18.357: Lecture 2

Surface tension:

History, motivation and physical origins

John W. M. Bush

Department of Mathematics
MIT

I. History

- surface tension in antiquity

II. Motivation

- who cares about surface tension?

III. Physical origins

- a heuristic discussion

I. The history of surface tension

Surface tension in antiquity

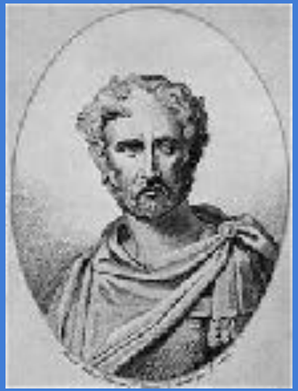


Hero of Alexandria (~ 100 AD)

- greek mathematician and engineer,
  ``the greatest experimentalist of antiquity"

- exploited capillarity in a number of inventions
  described in his book, *Pneumatics*
  , including the water clock

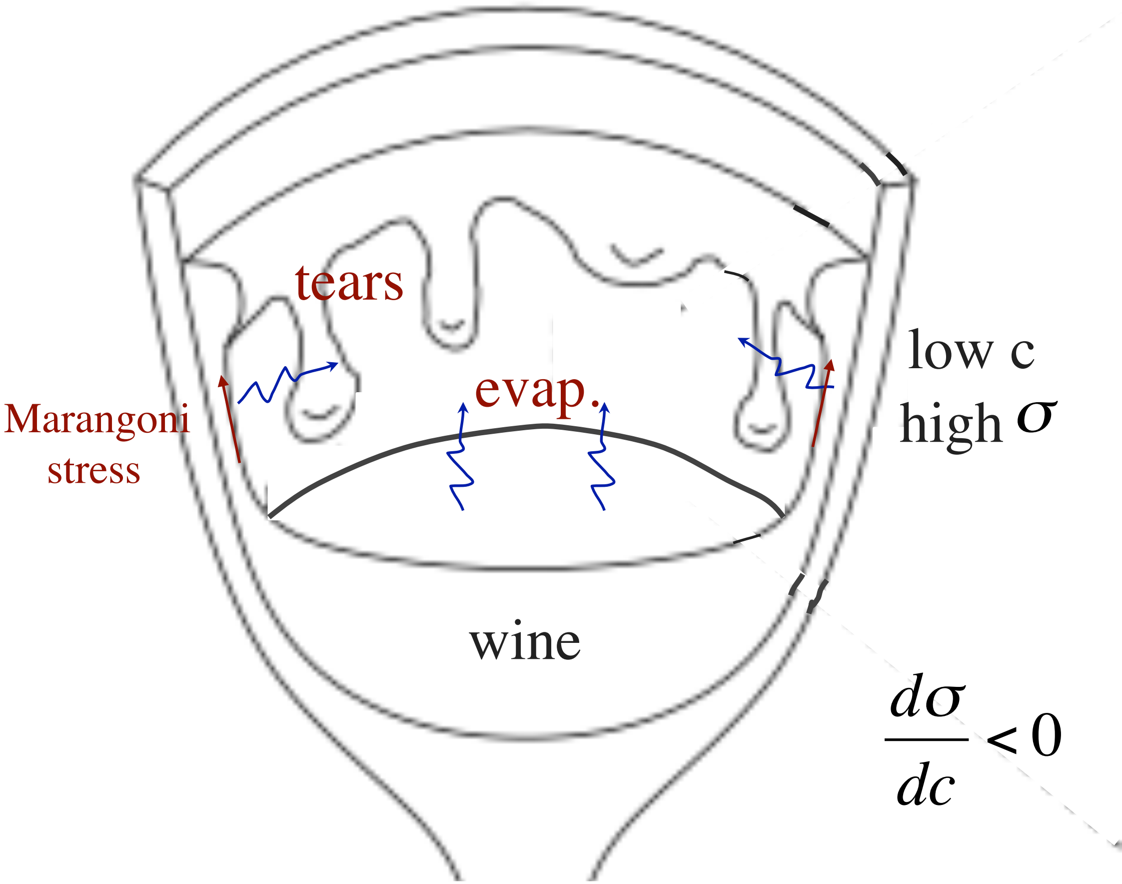
Pliny the Elder (~ 50 AD)



- author, natural philosopher, army and naval commander of the early Roman Empire
- described the glassy wakes of ships

"True glory comes in doing what deserves to be written; in writing what deserves to be read; and in so living as to make the world happier."

"Truth comes out in wine."



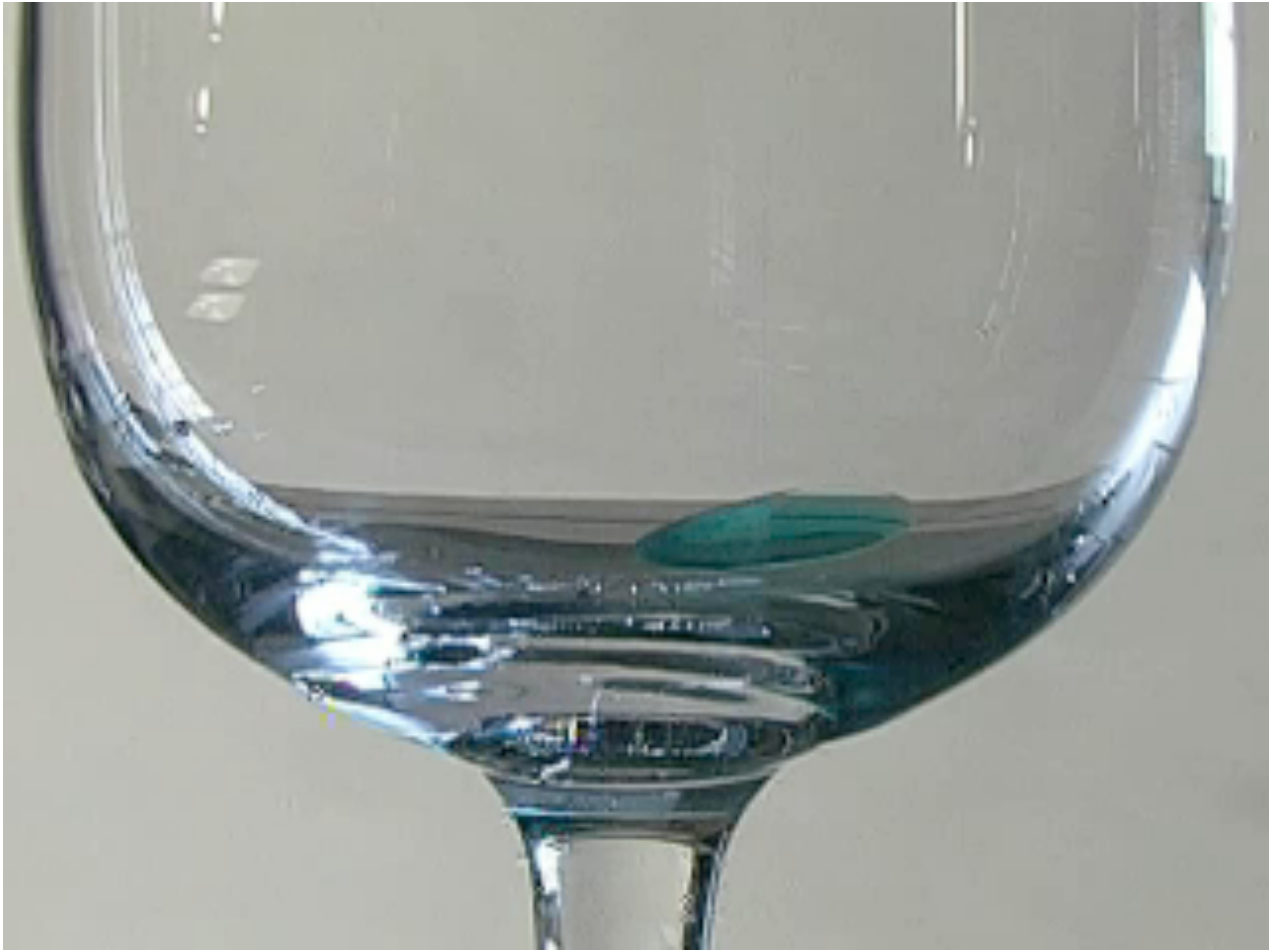
The tears of wine

"Who hath sorrow? Who hath woe? They that tarry long at the wine.
Look not though upon the strong red wine *that moveth itself aright*.
At the last it biteth like a serpent and stingeth like an adder."

- Proverbs 23: 29-32 (c.a. 950 BC)

King Solomon, "the wisest man that ever lived".

The tears of wine



The first `Marangoni flow' studied scientifically (Thomson 1855).

Surface tension in history

Leonardo da Vinci (1452-1519)

- reported capillary rise in his notebooks
- hypothesized that mountain streams are fed by capillary networks

Francis Hauksbee (1666-1713)

- conducted systematic investigation of capillary rise
- his work was described in Newton's *Opticks*, but no mention was made of Hauksbee

Benjamin Franklin (1706-1790)

- polymath: scientist, inventor, politician
- examined the ability of oil to suppress waves

Surface tension in history

Pierre-Simon Laplace (1749-1827)

- french mathematician and astronomer
- elucidated the concept and theoretical description of the meniscus: hence, *Laplace pressure*

Thomas Young (1773-1829)

- polymath, solid mechanician, scientist, linguist
- demonstrated wave nature of light with ripple tank expts
- described wetting of a solid by a fluid

Joseph Plateau (1801-1883)

- Belgian physicist, continued his expts after losing his sight
- extensive study of capillary phenomena, soap films, minimal surfaces, drops and bubbles

II. Motivation

- who cares about surface tension?

Motivation: who cares about surface tension?

As we shall soon see, surface tension dominates gravity on a scale less than the capillary length, ~2 mm.

Biology

- all small creatures live in a world dominated by surface tension
- surface tension important for insects for many basic functions
- weight support and propulsion at the water surface

- adhesion and deadhesion via surface tension
- the archer fish: hunting with drops (VIDEO)
- the pistol shrimp: hunting with bubbles (VIDEO)
- underwater breathing and diving via surface tension
- natural strategies for water-repellency in plants and animals
- the hydraulics of trees
- the dynamics of lungs and the role of surfactants and impurities

Hunting with drops

The Archer Fish

Motivation: who cares about surface tension?

As we shall soon see, surface tension dominates gravity on a scale less than the capillary length, ~2 mm.

Biology

- all small creatures live in a world dominated by surface tension
- surface tension important for insects for many basic functions
- weight support and propulsion at the water surface

- adhesion and deadhesion via surface tension
- the archer fish: hunting with drops (VIDEO)
- the pistol shrimp: hunting with bubbles (VIDEO)
- underwater breathing and diving via surface tension
- natural strategies for water-repellency in plants and animals
- the hydraulics of trees
- the dynamics of lungs and the role of surfactants and impurities

Hunting with bubbles

The Pistol Shrimp

Motivation: who cares about surface tension?

As we shall soon see, surface tension dominates gravity on a scale less than the capillary length, ~2 mm.

Biology

- all small creatures live in a world dominated by surface tension
- surface tension important for insects for many basic functions
- weight support and propulsion at the water surface
- adhesion and deadhesion via surface tension
- the archer fish: hunting with drops (VIDEO)
- the pistol shrimp: hunting with bubbles (VIDEO)
- underwater breathing and diving via surface tension
- natural strategies for water-repellency in plants and animals
- the hydraulics of trees
- the dynamics of lungs and the role of surfactants and impurities

Motivation: who cares about surface tension?

Geophysics and environmental science

- the dynamics of raindrops and their role in the biosphere
- most biomaterial is surface active, sticks to surface of drops/bubbles
- chemical, thermal and biological transport in the surf zone
- early life: early vessicle formation, confinement to an interface
- oil recovery, carbon sequestration, groundwater flows
- design of insecticides intended to coat insects, leave plant unharmed
- chemical leaching and the water-repellency of soils: *desertification*
- oil spill dynamics and mitigation (*e.g.* use of dispersants in BP spill)
- disease transmission via droplet exhalation (*e.g.* COVID-19)
- dynamics of magma chambers and volcanoes
- the exploding lakes of Cameroon

Ref. Drops and bubbles in the environment, Bourouiba & Bush (2012)

Motivation: who cares about surface tension?

Technology

- capillary effects dominant in microgravity settings: NASA (Video)
- cavitation-induced damage on propellers and submarines
- design of superhydrophobic surfaces

  e.g. self-cleaning windows, drag-reduction, erosion-resistant surfaces
- lab-on-a-chip technology: medical diagnostics, drug delivery
- microfluidics: continuous and discrete fluid transport and mixing (Video)
- tracking submarines with their surface signature
- inkjet printing
- the bubble computer (Video)

Drinking in space

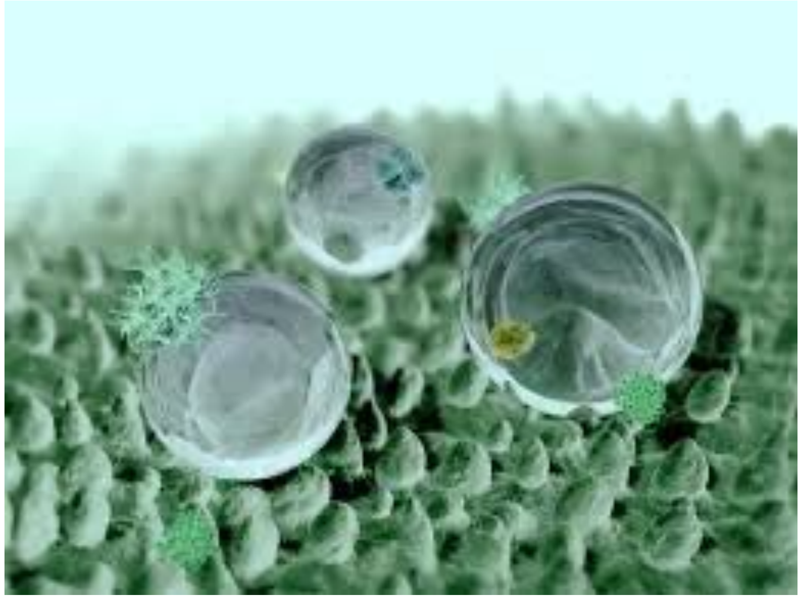
Motivation: who cares about surface tension?

