Lecture 23

A. Hydrodynamic interferometry

B. Revisitation of QM pilot-wave theory

C. New hydrodynamically-inspired p-w theories

Bohmian Mechanics (1952)

David Bohm

- equate quantum velocity of probability $\mathbf{u}$ and particle velocity $\dot{\mathbf{x}}_p$
- solve Schrodinger's equation for Ψ, from which Q is computed
- solve trajectory equation

$$\boxed{m\,\ddot{\mathbf{x}}_p \;=\; -\,\nabla Q \;-\; \nabla V}$$

NONLOCAL

- as an attempt to restore reality to QM, it drew the ire of Copenhagen adherents

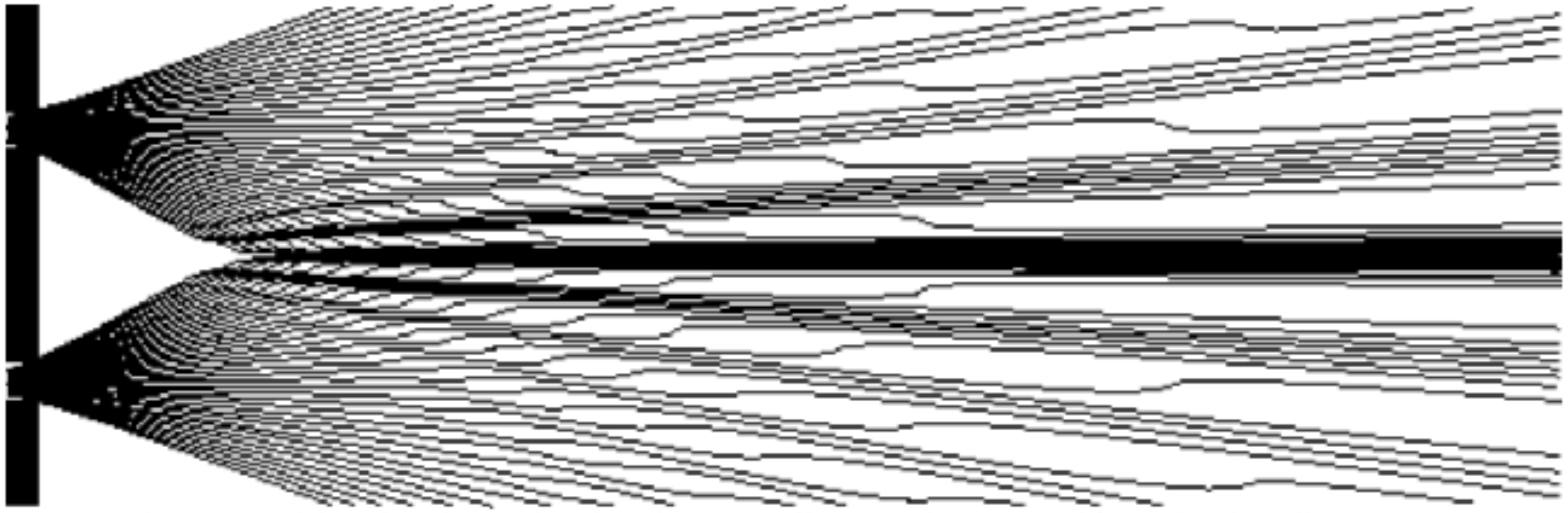


FIGURE 1: An ensemble of trajectories for the two-slit experiment, uniform in the slits. (adapted by Gernot Bauer from Philippidis, Dewdney, & Hiley 1979: 23, fig. 3)

- in double-slit diffraction, no particles cross the centerline owing to form of Q

Surreal trajectories: Theory

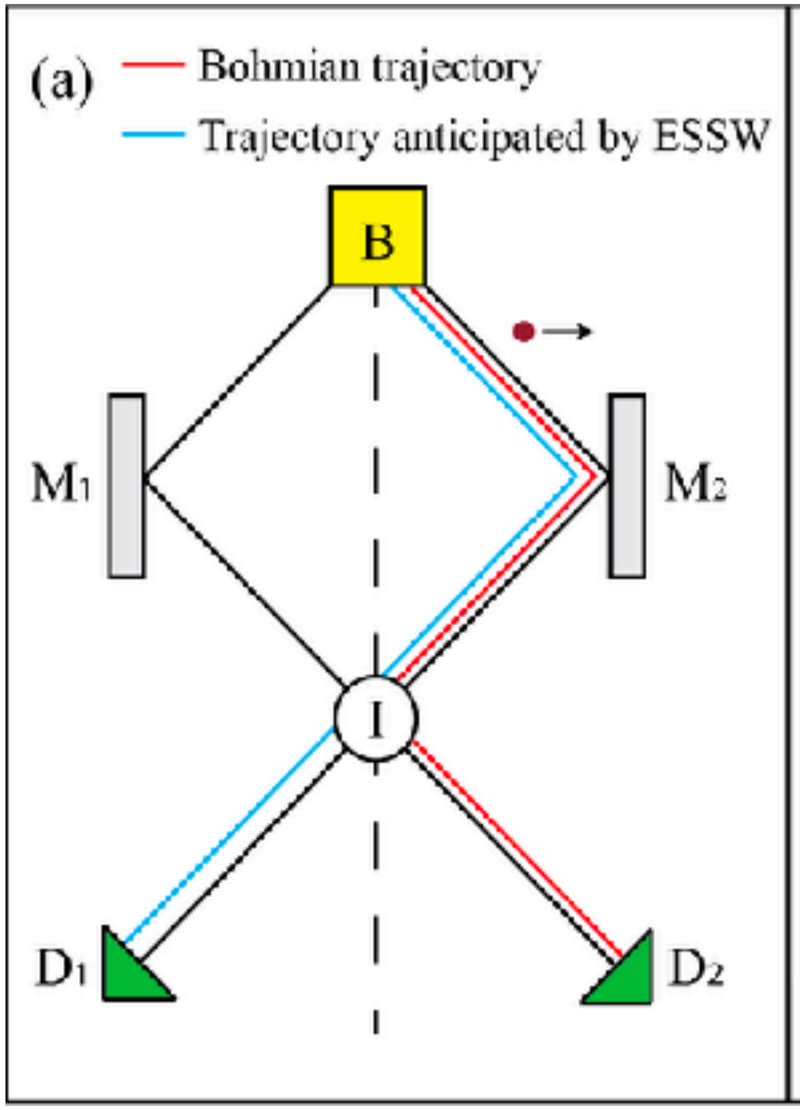
- Englert, Sully, Süssman and Walther (ESSW). 1992

- proposed an interference experiment intended to expose the shortcomings of Bohmian mechanics

- Bohmian trajectories (red) are redirected at Ⓘ by the quantum potential, so differ from expected paths (blue)

  `Bohmian trajectories are at odds with common sense: they are not real, they are surreal.'

- their reasoning was criticized by Aharanov & Vaidman (1996), who concluded:



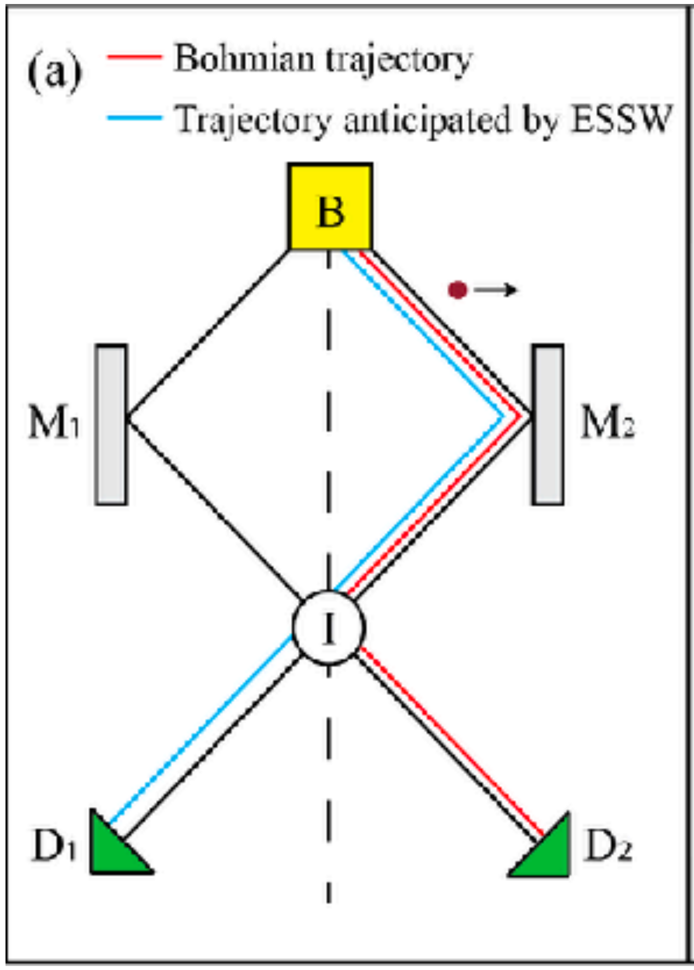
` ESSW does not show that Bohmian mechanics is inconsistent, only that Bohmian trajectories behave differently from what one would expect classically.'

➡ **What one expects classically depends on how much one knows about classical mechanics.**

Surreal trajectories: Experiments

- experimental investigations using `weak measurement' found mean trajectories consistent with the surreal trajectories *(Mahler et al., 2016)*

> *`We demonstrate that the trajectories seem surreal only if one ignores their manifest nonlocality.'*



Copenhagen

- surreal trajectories are taken as evidence of nonlocality
- the particle is entangled with the measurement device

Bohmian mechanics

- surreal trajectories are evidence of the nonlocal quantum potential Q

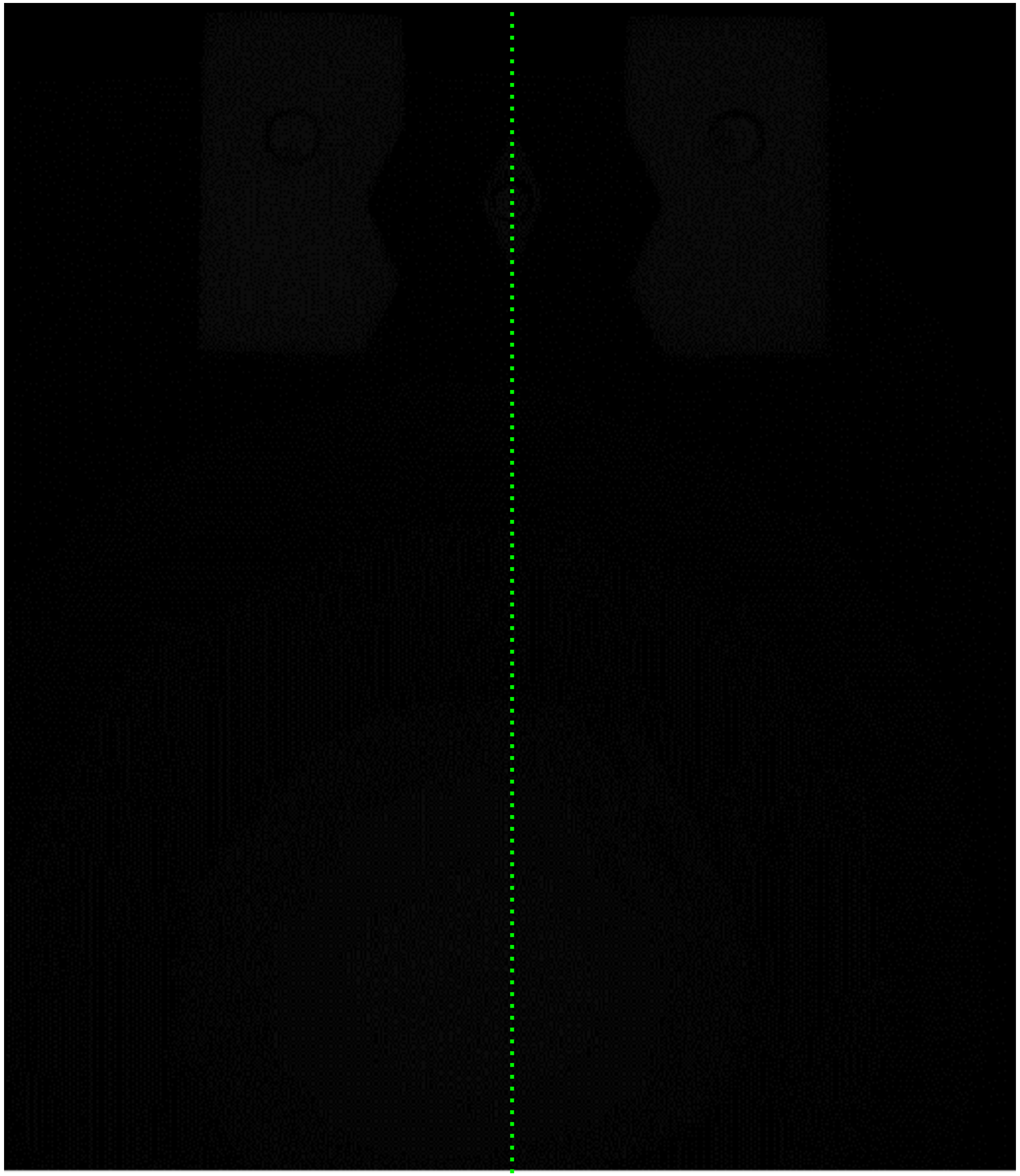
`Might we achieve an analogous effect with pilot-wave hydrodynamics, which is undeniably local?'

Real surreal trajectories

- drop launched toward the beam-splitter goes left/right 50% of the time

- owing to the interaction between the distant barrier and its pilot wave, the droplet never crosses the centerline, resulting in a real, “surreal” trajectory.

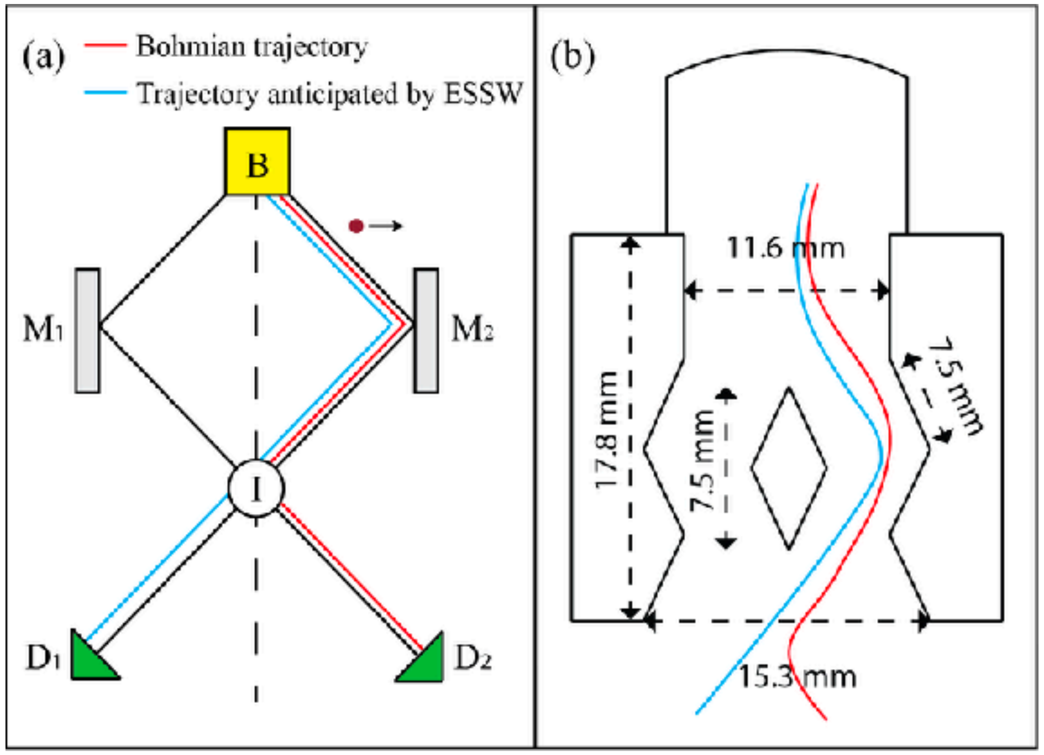
- if one barrier is removed, the pilot-wave is no longer effected by it, and the droplet follows the expected trajectory

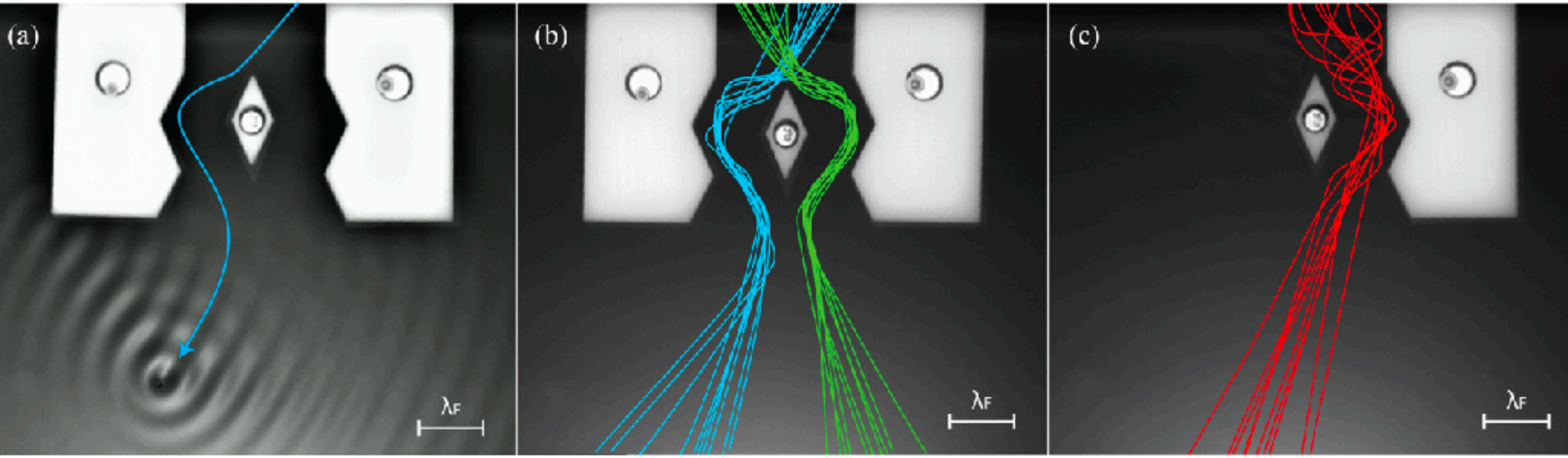


Real surreal trajectories

Experiments

- arise in the walker system at high Me
  - *Frumkin, Struyve, Darrow, JB (PRA 2022)*



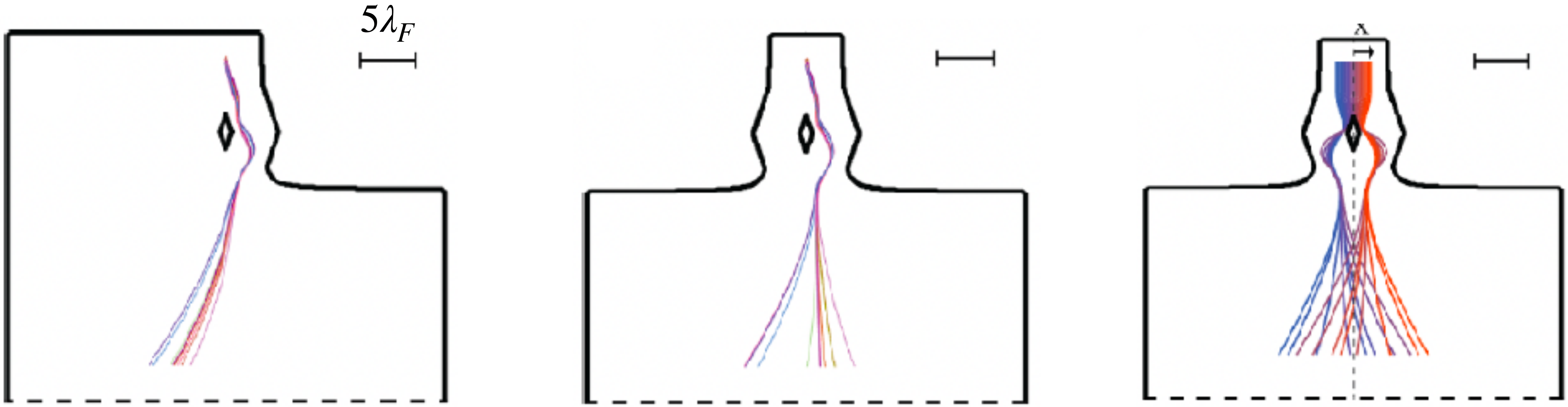


The designation of Bohmian trajectories as surreal is based on misconceptions concerning the limitations of classical mechanics and a lack of familiarity with pilot-wave hydrodynamics.

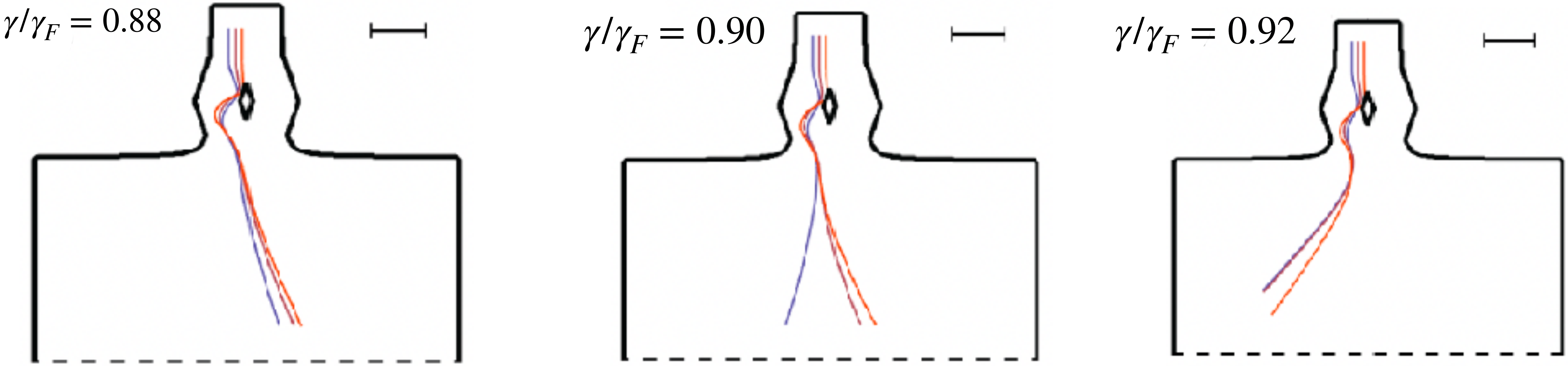
Real surreal trajectories: Simulations of Dave Darrow

- undertaken with the numerical model of Faria (2017)

Dependence on impact parameter at $\gamma/\gamma_F = 0.902$

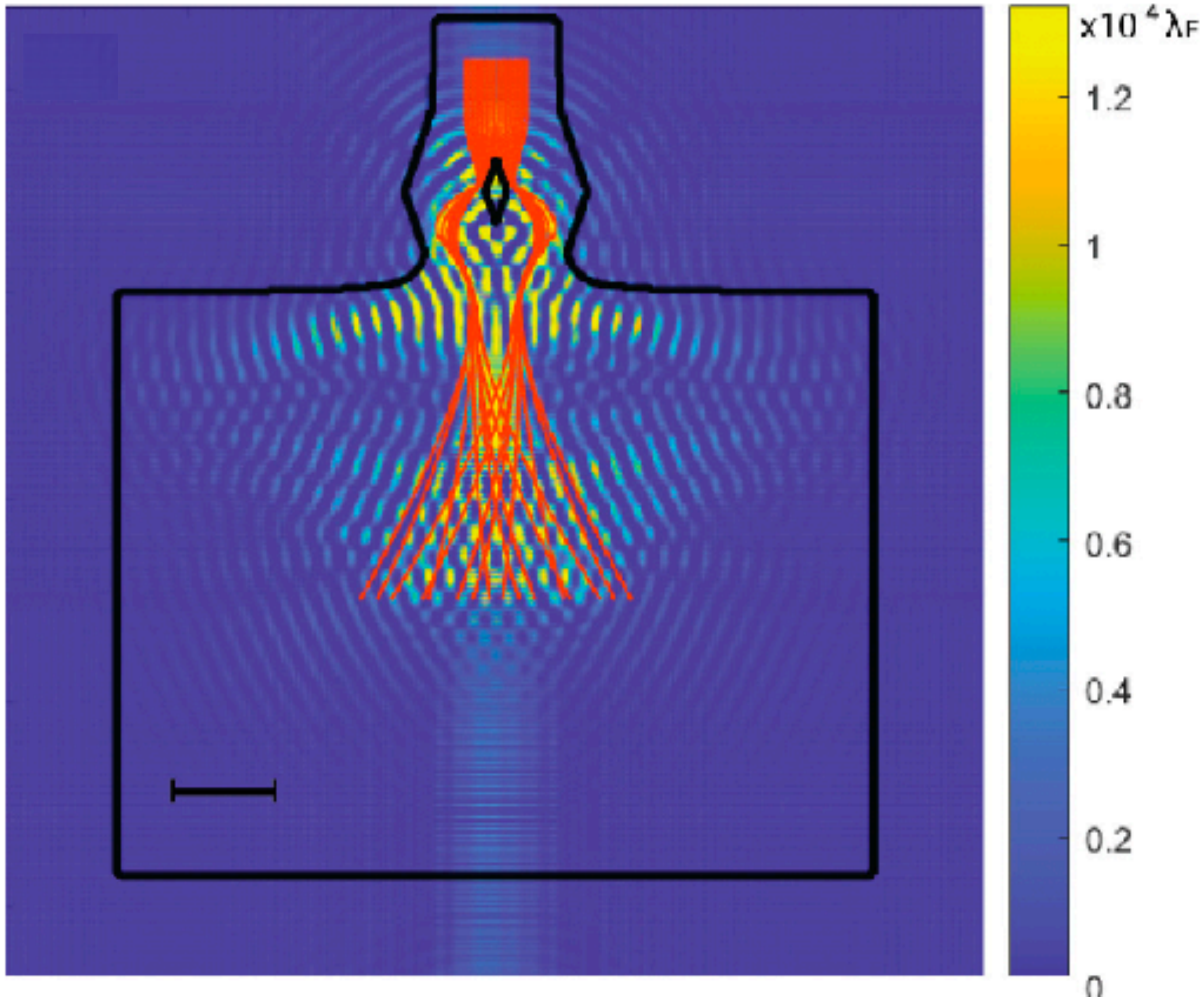


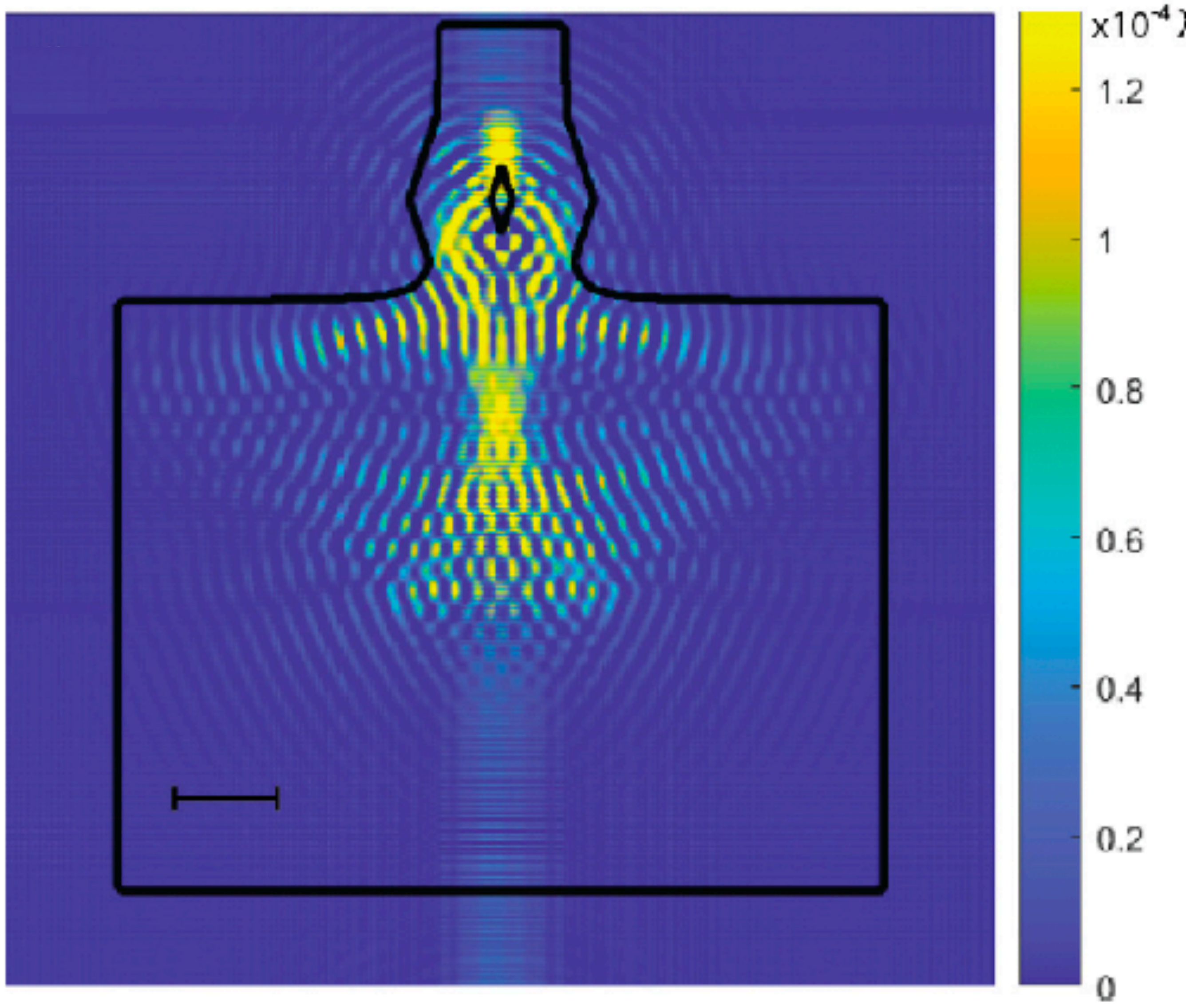
Dependence on memory



Real surreal trajectories: Simulations

- trajectories and an associated mean pilot-wave field as deduced from a Gaussian distribution of impact parameters, an ensemble of initial conditions

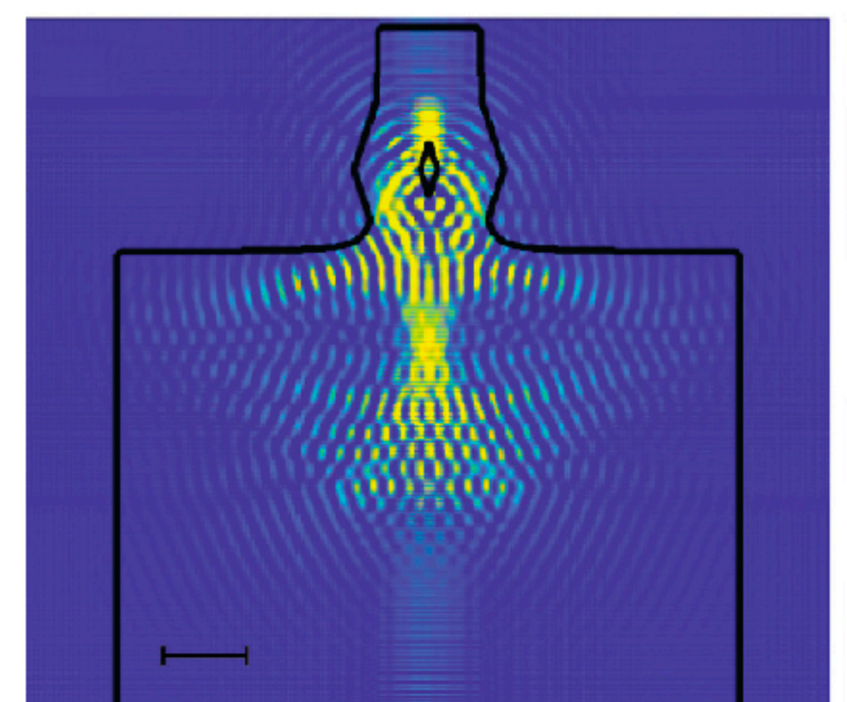




Relation to quantum potential?

