18.357: Lecture 21

The beating heart

Water waves

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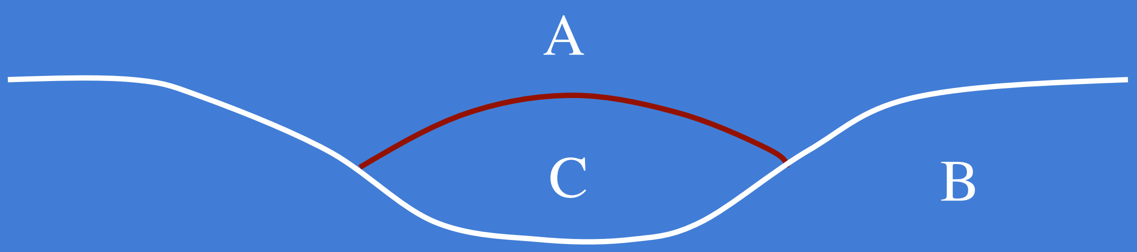
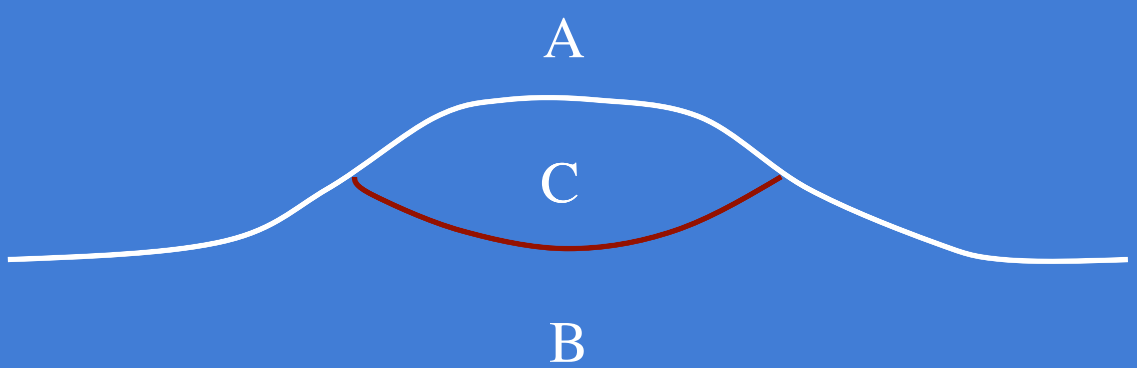
Static drops at interfaces (Pujado & Scriven 1972)

Pendant Lenses: stable only for drops small relative to capillary length ℓ_c

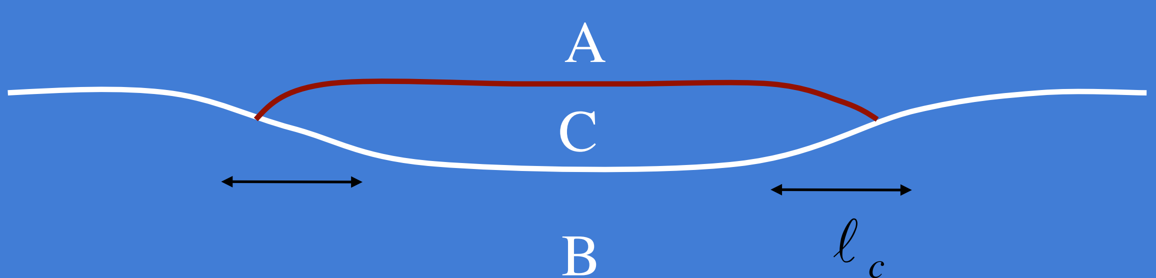


$\rho_A < \rho_B < \rho_C$



$\rho_C < \rho_A < \rho_B$

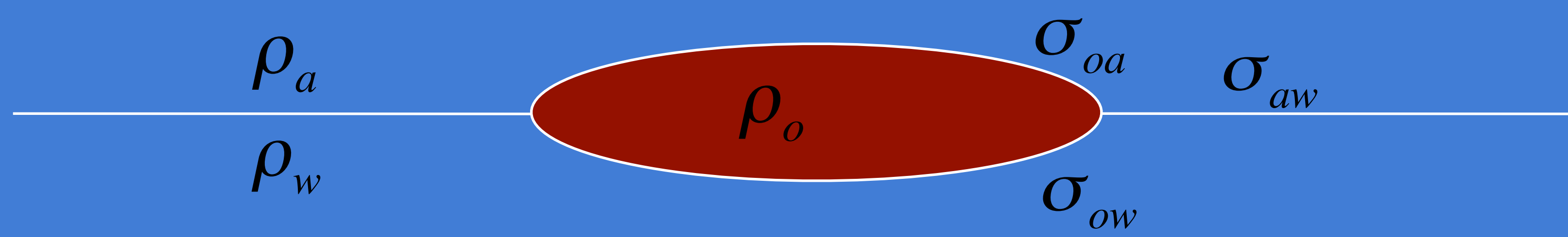
Sessile Lens: stable for drops of any size



