Lecture 6

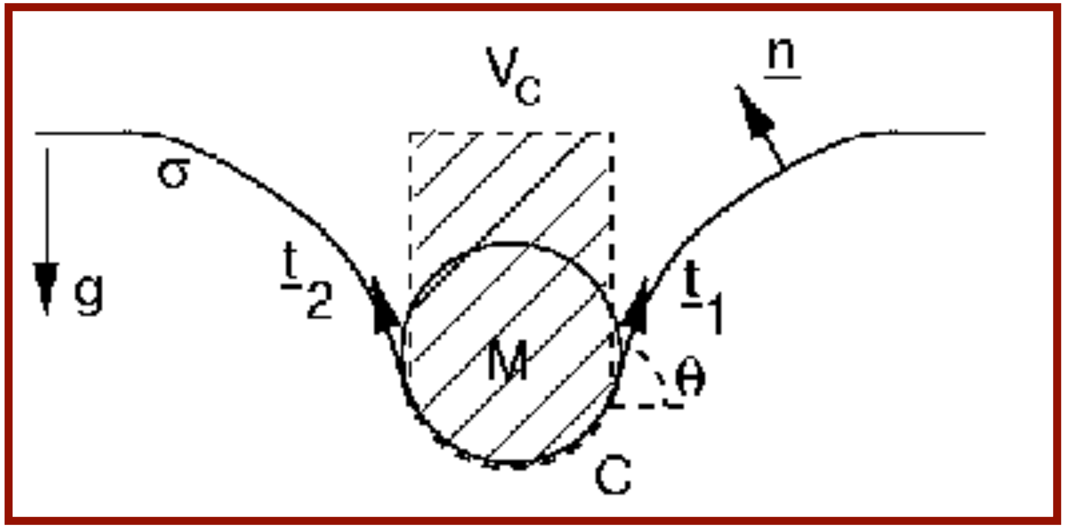
Floating bodies, capillary forces, rotating drops

On floating bodies

Heavy things sink, light things float.

Not exactly…..

Statics of floating bodies



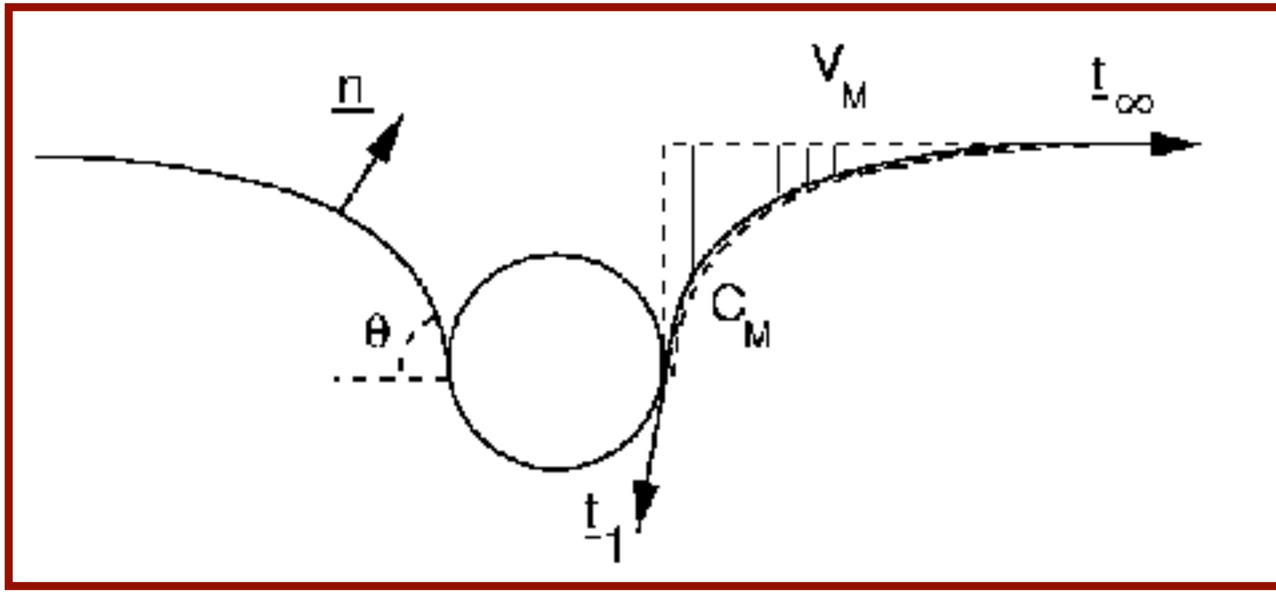
Force balance on body:

$$Mg = \hat{\mathbf{z}} \cdot \int_C -p\,\mathbf{n}\,d\ell + 2\sigma \sin\theta$$

Buoyancy: $F_b = \int_C \rho\, g\, z\, (\hat{\mathbf{n}} \cdot \hat{\mathbf{z}})\, d\ell = \rho\, g\, V_c$

Surface tension: $F_C = 2\sigma \sin\theta$

Force balance on meniscus



$$0 = \int_{C_M} -p\, \hat{\mathbf{n}} \cdot \hat{\mathbf{z}}\, d\ell = F_b + F_c$$

where
$$F_b = \int_{C_M} \rho\, g\, z\, \hat{\mathbf{n}} \cdot \hat{\mathbf{z}}\, d\ell = \rho\, g\, V_M$$

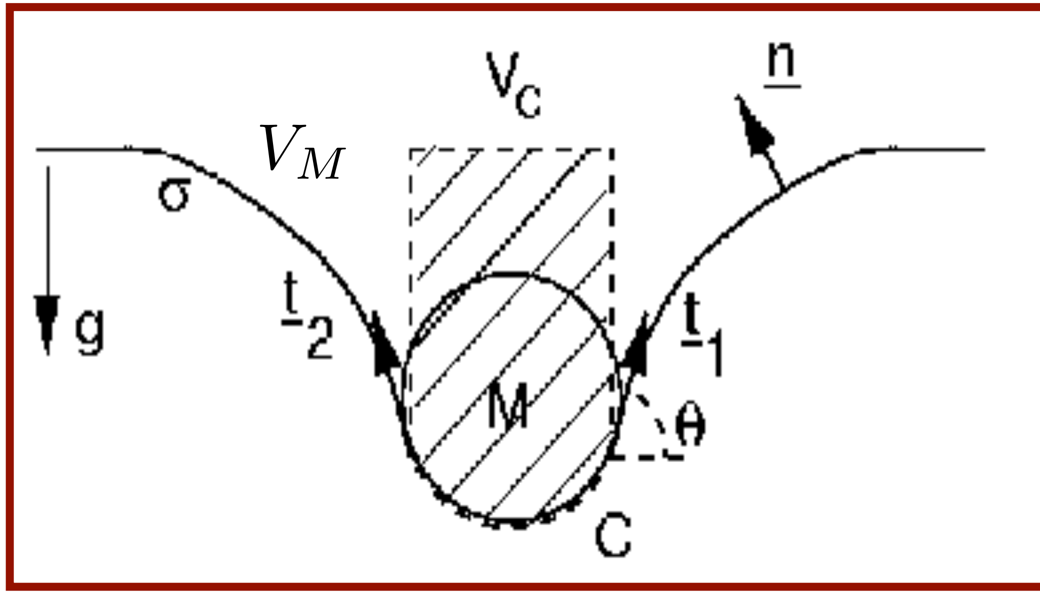
$$F_C = \int_{C_M} \sigma\, (\nabla \cdot \hat{\mathbf{n}})\, (\hat{\mathbf{n}} \cdot \hat{\mathbf{z}})\, d\ell = 2\sigma\left(\hat{\mathbf{t}}_\infty - \hat{\mathbf{t}}_1\right) \cdot \hat{\mathbf{z}} = -2\sigma \sin\theta$$

