18.357: Lecture 3

Surface tension:

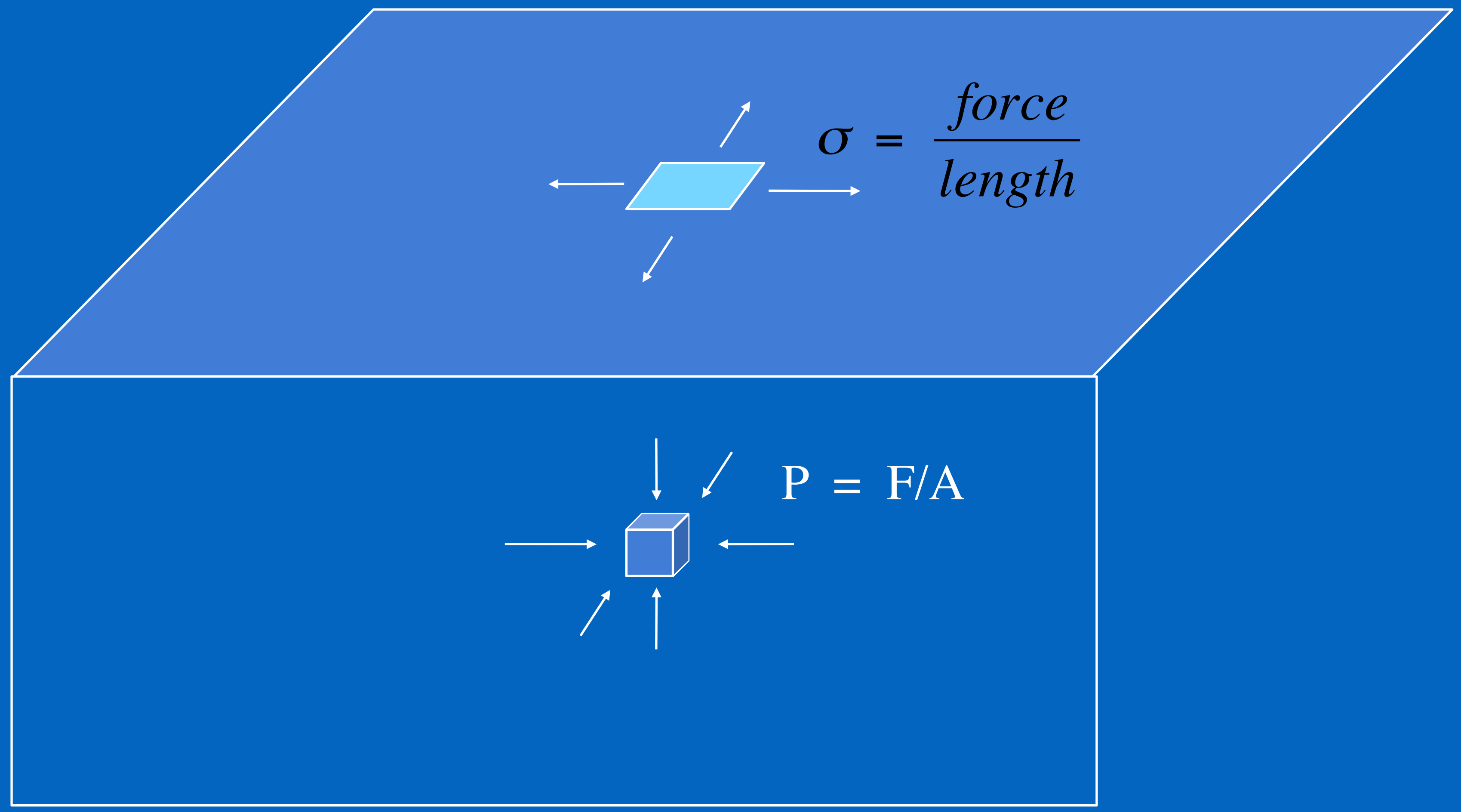
Capillary pressure, scaling, wetting

John W. M. Bush

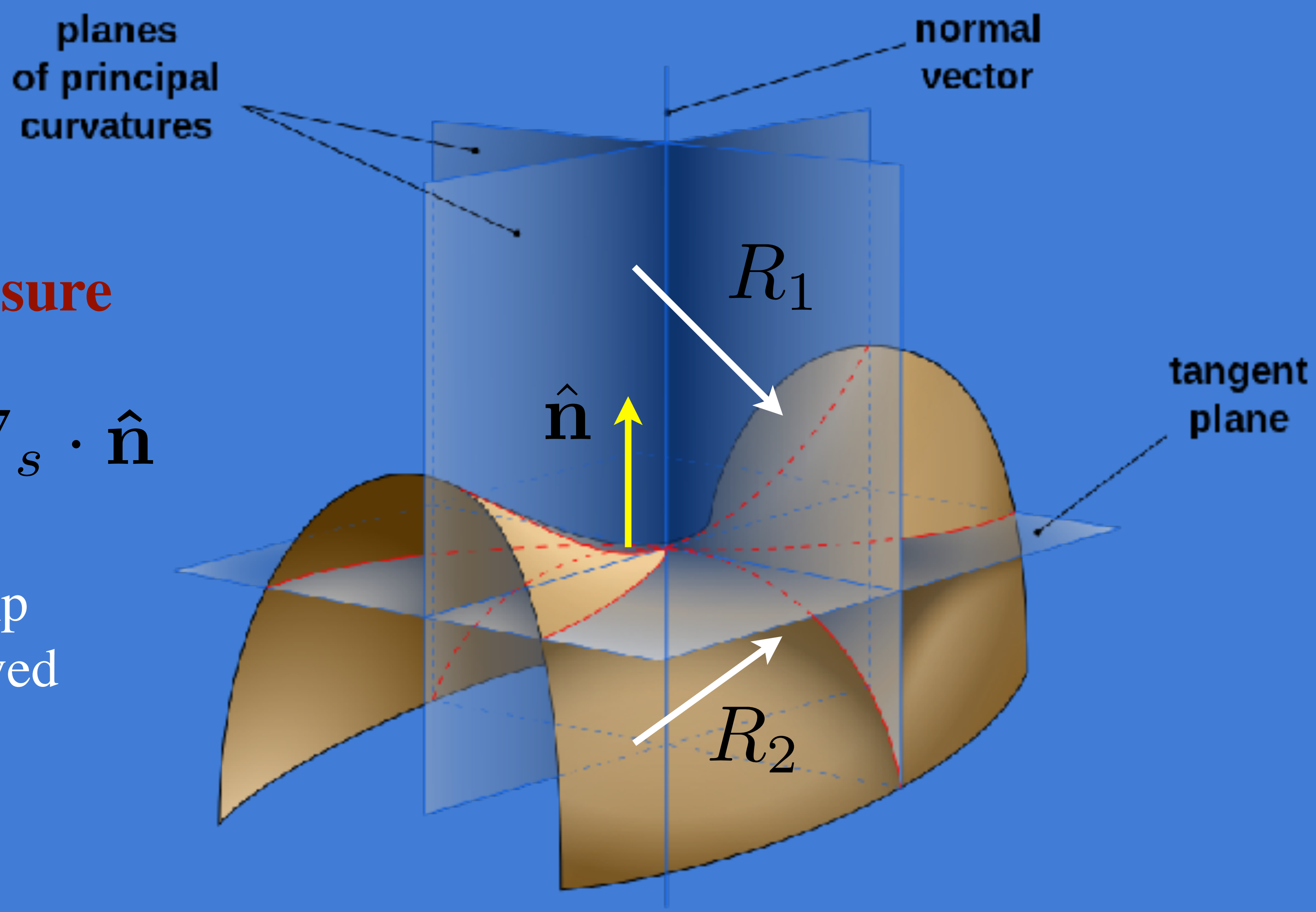
Department of Mathematics
MIT

Surface tension: analogous to a negative surface pressure

- gradients in surface tension necessarily drive surface motion



Curvature pressure



Laplace pressure

$$\Delta P = \sigma \; \nabla_s \cdot \hat{\mathbf{n}}$$

- pressure jump across a curved interface

Curvature: $\nabla \cdot \hat{\mathbf{n}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ where R_1 , R_2 are the principal radii of curvature

The scaling of surface tension

DIMENSIONAL ANALYSIS

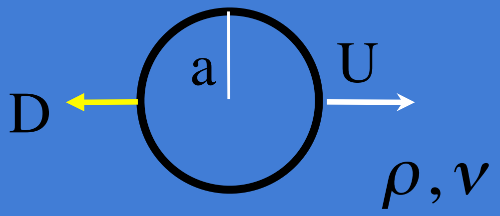
Fundamental Concept

The laws of Nature cannot depend on arbitrarily chosen system of units.
A system is most succinctly described in terms of dimensionless variables.

Deduction of Dimensionless groups: **Buckingham's Theorem**

For a system with M physical variables (e.g. density, speed, length, viscosity) describable in terms of N fundamental units (e.g. mass, length, time, temperature), there are M - N dimensionless groups that govern the system.

