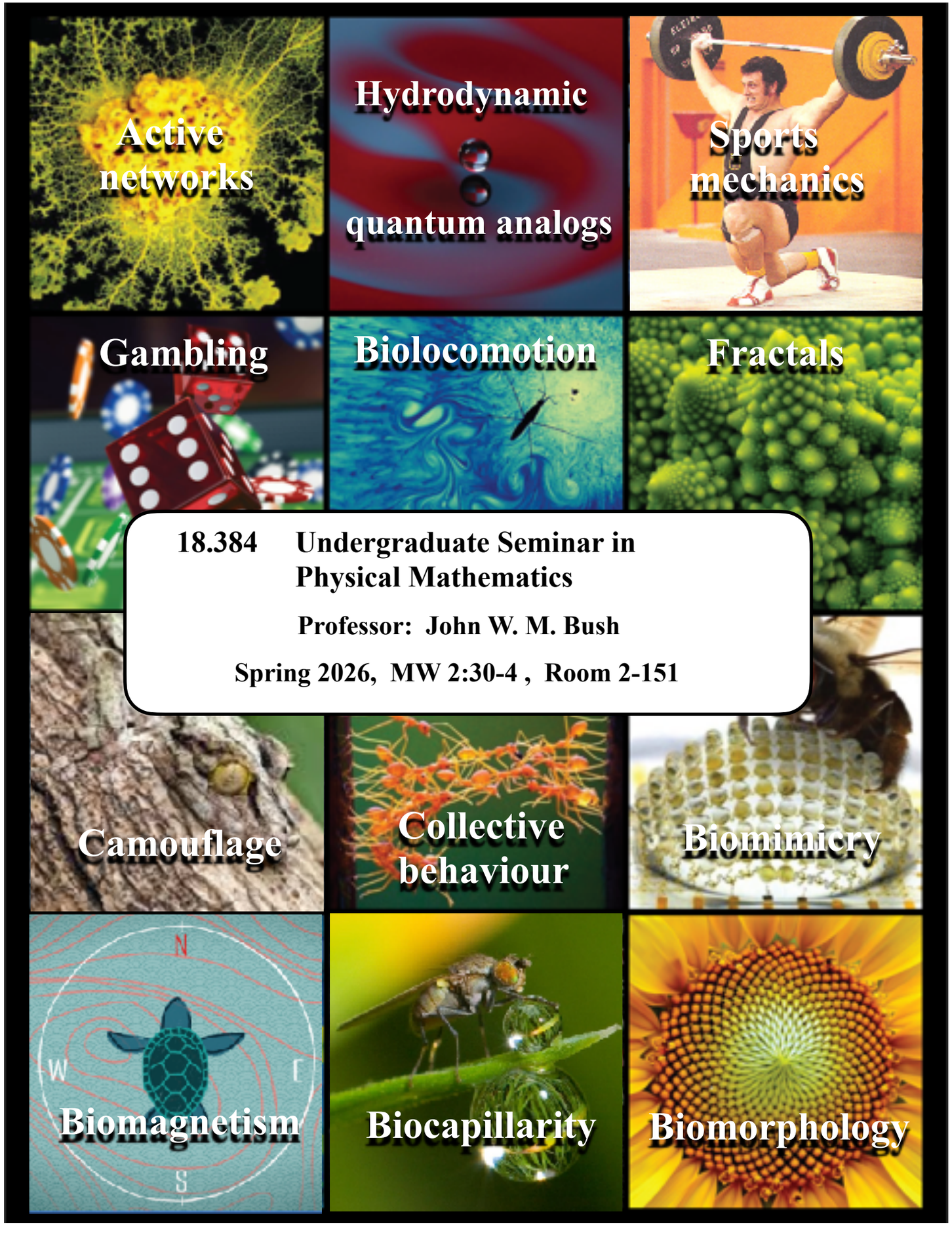

Active networks
Hydrodynamic quantum analogs
Sports mechanics
Gambling
Biolocomotion
Fractals
18.384 Undergraduate Seminar in Physical Mathematics
Professor: John W. M. Bush
Spring 2026, MW 2:30-4 , Room 2-151
Camouflage
Collective behaviour
Biomimicry
Biomagnetism
Biocapillarity
Biomorphology

18.384 : Undergraduate Seminar in Physical Applied Math

Goals: 1) to give you the chance to explore any subject of interest to you (and hopefully to me and the rest of us)

2) to give you the opportunity to improve your presentation skills (speaking and writing)

3) to teach you to write and speak in an academic style, and to respect academic norms of scholarship

Format: choose one overarching theme that can be plausibly split into 3 subtopics (each the subject of one talk, one paper)

e.g. mathematics of music, mathematics in botany, mathematics in sport, fractals in nature, the evolution of language/music, the evolution of evolution, *etc*…….

18.384 UNDERGRADUATE SEMINAR IN PHYSICAL MATHEMATICS

Professor John W.M. Bush
Room 2-446
Phone: 3-4387 (office)
bush@math.mit.edu
https:/thales.mit.edu/bush/
Office hours: after class; available upon request

Spring 2026
MW 2:30-4pm
Room 2-151

COURSE GOALS

- to improve your written and spoken communication, and your presentation style
- to instruct you in academic writing and integrity, critical thinking and reading
- to give you the opportunity to study something that you are excited about

COURSE FORMAT

- for the first few weeks, I will give lectures that will provide valuable material for several of the projects
- thereafter, you will take turns giving presentations, and collectively assessing them
- a syllabus with the lecture schedule will be posted in the coming days
- after each mini-lecture, you will hand in a short paper (1500 words). After your final presentation, you will combine these papers into a final paper (5000 words).

GRADING SCHEME

- 10% : first presentation and paper
- 10% : second presentation and paper
- 10% : third presentation and paper
- 50% : final presentation and paper
- 20% : class participation – half of which is for contributions to discussion, half of which is based on feedback requested on other presentations

READING MATERIAL

- there is **no required text** for the course
- materials of interest will be distributed as the class proceeds

COURSE POLICY

- attendance is mandatory barring illness approved by S^3; attendance will be taken
- AI may be used as a research tool, but writing must be done independently

18.384 Seminar in Physical Applied Mathematics: INFORMATION SHEET

Name:

email:

Major:

Year:

Previous relevant math/physics/engineering courses?

Most enjoyable course:

Least enjoyable course:

Scientific interests:

Non-scientific interests:

Other important facts:

What do you find interesting?

Give this question some thought in the next few weeks as you settle upon a research theme.

After my initial 3 weeks of lecturing, you will be asked to pitch a topic to the class (in 5 minutes) and submit a brief (< 1 page) proposal to me.

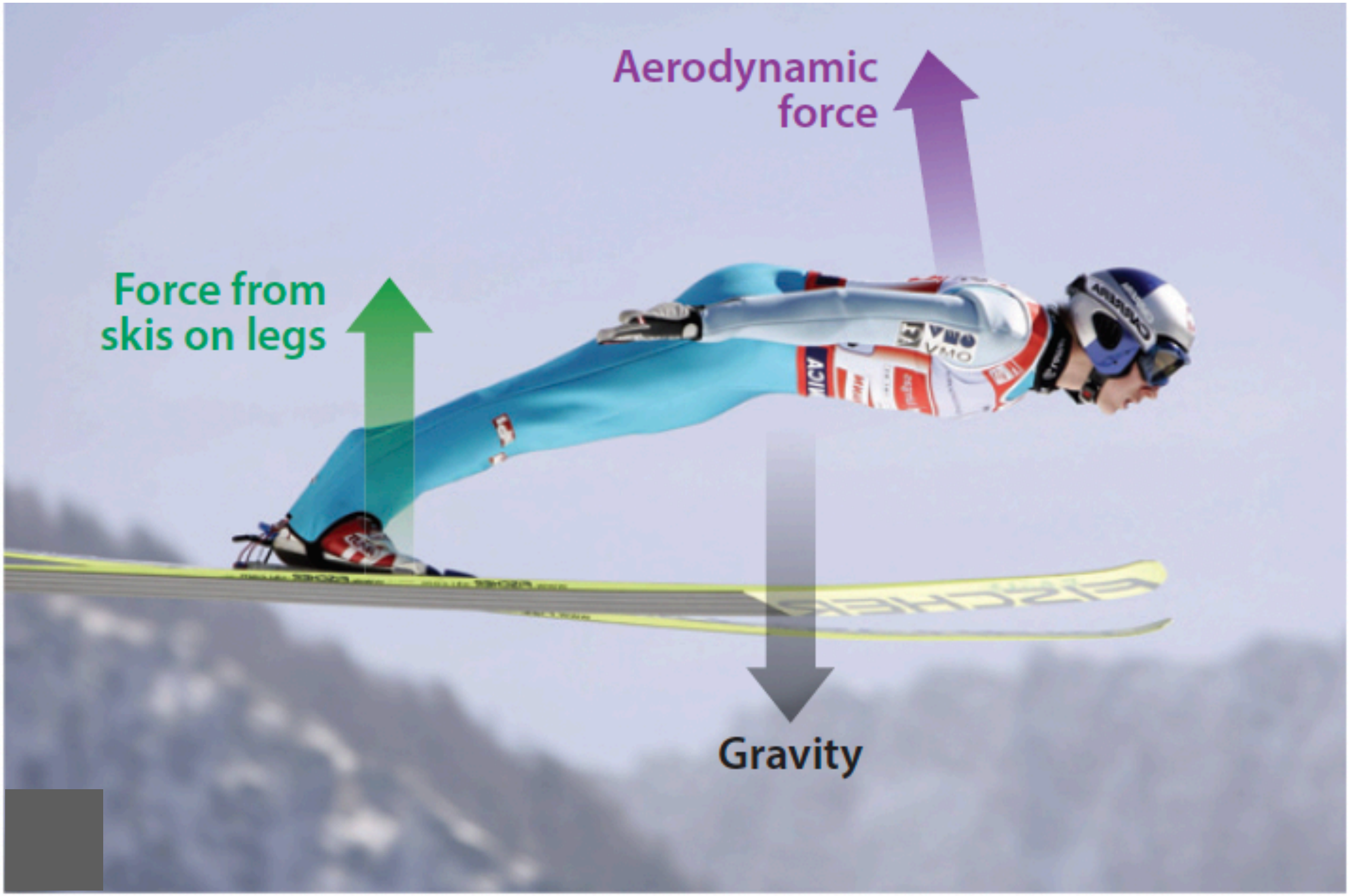
Roberto Carlos, Tournoi de France, 1997

Roberto Carlos, Tournoi de France, 1997

A world record throw

A world record throw

Sports mechanics

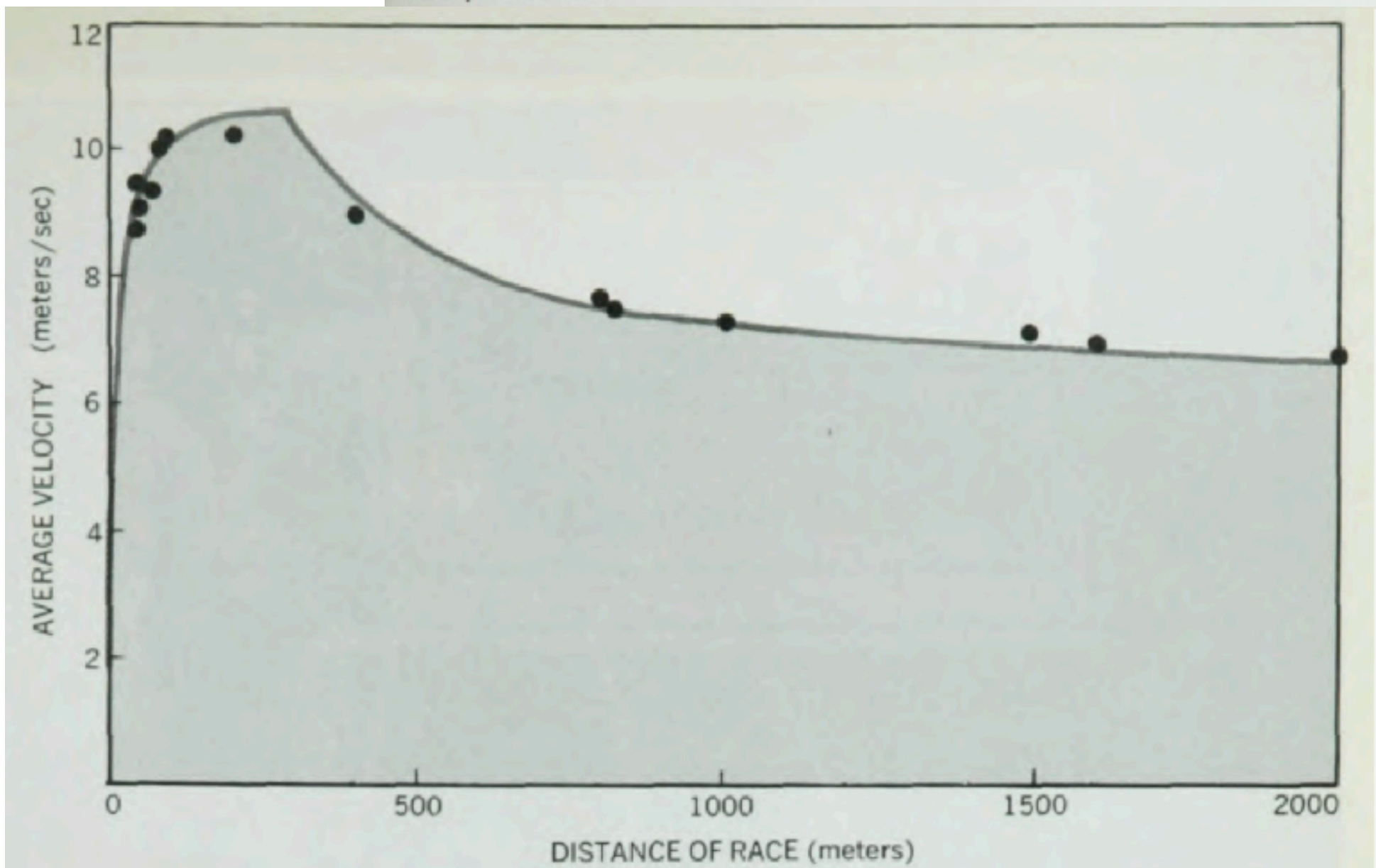


A favorite paper…

A theory of competitive running

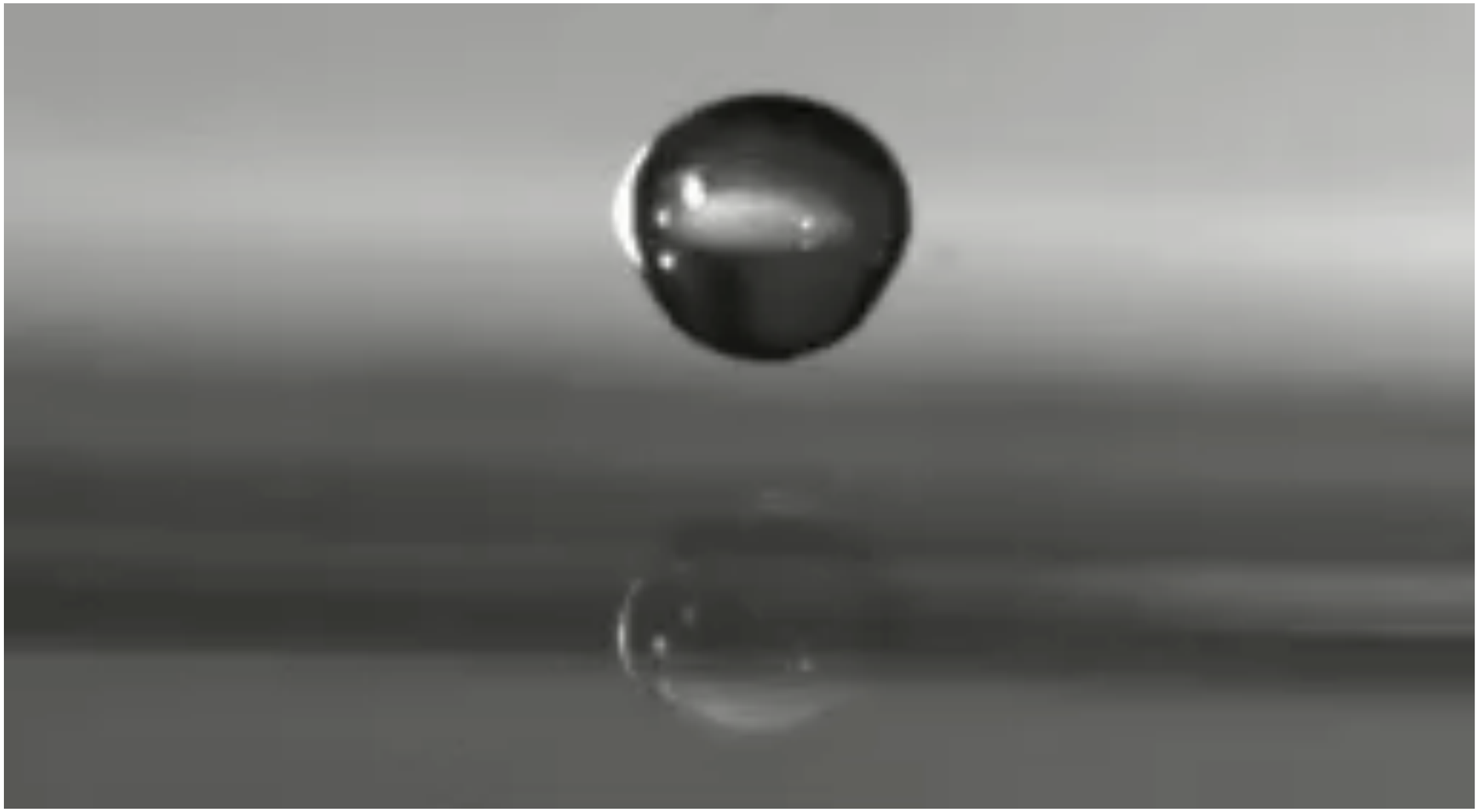
Using simple dynamics one can correlate the physiological attributes of runners with world track records and determine the optimal race strategy.