via Frenet-Serret

Generalized Archimedes Principle

- the weight of a floating body still equals that of the displaced fluid

$$Mg = F_B + F_C$$



Buoyancy: $$F_B = \rho g V_C$$

= wt of fluid displaced above body

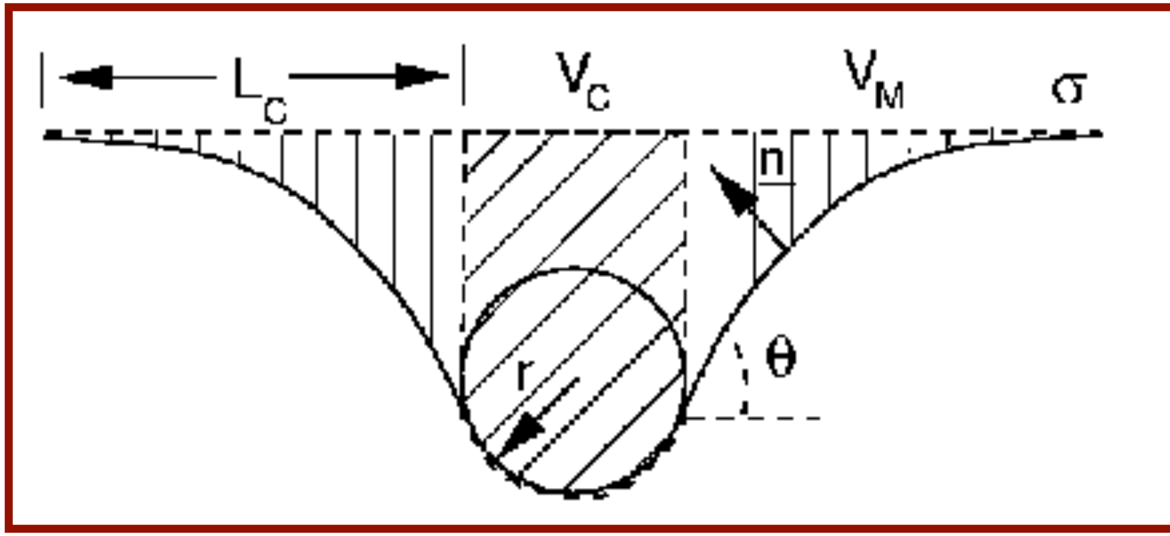
Surface tension: $$F_C = 2\sigma \sin\theta = \rho g V_M$$

= wt of fluid displaced above meniscus

Weight support:

statics of floating bodies

- **J. Keller (1998)**



$\Rightarrow$ $F_b = \rho\, g\, V_c =$ wt. of fluid displaced above body

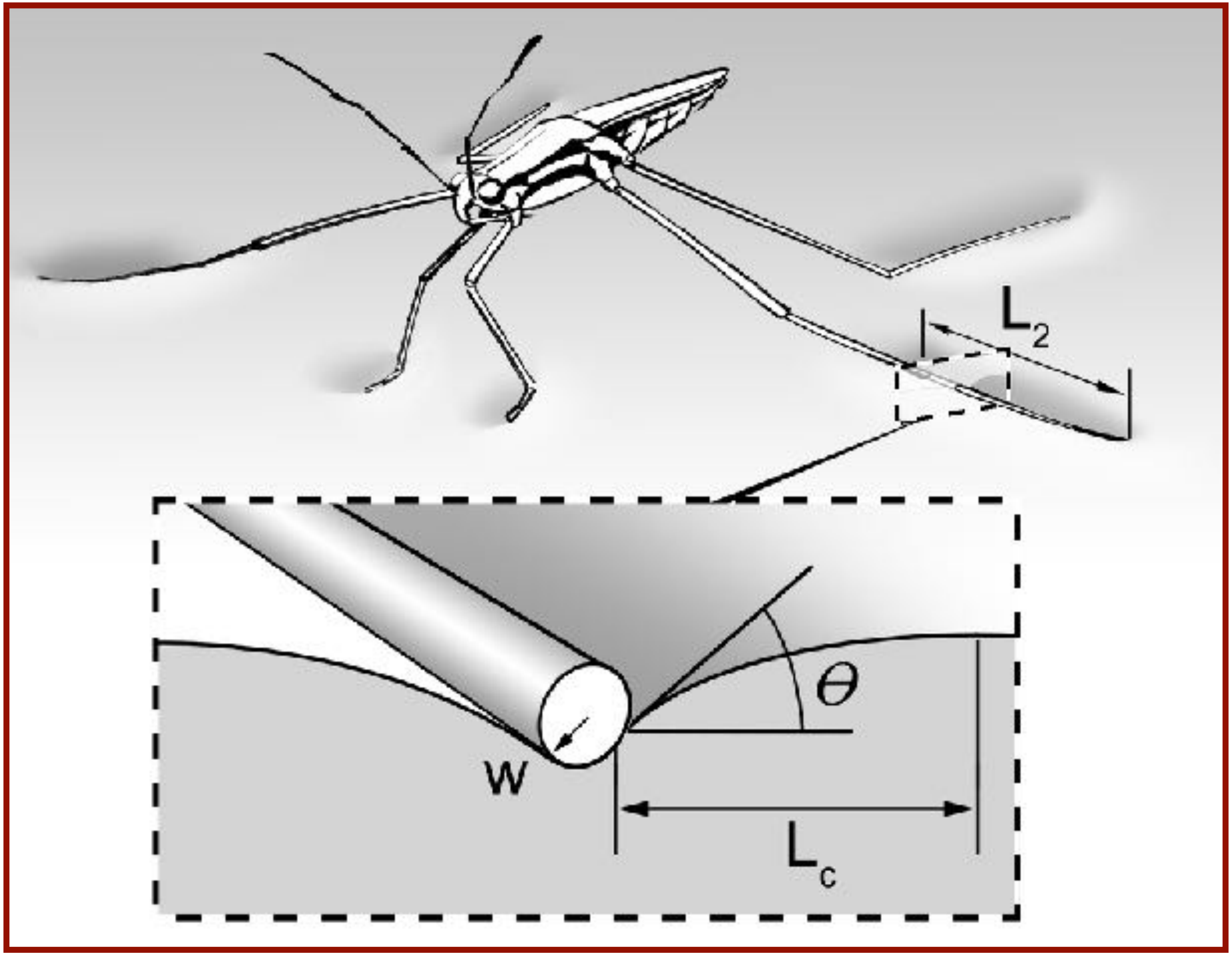
$\Rightarrow$ $F_c = 2\sigma \sin\theta = \rho g V_M =$ wt. of fluid above meniscus

$$\Rightarrow \boxed{\frac{F_b}{F_c} = \frac{V_c}{V_M} \approx \frac{r}{L_c}}$$

where $L_c = \left(\frac{\sigma}{\rho g}\right)^{1/2} \approx 0.3\ \text{cm}$

$\Rightarrow$ **small creatures (eg. insects) supported principally by σ**

Capillary forces support the weight of water-walking insects.



