18.357: Lecture 8

Capillary rise continued

Marangoni Flows I: Thermocapillarity

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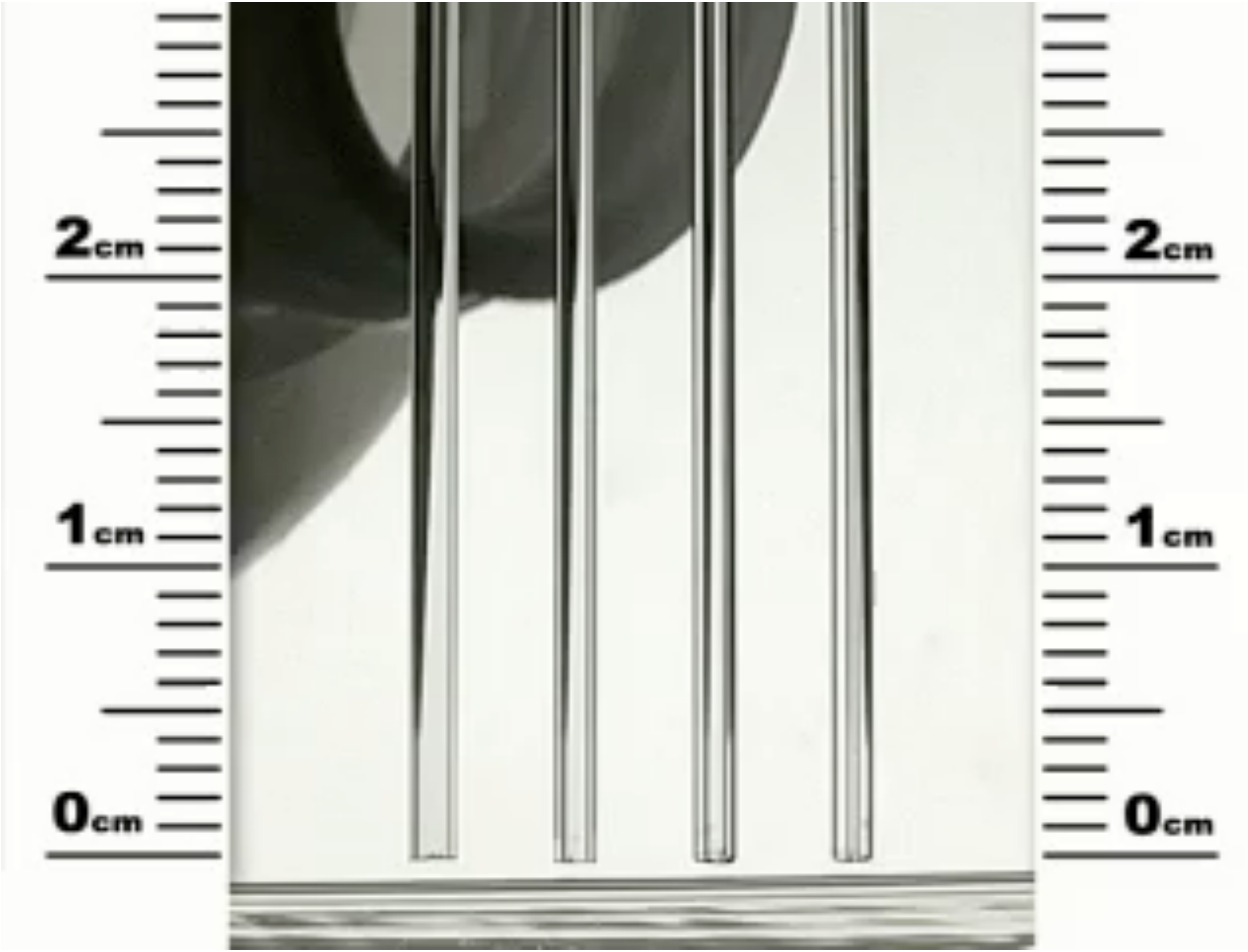
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Governing Equations

