Lecture 21

A. Crossing the threshold

B. Variable-phase stroboscopic model

A. Crossing the Faraday threshold

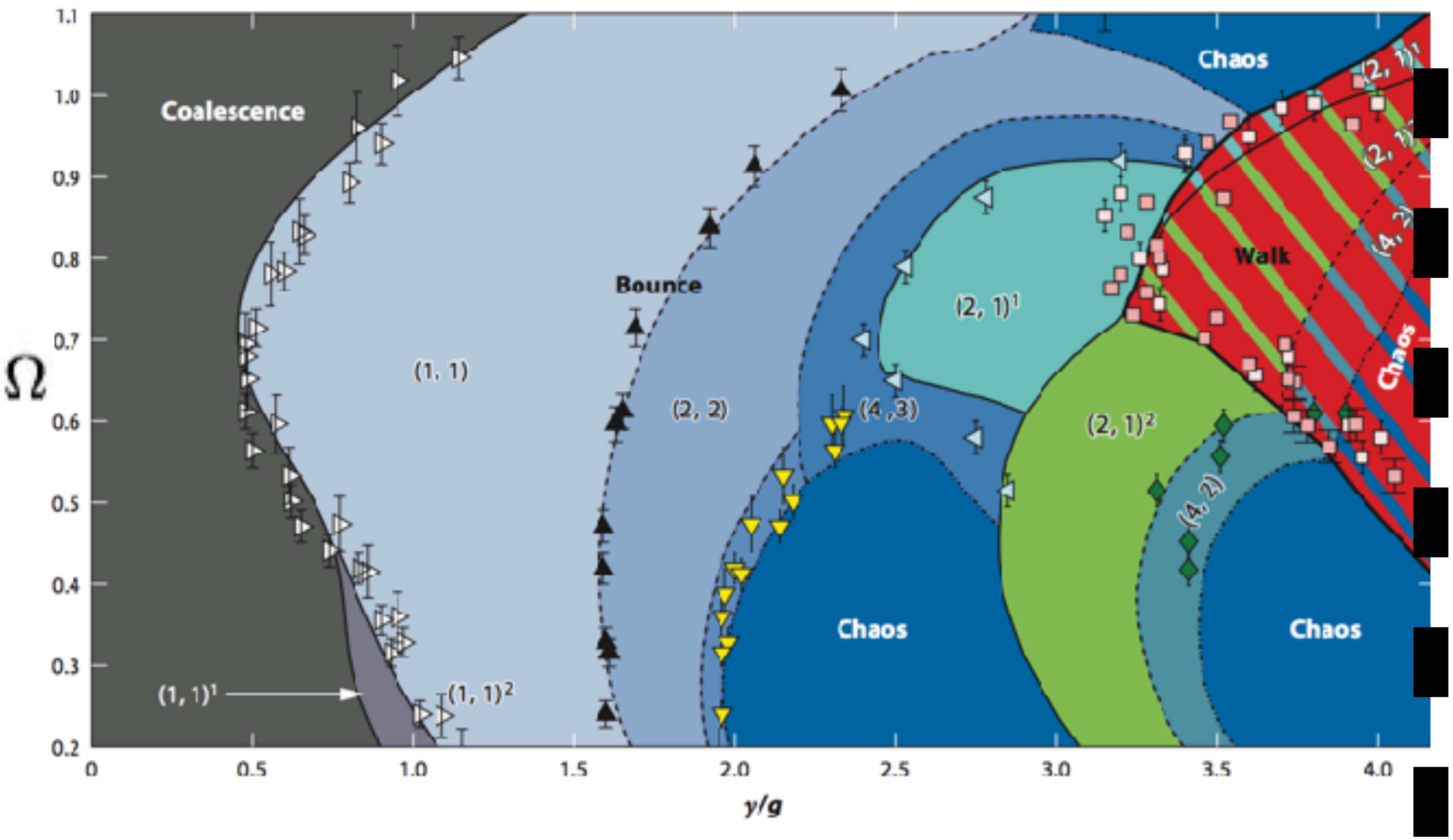
Bouncing droplet dynamics above the Faraday threshold

L. D. Tambasco, J. J. Pilgram, and J. W. M. Bush

Citation: Chaos **28**, 096107 (2018); doi: 10.1063/1.5031426
View online: https://doi.org/10.1063/1.5031426

Crossing the Faraday threshold

$$\Omega = \frac{2\pi f}{(\sigma/\rho R_0^3)^{1/2}} = \frac{\text{forcing frequency}}{\text{drop's natural frequency}}$$



