

**18.357: Lecture 15**

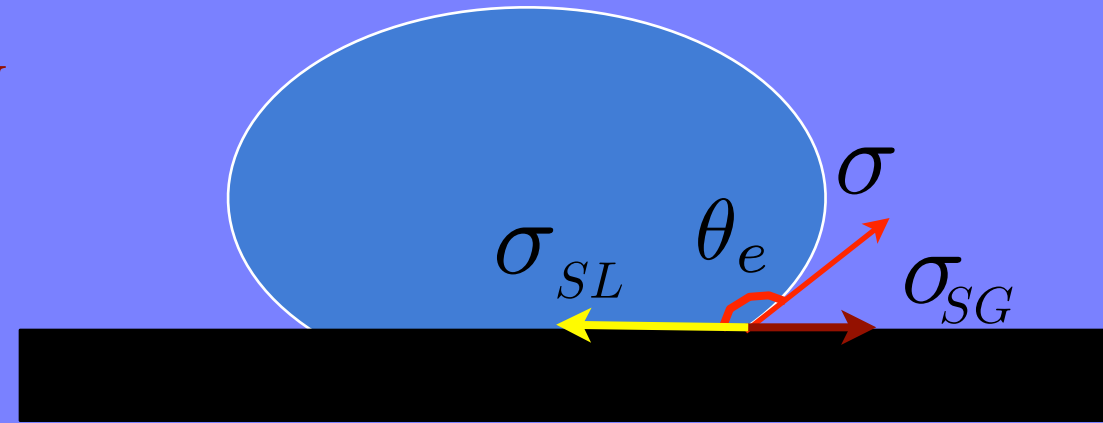
**Contact angle hysteresis,  
The wetting of textured solids**

**John W. M. Bush**

Department of Mathematics  
MIT

# Fluid-Solid Contact: WETTING

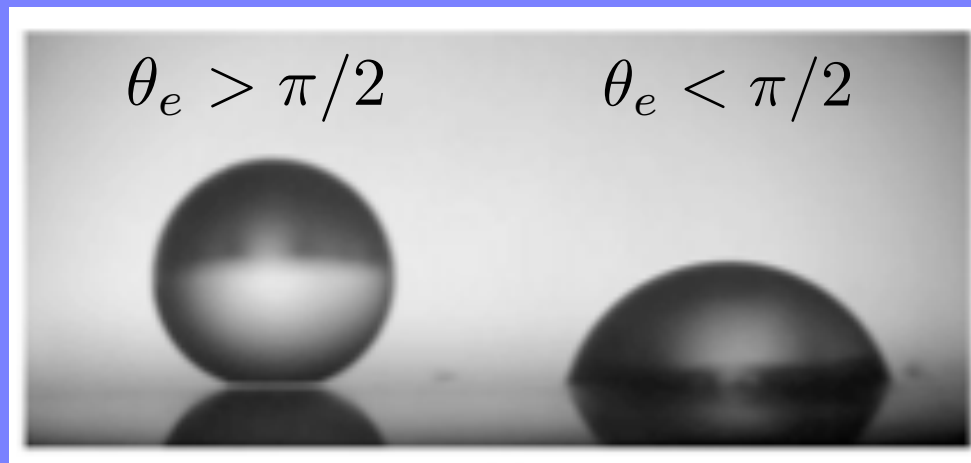
Equilibrium contact angle  $\theta_e$



Energy differential:  $dW = dx (\sigma_{SG} - \sigma_{SL}) - dx \sigma \cos\theta_e$

Young's relation:

$$\sigma \cos\theta_e = \sigma_{SL} - \sigma_{SG}$$

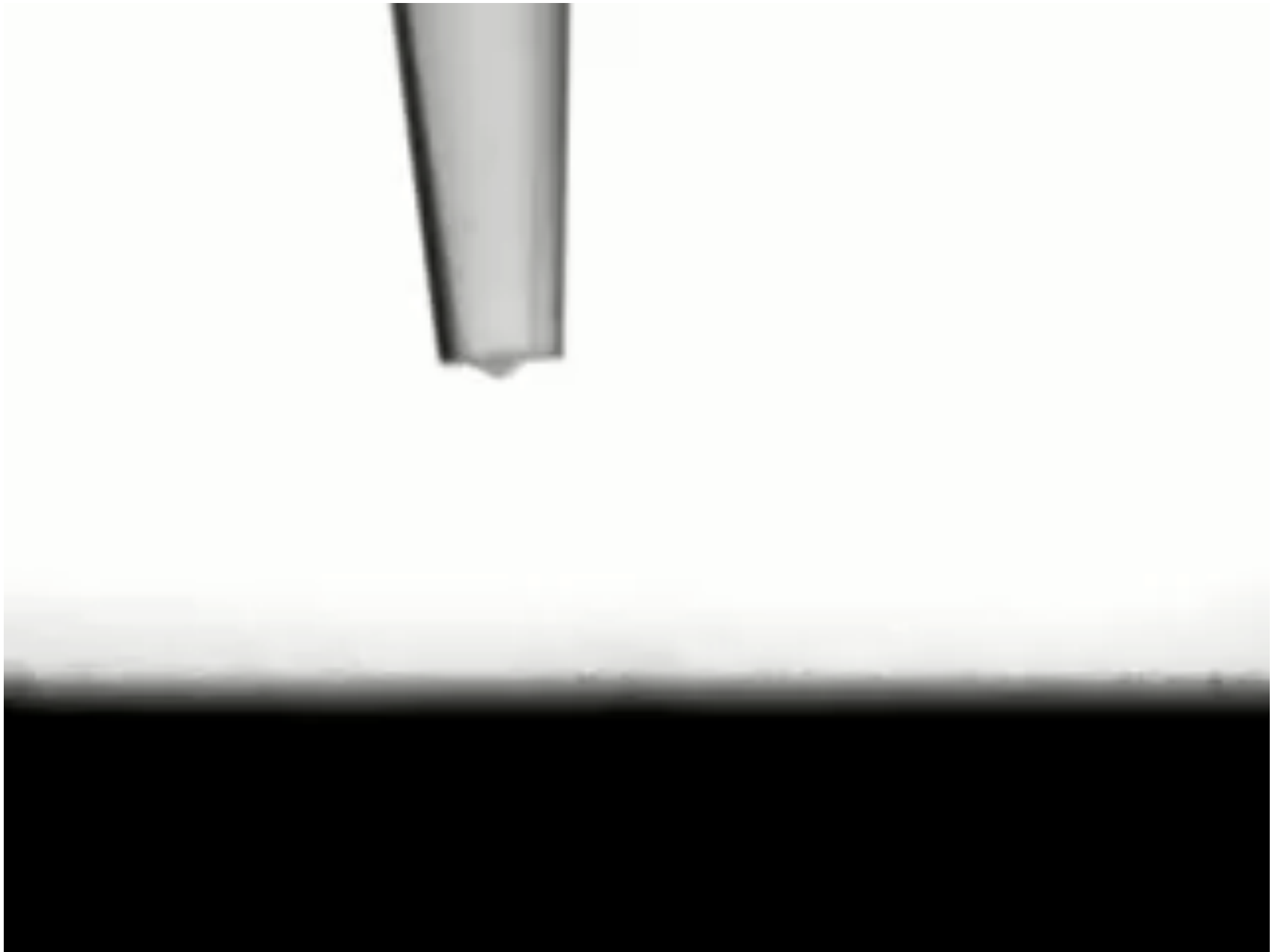


Hydrophobic  
surface

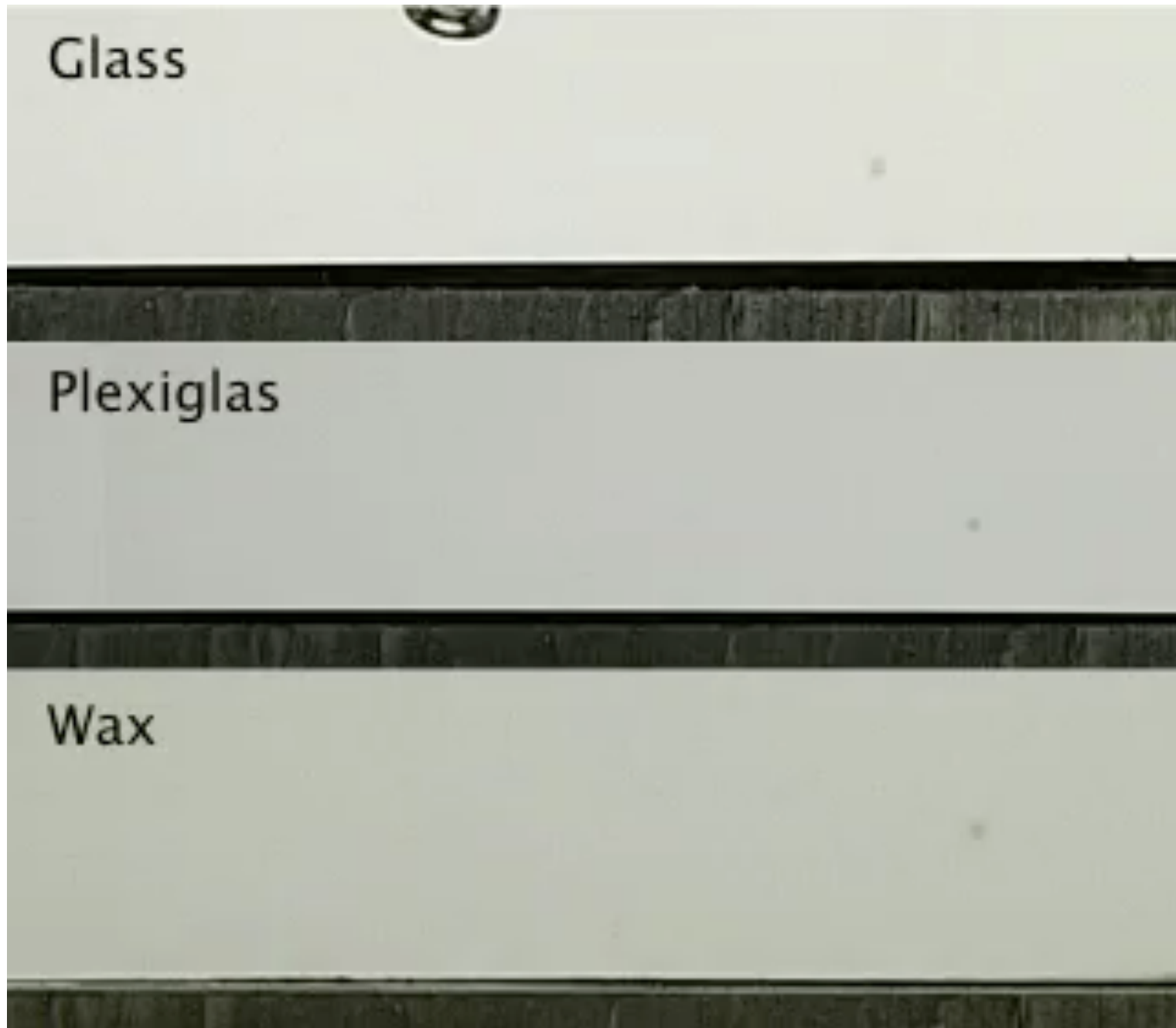
Hydrophilic  
surface



**Total wetting on a flat solid**



**Partial wetting on a flat solid**



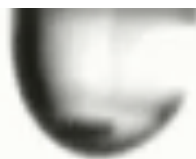
**Partial wetting**



$2\lambda$



methylnaphtalene



$$\gamma = 20 \text{ mN/m}$$

