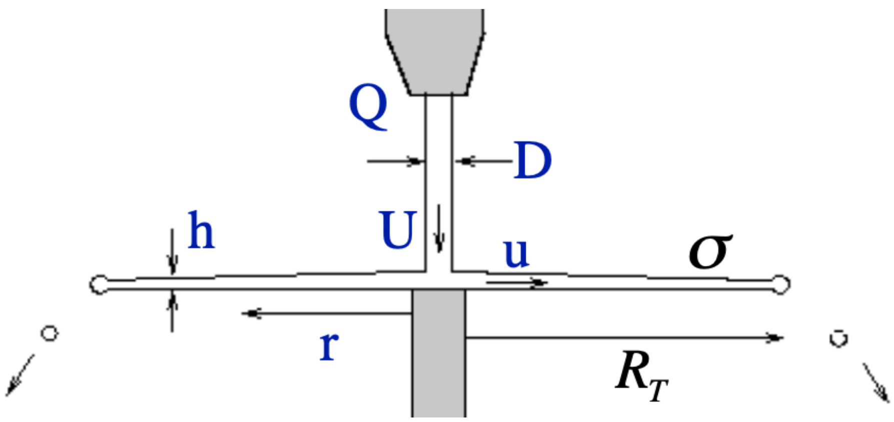
Lecture 13: Fluid sheets, chains and fishbones

Fluid sheets: generated by jets colliding with an impactor or another jet

Circular Sheet

- for $We = \frac{\rho U^2 D}{\sigma} > 10^3$, sheet flaps as on a soap film
- consider case of $We < 10^3$, where a stable circular sheet arises



Scaling: $Re \sim \frac{UR}{\nu} \sim \frac{30 \cdot 10}{.01} \sim 3 \times 10^4 \gg 1$

$\Rightarrow$ inertia dominates viscosity

$Fr = \frac{U^2}{gR} \sim \frac{30^2}{10^3 \cdot 10} \sim 0.1 \Rightarrow$ inertia dominates g

Dominant force balance: inertia vs. curvature

$$\rho u^2 h = 2\sigma$$

Continuity : $Q = 2\pi r u h$

$$\Rightarrow h(r) = \frac{Q}{2\pi r u} \sim \frac{1}{r}$$ since u = const (expts)

Subing $h(r)$ into ✱ yields sheet radius :

$$\boxed{R_T = \frac{\rho Q u}{4\pi\sigma}}$$ Taylor radius

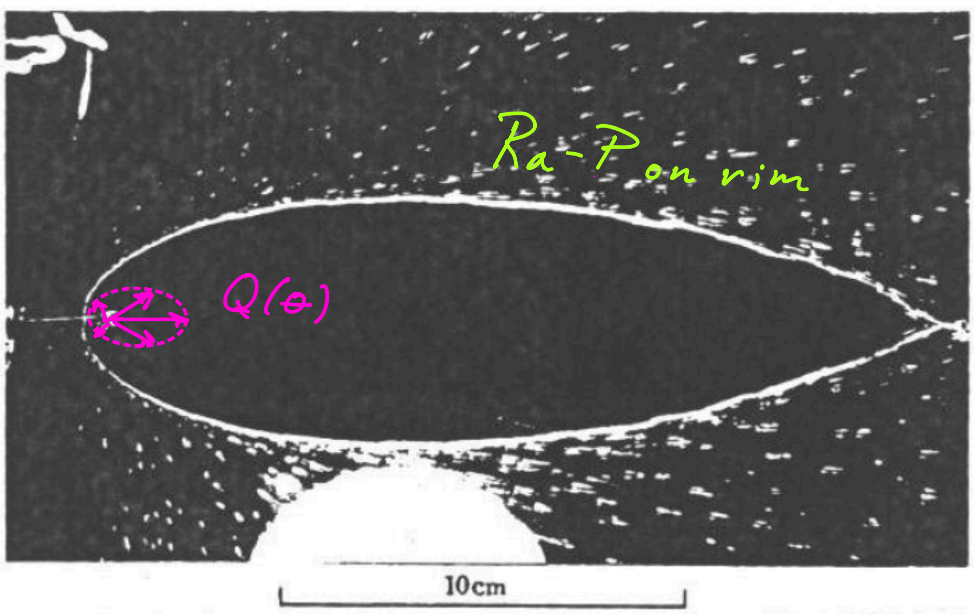
Note : 1. mass released from rim via Ra-P

2. same forms generated by colliding jets

3. polygonal sheets arise for more viscous sheets

Formation of thin flat sheets of water (Taylor 1960)

Case 1 : Unstable rims (eject droplets)



Sheet shape set by balance of curvature + inertia :