20 cS silicone oil, 80Hz

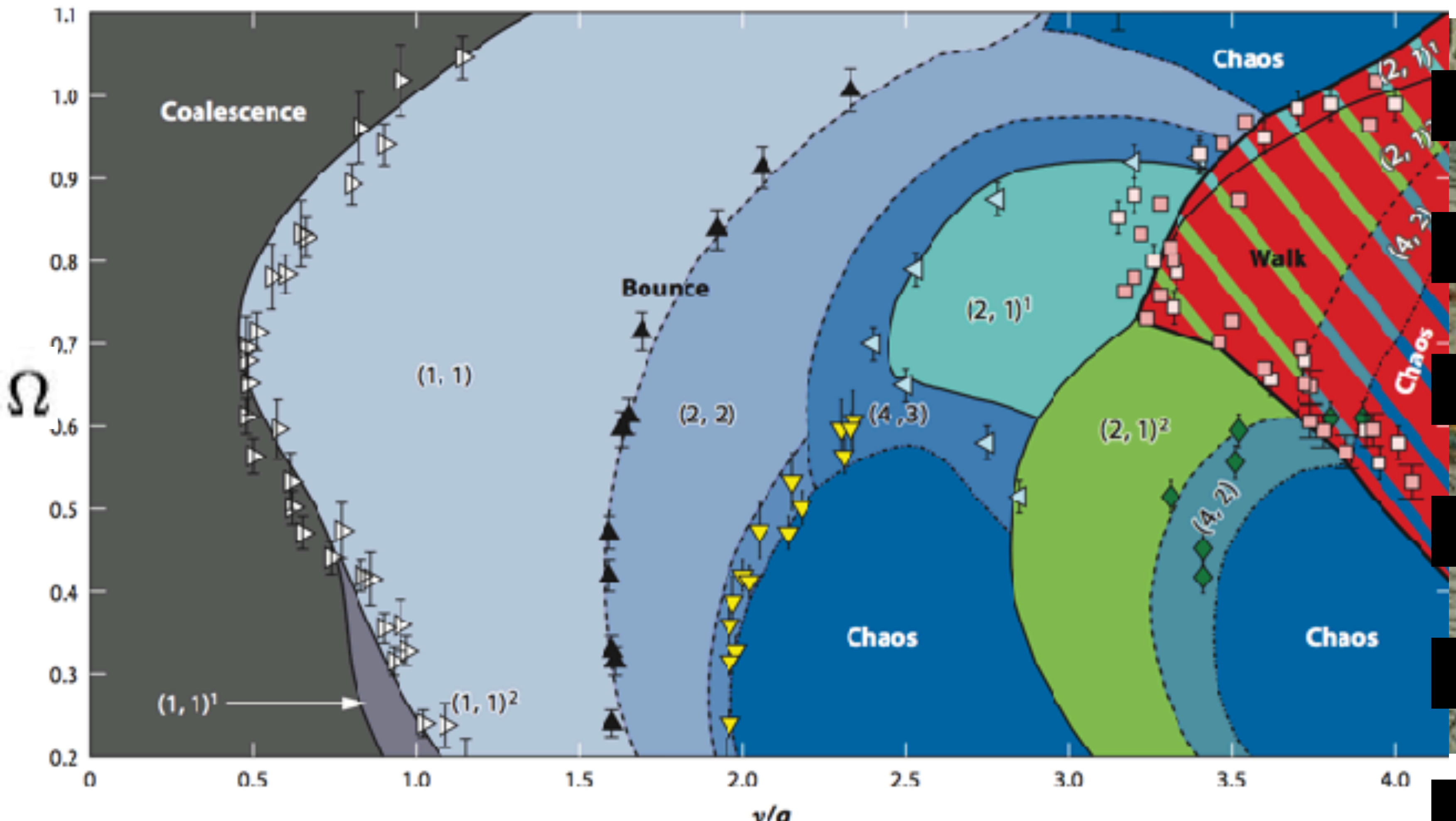
γ_F

Molacek & Bush, 2013

Crossing the Faraday threshold

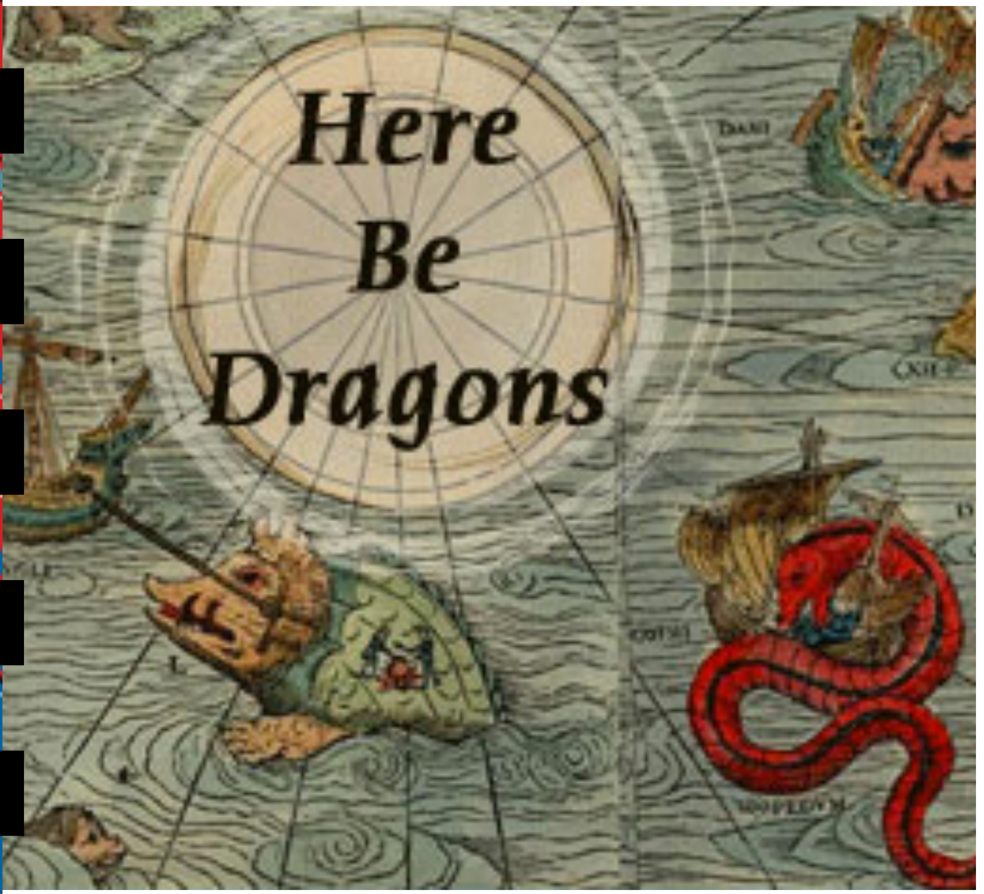
$$\Omega = \frac{2\pi f}{(\sigma/\rho R_0^3)^{1/2}} = \frac{\text{forcing frequency}}{\text{drop's natural frequency}}$$

What happens above the Faraday threshold?