Static weight support requires: $$Mg \ < \ 2\sigma P \sin\theta$$

where P is total contact length

Water striders

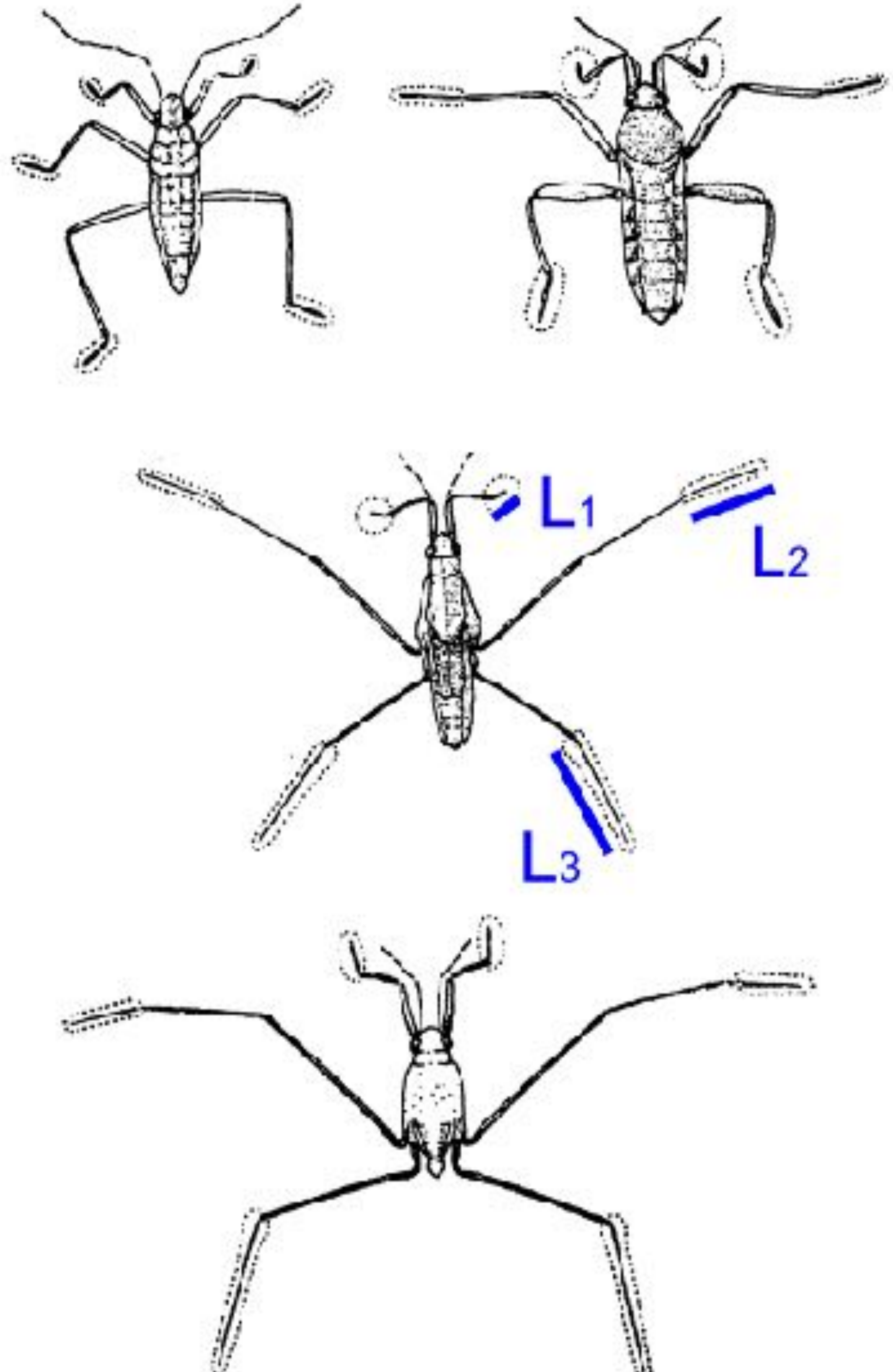
$$F_s = 2\sigma P$$

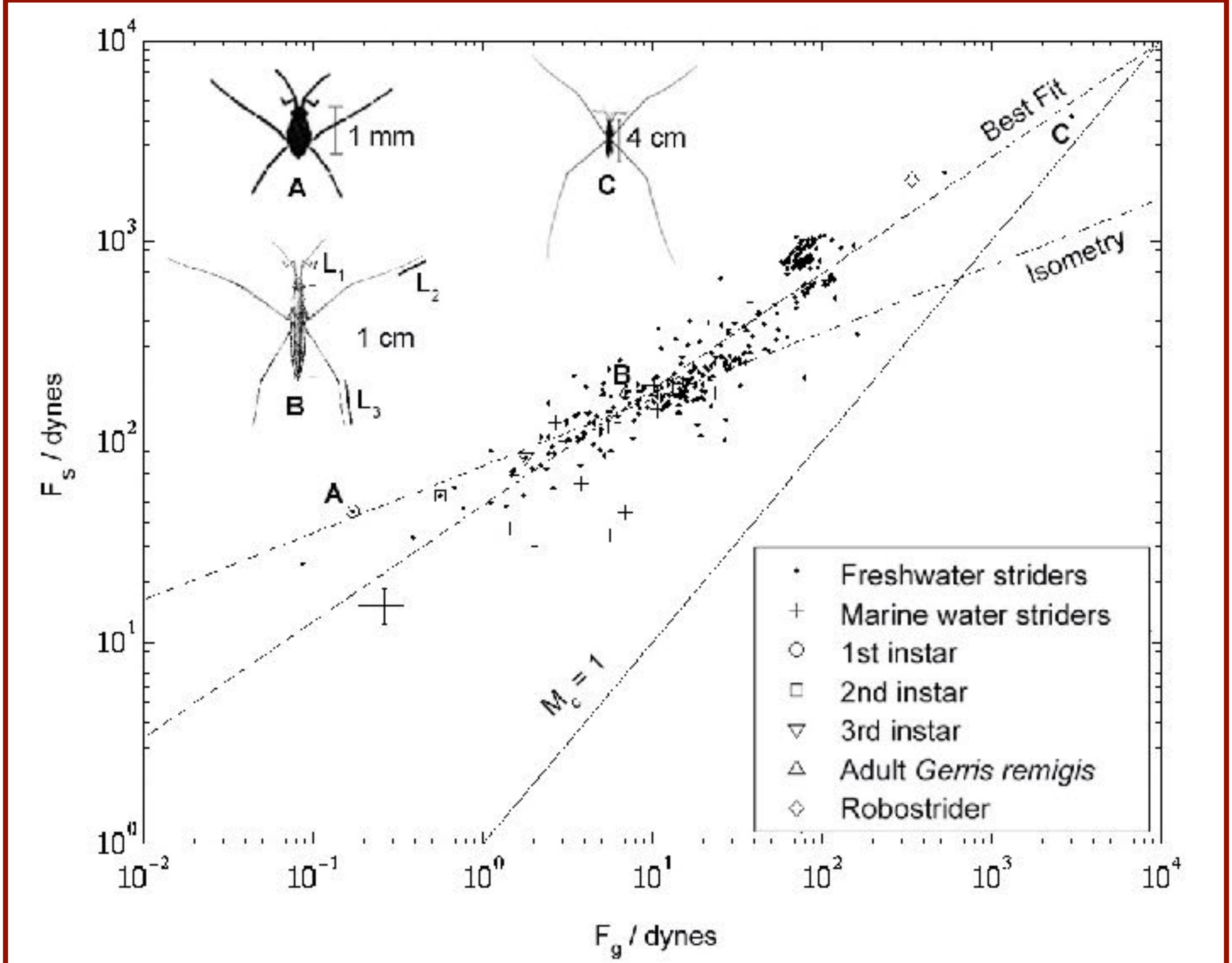
$$F_g = Mg$$

$$P = 2(L_1 + L_2 + L_3)$$

What is $F_s(F_g)$?

i.e. what is the dependence of form on size?



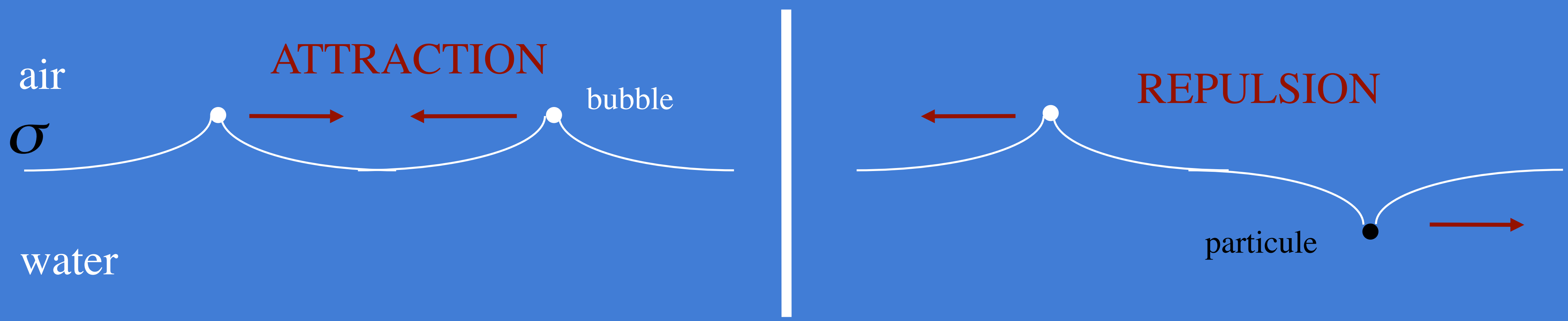


Hu, Chan & Bush (Nature, 2005)

Capillary forces

Capillary forces

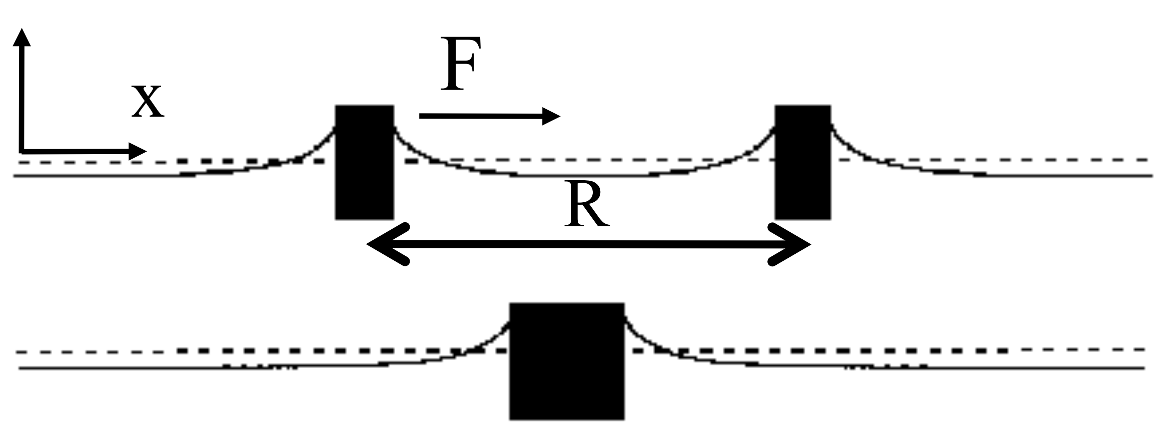
- exist by virtue of the interaction of the menisci of floating bodies
- attractive/repulsive if the menisci are of the same/opposite sense