E.g. Uniform translation of a sphere



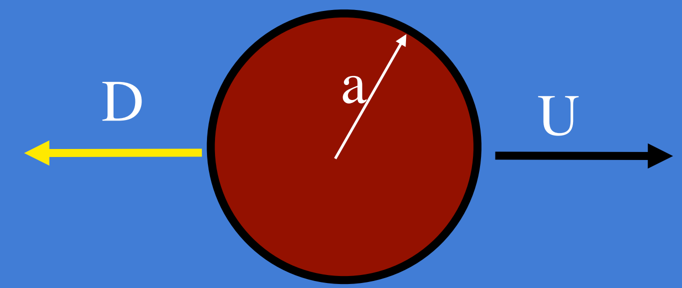
Physical variables: $U, \mathrm{a}, \nu, \rho, D \Rightarrow$ M = 5

Fundamental units: M , L , T $\Rightarrow$ N = 3

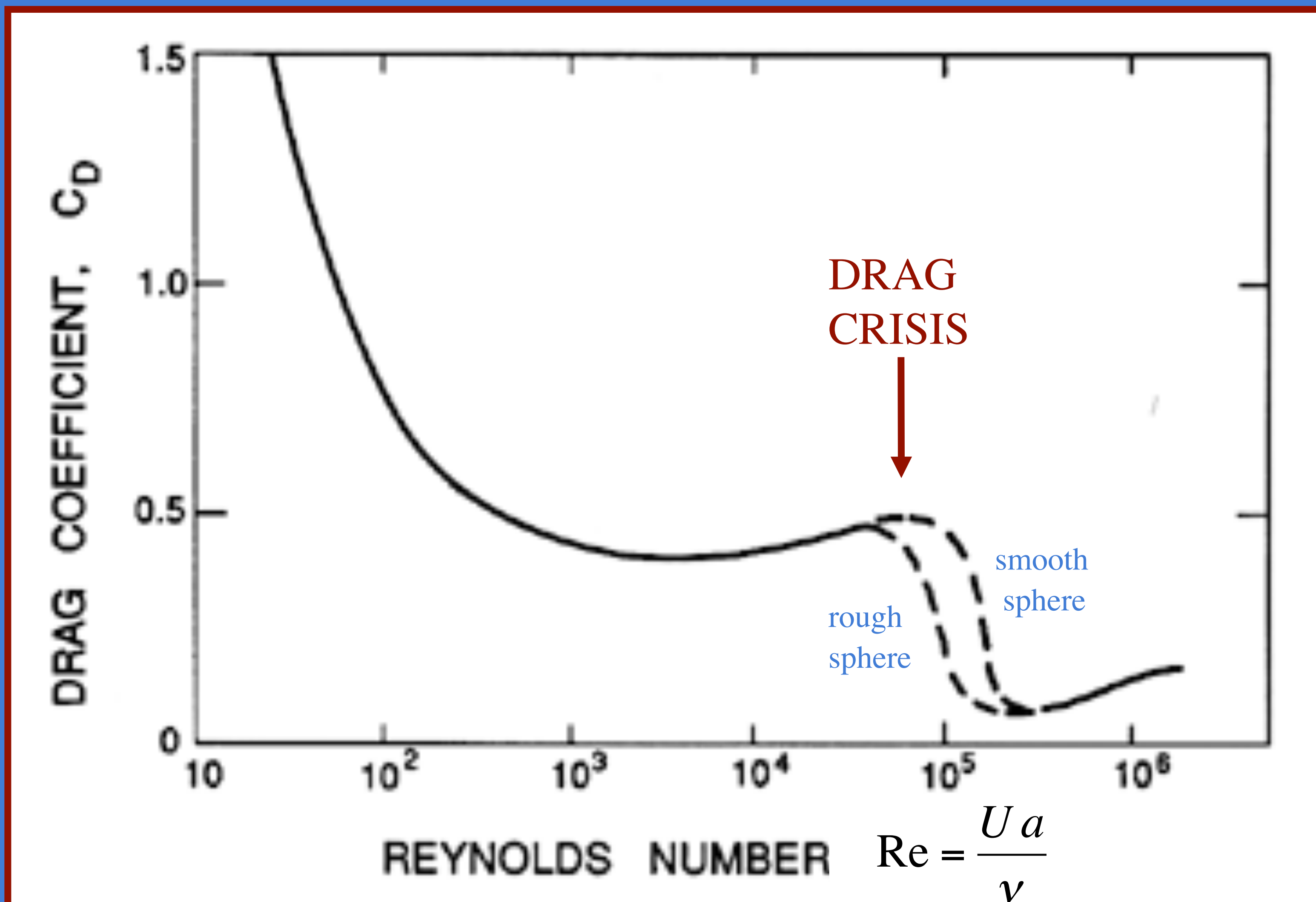
M - N = 2 dimensionless groups: $C_d = \dfrac{D}{\rho U^2}$, $\mathrm{Re} = \dfrac{U \mathrm{a}}{\nu}$

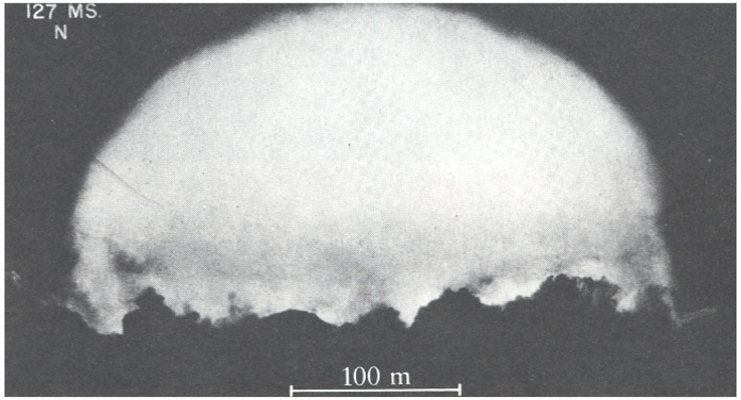
System uniquely determined by a single relation: $C_d = F(\mathrm{Re})$

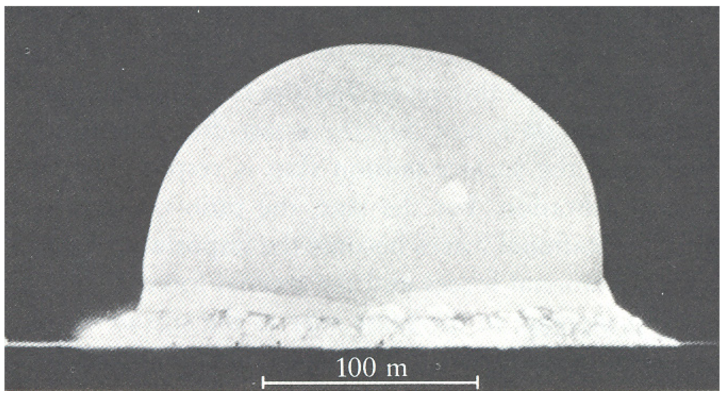
Drag: $D = C_D(\mathrm{Re})\,\rho\,U^2\,a^2$