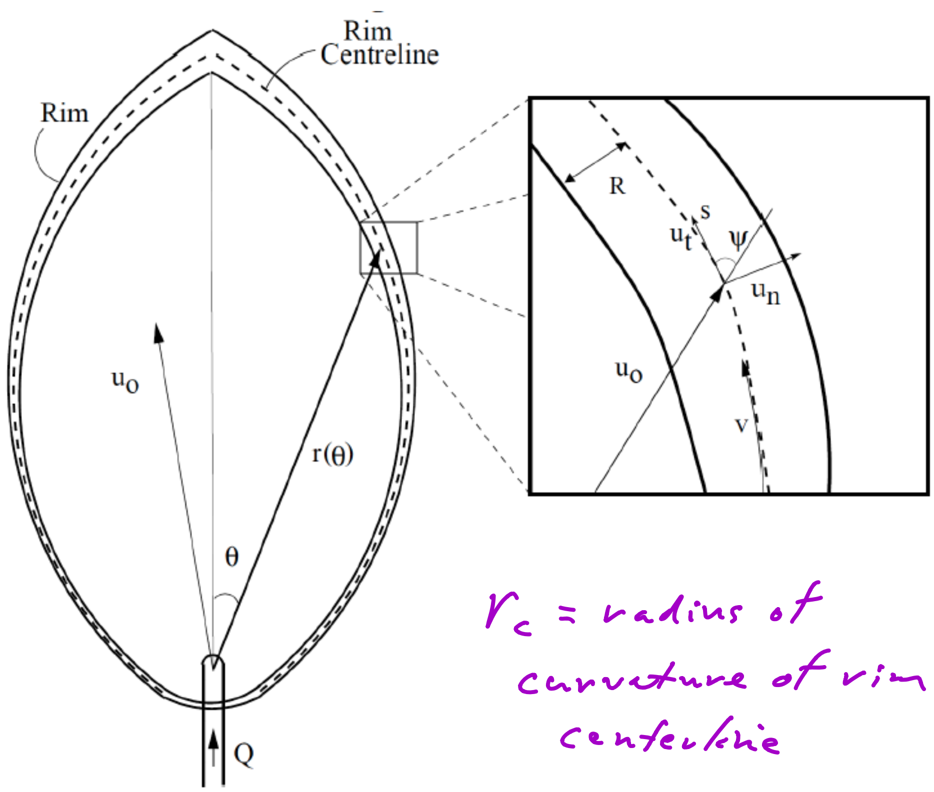
$\rho u_n^2 h \sim 2\sigma$ where u_n = speed normal to rim

Sheet thickness : $h = \dfrac{Q(\theta)}{2\pi r u}$

Taylor radius : $\boxed{R_T(\theta) = \dfrac{\rho u_n^2 Q(\theta)}{4\pi\sigma u}}$ now a fn of θ

Case 2 : Stable rims

- arise with more viscous fluids
- when rims are stable, fluid enters the rim and proceeds along it
- there is a centripetal force on rim associated with flow along it
  $\Rightarrow$ must be considered in shape calculation



r_c = radius of curvature of rim centerline

Normal Force Balance on Rim:

$$\rho u_n^2 h + \frac{\rho \pi R^2 V^2}{r_c} = 2\sigma$$

INERTIA CENTRIPETAL CURVATURE

Continuity in Rim:

$$0 = u_n h - \frac{\partial}{\partial s}\left(V \pi R^2\right)$$

VOL FLUX FROM SHEET

TANG. GRADIENT IN VOL FLUX ALONG RIM

Tangential Force Balance

if $R_e \ll 1$

$$\frac{\partial}{\partial s}\left(\pi R^2 V^2\right) = \rho h u_t u_n - \cancel{\pi R^2 \sigma \frac{\partial}{\partial s}\overbrace{(\nabla \cdot n)}^{\frac{1}{R}}} + \cancel{3\mu \frac{\partial}{\partial s}\left(R^2 \frac{dV}{ds}\right)}$$

TANG GRAD. IN TANG MOM FLUX

TANG MOM FLUX FROM SHEET

GRADIENT IN CURVATURE PRESSURE

VISCOUS RESISTANCE TO STRETCHING

Continuity in sheet:

$$h(r,\theta) = \frac{Q(\theta)}{u_0 r}$$

Geometry:

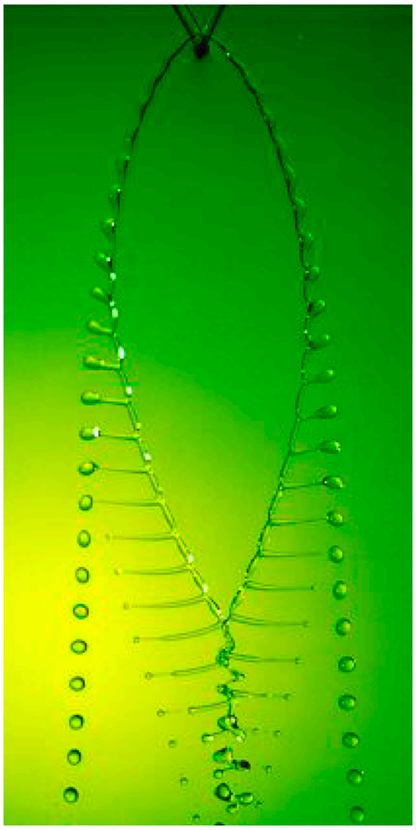
$$u_r = u_0 \sin\psi, \quad u_t = u_0 \cos\psi$$

$$\frac{1}{r_c} = \frac{\sin\psi}{r}\left(\frac{d\psi}{d\theta} + 1\right)$$

- this system may be nondimensionalized and reduced to a set of coupled ODEs in the four variables $r(\theta)$, $V(\theta)$, $R(\theta)$, $\psi(\theta)$.
- given $Q(\theta)$, these may be integrated to deduce the sheet shape

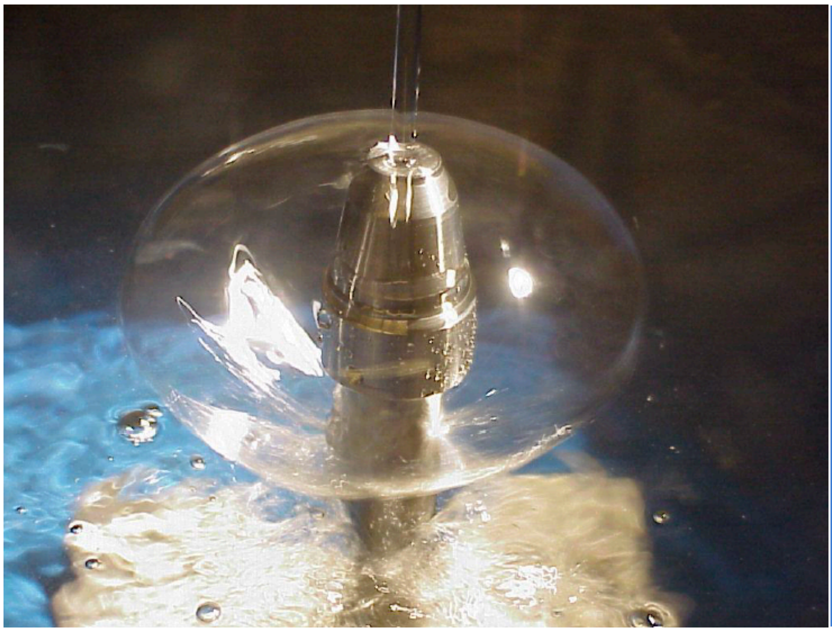
Fluid Fishbones

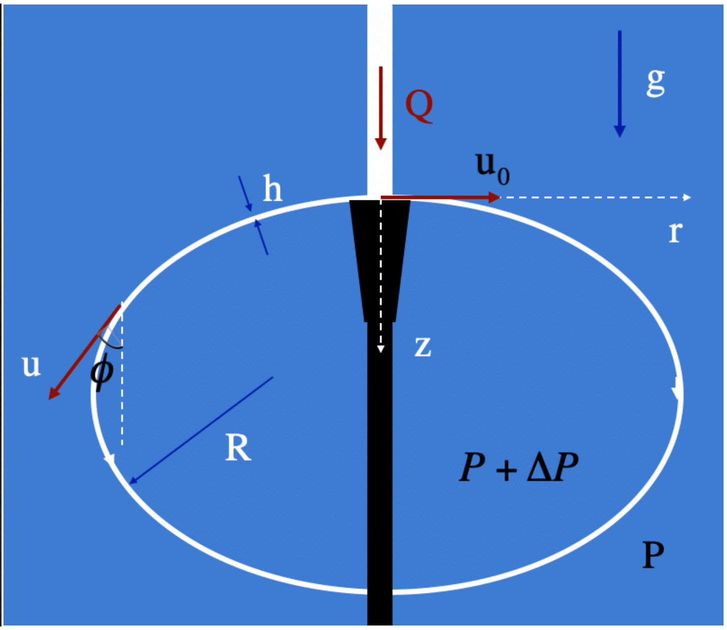
- arise when timescale of Ra-P of the rim is comparable to time spent in the rim
- Ra-P sets in ⇒ bulbous regions flung outwards
  ⟹ "fishbones"
- bones pinch off via Ra-P



Water Bells (Savart 1833, Taylor 1959, Clanet)