20 cS silicone oil, 80Hz

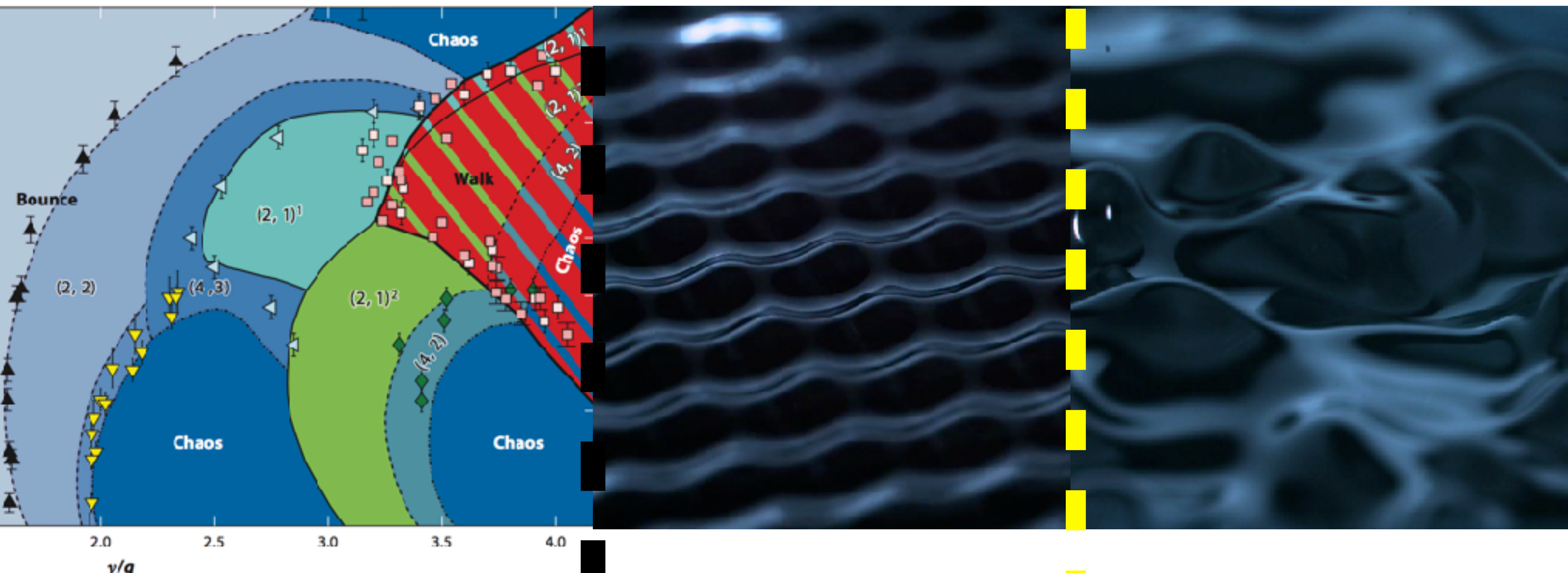
γ_F



Note: existing theoretical models break down here

Crossing the Faraday threshold

No waves | **Faraday waves** | **Drop generation**



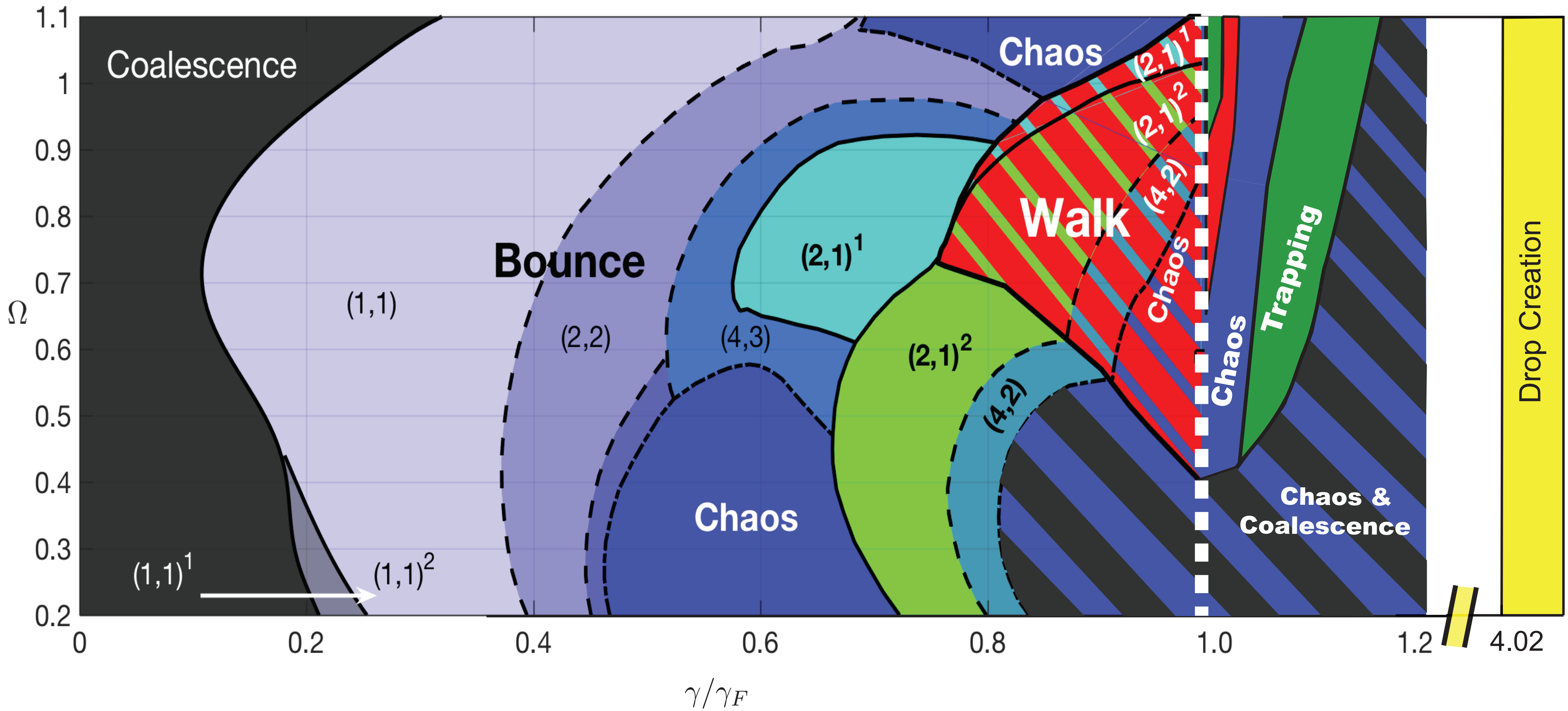