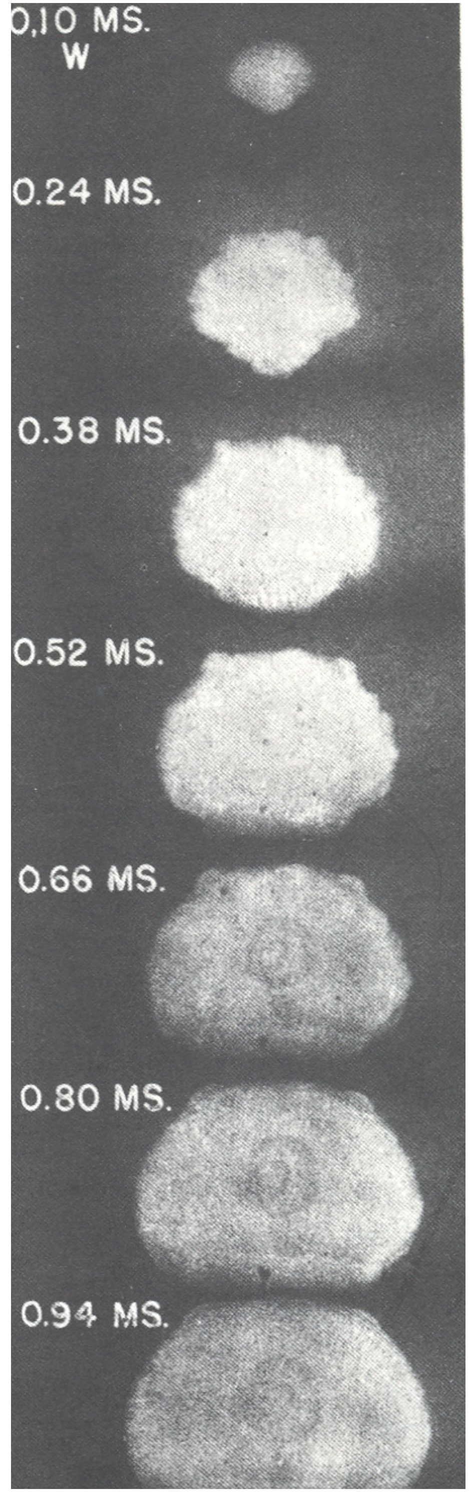


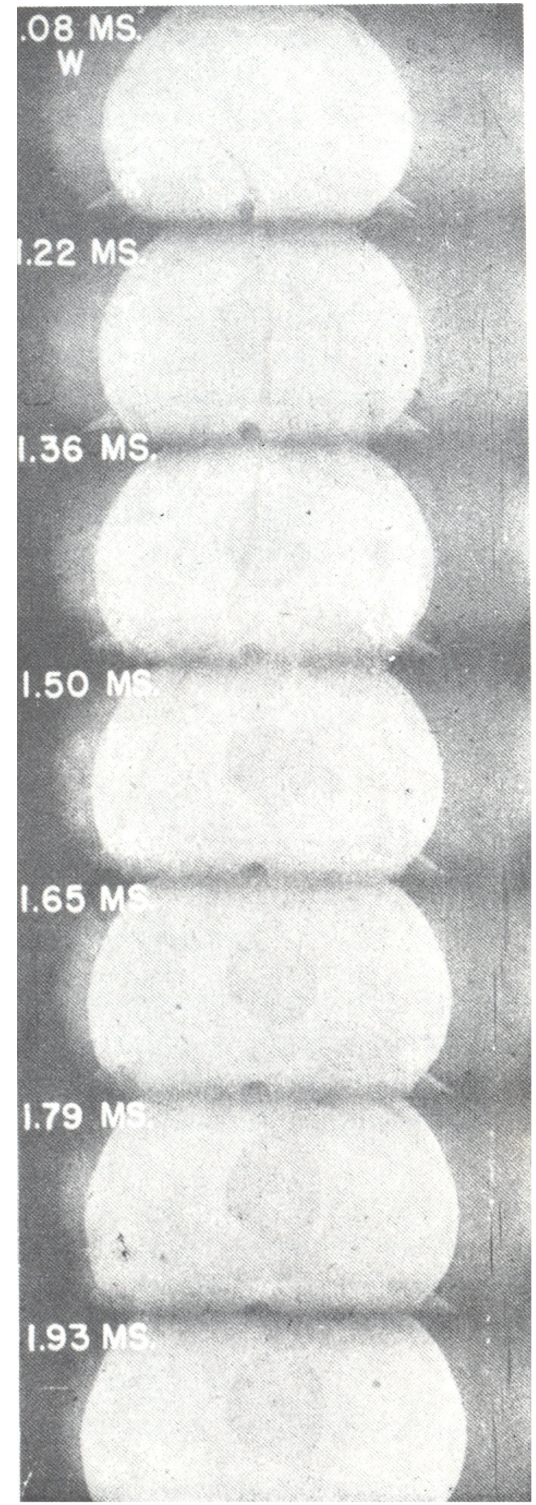
Variation of C_D with $\mathrm{Re} = \dfrac{U\,a}{\nu}$:









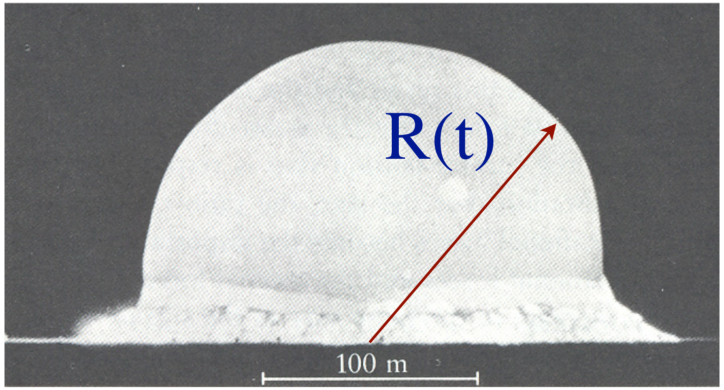


Can we predict R(t) ?

Given R(t), can we infer the energy released?

The scaling of atomic blast clouds

- G.I. Taylor, Sedov



Physical variables: R, t, E, ρ

Fundamental units: M, L, T

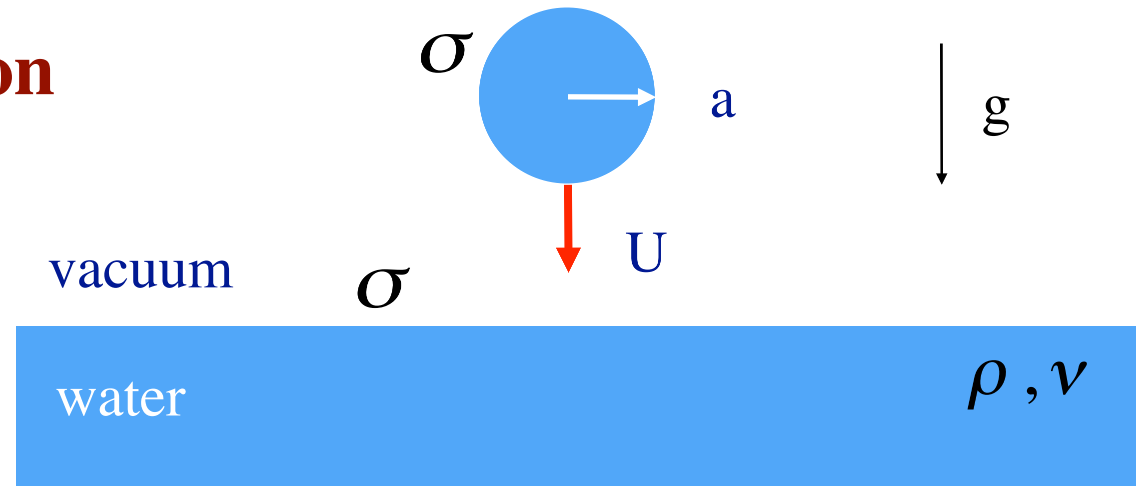
→ system prescribed by one dimensionless group:

$$\Pi = \frac{Et^2}{\rho R^5} = constant$$

→ yields desired scaling for radius of the blast cloud:

$$R \sim \left(\frac{E}{\rho}\right)^{1/5} t^{2/5}$$

The scaling of surface tension



$$W_e = \frac{\rho U^2 a}{\sigma} = \frac{\text{INERTIA}}{\text{CURVATURE}} = \text{Weber number}$$

$$C_a = \frac{\rho \nu U}{\sigma} = \frac{\text{VISCOSITY}}{\text{CURVATURE}} = \text{Capillary number}$$

$$B_o = \frac{\rho g a^2}{\sigma} = \frac{\text{GRAVITY}}{\text{CURVATURE}} = \text{Bond number}$$

Note: σ is dominant relative to gravity when $B_o < 1$

i.e. $a < \left(\frac{\sigma}{\rho g}\right)^{1/2} = \ell_c =$ capillary length $\sim$ 2mm for air-water

When is surface tension important relative to gravity?

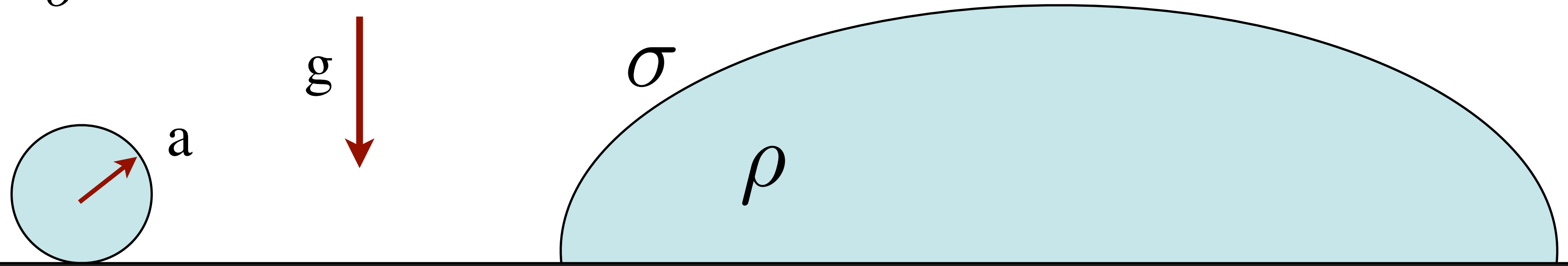
- when curvature pressures are large relative to hydrostatic:

Bond number: $$B_o = \frac{\rho g a}{\sigma / a} = \frac{\rho g a^2}{\sigma} < 1$$

i.e. for drops small relative to the capillary length:

$$a < l_c = \left(\frac{\sigma}{\rho g} \right)^{1/2} \sim$$ 2 mm for air-water $(\sigma = 70 \text{ dynes/cm})$

$B_o \ll 1$ $\quad$ $B_o > 1$



Surface tension dominates the world of insects - and of microfluidics.