Bohmian Mechanics (1952)

- a dynamical reformulation of a statistical theory deduced by equating the particle velocity with the quantum velocity of probability
- it would thus be natural that the Bohmian trajectories be average values
  - consistent with `weak measurement' expts (Steinberg *et al*. 2016)
- in response to Keller's criticism, Bohm retreated to the stance that Bohmian trajectories were mean trajectories
- Bohm & Vigier (1953) sought a stochastic element to describe the departures from the mean (like Brownian motion of gas molecules about streamlines)



- if bouncing drops are a quantum analog, then their mean motion should be an analog of Bohmian mechanics

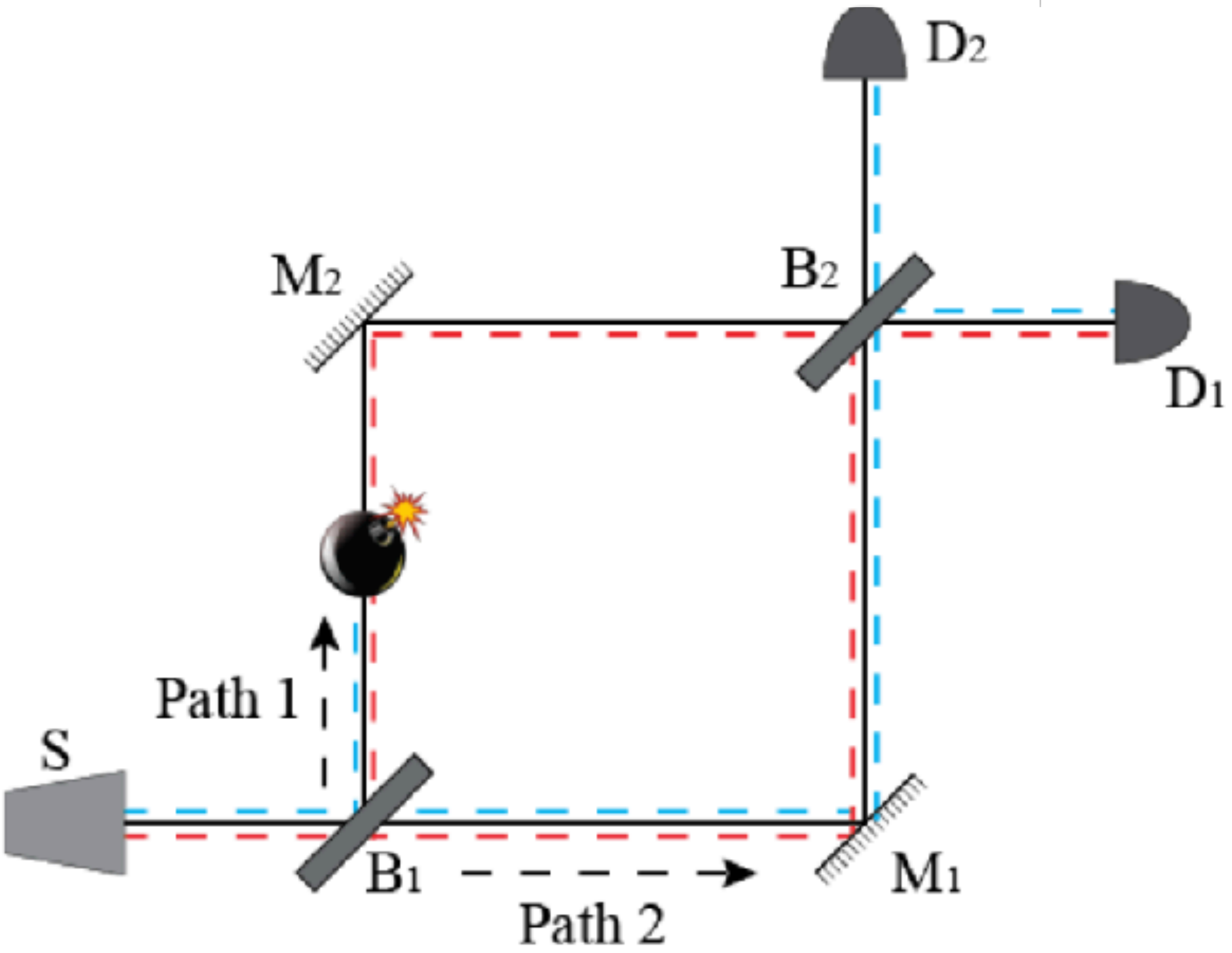
Does the mean pilot-wave field play the role of the quantum potential in our system?

NewScientist

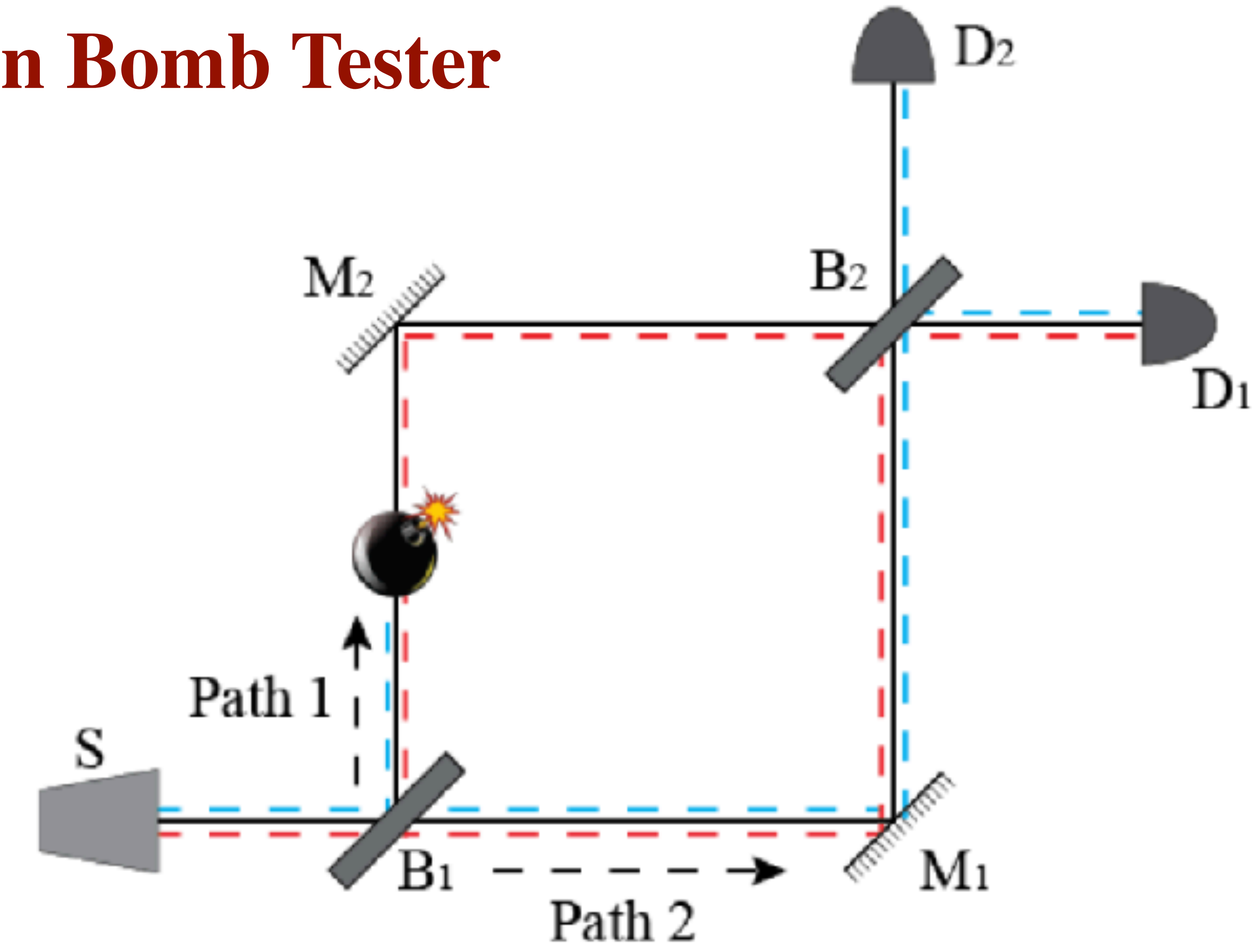
Seven wonders of the quantum world

From undead cats to particles popping up out of nowhere, from watched pots not boiling – sometimes – to ghostly influences at a distance, quantum physics delights in demolishing our intuitions about how the world works. Michael Brooks tours the quantum effects that are guaranteed to boggle our minds.

1. *Corpuscles and buckyballs*
2. *The Hamlet effect*
3. *Something for nothing*
4. *The Elitzur–Vaidman bomb tester*
5. *Spooky action at a distance*
6. *The field that isn't there*
7. *Superfluids and supersolids*

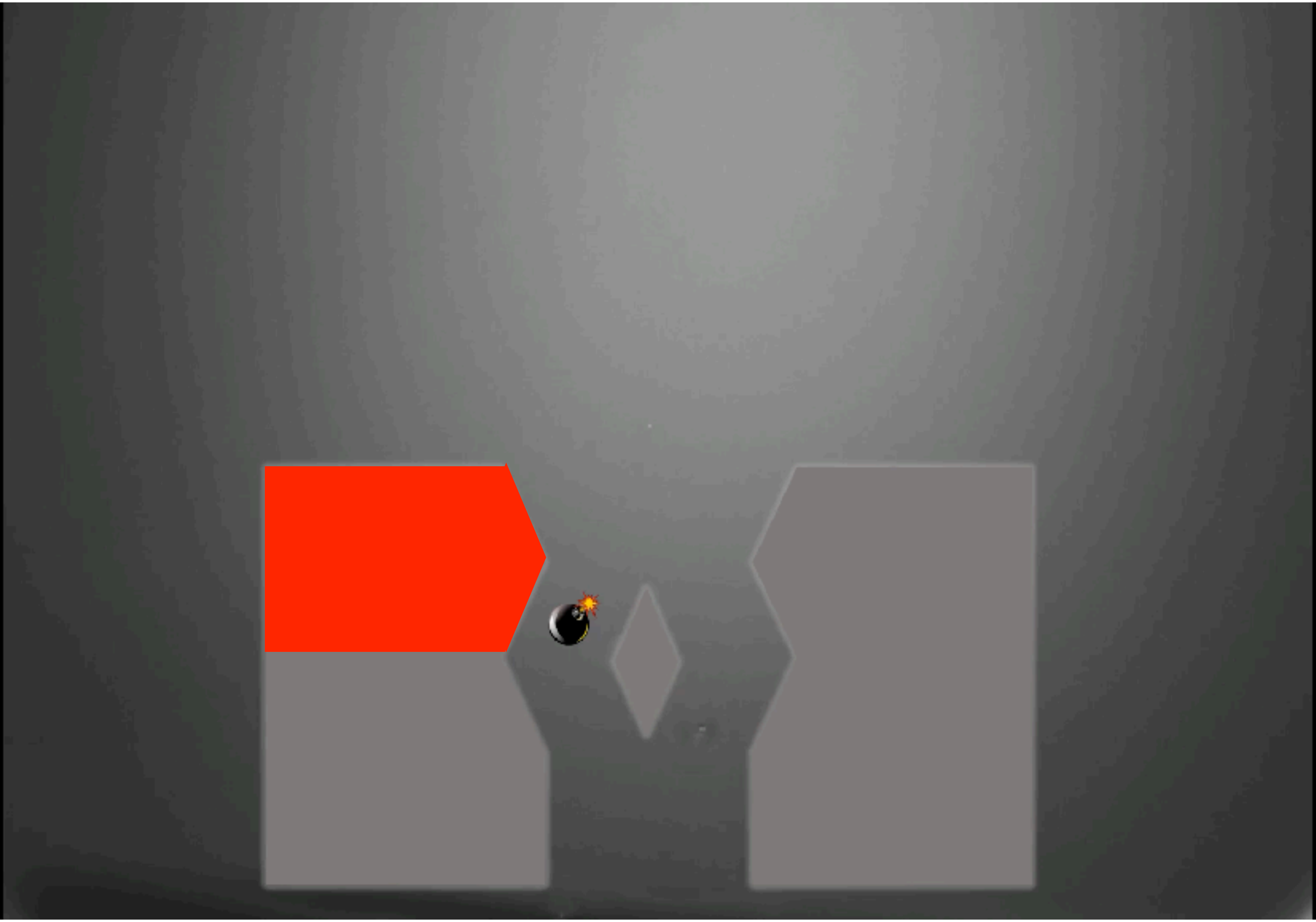


The Elitzur-Vaidman Bomb Tester



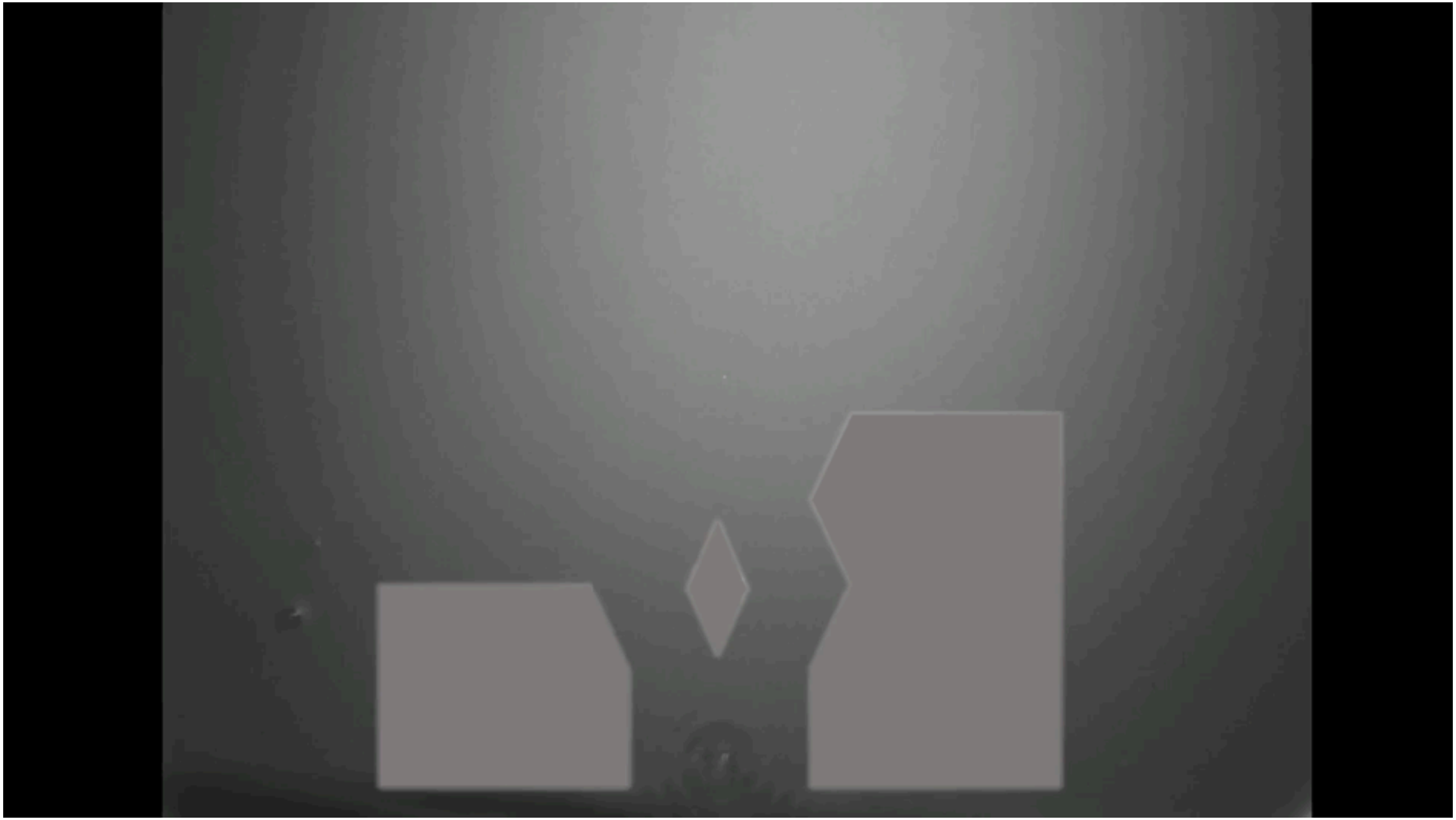
- in the absence of a bomb (red), interference always causes photon to arrive at D1
- with bomb (blue), this interference is destroyed, so particle either detonates bomb (Path 1) or arrives at D1 or D2 with equal probability
- if bomb is present 50% of the time, then you can detect it 25% of the time via a particle that took Path 2, so never interacted with it

A topographic bomb



- topography plays the role of the bomb, induces surreal trajectories
- 50% of particles explode bomb, 50% diverted to the right

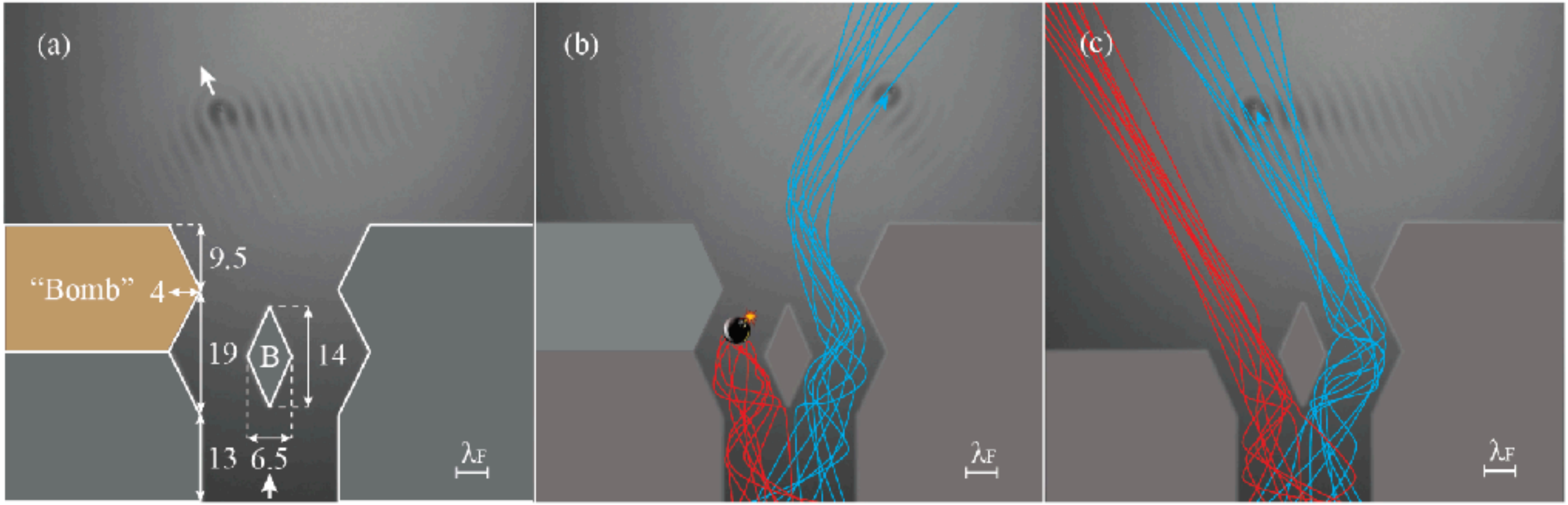
No bomb



- drops initially directed to right or left with equal probability by beam splitter
- without the `bomb’, all drops are directed to the left, no surreal trajectories arise

A hydrodynamic analog of the quantum Bomb tester

- *Frumkin & JB, PRA (2023)*

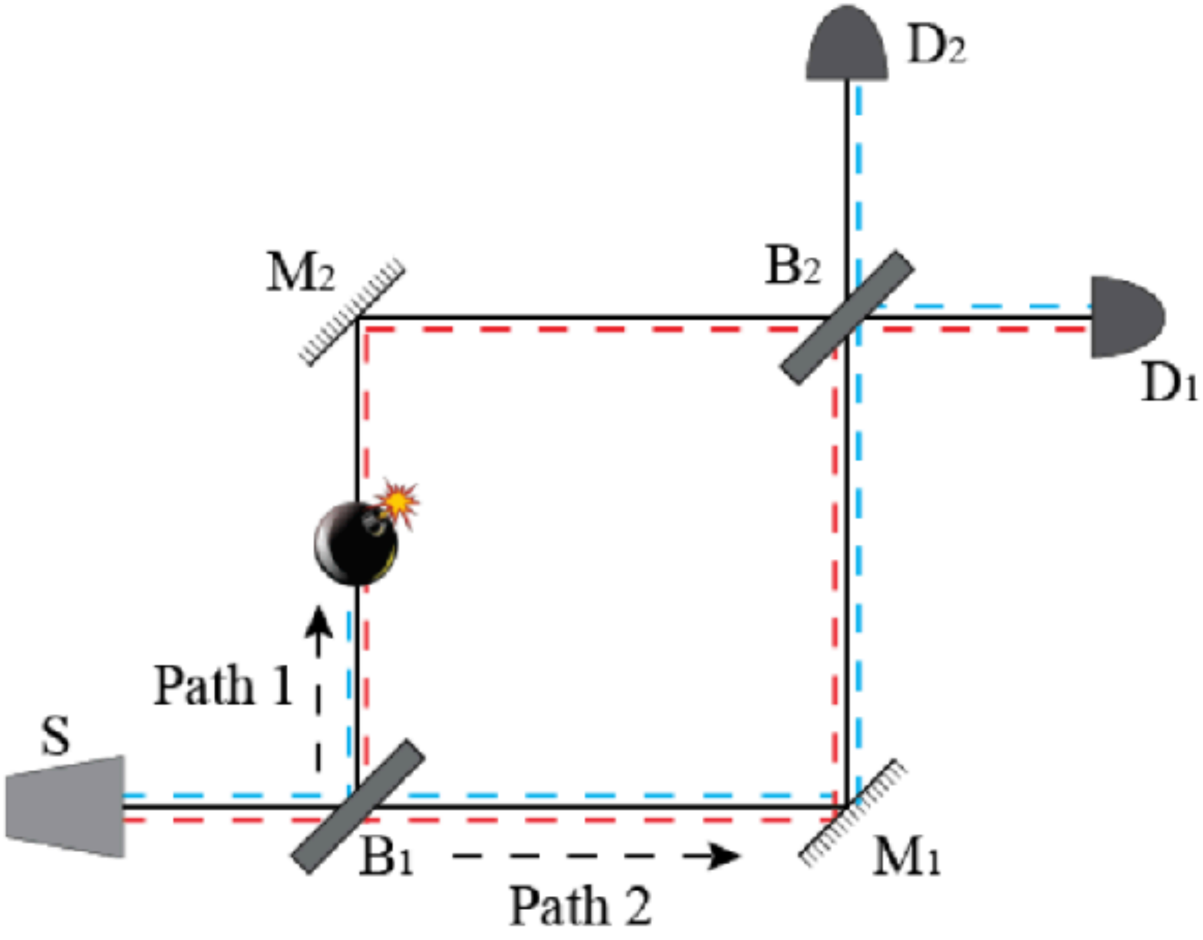


- submerged topography (orange) plays the role of the `bomb'
- in the absence of the bomb, all trajectories go to the left
- in the presence of the bomb, surreal trajectories may arise:
  - the droplet's pilot-wave interacts with the bomb, altering the droplet's path
- if bomb is present 50% of the time, then 25% of the time, the droplet detects a bomb along a path it didn't take

`Interaction-free measurement'

- one can seemingly measure an object without interacting with it…

… but how can that possibly be?



Copenhagenesque rationale

- bomb disrupts interference of wave function through alteration of Ψ

Bohmian mechanics

- guiding wave splits into 2 components at beamsplitter B_1, pilot plus empty wave
- when particle follows Path 2, its empty wave is disrupted by bomb on Path 1

Inference

- the effect can be simply rationalized if one concedes that the wave is real, in which case the measurement is *not* interaction-free

Hydrodynamic interferometry: Recap

Real surreal trajectories

` ESSW does not show that Bohmian mechanics is inconsistent, only that Bohmian trajectories behave differently from what one would expect classically.'