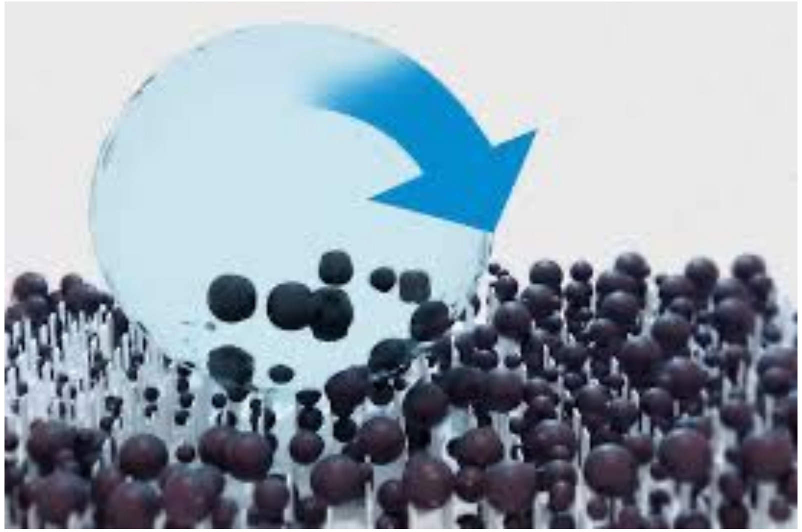
Technology

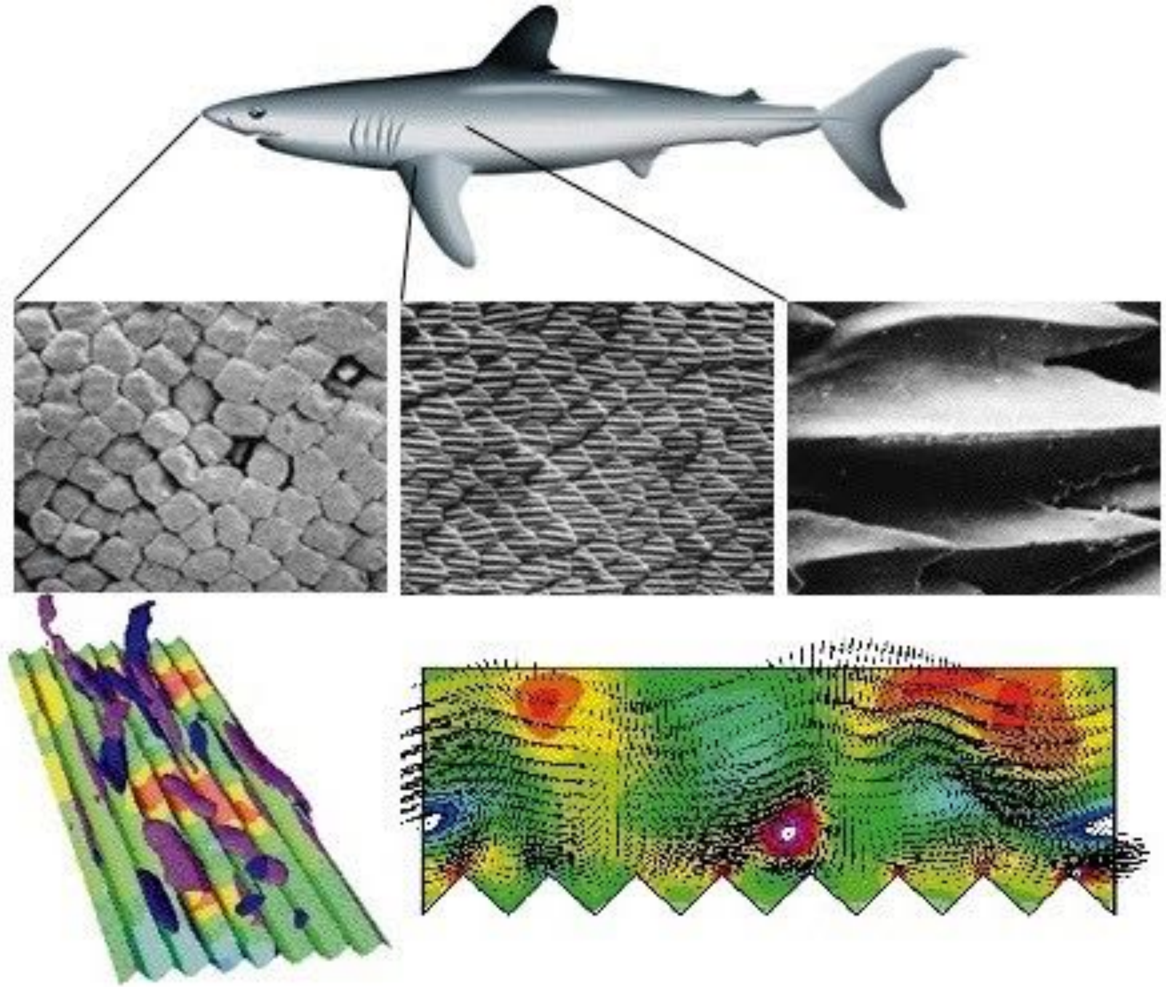
- capillary effects dominant in microgravity settings: NASA (Video)
- cavitation-induced damage on propellers and submarines
- design of superhydrophobic surfaces (Slides)

  e.g. self-cleaning windows, drag-reduction, erosion-resistant surfaces
- lab-on-a-chip technology: medical diagnostics, drug delivery
- microfluidics: continuous and discrete fluid transport and mixing (Slides)
- tracking submarines with their surface signature
- inkjet printing
- the bubble computer (Video)

Biomimetic surfaces: for water-repellency, self-cleaning, and drag reduction







Motivation: who cares about surface tension?

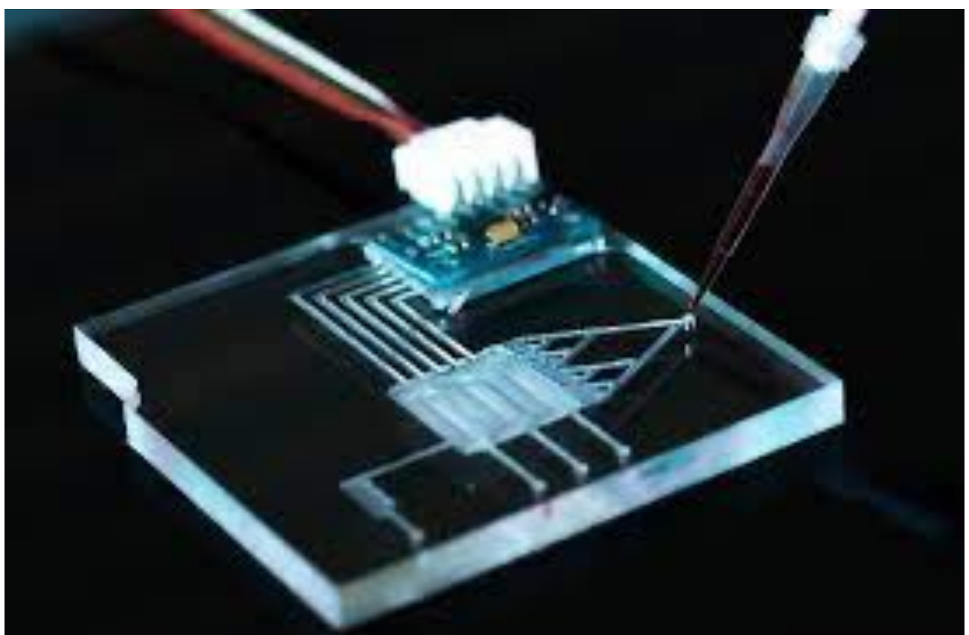
Technology

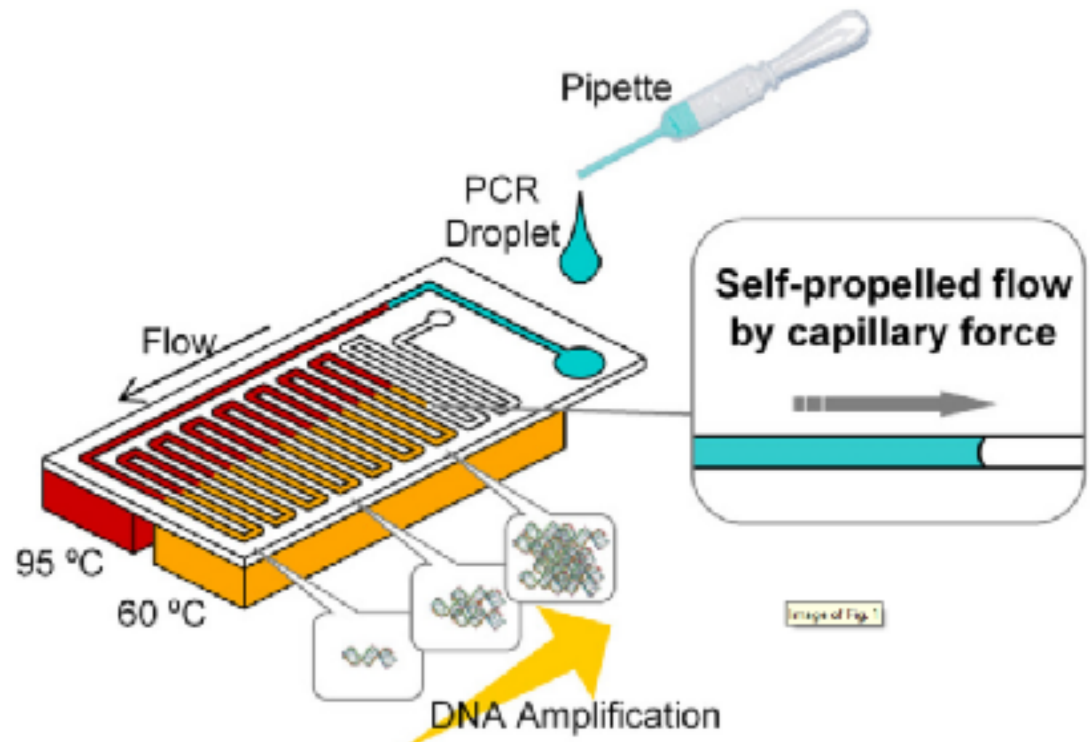
- capillary effects dominant in microgravity settings: NASA (Video)
- cavitation-induced damage on propellers and submarines
- design of superhydrophobic surfaces (Slides)

  e.g. self-cleaning windows, drag-reduction, erosion-resistant surfaces
- lab-on-a-chip technology: medical diagnostics, drug delivery
- microfluidics: continuous and discrete fluid transport and mixing (Slides)
- tracking submarines with their surface signature
- inkjet printing
- the bubble computer (Video)

Microfluidics: `*lab on a chip*'

- anticipated in Feynman's 1959 Lecture: `*Plenty of Room at the Bottom*'
- manipulation of fluid in channels of typical scale 10-100 microns
- applications: capillary electrophoresis, DNA analysis, cell sorting
- diagnostics: cancer and pathogen detection
- digital microfluidics: drops and bubbles play the role of vessels



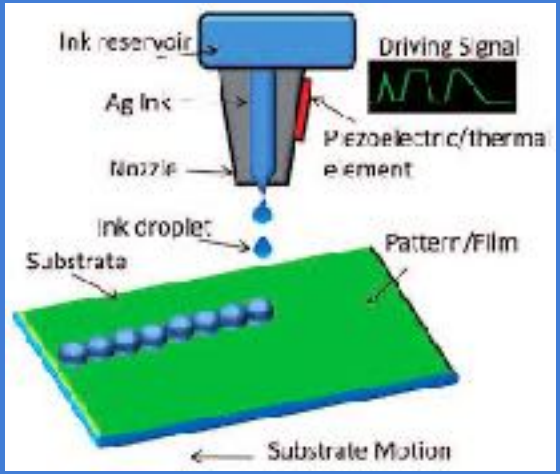


Motivation: who cares about surface tension?