$\rho_A < \rho_C < \rho_B$

e.g. oil on water

Oil on water: A brief review



- when an oil drop is emplaced on the surface of water, its behaviour will depend on the spreading coefficient

$$S = \sigma_{aw} - \sigma_{oa} - \sigma_{ow}$$

- for $S < 0$, an equilibrium configuration arises: the drop assumes the form of a sessile lens

Statics: Langmuir (1933); Pujado & Scriven (1972)

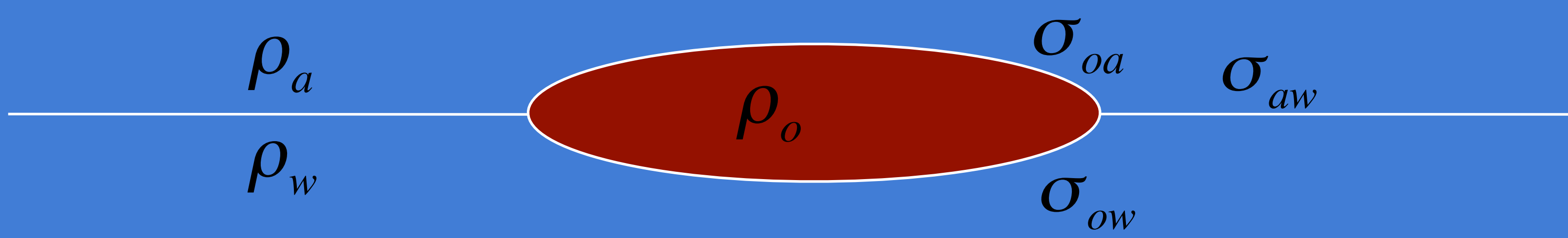
Dynamics: Wilson & Williams (1997), Miksis & Vanden-Broeck (2001)

- for $S > 0$, the droplet will completely wet the underlying liquid, and so spread to a layer of molecular thickness

Franklin (1760); Fay (1963); DePietro & Cox (1980); Foda & Cox (1980); Joanny (1987); Brochard-Wyart et al. (1996); Fraaije & Cazabat (1989)

The beating heart

Problem Statement



$$S = \sigma_{aw} - \sigma_{oa} - \sigma_{ow} < 0$$

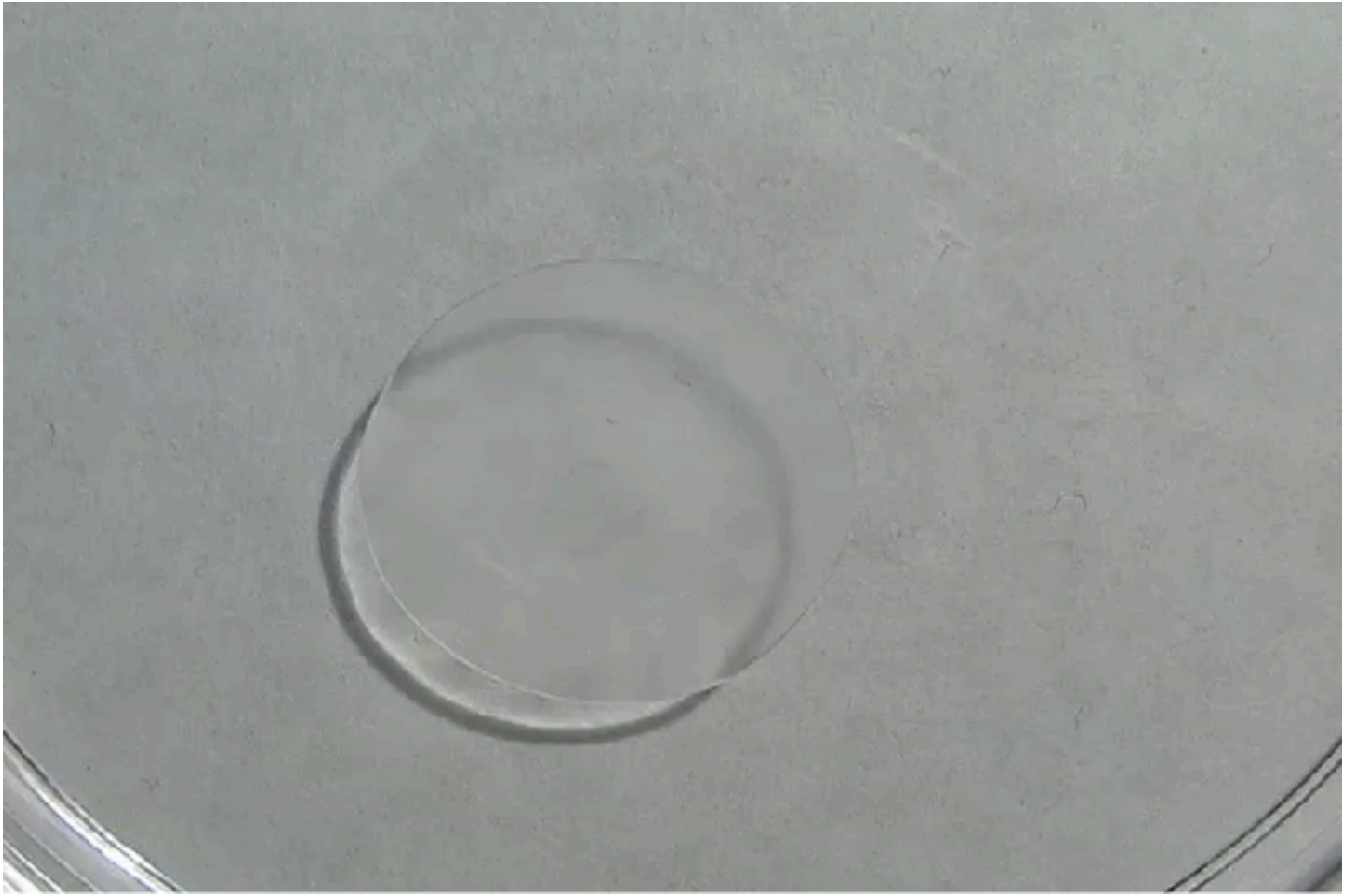
- when a drop of mineral oil containing a small quantity of nonionic, water-insoluble surfactant (Tergitol) is so emplaced, a sessile lens with $S<0$ is formed