Falling rain drops

Force balance:

$$\rho_a U^2 a^2 \sim Mg = \tfrac{4}{3}\pi a^3 \rho g$$

Fall speed: $U \sim \sqrt{\dfrac{\rho g a}{\rho_a}}$

Drop integrity requires:

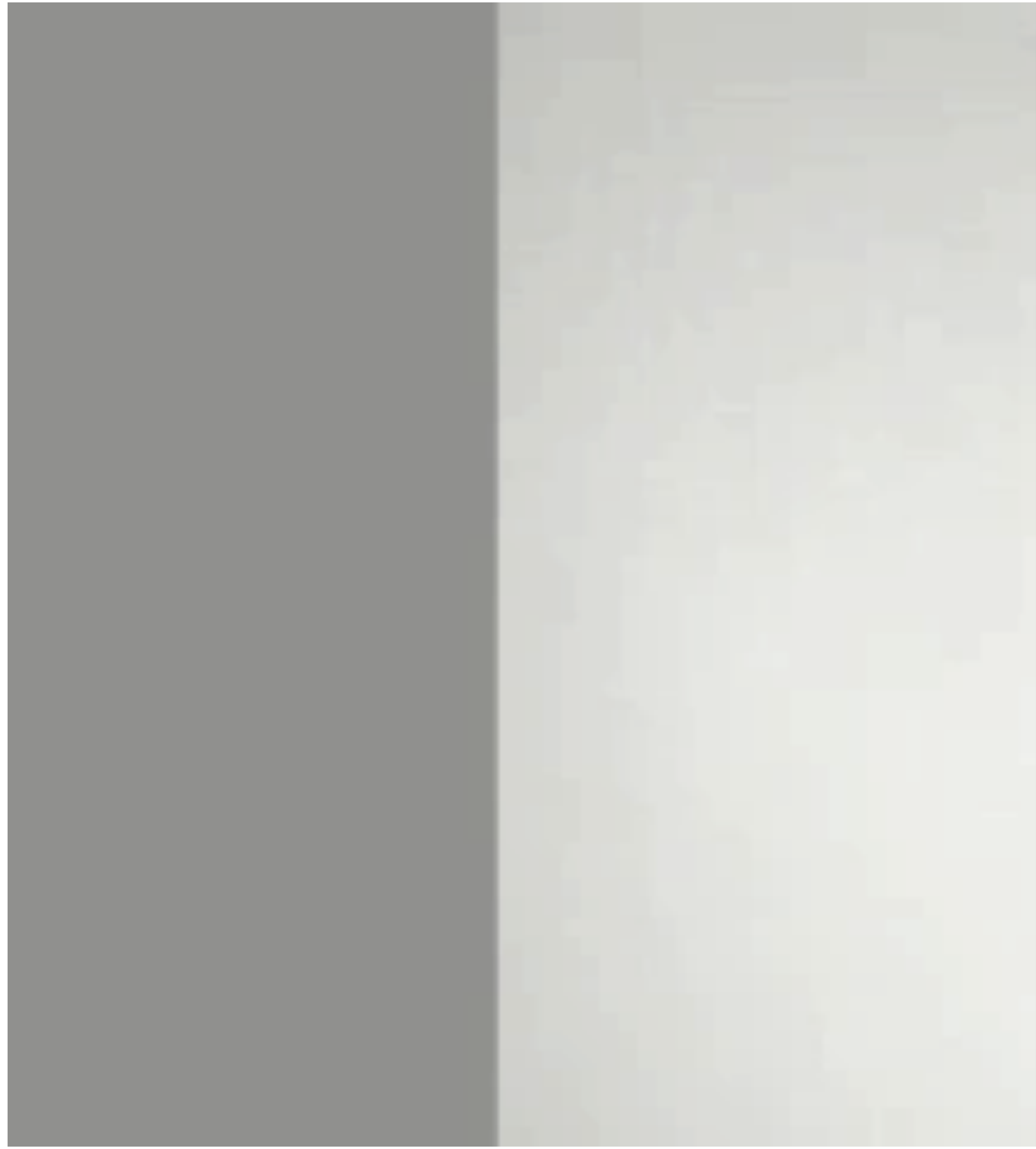
$$\rho_a U^2 \sim \rho g a < \sigma / a$$

Small drops

If a drop is small relative to the capillary length

$$a < \ell_c = \sqrt{\sigma/\rho g} \approx 2\text{mm}$$

σ maintains it against the destabilizing influence of aerodynamic stresses.