- explains the formation of bubble rafts on champagne
- explains the mutual attraction of Cheerios, and their attraction to the walls
- utilized for self-assembly on the microscale

Capillary attraction

$$E_{Tot} = E_S + GPE$$

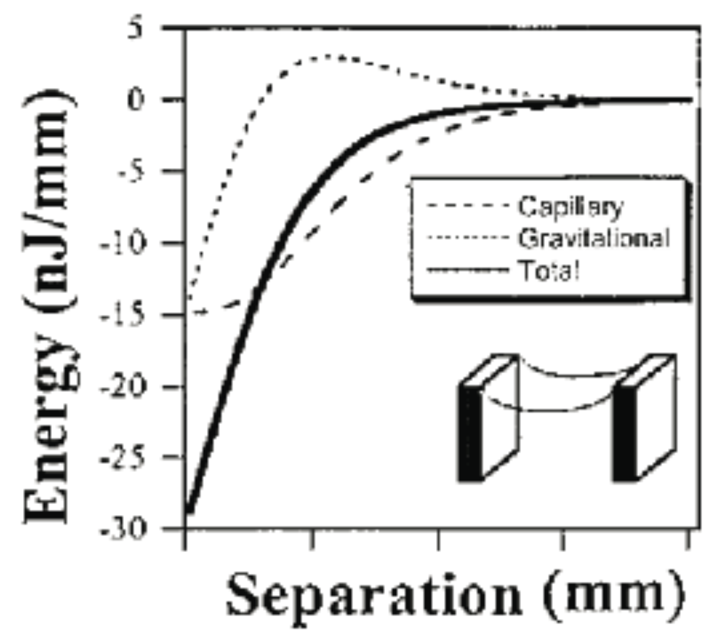


$$E_S = \sigma A(R)$$

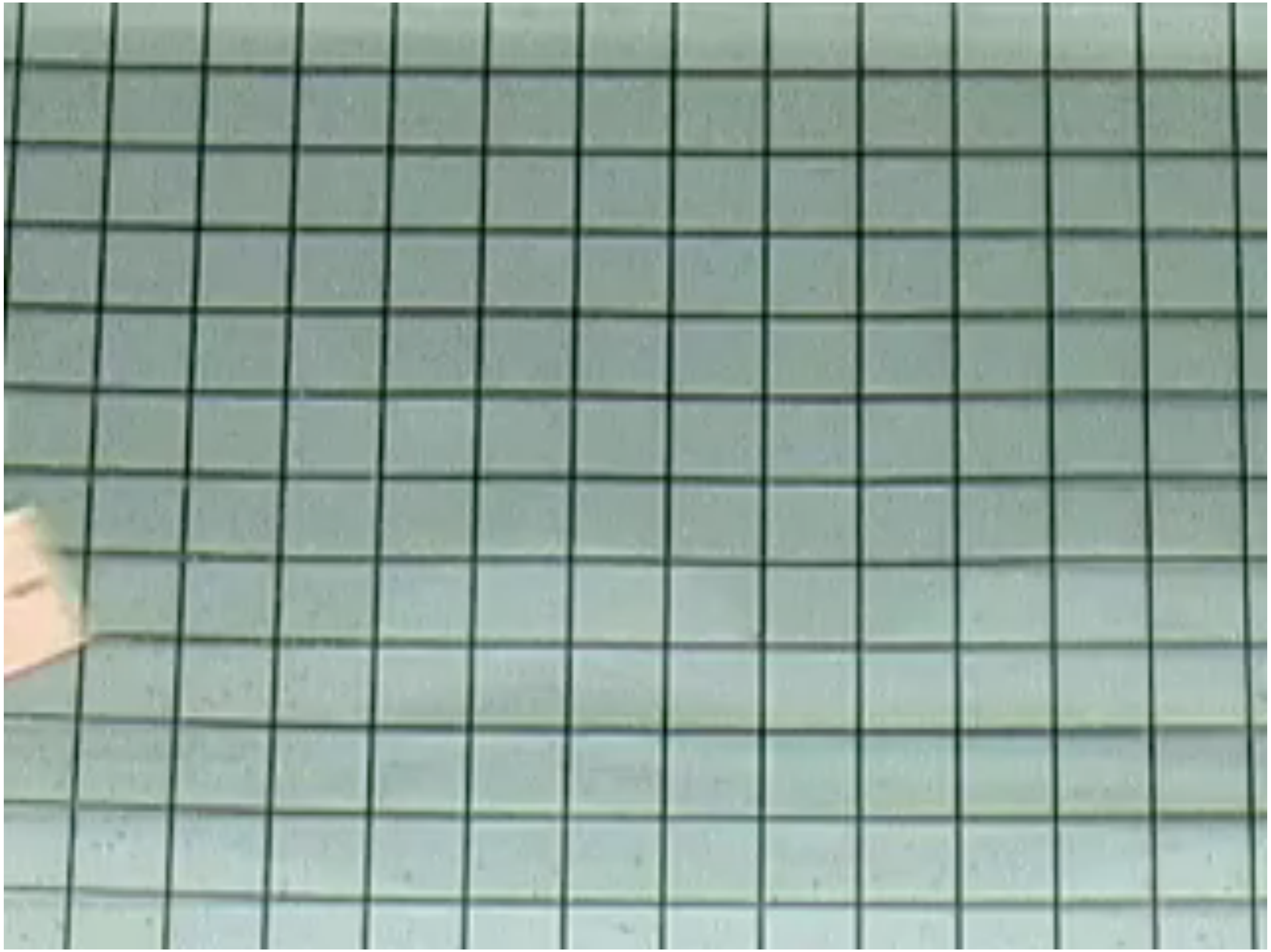
$$GPE(R) = \int_{x=-\infty}^{x=\infty} \int_{z=0}^{h(x)} \rho g \, dz \, dx$$

$$\Rightarrow \quad F(R) = -\frac{dE_{Tot}(R)}{dR}$$

Gryzbowski *et al* 2001



Floating copper



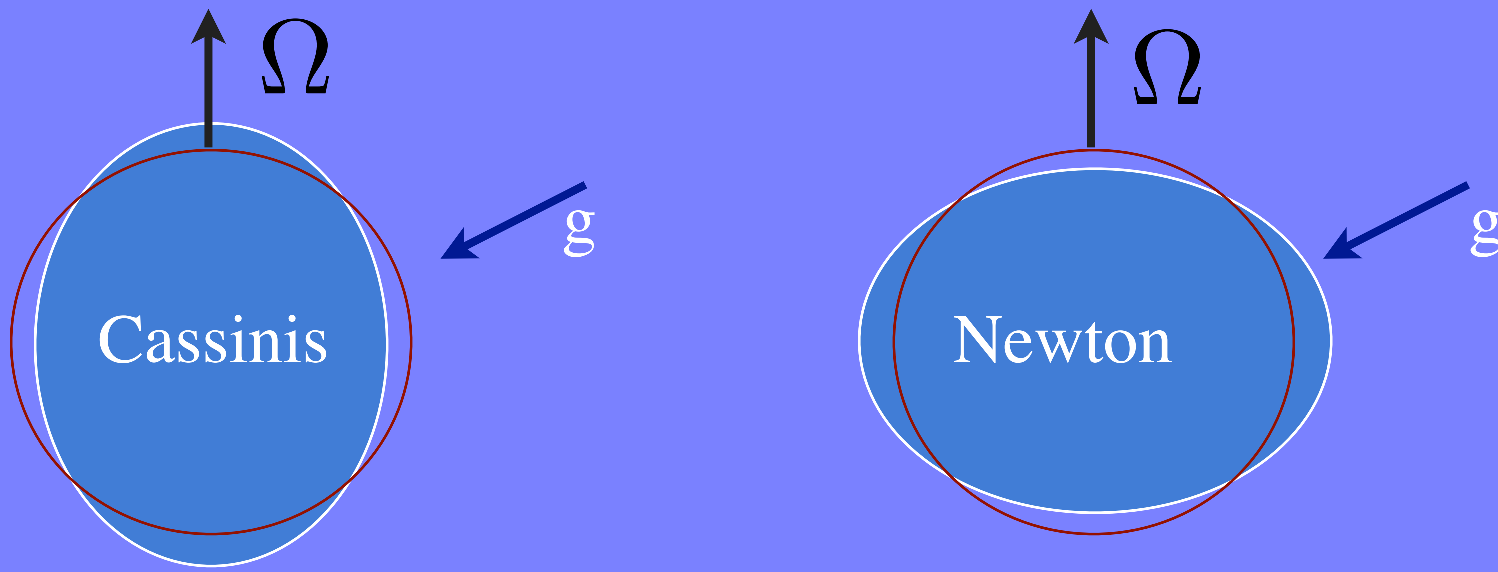
How does it float? Why the attractive force between floaters?