- however, NO EQUILIBRIUM SHAPE EMERGES; the lens is characterized by periodic fluctuations in radius, and so resembles…

THE BEATING HEART

Stocker & Bush (JFM, 2007)

The beating heart



The beating heart

- first reported by Buetschli (1894), a Professor of Zoology at the University of Heidelberg, in his treatise *Investigations on Protoplasm*
- subsequently described qualitatively by Sebba (1979, 1981)

Motivation

"The ultimate goal of physiologists is to be able to explain living behaviour in terms of physicochemical forces. Thus, any expansion of our knowledge of such forces, based on inanimate systems, should be examined to see whether this might not offer insight into biological behaviour".

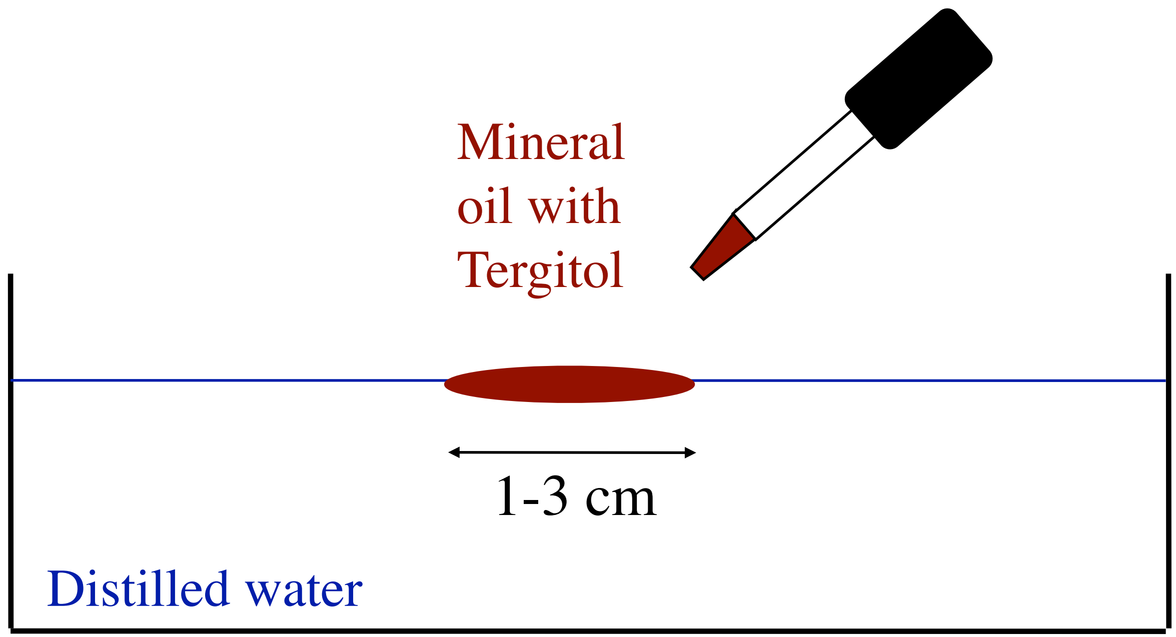
- Sebba (1979)

- many biological systems exhibit periodic behaviour; e.g. oscillations of cells of nerves and muscle tissue, oscillations in mitochondria, and biological clocks
- conversion of chemical into mechanical energy is one of the main processes in biological movements; e.g. chloroplast movements and muscle contraction
- fundamental interest $\Longrightarrow$ **What is going on?**