Joseph B. Keller



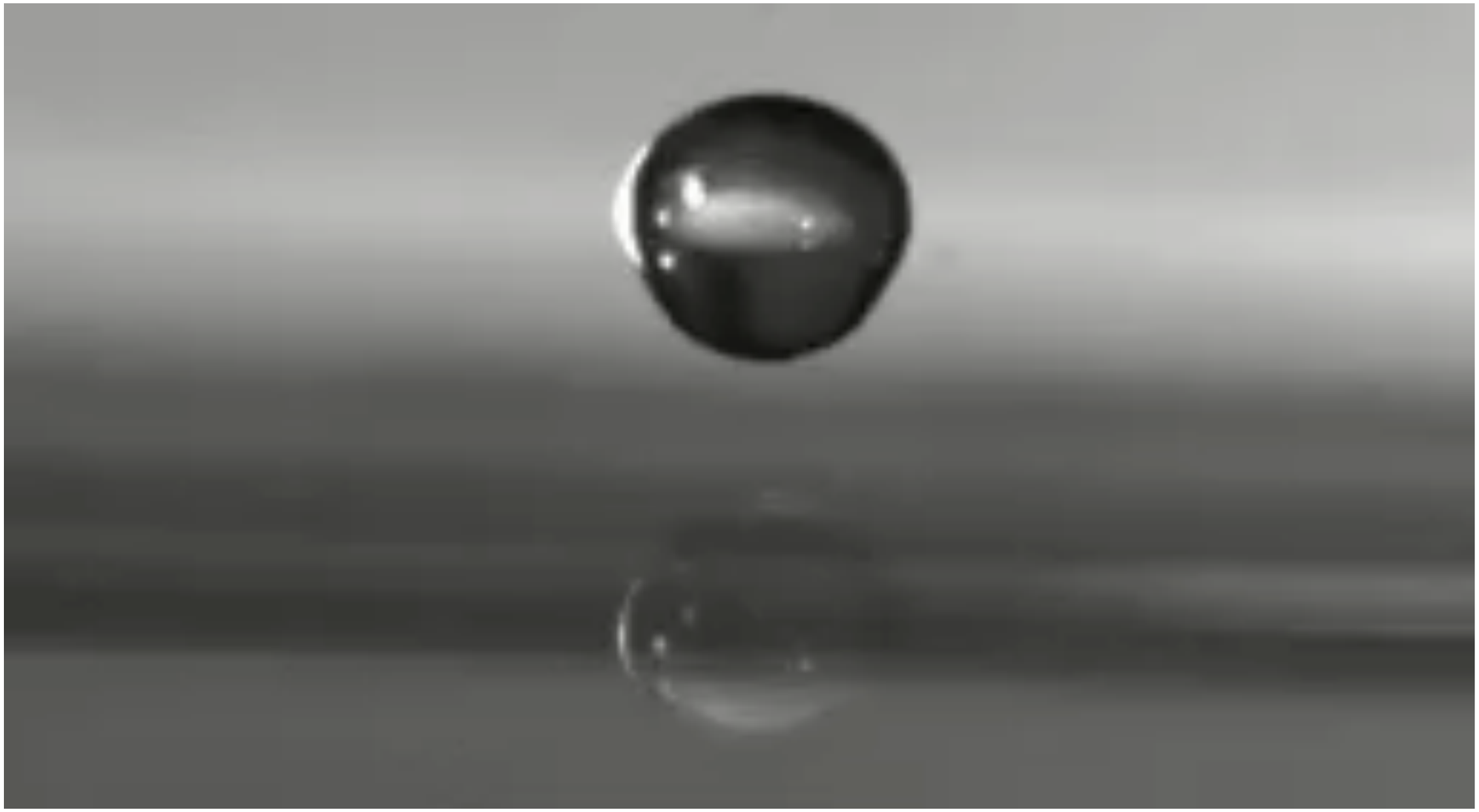
The average velocity for running a race. The theoretical curve is seen to agree with average velocities calculated from world records. Figure 3

A raindrop strikes a puddle



Or milk drop in a coffee cup...

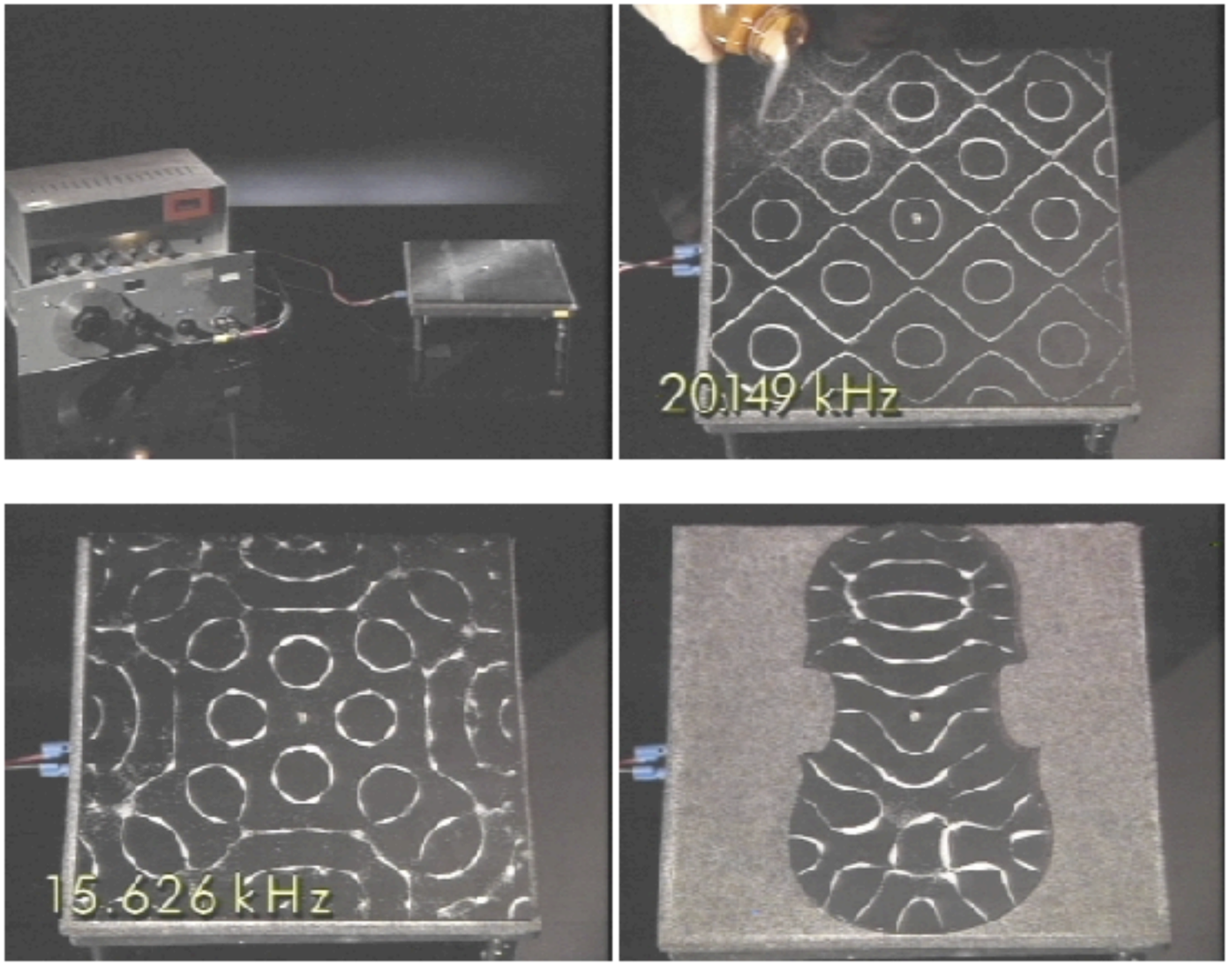
A raindrop strikes a puddle



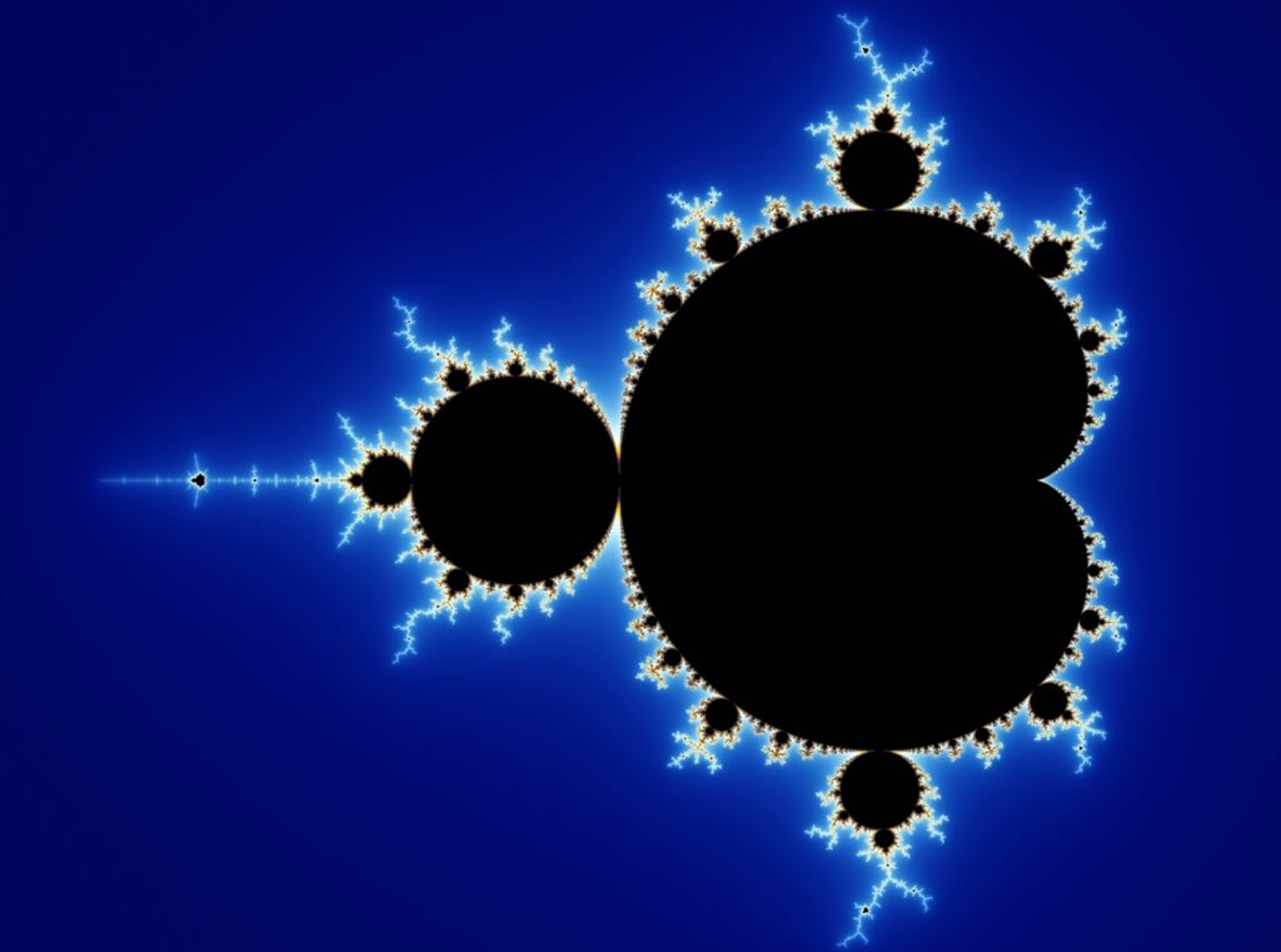
Or milk drop in a coffee cup...

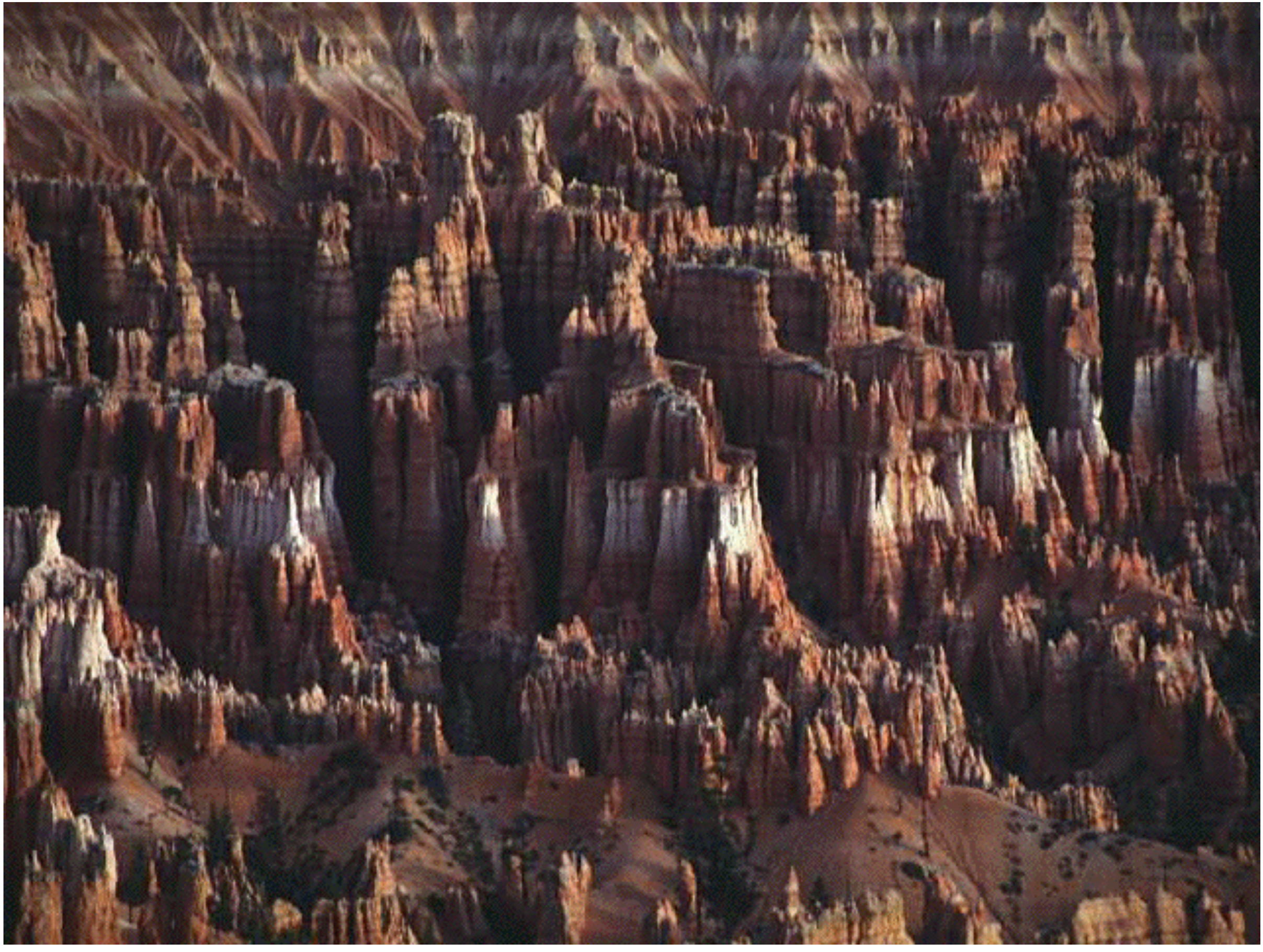
Mathematics in music

- the construction of the modern music scale
- mechanics of musical instruments: strings, reeds, brass instruments, drums, other…



Pattern formation in nature





If you dig deeply enough into anything, you will find mathematics.

If you dig deeply enough into anything, you will find mathematics.

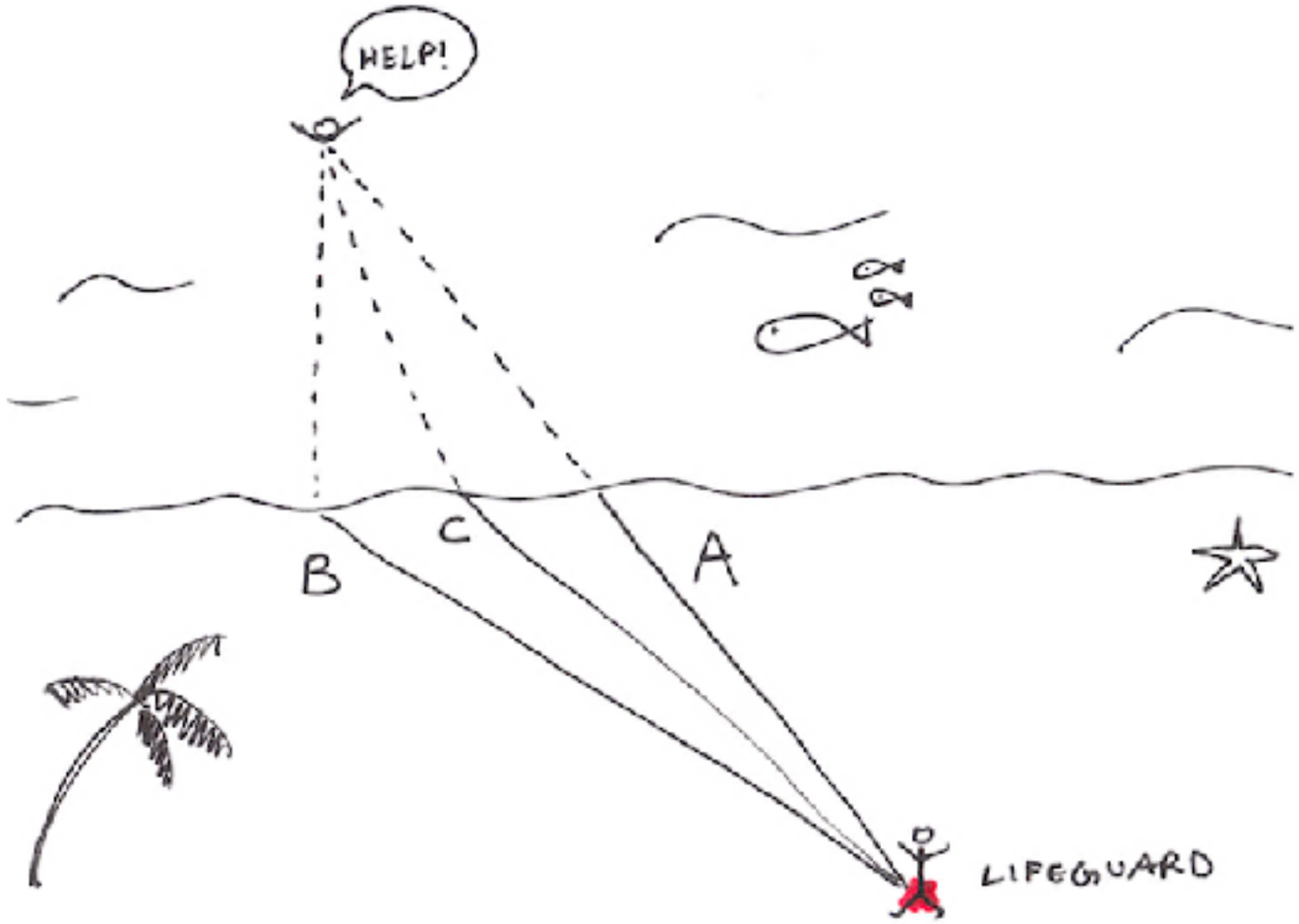
Mathematics in biology

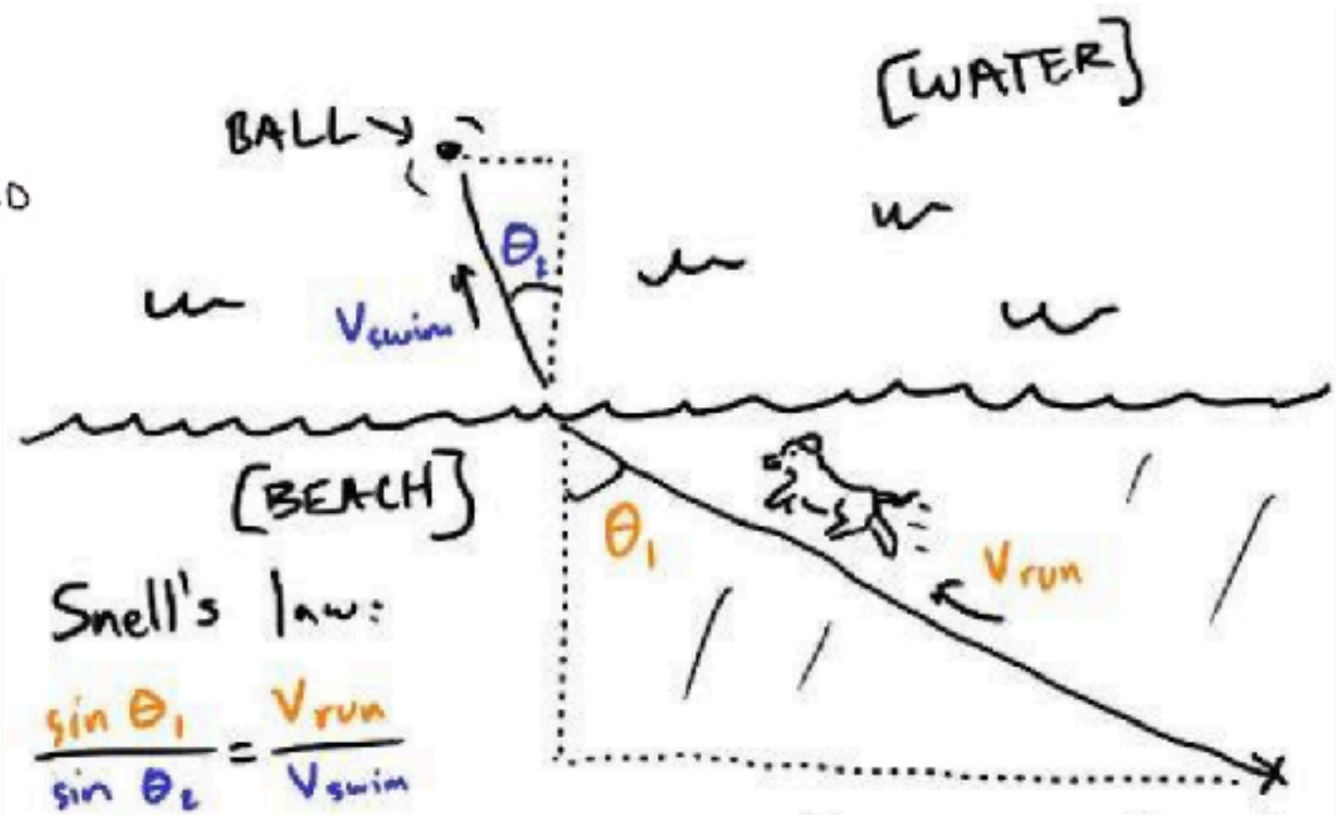
- rationalizing Nature's designs

- may be treated as constrained optimization problems

- but it is often difficult to identify what is being optimized and what the relevant constraints are