- `surreal' trajectories are not at odds with classical intuition informed by a familiarity with pilot-wave hydrodynamics

- may be readily understood as a manifestation of non-Markovian pilot-wave dynamics, with no need to invoke `quantum nonlocality'

Interaction-free measurement

- a misinference necessitated by the assumption that QM is complete, that there is no underlying reality

- an inference that may be obviated if one concedes the reality of a guiding wave that interacts with its environment

Brief review of quantum pilot-wave theories

- before we develop new classical pilot-wave theories for the microscope scale…

Bohmian Mechanics (1952)


David Bohm

- equate quantum velocity of probability $\mathbf{u}$ and particle velocity $\dot{\mathbf{x}}_p$
- solve Schrodinger's equation for Ψ , from which Q is computed
- solve trajectory equation

$$\boxed{m\,\ddot{\mathbf{x}}_p \;=\; -\,\nabla Q \;-\; \nabla V}$$

NONLOCAL

Shortcomings

- Einstein's objection: it is `*nonlocal'* by virtue of the quantum potential Q
- no mechanism for wave generation; no feedback of particle on field

Extensions (Bohm & Vigier 1954)

- invoke a stochastic forcing $\nabla\Phi_S$ from a `sub quantum realm':

$$\boxed{m\,\ddot{\mathbf{x}}_p \;=\; -\,\nabla Q \;-\; \nabla V \;+\; \nabla\Phi_S}$$

- particles jostle about $\mathbf{u}$ like Brownian motion of gas molecules about streamlines

	Bohmian mechanics	**Walkers**
WAVELENGTH	λ_B	λ_F
GUIDANCE	$m\,\ddot{\mathbf{x}}_p = -\nabla Q - \nabla V + \nabla\Phi_S$ **NONLOCAL** ***AD HOC***	$m\,\ddot{\mathbf{x}}_{\mathbf{p}} = -D\,\dot{\mathbf{x}}_{\mathbf{p}} + \nabla\eta(\mathbf{x},t) - \nabla V$ **LOCAL**
NON-LOCAL WAVE POTENTIAL	$\mathbf{Q} = -\frac{\hbar^2}{m^2}\frac{1}{\sqrt{\rho}}\nabla^2\sqrt{\rho}$ **QUANTUM POTENTIAL**	$\bar{\eta}(\mathbf{x}) = \eta_B * \mu(\mathbf{x})$ **MEAN WAVE FIELD**
STOCHASTIC FORCING	$\nabla\Phi_S$ **ARBITRARY, *ad hoc***	$-\nabla\eta^*(\mathbf{x},t)$ **PERTURBATION WAVE FIELD**
WAVE ORIGIN	**NONE**	**PARTICLE VIBRATION**

de Broglie's pilot-wave theory: The double-wave solution

> " A freely moving body follows a trajectory that is orthogonal
> to the surfaces of an associated wave guide".
>
> \- **Louis de Broglie (1892-1987)**

- Ψ_s is the probability wave, as prescribed by standard quantum theory
- $\Psi(x,t) = A(x,t)\ e^{i\Phi(x,t)}$ is a real physical wave responsible for guiding the particle according to his Guidance Equation: $\dot{\mathbf{x}}_{\mathbf{p}} = -c^2 \frac{\nabla \Phi}{\Phi_t}$

- wave generated by internal particle vibration (*Zitterbewegung*) at the Compton frequency: $\omega_c = \frac{m_0 c^2}{\hbar}$
- a solution of Klein-Gordon equation triggered by oscillations in rest mass
- particle follows point of constant wave amplitude: his guidance equation yields

$\mathbf{p} = \gamma m_0 \mathbf{v} = \hbar \mathbf{k}$ for a monochromatic wave $\Phi = \mathbf{k} \cdot \mathbf{x} - \omega t$

- **Harmony of Phases**: the particle oscillates in resonance with its guiding wave
- *incomplete*: wave generation mechanism, precise form of Ψ not specified

The Klein-Gordon Equation

$$\boxed{\frac{1}{c^2}\,\Psi_{tt} \;-\; \nabla^2\Psi \;+\; \frac{m_o^2c^2}{\hbar^2}\,\Psi \;=\; 0}$$

Seek 1D solution: $\Psi(x,t) = A(x,t)\; e^{i\Phi(x,t)}$ where $\Phi = kx - \omega t$

Imaginary component of KG: $\Phi_t A_t - c^2\Phi_x A_x = 0$ ★

Particle moves along a point of constant amplitude A(x,t) of its pilot wave:

$$\frac{DA}{Dt} = A_t + v A_x = 0 \longrightarrow v = -A_t / A_x \xrightarrow{\bigstar} \boxed{v = -c^2\frac{\Phi_x}{\Phi_t}}$$

Use: $\Phi_x = k$ and $\Phi_t = -\,\omega = -\,\gamma m_0 c^2/\hbar$

→ **De Broglie relation:** $\boxed{p = \gamma m_0 v = \hbar k}$

de Broglie’s Theorem of Phase Harmony

> “A periodic phenomenon with frequency ω_c is seen by a stationary observer to have a frequency ω_c/γ and remain constantly in phase with a wave having frequency $\omega_c\gamma$ propagating in the same direction with velocity $v = \beta\, c$.”

where $\gamma = (1-\beta^2)^{-1/2} > 1$

3 frequencies:

$\omega_c = \dfrac{m_0 c^2}{\hbar}$	$\omega_c\sqrt{1-\beta^2}$	$\dfrac{\omega_c}{\sqrt{1-\beta^2}}$
PARTICLE / CLOCK	**APPARENT CLOCK**	**WAVE**

de Broglie wave: $\Psi(x,t) = A(x,t)\ e^{i\Phi(x,t)}$

Phase variation in progressing wave

$$d\Phi = (\Phi_t + \mathbf{v}\cdot\nabla\Phi)\ dt = \omega\gamma^{-1}\ dt = \omega_{clock}\ dt \quad \text{via} \quad v = -c^2\,\frac{\Phi_x}{\Phi_t}$$

- the clock remains constantly in phase with its guiding wave

“Une grande loi de la Nature.”

de Broglie’s pilot-wave theory

- **fast** dynamics: mass oscillations at

$$\omega_c = \frac{m_0 c^2}{\hbar}$$ create wave field

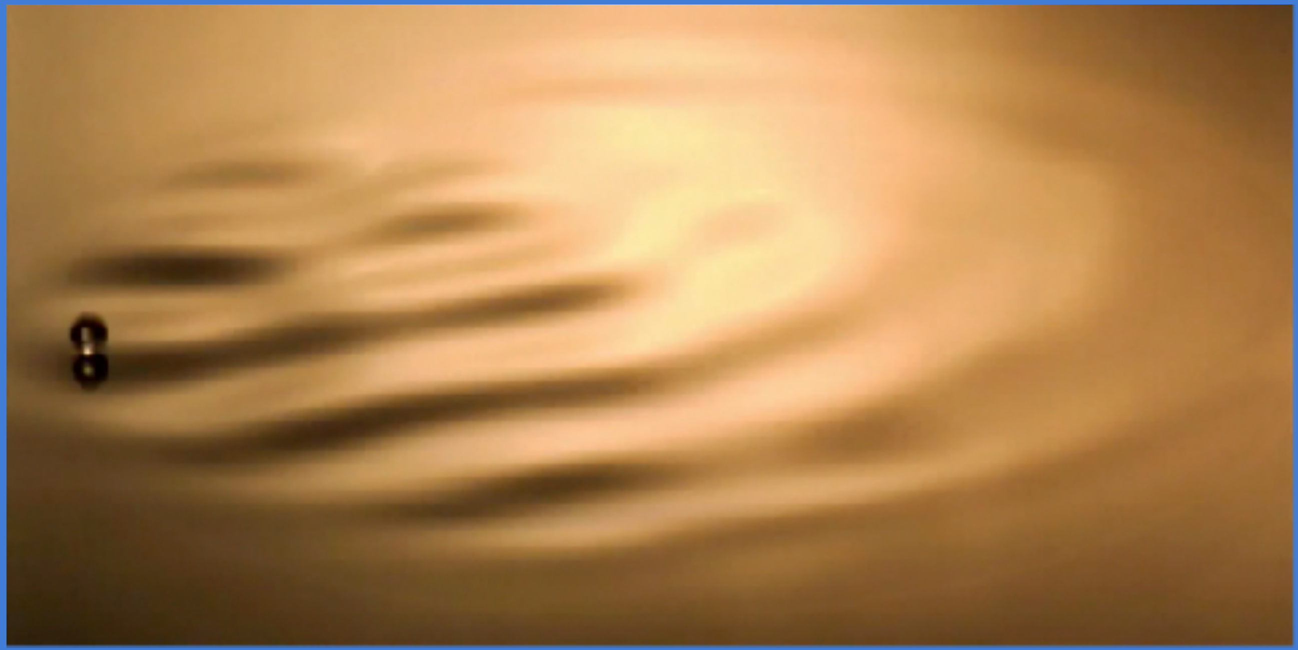
centered on particle

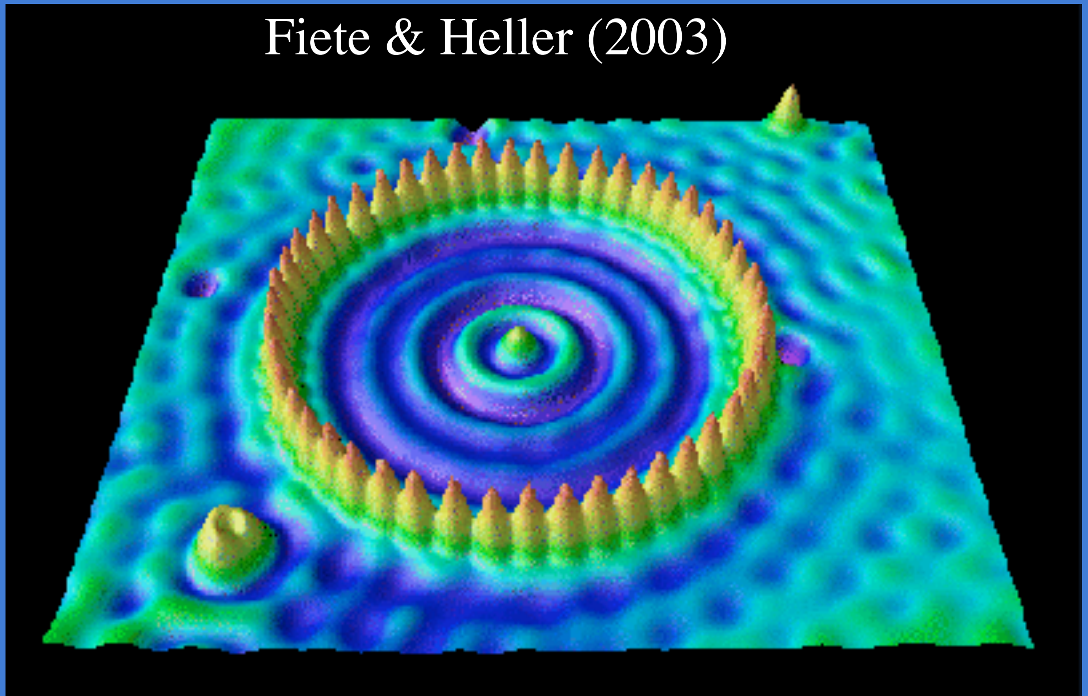
- **intermediate** pilot-wave dynamics:

particle rides its guiding wave field such that

$$\mathbf{p} = \hbar\, \mathbf{k}$$

- **long-term statistical** behaviour described by standard quantum theory

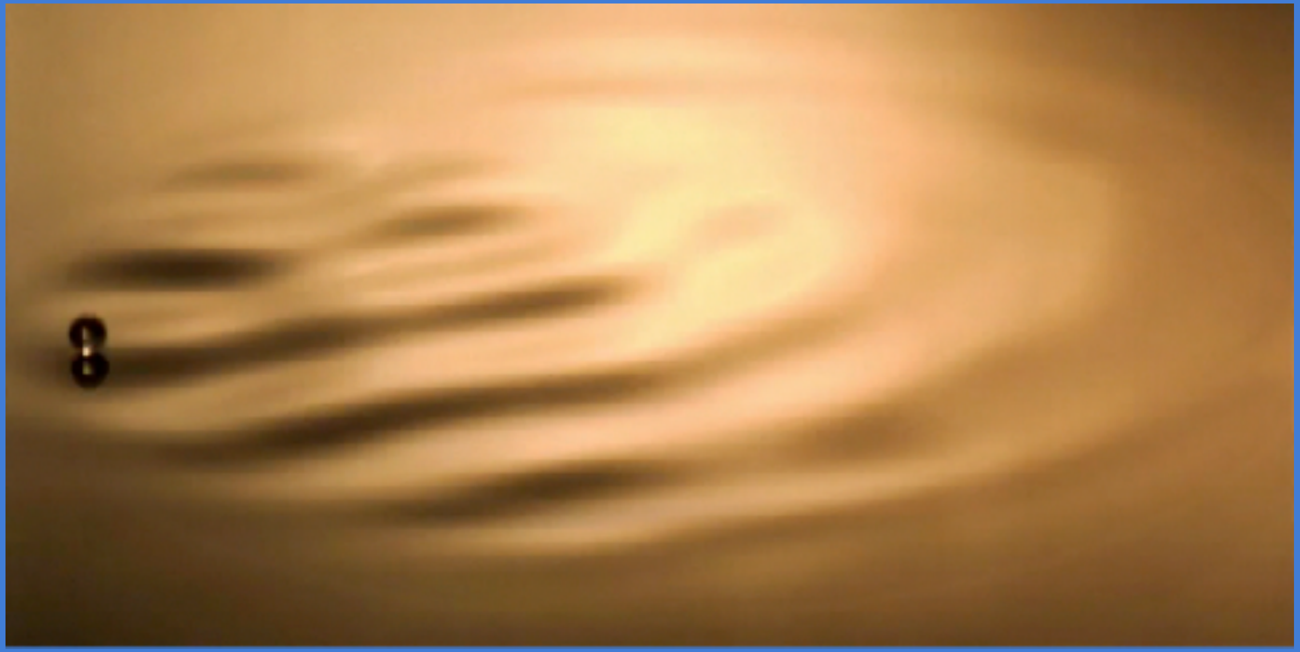




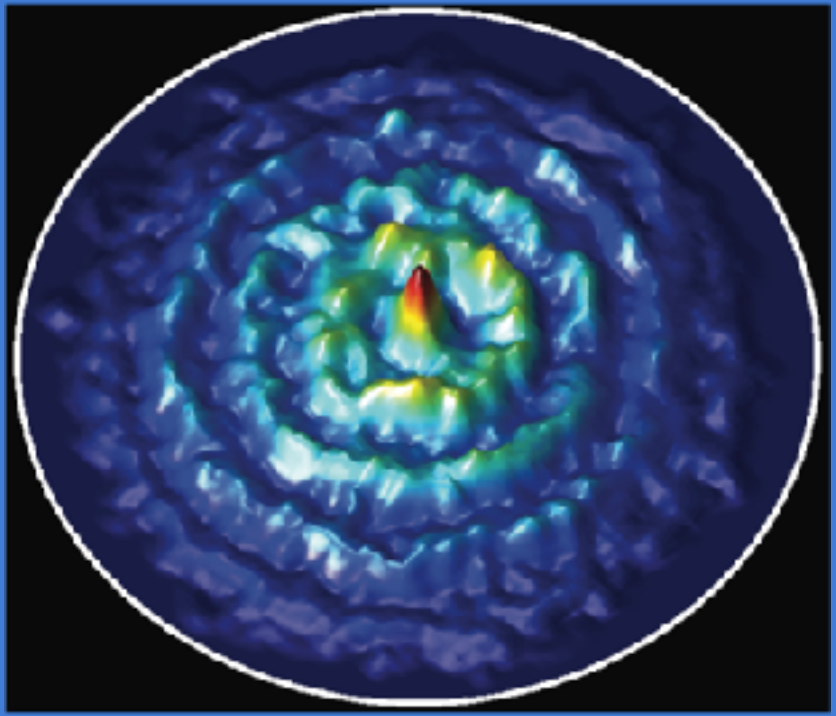
Fiete & Heller (2003)

Emerging physical picture:
3 time scales

- **fast** dynamics: bouncing at resonance creates monochromatic wave field



- **intermediate** (strobed) pilot-wave dynamics: droplet rides its instantaneous guiding wave



- **long-term statistical** behaviour described by Faraday wave modes

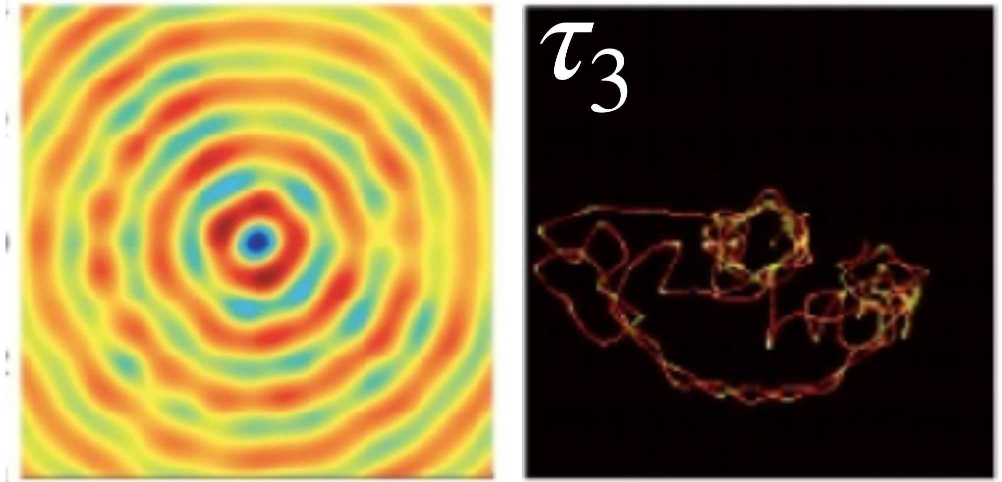
Emerging physical picture: 4 timescales

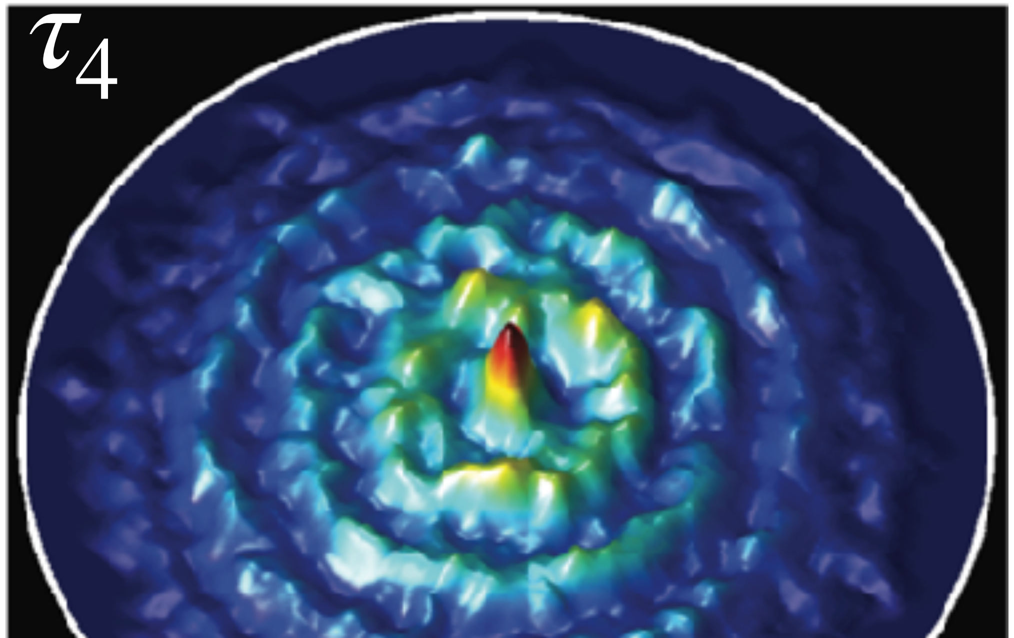
I. Wave generation

II. Pilot-wave dynamics

III. Establishment of mean pilot-wave

IV. Statistical convergence





Emerging physical picture: 4 timescales

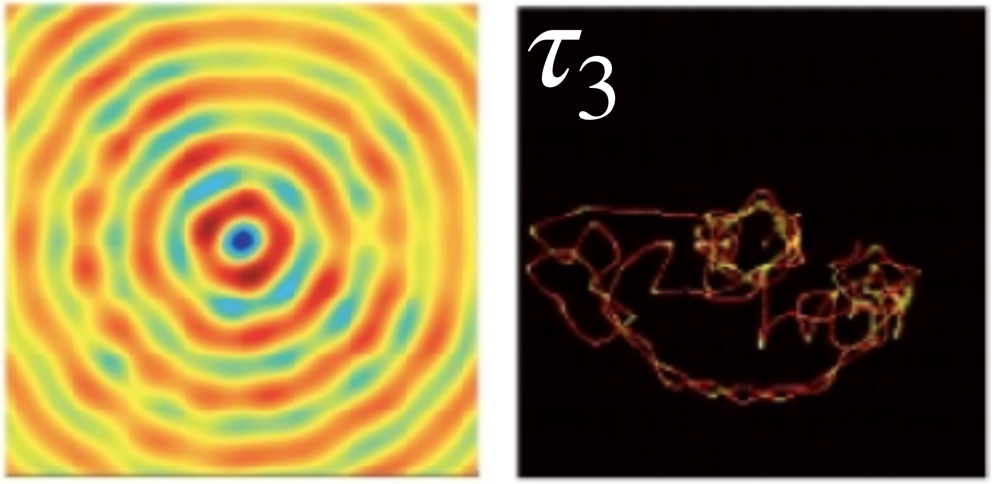
- clear separation of scales:

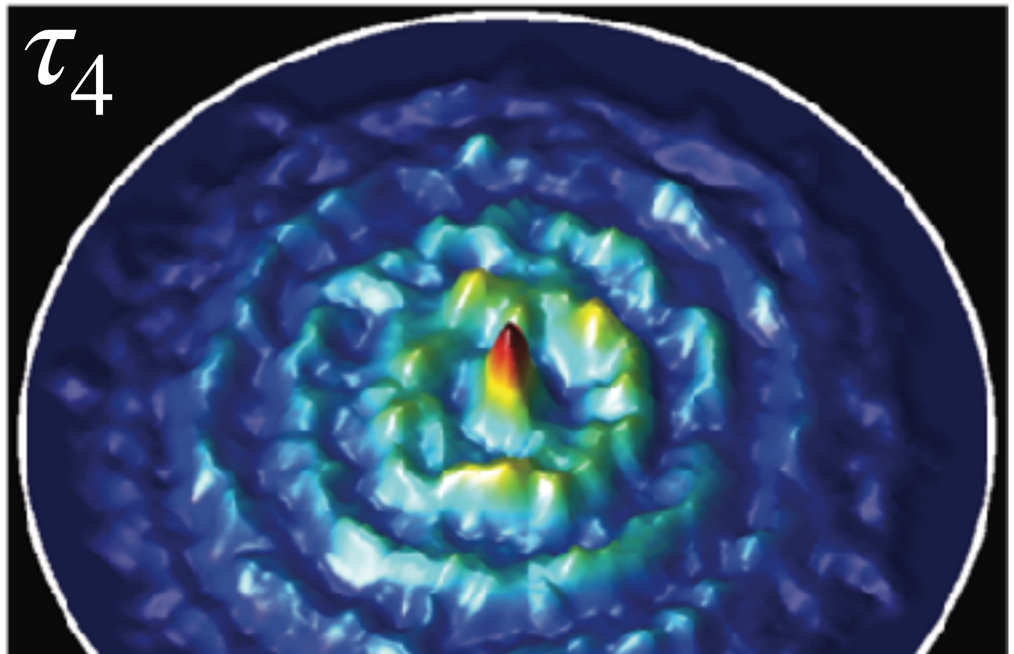
$$\tau_1 \ll \tau_2 \ll \tau_3 \ll \tau_4$$

- timescales τ_1, τ_2, τ_3 all unresolved in QM
- identify mean pilot-wave with Q: evolution for $\tau_3 < t < \tau_4$ looks like Bohmian mechanics

Note:

- if this system is a HQA, then we should be able to develop an analog of Bohmian mechanics
- QM does resolve the relaxation to a statistical equilibrium from an ensemble of ICs
- we should do likewise





	de Broglie	**Walkers**
WAVE TRIGGER	**ZITTERBEWEGUNG**	**Bouncing**
VIBRATION FREQUENCY	$\omega_c = \frac{m_o c^2}{\hbar}$	$\omega_d = \sqrt{\frac{\sigma}{m}}$
WAVES	**Matter waves**	**Capillary Faraday**
WAVE-PARTICLE RESONANCE	**Harmony of phases**	$\omega_d = \omega_F$
WAVE ENERGETICS	$mc^2 \longleftrightarrow \hbar\omega$	$mgH \longleftrightarrow$ Surface Energy
KEY PARAMETER	$\hbar$	σ
STATISTICAL WAVELENGTH	λ_B	λ_F
VIBRATION LENGTH	$\lambda_c = h/mc$	λ_F

Bush (2015)

Shortcomings of the quantum pilot-wave theories

Bohmian mechanics

- a dynamical reformulation of a statistical theory
- particle is piloted by a wave form Ψ_s of unspecified origins
- *nonlocal*: particle is guided by the non-local quantum potential

de Broglie's mechanics

- original double-solution theory distinguished between Ψ and Ψ_s
- form of pilot-wave Ψ unspecified: several options considered
- at one stage set $\Psi \propto \Psi_s$: reduces to Bohmian mechanics

→ two theories conflated into `de Broglie-Bohm theory'

The subquantum realm

In their later years, both de Broglie (1987) and Bohm appealed to a stochastic sub quantum realm, in what is now known as ….

The quantum vacuum

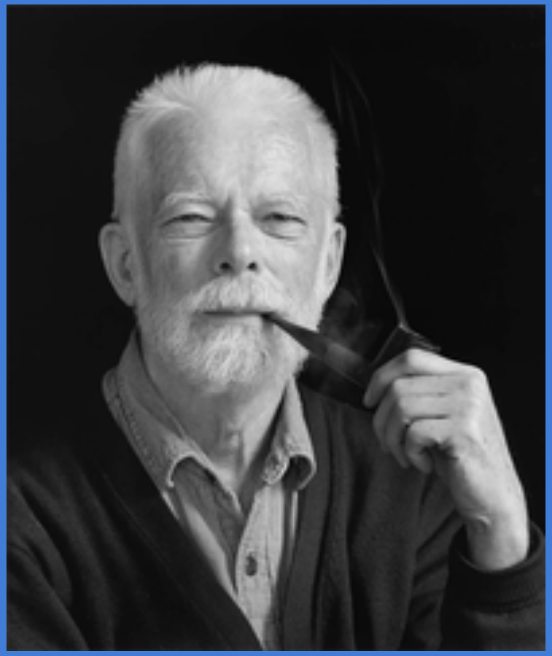
The quantum vacuum is seen as a turbulent sea, roiling with waves associated with a panoply of fields, including electromagnetic and Higgs fields, as well as those responsible for the weak and strong forces. Insofar as they interact with quantum particles, all such fields are candidates for de Broglie's pilot-wave.

Vacuum-based pilot-wave theories…

… seek de Broglie's pilot-wave theory in the quantum vacuum fields.

Stochastic Mechanics (1958)

Ed Nelson



- Bohm & Vigier (1954) posited a stochastic subquantum motion complementing the Madelung flow

- Nelson (1958) showed that LSE describes Brownian motion of a mass m with diffusivity $\hbar/m$

$$\boxed{D \sim \frac{\hbar}{m} \sim \frac{\hbar k}{mk} \sim U\lambda_B}$$

- a random walk with characteristic velocity U and length scale λ_B

- Surdin (1972) proposed the EM zero-point field as the source of the stochasticity

- this approach has been forwarded by the group of Gerhard Groessing (2013 onwards), who took inspiration from the walking droplets

The QuantumVacuum and Stochastic Electrodynamics

— *Boyer 2011, Milonni 2013*

- at zero temperature, there is electromagnetic (`zero-point') energy
- provides alternative explanations for Casimir forces, van der Waals forces, the blackbody radiation spectrum
- there is only one spectral form that is homogeneous, isotropic, scale invariant and Lorentz invariant

ENERGY PER NORMAL MODE: $U(\omega) = C\omega/c$ where C is a constant

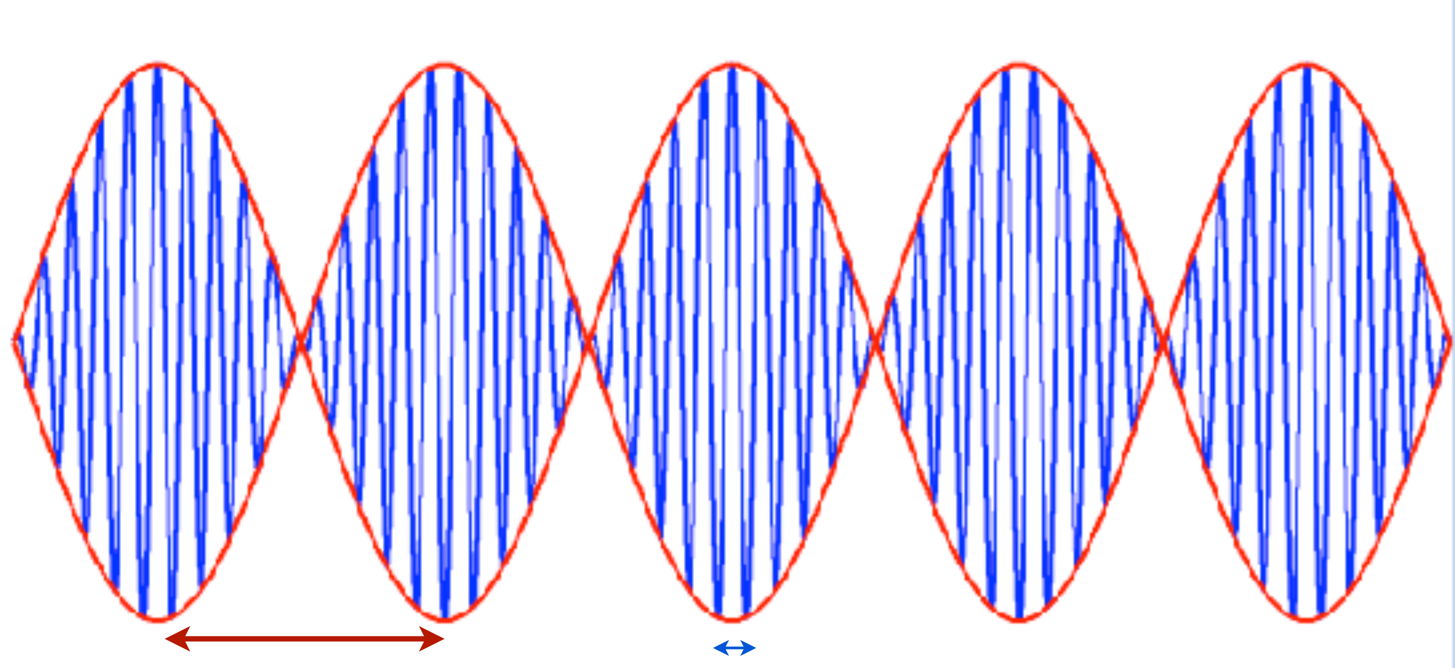
- empirical fact deduced from experiments on Casimir effect: $C = \hbar c/2$

Zero point energy: $$\boxed{U(\omega) = \hbar\omega/2}$$

- provides a natural means of introducing $\hbar$ into a classical theory
- might provide the seed field for de Broglie's matter waves, which would thus be of electromagnetic origin (*de la Pena, Cetto & Valdes-Hernandes, 2015*)

The Quantum Pilot Wave (according to SED)

- EM wave generated by resonant interaction between particle ZTB and the vacuum fluctuations



$$\lambda_B = \frac{h}{mv} \qquad \lambda_c = \frac{h}{m_o c}$$

"The de Broglie wave is the wave formed by the modulation of the Lorentz-transformed, Doppler-shifted superposition of the whole set of random, stationary EM waves with the Compton frequency with which the particle interacts."