The breaking heart : high surfactant concentration

Experiments

- mineral oil with 0-10% Tergitol 15-S-3 (nonionic surfactant; insoluble in water, soluble in oil)
- lens emplaced on water surface with eye-dropper



Observations

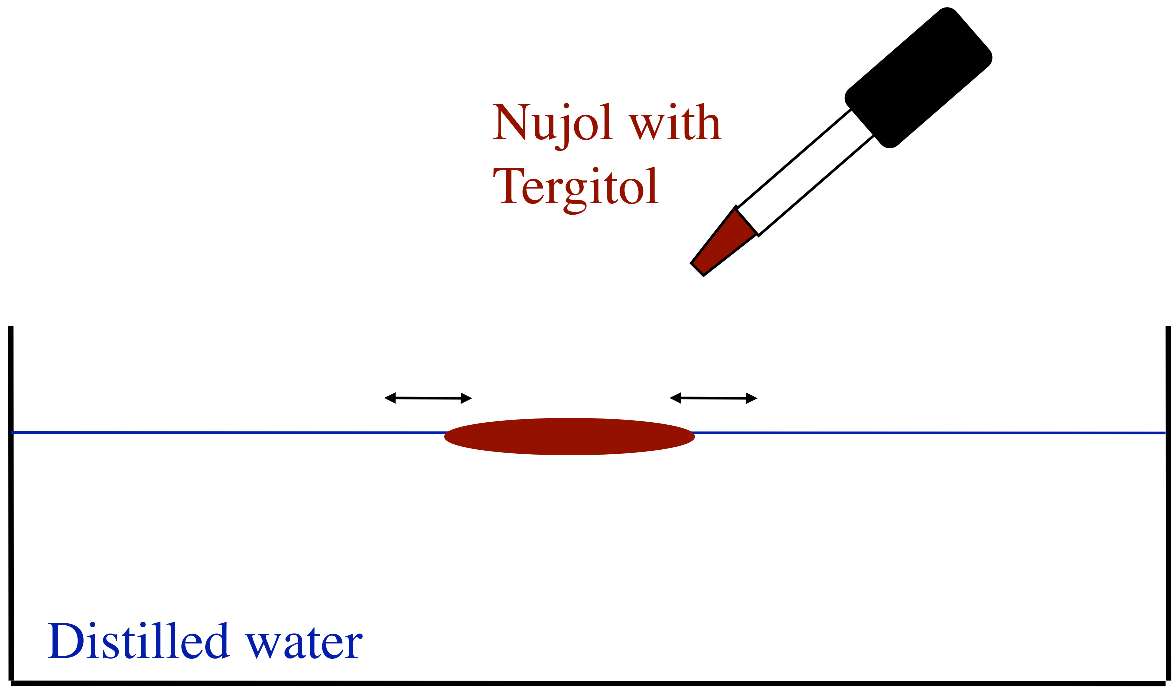
- lens behaviour independent of water depth
- lens behaviour strongly dependent on surfactant concentration Γ

$\Longrightarrow$ for $\Gamma = 0$: no beating; stable sessile lens

$\Longrightarrow$ for moderate Γ, steady beating observed

$\Longrightarrow$ for high Γ, drop edges become unstable to fingers

$\Longrightarrow$ for highest Γ, lens explodes into series of smaller beating lenses



Key Observations

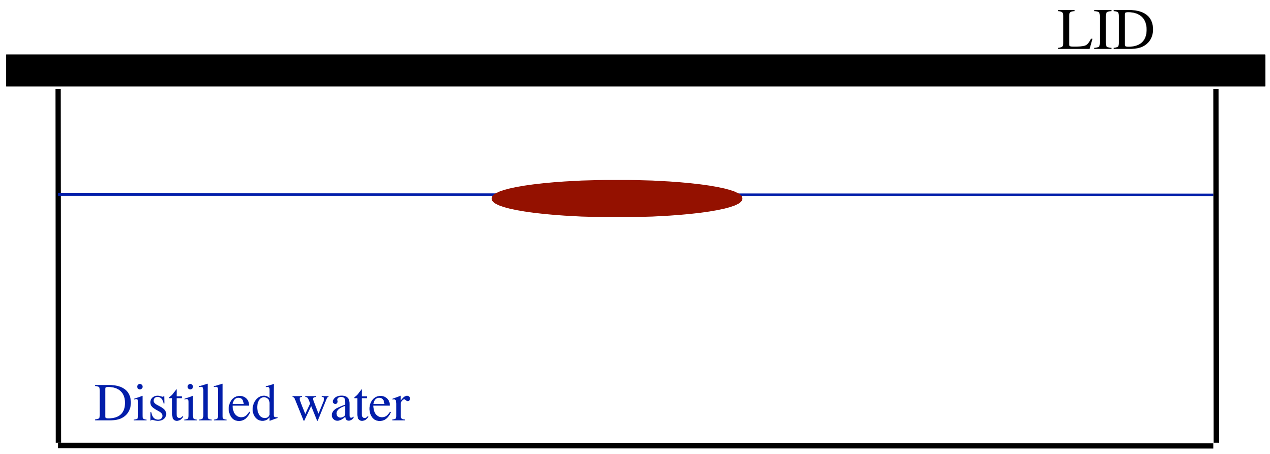
- beating marked by slow expansion, rapid retraction
- odour of Tergitol always accompanies beating modes
- placing lid on the chamber suppresses the oscillations