Large drops

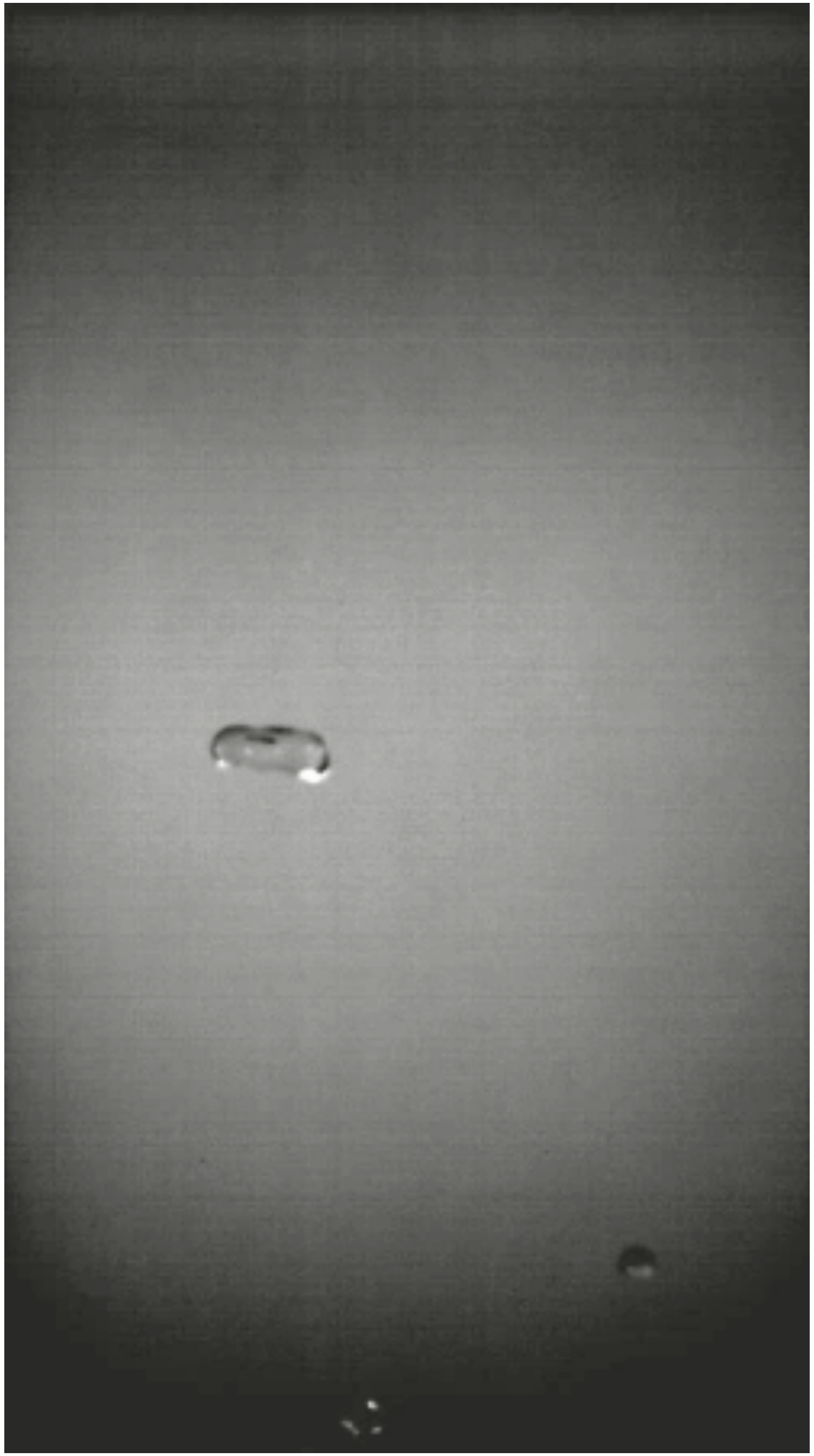
Drops larger than the capillary length

$$a > \ell_c \approx 2\text{mm}$$

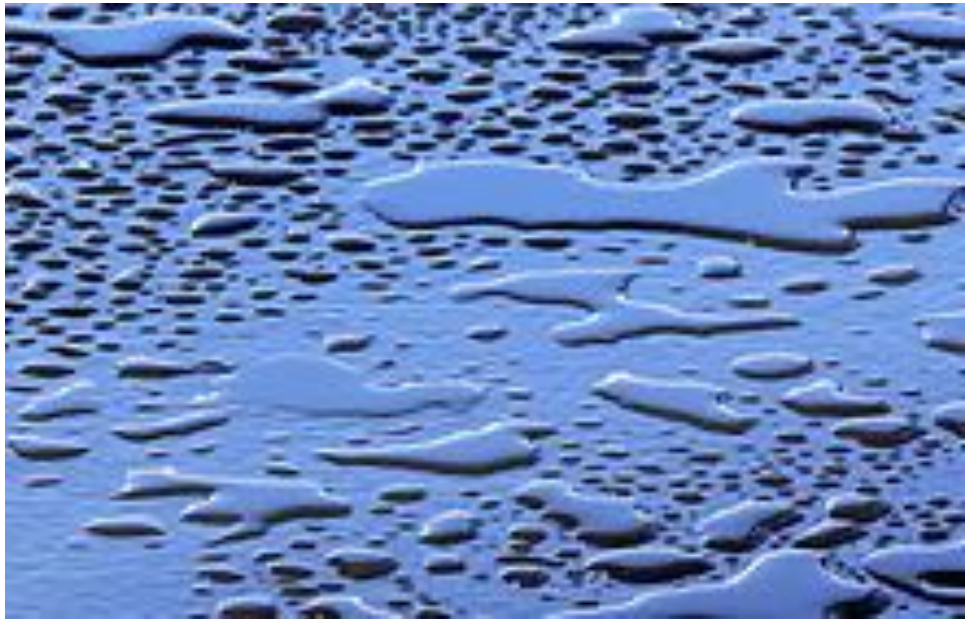
break up under the influence of aerodynamic stresses.

The break-up yields drops with size of order:

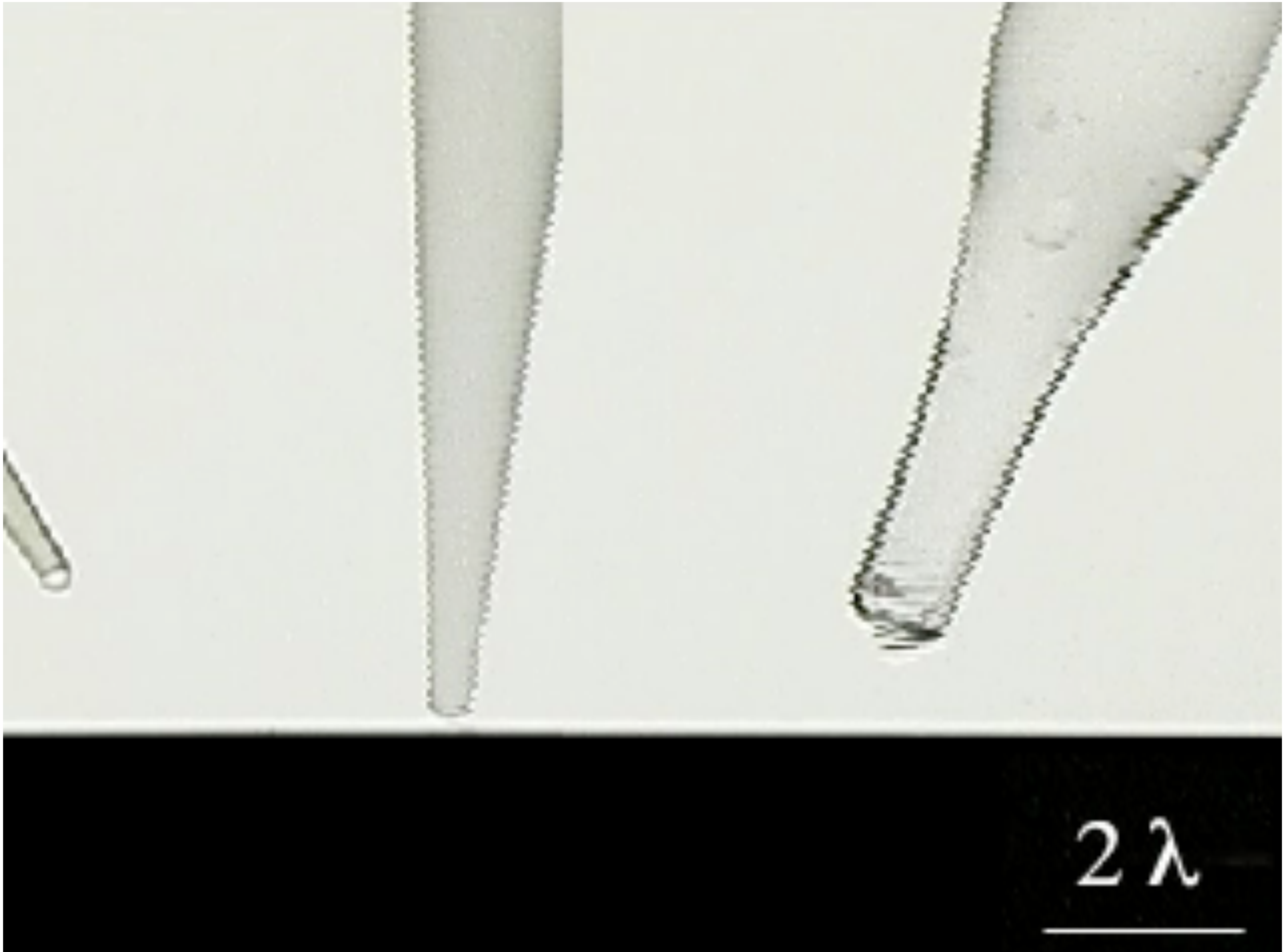
$$\ell_c \approx 2\text{mm}$$



Puddles on flat surfaces



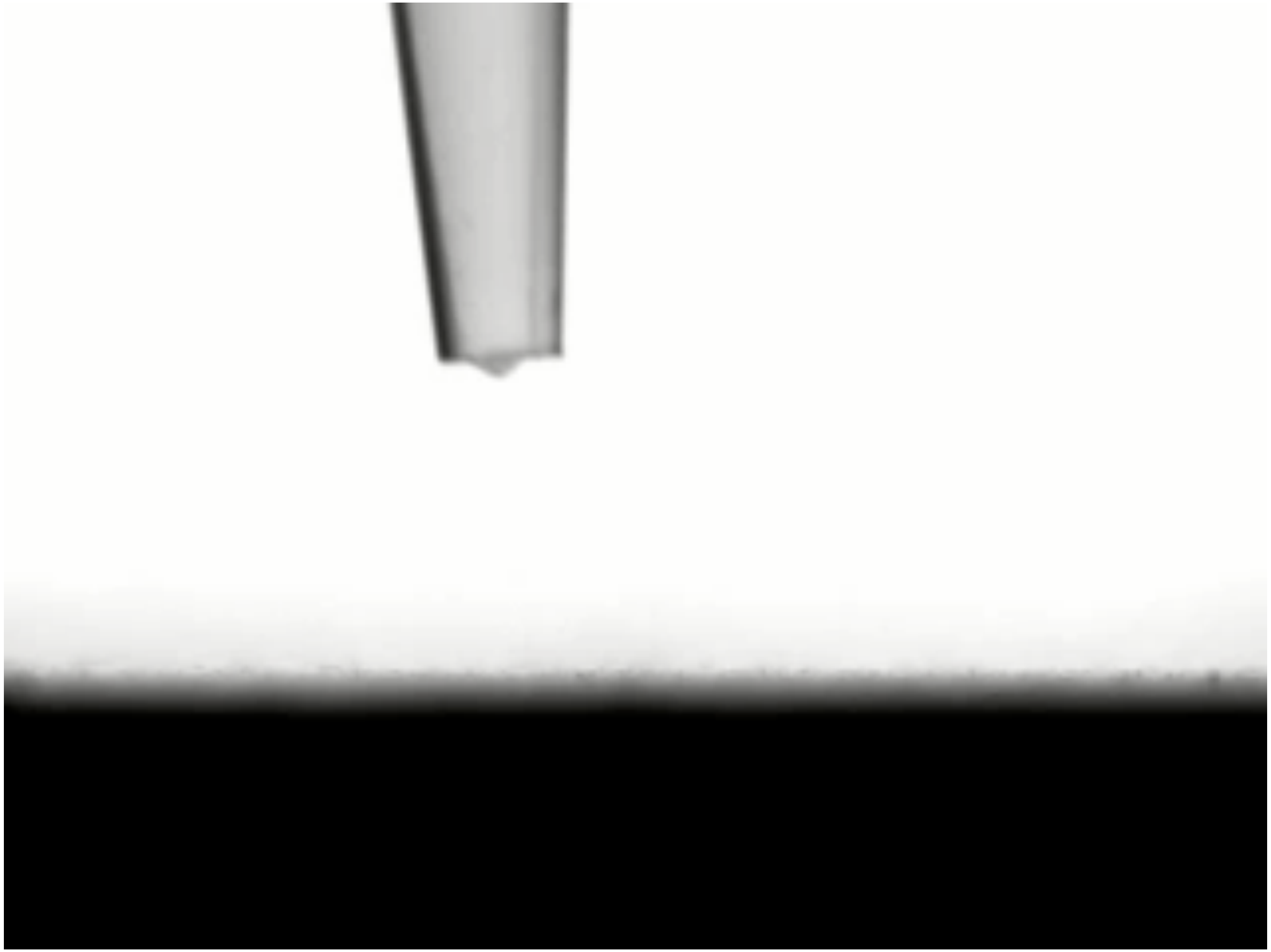
What sets their size?