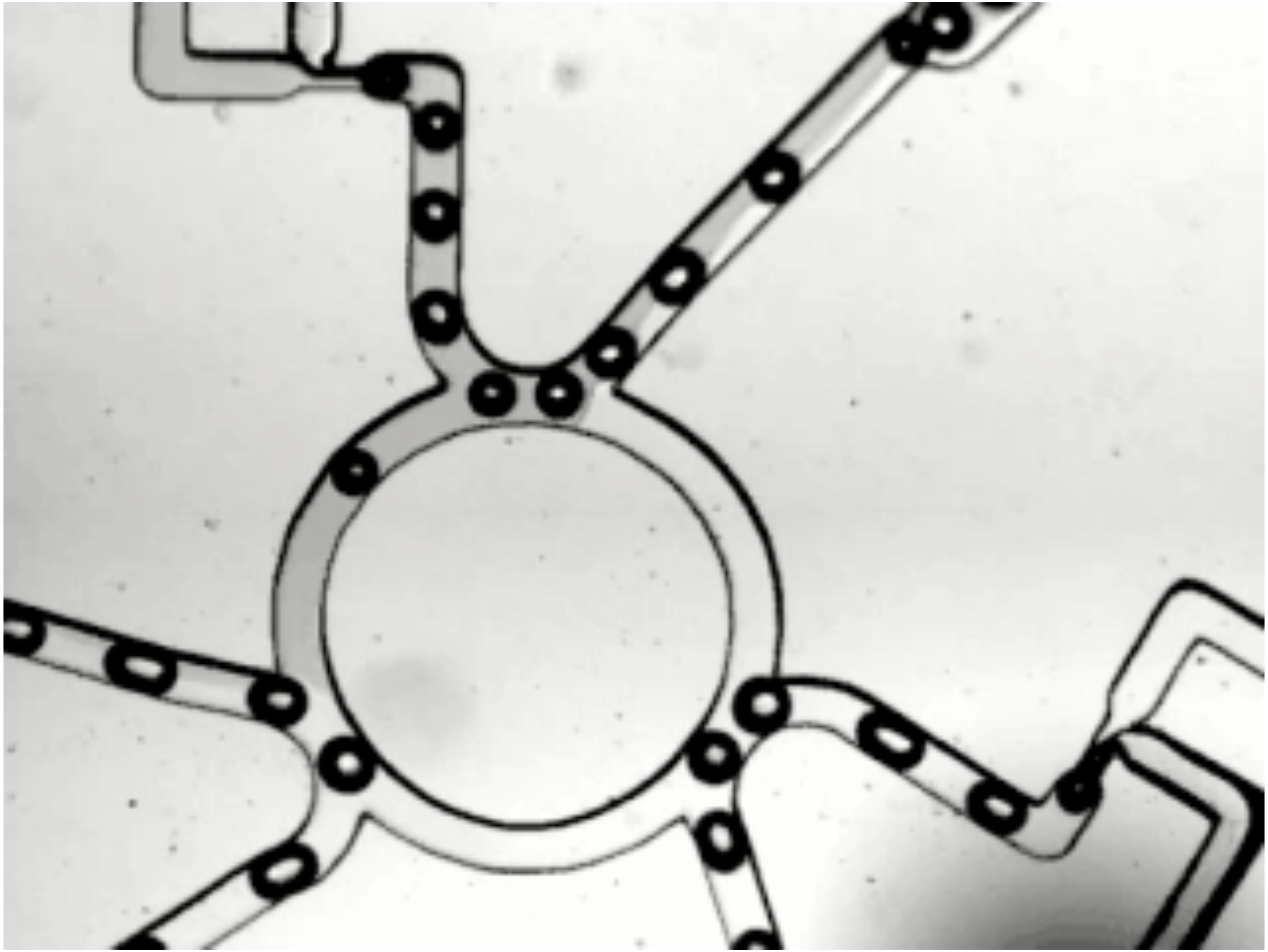
Technology

- capillary effects dominant in microgravity settings: NASA
- cavitation-induced damage on propellers and submarines
- design of superhydrophobic surfaces

  e.g. self-cleaning windows, drag-reduction, erosion-resistant surfaces
- lab-on-a-chip technology: medical diagnostics, drug delivery
- microfluidics: continuous and discrete fluid transport and mixing
- tracking submarines with their surface signature
- inkjet printing
- the bubble computer (Video)



The bubble computer

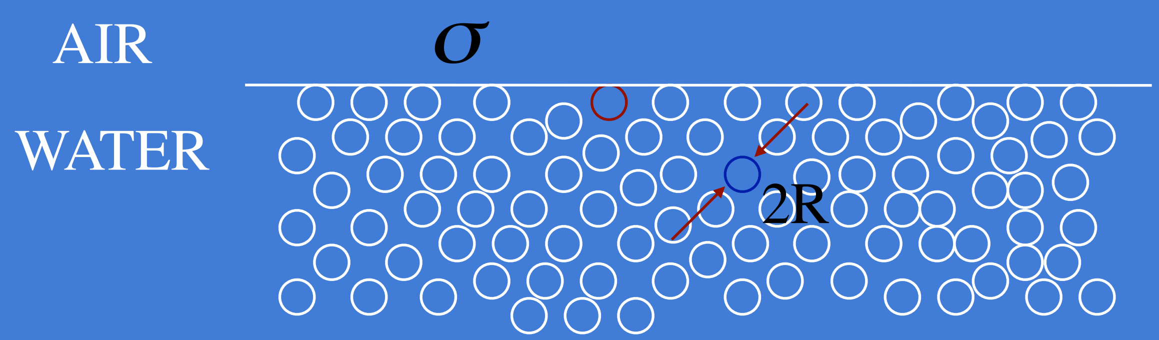


- bubbles in channels act as electrons in circuits

III. The physical origins of surface tension

Surface Tension: molecular origins

- each molecule in a fluid feels a cohesive force with surrounding molecules
- molecules at interface feel half this force; are in an energetically unfavourable state
- the creation of new surface is thus energetically costly



- cohesive energy per molecule of radius R in bulk is U, at surface is U/2
- surface tension is this loss of cohesive energy per unit area:

$$\sigma \sim \frac{U}{R^2}$$

Units: $$[\sigma] = \frac{\text{ENERGY}}{\text{AREA}} = \frac{\text{FORCE}}{\text{LENGTH}}$$

- air-water $\sigma \sim 70$ dyne/cm; oils $\sigma \sim 20$ dyne/cm; liquid metals $\sigma \sim 500$ dyne/cm

Surface Tension

- a tensile force per unit length acting at gas-liquid interfaces

Nomenclature

- σ denotes surface tension
- γ denotes interfacial tension (as arises at *any* interface: liquid-liquid, solid-liquid, solid-gas)

A note on units: *I prefer cgs*

$$1 \text{ dyne} = 1 \text{ g cm/s}^2 = 10^{-5} \text{ N} = \text{cgs unit of force}$$

For comparison,

$$1 \text{ atm} = 101 \text{ kPa} = 10^5 \text{ N/m}^2 = 10^6 \text{ dynes/cm}^2$$

$$\rightarrow \quad [\sigma] = \text{dynes/cm} = \text{mN/m}$$

Some numbers for

$$\sigma \sim \frac{U}{R^2}$$

H_20 : hydrogen bonds
— high U

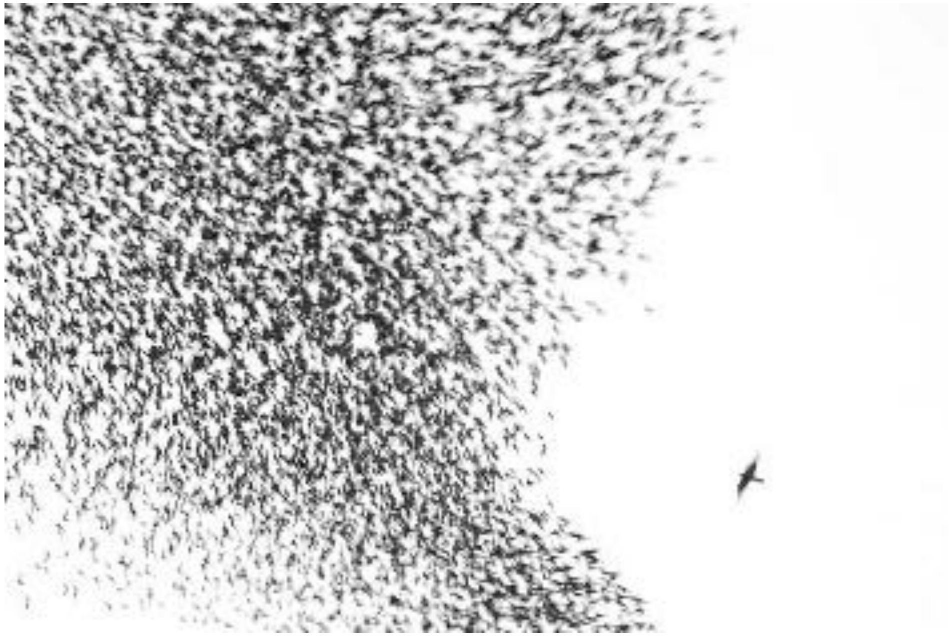
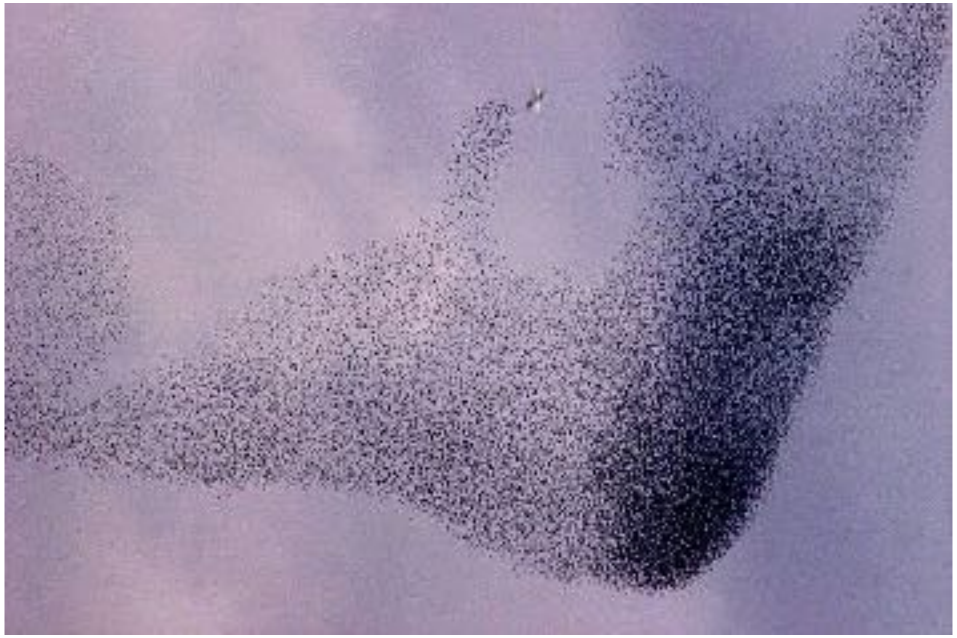
Oils: U ~ van de Waals
~ kT ~ 1/40 eV at 25^{o}C

Mercury: strongly cohesive
liquid metal: U ~ 1eV

Interface	σ (dynes/cm)
air - water (20 ^{o}C)	72
air - soapy water	30 - 35
air - water w/ superwetting agent	20
air - water (100 ^{o}C)	58
air - ethanol	23
oil - water	$\gamma \sim 40$
air - olive oil	30
air - Si oil	20
air - He (4 ^{o}K)	0.1
air - molten glass (800 ^{o}K)	500
air - mercury	415
air - glycerol	63

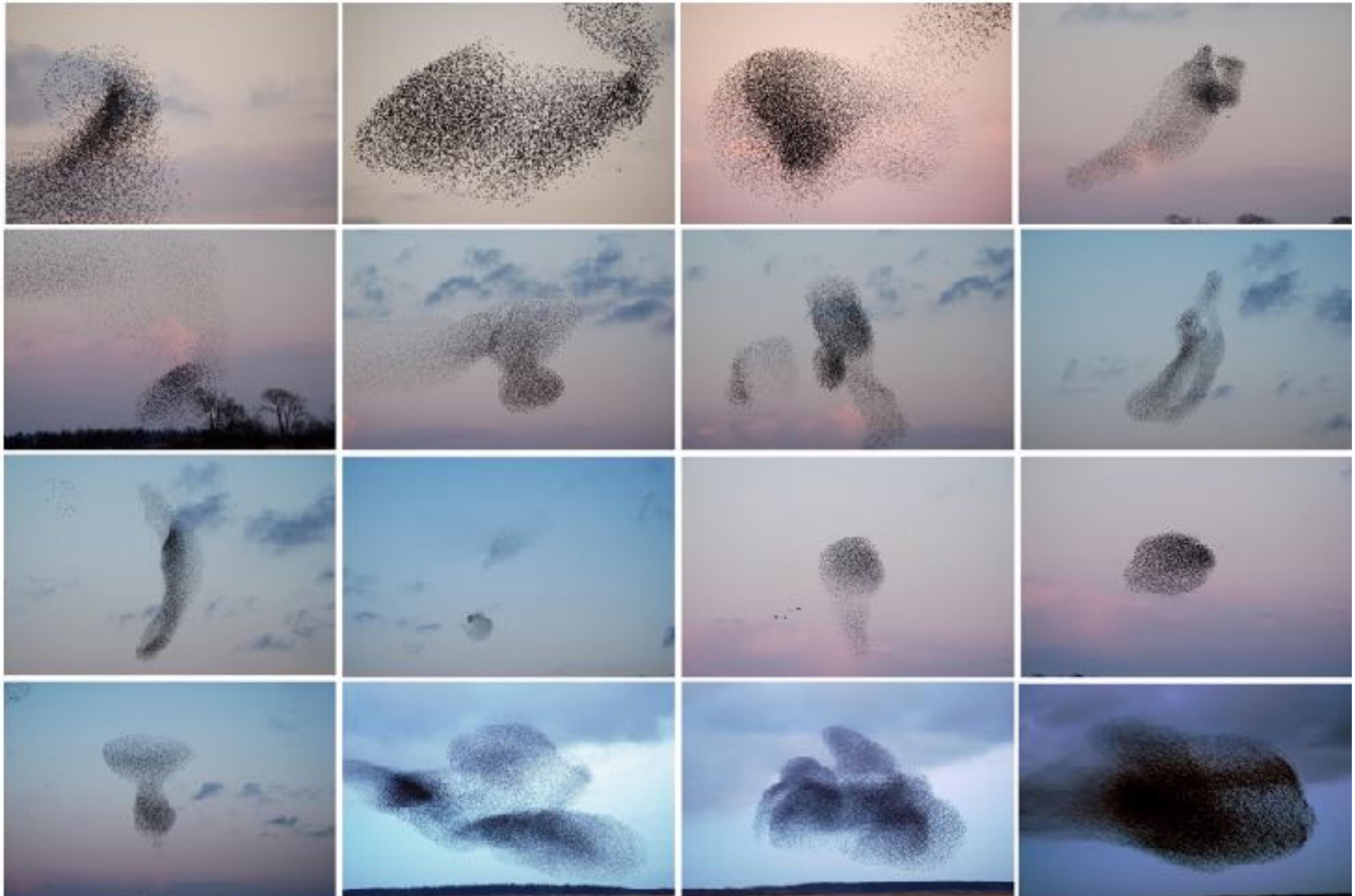
Surface tension in flocks, schools and swarms?