- De la Pena & Cetto (Quantum Dice, 1997)

- mass is simply a place holder for electromagnetic energy (Haesch & Rueda 2001)
- particle mass increases with speed due to increased interaction with vacuum field

Suggested analogy: how quantum dynamics *might* work

vacuum fluctuations

+

Zitterbewegung

+

resonant interaction

+

de Broglie's relativistic pilot-wave theory

Relation between the hydrodynamic and QM pilot-wave theories

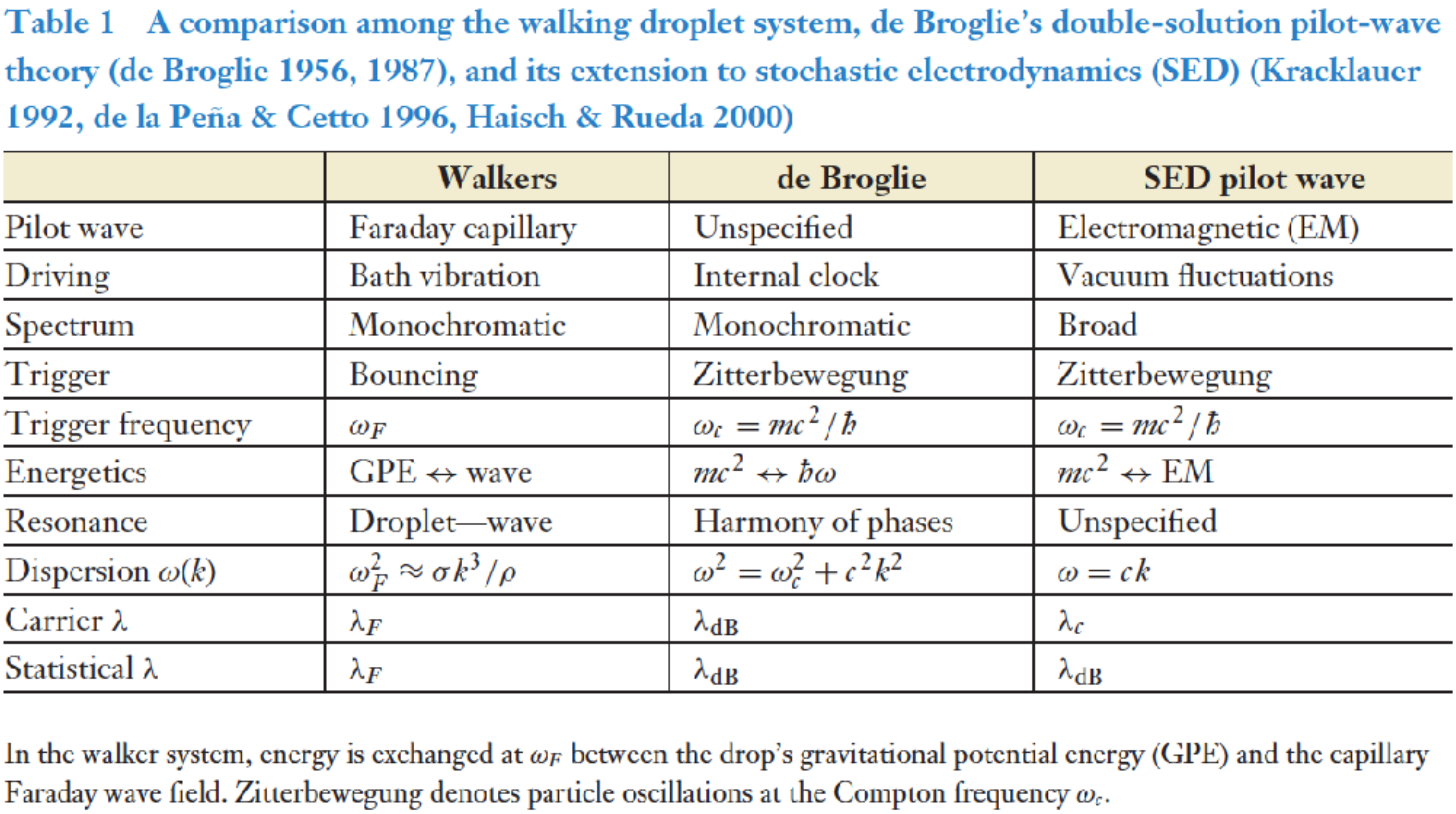


Table 1 A comparison among the walking droplet system, de Broglie's double-solution pilot-wave theory (de Broglie 1956, 1987), and its extension to stochastic electrodynamics (SED) (Kracklauer 1992, de la Peña & Cetto 1996, Haisch & Rueda 2000)

	Walkers	de Broglie	SED pilot wave
Pilot wave	Faraday capillary	Unspecified	Electromagnetic (EM)
Driving	Bath vibration	Internal clock	Vacuum fluctuations
Spectrum	Monochromatic	Monochromatic	Broad
Trigger	Bouncing	Zitterbewegung	Zitterbewegung
Trigger frequency	ω_F	$\omega_c = mc^2/\hbar$	$\omega_c = mc^2/\hbar$
Energetics	GPE ↔ wave	$mc^2 \leftrightarrow \hbar\omega$	$mc^2 \leftrightarrow$ EM
Resonance	Droplet—wave	Harmony of phases	Unspecified
Dispersion $\omega(k)$	$\omega_F^2 \approx \sigma k^3/\rho$	$\omega^2 = \omega_c^2 + c^2k^2$	$\omega = ck$
Carrier λ	λ_F	λ_{dB}	λ_c
Statistical λ	λ_F	λ_{dB}	λ_{dB}

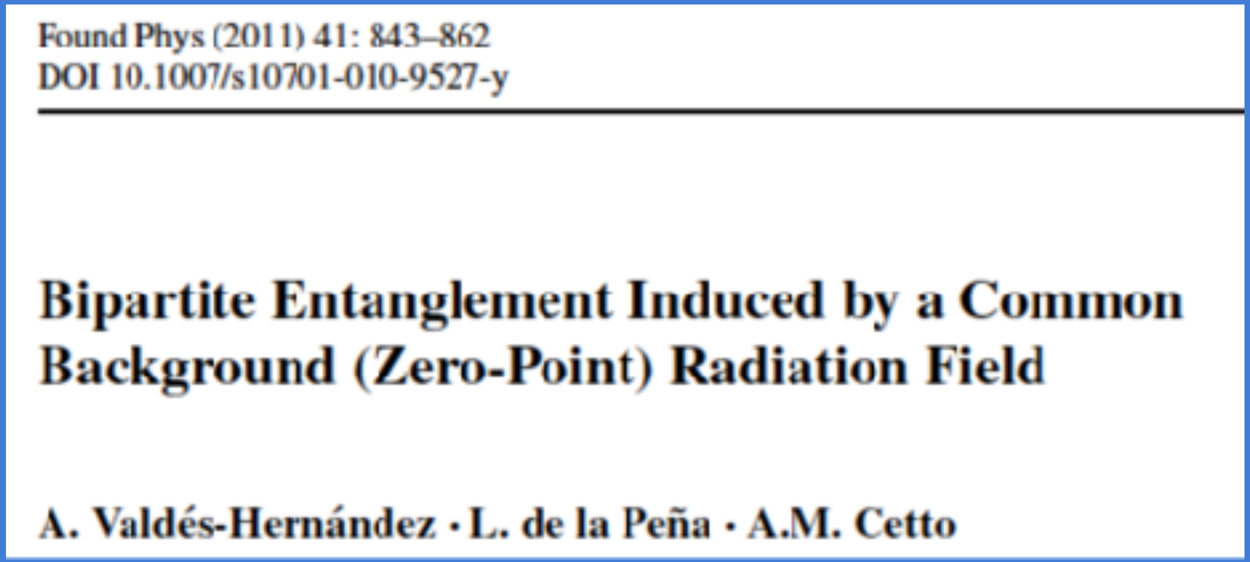
In the walker system, energy is exchanged at ω_F between the drop's gravitational potential energy (GPE) and the capillary Faraday wave field. Zitterbewegung denotes particle oscillations at the Compton frequency ω_c.

Bush, *ARFM* (2015)

Vacuum-based pilot-wave models

- modern extensions of de Broglie's pilot-wave theory, reminiscent of walkers
- suggests that QM paradoxes may be resolved by elucidating dynamics on the Compton scale

- in quantum field theory, the Compton frequency sets the time and length scales of particle pair production from the vacuum, which poses a challenge to experimental probing of such scales.



Found Phys (2011) 41: 843–862
DOI 10.1007/s10701-010-9527-y

Bipartite Entanglement Induced by a Common Background (Zero-Point) Radiation Field

A. Valdés-Hernández · L. de la Peña · A.M. Cetto

And quantum non-locality?

- identical particles interact through a common EM pilot wave
- rationalize entanglement in terms of classical, wave-induced correlations

The Klein-Gordon Equation

$$\boxed{\frac{1}{c^2}\Psi_{tt} \;-\; \nabla^2\Psi \;+\; \frac{m^2c^2}{\hbar^2}\,\Psi \;=\; 0}$$

Dispersion relation: $\omega = \omega_c(1+\beta^2)^{1/2}$

where $\omega_c = \dfrac{mc^2}{\hbar}$, $\beta = \dfrac{v}{c} = \dfrac{\hbar k}{mc} = \dfrac{k}{k_c}$, $k_c = \dfrac{mc}{\hbar}$

COMPTON

Seek solution: $\Psi(\mathbf{x},t) \;=\; e^{-i\frac{mc^2}{\hbar}t}\;\Psi^s(\mathbf{x},t) \longrightarrow$ \`Strobed' solution

FAST SLOW

$$\cancel{\frac{\hbar^2}{2mc^2}\,\Psi^s_{tt}} \;+\; i\hbar\;\Psi^s_t \;=\; -\;\frac{\hbar^2}{2m}\;\nabla^2\Psi^s$$

LSE

if $\Psi^s(\mathbf{x},t)$ is slowly varying, with a frequency ω s.t. $\omega/\omega_c \ll 1$

→ strobing at ω_c would reveal a slow wave that is a solution to the LSE.

The Klein-Gordon Equation

$$\boxed{\frac{1}{c^2}\Psi_{tt} \; - \; \nabla^2\Psi \; + \; \frac{m^2c^2}{\hbar^2}\,\Psi \; = \; 0}$$

Dispersion relation: $\omega = \omega_c(1+\beta^2)^{1/2}$

where $\omega_c = \dfrac{mc^2}{\hbar}$, $\beta = \dfrac{v}{c} = \dfrac{\hbar k}{mc} = \dfrac{k}{k_c}$, $k_c = \dfrac{mc}{\hbar}$

- for particle to be dressed in a monochromatic KG wave field, their speeds must match

Equate group velocity and particle velocity: $$\boxed{\frac{d\omega}{dk} = v}$$

- this is satisfied uniquely by the de Broglie wavelength: $\boxed{\mathbf{p} = \gamma m_0 \mathbf{v} = \hbar \mathbf{k}}$

- superluminal phase velocity: $$\boxed{v_\phi = \frac{\omega(k)}{k} = \frac{c^2}{v}}$$

The Klein-Gordon Equation

$$\boxed{\frac{1}{c^2}\,\Psi_{tt} \;-\; \nabla^2\Psi \;+\; \frac{m_o^2c^2}{\hbar^2}\,\Psi \;=\; 0}$$

COMPTON

Dispersion relation: $\omega = \omega_c(1+\beta^2)^{1/2}$, $\omega_c = \dfrac{m_oc^2}{\hbar}$

where $\beta = \dfrac{v}{c} = \dfrac{\hbar k}{mc} = \dfrac{\hbar k}{\gamma m_o c} = \dfrac{1}{\gamma}\dfrac{k}{k_c}$, $k_c = \dfrac{m_oc}{\hbar}$, $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$

Relation between de Broglie and Compton wavelengths:

$$\boxed{\frac{\lambda_B}{\lambda_c} = \frac{k_c}{k} = \frac{1}{\gamma\beta} = \frac{\sqrt{1-\beta^2}}{\beta}}$$

where $\lambda_c = \dfrac{h}{m_oc}$

Special cases

1. Stationary particle: $v=0,\ \beta=0,\ \gamma=1$ ⟶ $\lambda_B \to \infty$
2. Relativistic limit: $v\to c,\ \beta=1,\ \gamma\to\infty$ ⟶ $\lambda_B \to 0$
3. $\lambda_B = \lambda_c$ when $v = c/\sqrt{2},\ \beta = 1/\sqrt{2},\ \gamma = \sqrt{2}$

The Compton wavelength

- the wavelength of a photon whose kinetic energy is the same as the rest-mass energy of the particle:

$$E = \hbar\omega = \frac{hc}{\lambda_c} = mc^2 \longrightarrow \boxed{\lambda_c = \frac{h}{mc}}$$

$$\frac{1}{c^2}\Psi_{tt} - \underset{\substack{\uparrow \\ \text{KINETIC ENERGY}}}{\nabla^2\Psi} + \underset{\substack{\uparrow \\ \text{REST-MASS ENERGY}}}{\frac{m^2c^2}{\hbar^2}}\Psi = 0$$

- the length at which kinetic energies become comparable to rest-mass energy

Note: $\beta^2 = \left(\frac{v}{c}\right)^2 = \frac{\omega}{\omega_c} = \frac{\hbar^2 k_{dB}^2}{m^2c^2} = \frac{k_{dB}^2}{k_c^2} < 1$

where $\omega_c = \dfrac{mc^2}{\hbar}$ is the Compton frequency

What is the Compton scale?

Compton frequency

$$\omega_c = \frac{mc^2}{\hbar} \qquad \ell_c = \frac{c}{\omega_c} = \frac{\hbar}{mc}$$

Compton length

- it is the timescale of breakdown of the Lorentz-Dirac equation, as describes a charge moving in its own EM wave
- it sets the scales of the classical model of the electron (Burinskii 2008)
  - a charge orbits the Compton length at the Compton frequency
  - failure to resolve such a dynamics renders spin an intrinsic property

- sets the scale of particle creation and annihilation in the quantum vacuum
- measurements on this scale are difficult but currently underway in electron diffraction experiments

Time-Energy Uncertainty

$$\boxed{\Delta E \ \Delta t \geq \hbar/2}$$

A simple interpretation (due to de Broglie)

- if the particle is oscillating at the Compton frequency $\omega_c = mc^2/\hbar$

  exchanging rest mass energy mc^2 and field energy $\hbar\omega$

- unresolved dynamics on the Compton time scale implies unresolved energies on the order of the particle's rest mass energy; hence, uncertainty as to whether or not a particle actually exists…

- HQA: resolving fast (bouncing) dynamics may change system from unpredictable to predictable, deterministic to seemingly random

So, what is the matter wave field in QM?

> *`What is it that waves in wave mechanics? We have no idea…'*
> — J.S. Bell (1993)

- de Broglie suggested that the field satisfies the Klein-Gordon equation, as describes the Higgs field (*viz* Einstein's last paper, on `*Ghost waves*')

- workers in Stochastic Electrodynamics (SED) suggest an EM pilot wave (*de la Pena, Cetto, Valdes-Hernandes 2015*)

- several have suggested *gravitational waves*: matter waves arise in the fabric of space time *(Feoli & Scarpetta 1998, D'Errico 2023)*

Walker-inspired pilot-wave theories

- *Andersen et al*. (2015) considered a particle exciting a waveform that satisfies the LSE, then moving in response to that field

- *Borghesi* (2017) explored a pilot-wave in which a particle moves within a non-dissipative elastic substrate, supporting waves satisfying KG equation

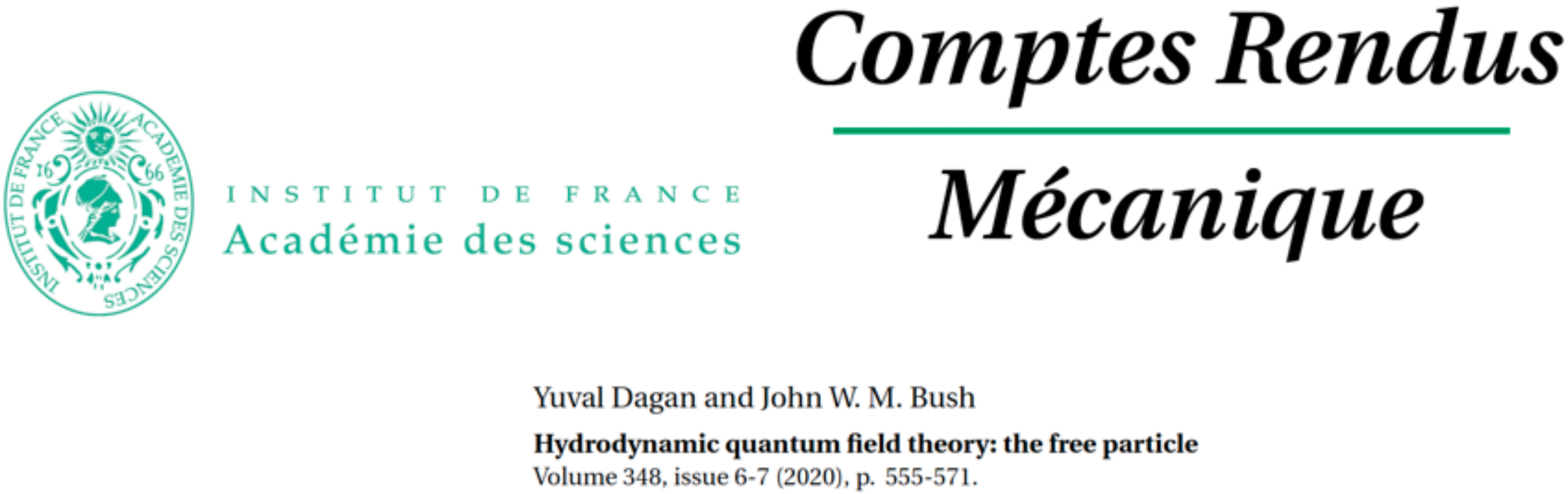
- *Drezet et al*. (2020) considered a vibrating particle self-propelling along a frictionless spring

- *Drezet et al*. (2024) is currently making a more direct attempt to complete de Broglie's mechanics

- *Dagan & Bush* (2020) and *Durey & Bush* (2020) proposed and explored a \`Hydrodynamic quantum field theory'

- *Darrow & Bush* (2024) developed a Lagrangian-based, relativistic pilot-wave theory

Hydrodynamically-inspired quantum field theory



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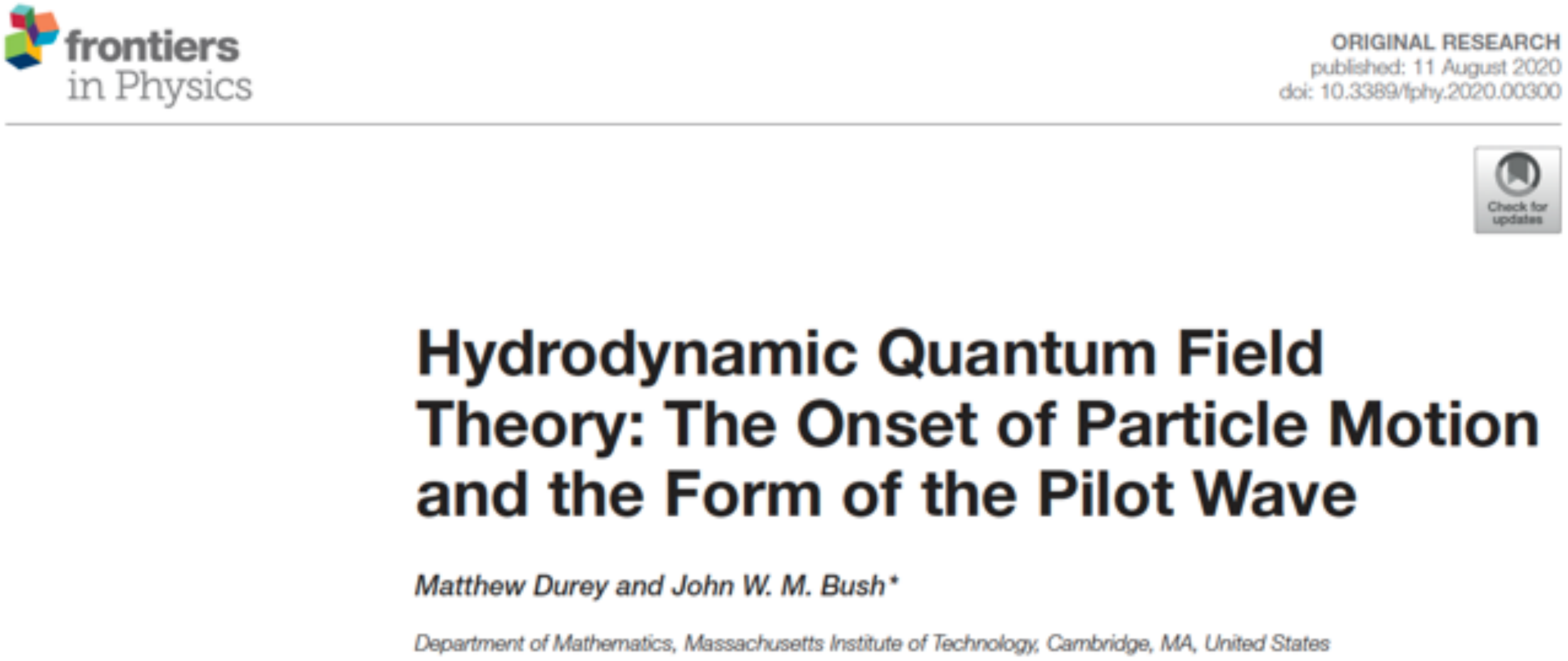
Yuval Dagan and John W. M. Bush

Hydrodynamic quantum field theory: the free particle
Volume 348, issue 6-7 (2020), p. 555-571.



frontiers
in Physics

ORIGINAL RESEARCH
published: 11 August 2020
doi: 10.3389/fphy.2020.00300

Hydrodynamic Quantum Field Theory: The Onset of Particle Motion and the Form of the Pilot Wave

*Matthew Durey and John W. M. Bush**

Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA, United States

Hydrodynamically-inspired quantum field theory

Dagan & Bush (2020)

- extend de Broglie's mechanics, informed by the pilot-wave hydrodynamics

> ``…*we can envisage a more active role for the particle, something which is not even admitted as conceivable in the conventional view. This may, for instance, enter as a 'source' of the pilot-wave field through an inhomogeneous term in the wave equation…*"
>
> **— *Holland (1995)***

- model particle as wave source, an oscillation at twice the Compton frequency

Forced Klein-Gordon equation

$$\phi_{tt} - c^2 \phi_{xx} + \omega_c^2 \phi = \epsilon \sin(2\omega_c t)\, e^{-[(x-x_p)/\lambda_c]^2}$$

`particle': a localized excitation
in the Higgs field

- consider the zero-particle-inertia, no-wave-damping limit
- particle moves in response to gradients in wave amplitude

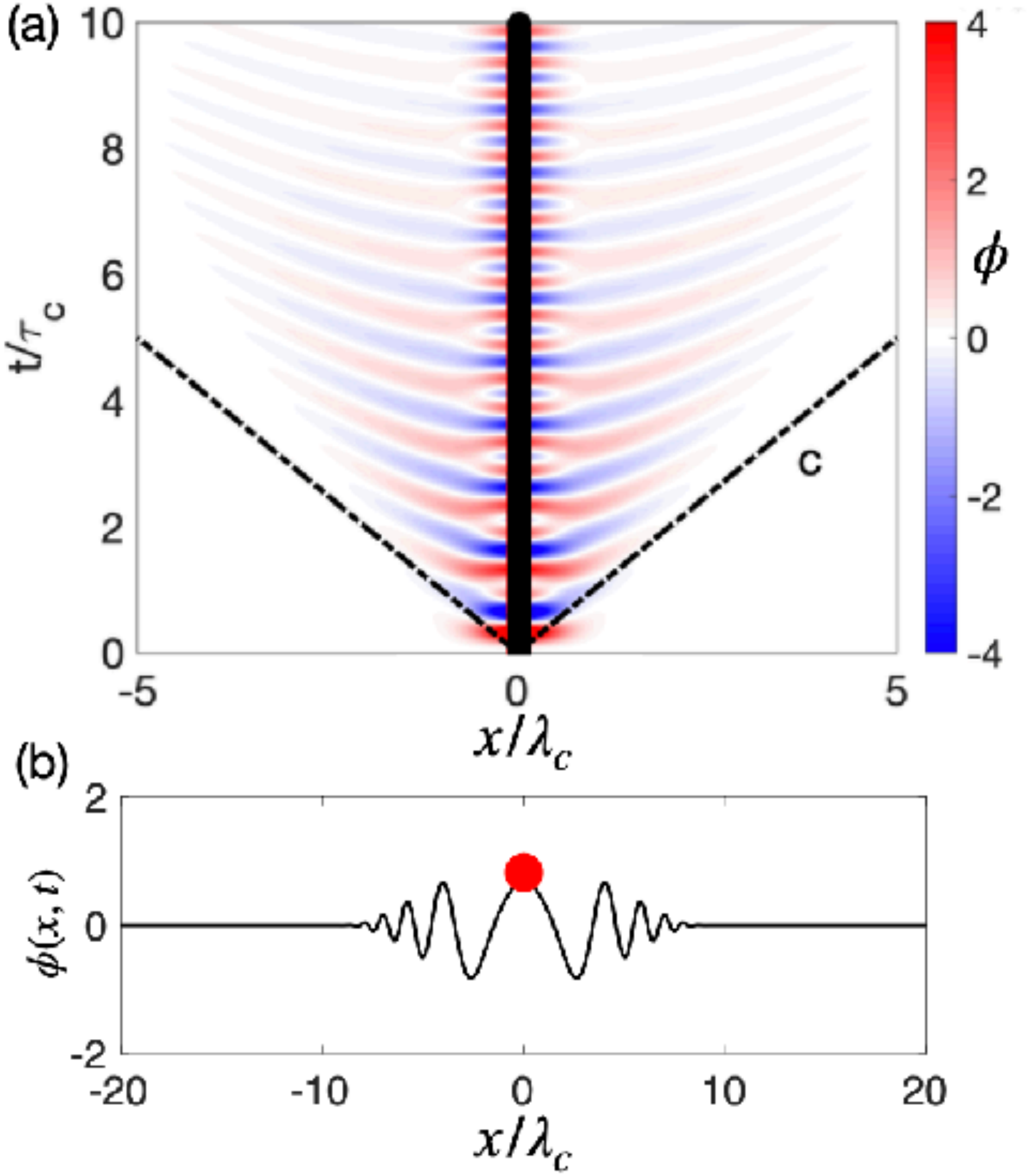
Coupling constant

Relativistic guidance equation:

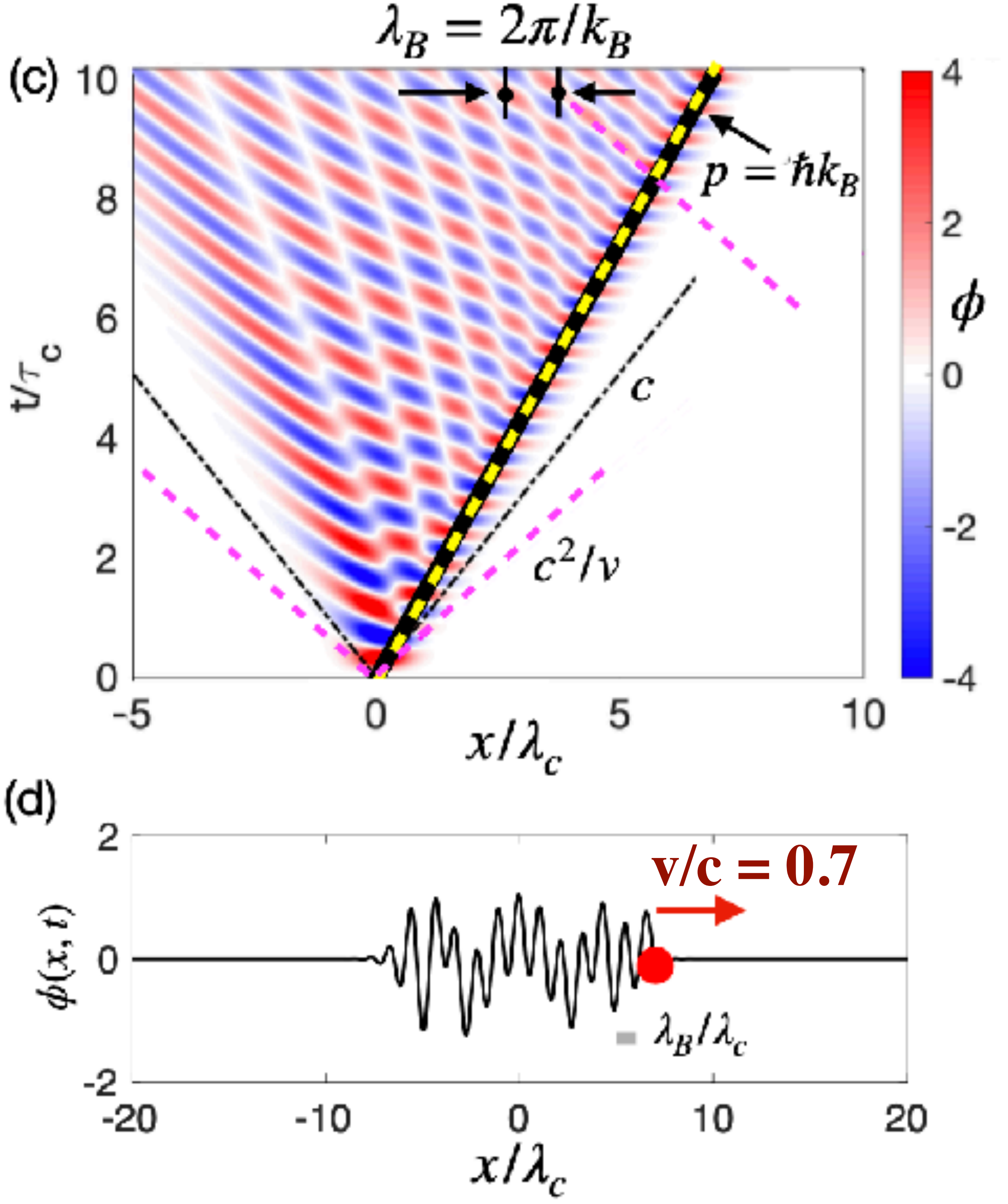
$$\gamma\ \dot{x}_p = -\overset{\downarrow}{\alpha}\ \frac{\partial \phi}{\partial x}$$

Hydrodynamic quantum field theory: Kinematics

Stationary particle

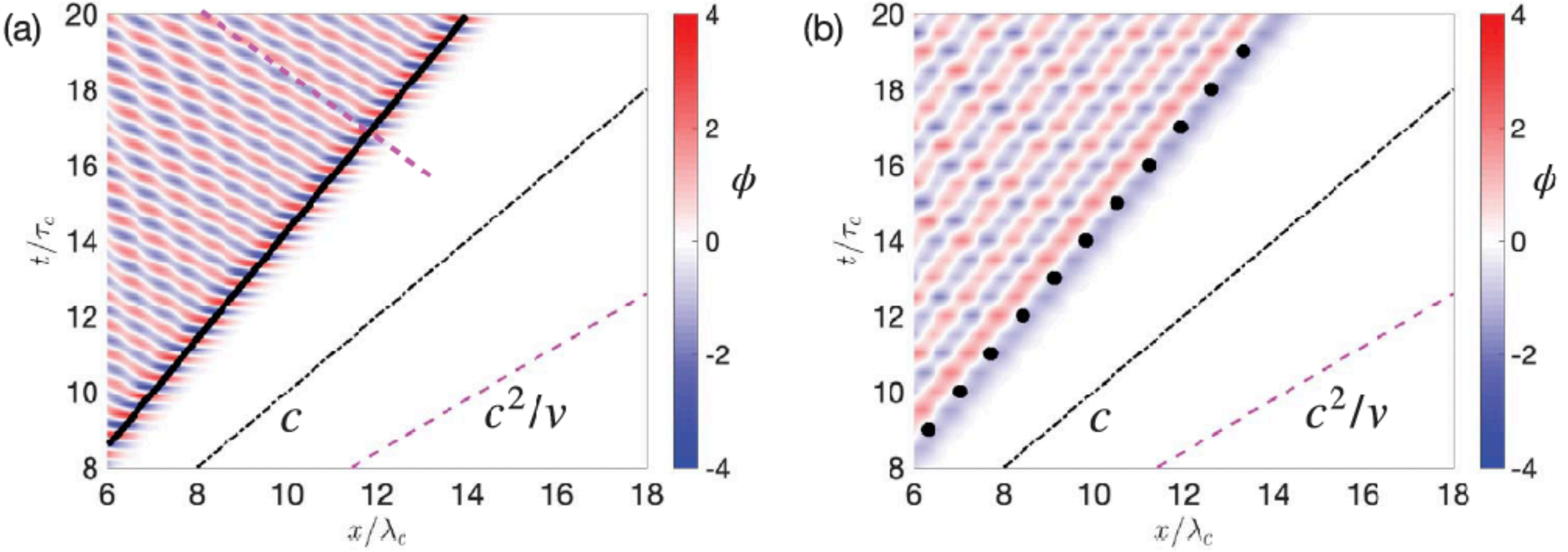


Uniformly translating particle



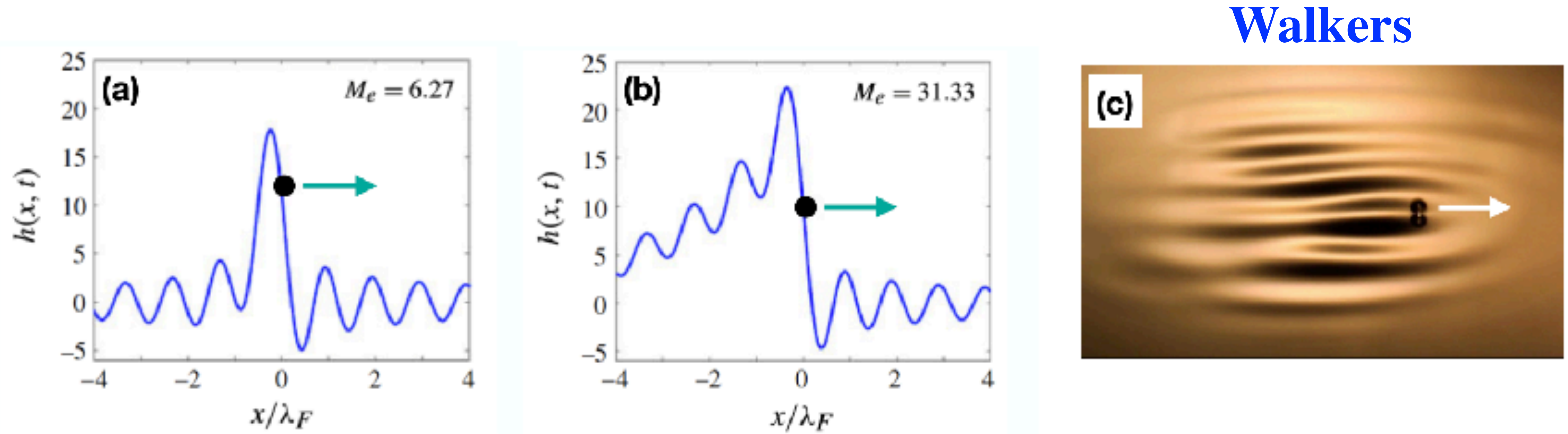
Hydrodynamic quantum field theory: Kinematics