The courtship of the Anurida

On rotating fluid bodies

Plateau droplets

The shape of a rotating, self-gravitating body



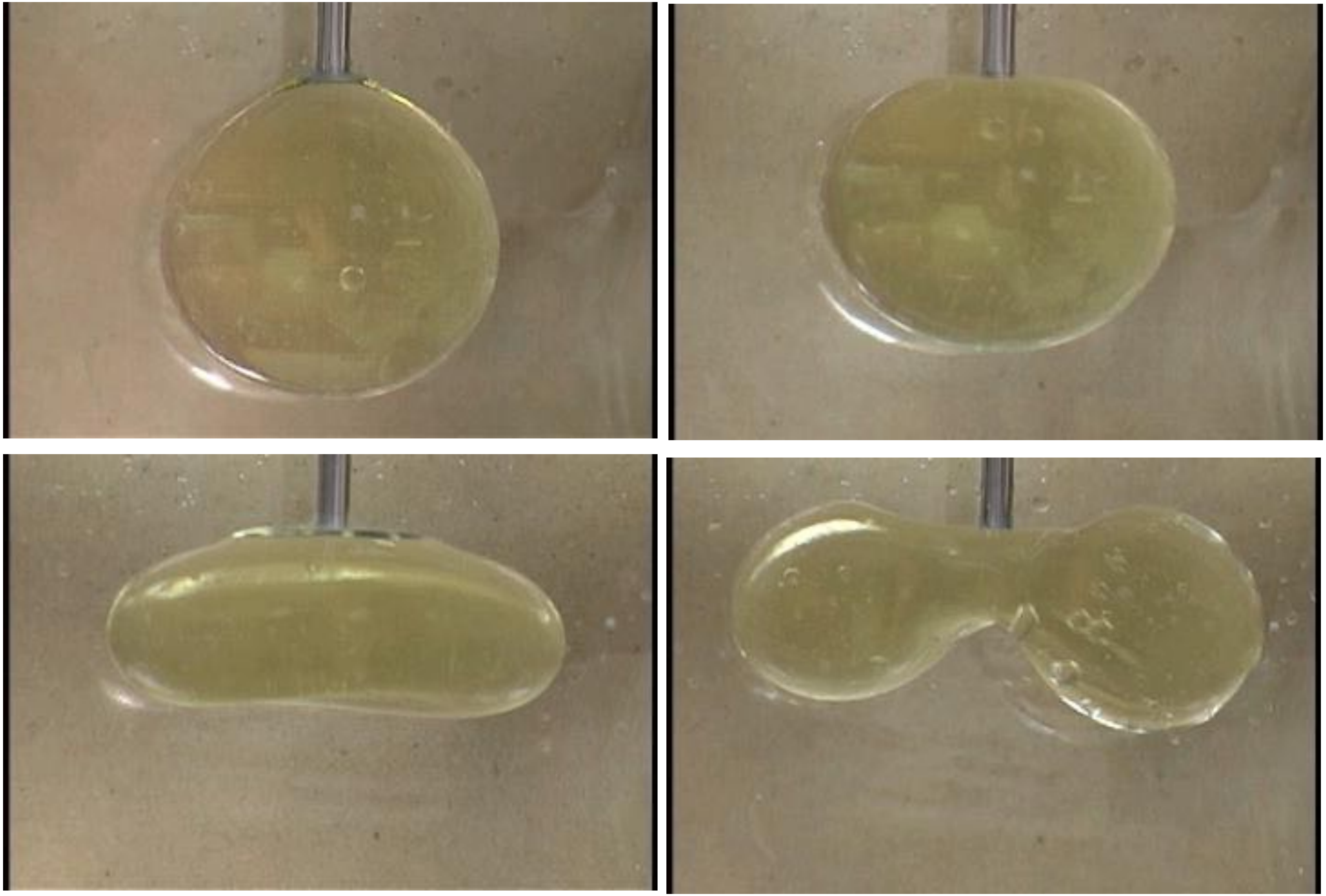
- debate was settled by geodetic measurements made in Lapland by Maupertuis, who was celebrated by Voltaire for having

  “hammered down the poles and the Cassinis”

- later, Voltaire and Maupertuis had a falling out (over a woman, Emilie du Chatelet), and the former taunted the latter:

  “You have gone to the ends of the Earth to confirm
  What Newton knew without leaving his home.”

Plateau's rotating drop experiments (1863)



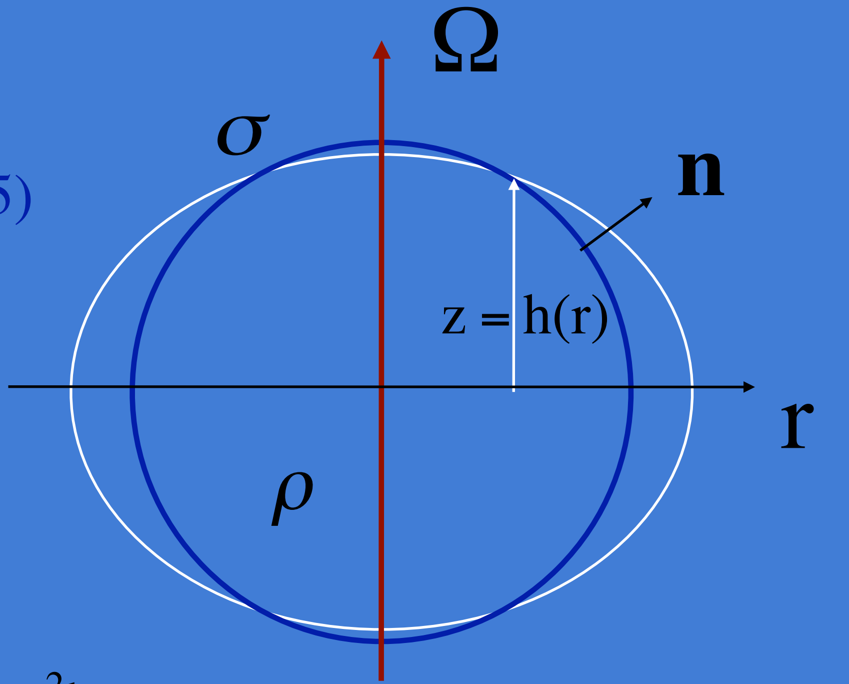
The confining role of self-gravitation is played by surface tension.

Rotating drops: a model of celestial bodies

Plateau (1863)
Chandrasekhar (1965)

Normal force balance

$$\Delta p + \tfrac{1}{2}\rho\Omega^2 r^2 = \sigma \nabla\cdot\mathbf{n}$$

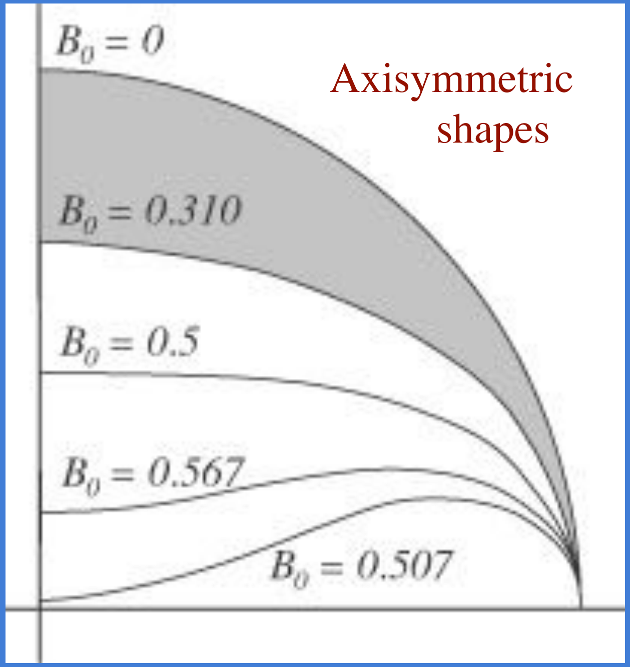


Define: $f(r,\theta) = z - h(r)$

so that $\mathbf{n} = \dfrac{\nabla f}{|\nabla f|} = \dfrac{\hat{\mathbf{z}} - h_r(r)\hat{\mathbf{r}}}{[1+h_r^2(r)]^{1/2}}$ $\quad \nabla\cdot\mathbf{n} = \dfrac{-rh_r - r^2 h_{rr}}{r^2(1+h_r^2)^{3/2}}$