Evaporation rates:

$$\varphi_{\text{oil}} = 0$$

$$\varphi_{\text{water}} = 2.3\times10^{-5}\ \text{kg m}^{-2}\text{s}^{-1}$$

$$\varphi_{\text{tergitol}} = \frac{\varphi_{\text{water}}}{50}$$



Key Observations

- beating marked by slow expansion, rapid retraction
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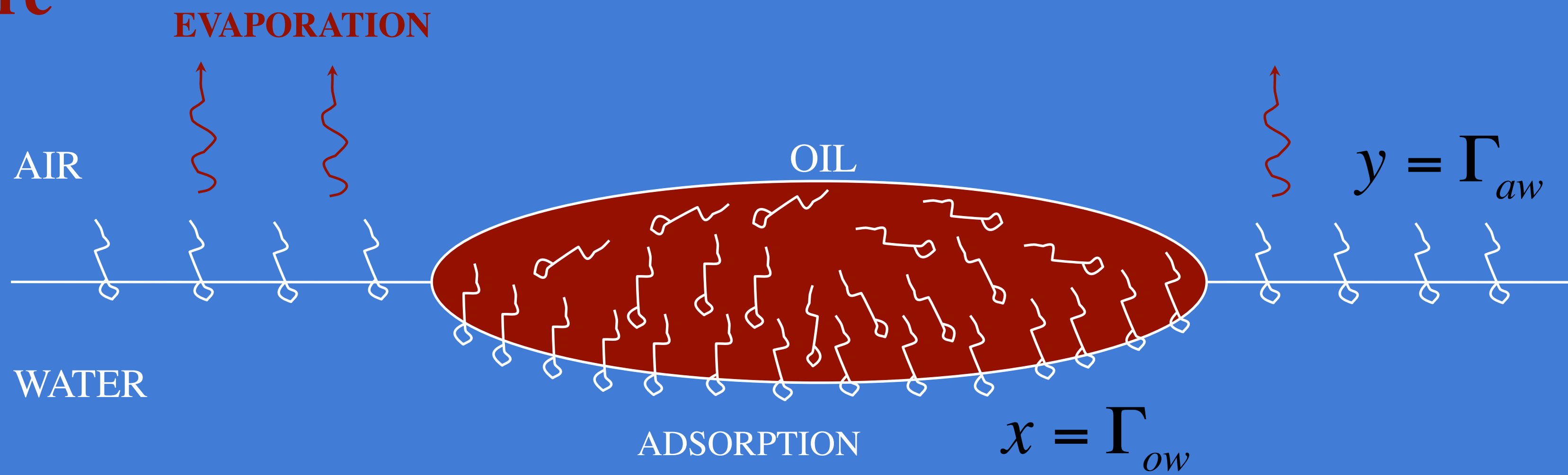
→ **evaporation is a critical ingredient**

Physical Picture



Stage 1: Slow expansion of drop

- adsorption of surfactant onto oil-water interface $\Rightarrow$ σ_{ow} decreases
- evaporation of surfactant from air-water surface $\Rightarrow$ σ_{aw} increases