20 cS silicone oil, 80Hz

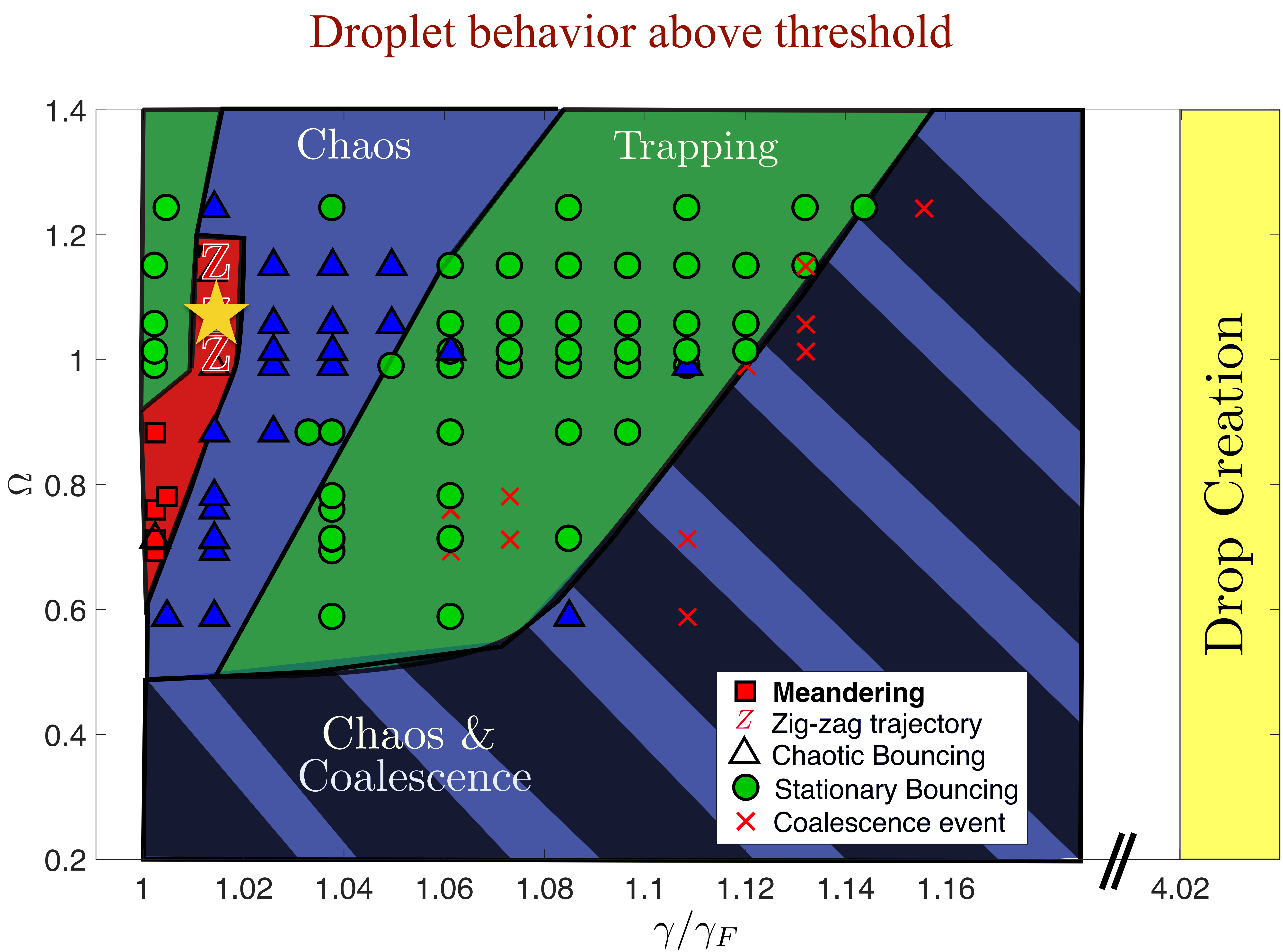
γ_F

- can tune the relative magnitudes of the self-induced and ambient wave fields

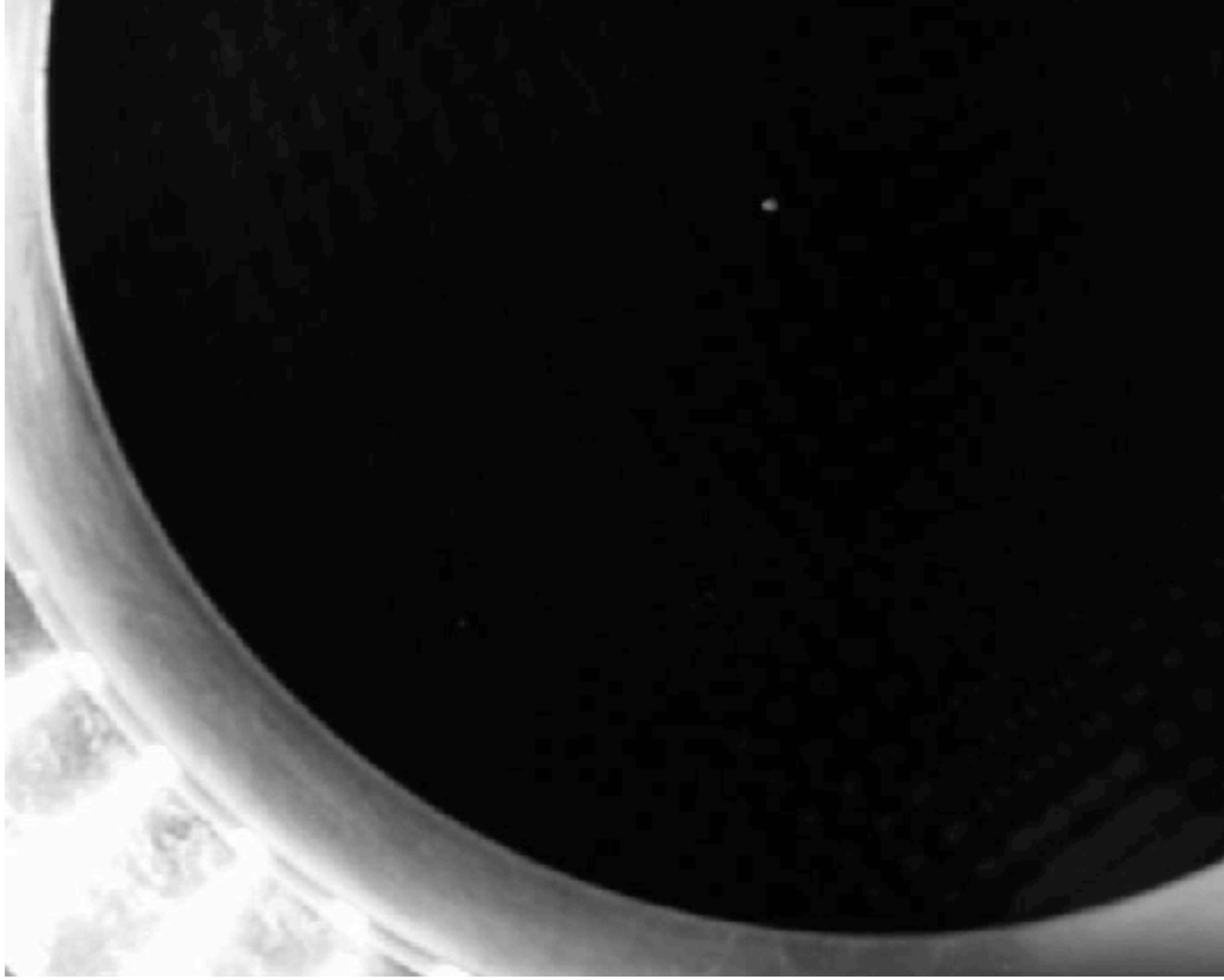
Crossing the Faraday threshold

20 cS, 80 Hz





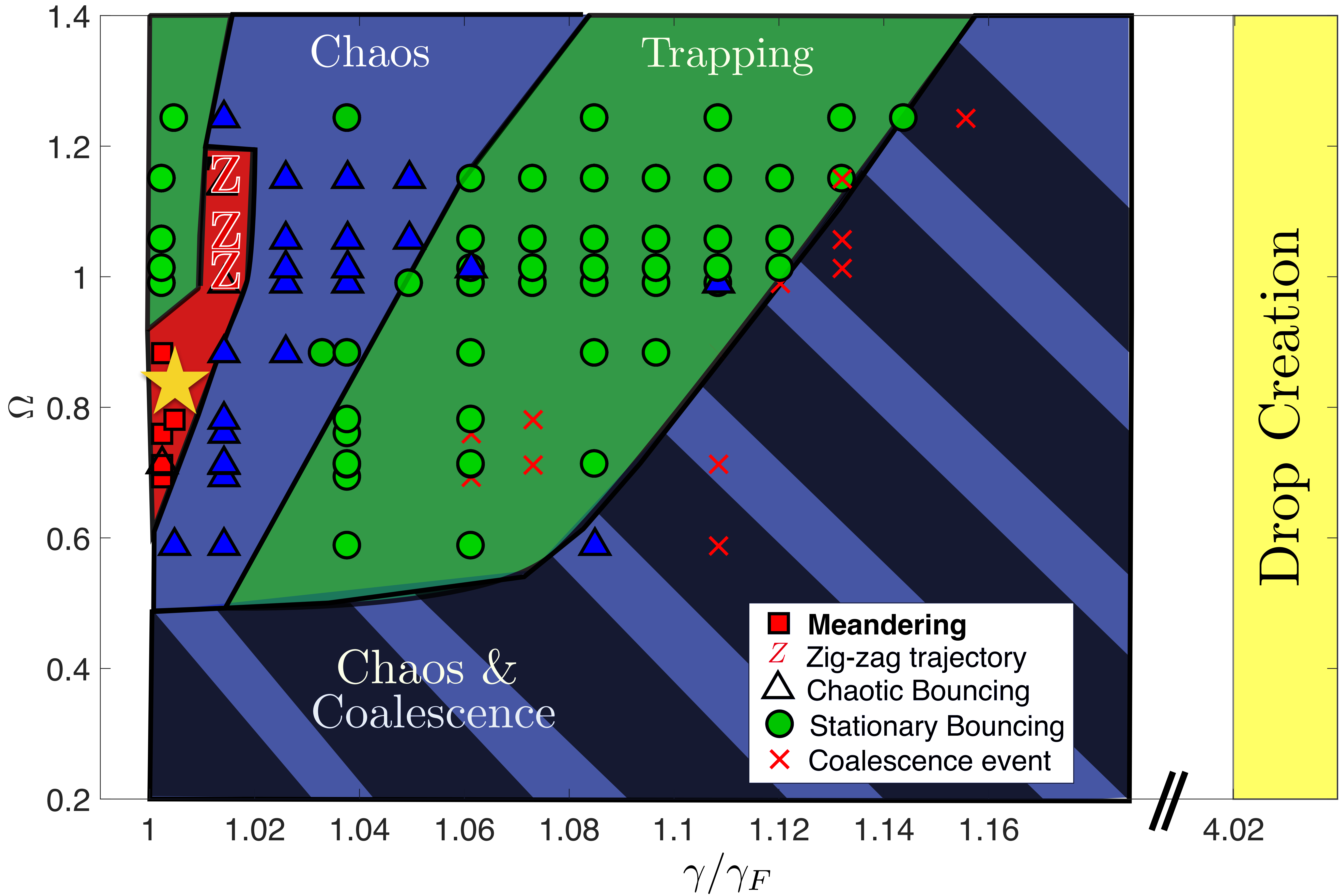
Droplet behavior above threshold
Chaos
Trapping
Chaos &
Coalescence
Drop Creation
Meandering
Zig-zag trajectory
Chaotic Bouncing
Stationary Bouncing
Coalescence event
Z
Ω
γ/γ_F
1.4
1.2
1
0.8
0.6
0.4
0.2
1
1.02
1.04
1.06
1.08
1.1
1.12
1.14
1.16
4.02