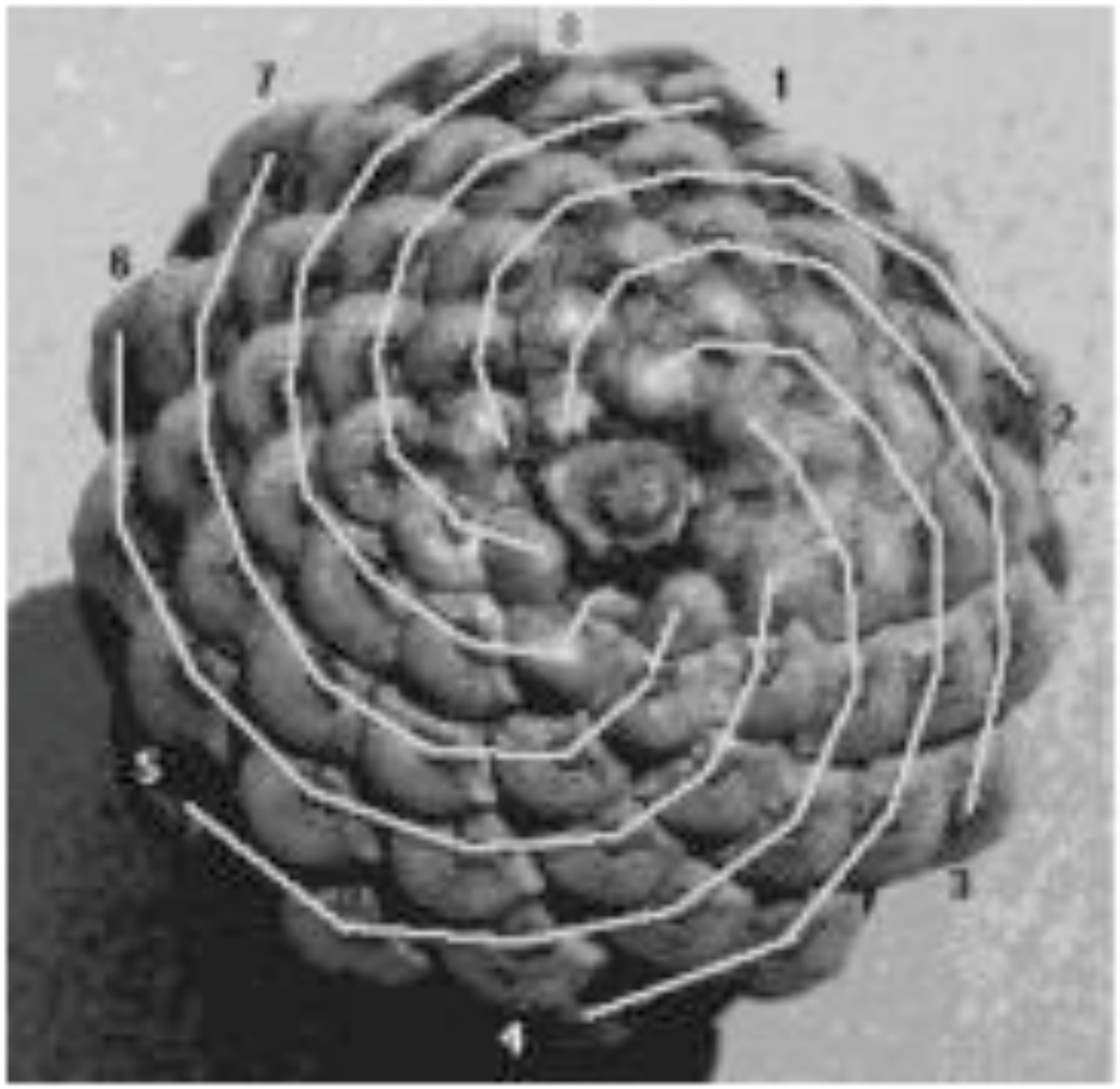
Snell's Law at the beach

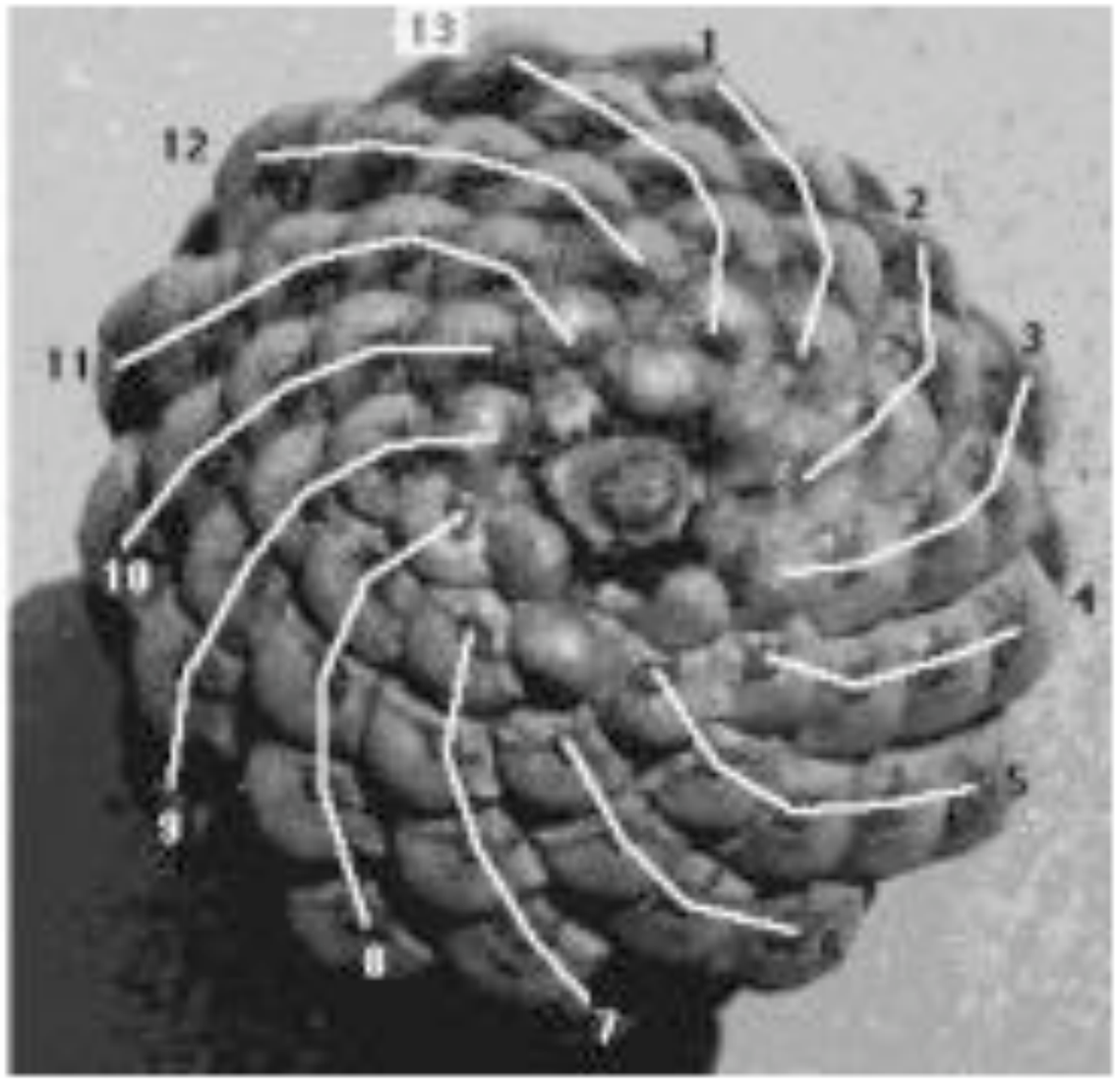




If a beam of light goes from a material w/ speed of light v_1 into a material w/ s.o.l. v_2, its refraction is also θ_1 and θ_2!

The Fibonacci Numbers Found in Nature





Maxwell on physical analogy

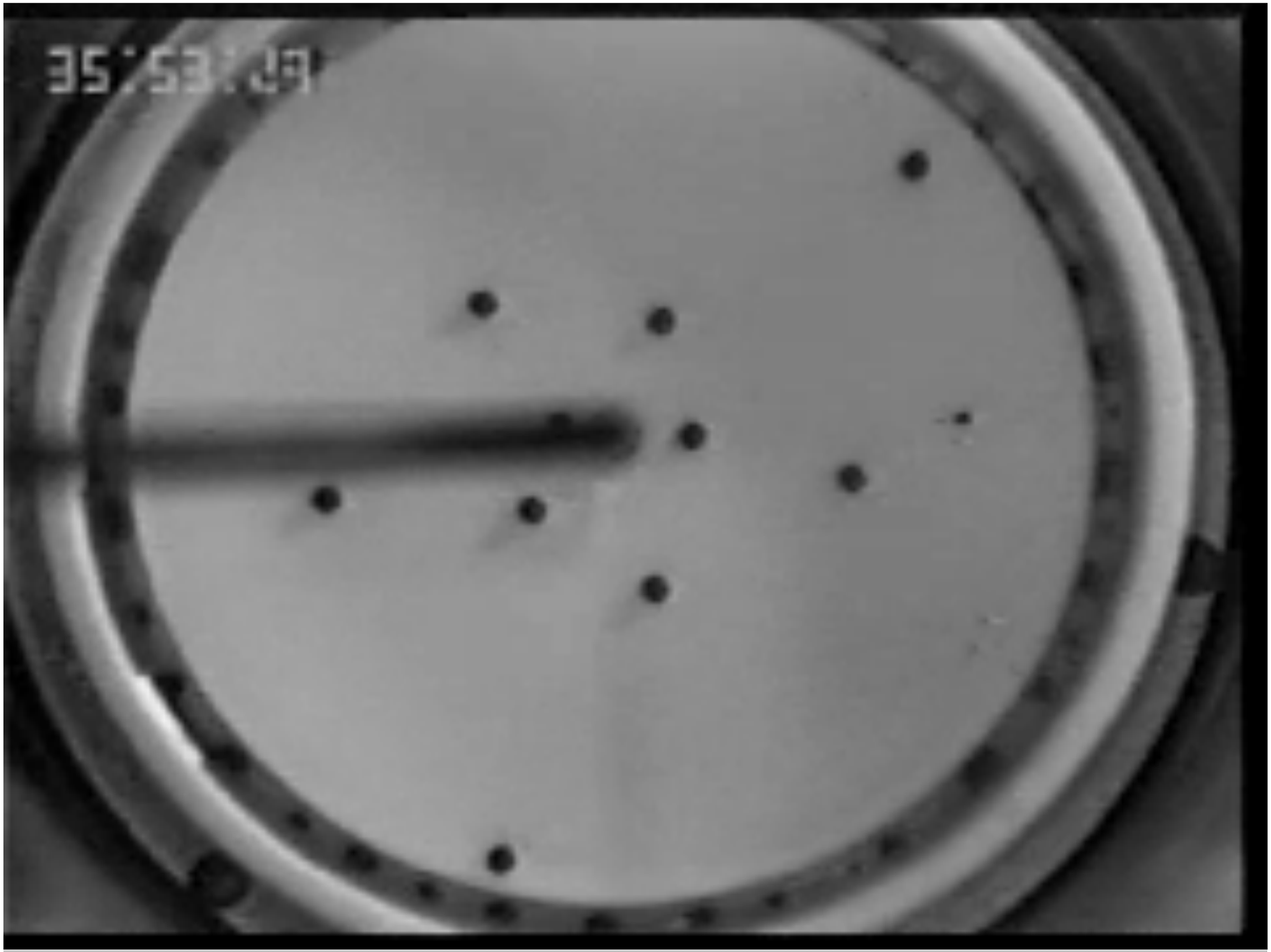
``By a physical analogy I mean that partial similarity between the laws of one science and those of another which makes each of them illustrate the other . . . . We find the same resemblance in mathematical form between two different phenomena."

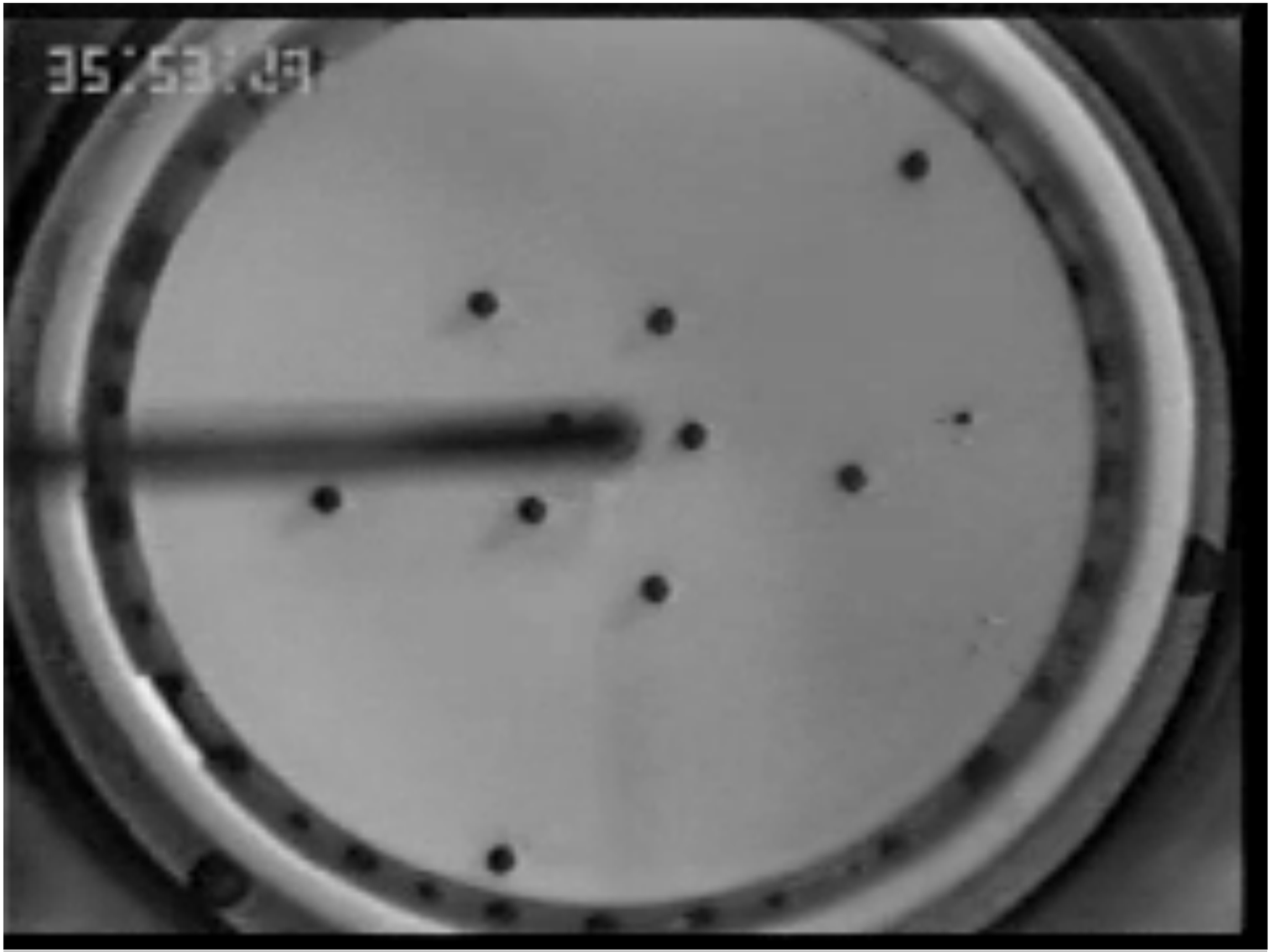
—James Clerk Maxwell, On Faraday's Lines of Force (1855).

``Analogy seems to have a share in most discoveries, but in some, it has the lion's share."

— Maxwell

- mathematics provides the linguistic basis for analogizing in science





Constrained optimization in nature

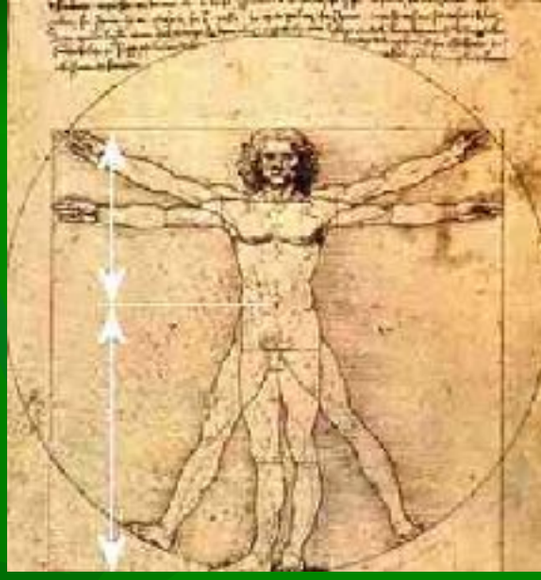
The Fibonacci sequence:

$$F_n = 1, 1, 2, 3, 5, 8, 13, 21, 34, 55...$$

The Fibonacci spiral minimizes the energy of the system.

