18.357: Lecture 5

I. Interfacial boundary conditions

II. Fluid statics: menisci, floating bodies

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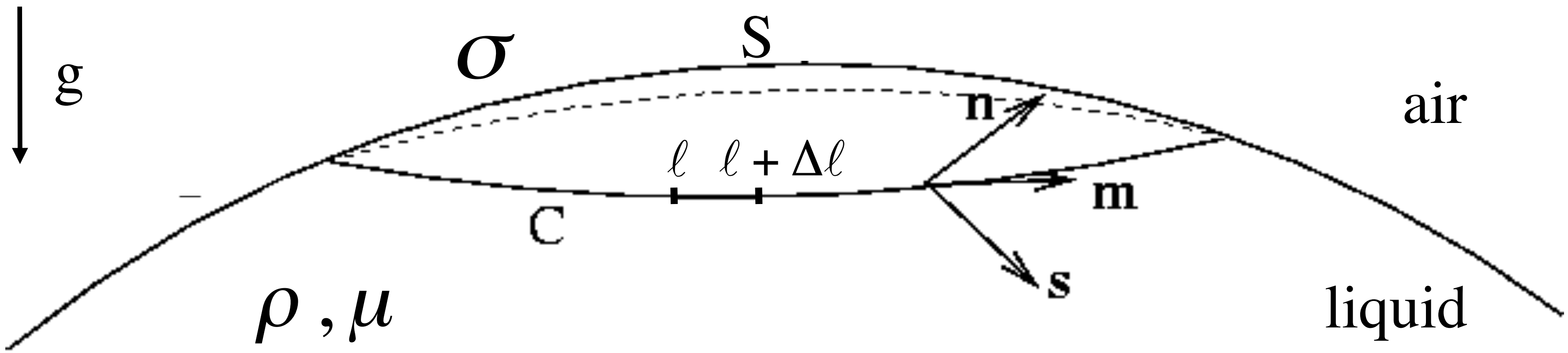
Surface tension: Geometry

Along a contour C bounding a surface S there is a tensile force per unit length σ acting in the $\mathbf{s}$ direction



Net force on S:

$$\oint_C \sigma \, \mathbf{s} \, d\ell = \iint_S \sigma \, (\nabla \cdot \mathbf{n}) \, \mathbf{n} \, dS + \iint_S \nabla\sigma \, dS$$

curvature pressure

Marangoni stress

1) normal curvature pressure $\sigma \nabla \cdot \mathbf{n}$ resists surface deformation

2) tangential Marangoni stresses may arise from $\nabla\sigma$

Governing Equations

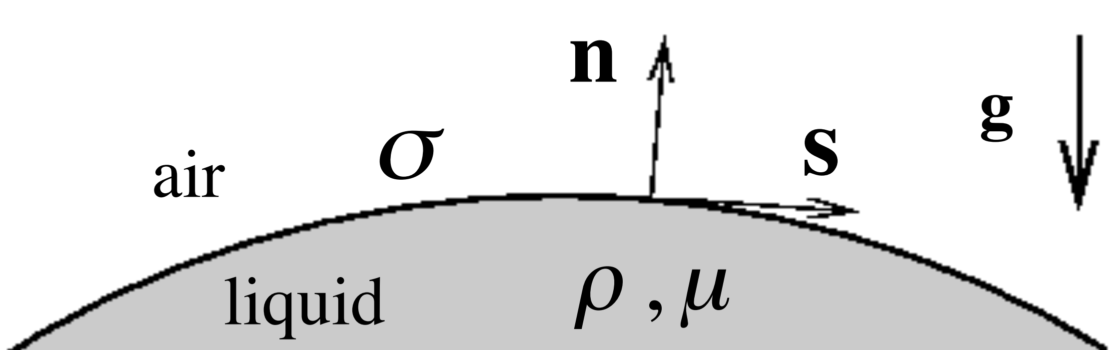
Navier-Stokes equations:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \rho\mathbf{g} + \mu\nabla^2\mathbf{u} \quad , \qquad \nabla\cdot\mathbf{u} = 0$$

Boundary Conditions

Normal stress: $\Delta\ \mathbf{n}\cdot\mathbf{T}\cdot\mathbf{n} = \sigma\,\nabla\cdot\mathbf{n}$

Tangential stress: $\Delta\ \mathbf{n}\cdot\mathbf{T}\cdot\mathbf{s} = \nabla_s\sigma$