Might the cost of being on the edge give rise to analogous behavior?

Starling flocks

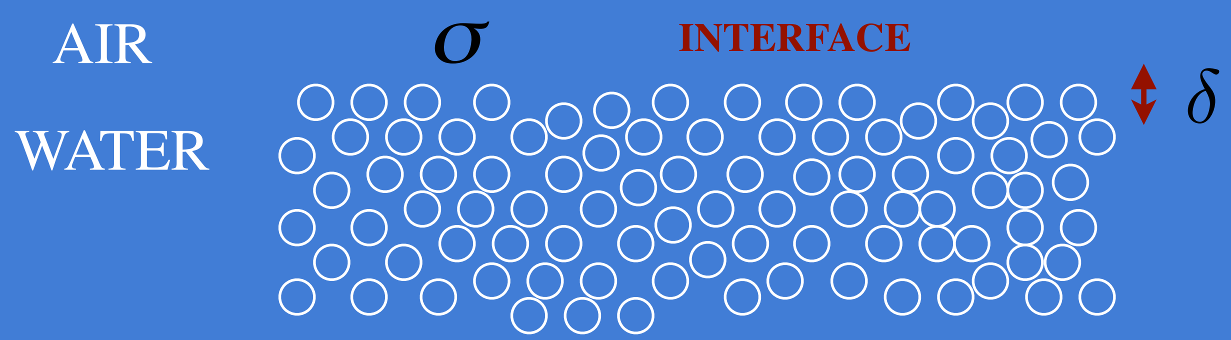


Might the cost of being on the edge give rise to analogous behavior?

Related question: the dynamics of cycling pelotons?

What is an interface?

- an idealized surface between two *immiscible* fluids; e.g. oil-water, air-water, oil-air
- there is no surface tension between miscible fluids, e.g. water-salt water
- in reality, the interface is rough on a molecular scale



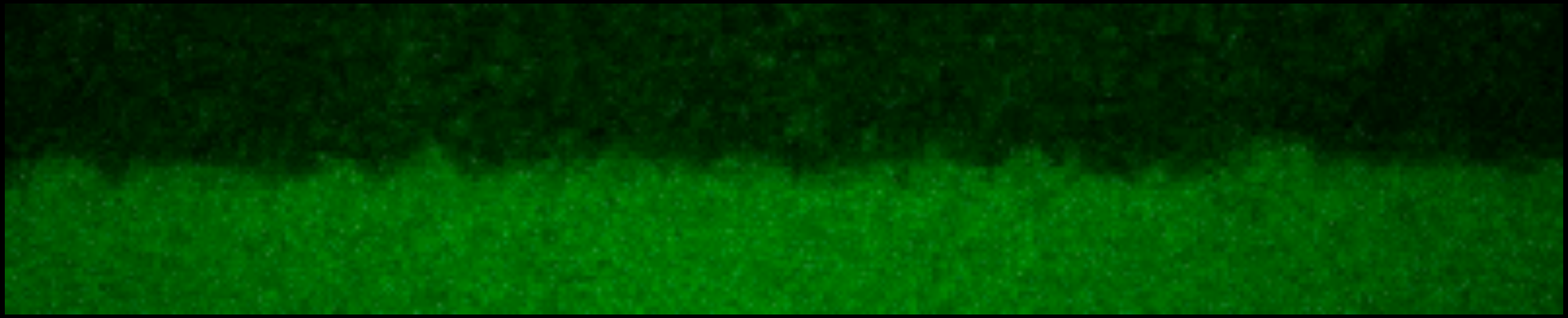
Roughness scale δ

- equate anomalous surface energy with thermal agitation energy

$$\sigma\delta^2 \sim kT \quad \longrightarrow \quad \boxed{\delta \sim (kT/\sigma)^{1/2}}$$

- treating the interface as sharp is consistent with the continuum hypothesis, wherein one assumes fluids are smooth beyond 10 molecular dimensions

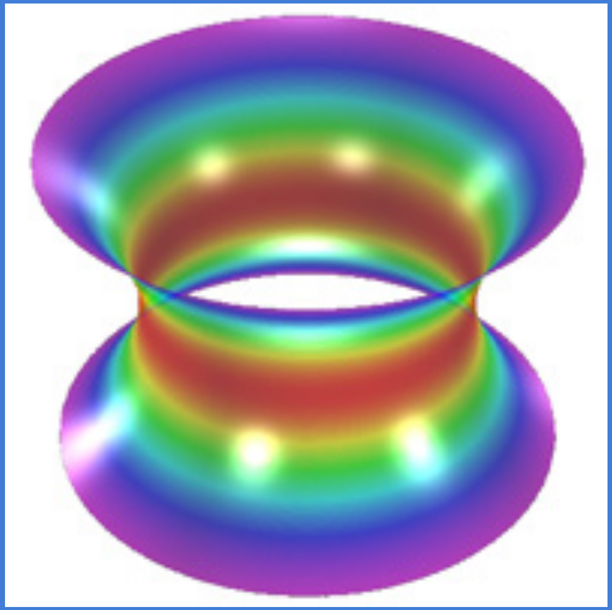
Evaporation across a fluid interface



• thermal agitation overcomes interfacial tension

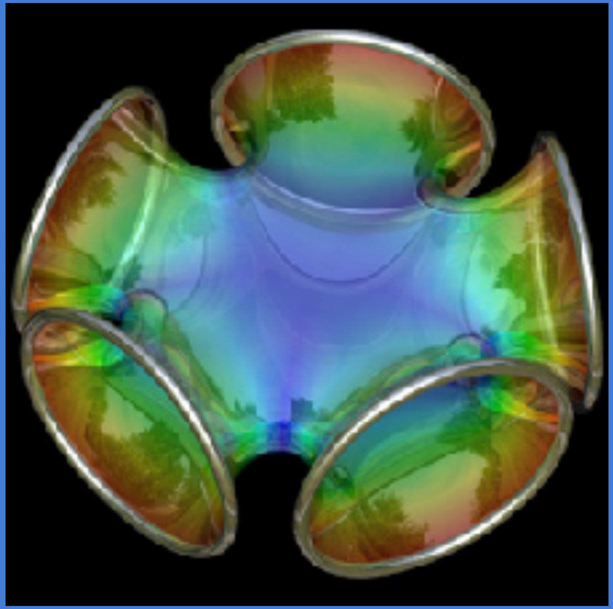
The creation of surface is energetically costly

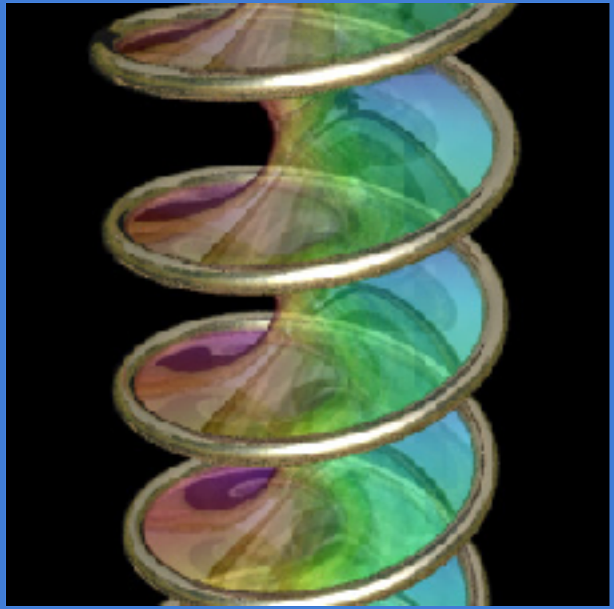
- quasi-static soap films (for which gravity, inertia are negligible) take the form of minimal surfaces

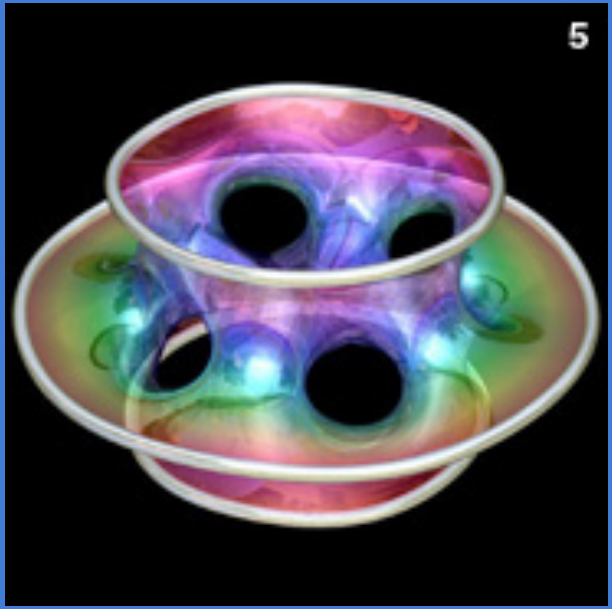


- hence their interest to mathematicians:

"Find the minimal surface bound by the multiply connected curve C, where C ...."



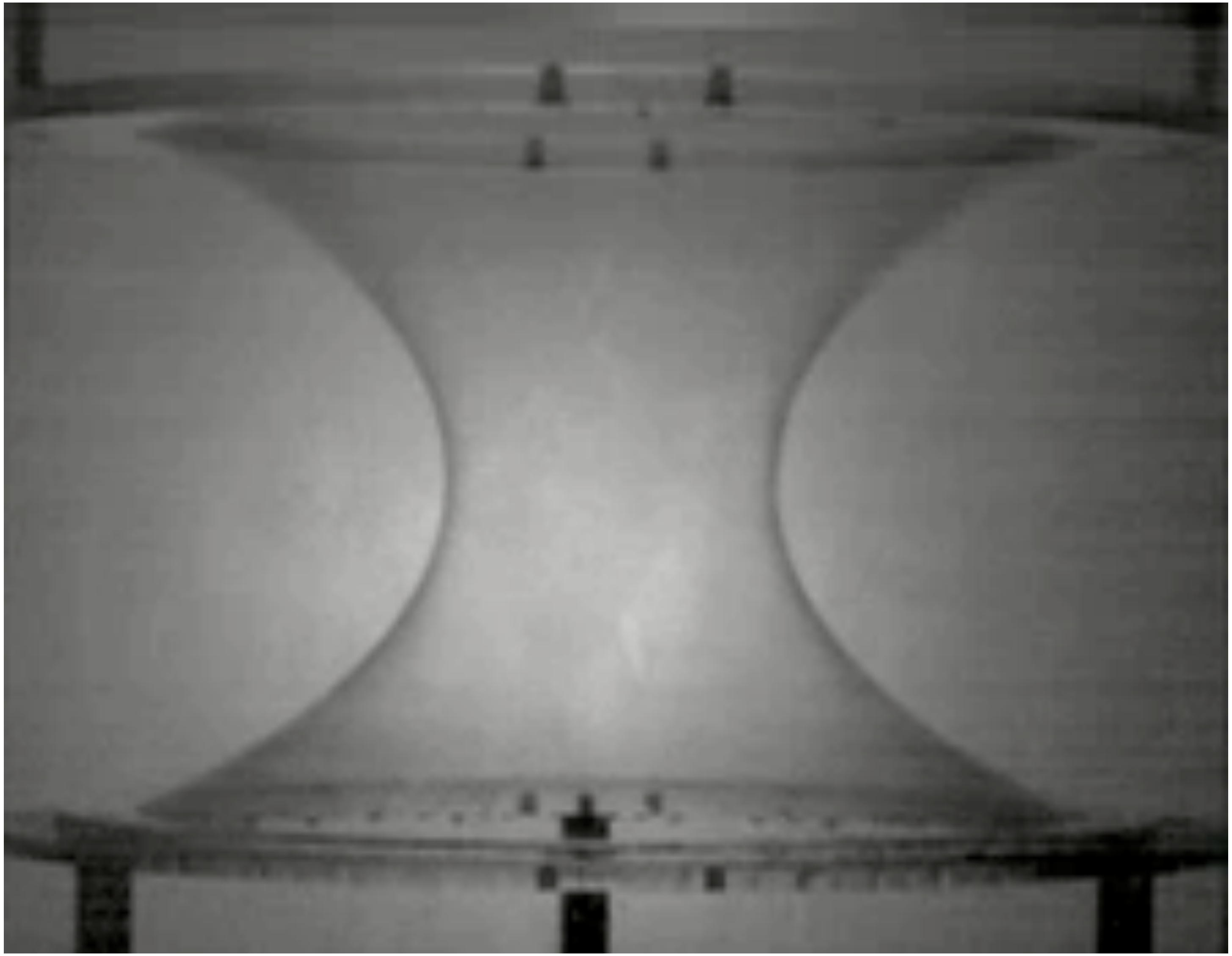




The minimal surface between a pair of rings

A catenoid when the rings are close,
a pair of circles when they are far apart.