$$\Delta P + 4B_0\,(r/a)^2 = \nabla\cdot\mathbf{n}$$

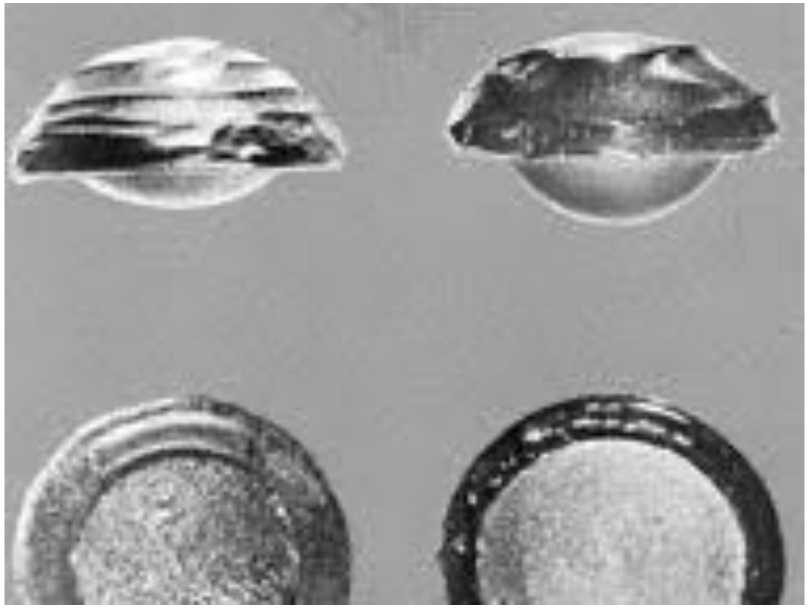
where $\Delta P = a\Delta p/\sigma$, $B_0 = \dfrac{\rho\Omega^2 a^3}{8\sigma} = \dfrac{\text{centrifugal}}{\text{curvature}}$



- for $0.09 < B < 0.31$, axisymmetric and lobed solutions possible
- for $B > 0.31$, only lobed forms obtain (Brown & Scriven, 1980)

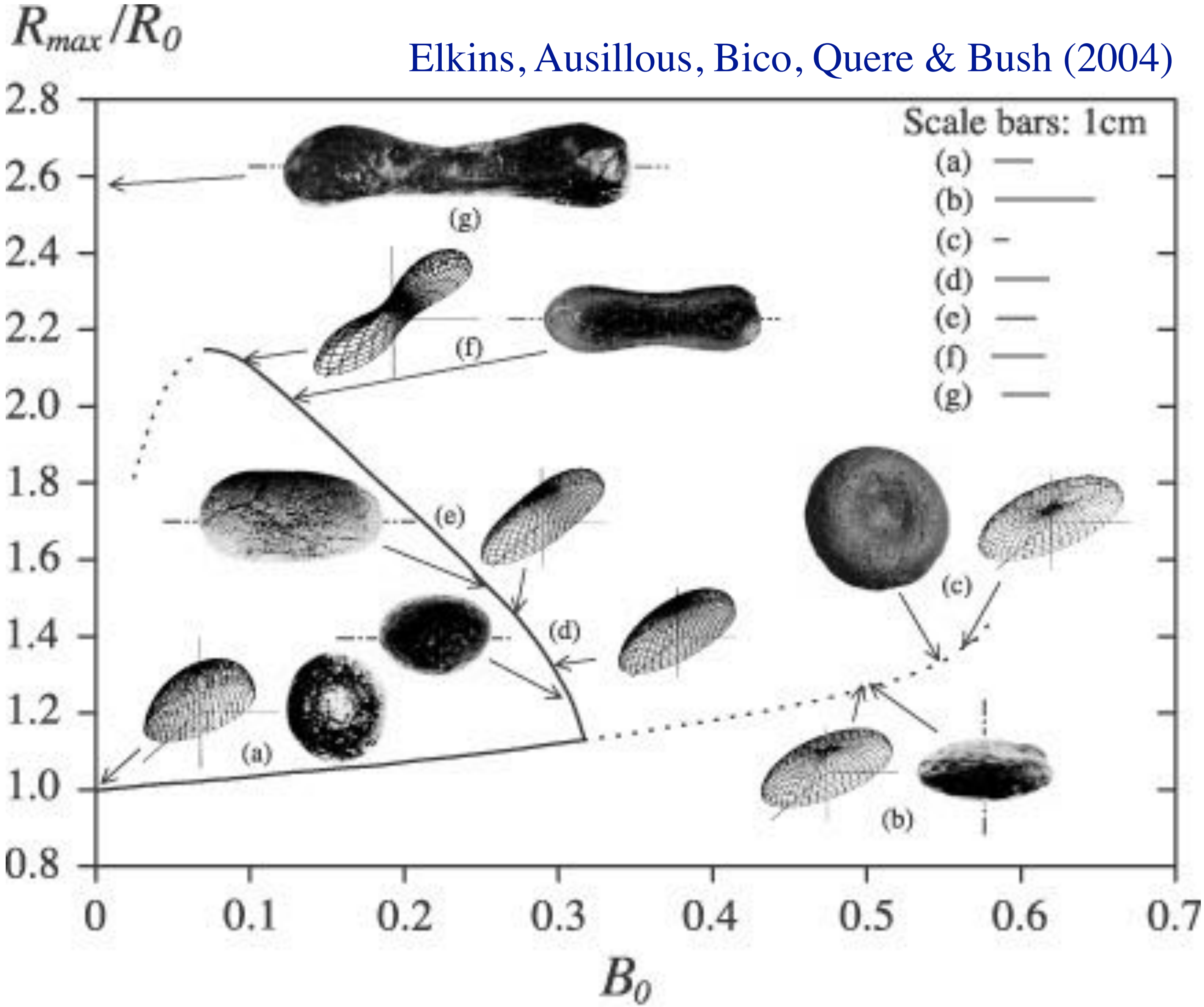
Tektites

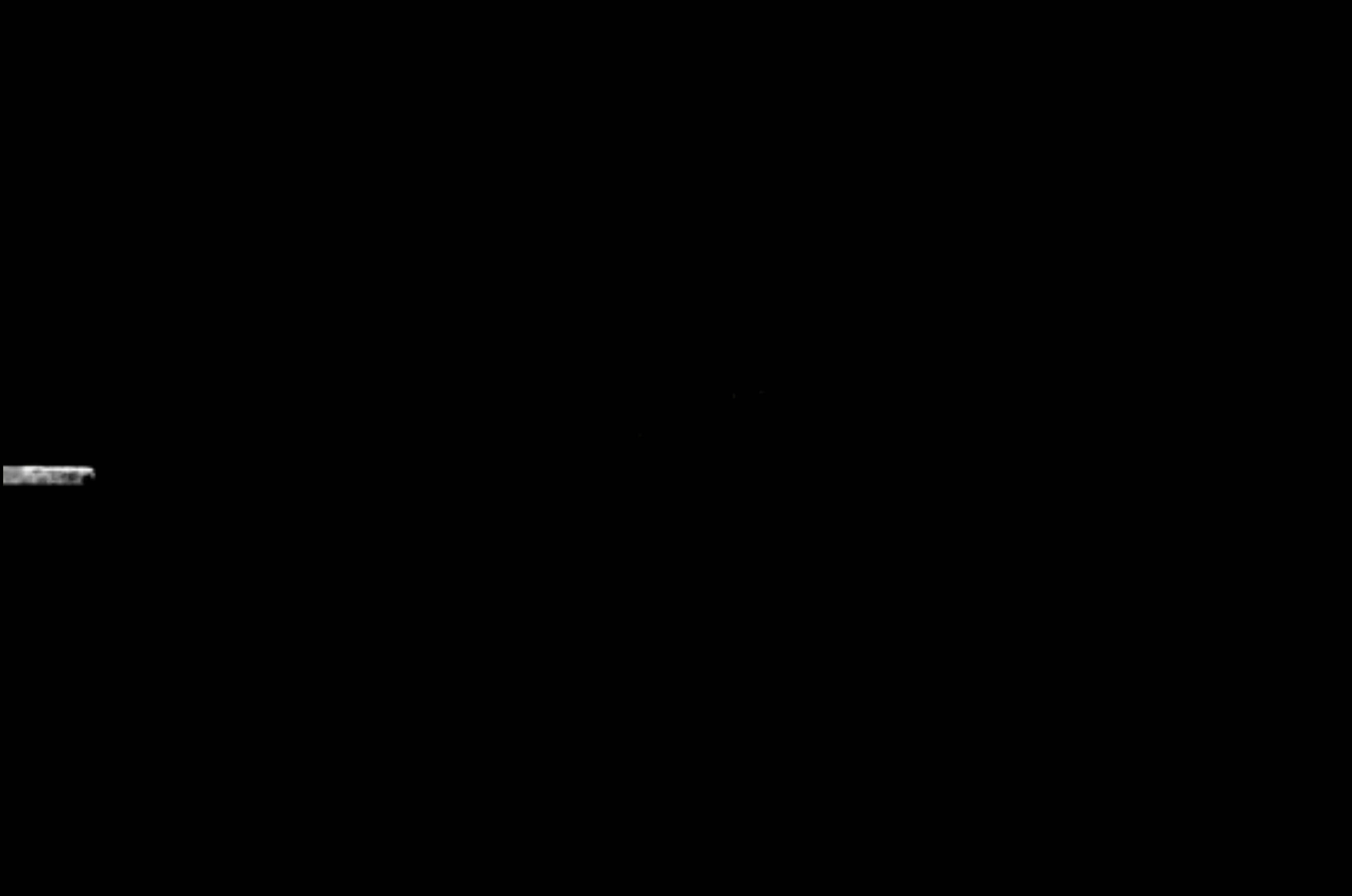
- **centimetric rock forms produced by splash from ancient meteor impacts**