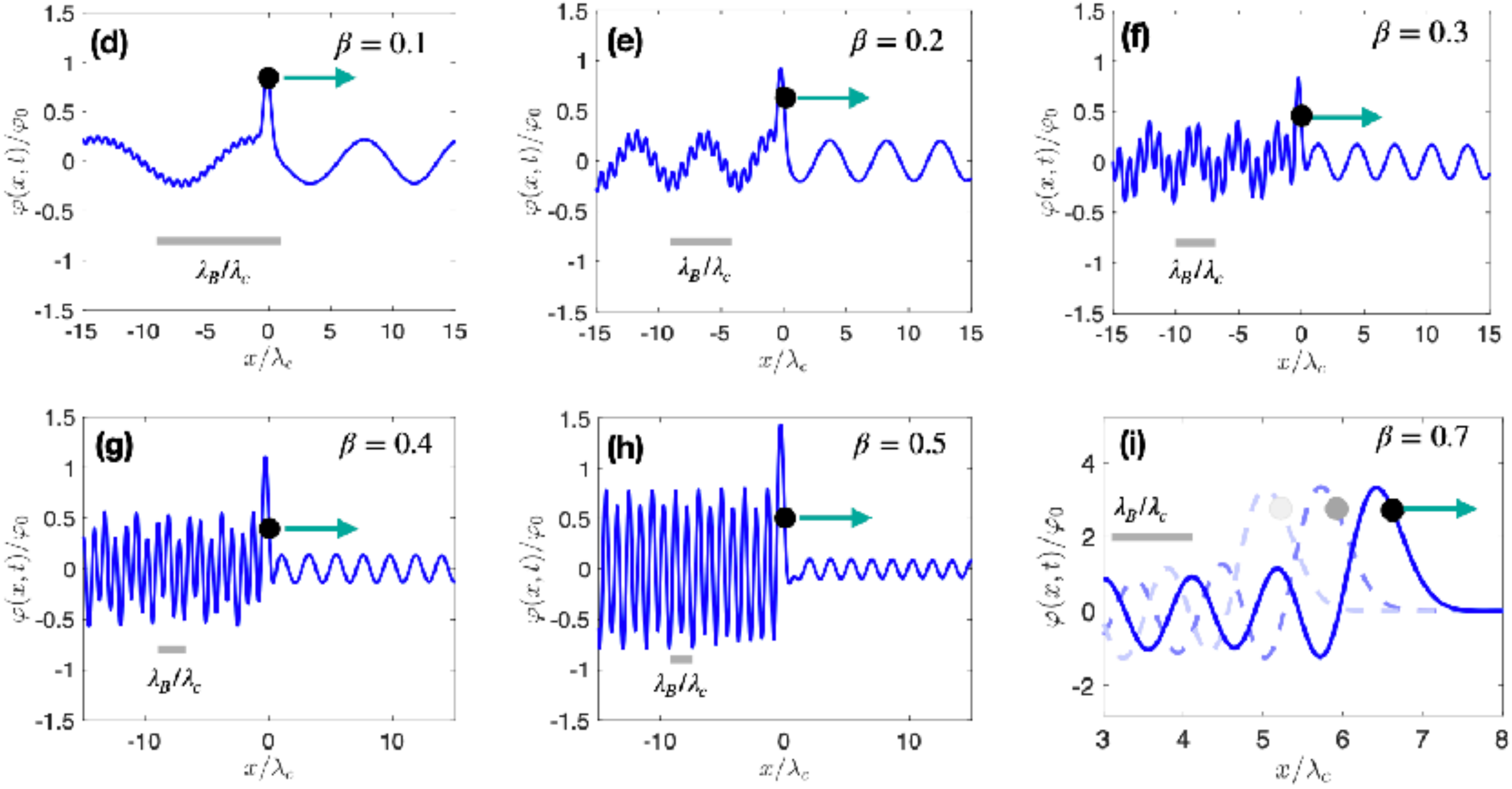
Pilot wave form **$v/c = 0.7$** **Wave form strobed at ω_c**



- phase speed comparable to c^2/v
- phase speed no longer apparent

HQFT: Kinematics





$$\beta = v/c$$

$$\lambda_B = \lambda_c/(\beta\gamma)$$

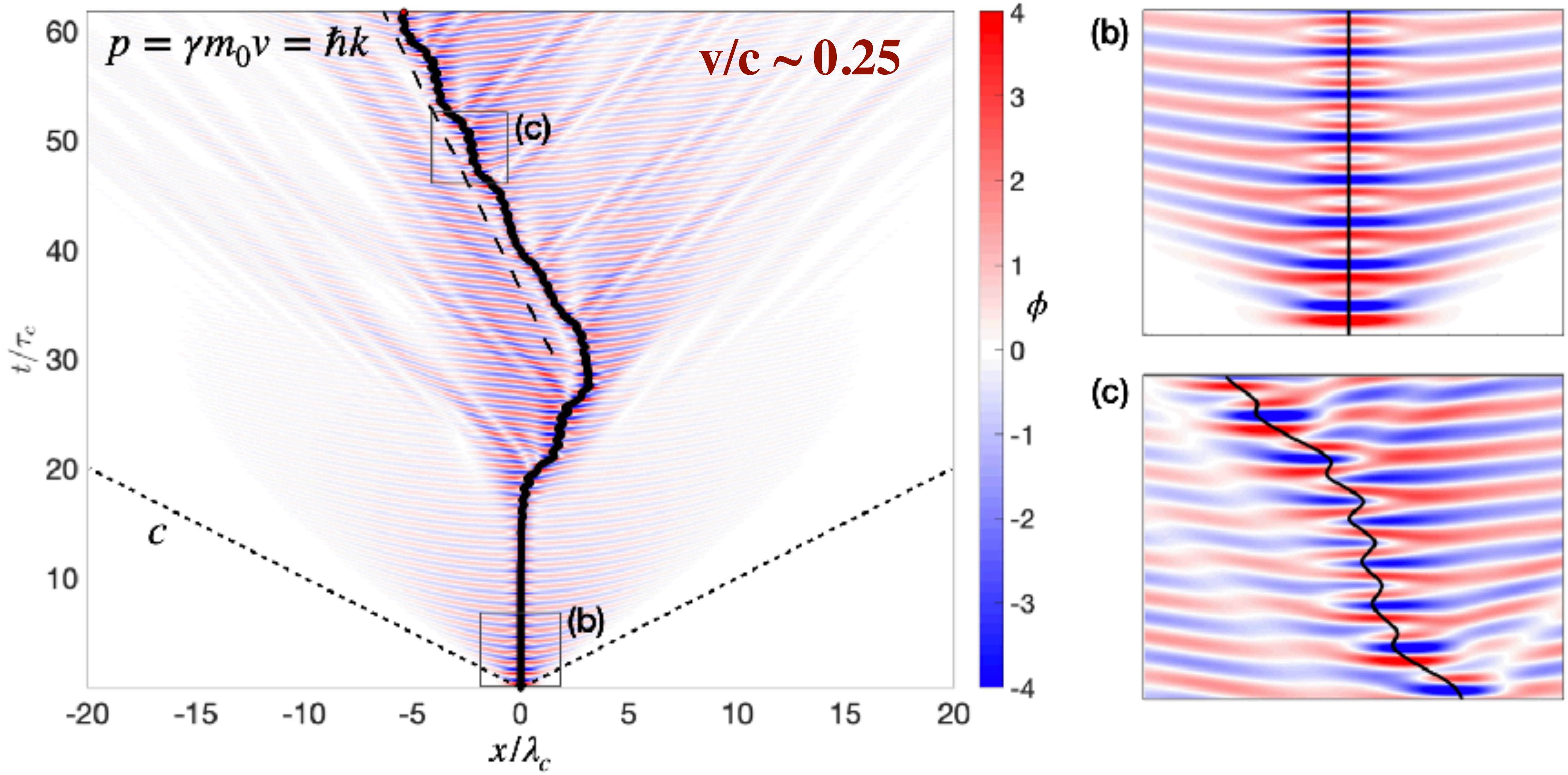
- strobed dynamics similar to that of walker system: particle rides steady wave form

HQFT Dynamics, The free particle: from *Jitter* to *Zitter*

- stationary state destabilizes into self-propelling state with mean momentum:

$$\boxed{\bar{p} = \gamma m v = \hbar k}$$

- changing coupling constant α changes $\bar{p}$ and k in concert

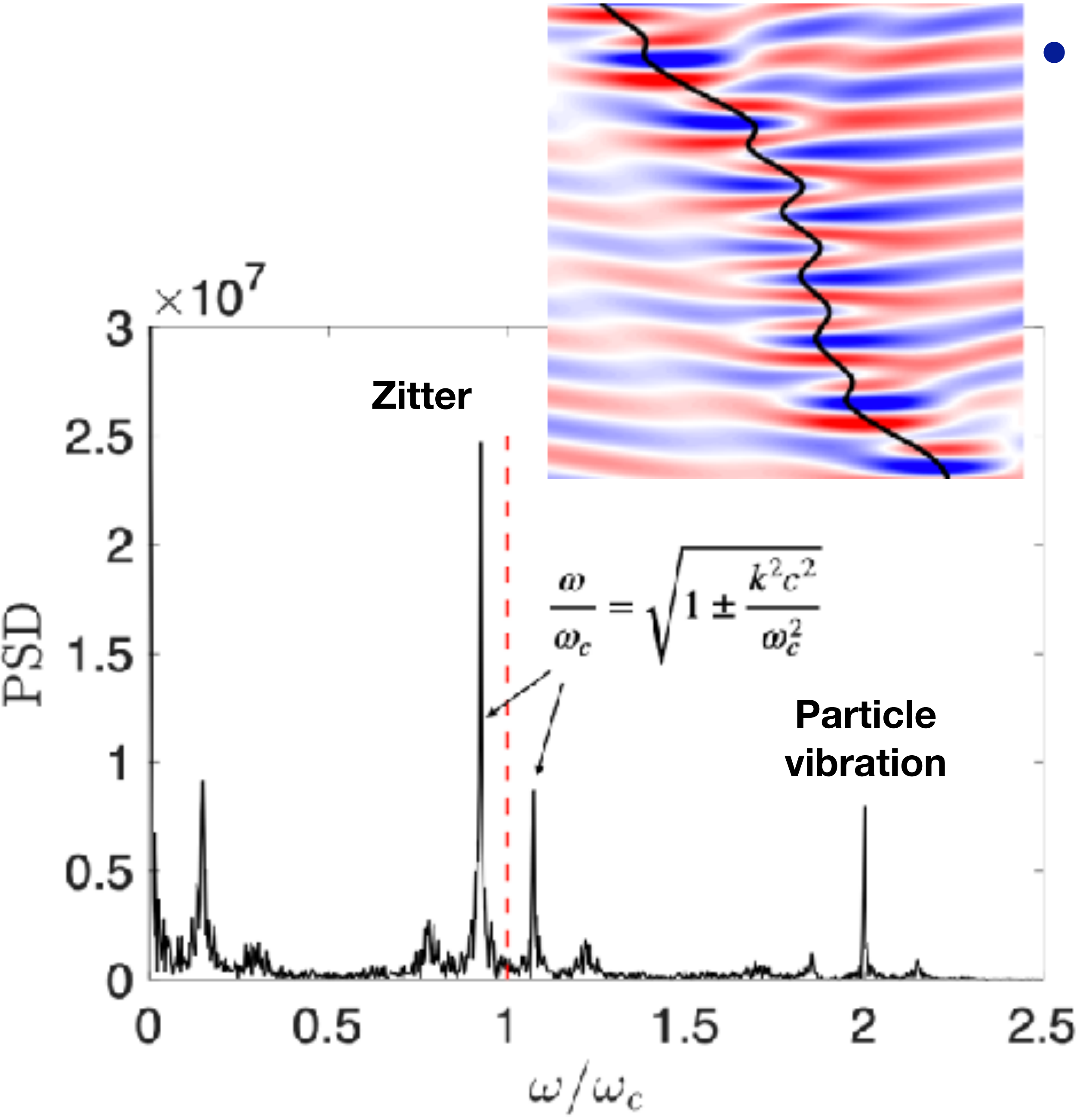


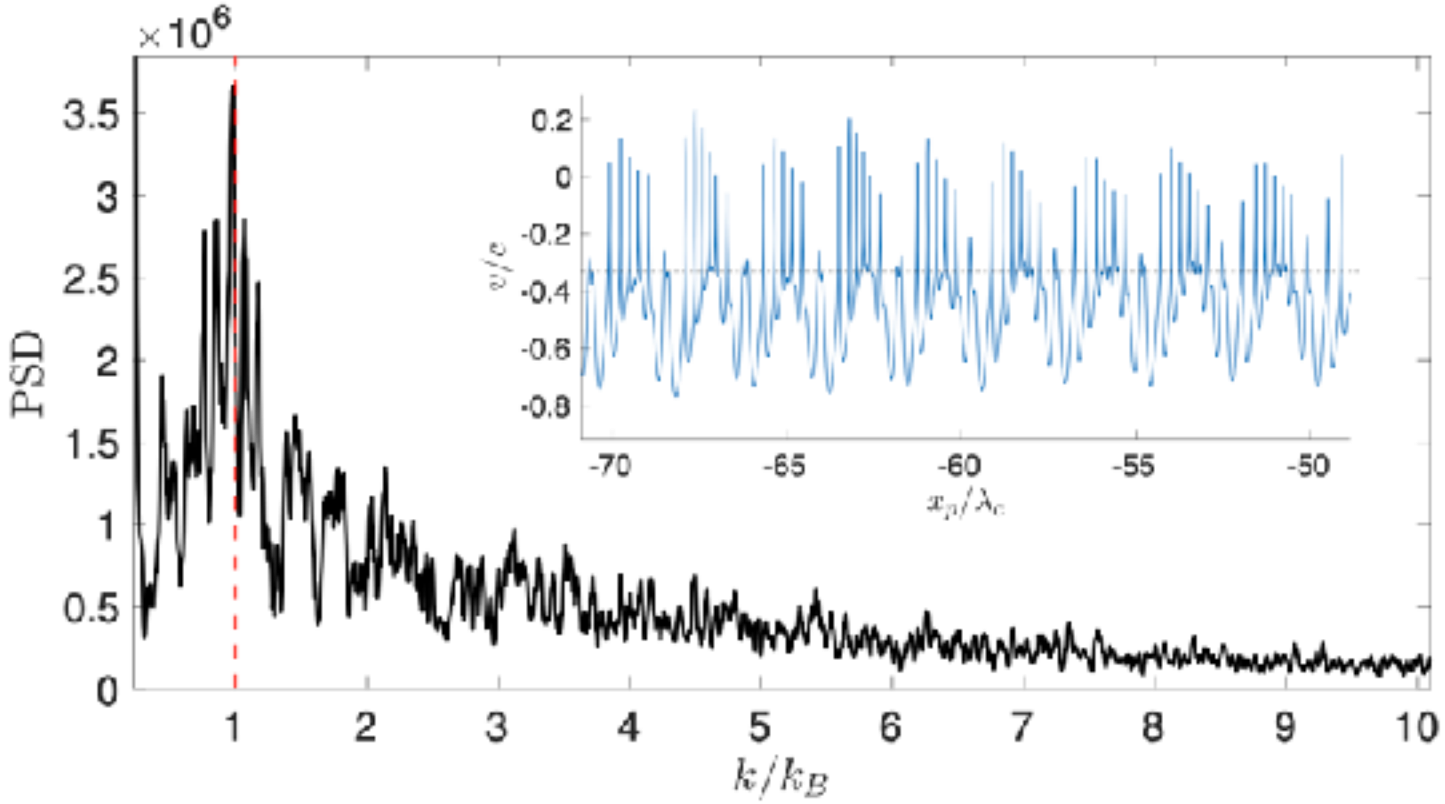
- mean motion complemented by in-line oscillations with de Broglie wavelength

Hydrodynamic quantum field theory: in-line Zitter

- mean motion consistent with de Broglie's relation

$$\boxed{\bar{p} = \gamma m v = \hbar k}$$





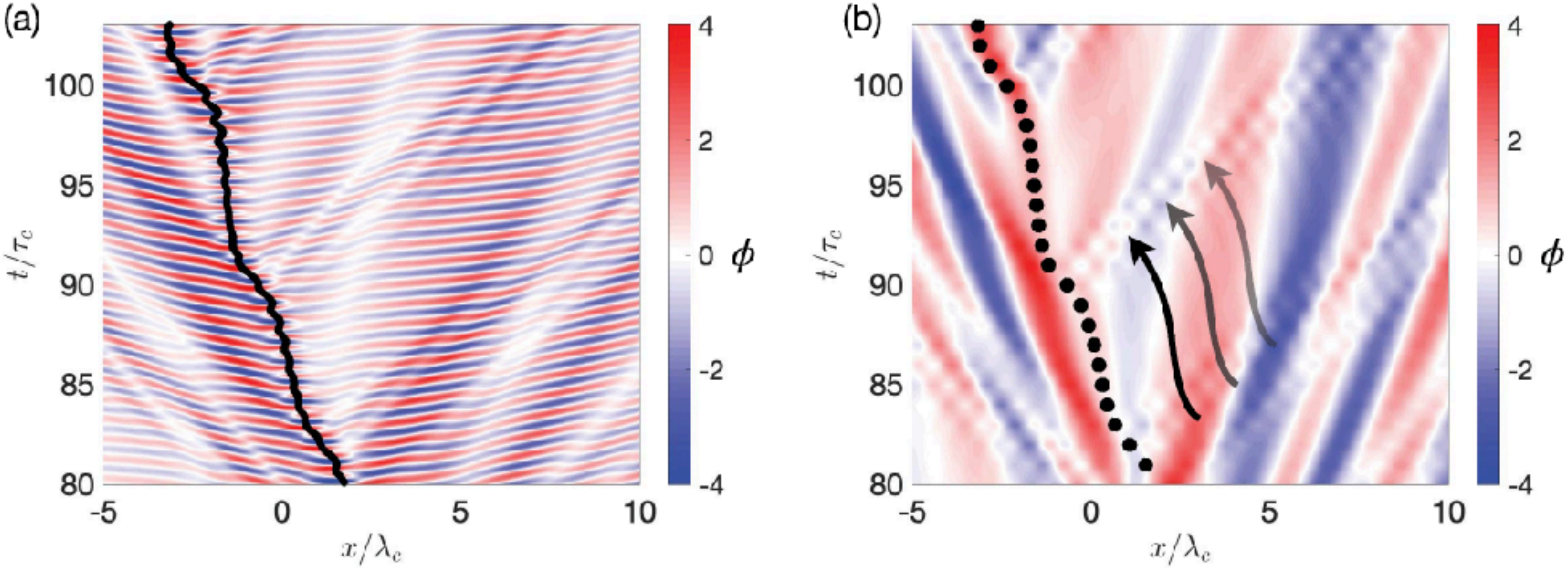
- motion accompanied by in-line oscillations with wavelength λ_B , frequency $\omega_{mod} = ck$
- suggests a dynamical interpretation of relativistic quantum dispersion relation

$$\omega^2 = \omega_c^2 + c^2k^2$$

HQFT Dynamics, The free particle: from *Jitter* to *Zitter*

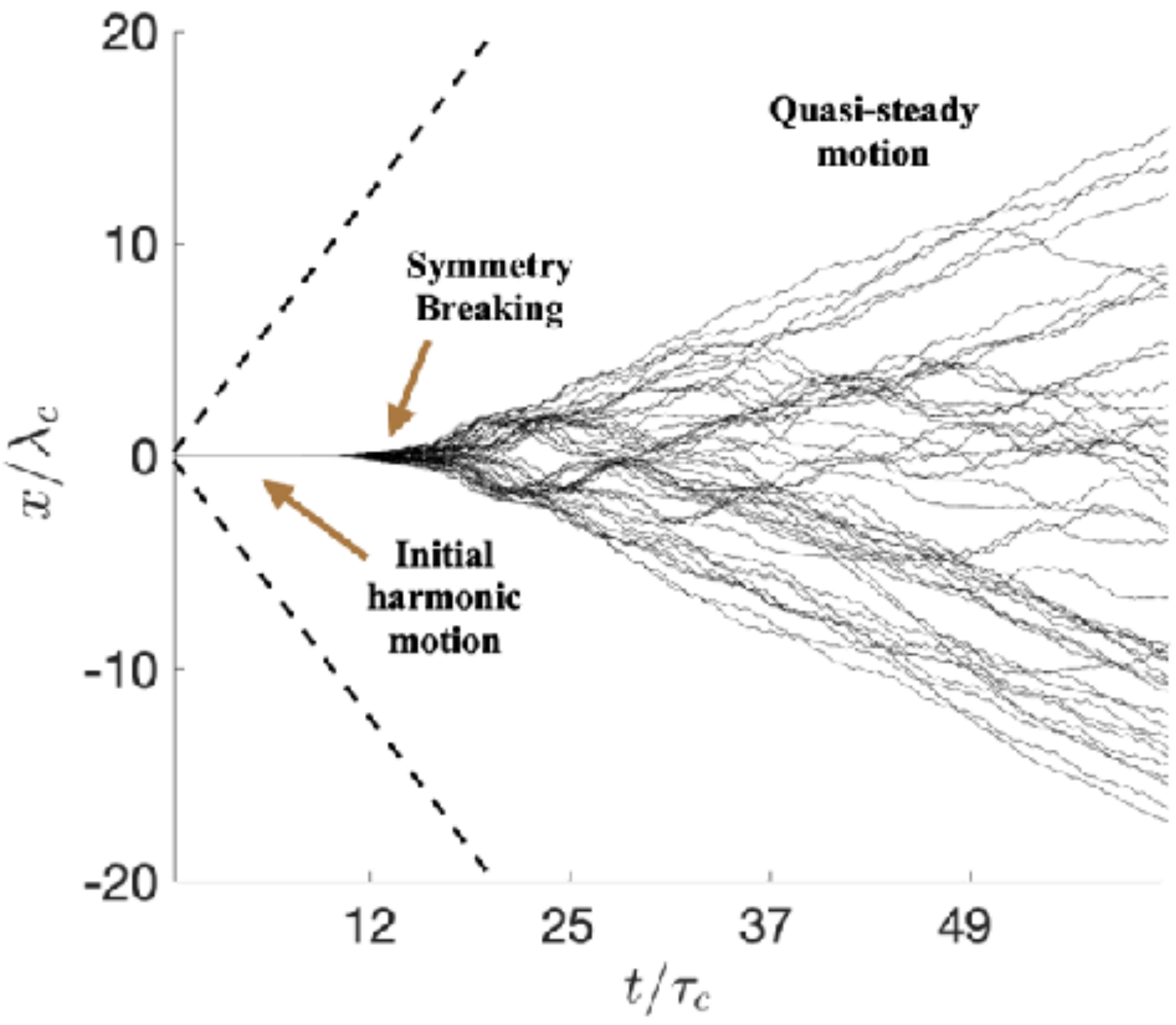
- wave form accompanying a free, self-propelling state

Pilot wave form **v/c ~ 0.25** **Wave form strobed at ω_c**



- motion accompanied by in-line oscillations with wavelength λ_B , frequency $\omega_{mod} = ck$
- strobed wave form dominated by structure on the de Broglie wavelength

HQFT: Evolution of an ensemble



Suggested direction

- couple dynamics of HQFT to Ensemble Interpretation of quantum mechanics
- consider an ensemble of initial conditions, see that system statistics evolve according to predictions of quantum mechanics
- start with systems with robust HQAs: Friedel oscillations, corrals, orbital PWH
- develop a trajectory-based description of quantum dynamics and statistics

The Physical Analogy

	Pilot-wave hydrodynamics	HQFT
Driving	Bath vibration	Zitterbewegung
Driving frequency	$2\ \omega_F$	$2\ \omega_c$
Particle vibration	Droplet bouncing	Zitterbewegung
Particle vibration frequency	ω_F	$\omega_c = \frac{m_0 c^2}{\hbar}$
Waves	Faraday Waves	Matter Waves
Pilot wavelength	λ_F	λ_B
Dispersion relation	$\omega_F^2 = gk_F + \frac{\sigma}{\rho} k_F^3$	$\omega^2 = \omega_c^2 + c^2 k^2$
Wave Energetics	$MgH \longleftrightarrow$ Surface energy	$mc^2 \longleftrightarrow \hbar\omega$
Wave energy parameter	σ	$\hbar$
Mean velocity	Free walking speed: u_0	$\bar{v} = \hbar k/(\gamma m_0)$
Vibration length	Step size: u_0/ω_F	$\lambda_c = h/mc$
In-line oscillation frequency	$u_0 k_F$	$\omega_{mod} = ck$
In-line oscillation length	λ_F	λ_B

HQFT: Analytics

Durey & Bush (2020)

$$\frac{\partial^2\phi}{\partial t^2} - c^2\frac{\partial^2\phi}{\partial x^2} + \omega_c^2\phi = \epsilon_p f(t)g(x - x_p(t))$$

$$\gamma(x_p')x_p' = -\alpha\left.\frac{\partial\phi}{\partial x}\right|_{x=x_p}$$

- recast trajectory equation in more familiar, integro-differential form

$$\gamma(x_p')x_p' = -\alpha\int_{-\infty}^{t} f(s)\partial_x\varphi(x_p(t) - x_p(s), t - s)\,\mathrm{d}s.$$

- reduces to a tractable form in the nonrelativistic limit, $v/c \ll 1$:

$$\frac{\mathrm{d}\hat{x}_p}{\mathrm{d}\hat{t}} = -\kappa\int_{-\infty}^{\hat{t}} \hat{f}(s)\big(\hat{x}_p(\hat{t}) - \hat{x}_p(s)\big)\frac{J_1(\hat{t} - s)}{\hat{t} - s}\,\mathrm{d}s, \qquad \text{where} \qquad \kappa = \alpha\epsilon_p/2c^3$$

- deduced analytically the critical coupling parameter for self-propulsion, $\kappa_c = 2.98$

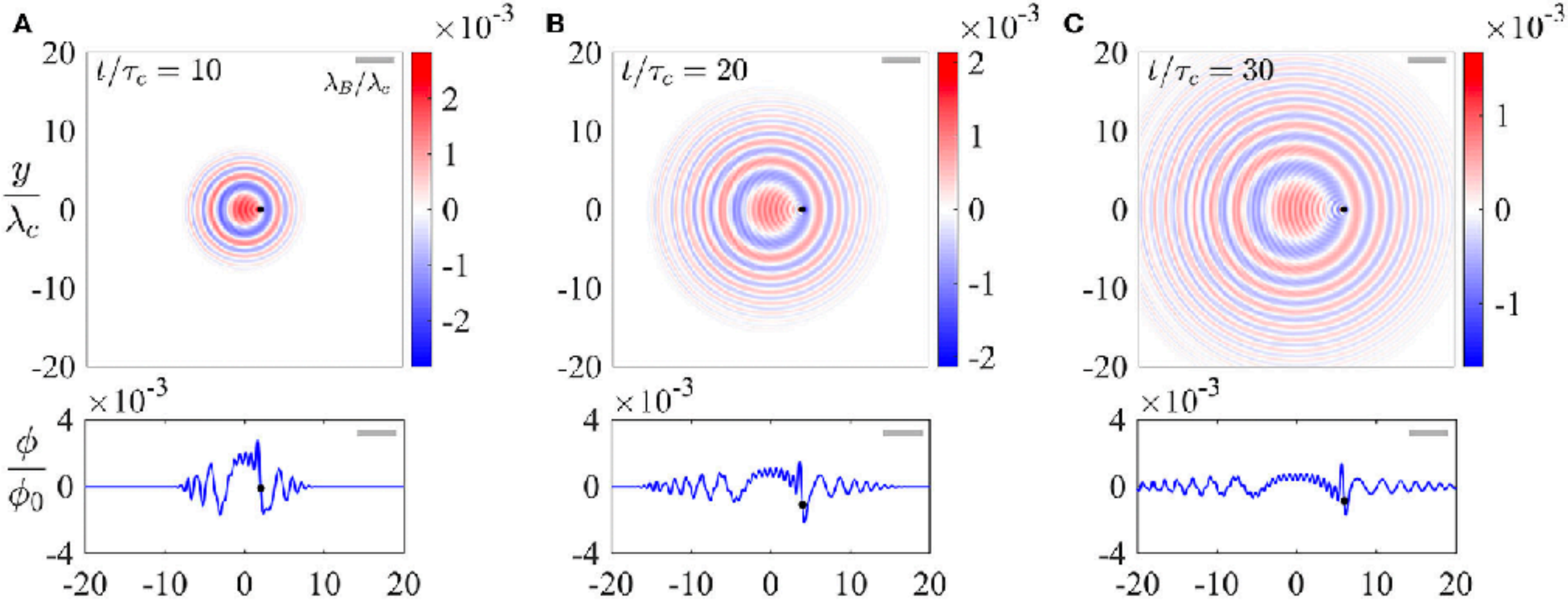
- onset of motion marked by in-line oscillations at frequency ω_c, the *Zittering* motion apparent in the simulations of Dagan & Bush