$\Phi \approx 1.6180339887\ldots$ is called the "Golden Ratio"

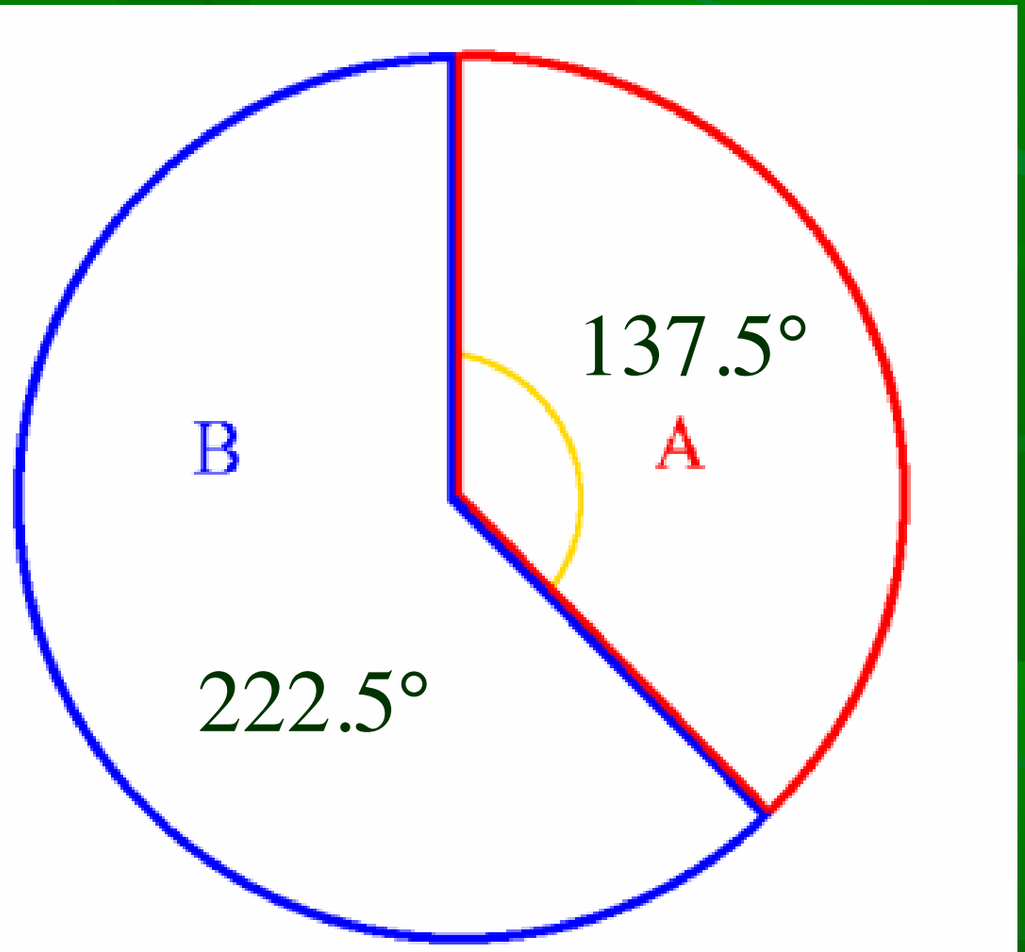
What do you think is called the "Golden Angle?"



$360° \times \Phi \approx 582.5°$

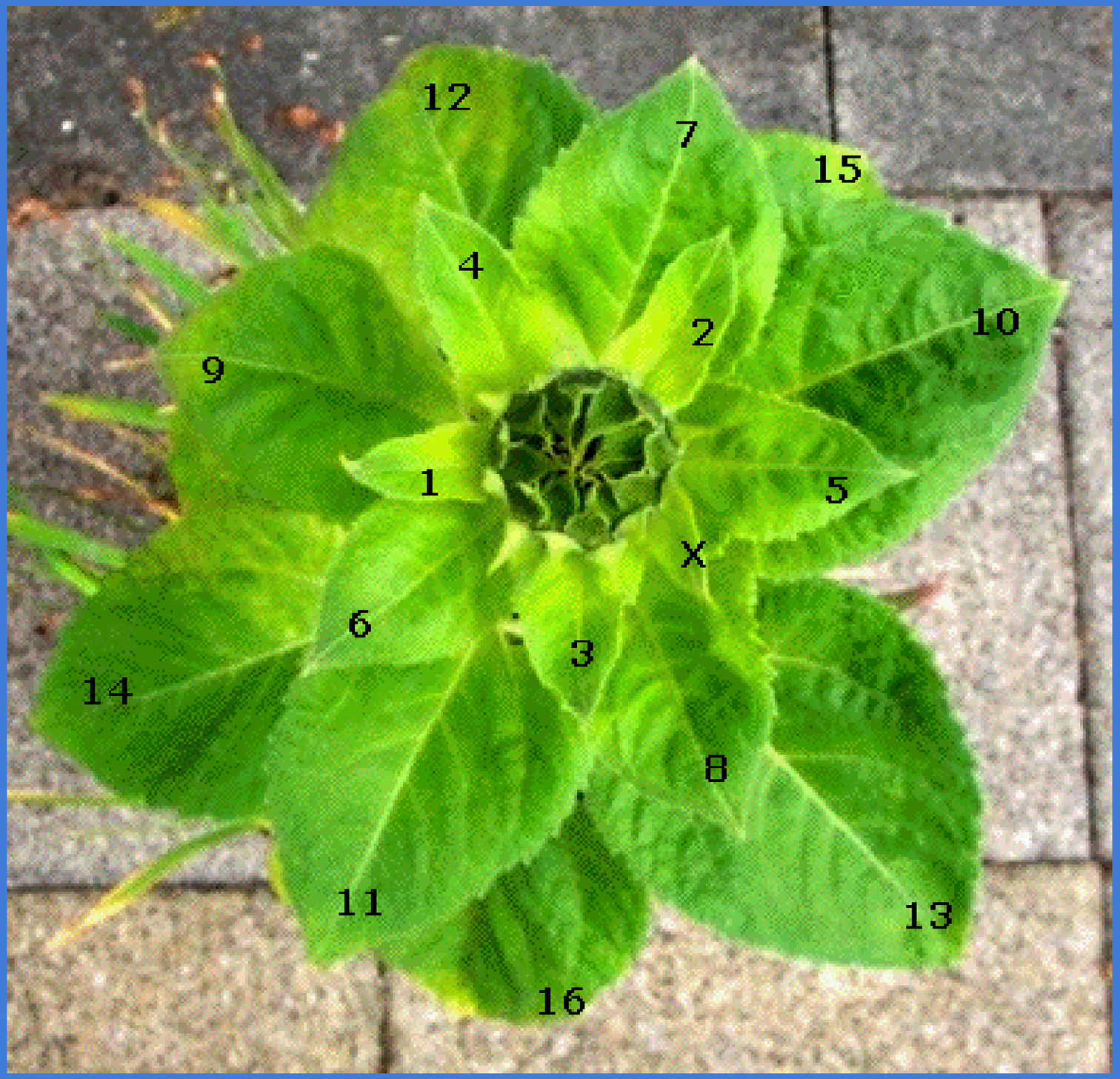
$= 222.5°$

$F_n = 1, 1, 2, 3, 5, 8, 13, 21, 34, 55\ldots$



$$\Phi = \lim_{n \to \infty} \frac{F_{n+1}}{F_n}$$

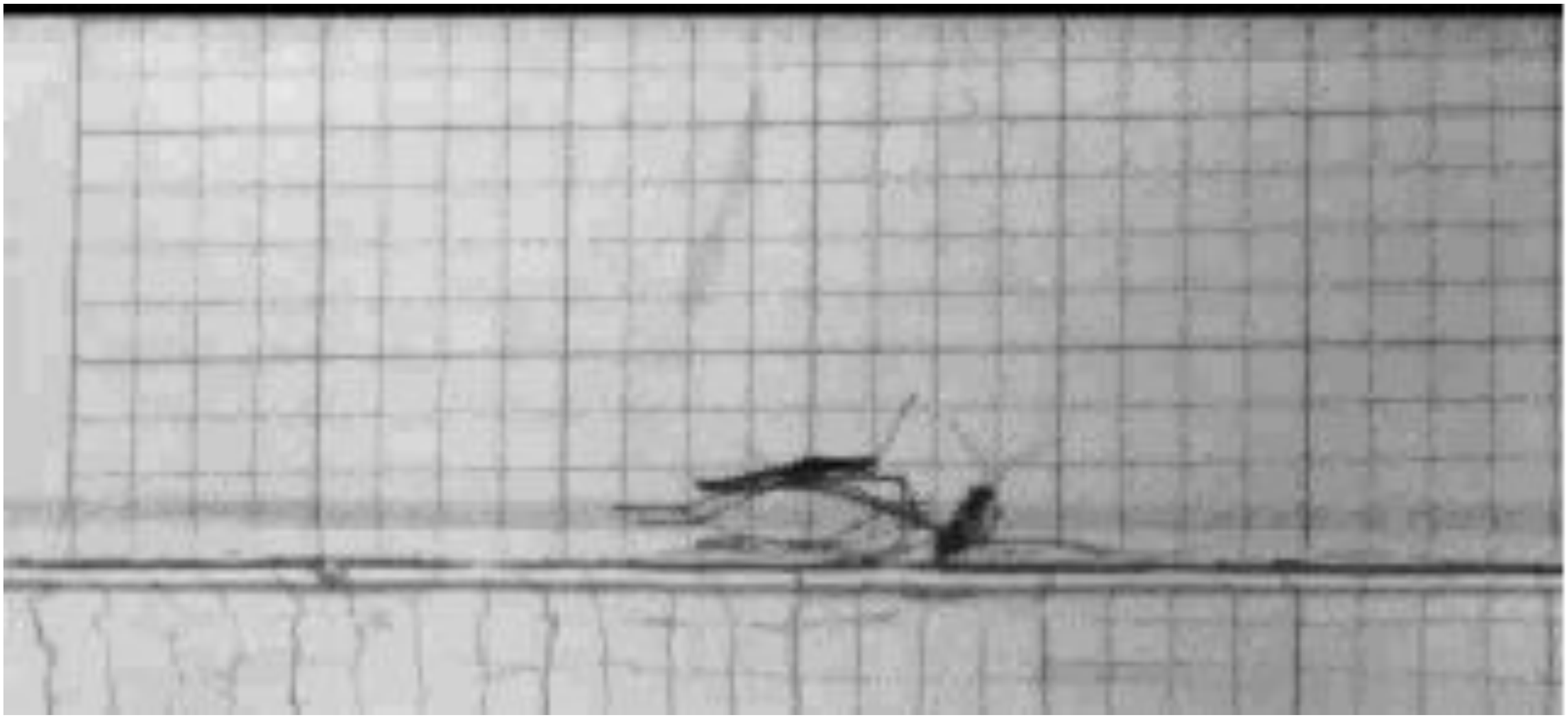
Plant leaves on stems emerge at this golden angle.



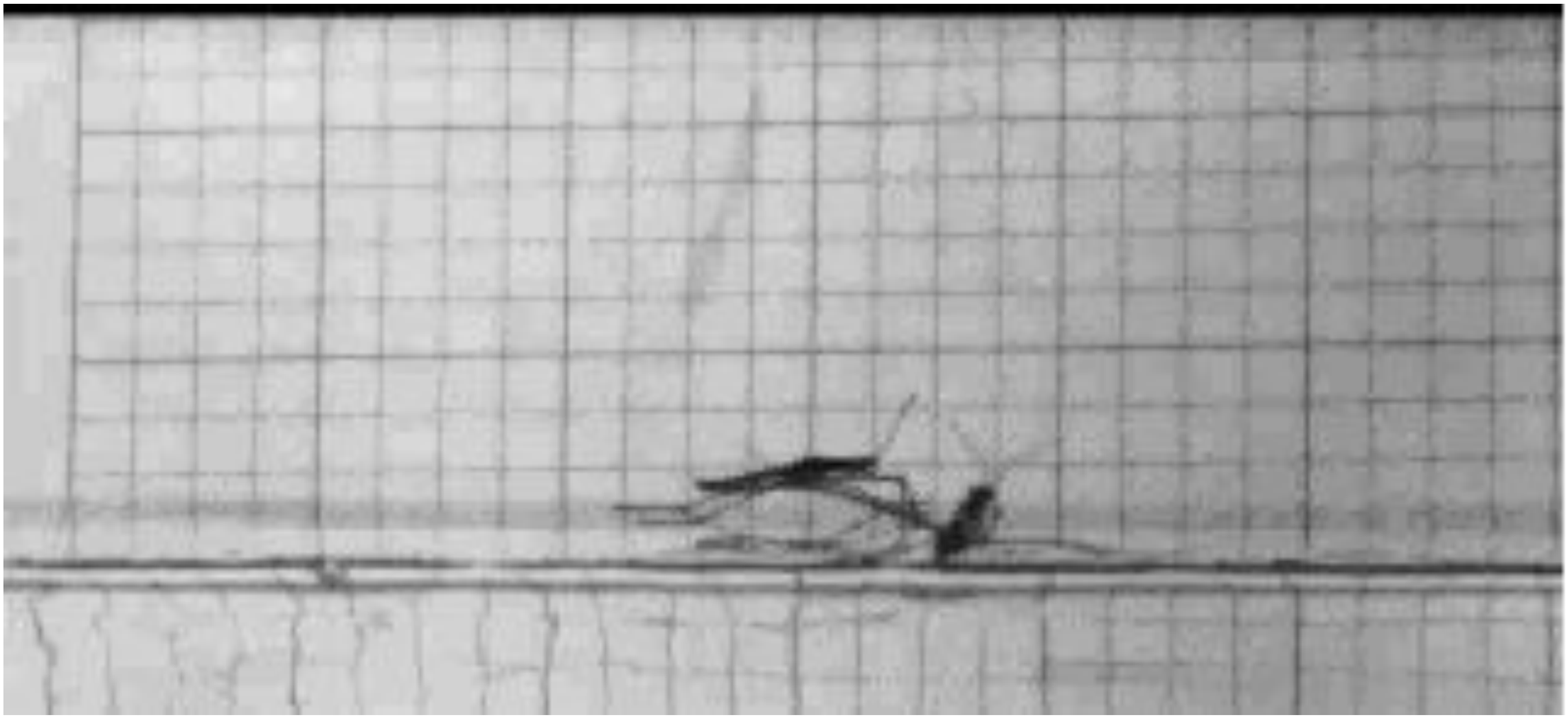
Why? To maximize the sunlight received.

Biocapillarity

Who cares about surface tension?



Water strider combat



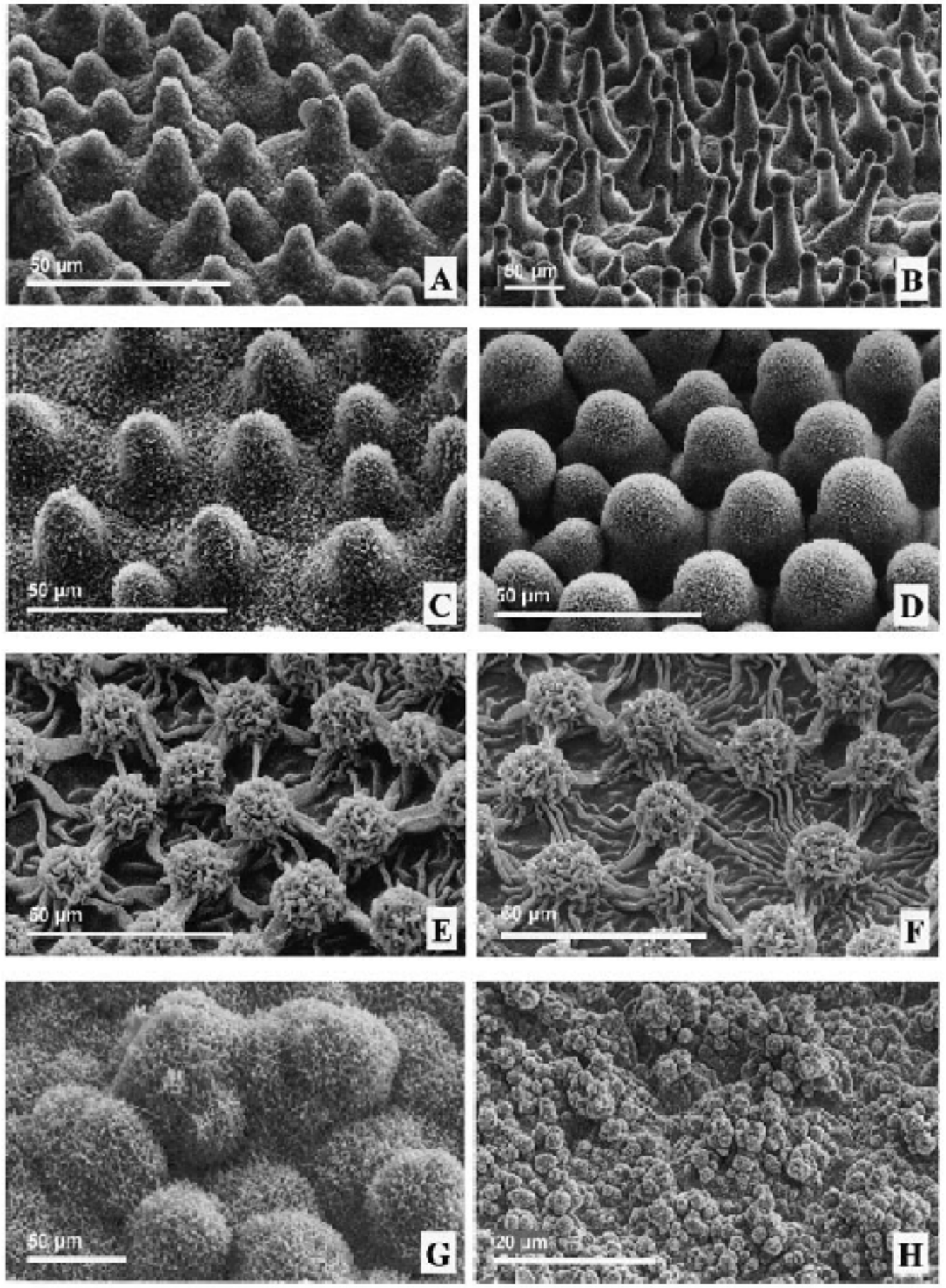
Water strider combat

Biomimicry


Hair area
Nano-hairs
10 µm
Micro-hairs
20.0 kV 10µm 12 mm CL:10.0
Spatula

Water-repellent surfaces in nature and technology

Can we copy nature's designs?



The pitcher plant

Kripa Varanasi's Liquiglide

Horizontal gene transfer and camouflage

Quirky mechanisms in biology

Quirky mechanisms in biology

Quirky mechanisms in biology

Quirky mechanisms in biology

Hemisphaerota cyanea (Chrysomelidae; Cassidinae)

How do these beetles use capillary adhere to surfaces?

(Credit: Getty Images)

TINY DROPLETS MAY HAVE KICKSTARTED LIFE ON EARTH

OCTOBER 24TH, 2017

POSTED BY NATHAN COLLINS-STANFORD

Tiny droplets of water may have been vital for the start of life on Earth, new research suggests.