Tektites

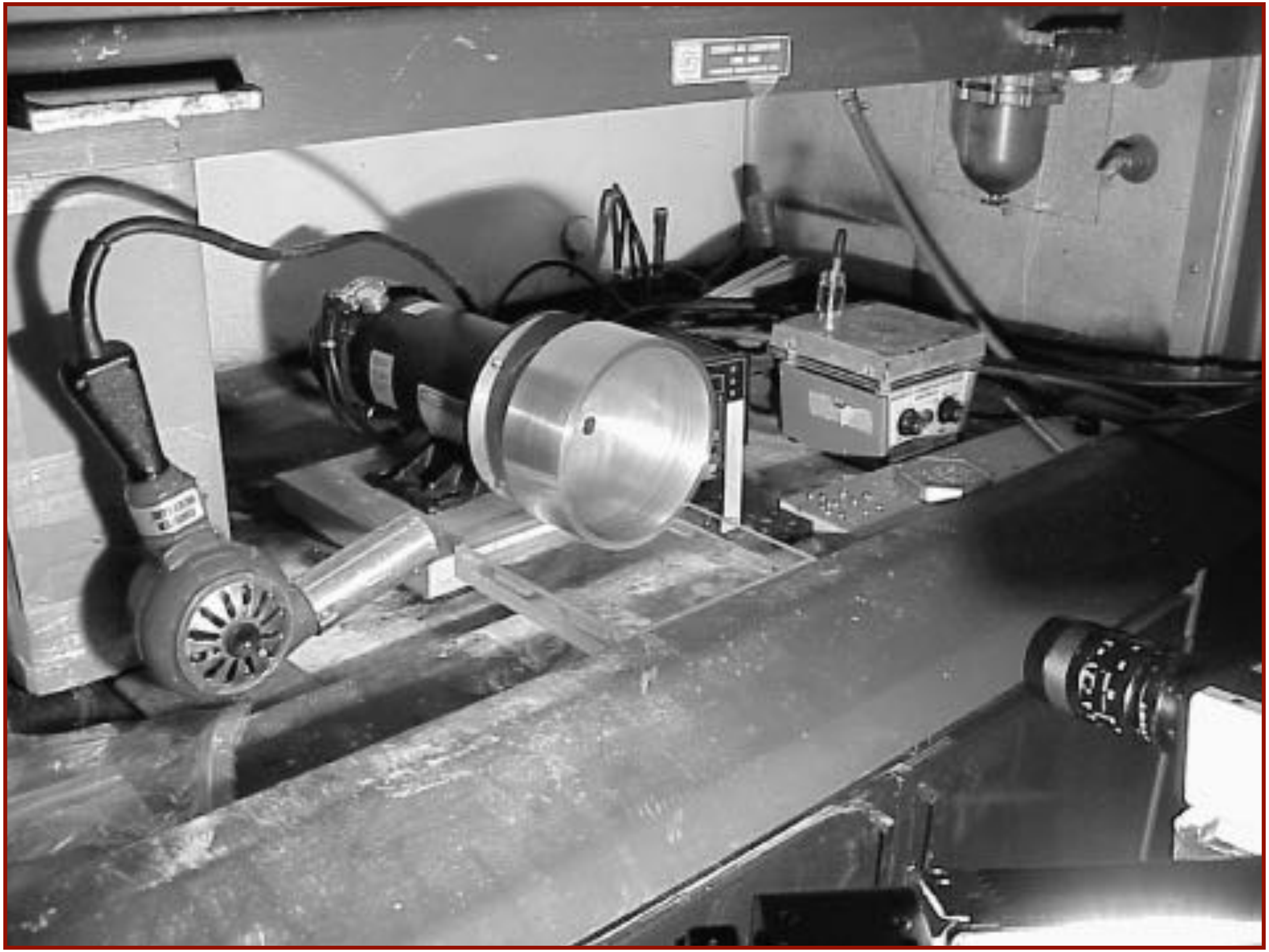
Elkins, Ausillous, Bico, Quere & Bush (2004)