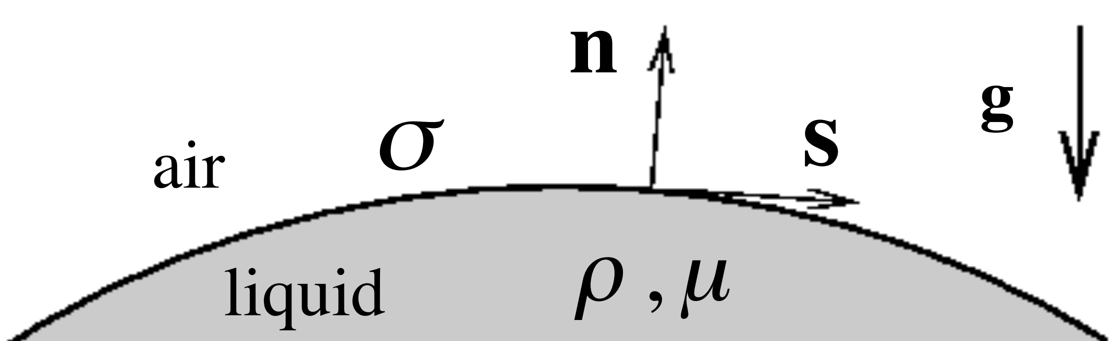
Navier-Stokes equations:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \rho\mathbf{g} + \mu\nabla^2\mathbf{u} \quad , \qquad \nabla\cdot\mathbf{u} = 0$$

Boundary Conditions

Normal stress: $\Delta\ \mathbf{n}\cdot\mathbf{T}\cdot\mathbf{n} = \sigma\,\nabla\cdot\mathbf{n}$

Tangential stress: $\Delta\ \mathbf{n}\cdot\mathbf{T}\cdot\mathbf{s} = \nabla_s\sigma$



Stress tensor

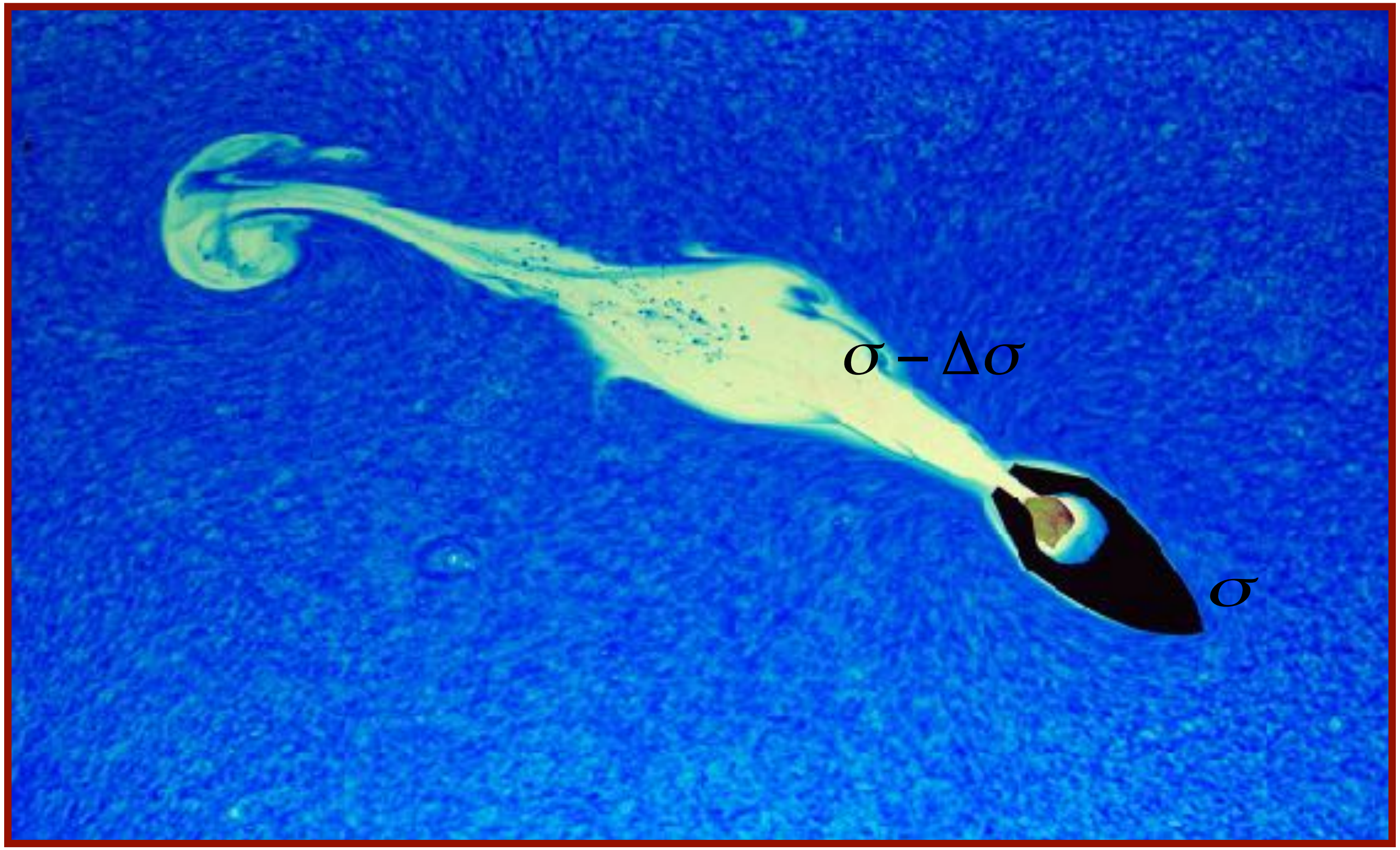
$$\mathbf{T} = -p\mathbf{I} + \mu\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right)$$