The rupture of a catenoidal soap film

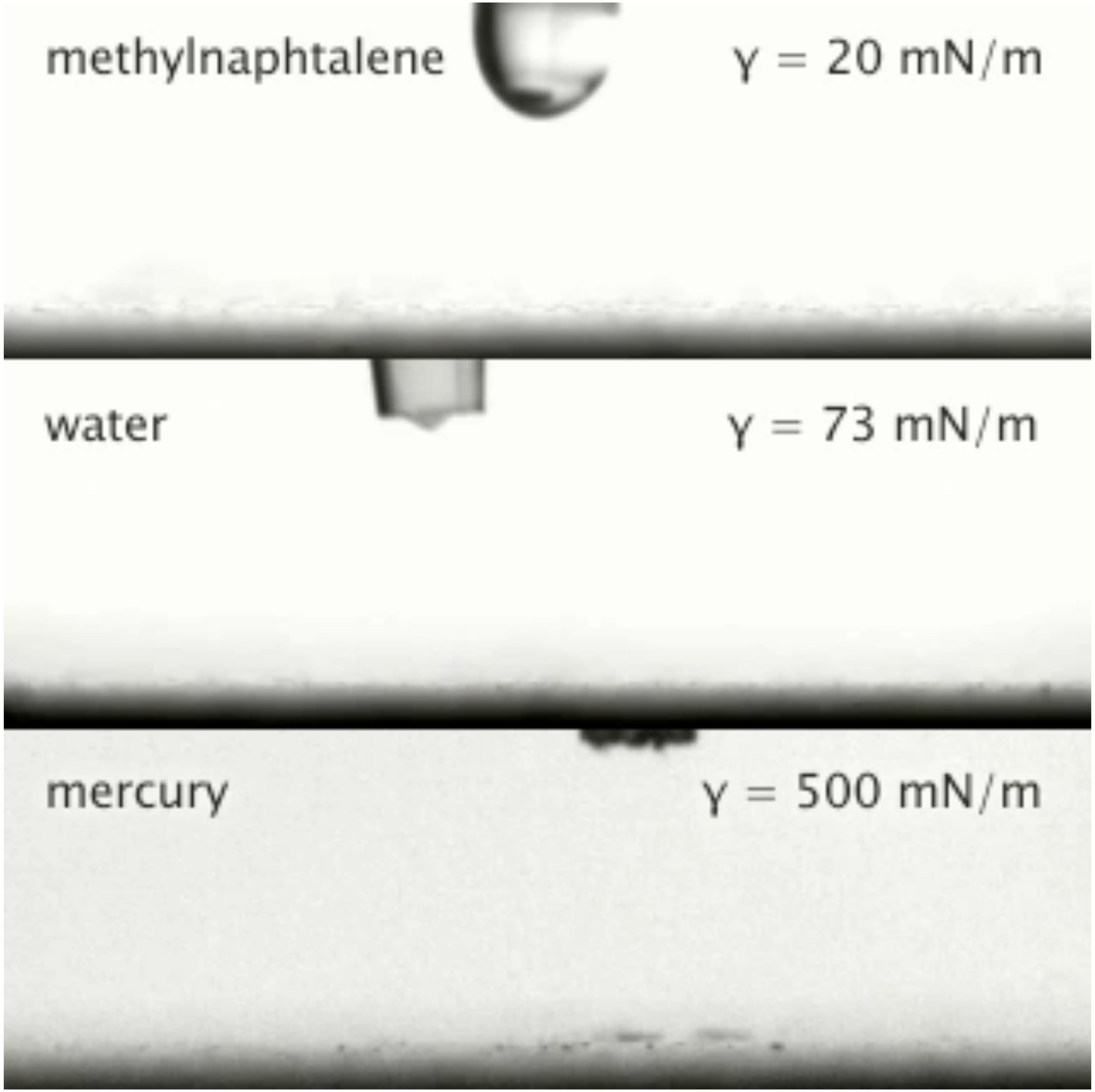


There is a critical distance at which a catenoid breaks.

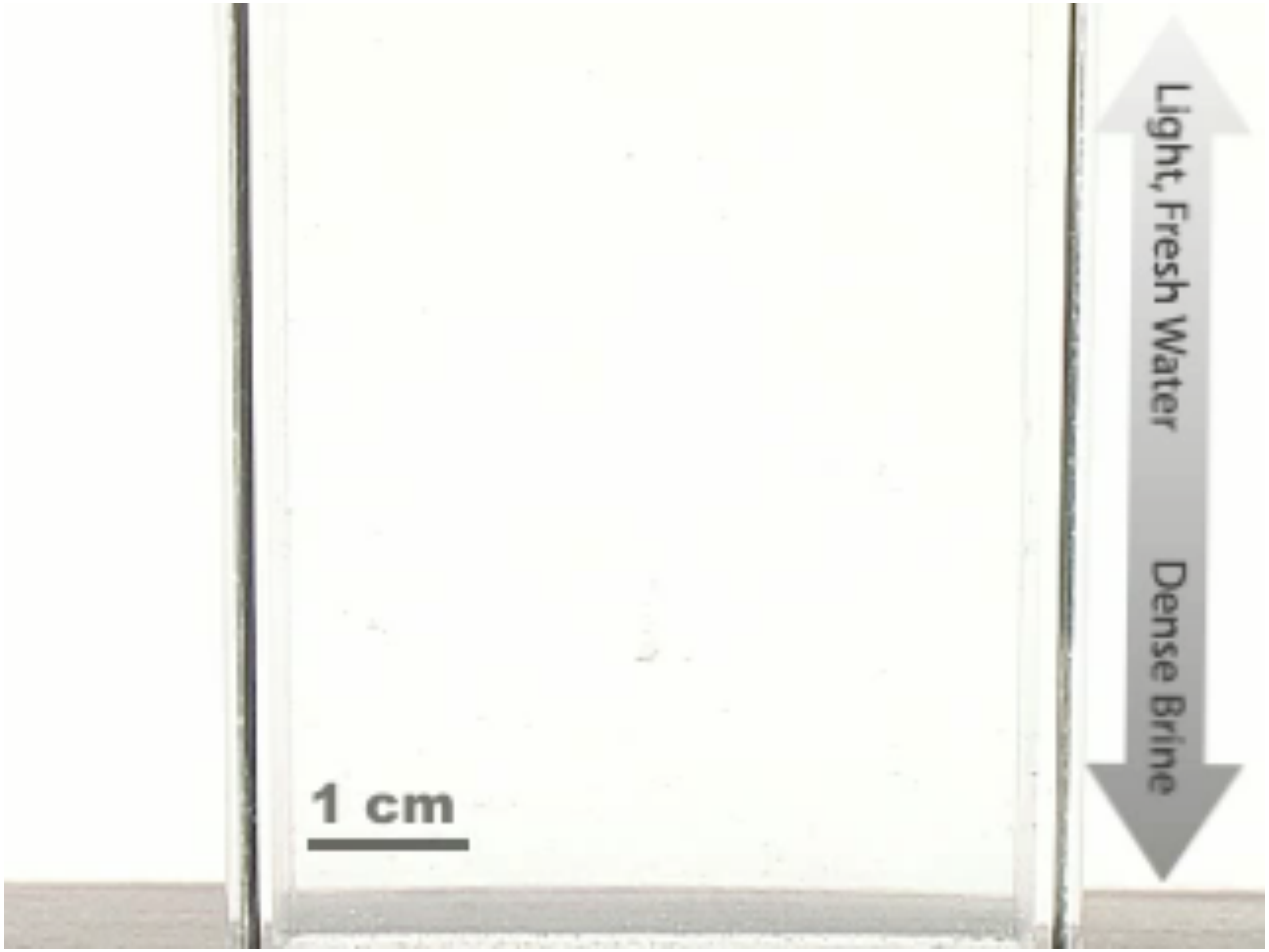
The creation of surface is energetically costly

Thus:

- small drops are nearly spherical (MFM)
- fluid jets pinch off into droplets (MFM)
- fluid atomization results in spherical drops (MFM)
- wet hair sticks together: the “wet look” (MFM)
- bubbles and films are fragile (MFM)



Form here influenced by gravity, wettability of substrate.

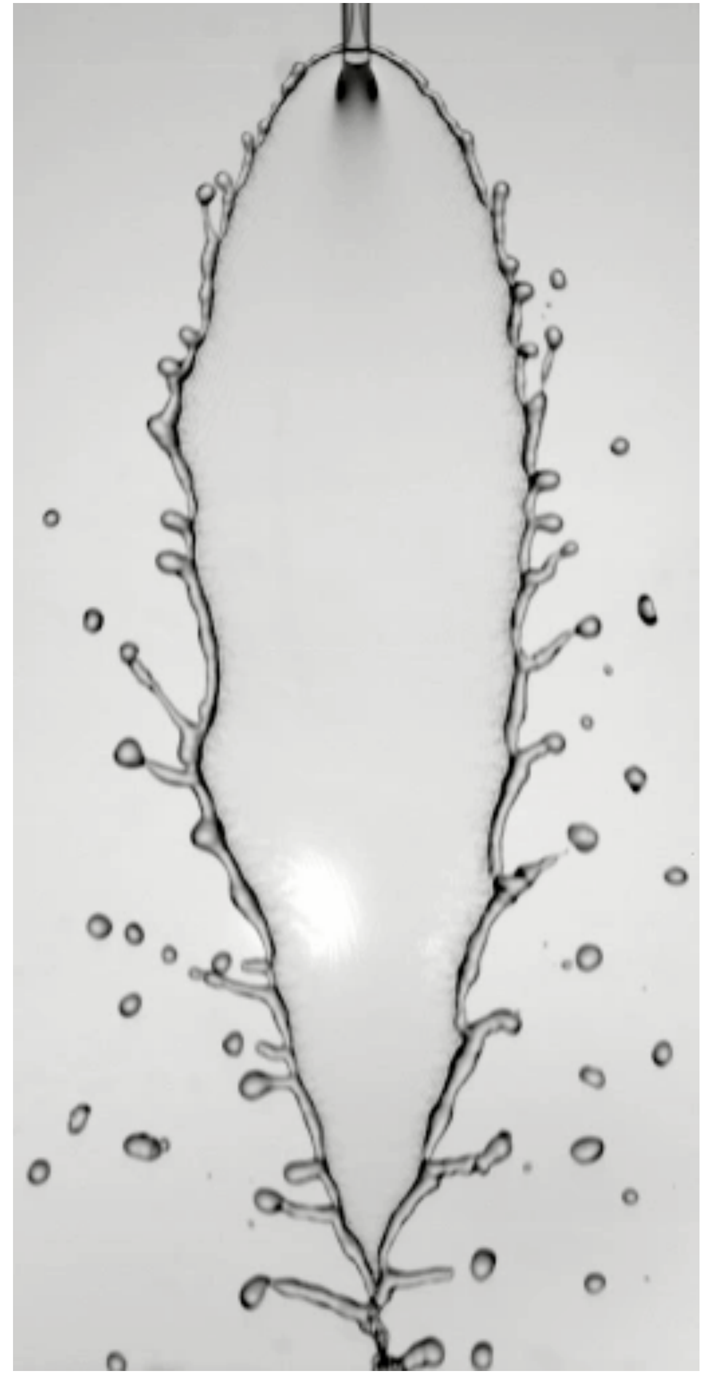


- large spherical drops deduced by eliminating distorting influence of gravity

The creation of surface is energetically costly

Thus:

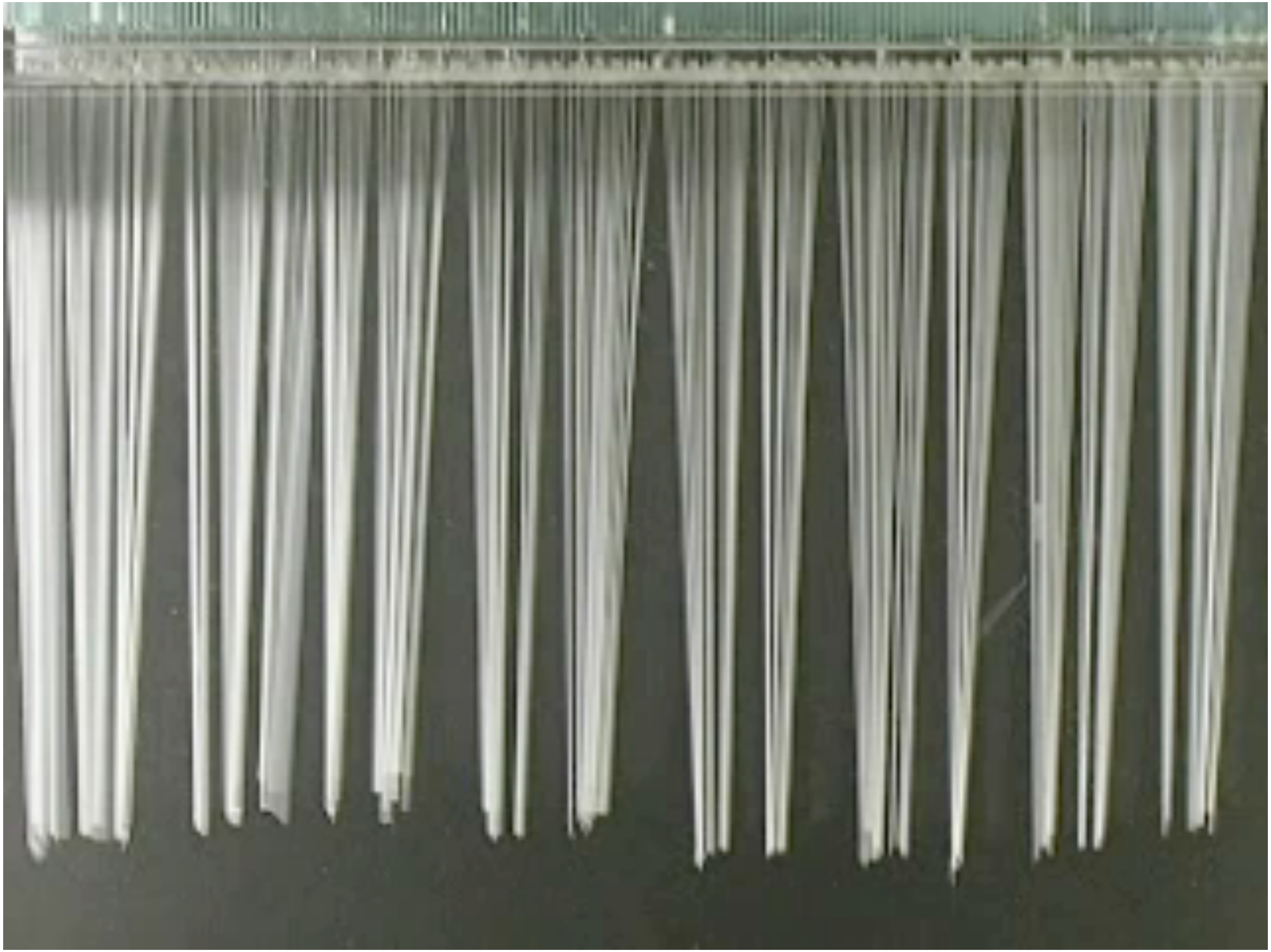
- small drops are nearly spherical (MFM)
- fluid jets pinch off into droplets (MFM)
- fluid atomization results in spherical drops (MFM)
- wet hair sticks together: the “wet look” (MFM)
- bubbles and films are fragile (MFM)

The creation of surface is energetically costly

Thus:

- small drops are nearly spherical (MFM)
- fluid jets pinch off into droplets (MFM)
- fluid atomization results in spherical drops (MFM)
- wet hair sticks together: the "wet look" (MFM)
- bubbles and films are fragile (MFM)

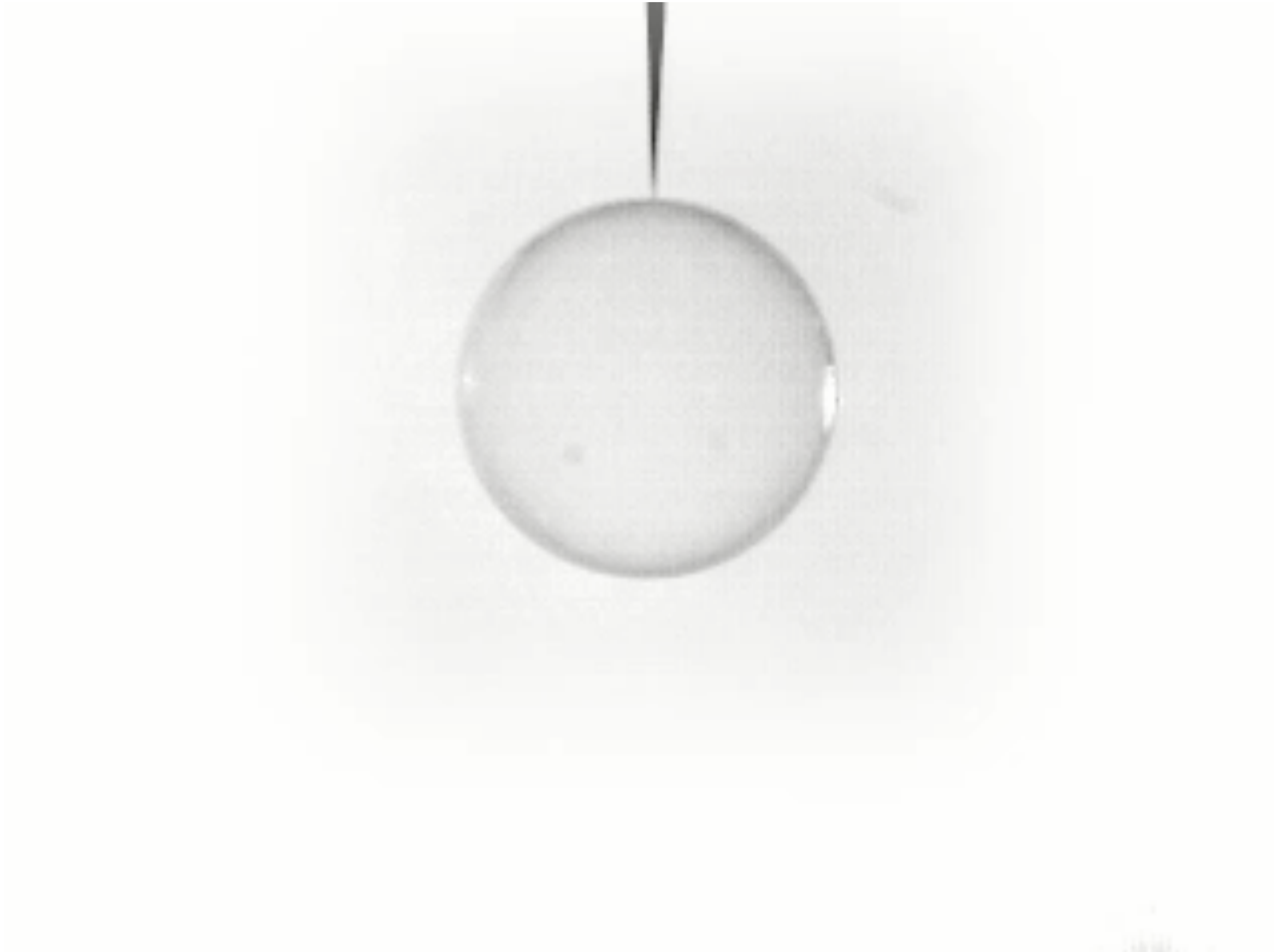


The wet hair instability: threads clump to minimize surface energy

The creation of surface is energetically costly

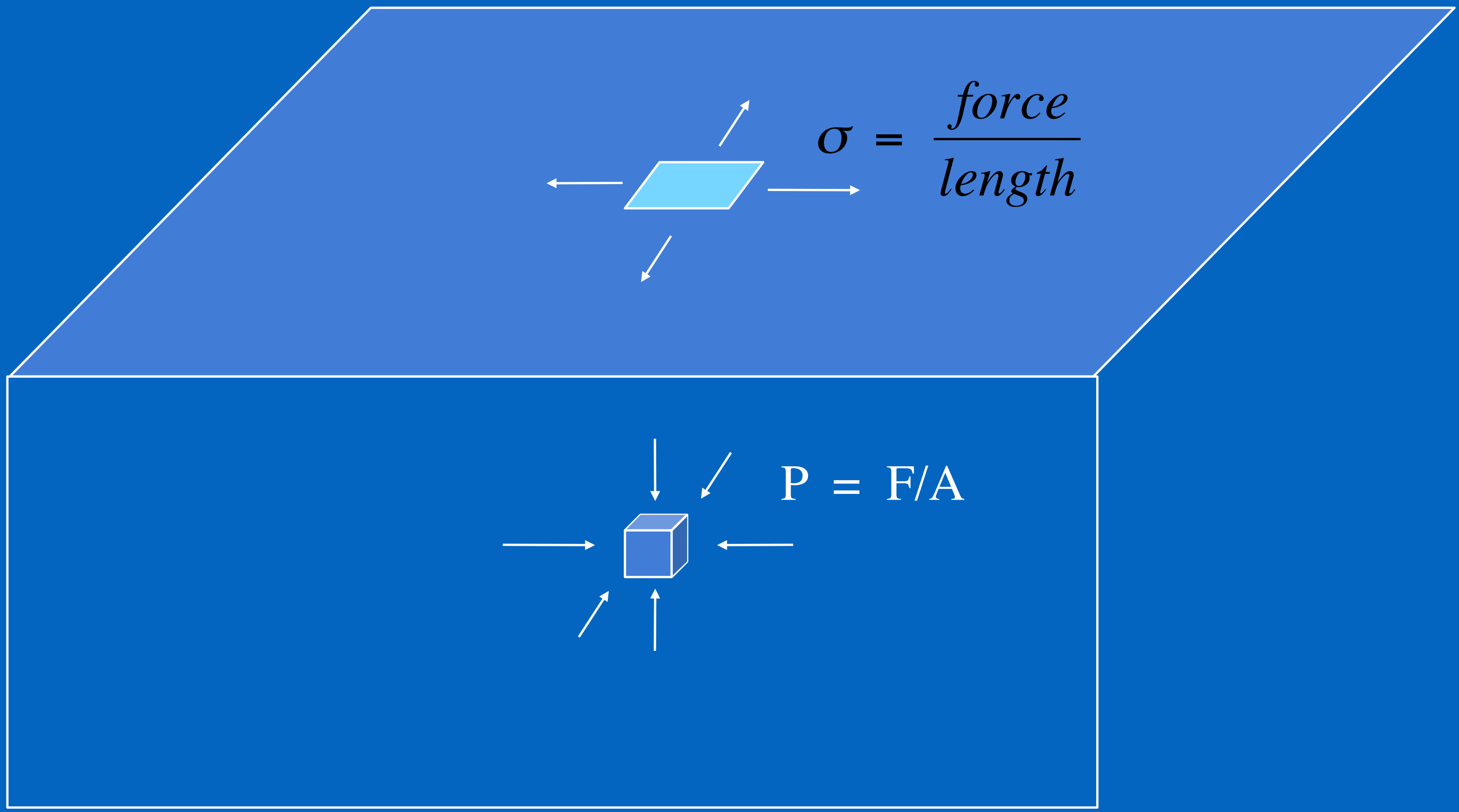
Thus:

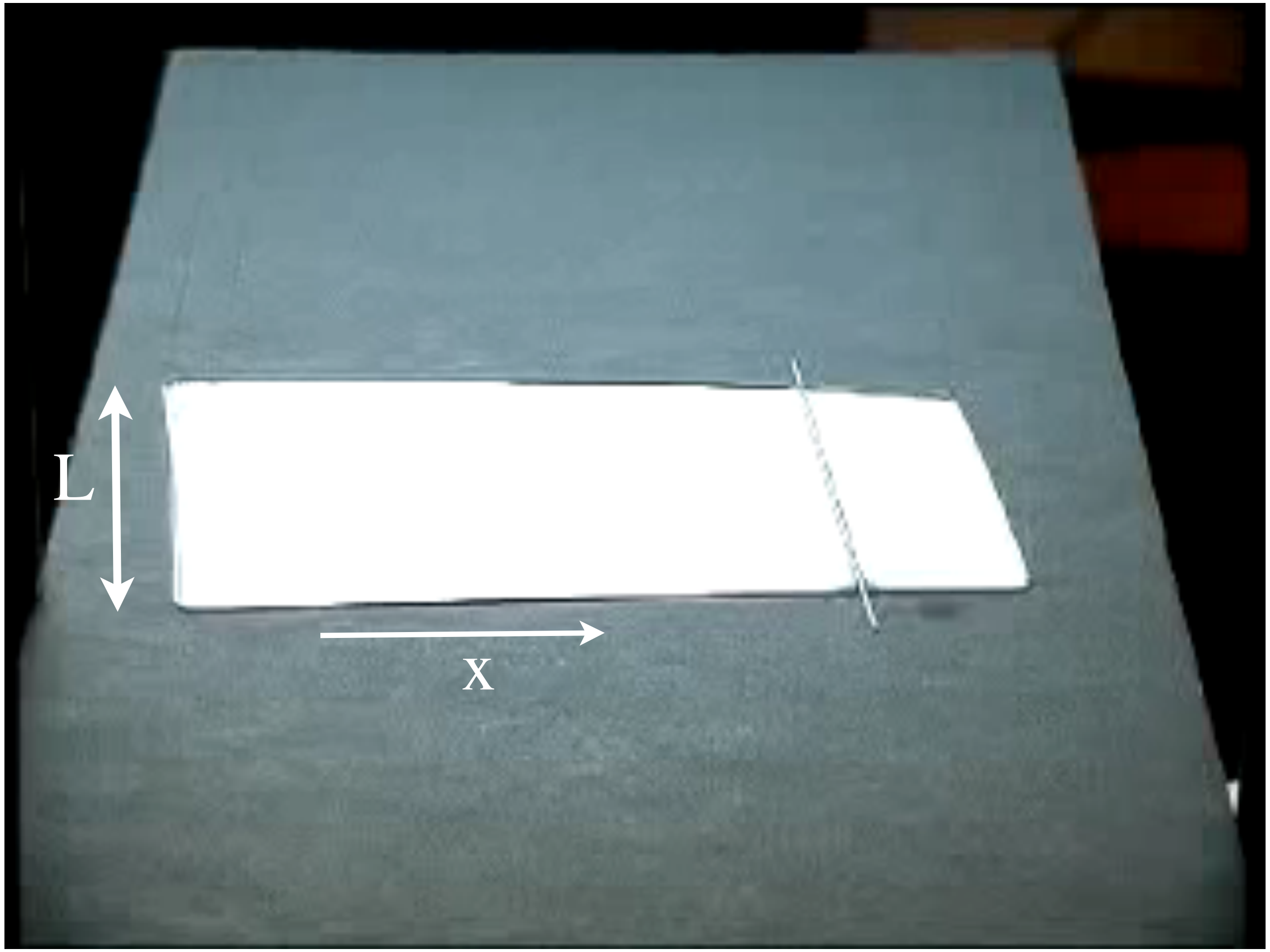
- small drops are nearly spherical (MFM)
- fluid jets pinch off into droplets (MFM)
- fluid atomization results in spherical drops (MFM)
- wet hair sticks together: the "wet look" (MFM)
- bubbles and films are fragile (MFM)