- gravity causes sheets to sag ⇒ curvature causes sheet to close





Bell Shape : prescribed by balance of inertia, gravity + curvature

Continuity : $Q = 2\pi r\, h(r)$

Energetic conservation : $u^2 = 2gz + u_0^2$

Sheet curvature : $\underline{\nabla} \cdot \underline{n} = \underbrace{\frac{1}{R}}_{\text{IN PLANE}} + \underbrace{\frac{\cos\phi}{r}}_{\text{OUT-OF-PLANE}}$

Normal force balance :

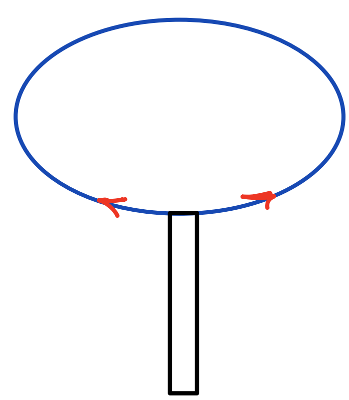
$$\underbrace{\frac{2\sigma}{R} + \frac{2\sigma\cos\phi}{r}}_{\text{CURVATURE}} + \underbrace{\rho g h \sin\phi}_{\text{GRAVITY}} = \Delta P + \underbrace{\frac{\rho h u^2}{R}}_{\text{INERTIA}}$$

Note: 1. bell closes due to out-of-plane curvature

2. influence of g reflected in top-bottom asymmetry

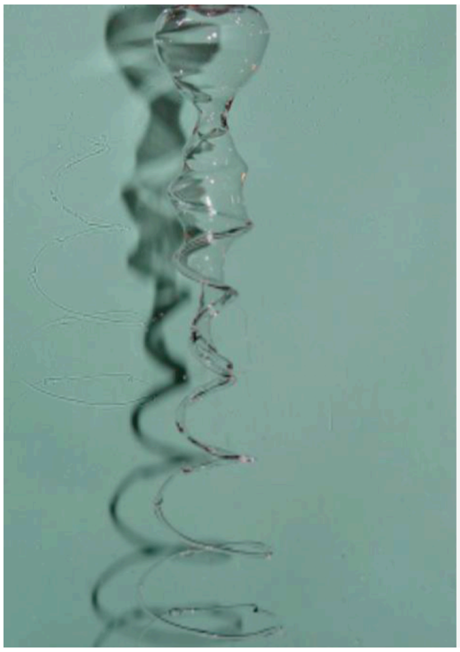
Recall: $Fr = \frac{\text{INERTIA}}{\text{GRAVITY}} \sim \frac{U^2}{gL} \sim 1$

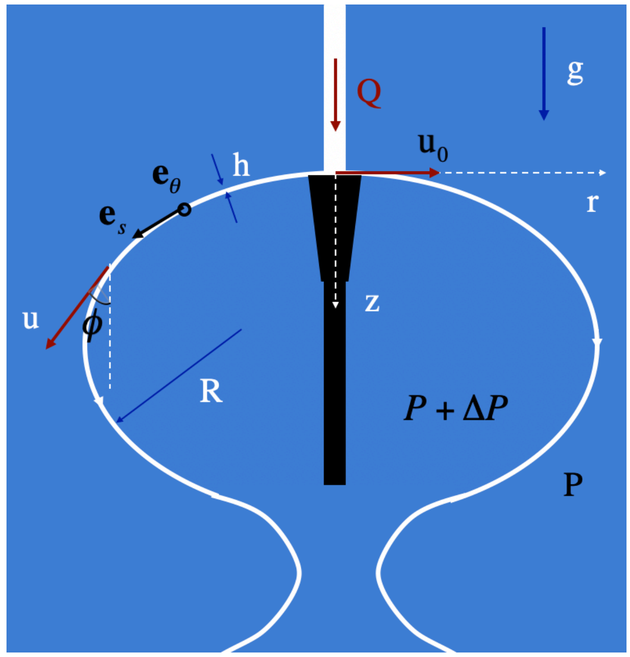
3. if $Fr \gg 1$ and sheet deflected upwards $\Rightarrow$ bell will still close on itself



4. viscous effect cause various exotica

Swirling Water Bells : formed from a swirling jet





Why don't they close ?

Sheet velocity : $\underline{V} = u\,\hat{e}_s + V\,\hat{e}_\theta$

SWIRL

Continuity : $Q = 2\pi r h u$

Conservation of Ang. Mom : $V r = V_0 r_0$

Energy Conservation : $u^2 + V^2 = 2gz + u_0^2 + V_0^2$

Normal Force Balance :

$$\frac{2\sigma}{R} + \frac{2\sigma\cos\phi}{r^2} + \rho g h \sin\phi = \Delta P + \frac{\rho h u^2}{R} + \rho\frac{h V^2 \cos\phi}{r}$$

CENTRIPETAL

$\Longrightarrow$ bell fails to close owing to centripetal force becoming large as $r \to 0$