Marangoni Flows

- flows dominated by the influence of surface tension gradients

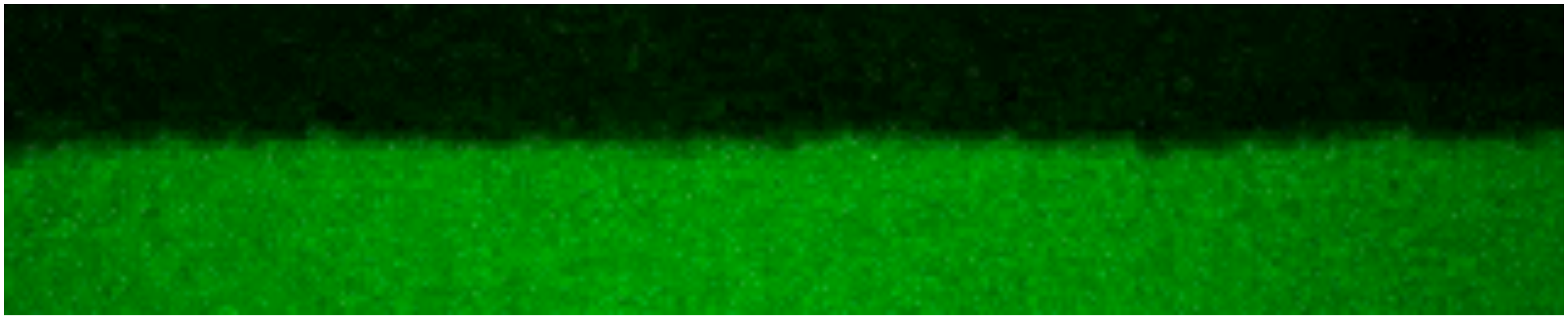
Recall tangential stress BC: $\Delta\, \mathbf{n}\cdot\mathbf{T}\cdot\mathbf{s} = \nabla\sigma$