Stress tensor

$$\mathbf{T} = -p\mathbf{I} + \mu\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right)$$

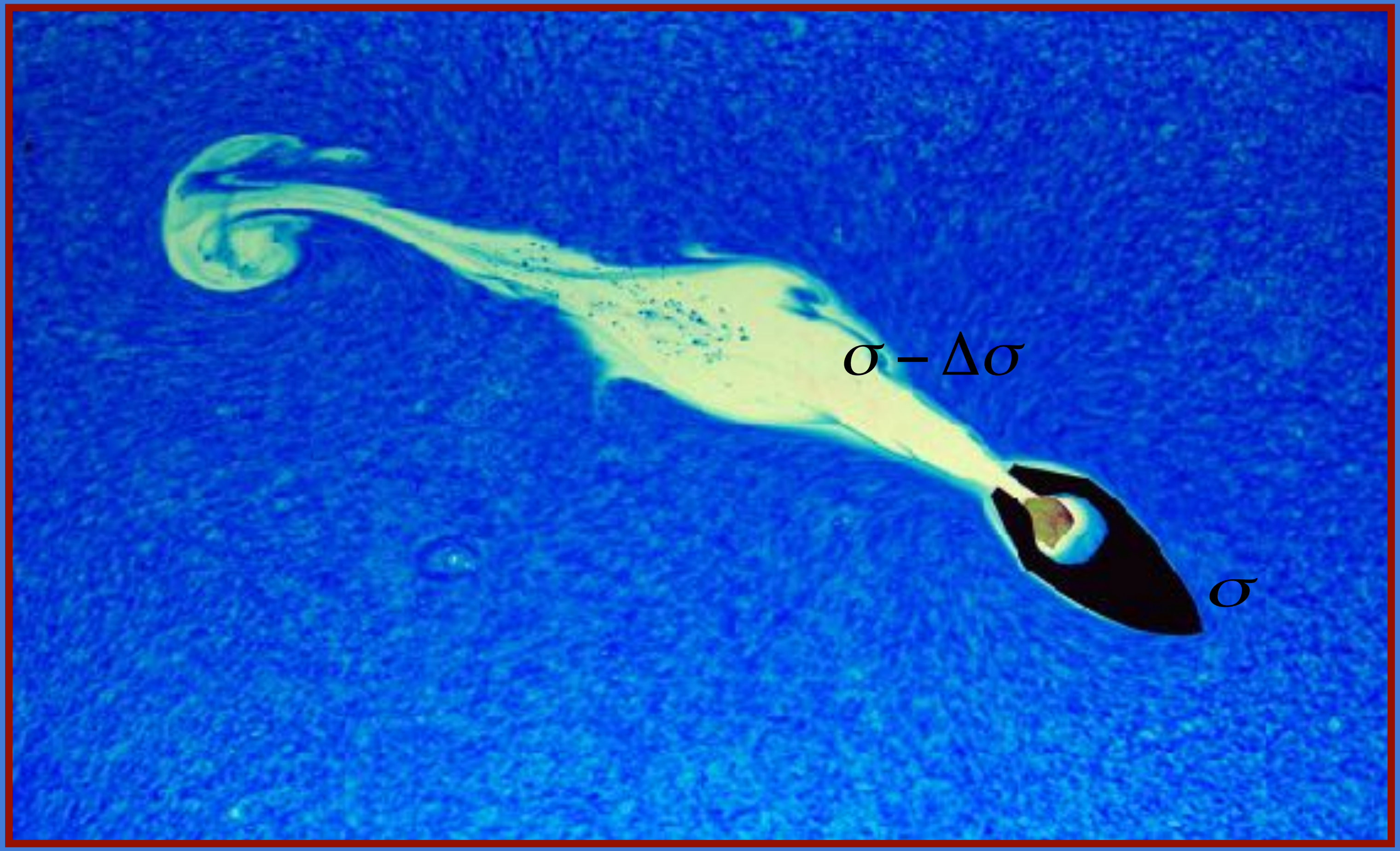
Capillary forces support the weight of water-walking insects.