HQFT I

Durey & Bush (2020)

- deduced analytically the form of the 1D and 2D pilot-wave field by solving an IVP

Time evolution of pilot-wave form for $v/c = 0.2$



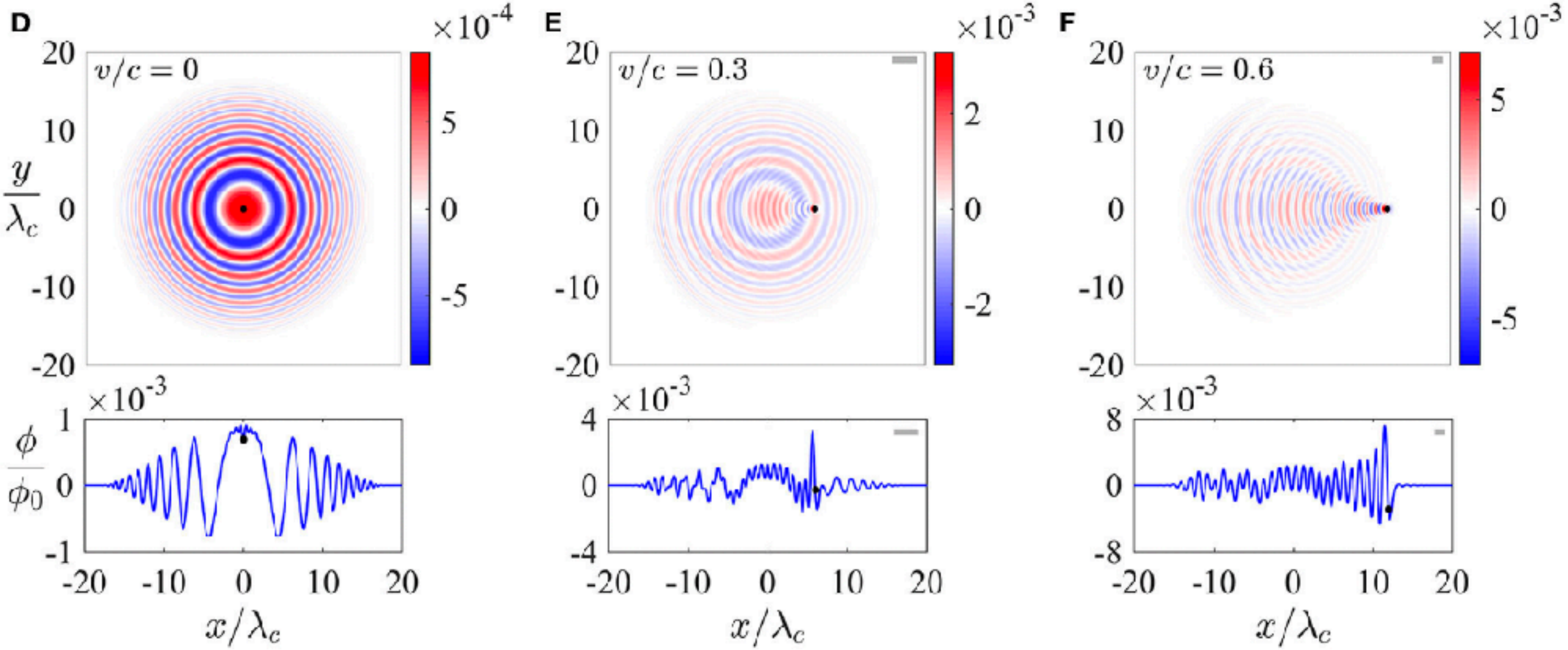
- pilot wave and particle momentum related through: $\bar{p} = \gamma m v = \hbar k$
- superposition of radially propagating waves with λ_c and carrier wave with λ_B

HQFT I

Durey & Bush (2020)

- deduced analytically the form of the 1D and 2D pilot-wave field by solving an IVP
- pilot wave and particle momentum related through: $\bar{p} = \gamma m v = \hbar k$

Dependence of waveform at $t = 20\tau_c$ on v/c

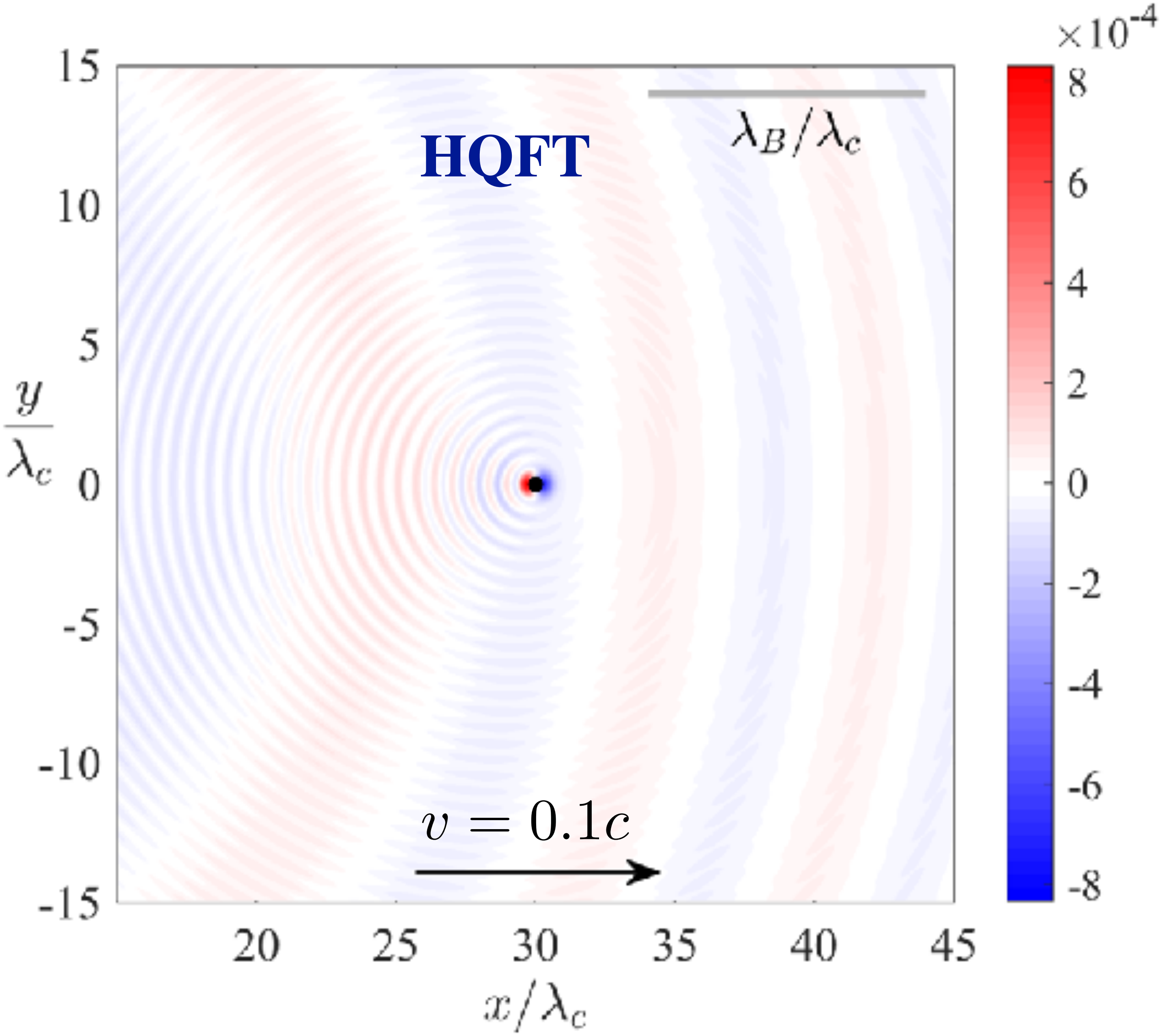


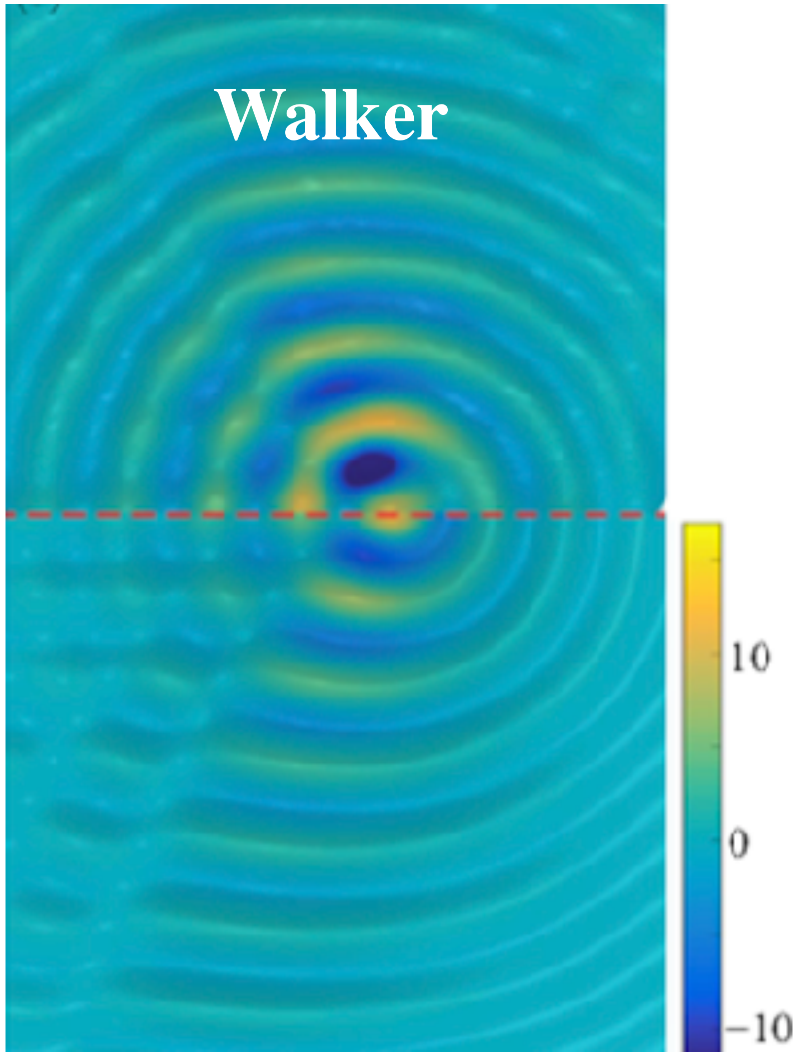
- superposition of radially propagating waves with λ_c and carrier wave with λ_B

HQFT: Analytics

Durey & Bush (2020)

- extended model to 2D, allowing comparison with walker wave field



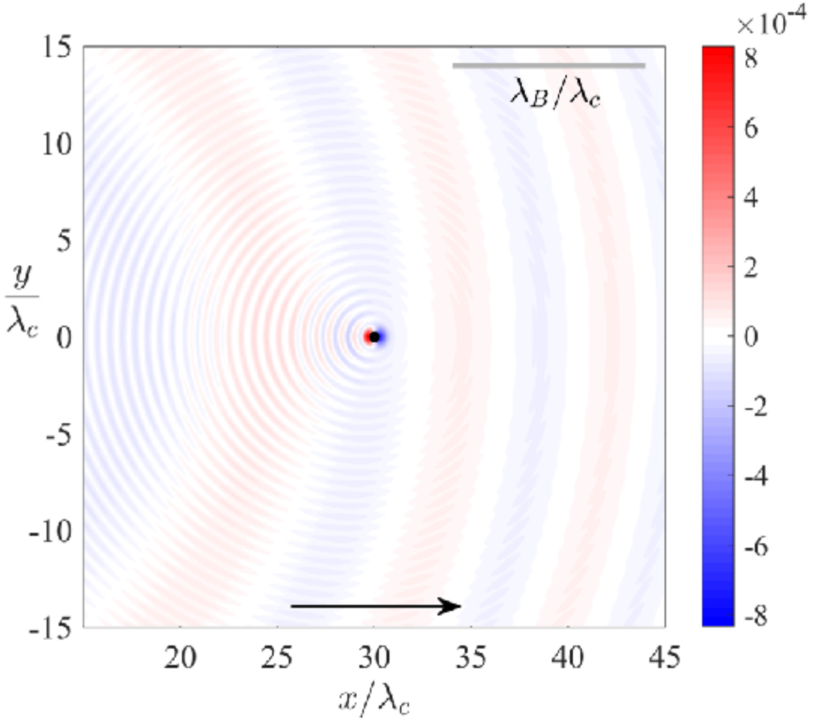


- superposition of radially propagating waves with λ_c and carrier wave with λ_B
- markedly different from the horseshoe-like form of the walker wave field
- for v << c, the 2D pilot-wave field takes the form of a plane wave with λ_B

Hydrodynamic quantum field theory

Dagan & Bush (2020)

Durey & Bush (2020)

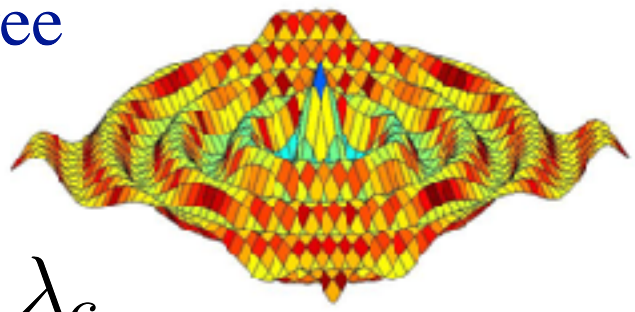


- pilot wave markedly different from that of walkers
- expect markedly different *slit diffraction* patterns ★
- 1D motion marked by in-line oscillations

HQFT wave form

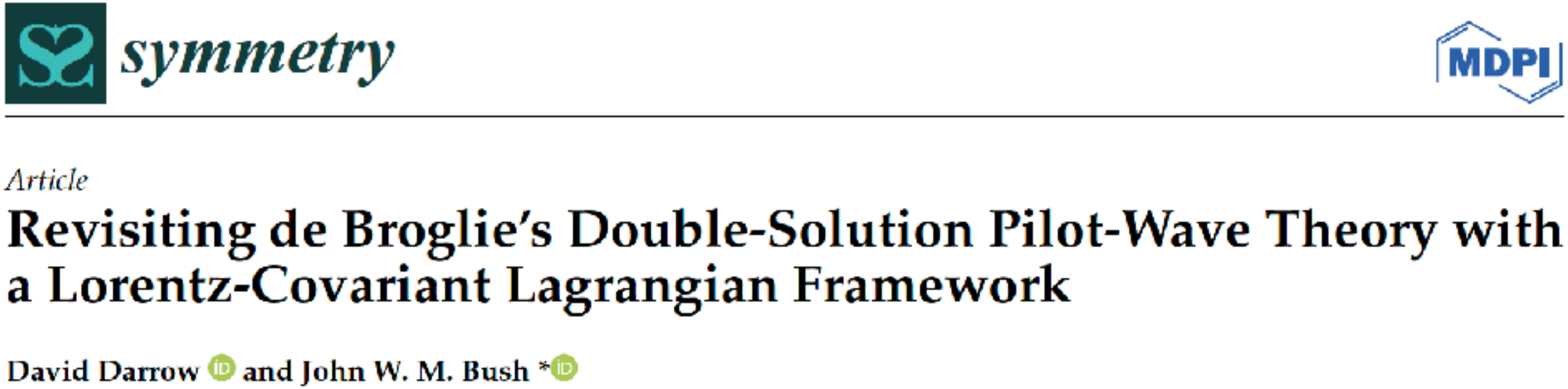
$$\frac{\lambda_B}{\lambda_c} = \frac{\sqrt{1-\beta^2}}{\beta}$$

- for v << c, pilot wave is monochromatic, with the de Broglie wavelength λ_B
  — one anticipates a statistical signature of λ_B via one of the three paradigms elucidated in pilot-wave hydrodynamics



- for v —> c, pilot-wavelength approaches the Compton wavelength λ_c
  — one anticipates structure on the Compton scale; e.g. *spin states* would correspond to the classical model of the electron (Burinskii 2008; Hestenes 2008)
- the Faraday wavelength in pilot-wave hydrodynamics plays the role of λ_B in nonrelativistic QM, and λ_c in relativistic QM

Hydrodynamically-inspired quantum field theory II



symmetry

MDPI

Article

Revisiting de Broglie's Double-Solution Pilot-Wave Theory with a Lorentz-Covariant Lagrangian Framework

David Darrow and John W. M. Bush *

HQFT II

David Darrow & JB

- combine particle and field Lagrangians at the level of actions

$$\mathcal{S} = \mathcal{S}_{\text{field}} + \mathcal{S}_{\text{particle}} + \mathcal{S}_{\text{interaction}}$$

$$\downarrow$$

$$\mathcal{S}_{\text{field}} = \frac{1}{2}\int_\Omega d^4q\ \left(\partial^\mu\phi\partial_\mu\phi - \omega^2\phi^2\right)$$

$$\mathcal{S}_{\text{particle}} = -\int_0^{t'} dt\ mc^2\gamma^{-1}$$

$$\mathcal{S}_{\text{interaction}} = \int_0^{t'} dt\ \gamma^{-1}\left(a_\tau\phi(q_p) + b_\tau\gamma\dot q_p^\mu\partial_\mu\phi(q_p)\right)$$

- one is free to choose the manner of the wave-particle coupling via the interaction action

Coupled wave and guidance equations

HQFT II

$$\boxed{\begin{gathered}(\partial_\mu\partial^\mu + \omega^2)\phi = \gamma^{-1}(a_\tau - \dot b_\tau)\delta^3(q - q_p)\\ \frac{d}{dt}((m - a_\tau\phi(q_p))\gamma\dot q_p) = \gamma^{-1}(a_\tau - \dot b_\tau)\nabla\phi(q_p)\end{gathered}} \longrightarrow \boxed{\begin{gathered}(\partial_\mu\partial^\mu + m^2)\phi = \gamma^{-1}b\delta^3(q - q_p)\\ \frac{d}{dt}(m\gamma\dot q_p) = \gamma^{-1}b\nabla\phi(q_p)\end{gathered}}$$

Coupling constants a, b

Single coupling constant b

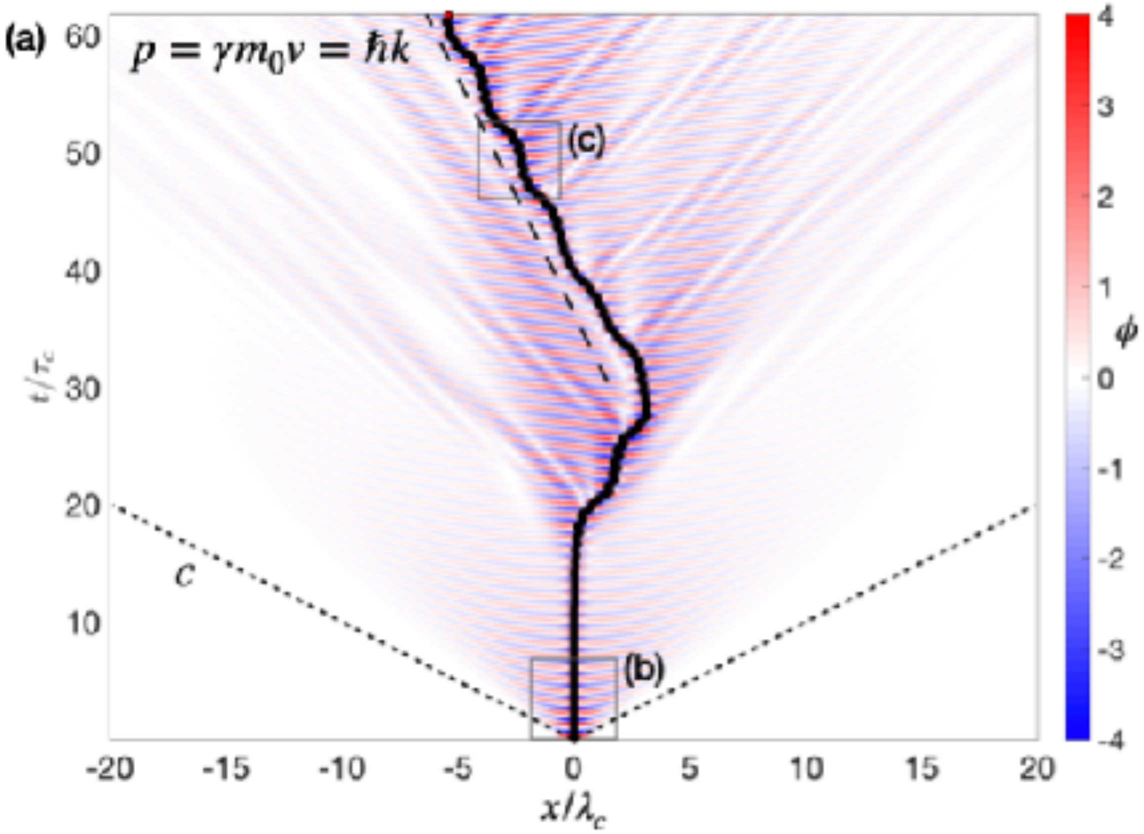
HQFT I

Dagan & Bush (2020)

$$(\partial_\mu \partial^\mu + \omega_c^2)\phi = -\sin(2\omega_c t)\delta^3(q - q_p)$$

$$\gamma \dot{q}_p = -\alpha \nabla \phi$$

1. Frame-dependent
2. Non-inertial dynamics
3. Forced oscillations at ω_c
4. No steady rectilinear state



HQFT II

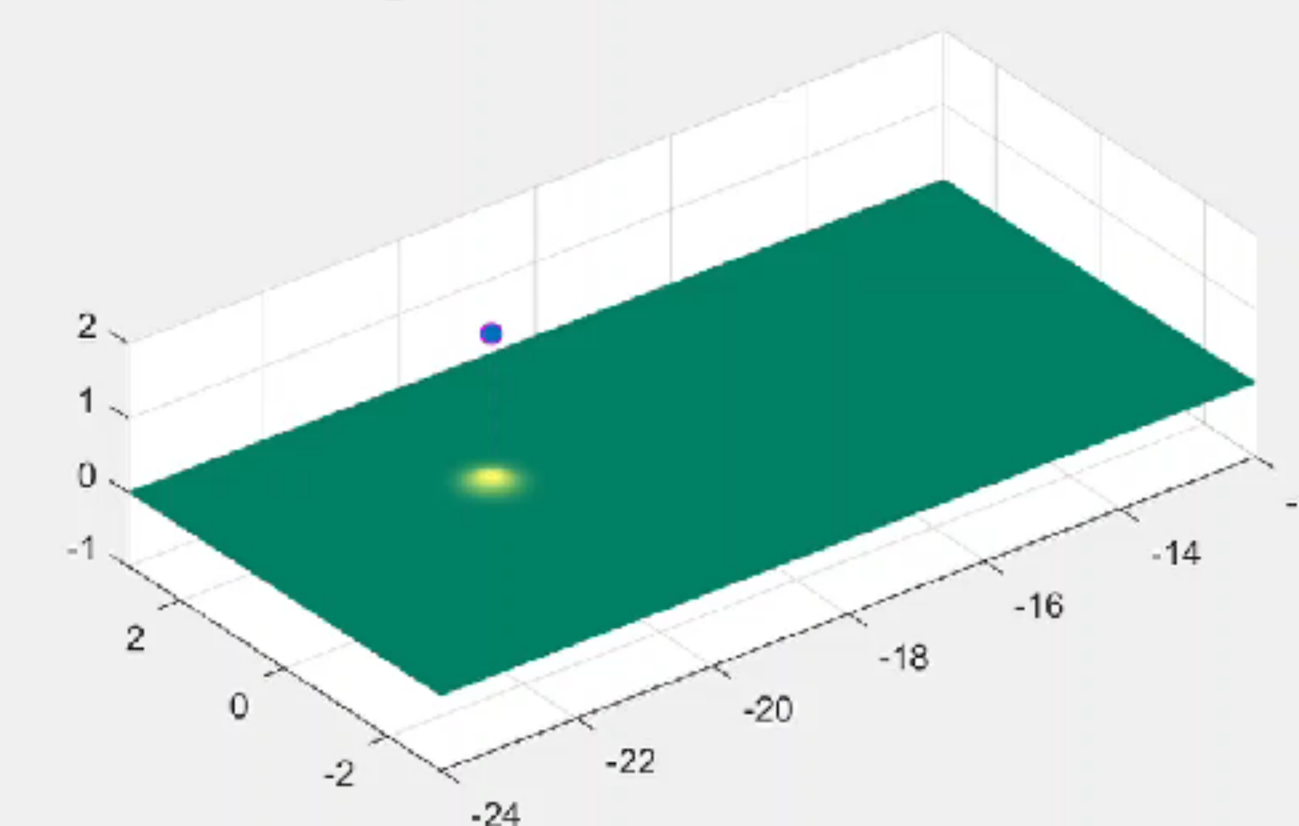
Darrow & Bush (2024)

$$(\partial_\mu \partial^\mu + m^2)\phi = \gamma^{-1} b \delta^3(q - q_p)$$

$$\frac{d}{dt}(m\gamma \dot{q}_p) = \gamma^{-1} b \nabla \phi(q_p)$$

1. Lorentz invariant
2. Inertial dynamics
3. Time independent

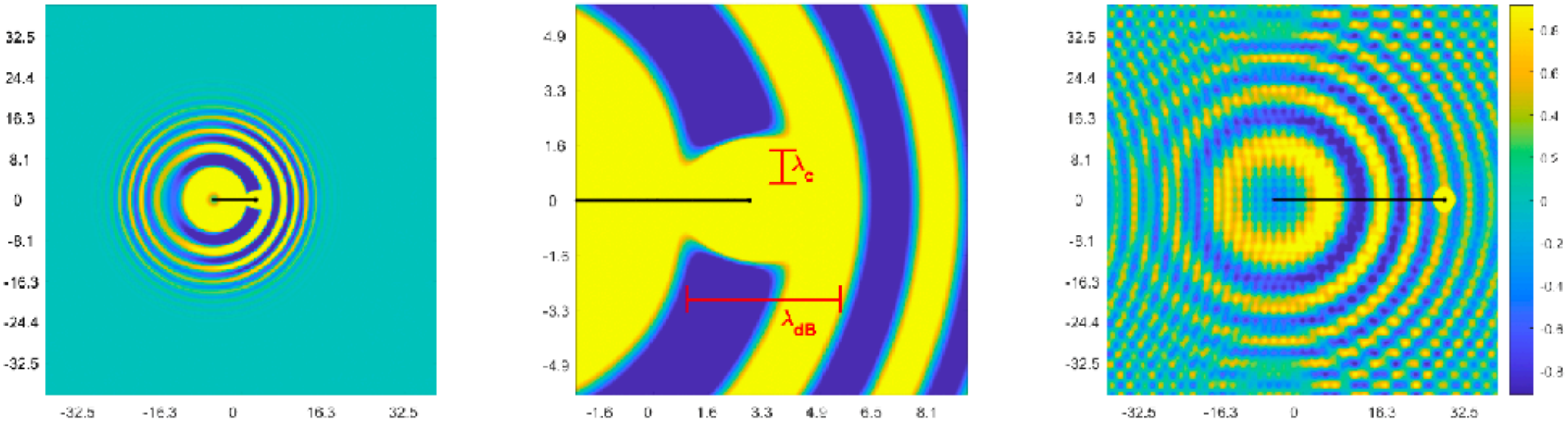
$\rightsquigarrow$ Emergent oscillations at ω_c