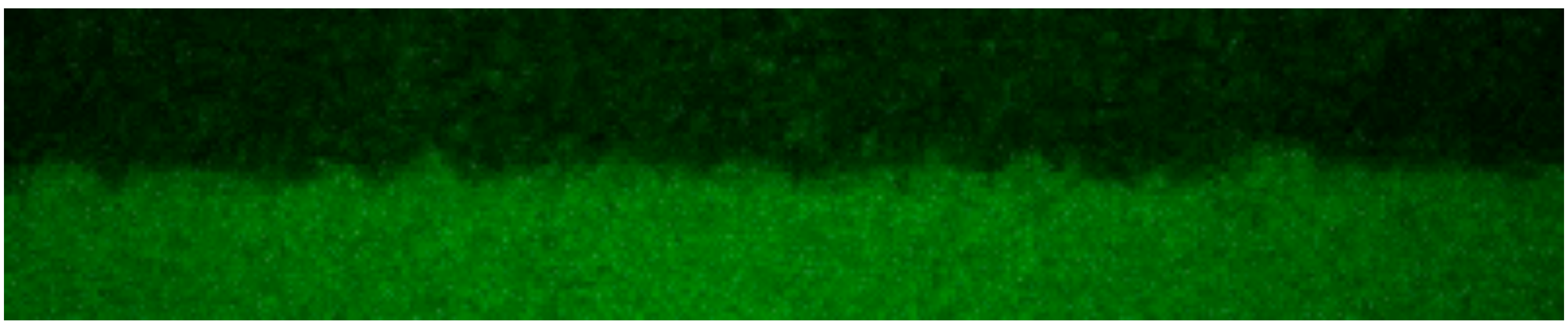
- $\nabla\sigma$ may arise due to dependence of $\sigma(T, c, \Gamma)$



Thermocapillary flows

- Marangoni flows induced by temperature gradients $\sigma(T)$





- surface tension decreases at higher temperature since gas phase then has more liquid phase molecules

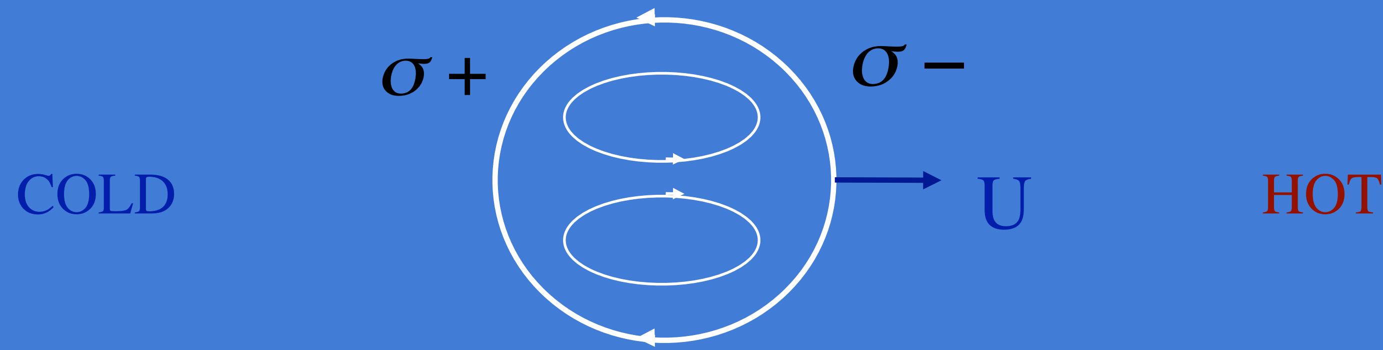
Thermocapillary Flows

- Marangoni flows induced by temperature gradients $\sigma(T)$
- Navier-Stokes coupled to heat equation through BCs