Zig-zaging

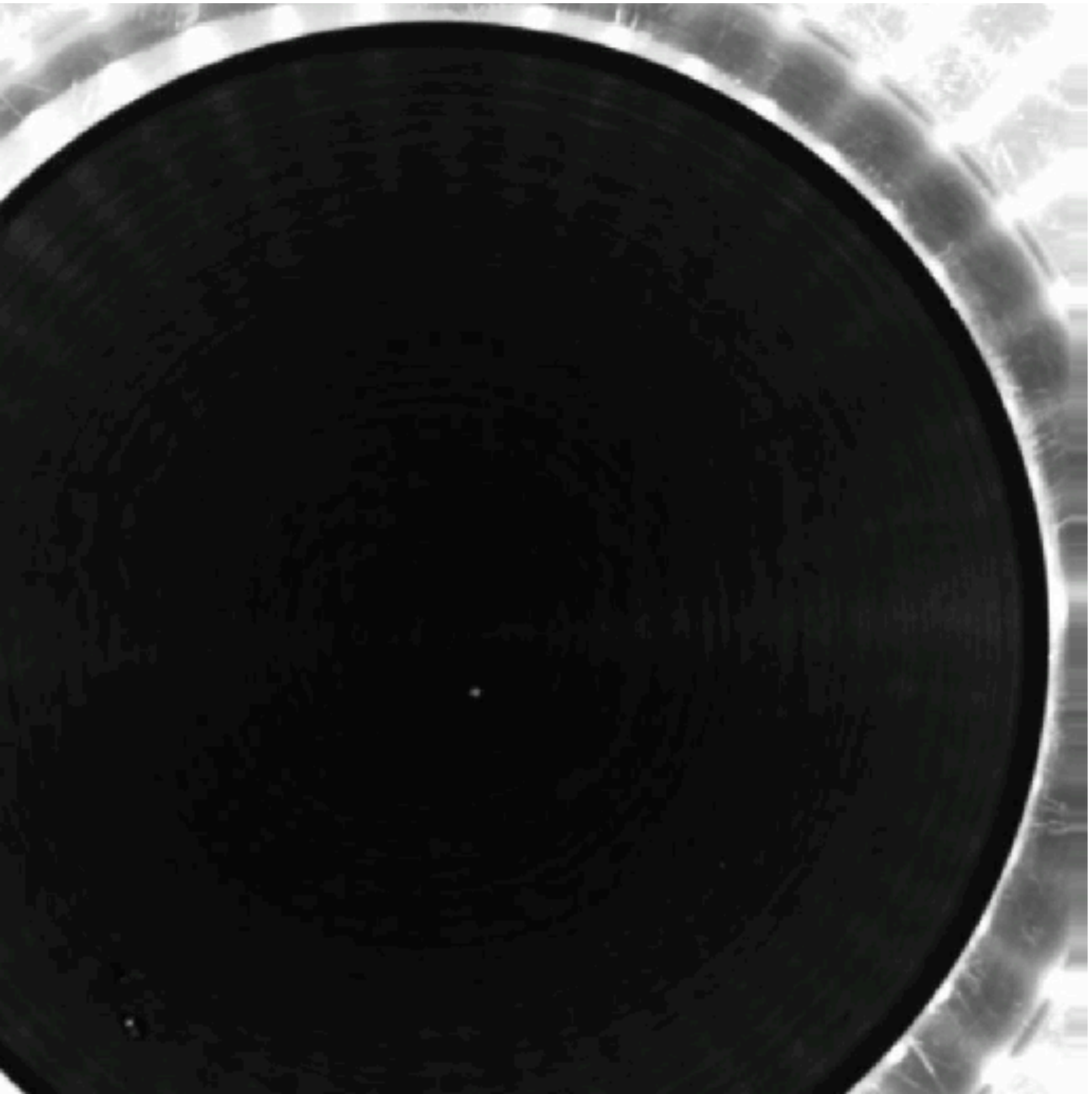
- rectilinear walking state weakly perturbed by ambient wave field
- drops zig-zag along troughs of the square Faraday wave field

Droplet behavior above threshold

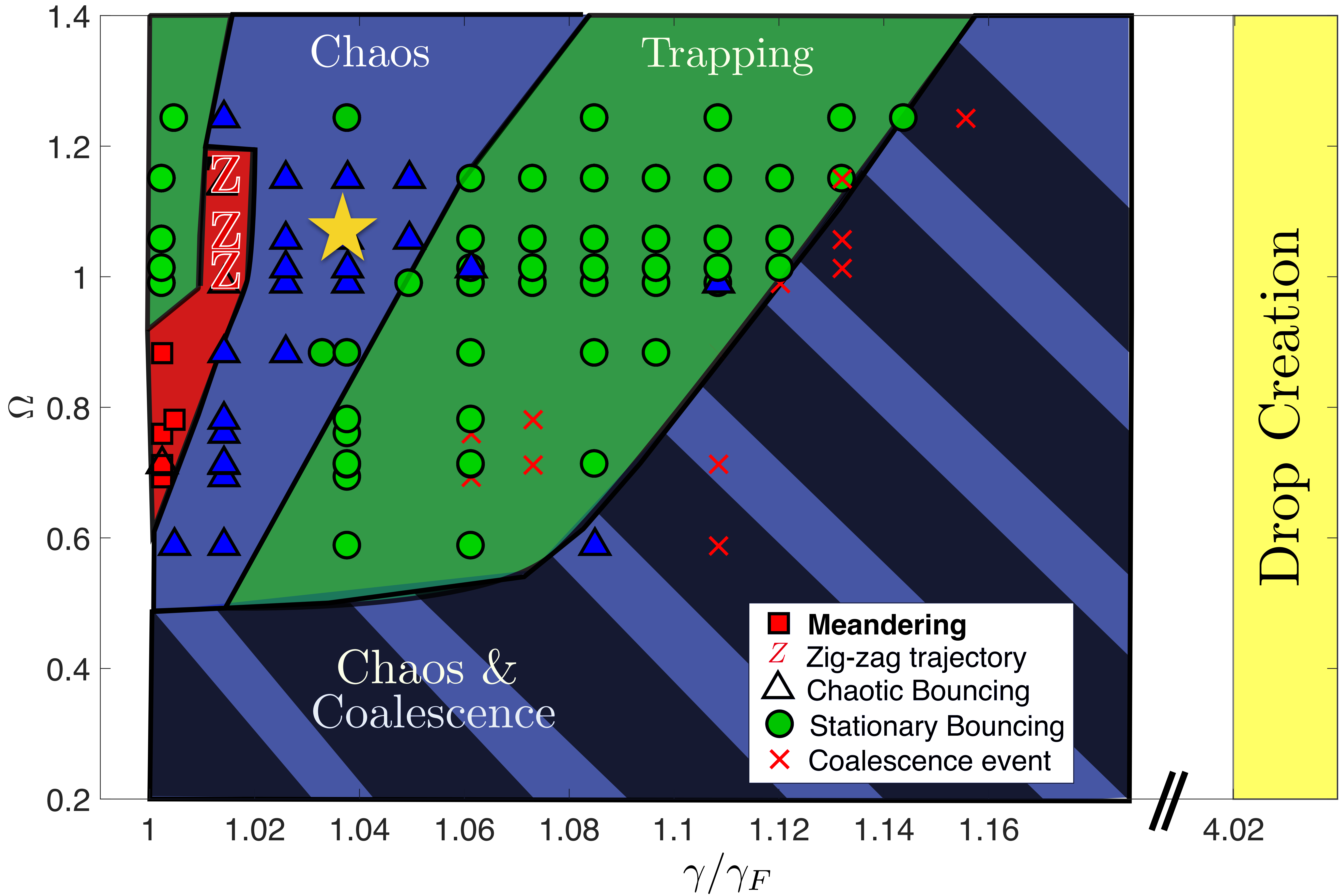


Meandering

- rectilinear walking state weakly perturbed by ambient wave field
- drop changes direction on a time scale long relative to the Faraday period