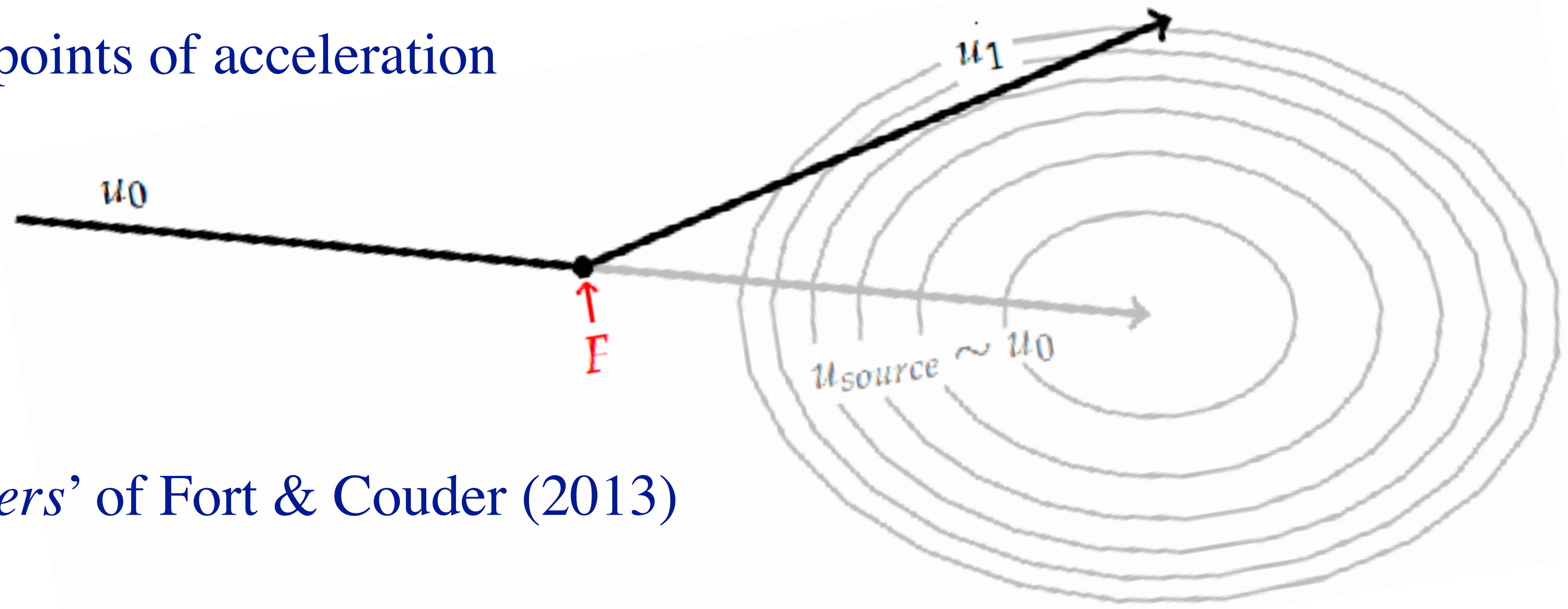
HQFT II: The free particle

David Darrow

$b = 53.3, \quad u/c = 0.35$

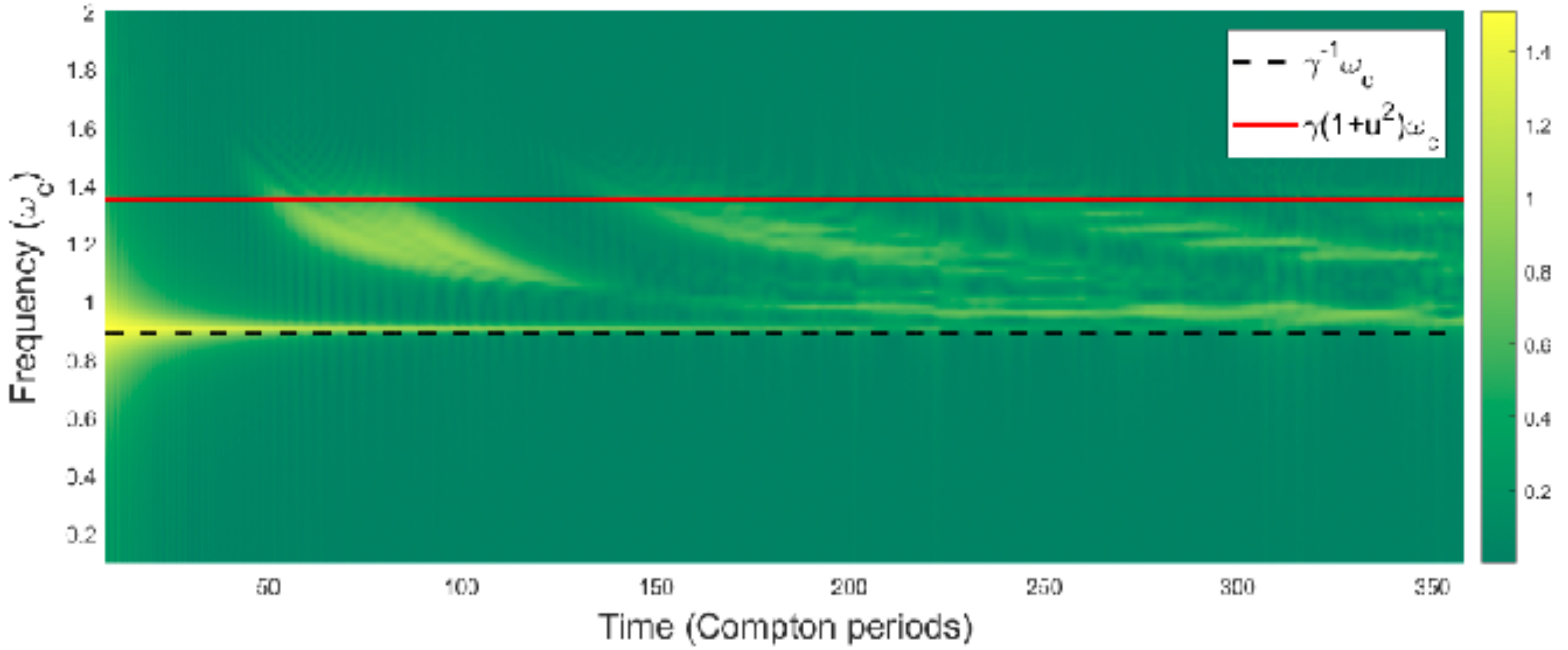


- Compton-scale Yukawa wave packet adjoining particle, de Broglie wave beyond it
- particle rides wave crest with the same group velocity: $p = \hbar k$
- particle only radiates at points of acceleration



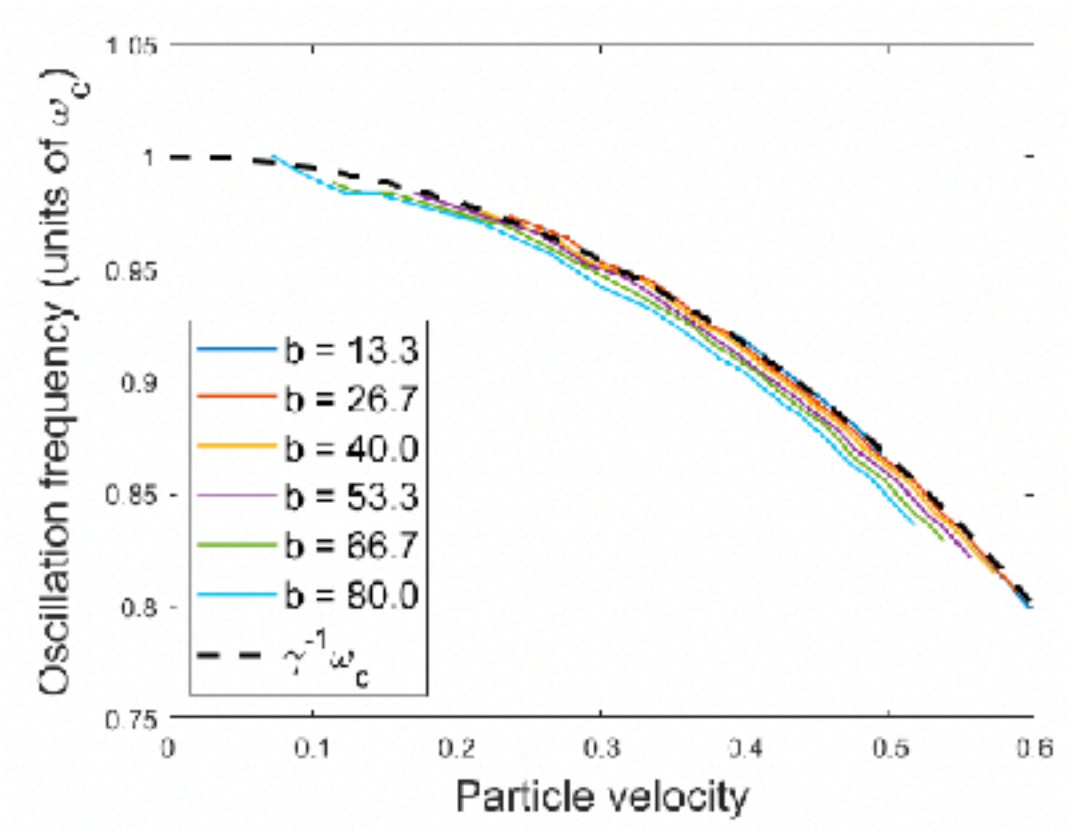
- resembles `*inertial walkers*' of Fort & Couder (2013)

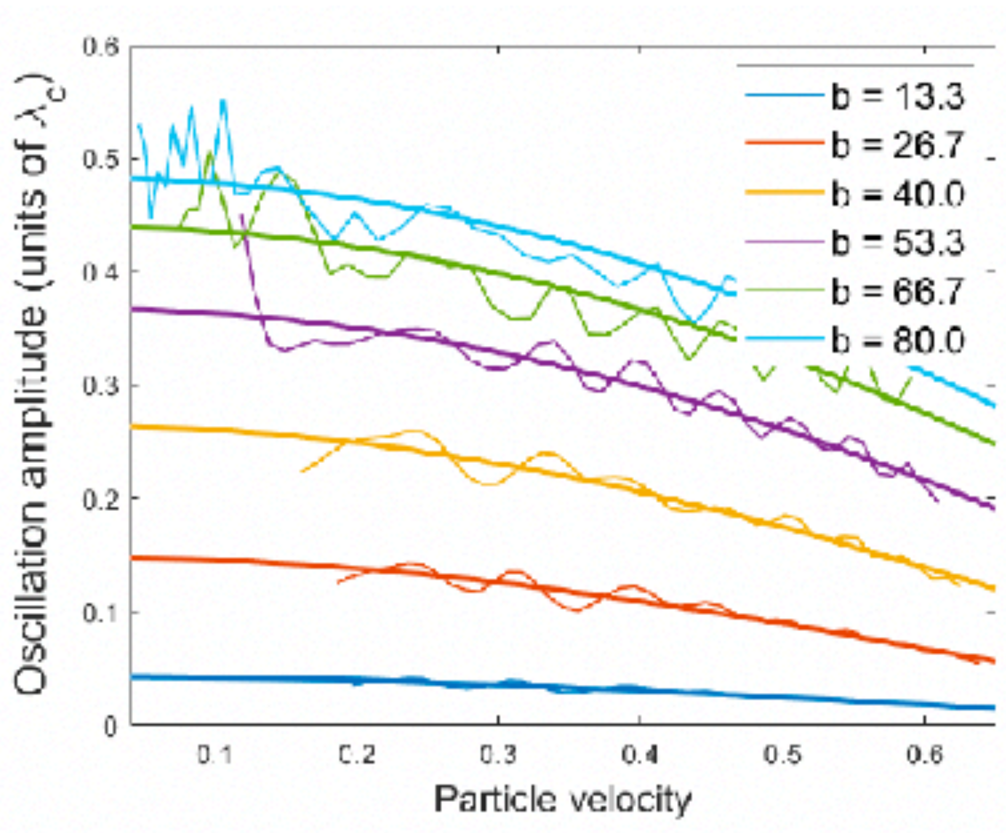
HQFT II: In-line oscillations of the free particle



$b = 53.3, \;\; u_0/c = 0.5$

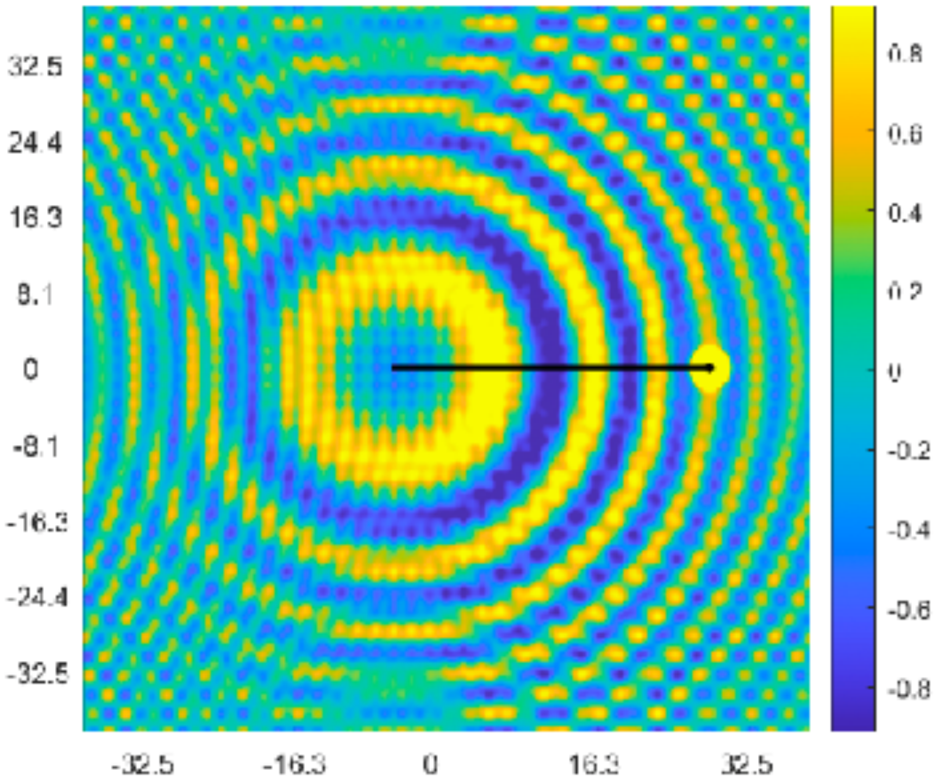
- initial motion accompanied by Compton-scale in-line oscillations at frequency $\gamma^{-1}\omega_c$

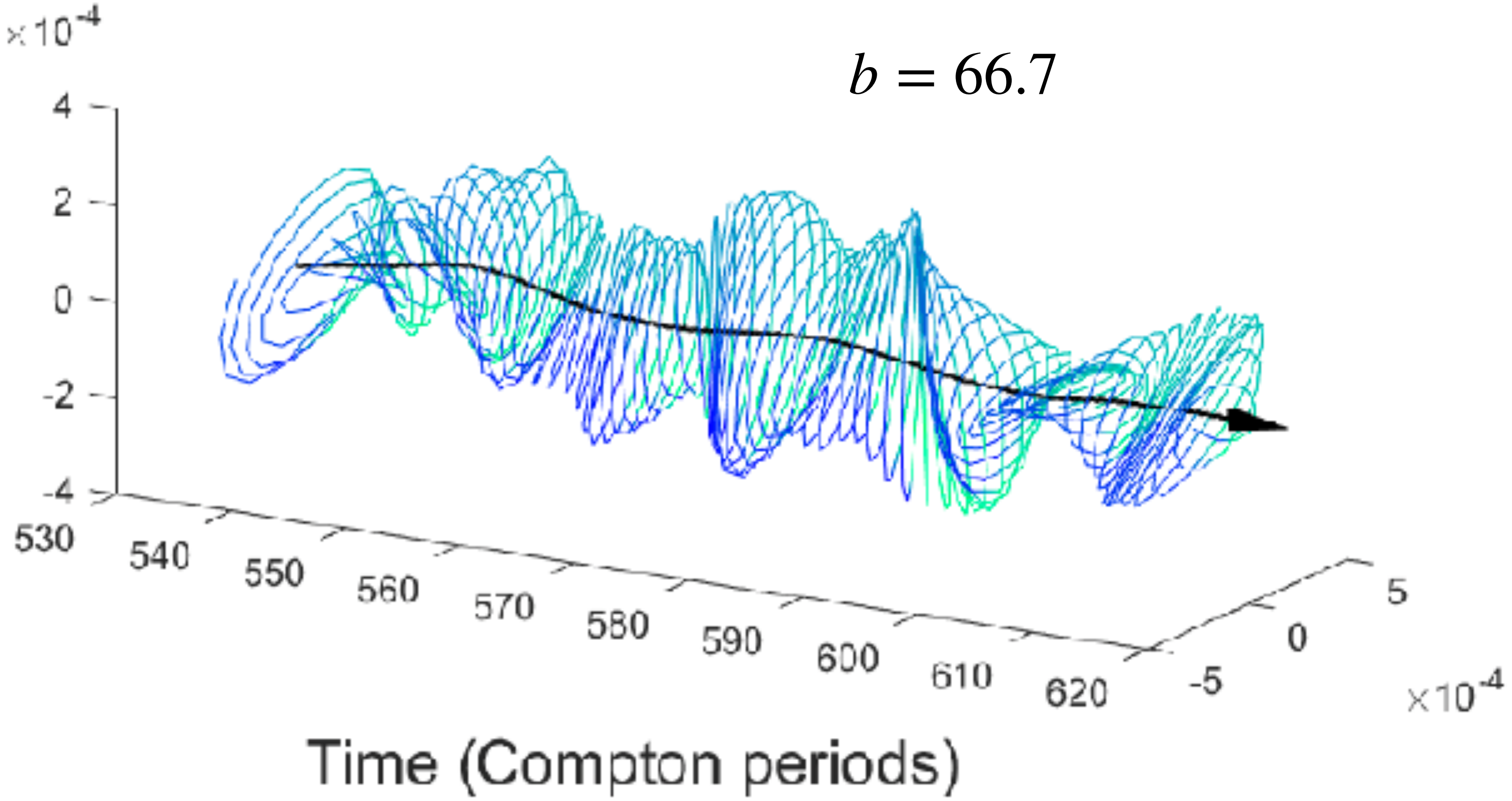




HQFT II: The free particle near boundaries

$b = 53.3, \;\; u/c = 0.35$





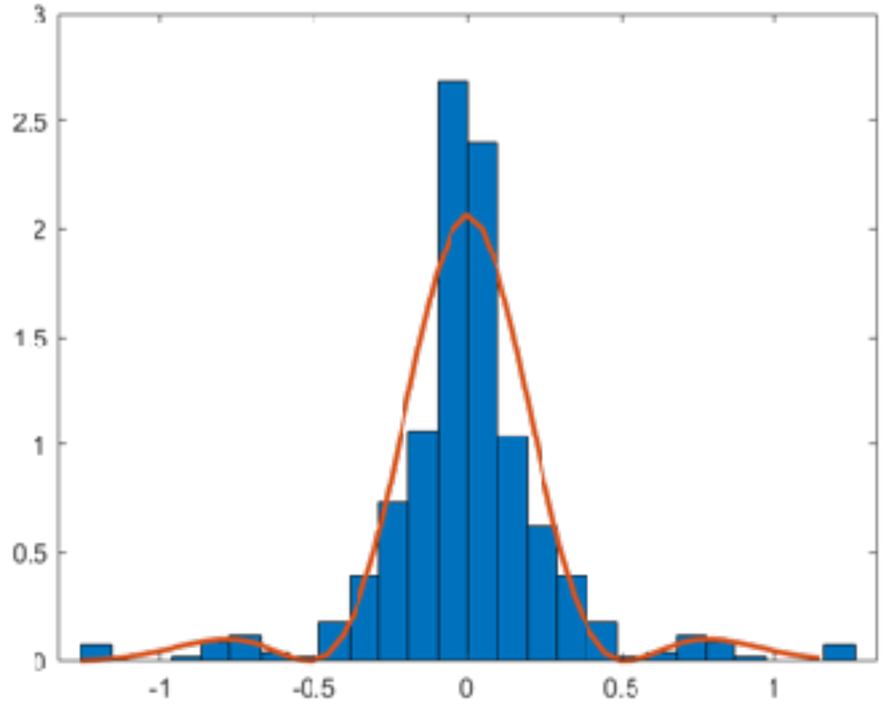
$b = 66.7$

- interaction of pilot wave with boundaries induces Compton-scale Zitter about mean path
- Zittering motion satisfies the Heisenberg Uncertainty relation: $\Delta p \Delta x \geq \hbar/2$
- measurement devices will necessarily introduce such Zittering motion
- suggests possibility of Uncertainty Relations having dynamic origins, being associated with Compton-scale dynamics