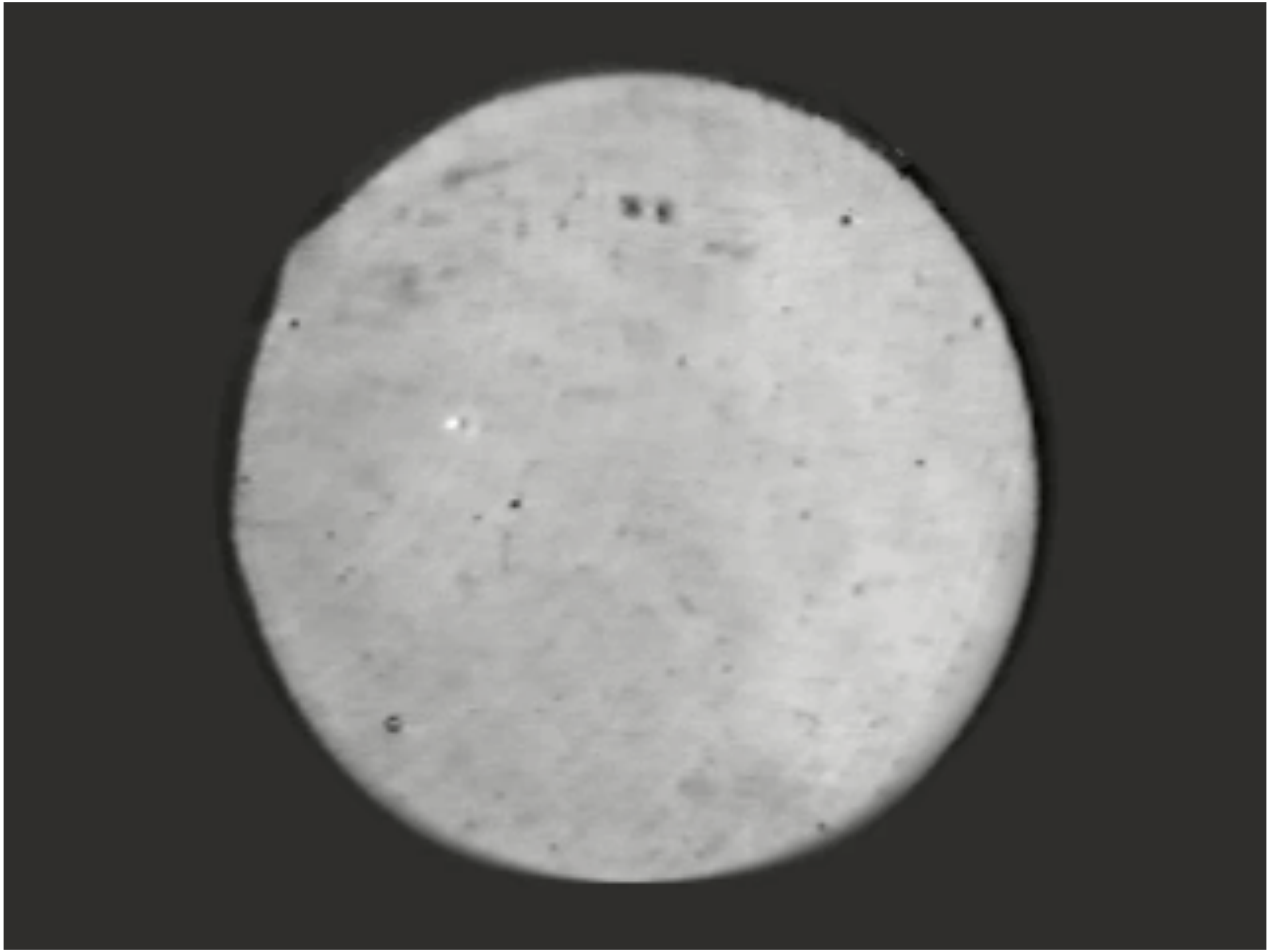
$$\frac{\partial T}{\partial t} + u \cdot \nabla T = \kappa \nabla^2 T$$