water



$$\gamma = 73 \text{ mN/m}$$

mercury



$$\gamma = 500 \text{ mN/m}$$

Fluorinated Oil



Oil



Water

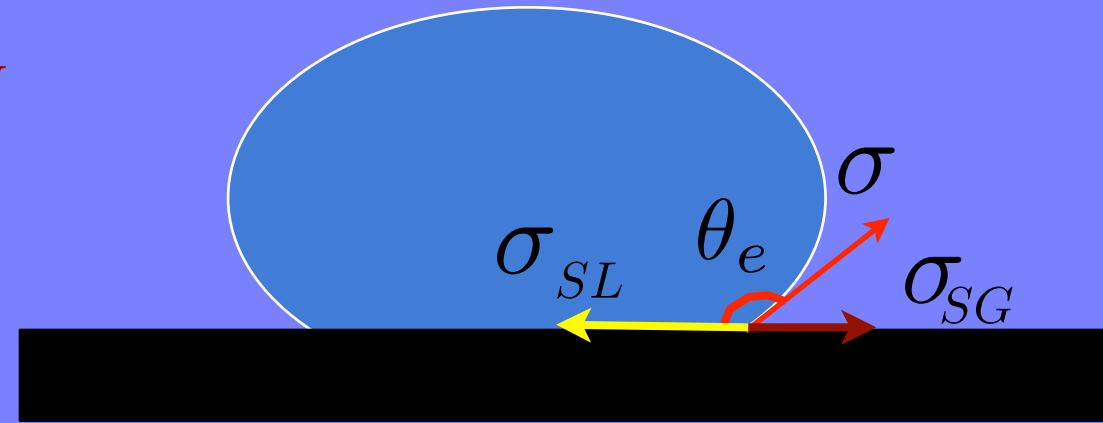


Mercury



# Fluid-Solid Contact: WETTING

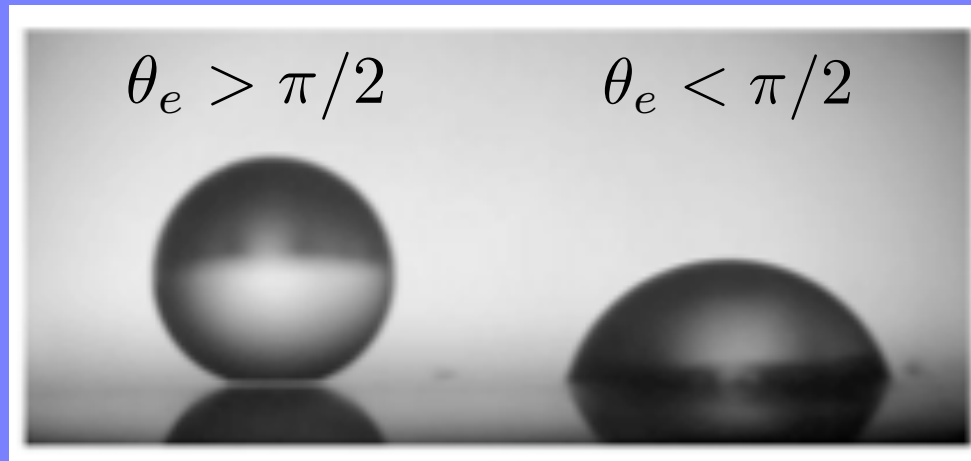
Equilibrium contact angle  $\theta_e$



Energy differential:  $dW = dx (\sigma_{SG} - \sigma_{SL}) - dx \sigma \cos\theta_e$

Young's relation:

$$\sigma \cos\theta_e = \sigma_{SL} - \sigma_{SG}$$



Hydrophobic  
surface

Hydrophilic  
surface

## The raindrop paradox



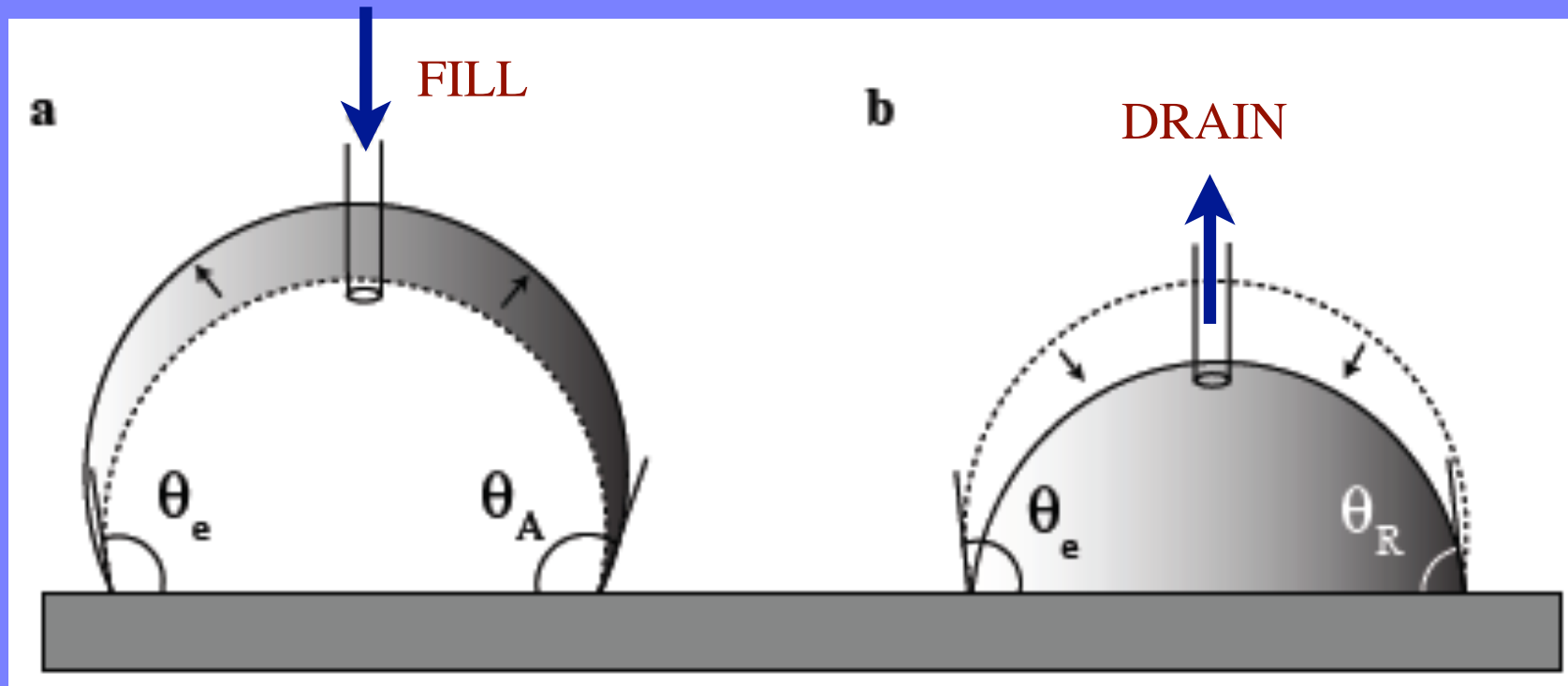
**Why do drops stick?**



## Contact angle hysteresis

Static contact angle is not uniquely  $\theta_e$

**Reality:** drop is stable over a range of  $\theta_r < \theta < \theta_a$



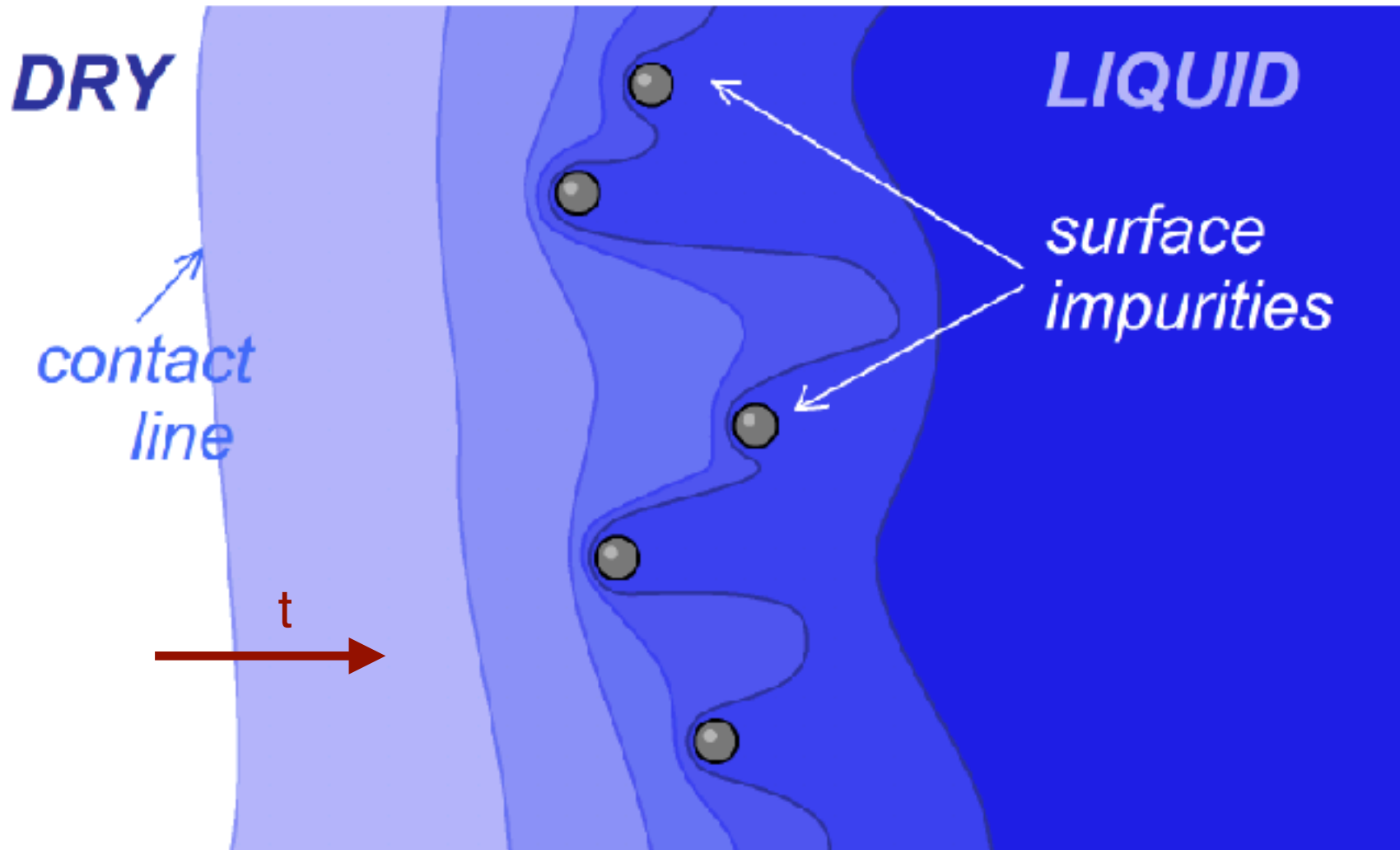
➔ **FORCE of ADHESION** resists drop motion