Surface tension: analogous to a negative surface pressure

- gradients in surface tension necessarily drive surface motion



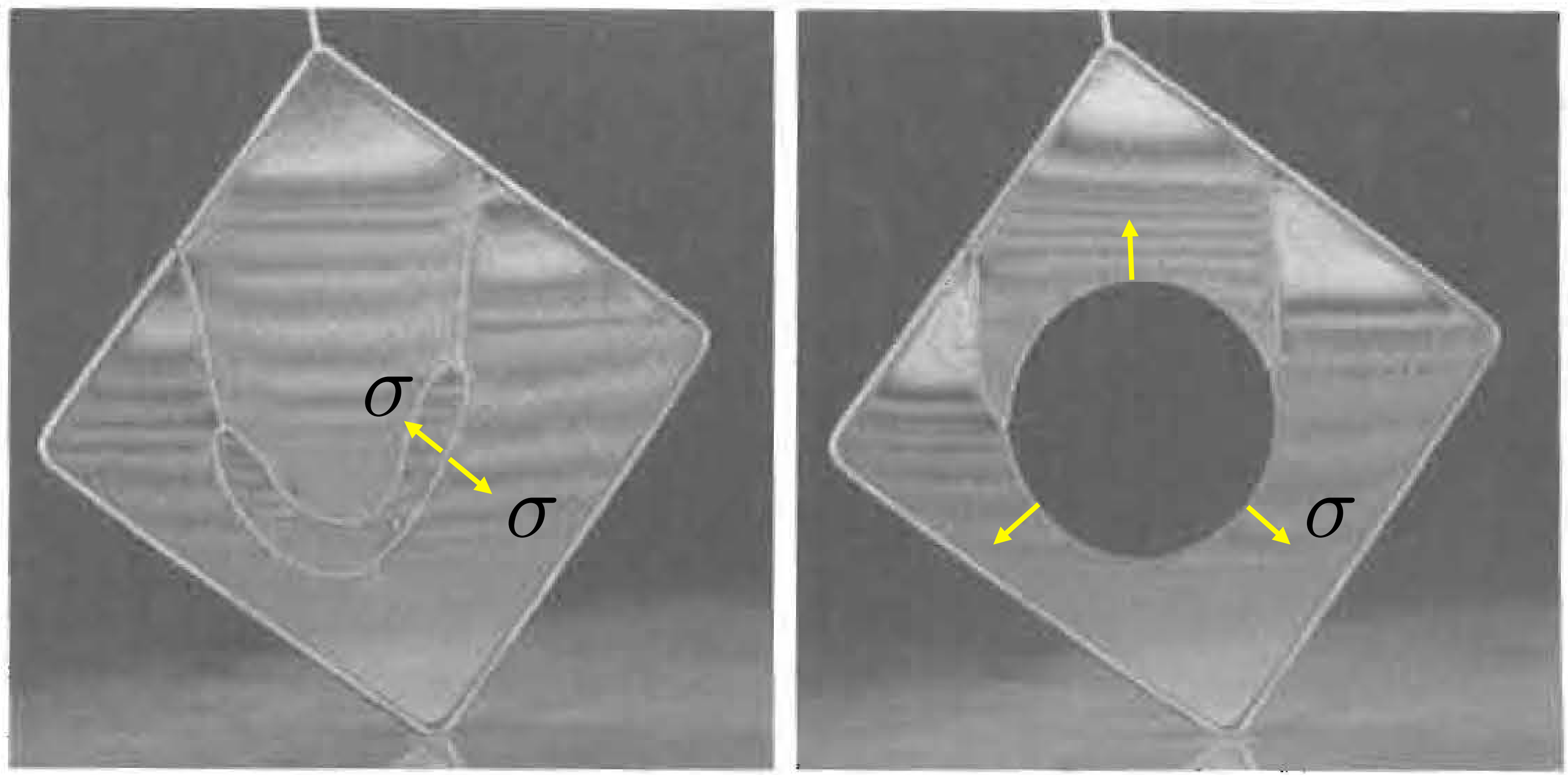


Surface tension: $[\,\sigma\,] \;=\; \dfrac{FORCE}{LENGTH} \;=\; \dfrac{ENERGY}{AREA}$

Surface energy: $E_\sigma \;=\; \displaystyle\int_S \sigma\, dA \;=\; 2\,\sigma\, L\, x$

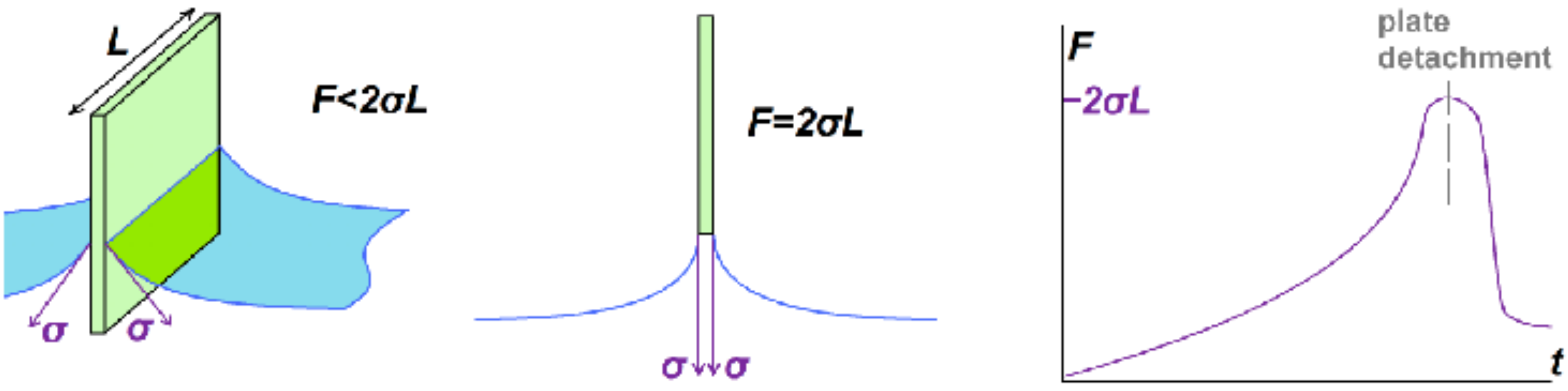
Force acting on rod: $F \;=\; \dfrac{dE_\sigma}{dx} \;=\; 2\,\sigma\, L$

A string in a soap film



If you use a hair, you can achieve neutral buoyancy, and so have a two-dimensional model of a balloon in the atmosphere.

A simple way to measure surface tension



- measure the force required to withdraw a plate from a free surface

NOTES

A floating paper clip…

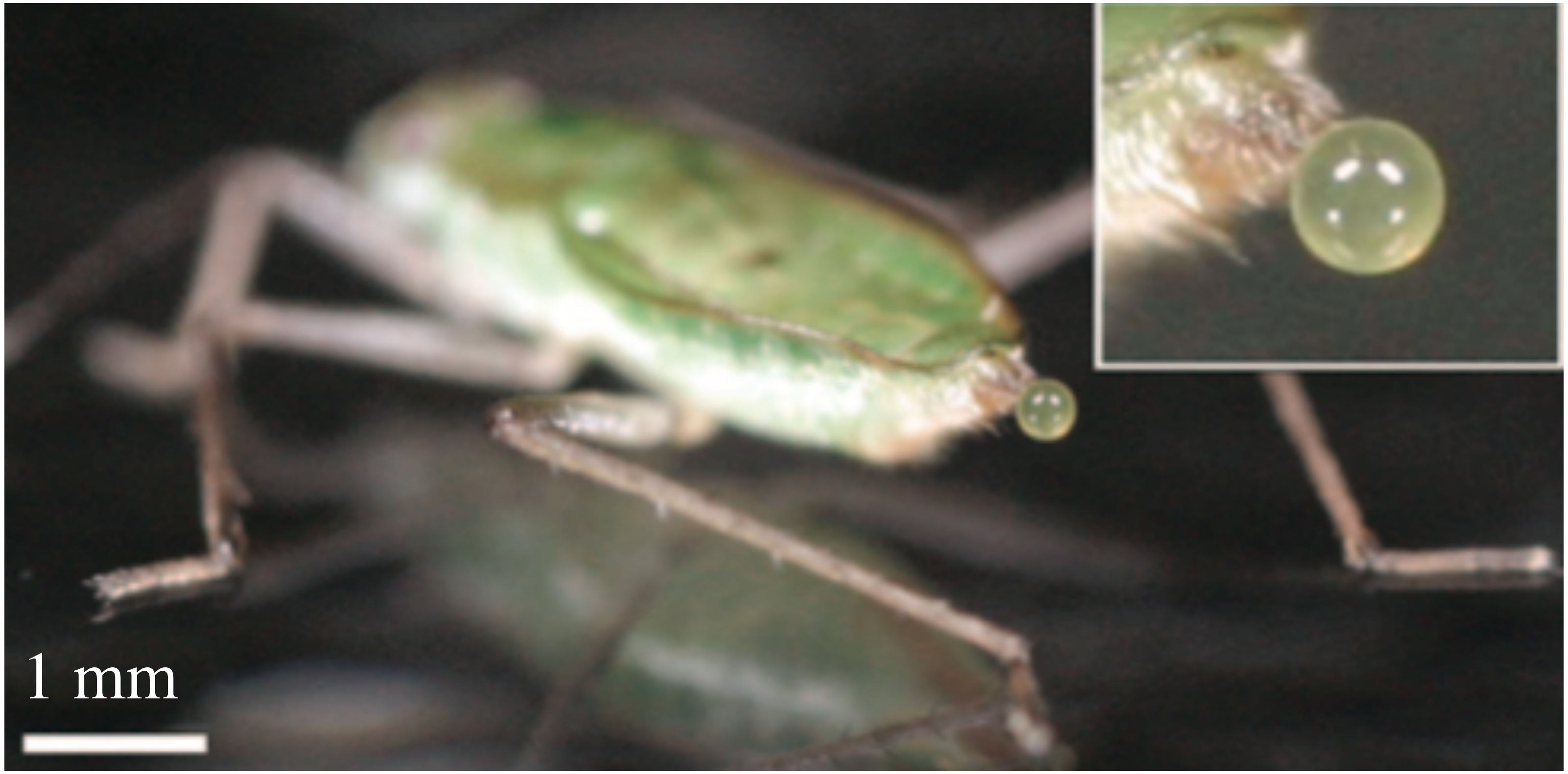
Capillary forces support the weight of water-walking insects.

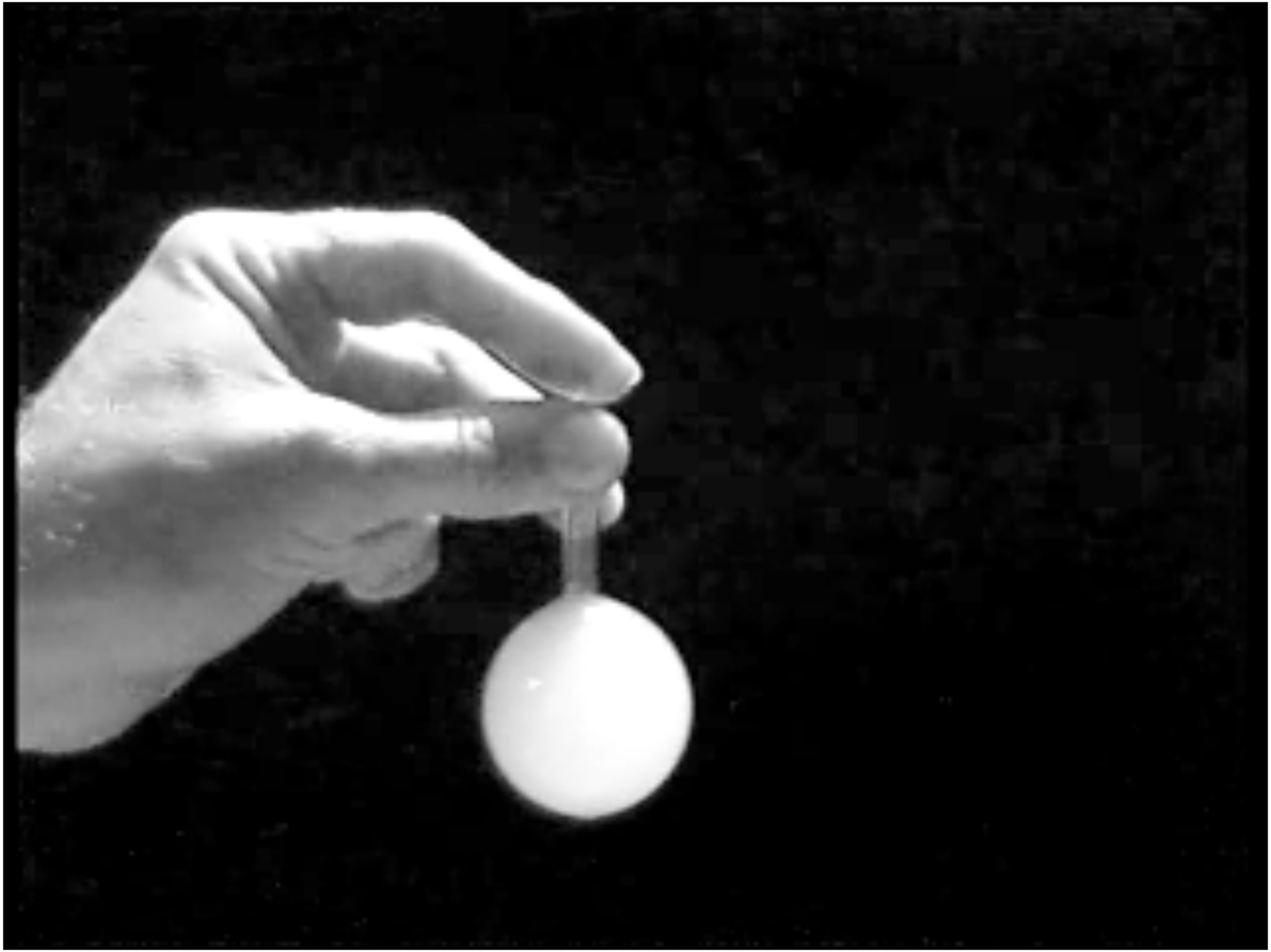
Capillary pressure

Which way does the air go?