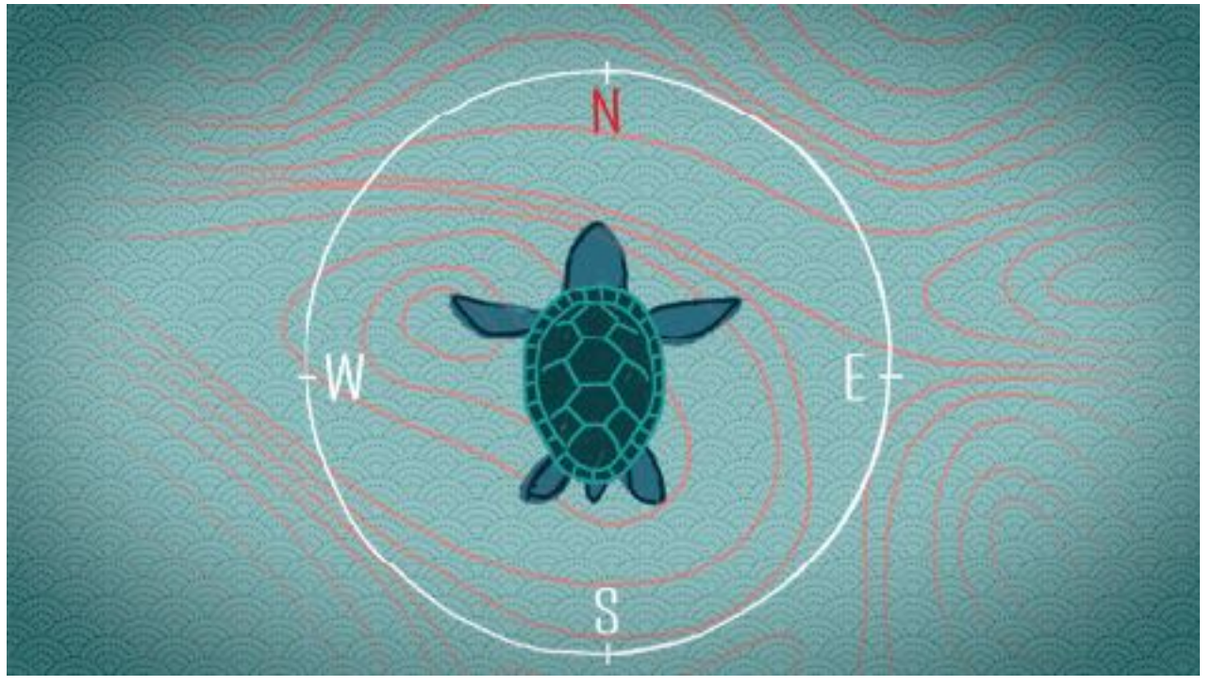
SHARE THIS ARTICLE

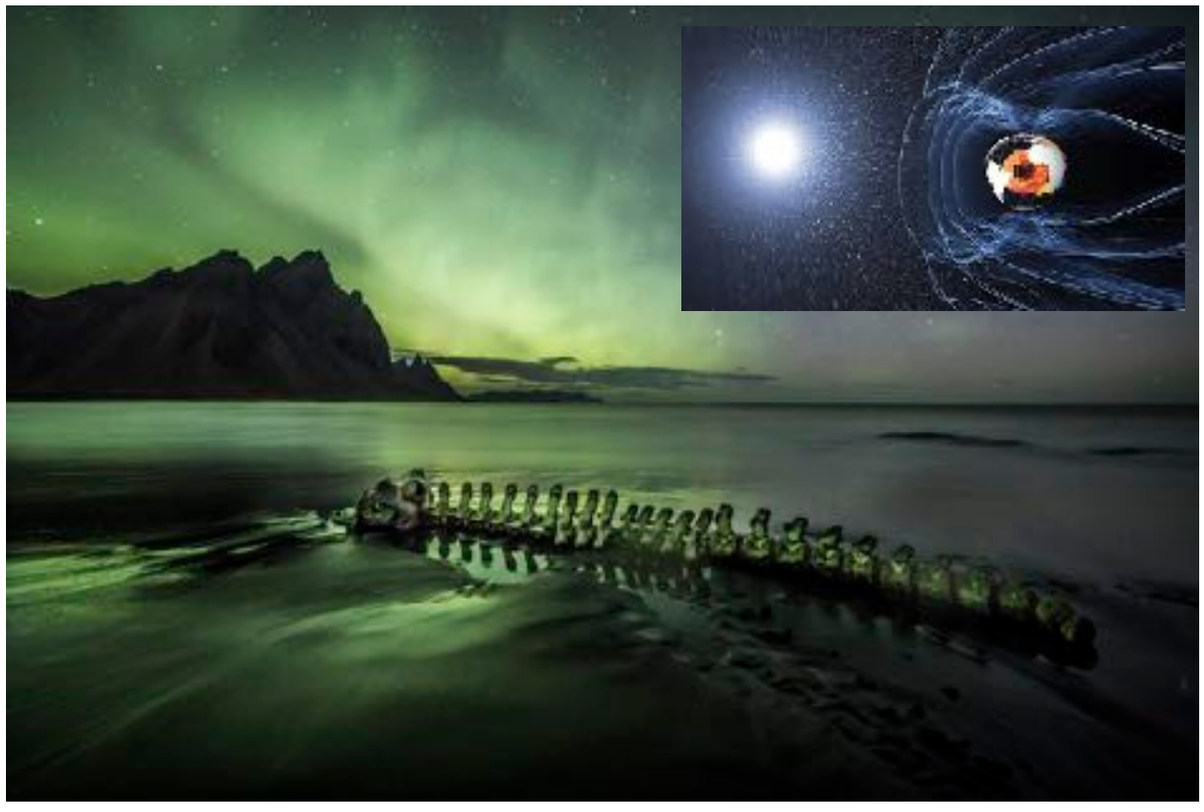
Droplets are relevant to astrobiology because **they may have played a role in the formation of life**. For example, droplets may have concentrated organic reactants, acted as bioreactors, or helped form the first protocell walls.

The rain-oasis effect

Why is rain more likely over an oasis?

Electromagnetic effects in biology





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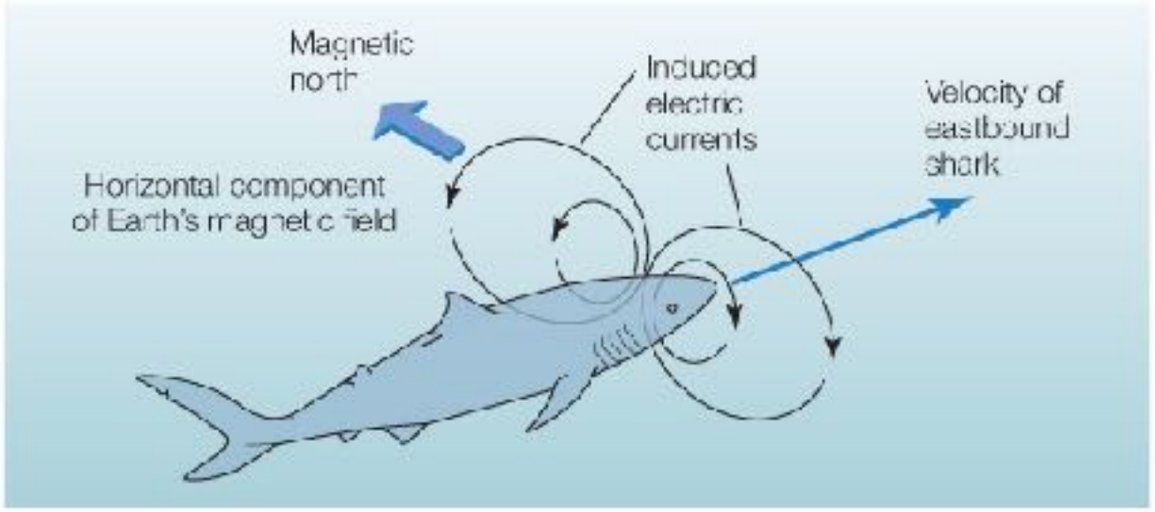
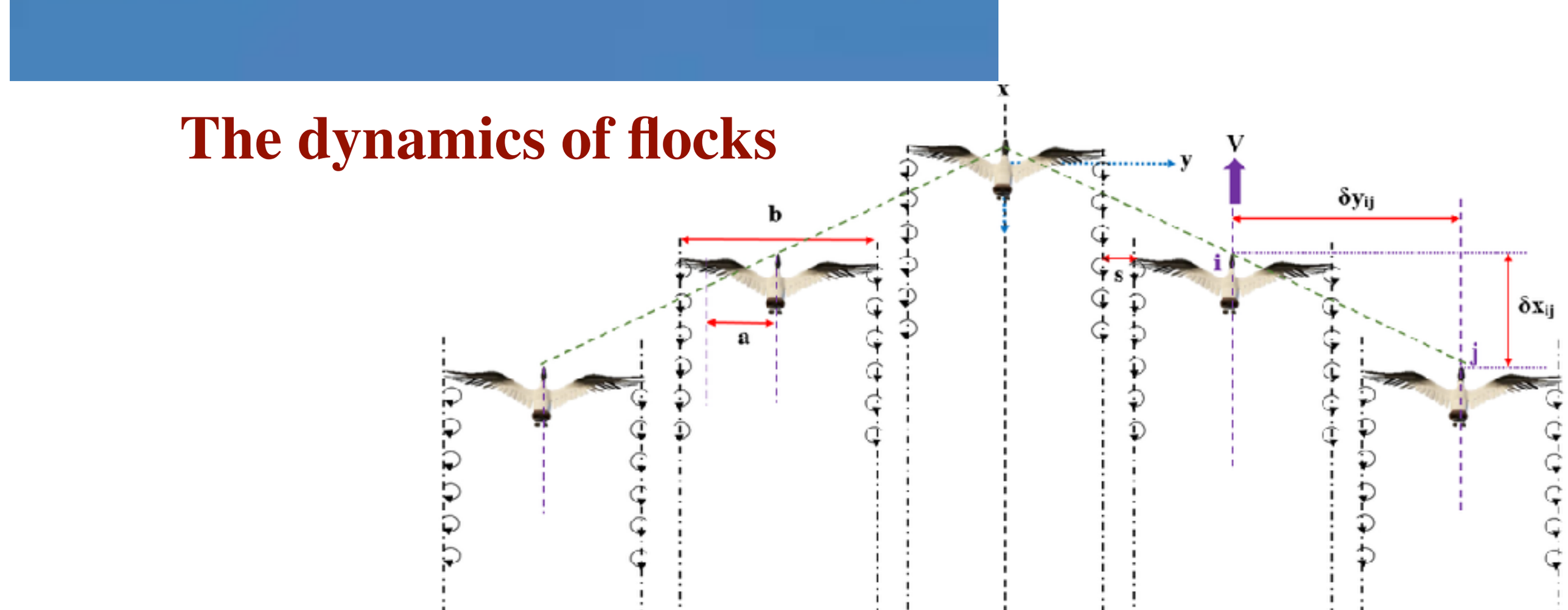