increases with  $\Delta\theta = \theta_a - \theta_r$

**Origins:** advancing contact lines pinned on surface irregularities

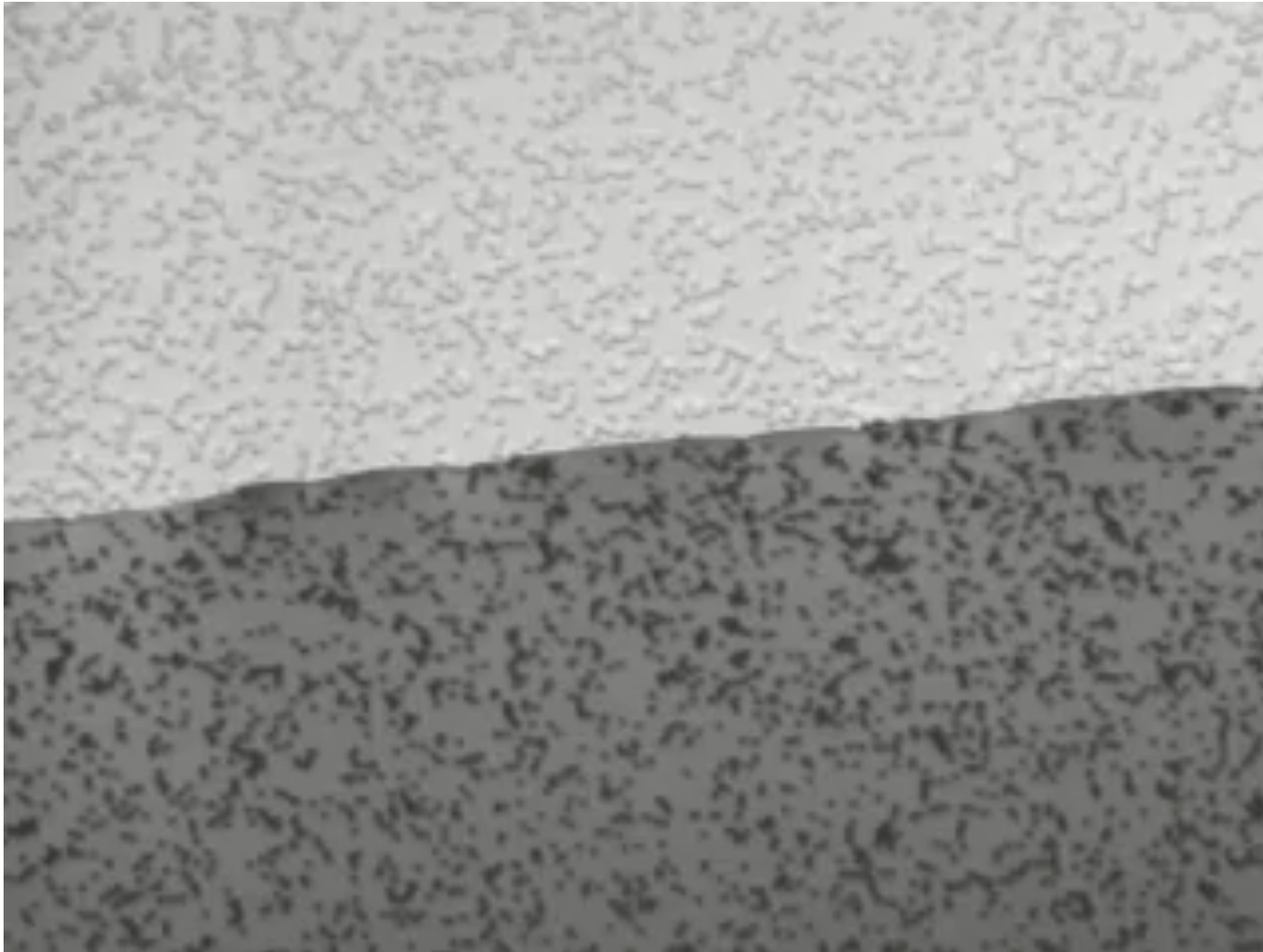
## The origins of contact angle hysteresis