e.g. thermocapillary drop motion: drops migrate toward heat source

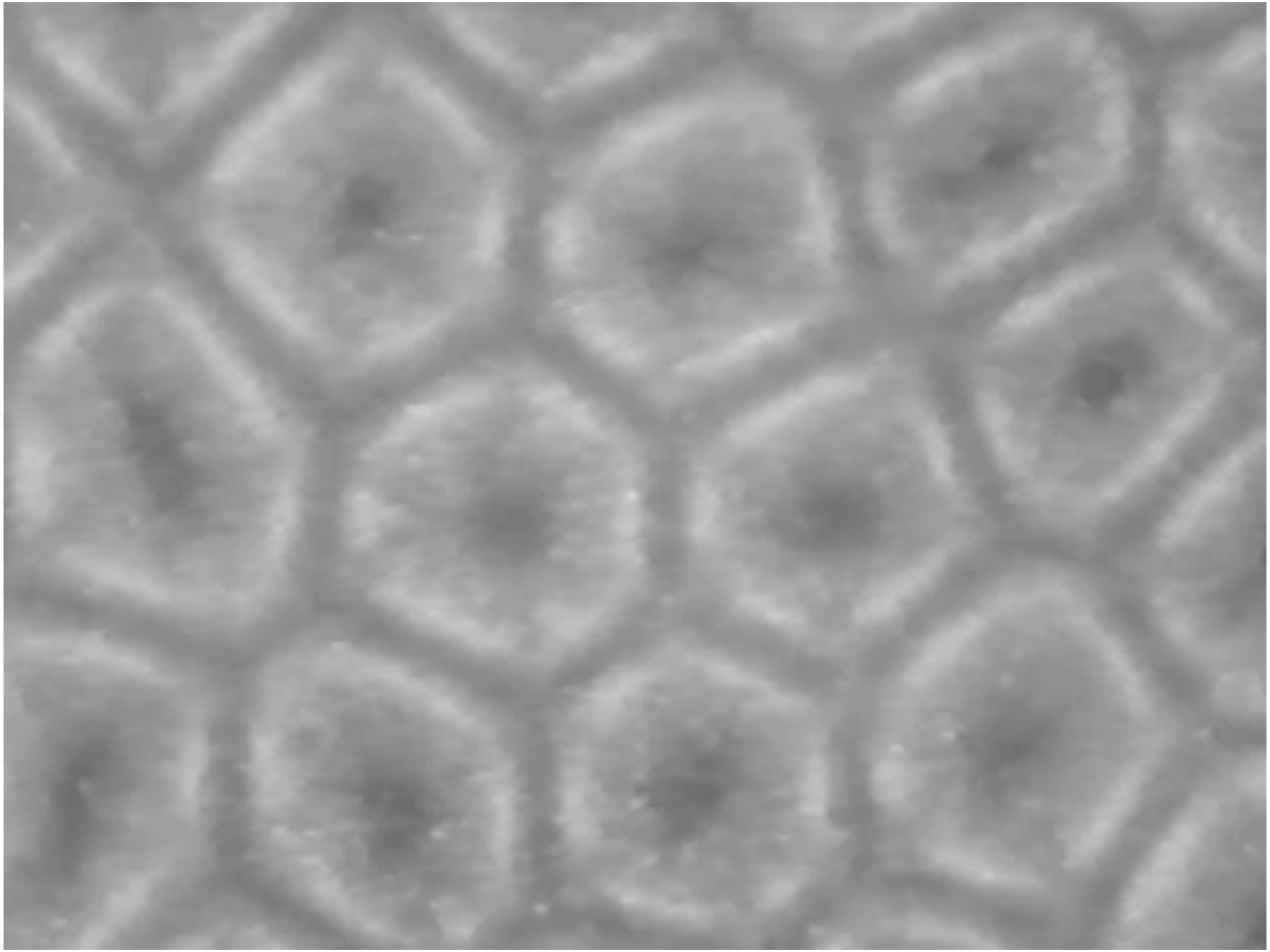
Young, Goldstein & Block (1962)