Marangoni Flows

- flows dominated by the influence of surface tension gradients

Recall tangential stress BC: $\Delta\, \mathbf{n}\cdot\mathbf{T}\cdot\mathbf{s} = \nabla\sigma$

- $\nabla\sigma$ may arise due to dependence of $\sigma(T, c, \Gamma)$



Marangoni Flows

- flows dominated by the influence of surface tension gradients
- $\nabla\sigma$ may arise due to dependence of $\sigma(T, c, \Gamma)$

The cocktail boat: fueled by alcohol

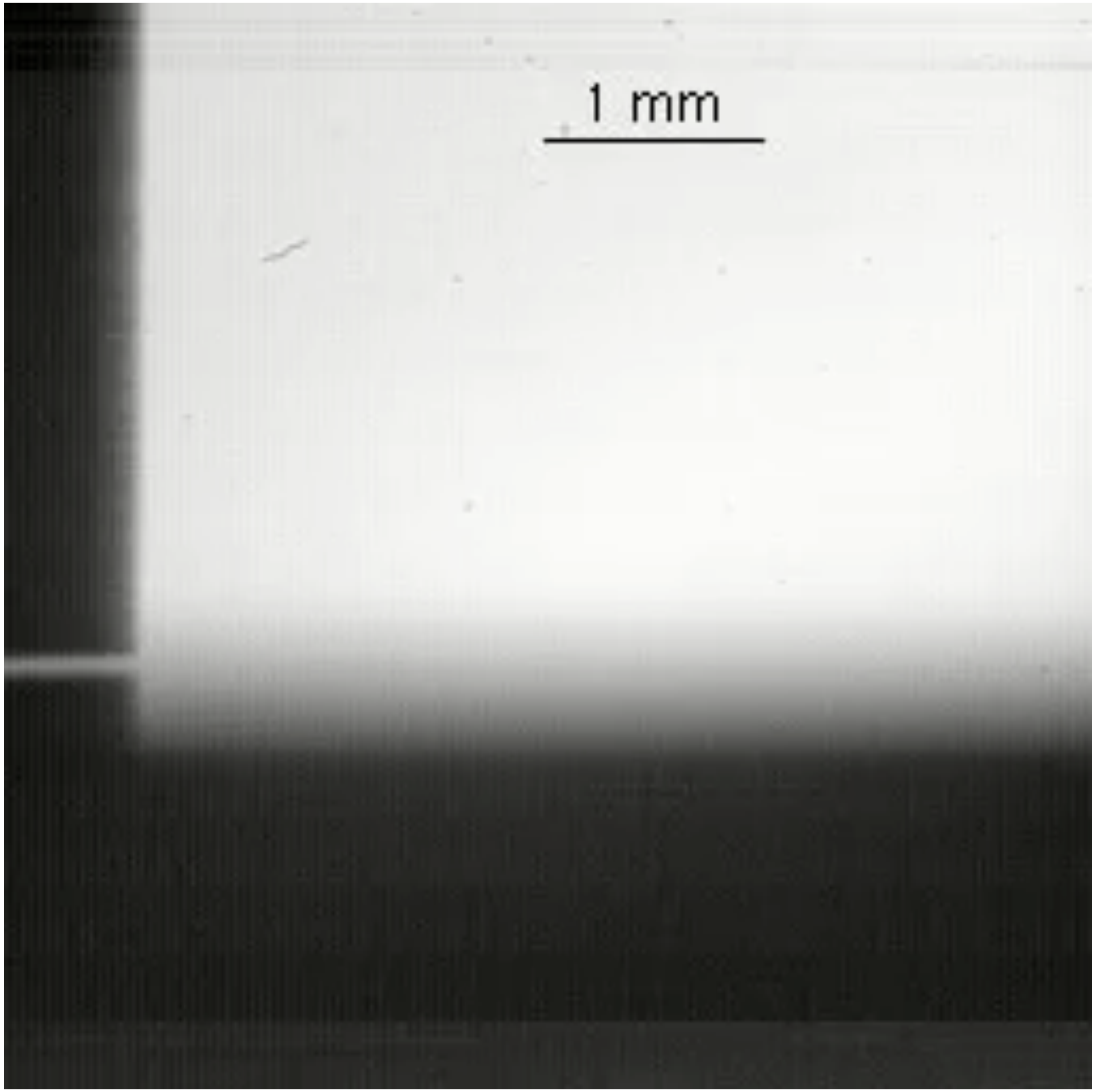
Some notes on geometry

I. Computing curvatures: a quick recap

II. Frenet-Serret equations

Interfacial statics

Capillary rise: the planar meniscus



On floating bodies

Heavy things sink, light things float.

Not exactly…..