- motion of contact line past chemical/textural irregularities is energetically costly

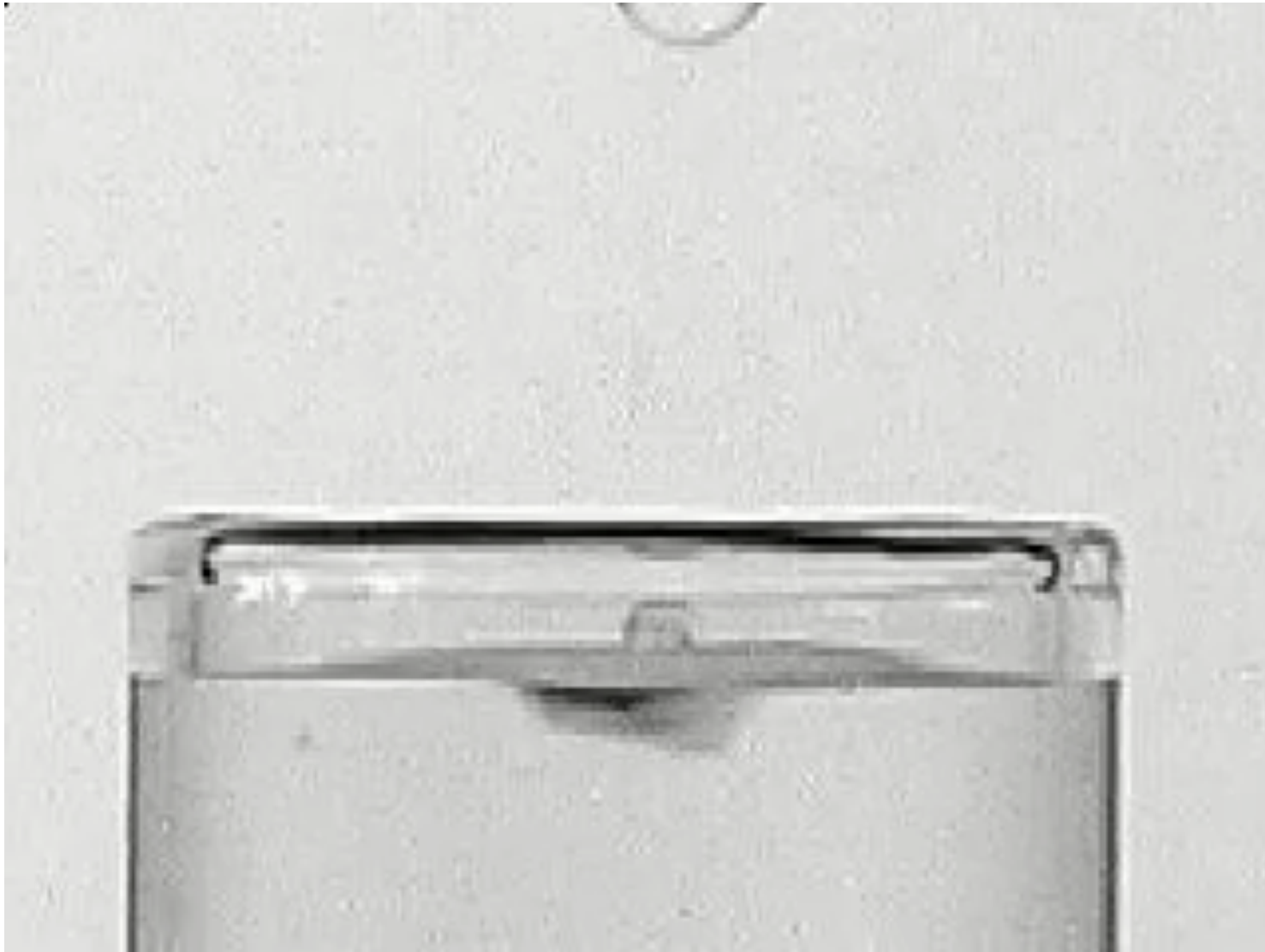


## The origins of contact angle hysteresis

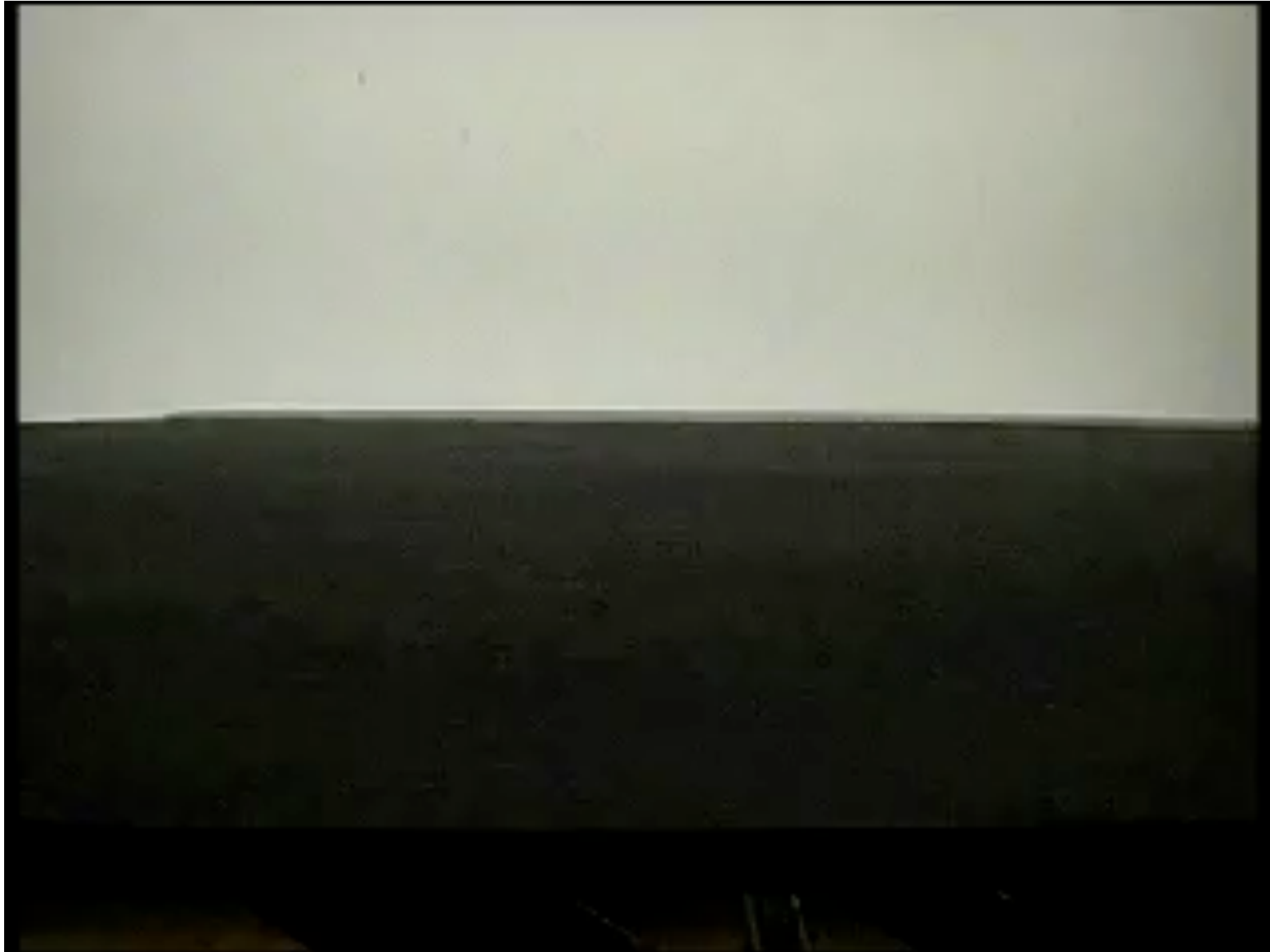
- motion of contact line past chemical/textural irregularities is energetically costly



## Contact angle pinning on corners



## Reduce contact angle hysteresis via