2 λ

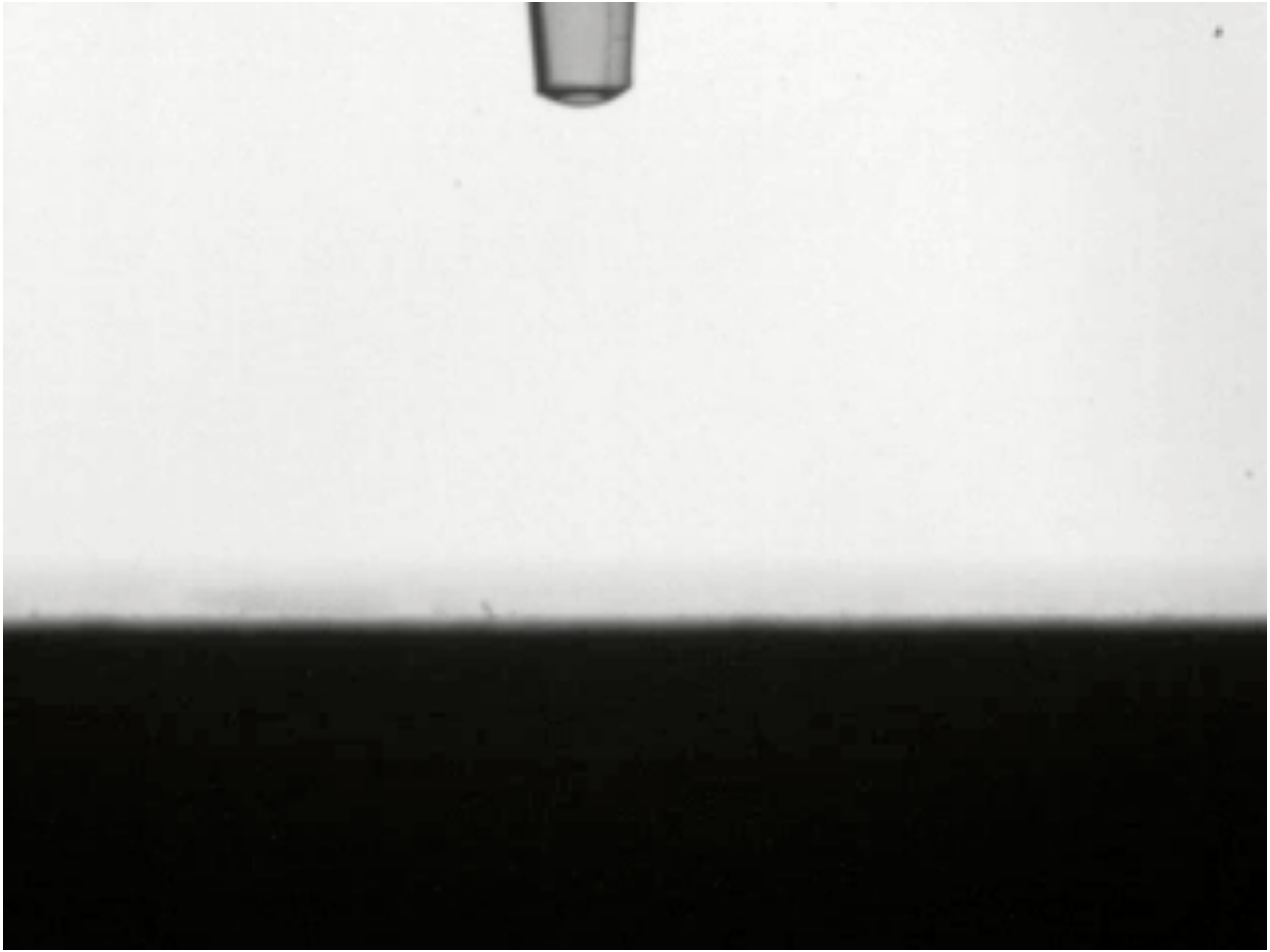
Wetting

Who cares about wetting?

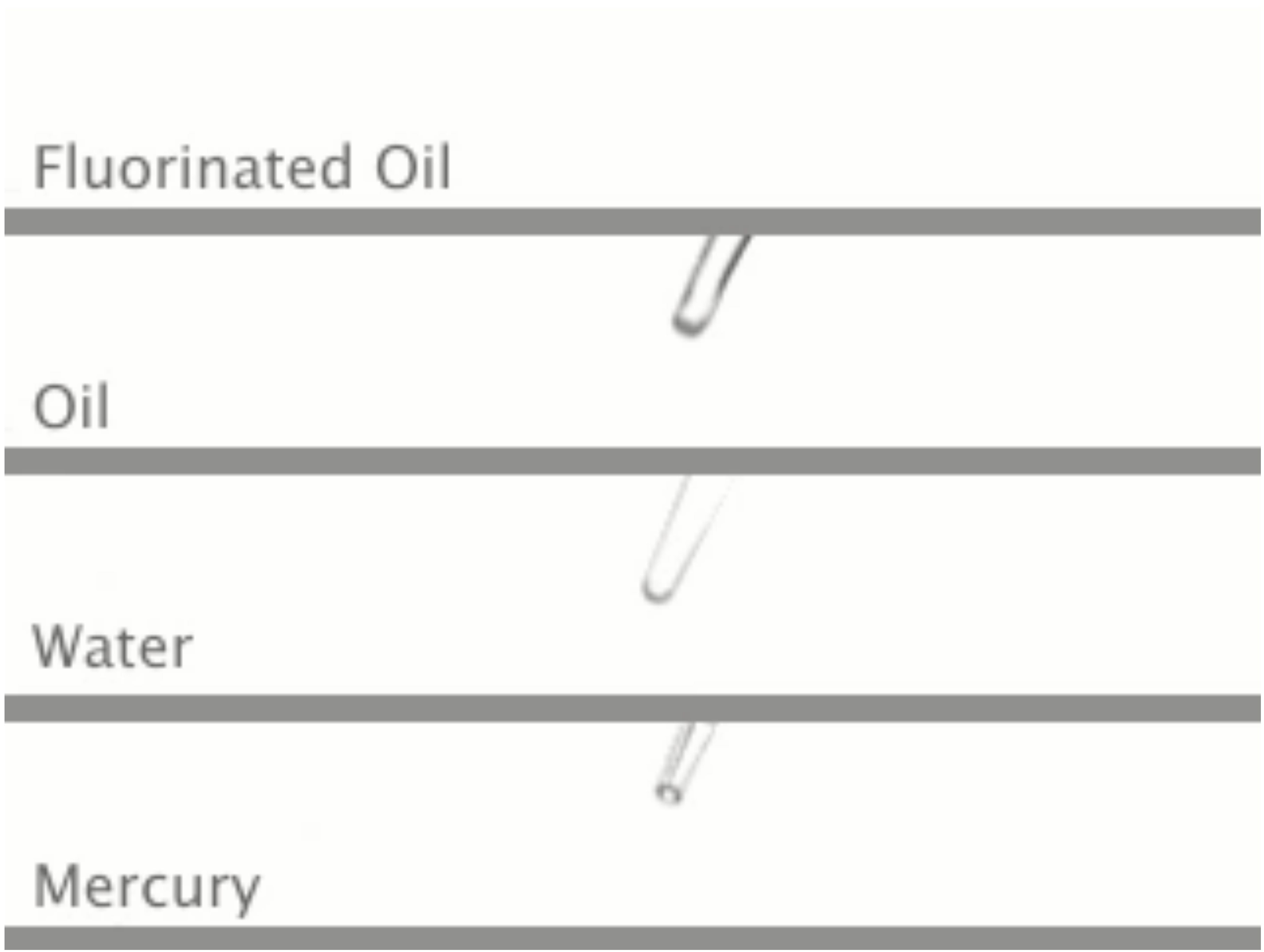
The world's smallest lizard: the Brazilian Pygmy Gecko