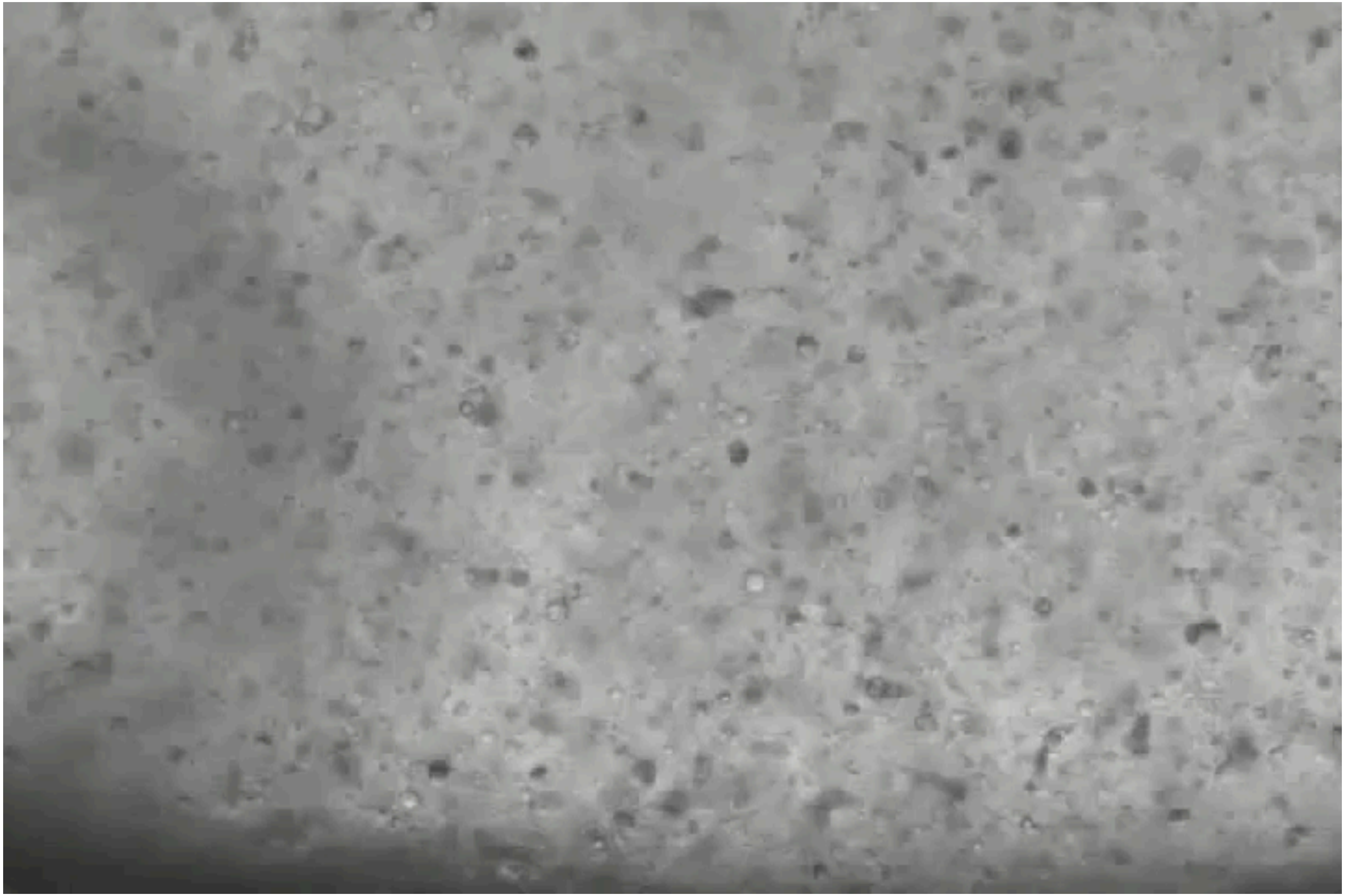
Stage 2: Rapid retraction

- flushing of surfactant onto air-water interface $\Rightarrow$ σ_{aw} decreases

$\Rightarrow$ σ_{ow} increases

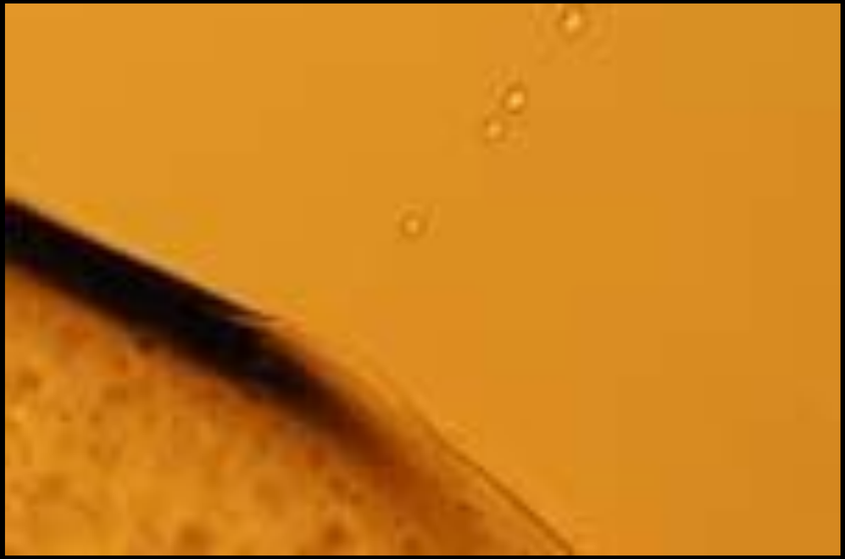
BUT WHY?