cleaning



## **Manifestations of contact angle hysteresis**

- liquid slug in a capillary tube
- drops stick to solids



## The raindrop paradox



**Why do drops stick?**



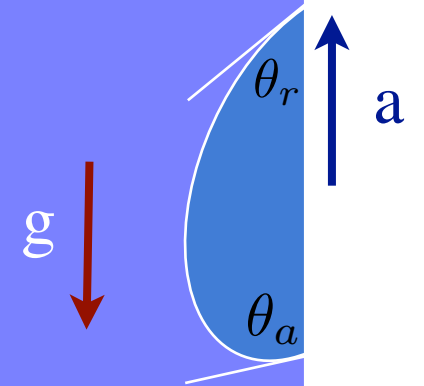
## The force of adhesion (Dussan & Chow 1983)

Raindrop stuck on a window

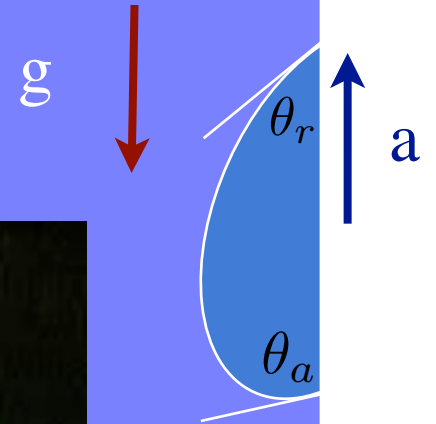
- small drops supported by contact line resistance