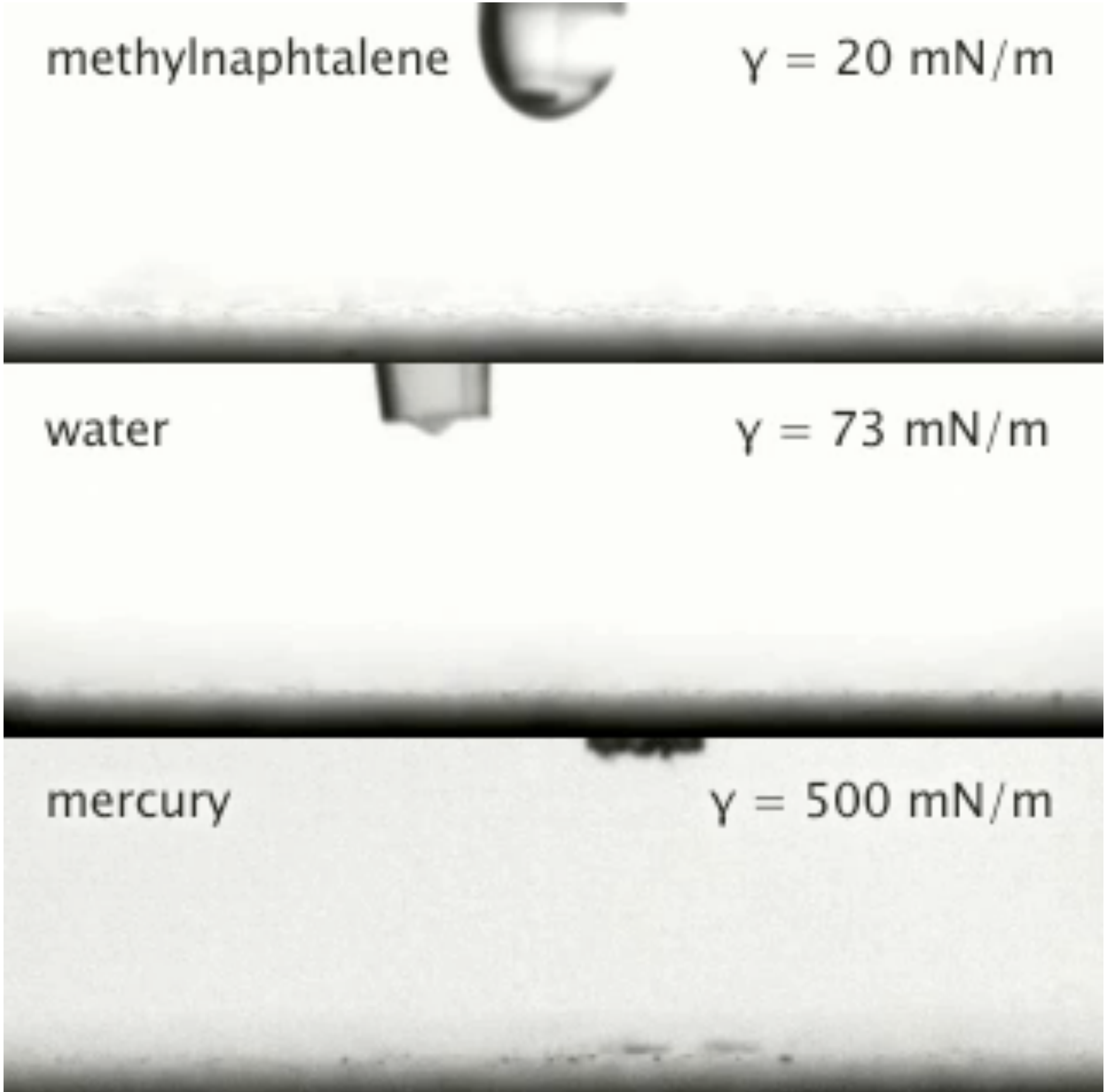
Partial wetting



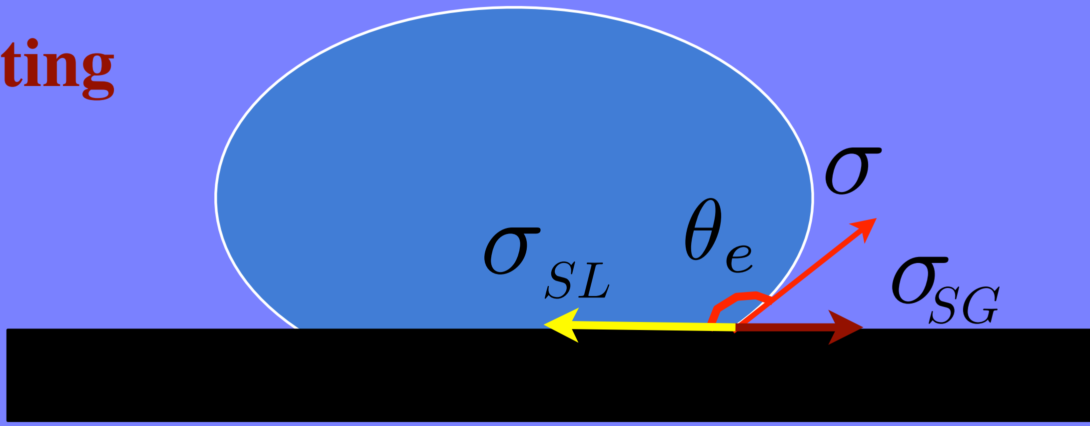
Total wetting


Fluorinated Oil
Oil
Water
Mercury

Water on glass


methylnaphtalene
γ = 20 mN/m
water
γ = 73 mN/m
mercury
γ = 500 mN/m

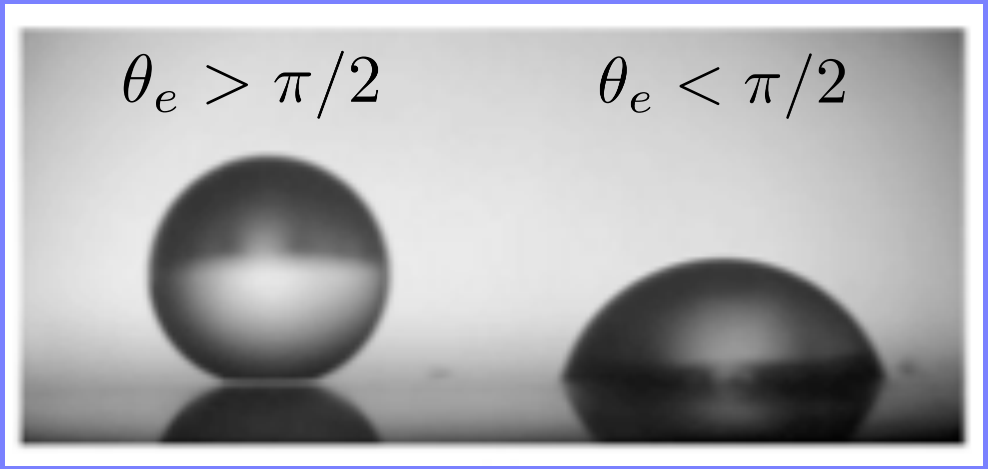
Fluid-Solid Contact: Partial wetting



Equilibrium contact angle θ_e

Energy differential: $dW = dx\,(\sigma_{SG} - \sigma_{SL}) - dx\,\sigma\,\cos\theta_e$

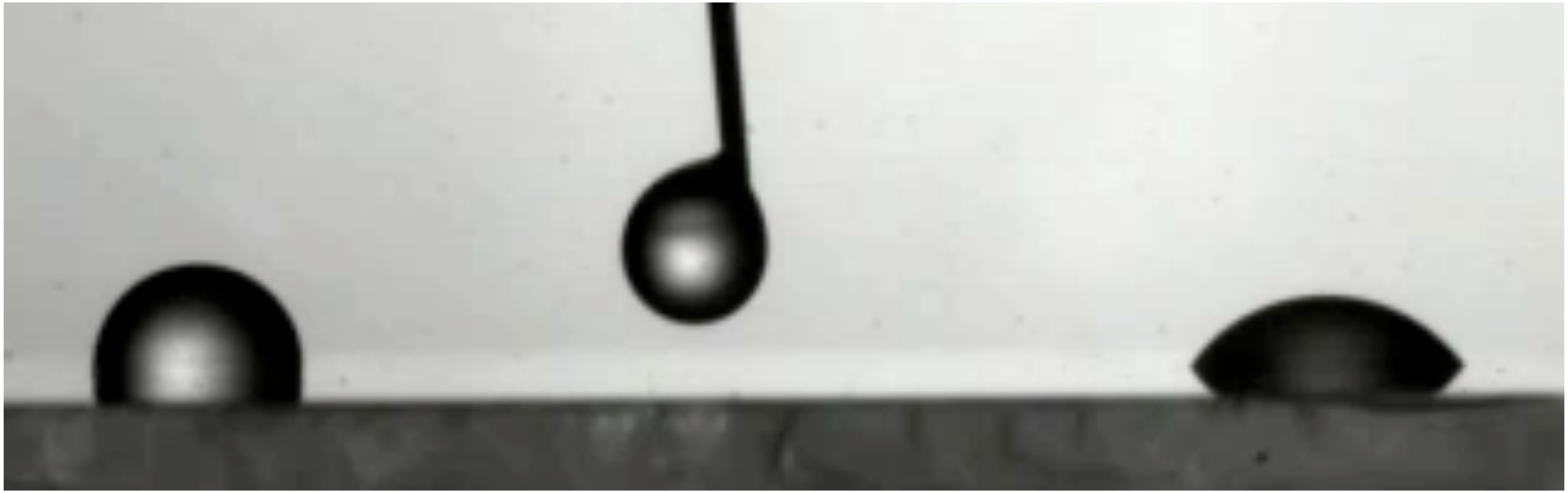
Young's relation: $$\sigma\ \cos\theta_e = \sigma_{SL} - \sigma_{SG}$$