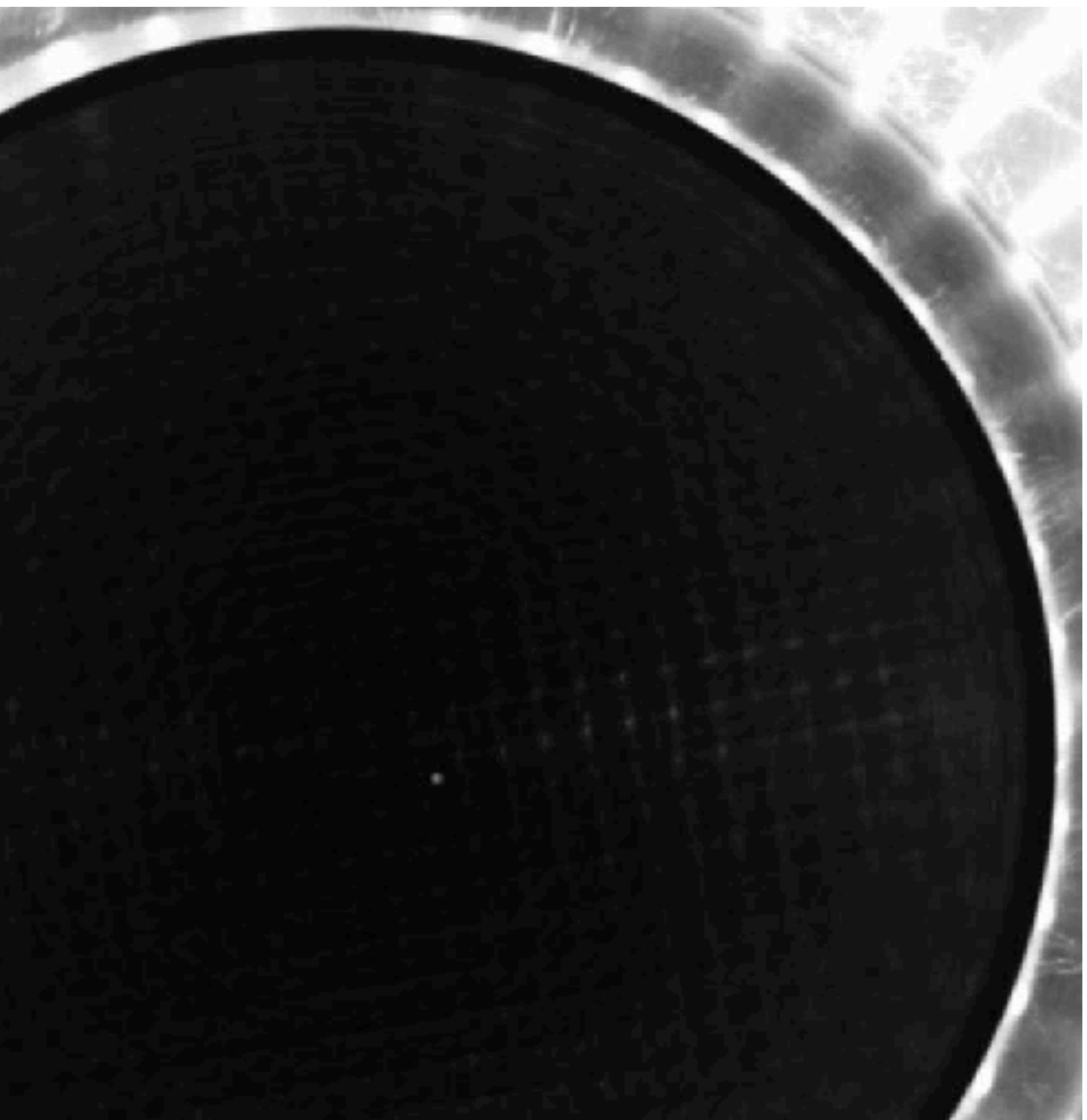


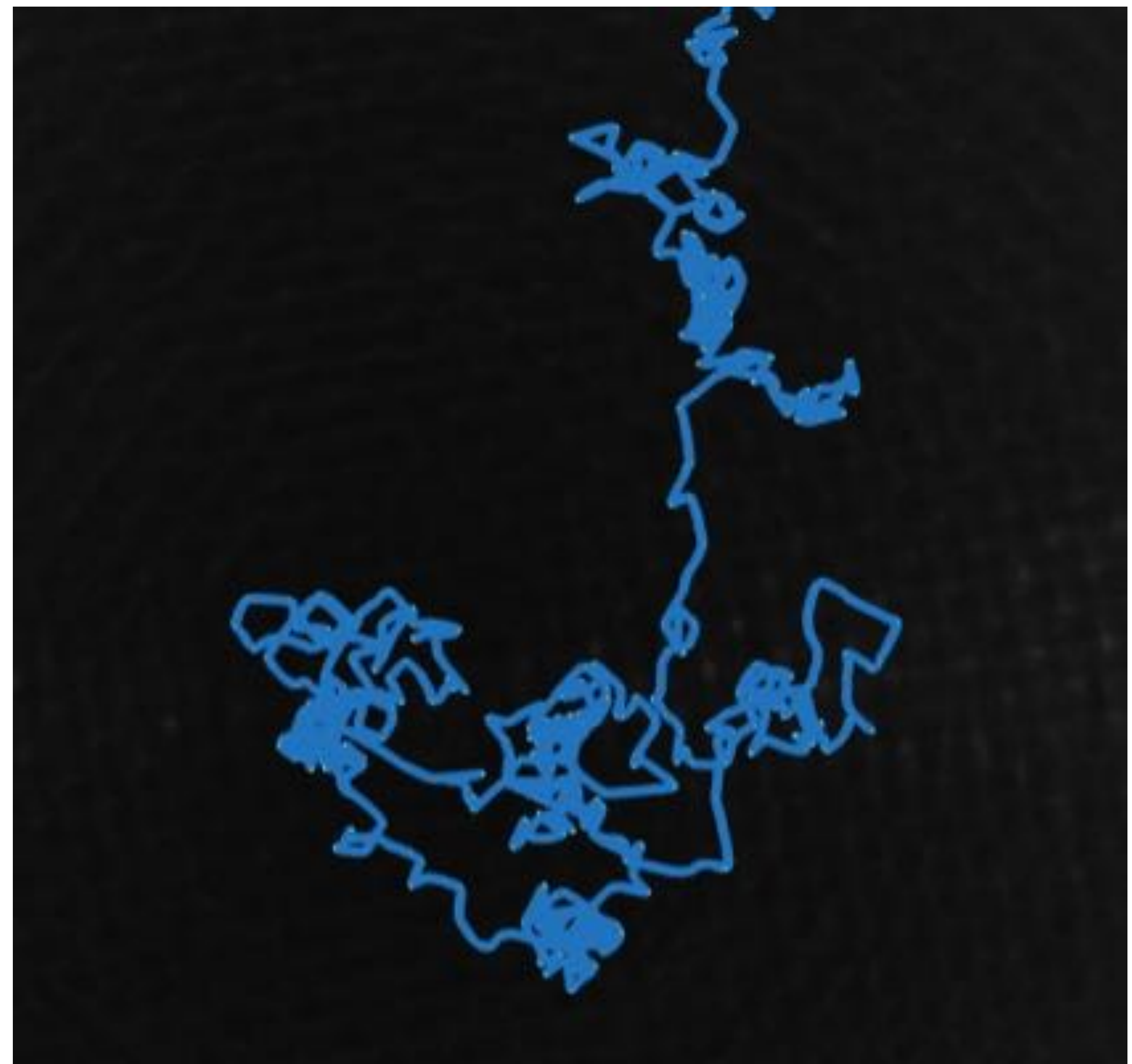
Droplet behavior above threshold



Chaotic bouncing

- droplet motion strongly perturbed by ambient wave field