Collective behavior in nature

The dynamics of flocks







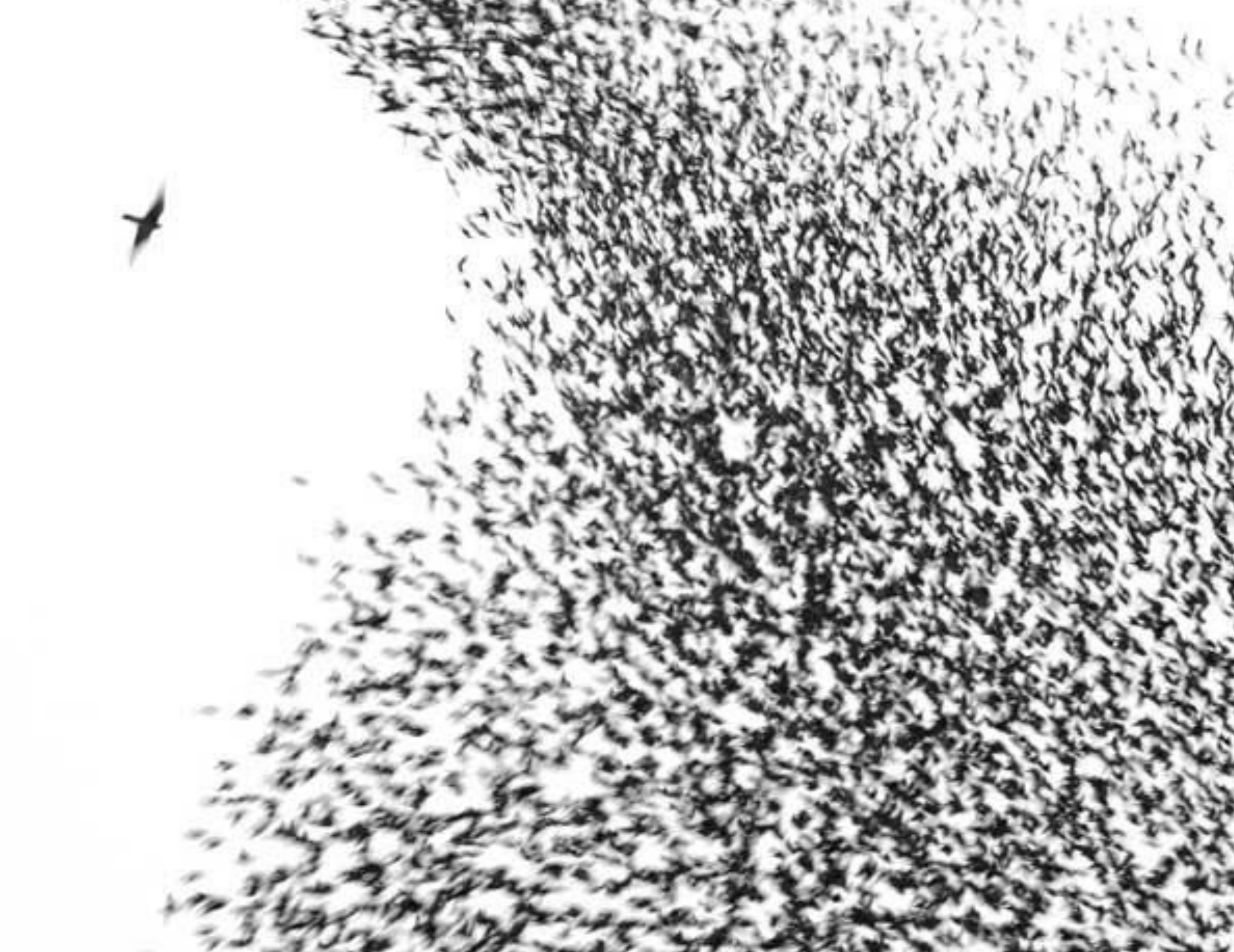






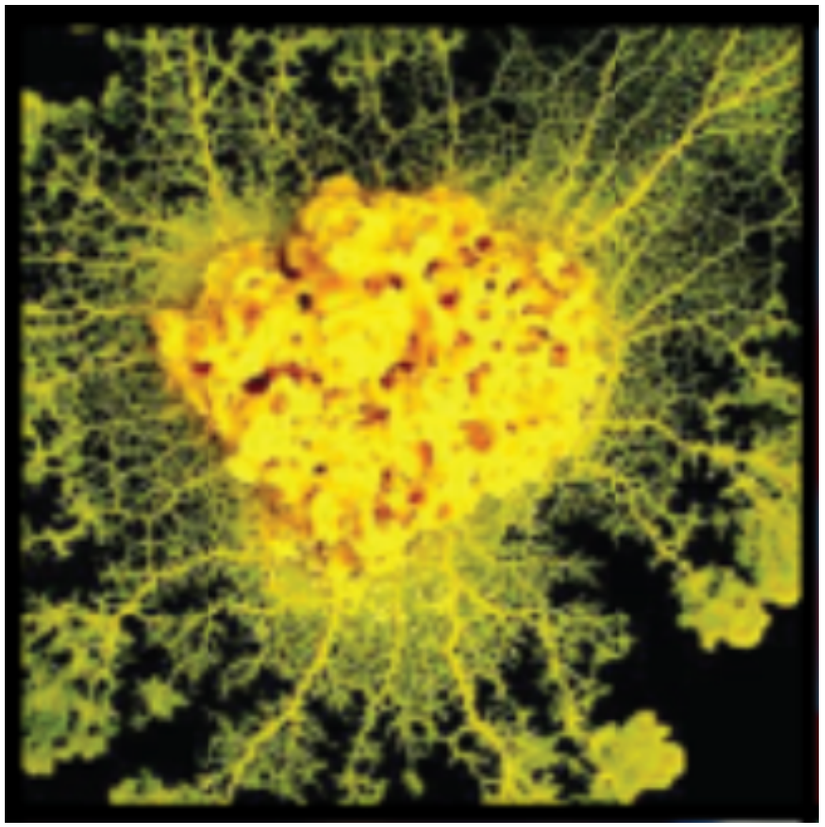




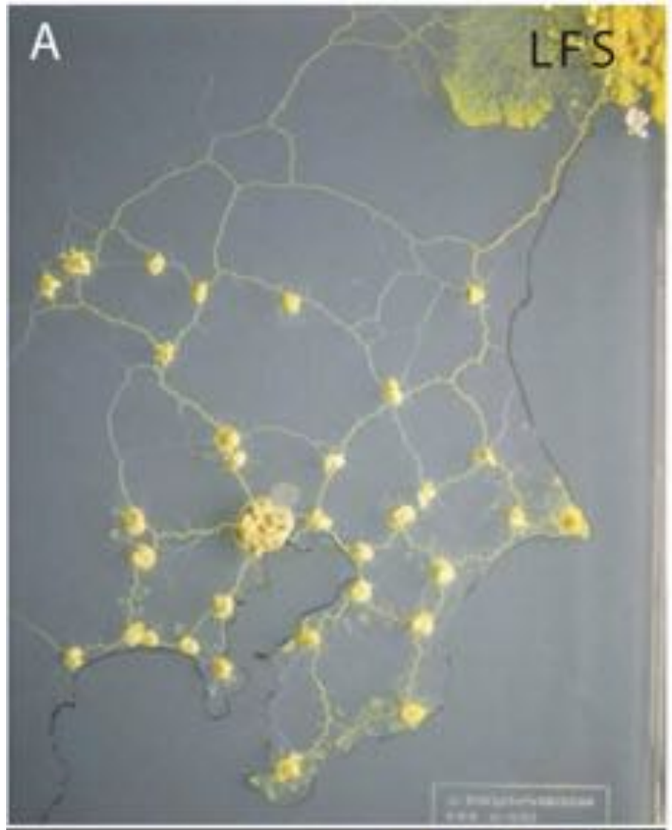



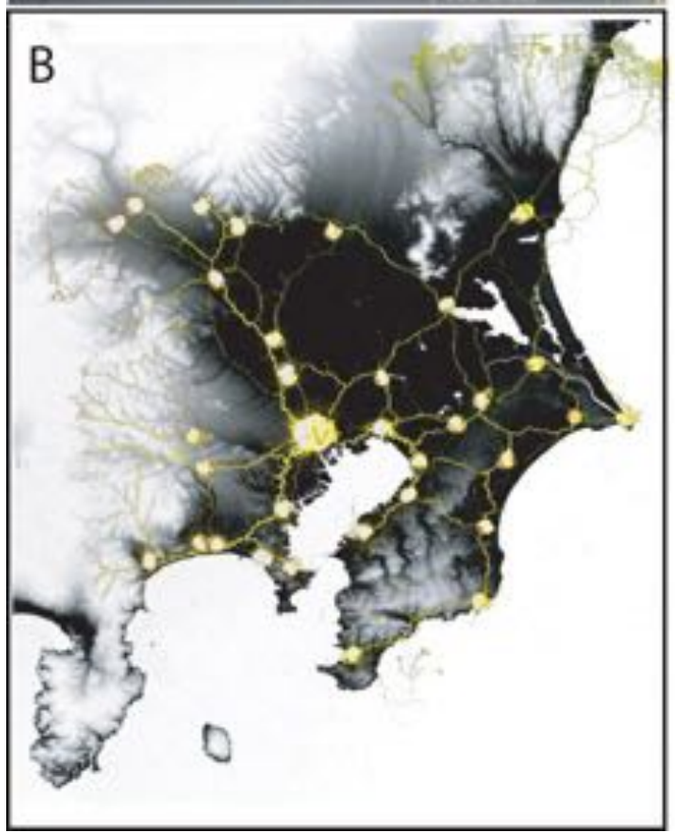
Ant bridges and ant balls

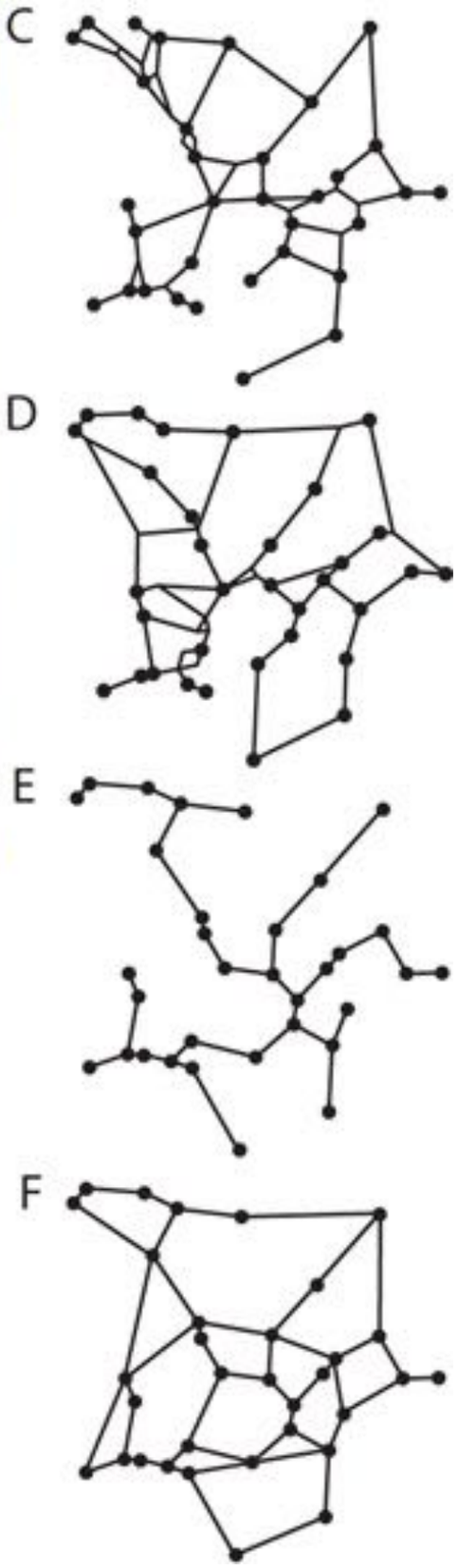
Active networks



- brainless slime mold postdicts Tokyo subway map



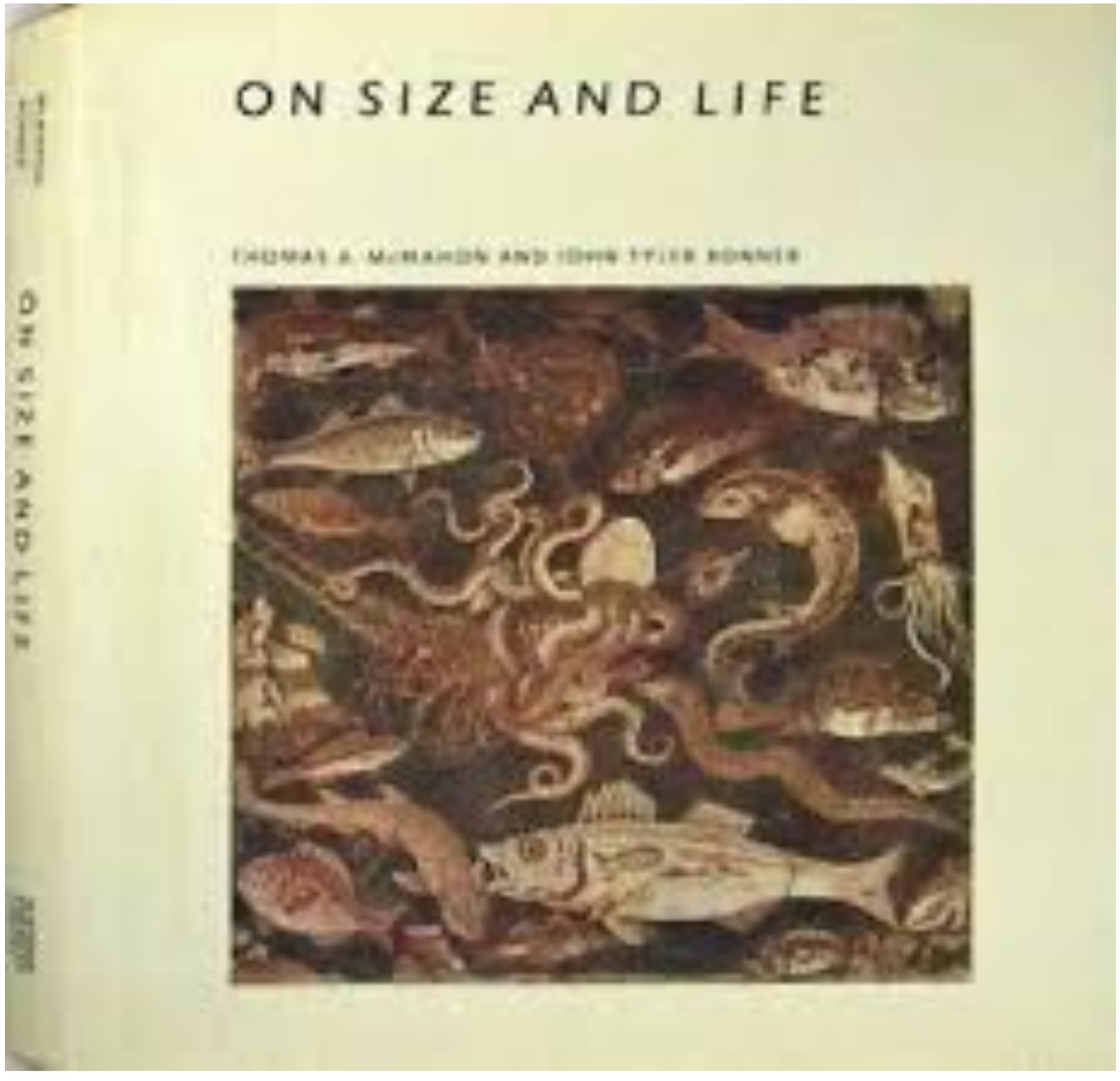




Fluid dynamics and the pandemic

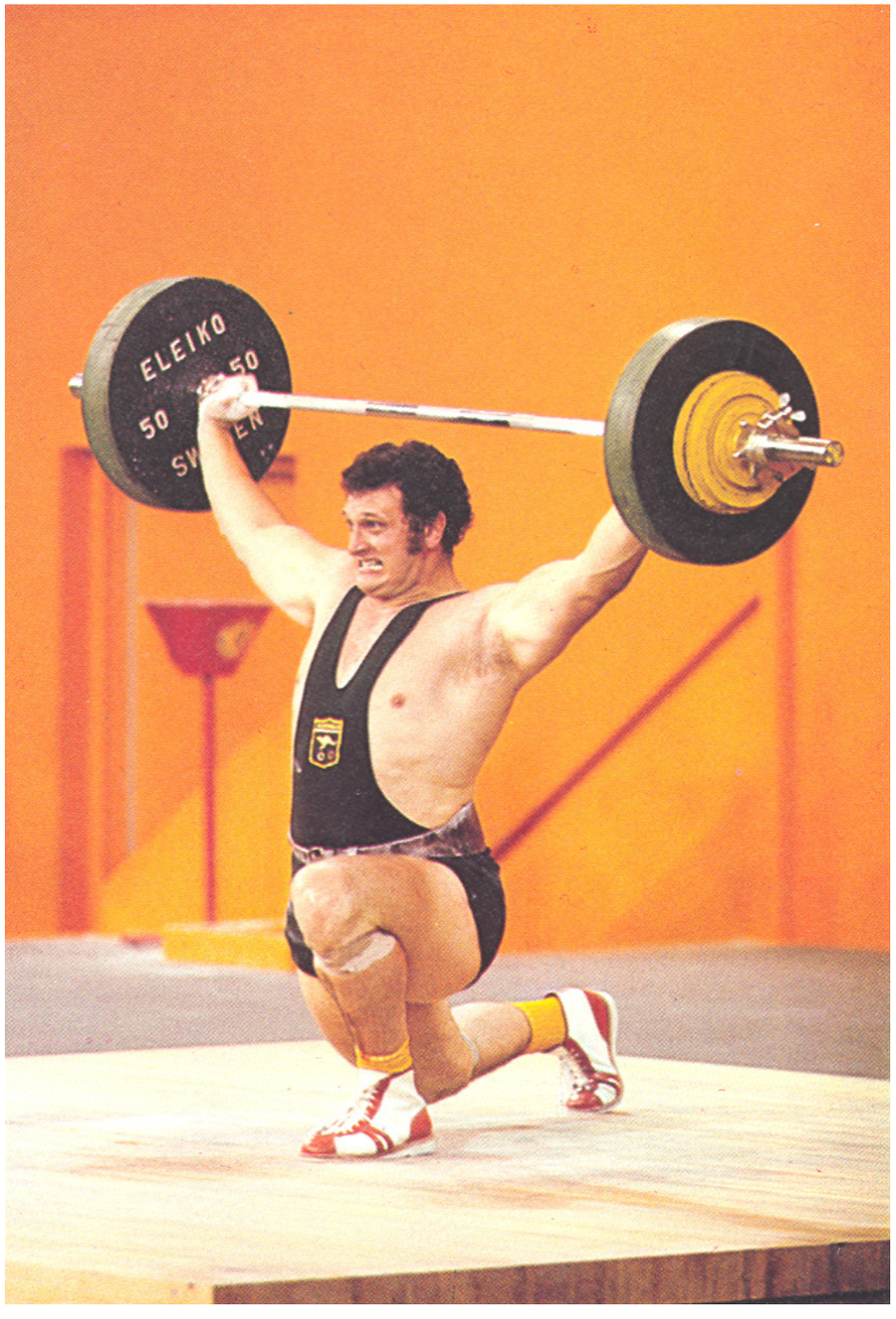
Doc Edgerton's ``The sneeze''

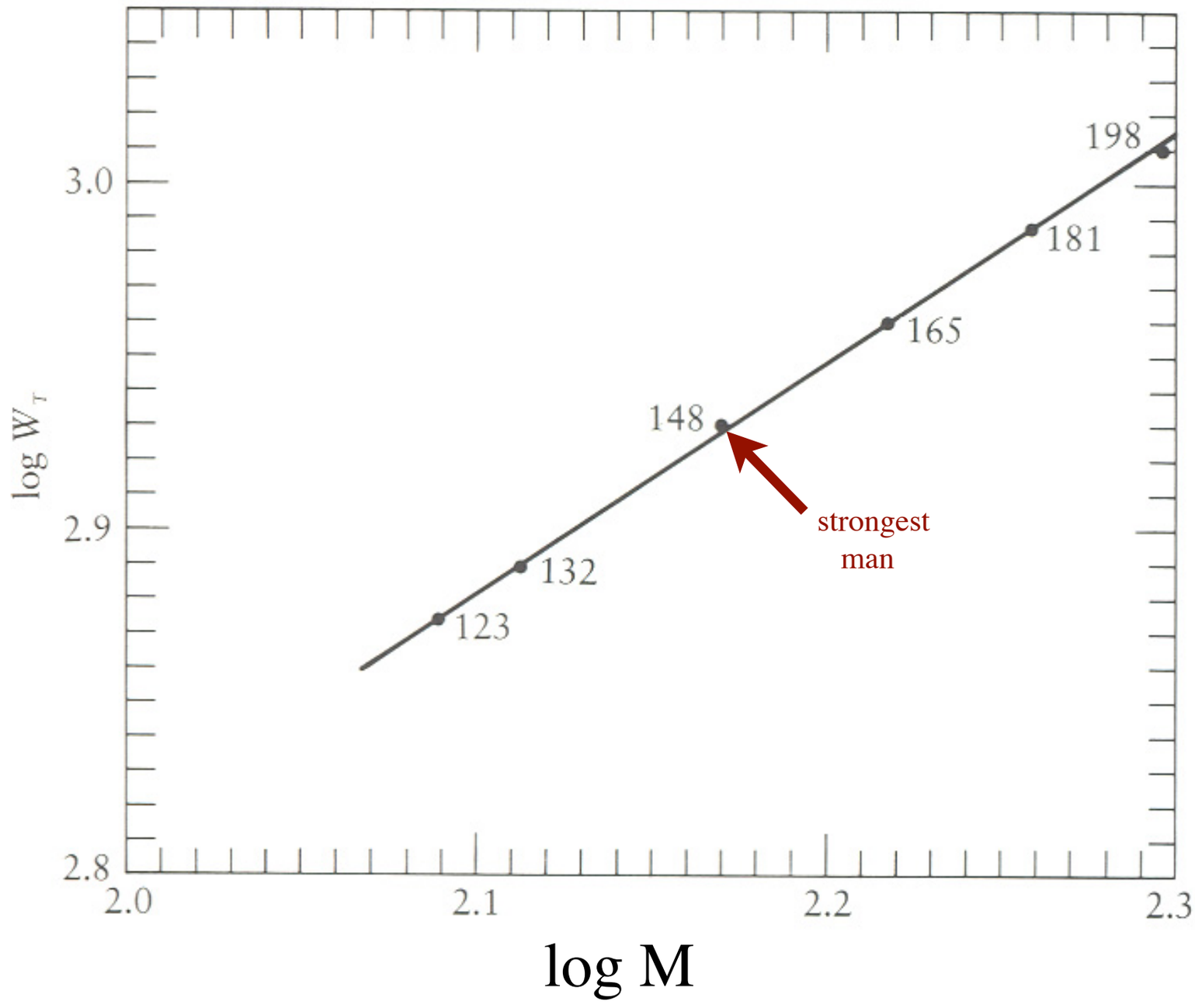
On mechanism and scale in biology



How much force can a creature of length L and mass M generate/sustain?

$$W_T \sim M^{2/3} \sim L^2$$



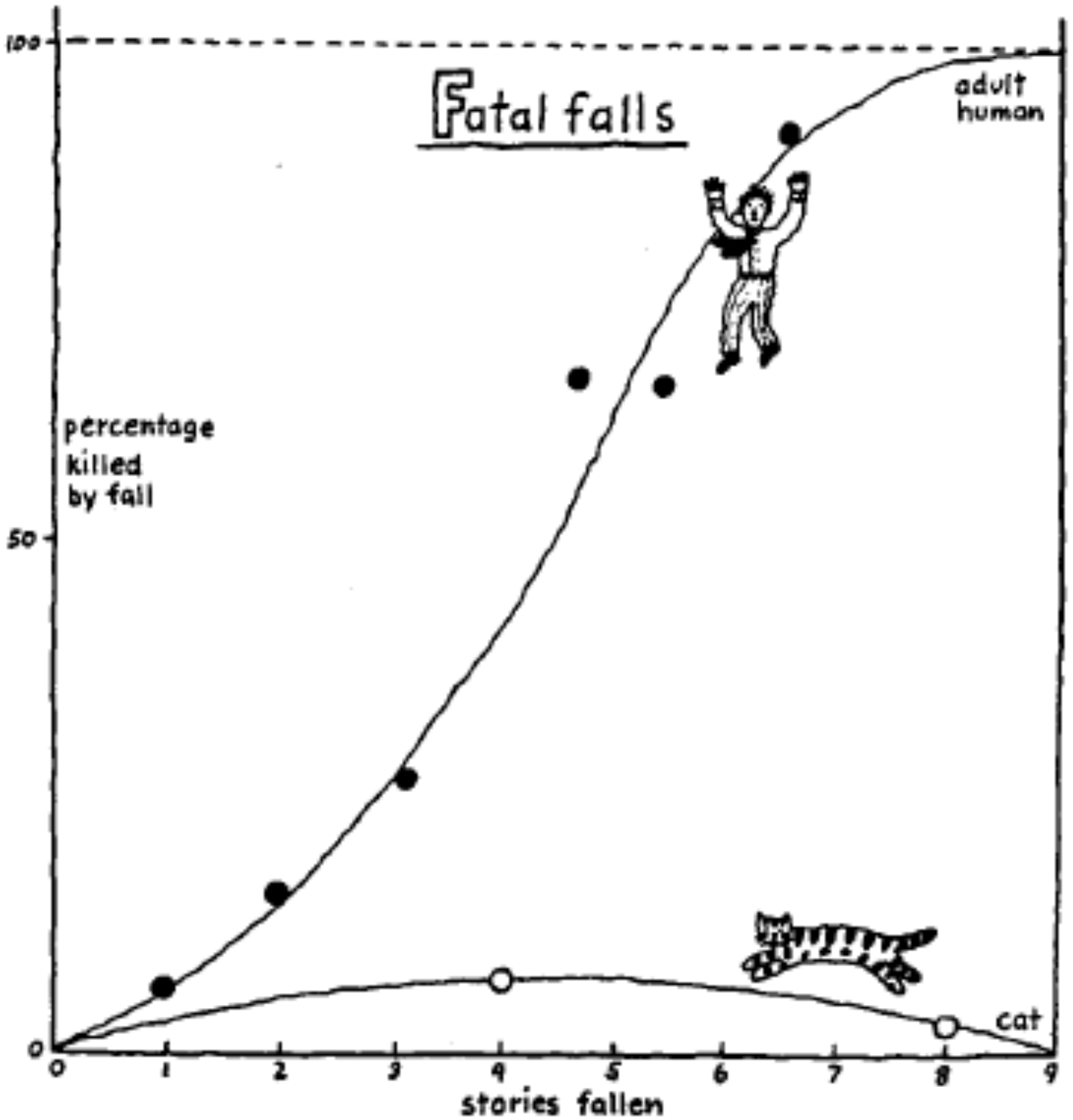


Ratio of force generated to size: $\frac{W_T}{M} \sim \frac{M^{2/3}}{M} \sim M^{-1/3}$

“You can drop a mouse down a thousand-yard mine shaft and, on arriving at the bottom, it gets a slight shock and walks away. A rat is killed, a man is broken, a horse splashes."