$$F_c \sim 2\pi a \sigma (\cos \theta_r - \cos \theta_a)$$

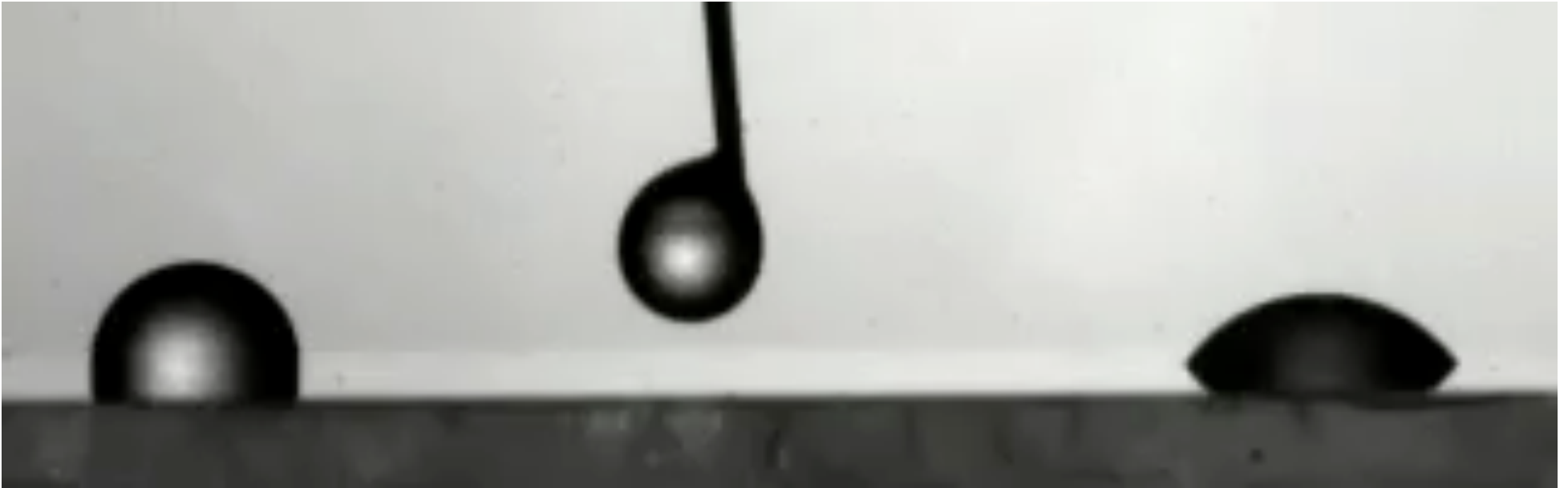
- drops grow by accretion until weight prompts rolling



# The triumph of gravity over contact forces



## Spontaneous motion in response to a wettability gradient



- lateral chemical force must overcome contact force

# Spontaneous motion in response to a chemical gradient



- lateral chemical force must overcome contact force

# Overcoming contact forces via vibration



- force at drop's natural frequency

$$\rho U^2 \approx \sigma / R$$

→ 
$$\omega \sim \left( \frac{\sigma}{\rho R^3} \right)^{1/2}$$

## Propulsion via contact angle hysteresis and vibration



- exploited by a class of shorebirds for feeding





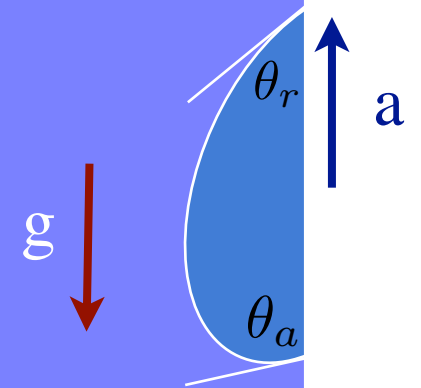
## The force of adhesion (Dussan & Chow 1983)

Raindrop stuck on a window

- small drops supported by contact line resistance

$$F_c \sim 2\pi a \sigma (\cos \theta_r - \cos \theta_a)$$

- drops grow by accretion until weight prompts rolling



Water-repellency

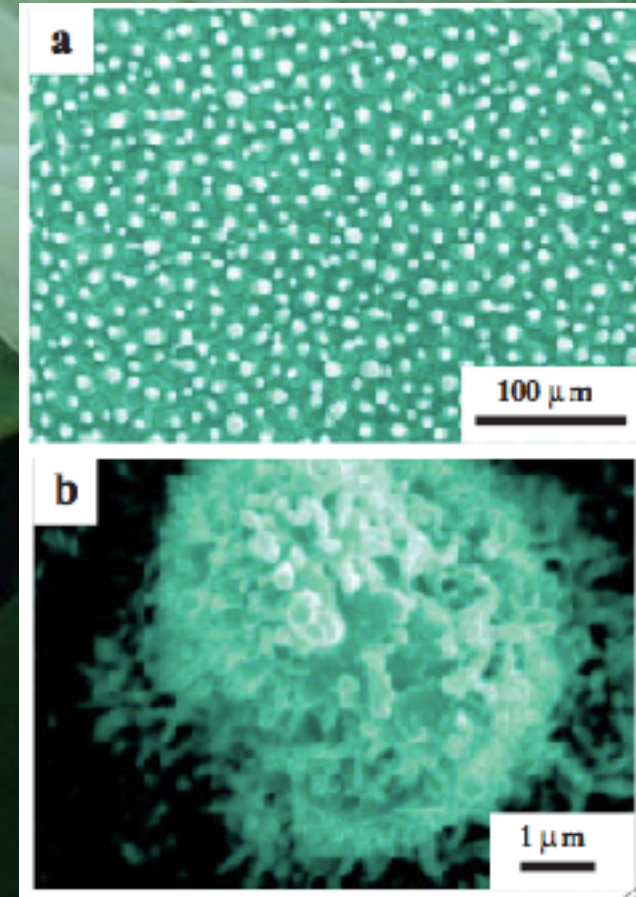
- impinging drops roll off rather than adhering
- requires large  $\theta_e$ , small  $\Delta\theta = \theta_a - \theta_r$

**How can we reduce the force of adhesion?**

## Water repellency in nature

“One who performs his duty without attachment, surrendering the results unto the Supreme Being, is unaffected by sinful action, as the lotus leaf is untouched by water.”

Bhagavad Gita 5.10



Feng et al. (2004)

- the lotus leaf is superhydrophobic and self-cleaning by virtue of its waxy surface roughness