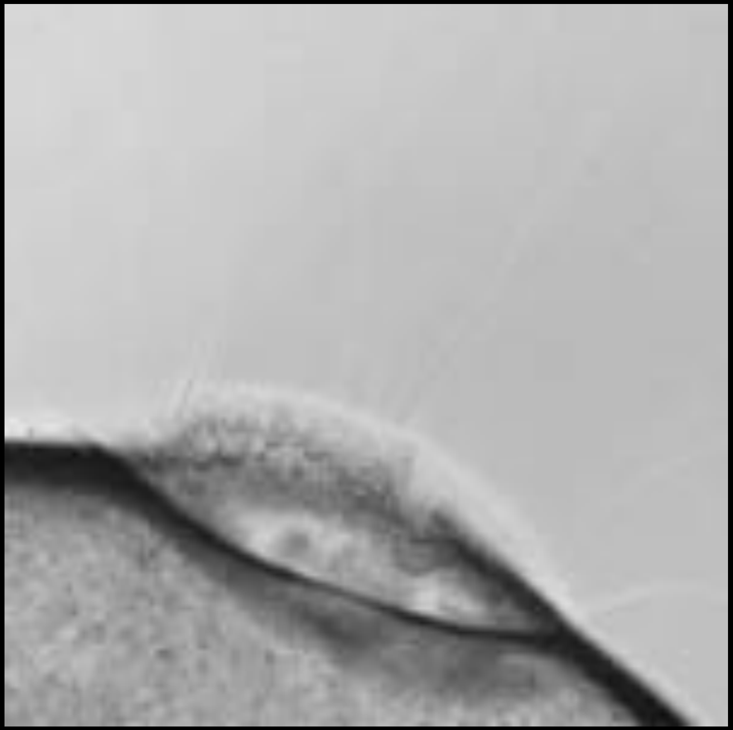
Flushing events: breaking waves dump surfactant onto surface

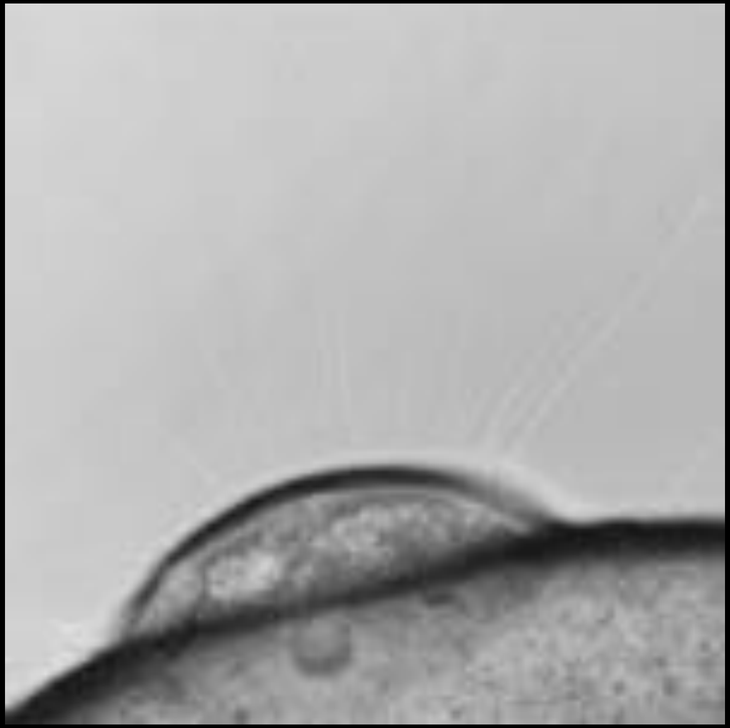


Flushing events

- emulsification set by surfactant's HLC (hydrophobicity-lipophobicity coefficient)

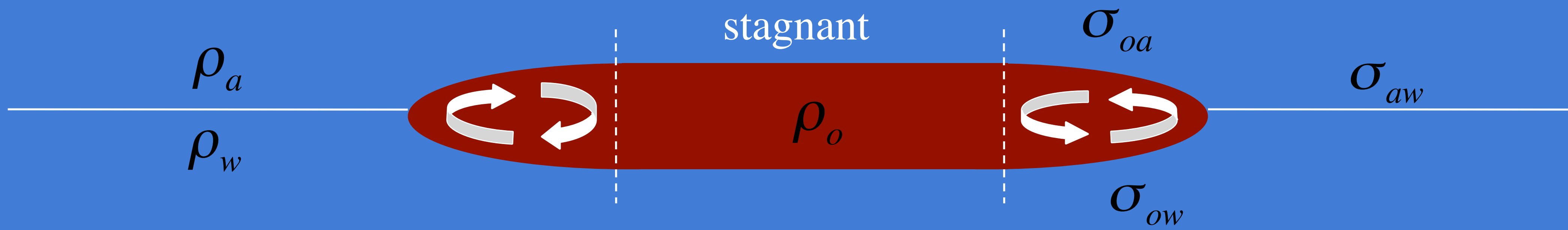






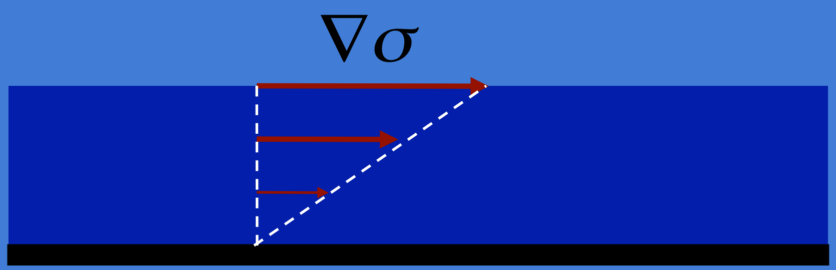
Internal circulation: confined to outer extremities of the lens