— J.B.S. Haldane, biologist

How Cats Survive Falls from New York Skyscrapers



Wingsuit base jumping…

What determines the shape of trees?

Constrained optimization in nature

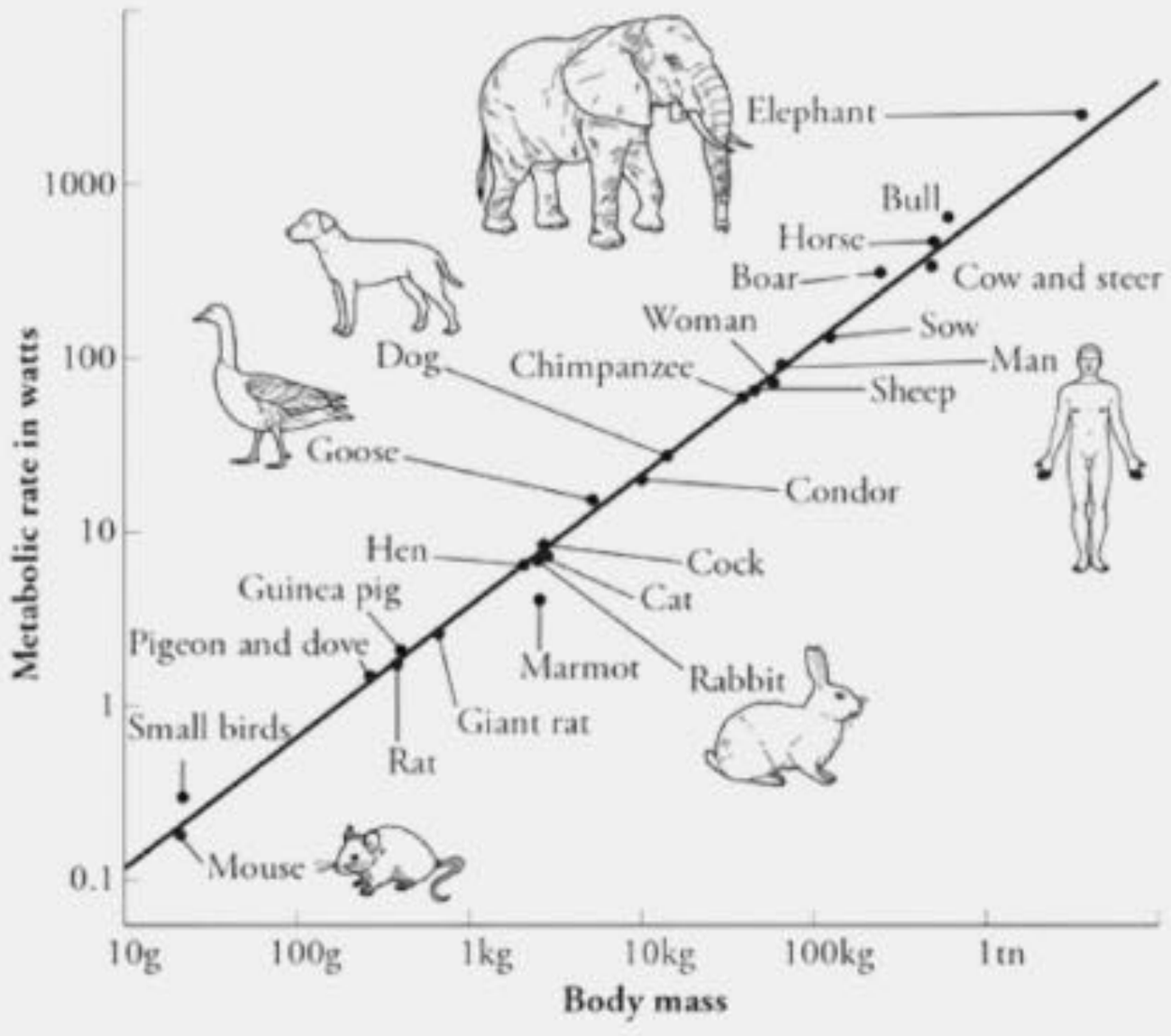
What sets the height of the tallest tree?

How does the trunk thickness scale with tree height?

On scaling in biology

Kleiber's law

$$metabolic\ rate \approx k \cdot (mass)^{\frac{3}{4}}$$

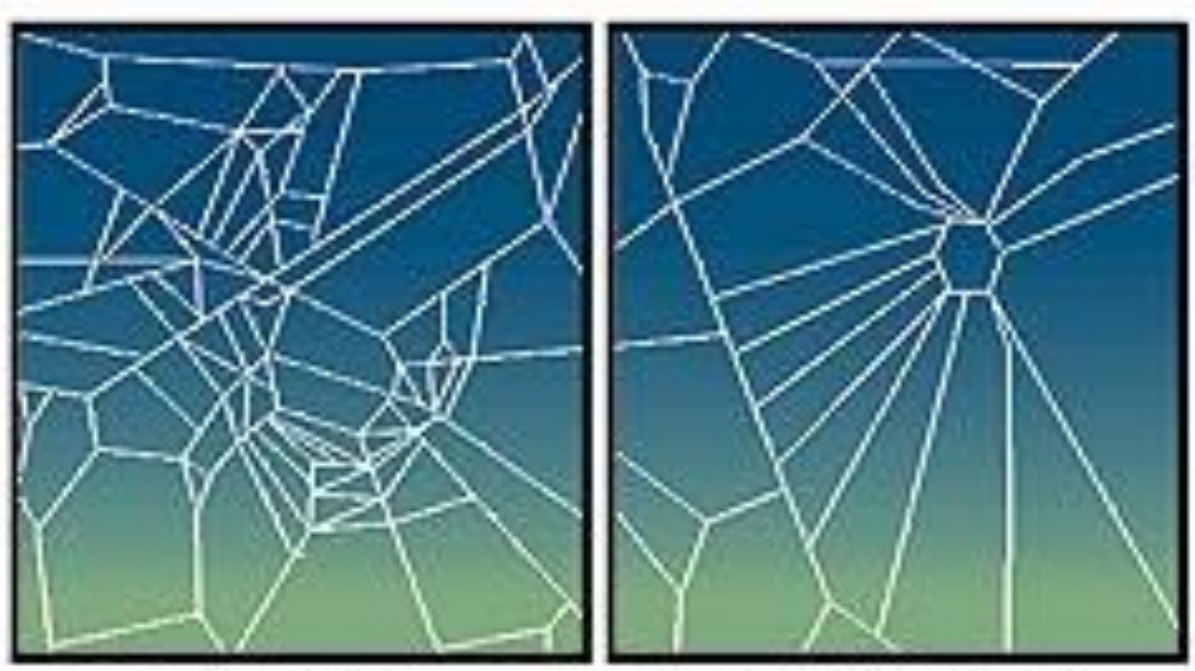


How does one rationalize tree shapes?

How does lightning choose its path?

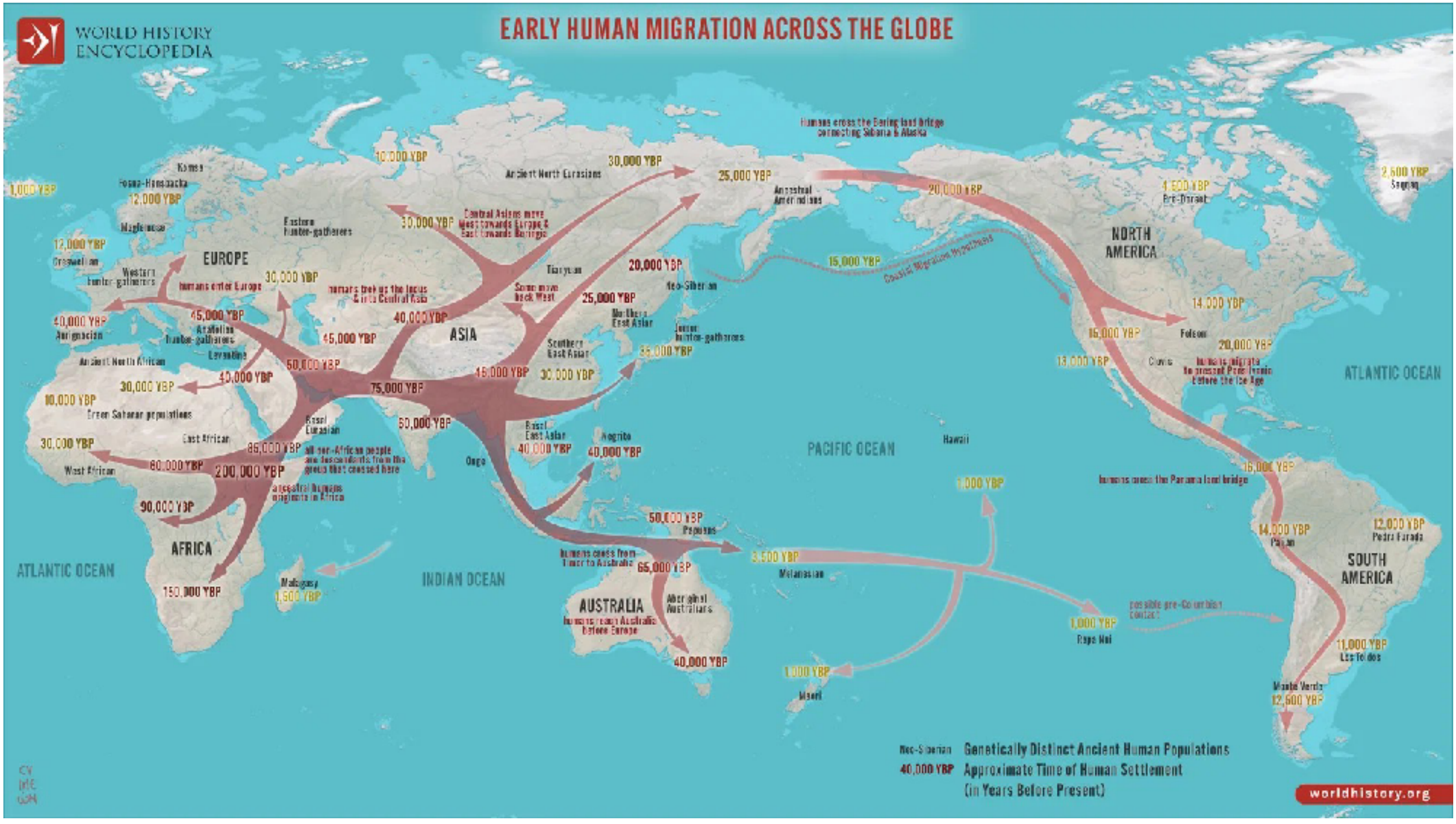
What determines the geometry of spider webs?