Thermal convection in a plane layer

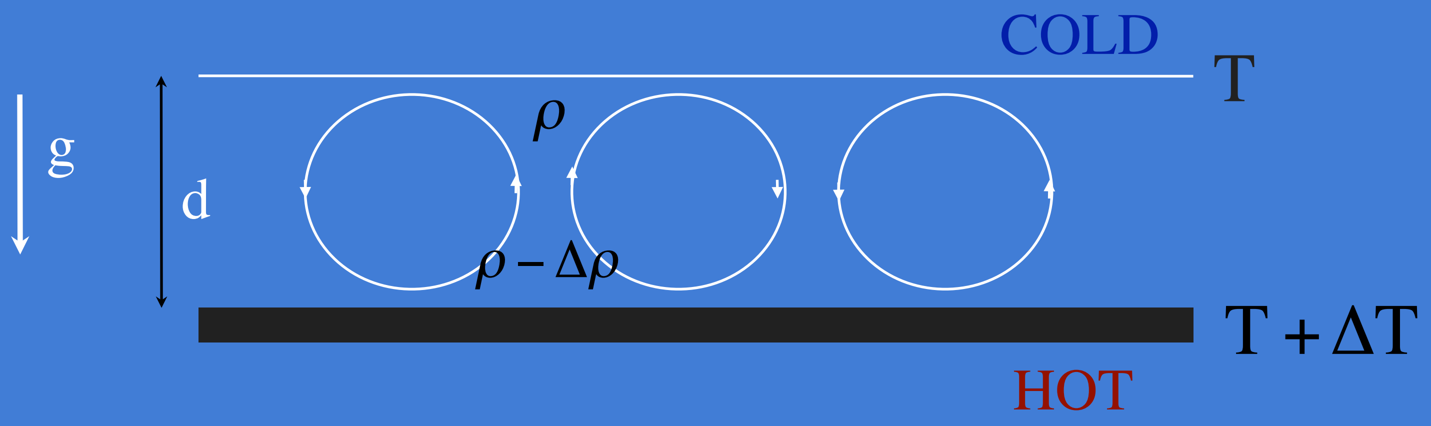


Thermal convection in a plane layer



THERMAL CONVECTION

Rayleigh-Benard $\rho(\mathrm{T}) = \rho_0\left[1 + \alpha(\mathrm{T} - \mathrm{T}_0)\right]$

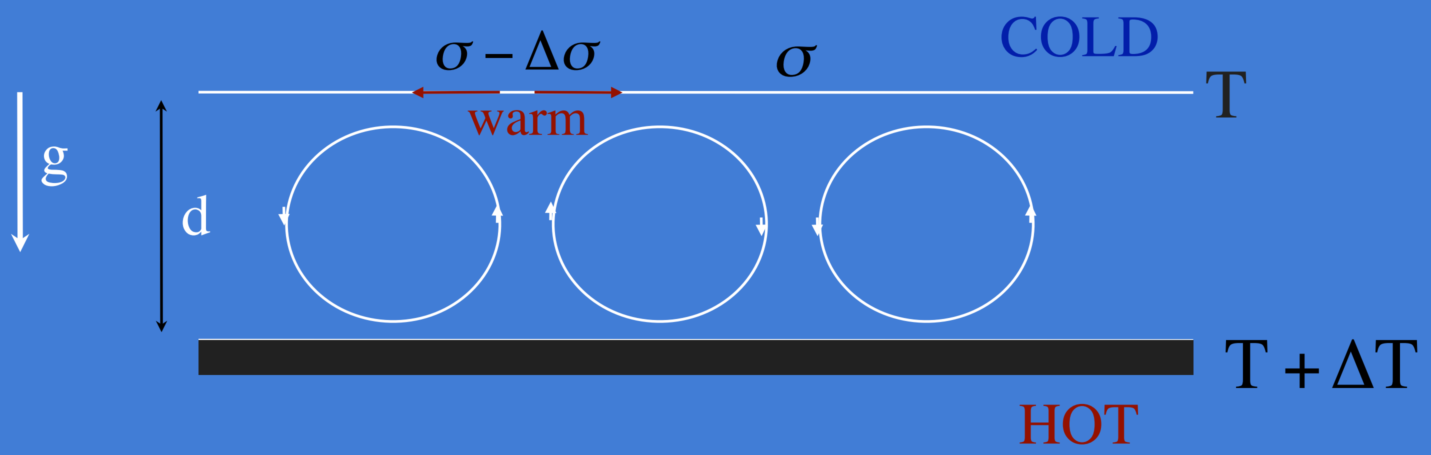


Stability prescribed by:

$$Ra = \frac{g\alpha\Delta\mathrm{T}\,\mathrm{d}^3}{\kappa\nu}$$

Rayleigh number

Marangoni-Benard $\sigma(\mathrm{T}) = \sigma_0 - \Gamma(\mathrm{T} - \mathrm{T}_0)$



$$Ma = \frac{\Gamma\Delta\mathrm{T}\,\mathrm{d}}{\kappa\mu}$$

Marangoni number

Note: Marangoni convection dominates for thin films