Internal circulation



- internal circulation confined to drop edges, absent in flat central region
- Marangoni flow associated with gradient in Γ: indicates Γ lowest at drop edge
- consistent with radial gradient in adsorption flux along surface
- reflects geometric constraint: less surfactant available to corners from bulk

Marangoni shear layer



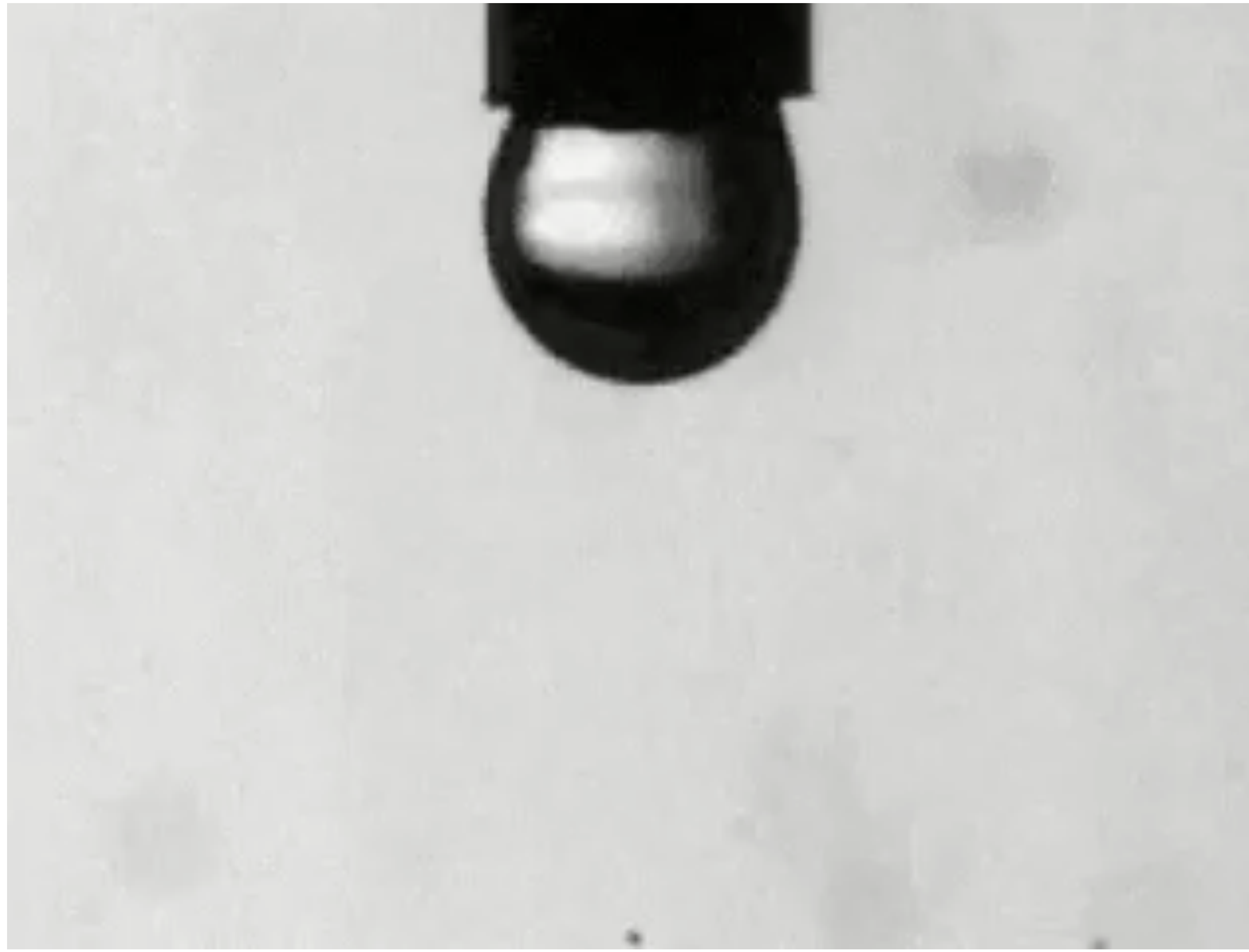
- unstable to longitudinal rolls or transverse waves (as in wine glass)
- flushing events associated with breaking Marangoni waves

(Frenkel and Halpern 2005)

Variations on the same theme

Variations on the same theme

Pinch-off



The spitting drop

Fernandez & Homsy (2004)

Waves

What relates the frequency to the wavelength?

Why does one only surf near the beach?

Big splash

Small splash