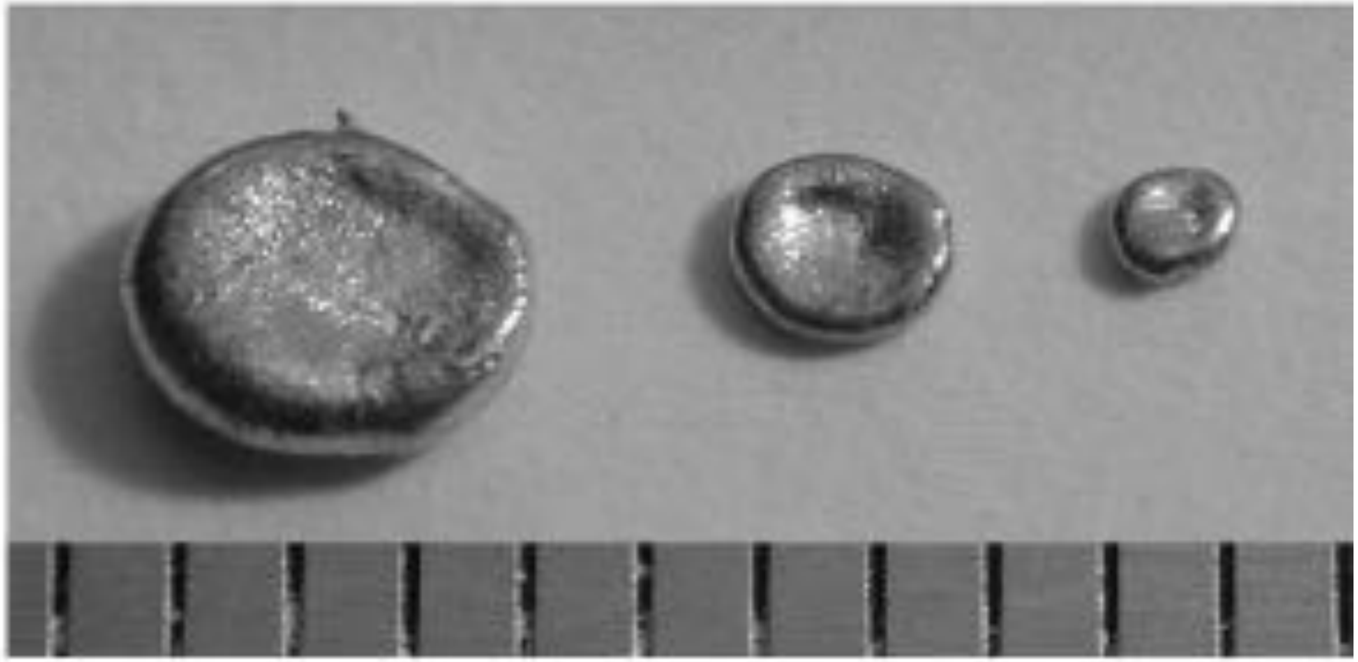


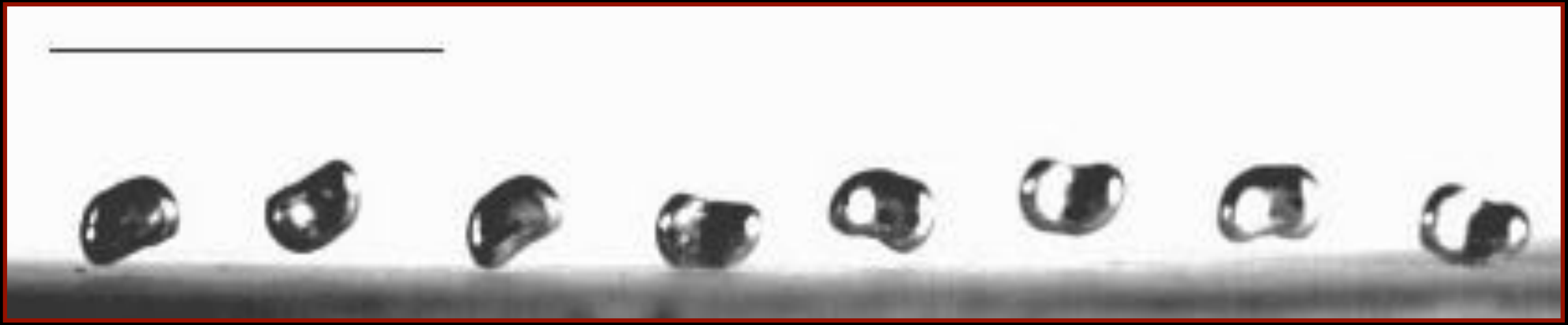
Aussillous and Quere (2003)

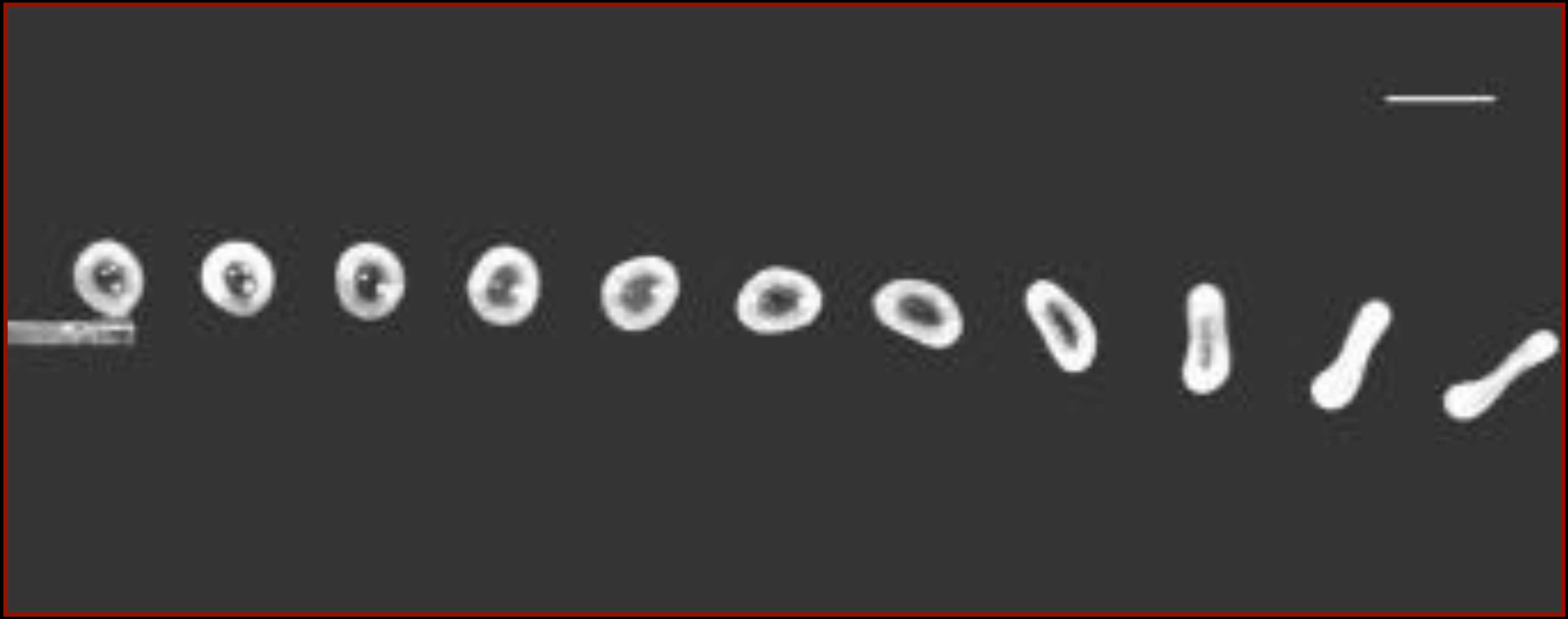
A lab model of splash-form tektites



Synthetic tektites

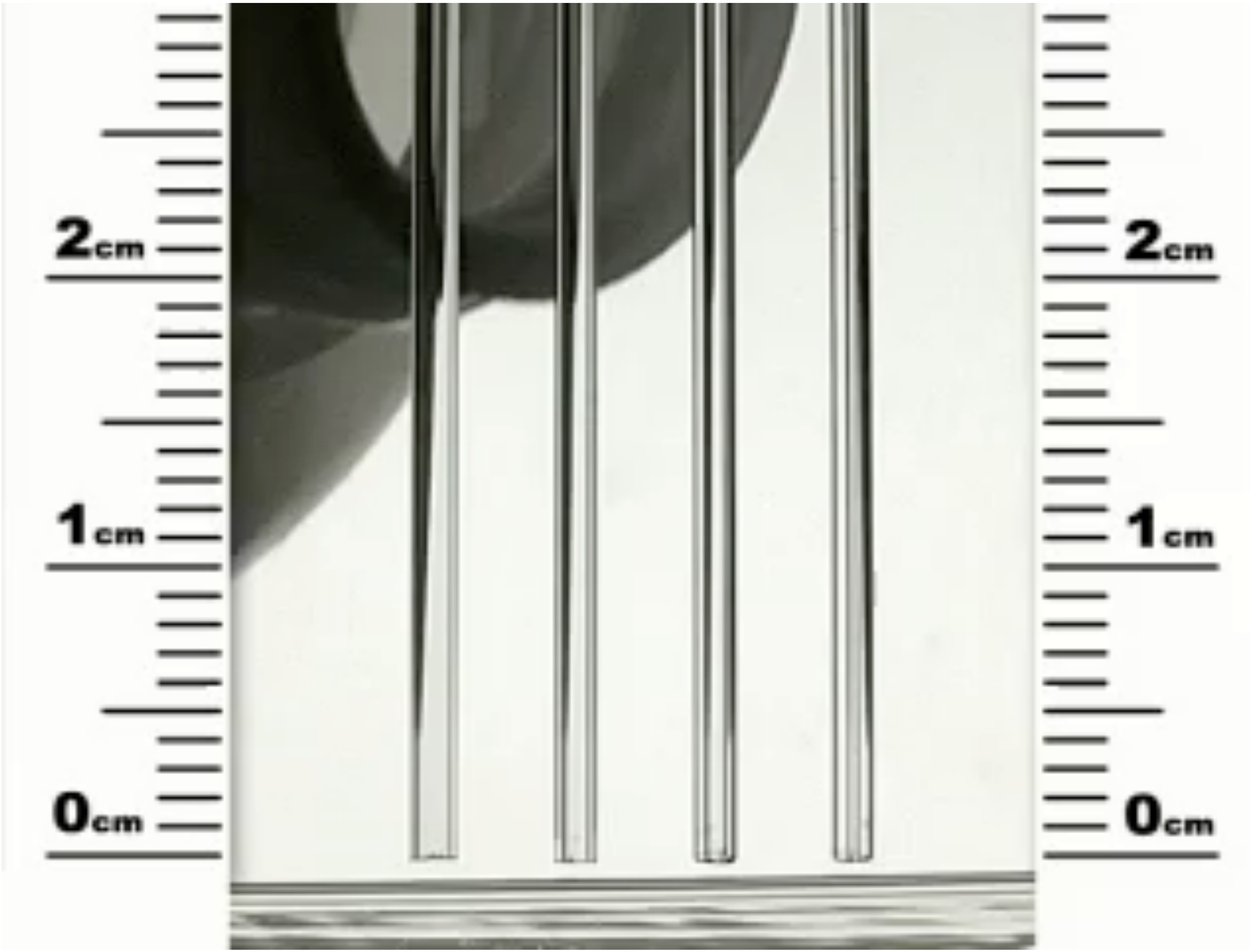






The tumbling tektite

The spinning drop tensiometer


2cm
1cm
0cm
2cm
1cm
0cm