Normal (no chemical) Marijuana Benzedrine

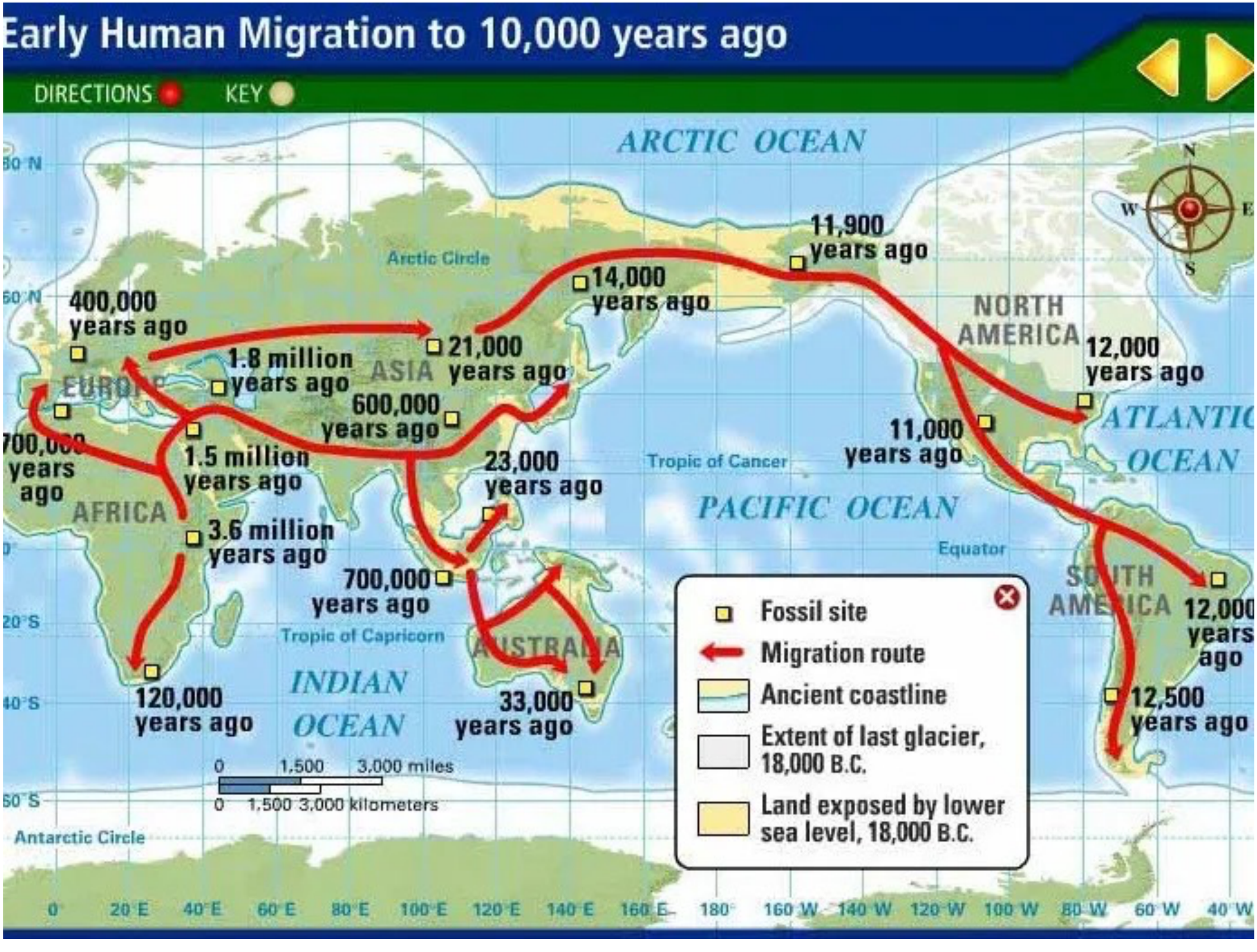


Caffeine Chloral Hydrate

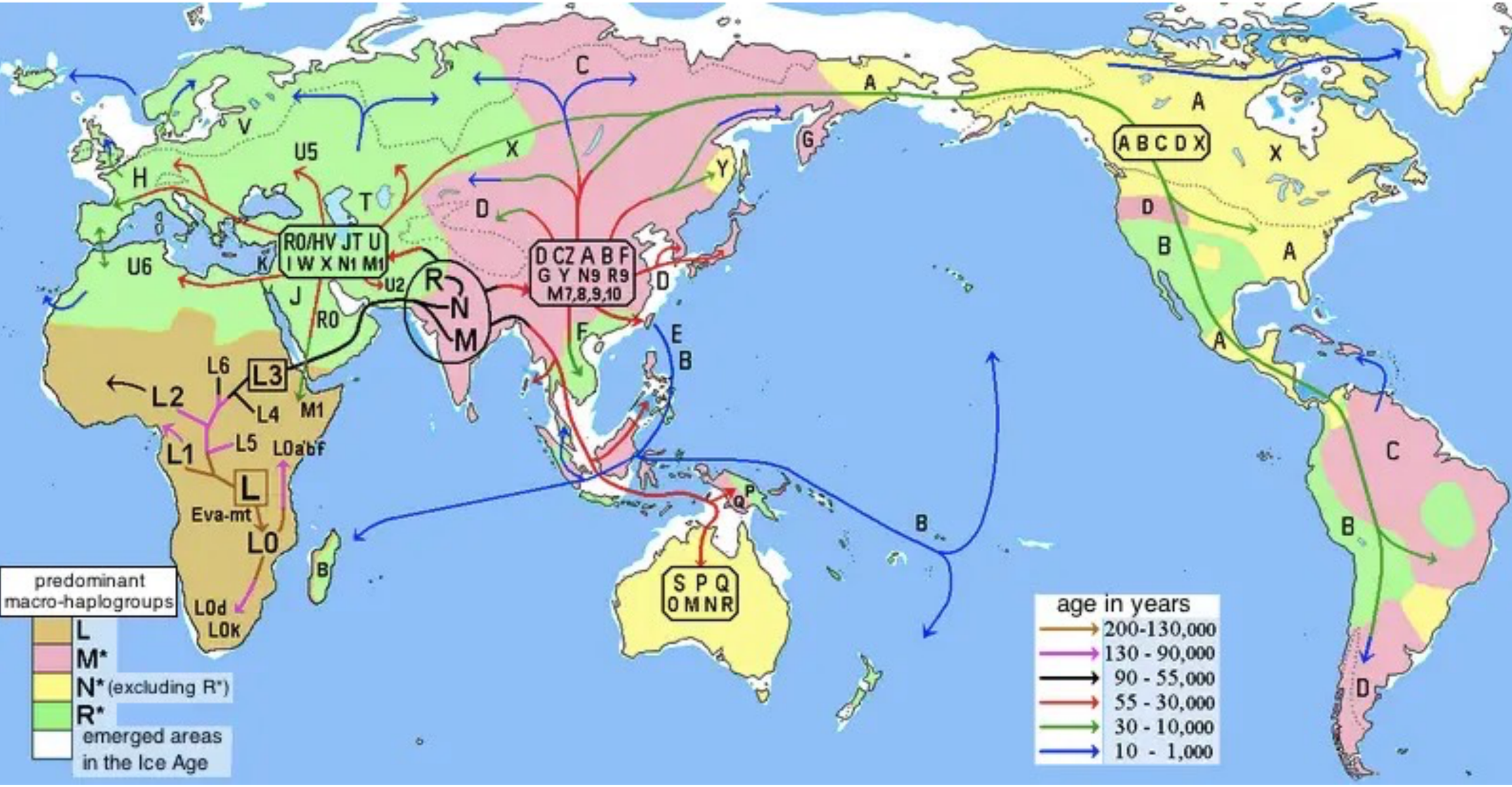
Paths of human migration



Paths of human migration



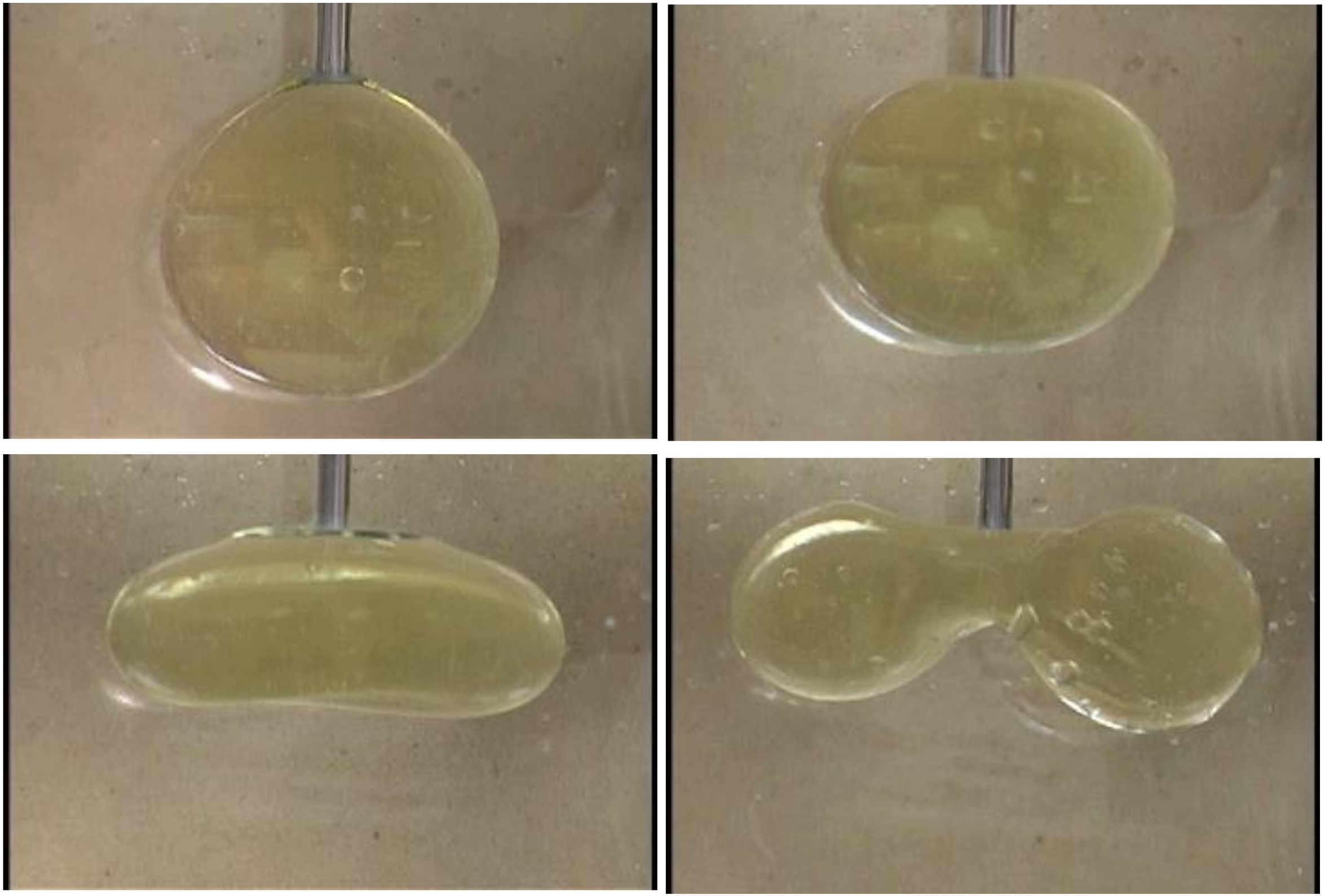
Paths of human migration



The mathematics of democracy

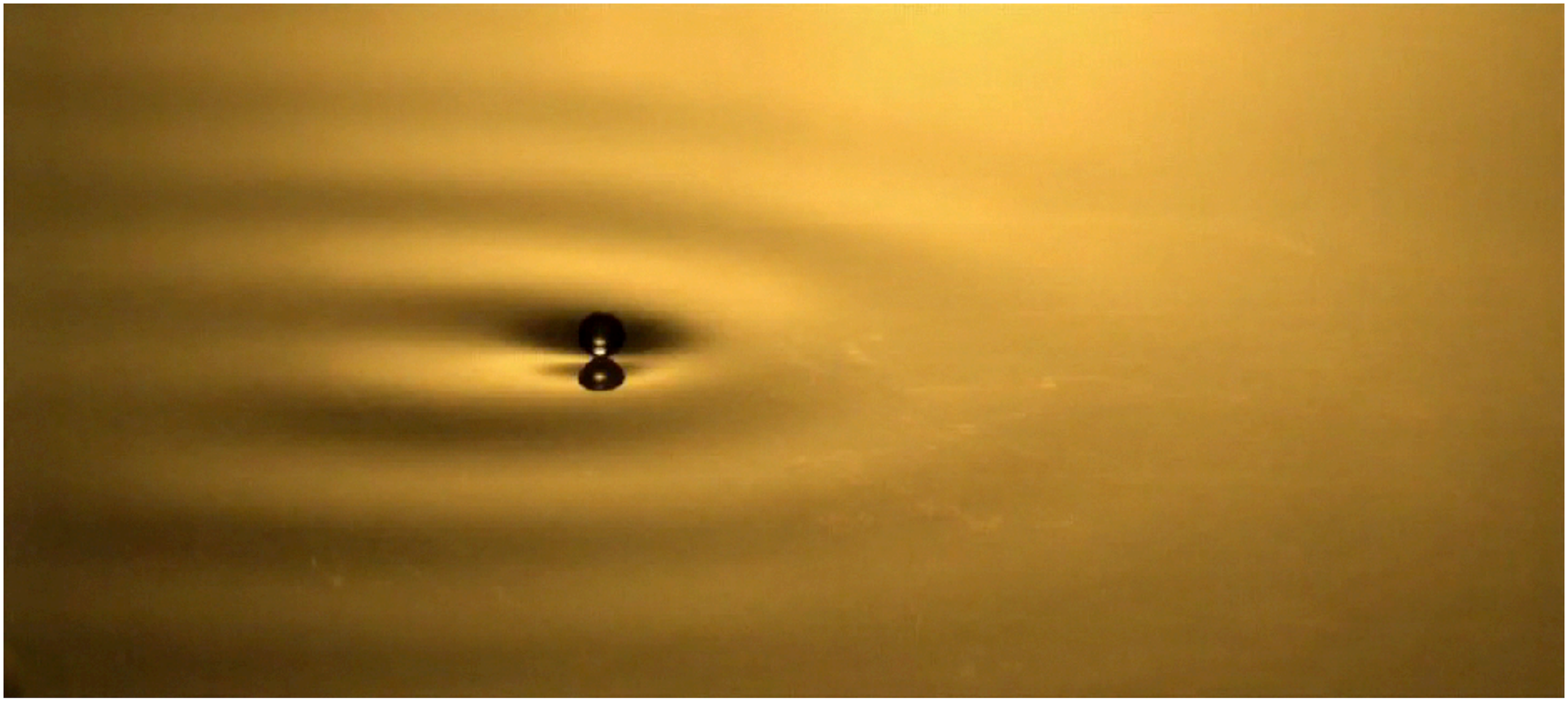
- what are the contributors to *political polarization?*
- can its evolution be modeled mathematically?

Plateau's rotating drop experiments (1863)



How can we analogize gravity with surface tension?

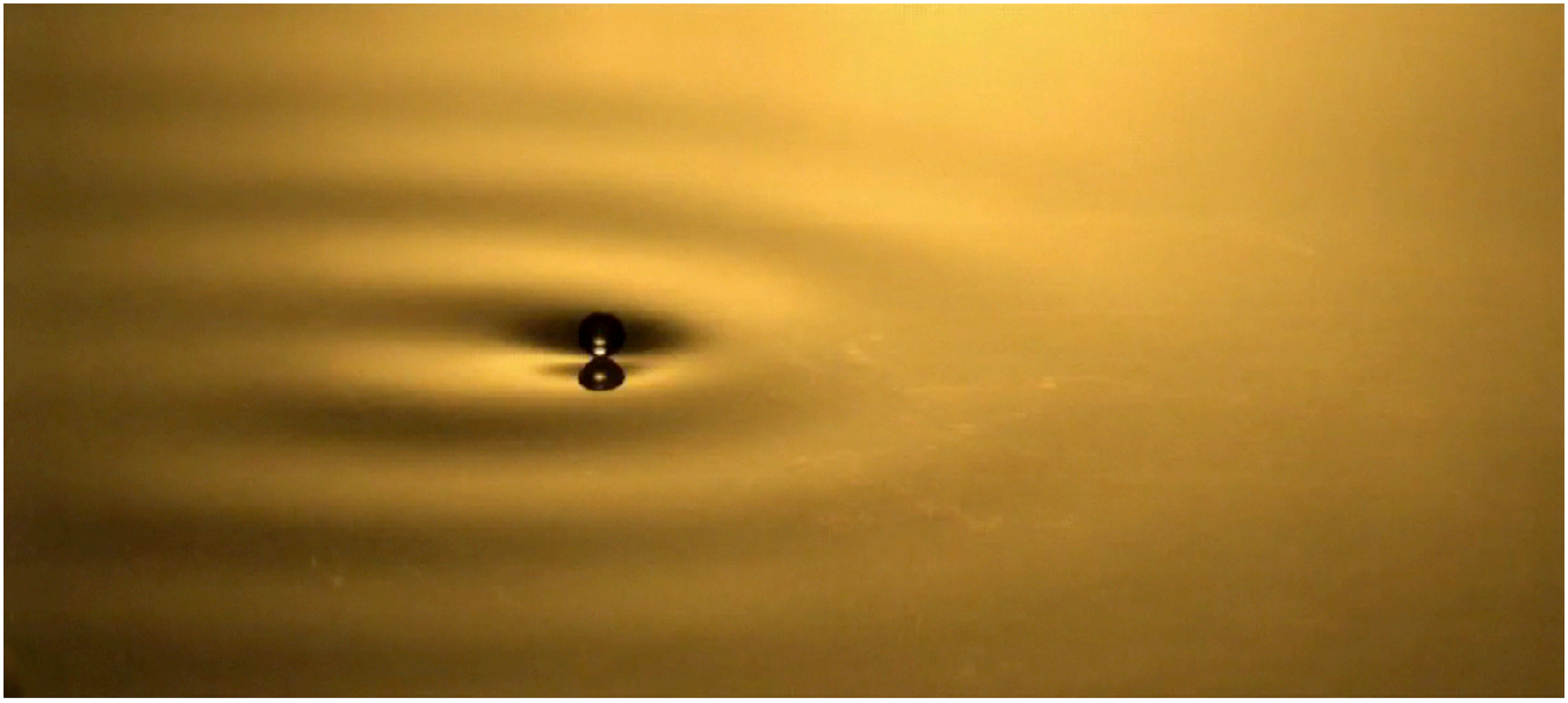
Hydrodynamic quantum analogs



20cS Si oil, 50 Hz

- a millimetric droplet bounces on the surface of a vibrating bath
- the droplet self-propels by virtue of a resonant interaction with its own wave
- walkers exhibit many features previously thought to be peculiar to the quantum realm

Hydrodynamic quantum analogs



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Wave-particle duality at the classical scale

"Both matter and radiation possess a remarkable duality of character, as they sometimes exhibit the properties of waves, at other times those of particles.

Now it is obvious that a thing cannot be a form of wave motion and composed of particles at the same time - the two concepts are too different."

- Heisenberg, *On Quantum Mechanics (1930)*

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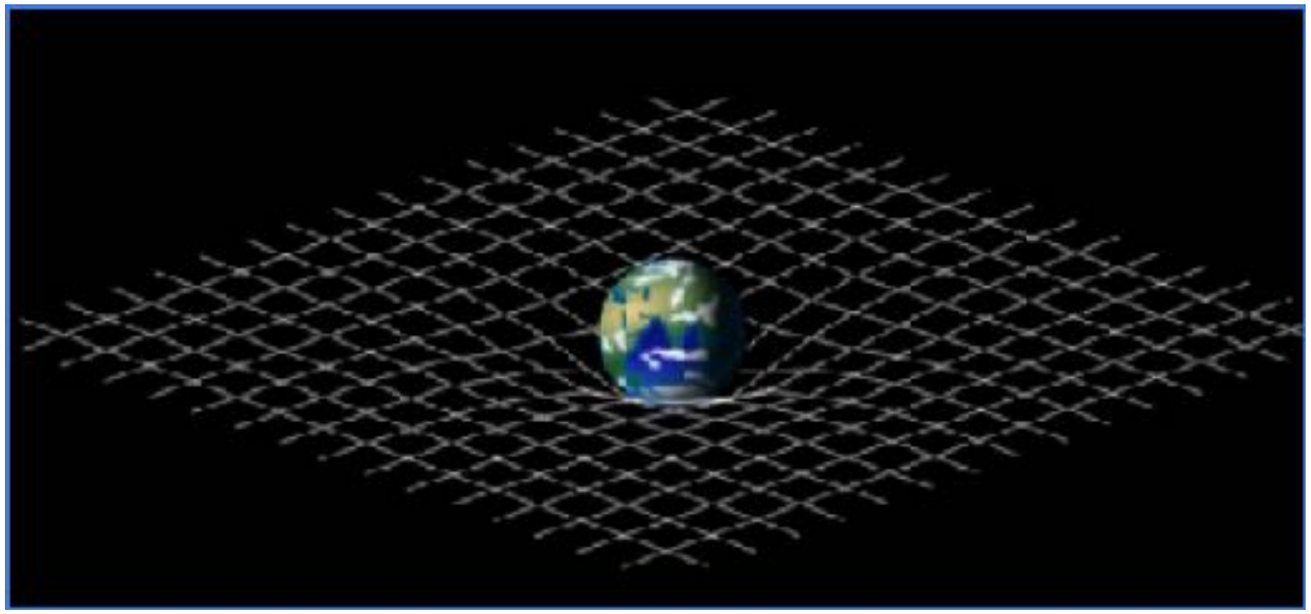
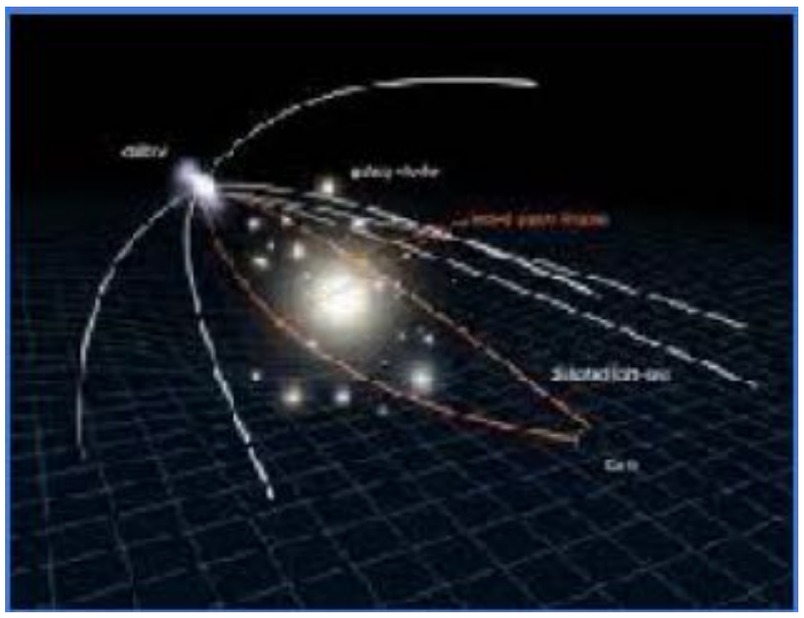
- Heisenberg, *On Quantum Mechanics (1930)*

General relativity (Einstein 1915)



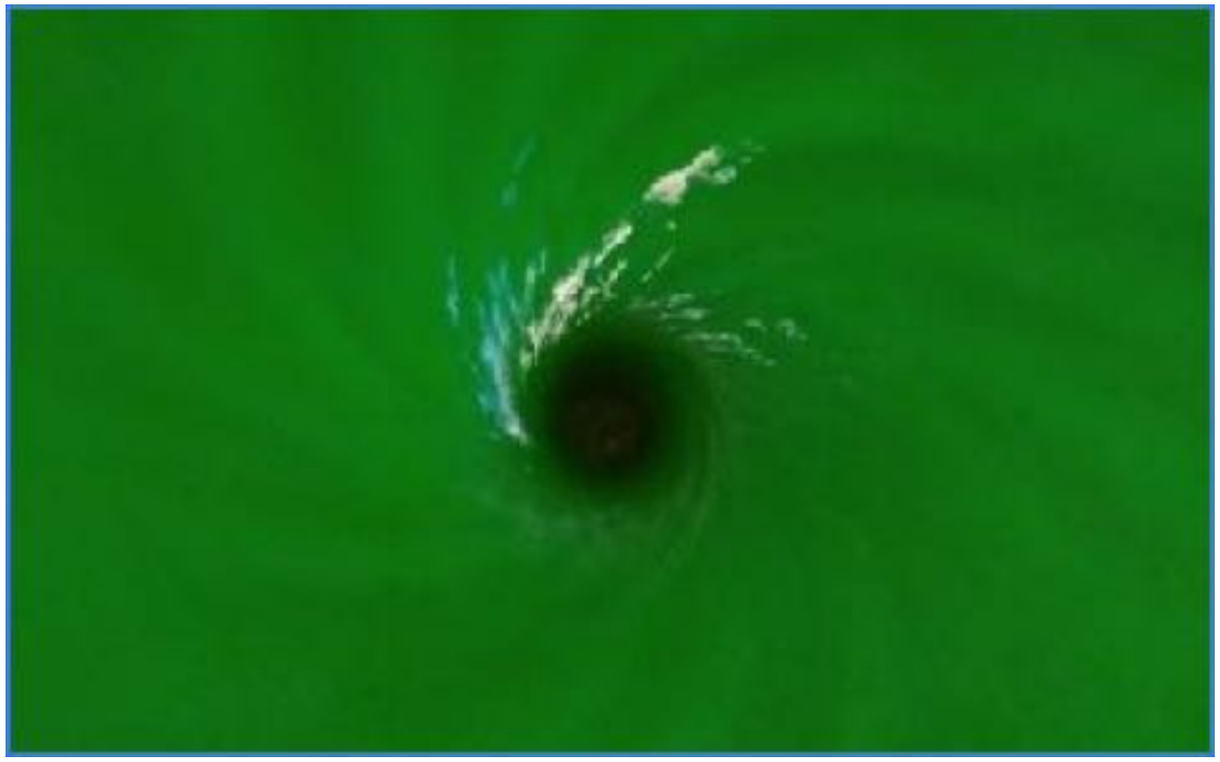
- describes gravity as the curvature of space-time caused by the presence of mass and energy
- massive objects like planets and stars curve the spacetime around them
- other objects, including photons, follow geodesics as they traverse this curved spacetime
- successful in rationalizing gravitational lensing



Analog gravity (Unruh *et al.* 1980s —)

- attempts to model phenomena of GR (e.g. black holes) using fluidic systems, e.g. acoustics in moving fluids, gravity waves, superfluid He, BECs

- in hydrodynamic analogs, the interface plays the role of the fabric of space-time



- established analogs between wave interactions with vortices and black holes
- ditto between wave interactions with hydraulic jumps and white holes

Hydrodynamic analog: a photon approaches a rotating black hole

Hydrodynamic analog: a photon approaches a rotating black hole

Gravitoelectromagnetism

- in limit of weak spacetime curvature (weak gravitational fields)

GEM equations	Maxwell's equations
$\nabla \cdot \mathbf{E}_g = -4\pi G \rho_g$	$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$
$\nabla \cdot \mathbf{B}_g = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E}_g = -\frac{\partial \mathbf{B}_g}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{B}_g = -\frac{4\pi G}{c^2}\mathbf{J}_g + \frac{1}{c^2}\frac{\partial \mathbf{E}_g}{\partial t}$	$\nabla \times \mathbf{B} = \frac{1}{\epsilon_0 c^2}\mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$

Lorentz force

GEM equation	EM equation
$\mathbf{F_g} = m\left(\mathbf{E}_g + \mathbf{v} \times 4\mathbf{B}_g\right)$	$\mathbf{F_e} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$

Can we reconcile QM and GR via hydrodynamic analogs?