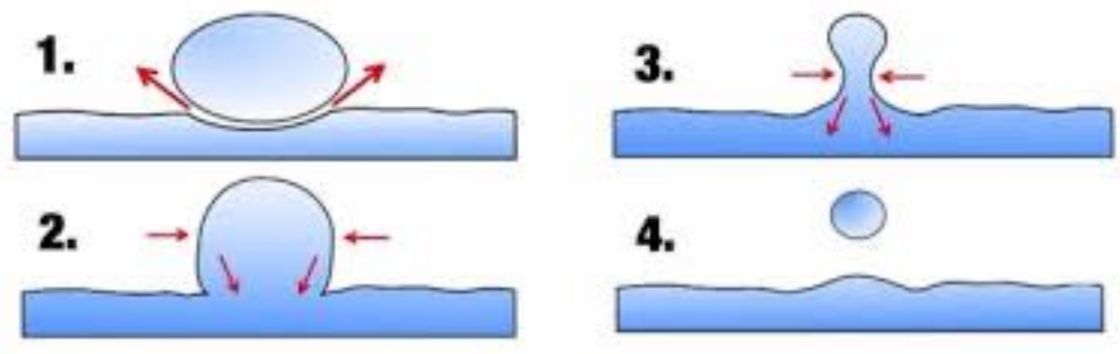
Who cares about surface tension?

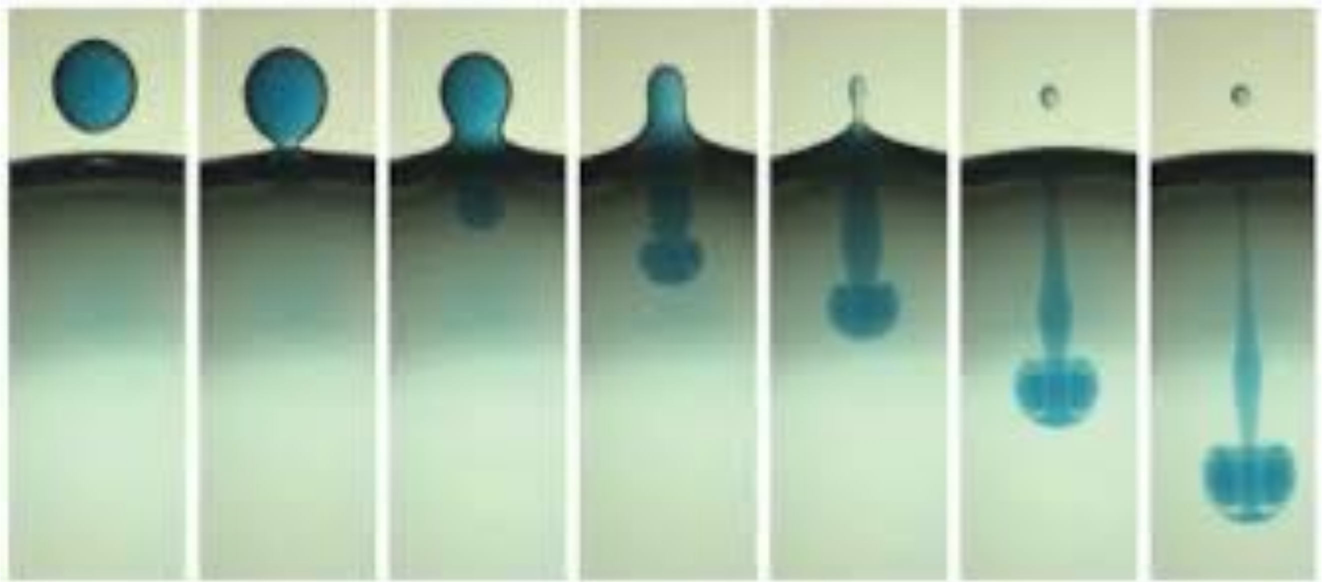
Capillary pressures in biology



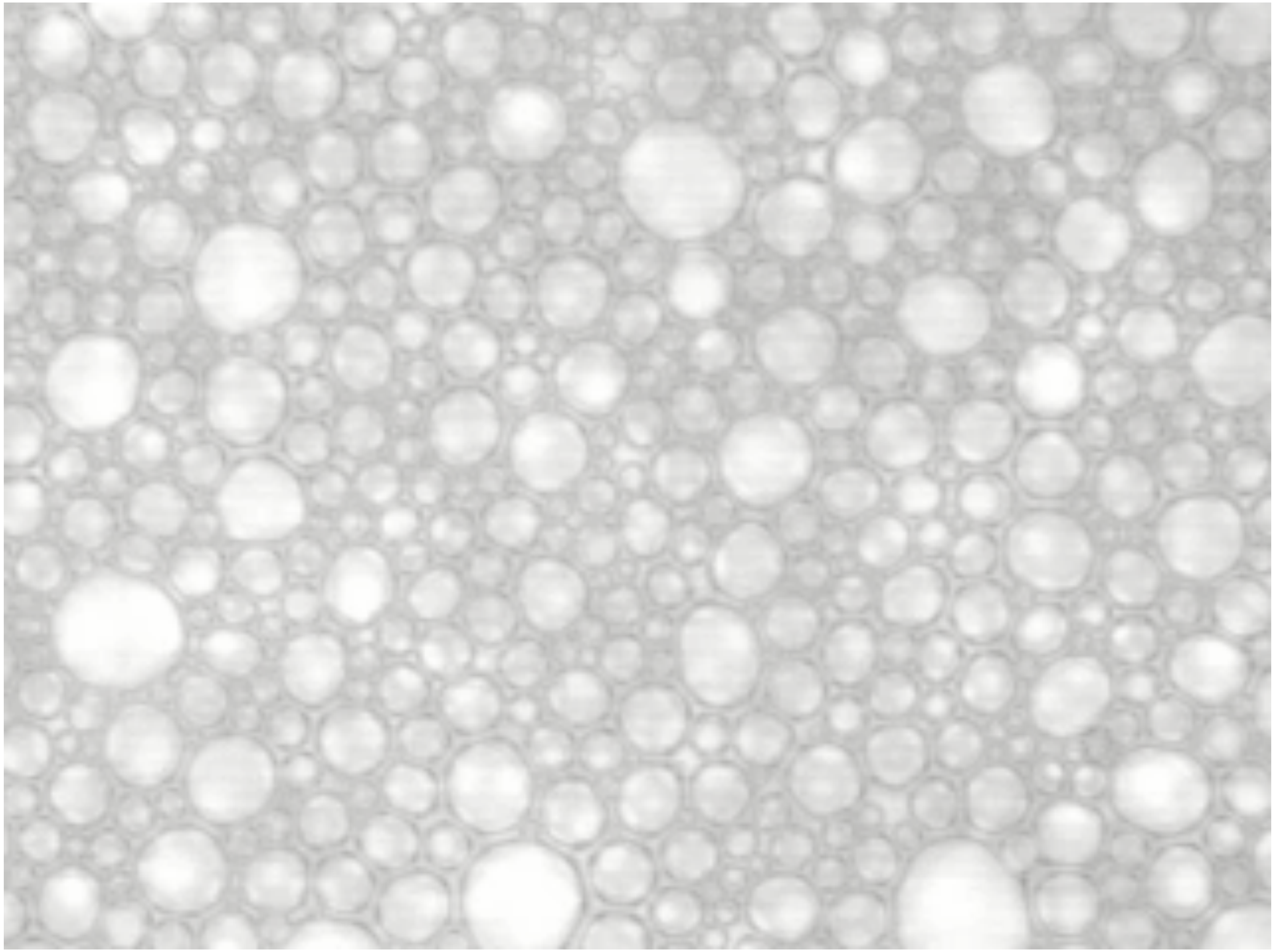


Vortex generation following drop coalescence



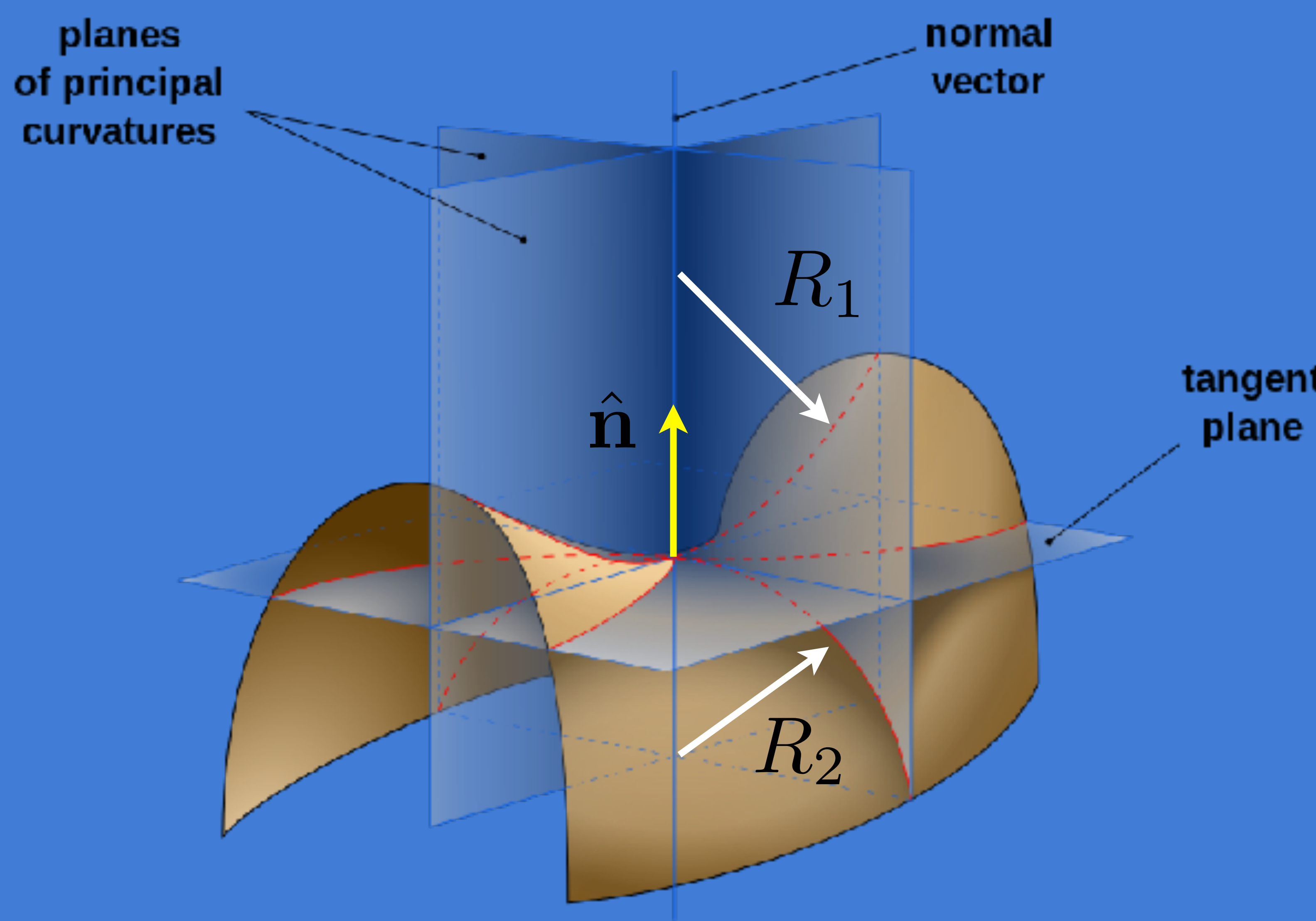


Ostwald ripening



- foam coarsens in response to diffusion of gas from small to large bubbles

Curvature



$$\nabla \cdot \hat{\mathbf{n}} = \frac{1}{R_1} + \frac{1}{R_2}$$

where R_1 , R_2 are the principal radii of curvature

Curvature: $\nabla \cdot \hat{\mathbf{n}} = \frac{1}{R_1} + \frac{1}{R_2}$ = 2/R for a sphere

Capillary pressure: $\Delta p = \frac{4\sigma}{R}$

Which way does the air go?

End