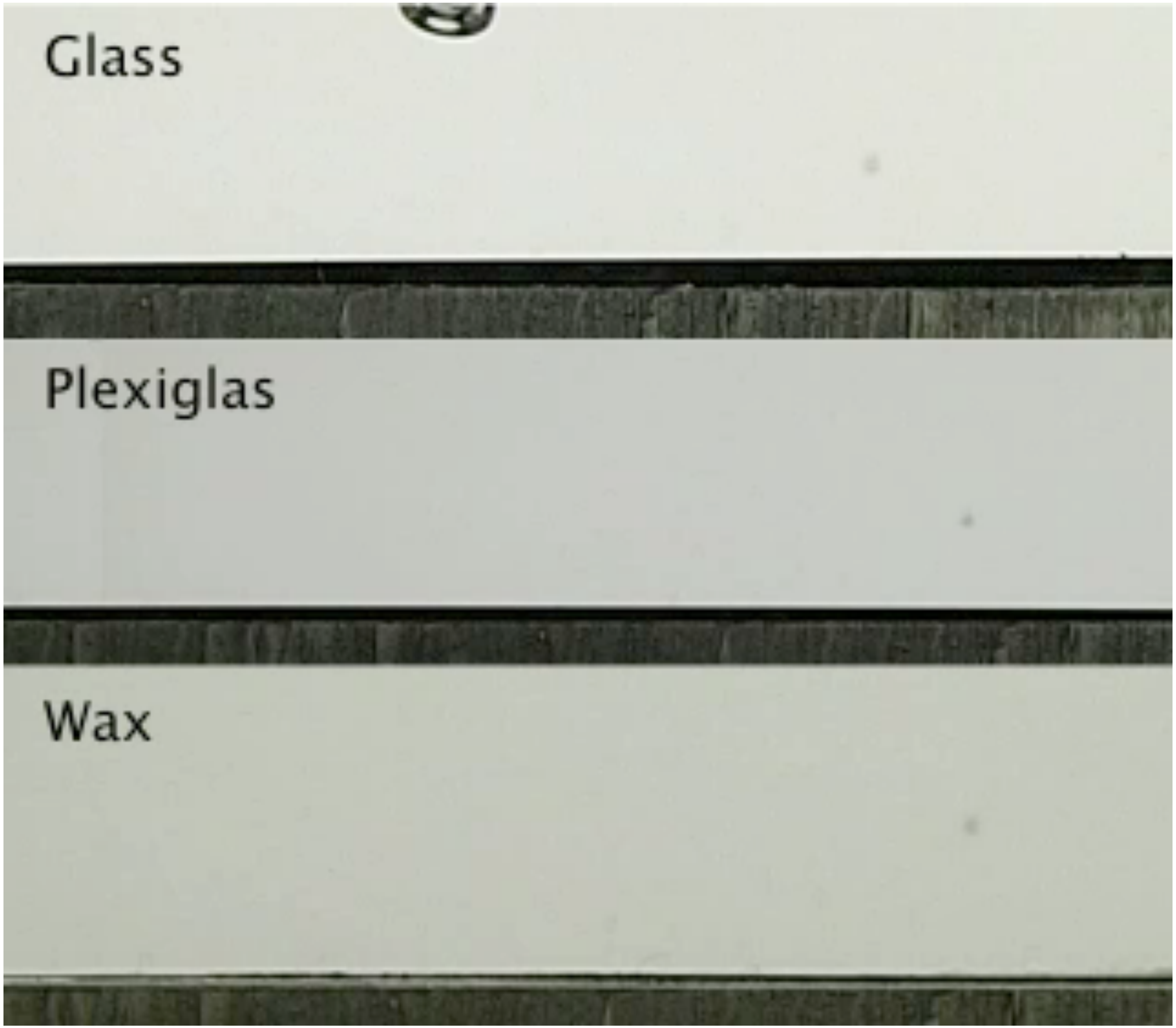
Hydrophobic surface

Hydrophilic surface

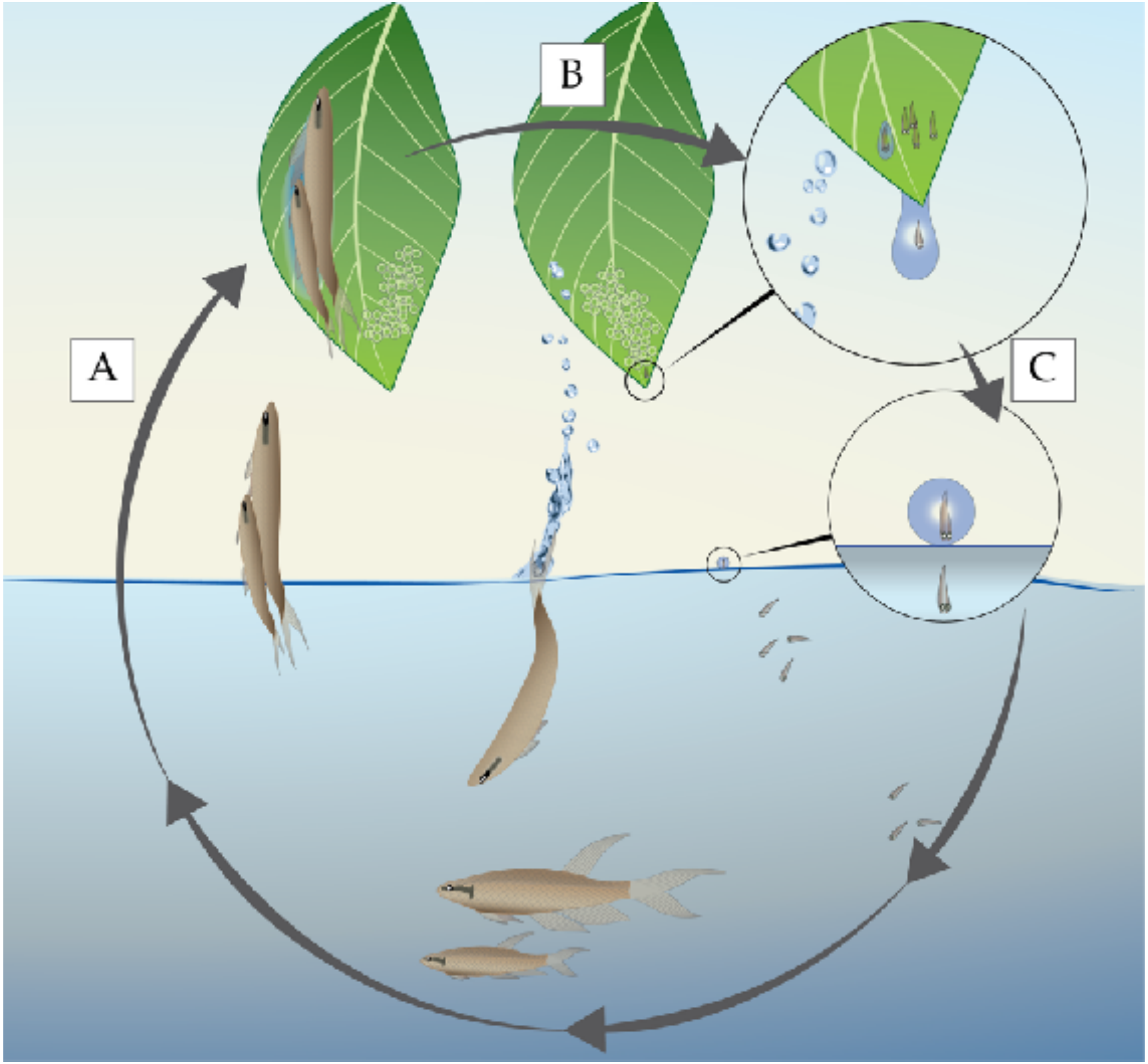
Spontaneous motion in response to a wettability gradient



Puddles revisited



Plenty of fish in the trees



Capillary adhesion in nature