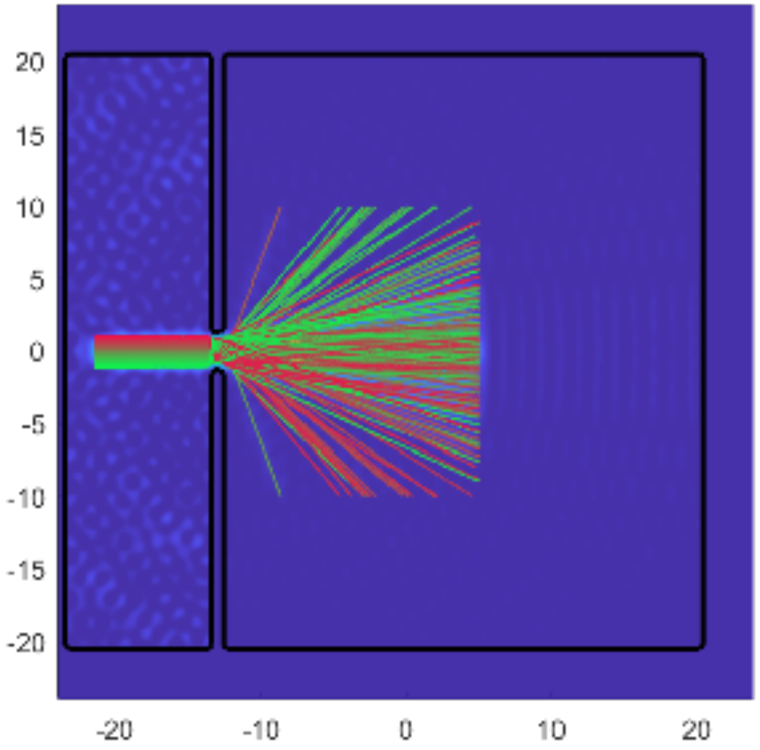
Slit diffraction with HQFT II

David Darrow

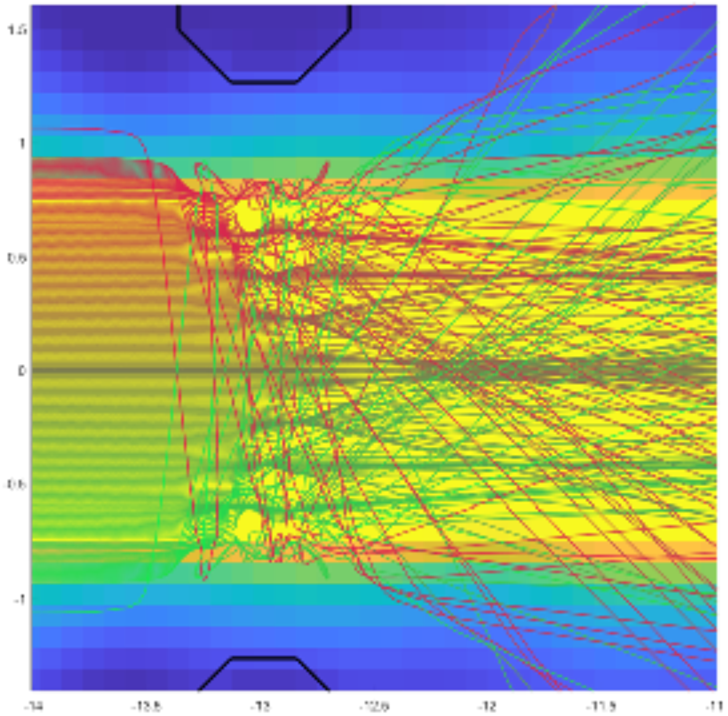
Single slit diffraction pattern



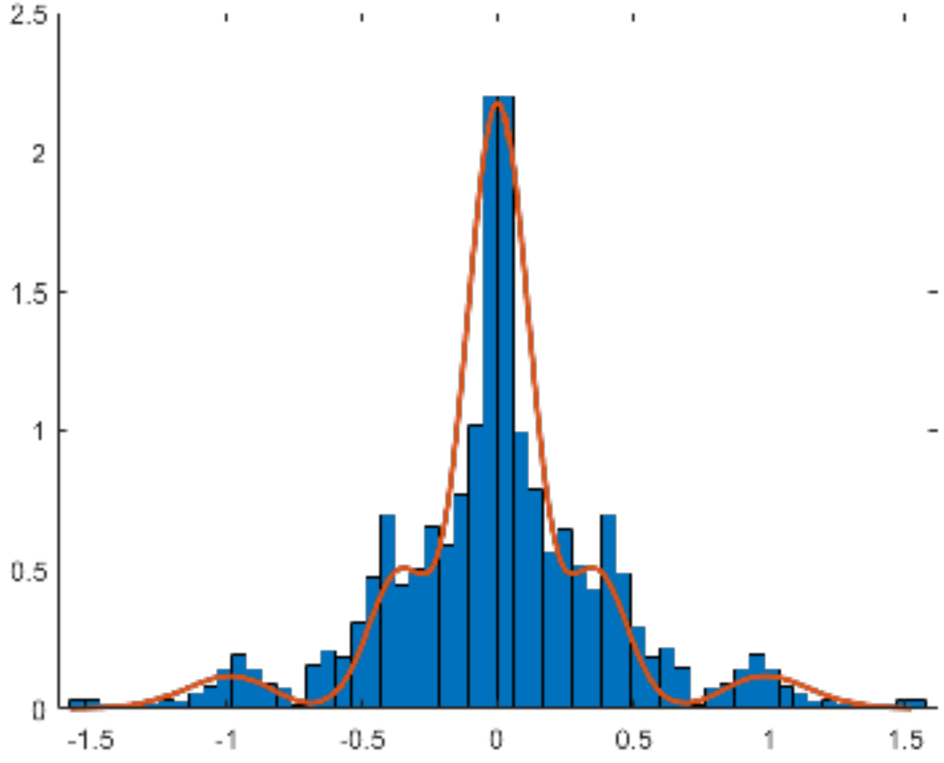
Trajectories



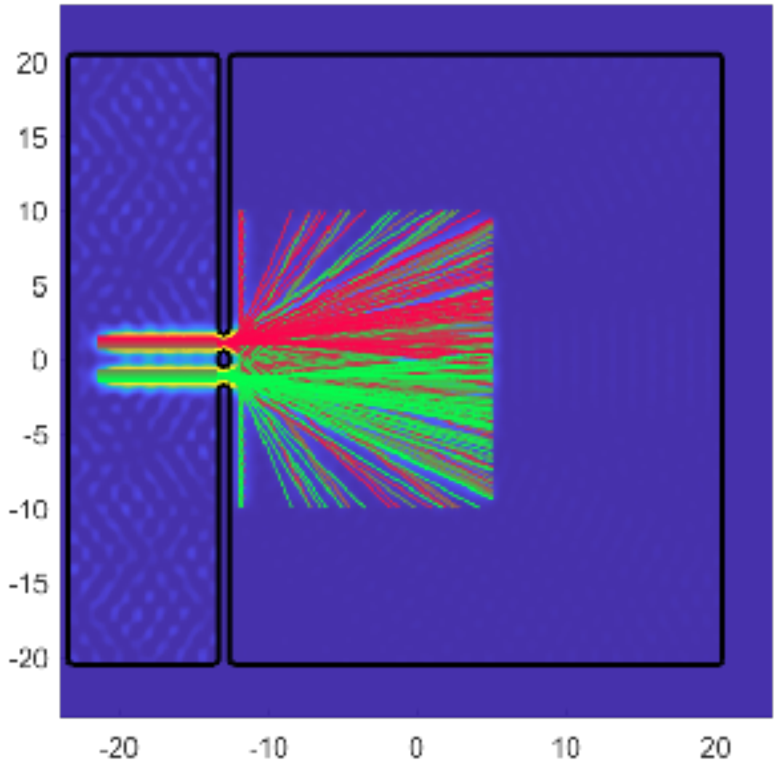
Trajectories inside slit



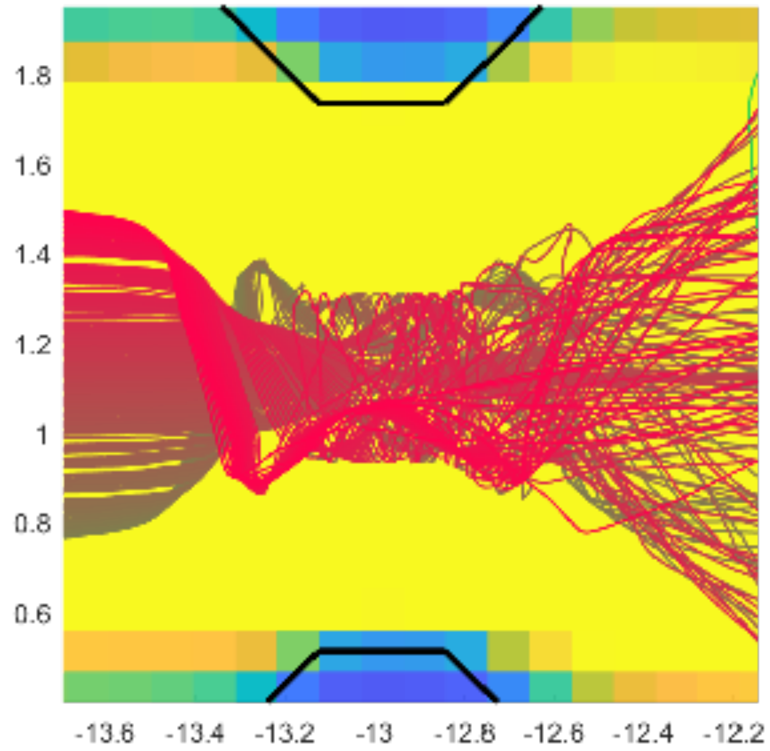
Double slit pattern



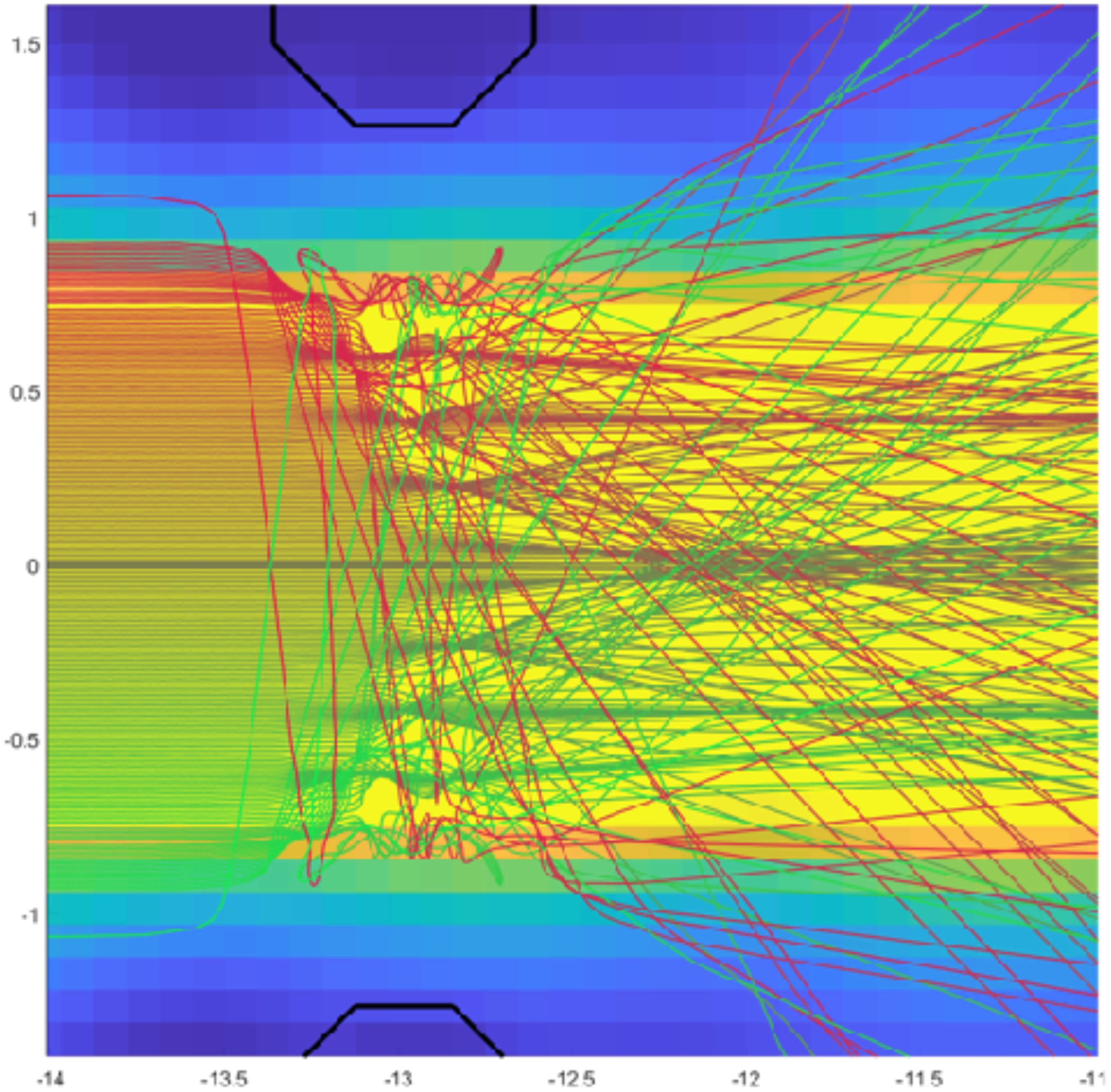
Trajectories

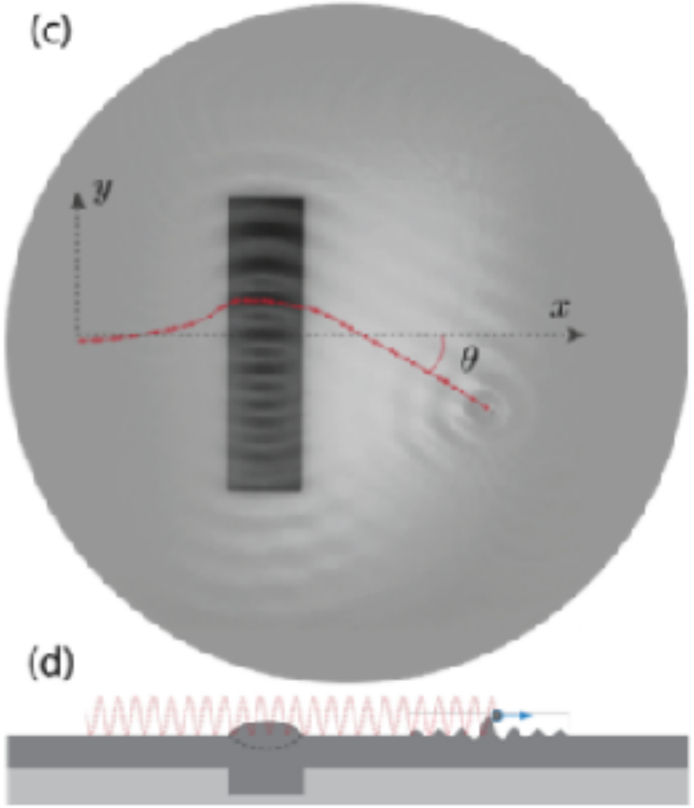


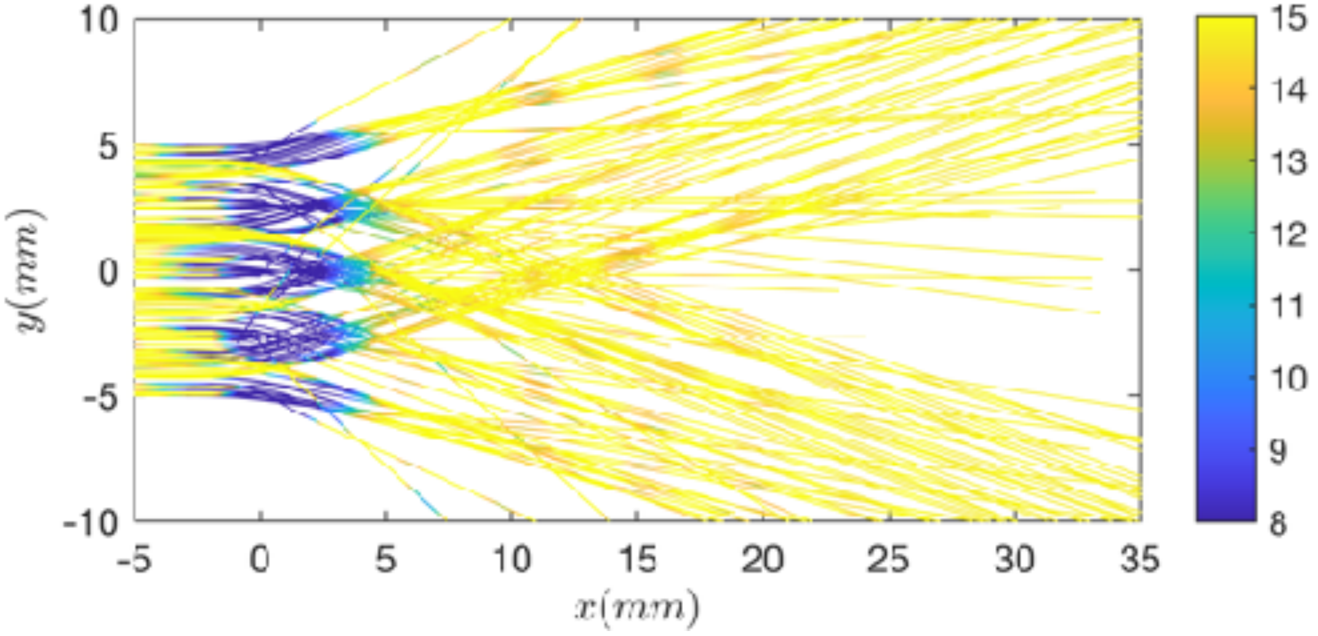
Trajectories inside slit



Trajectories inside slit



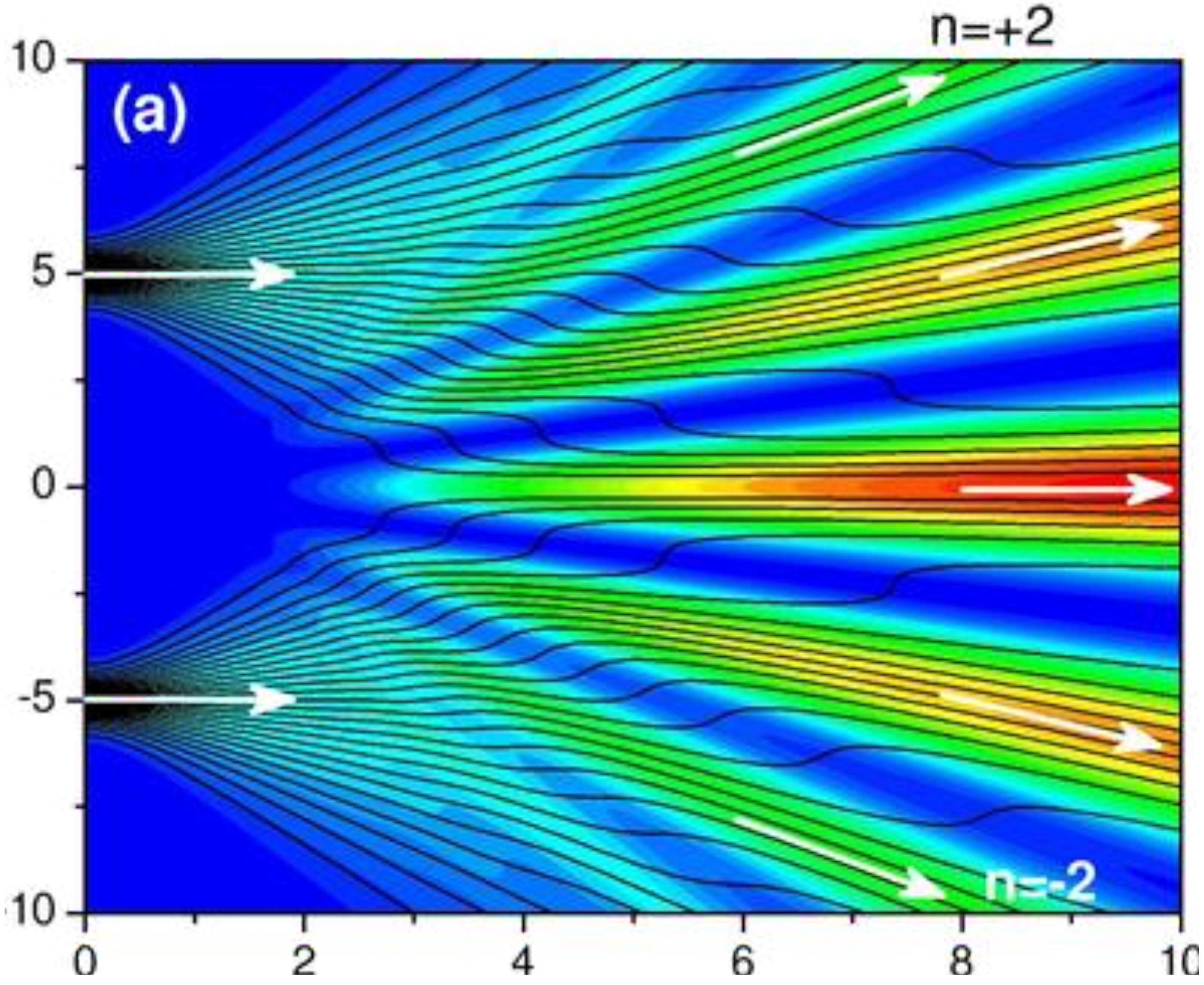




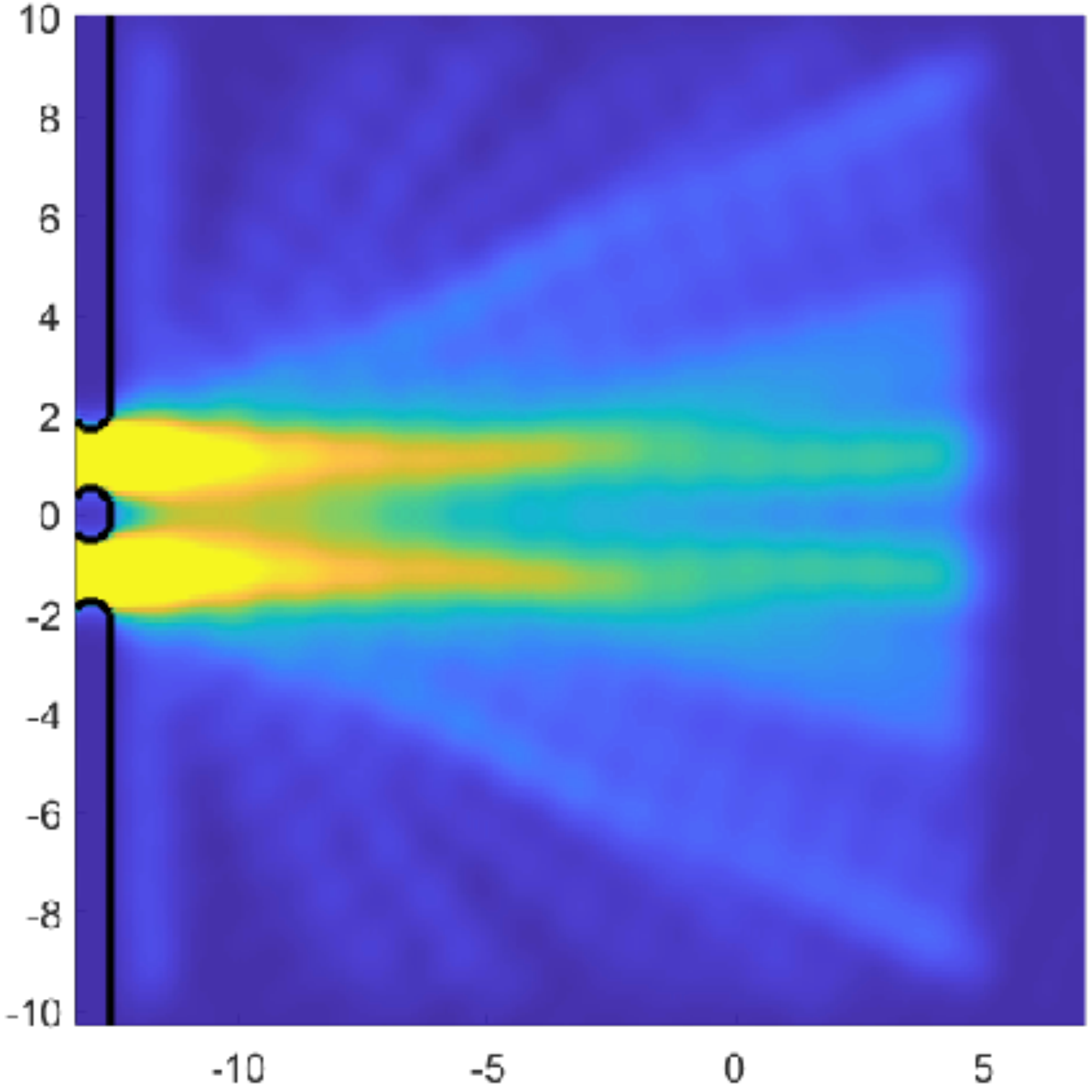
- evidence of channeling, with channel width prescribed by λ_c
- reminiscent of patterns arising in HQA of the Kapitza-Dirac effect ⟶ ***ponderomotive?***
- reminiscent of physical picture suggested in SED: resonance excites EM waves in gap

Double-slit diffraction with HQFT II

Quantum potential and Bohmian trajectories



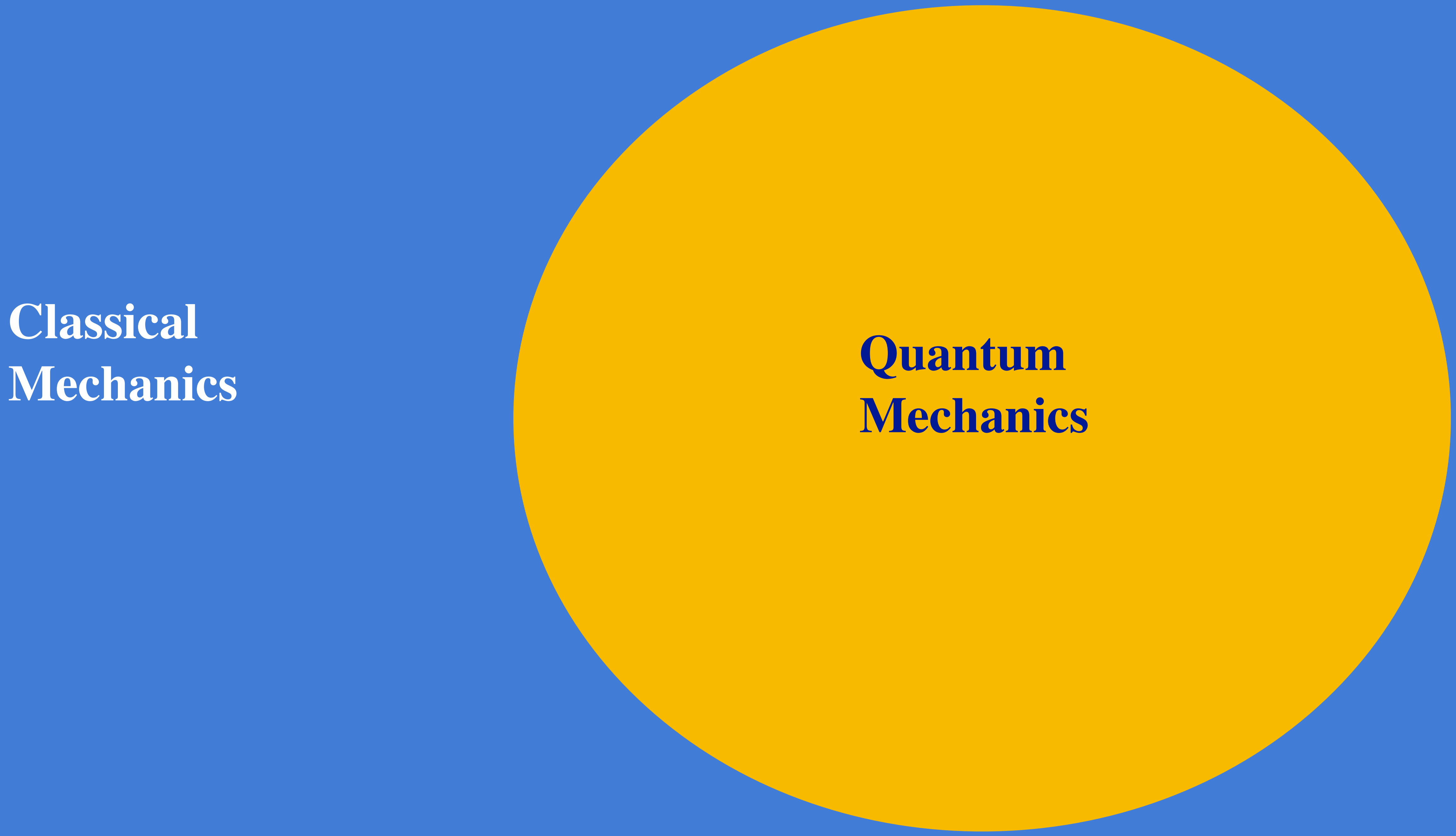
Mean-pilot-wave potential



David Darrow

BIG PICTURE

- the landscape before PWH: classical mechanics and quantum mechanics

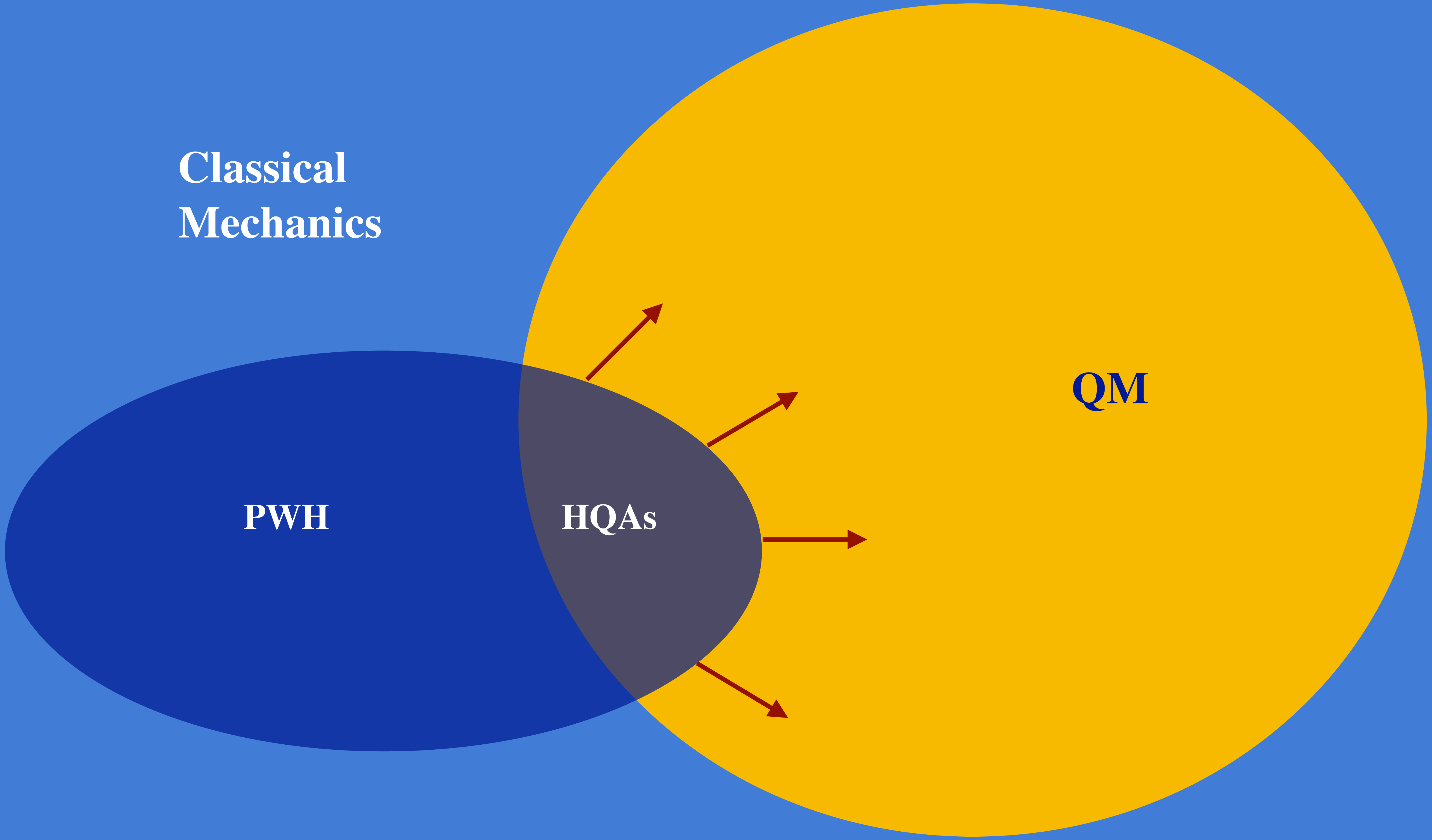


Classical Mechanics

Quantum Mechanics

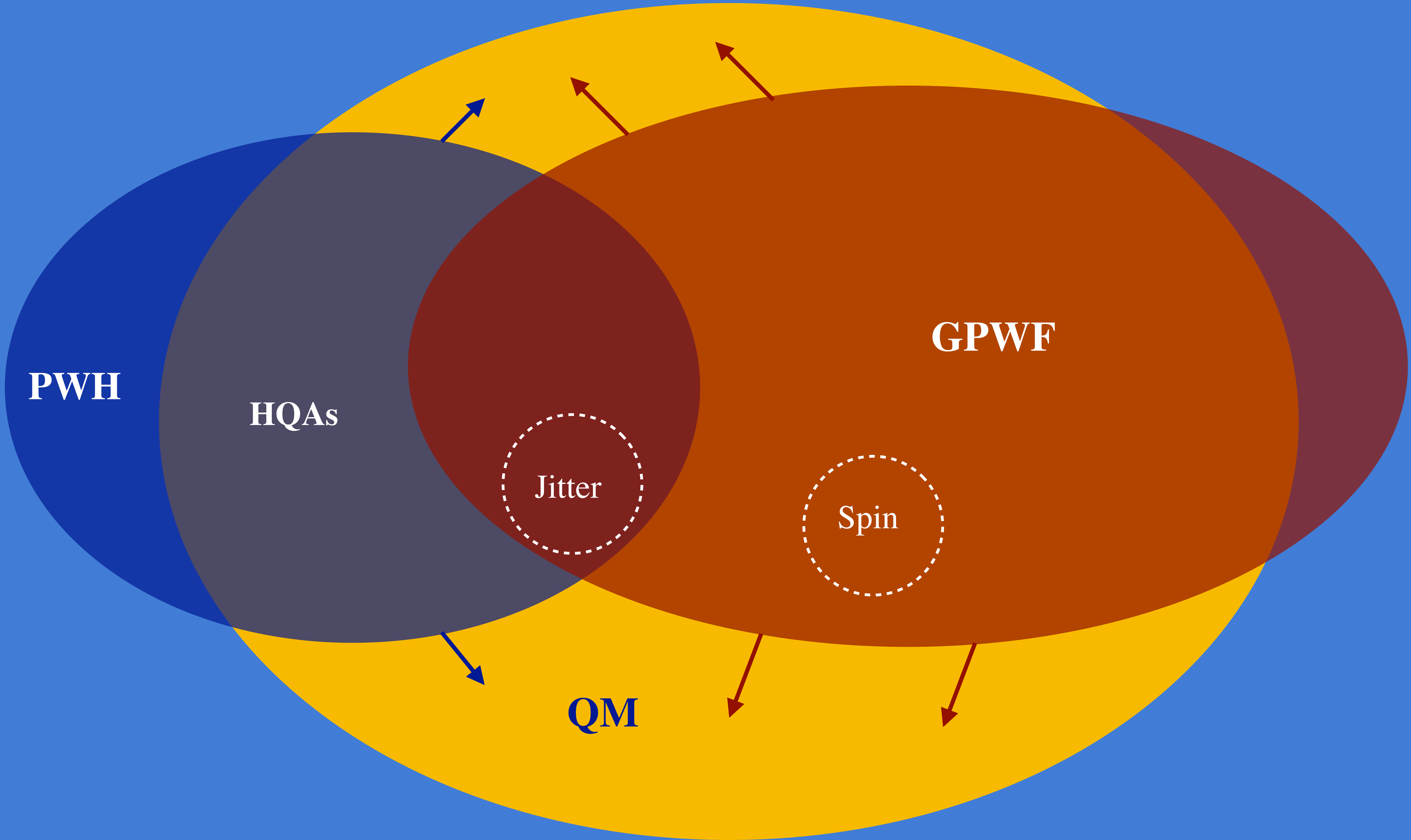
BIG PICTURE

- enter pilot-wave hydrodynamics



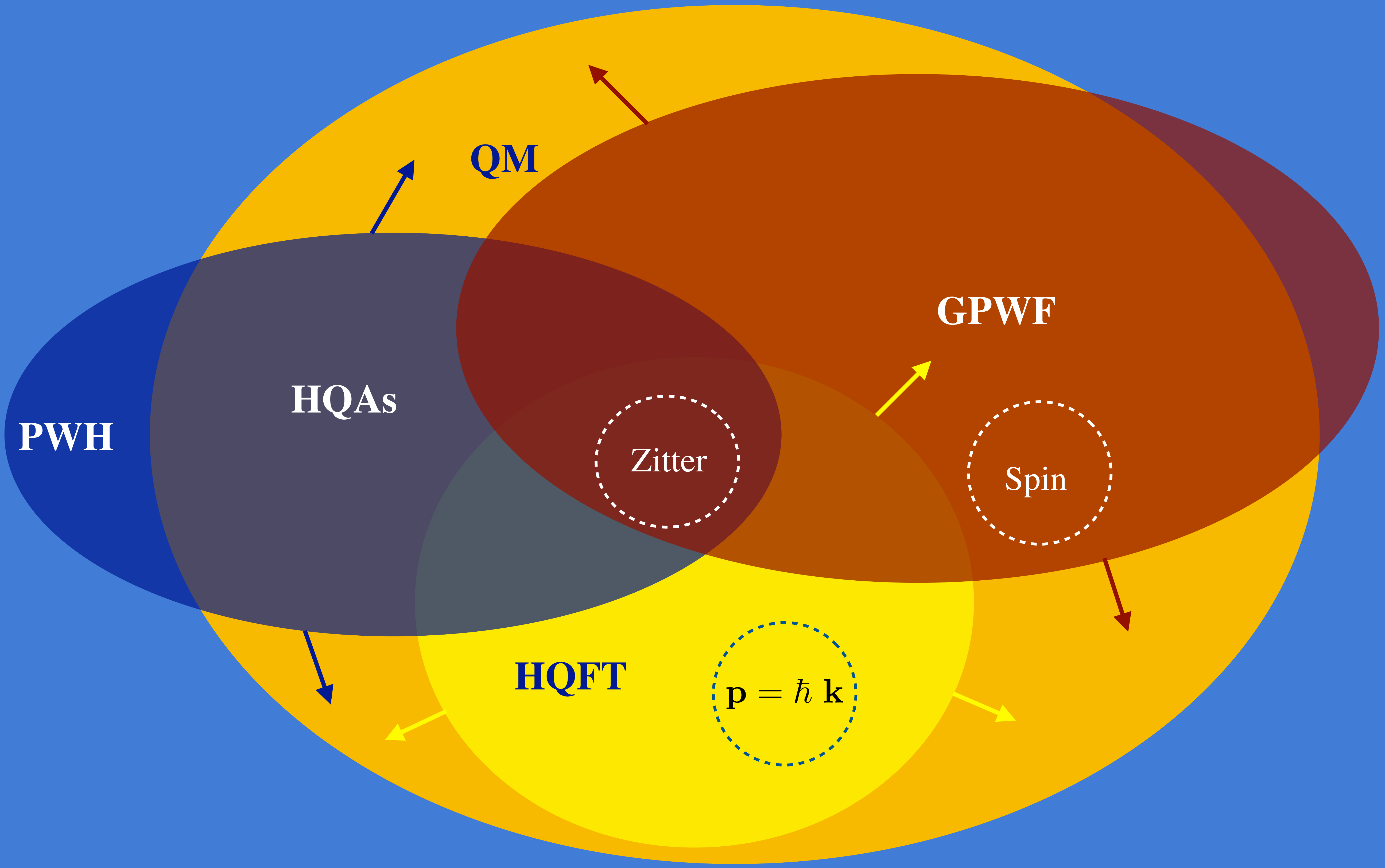
BIG PICTURE

- has motivated exploration of the Generalized Pilot-wave Framework



BIG PICTURE

- has inspired the development of a number of HQFTs



Summary

- there are striking similarities between pilot-wave hydrodynamics (PWH) and the extant quantum pilot-wave theories of Bohm, de Broglie, Nelson

- the Generalized Pilot-wave Framework (GPWF) captures even richer dynamics *e.g.* spin states; in-line jitter; erratic, diffusive motion with $D \sim \lambda U$

- the GPWF predicts features that were only later identified in PWH; e.g. jitter, chaotic walking, spin

- hydrodynamically-inspired QFT captures key features of de Broglie's mechanics (e.g. de Broglie's harmony of phases, $p = \hbar k$) and early models of QM (*Zitter*)

- many of the features of PWH and the GPWF are also evident in HQFT; *e.g.* a quasi-monochromatic wavefield revealed through strobing, in-line Zitter, ponderomotive effects

- there is a compelling resonance between PWH, GPWF, HQFT and extant realist models of QM

Bohmian mechanics

- an invaluable touchstone for HQA, as it involves real particles, real trajectories
- a successful attempt to restore reality to quantum mechanics
- its nonlocality is seen by its adherents as a strength in light of Bell violations
- criticized by the Copenhagen adherents as going too far
- criticized by HQA adherents as not going far enough

HQA Perspective

- in neglecting the Compton timescale, Bohmian mechanics discards the source of the pilot-wave field, rendering the theory both non-relativistic and nonlocal
- Bohmian mechanics describes only a mean dynamics, whose behavior is prescribed by Q, which plays a role analogous to our mean pilot wave
- consideration of ensembles of ICs should allow us to develop a Bohmian description of the evolving statistics in PWH
- we seek to restore locality to QM through adopting a de Broglie-like perspective: the particle is the source of its own pilot-wave field