- erratic changes in direction on the Faraday/bouncing period

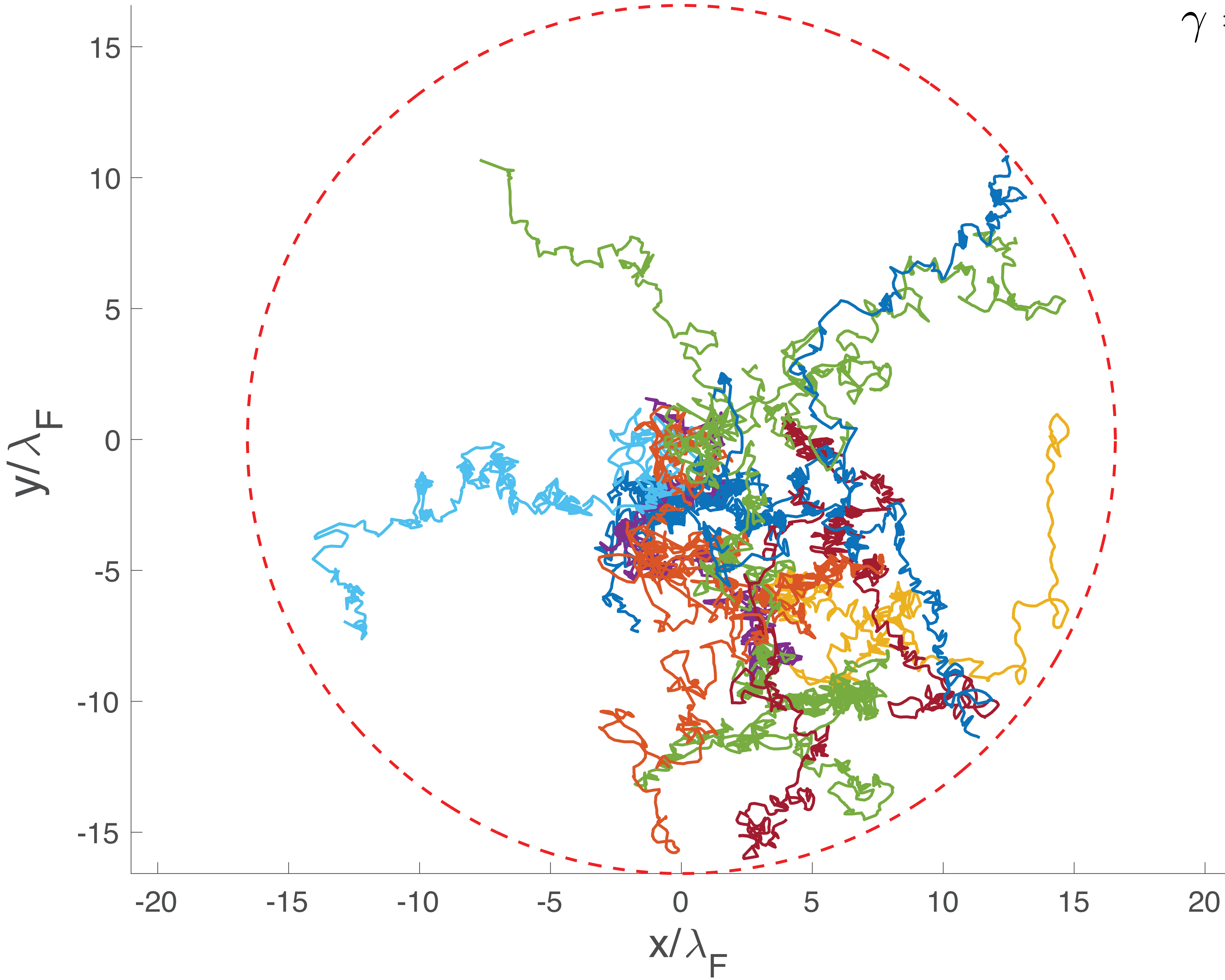




Diffusion in the chaotic regime

R = 0.456 mm

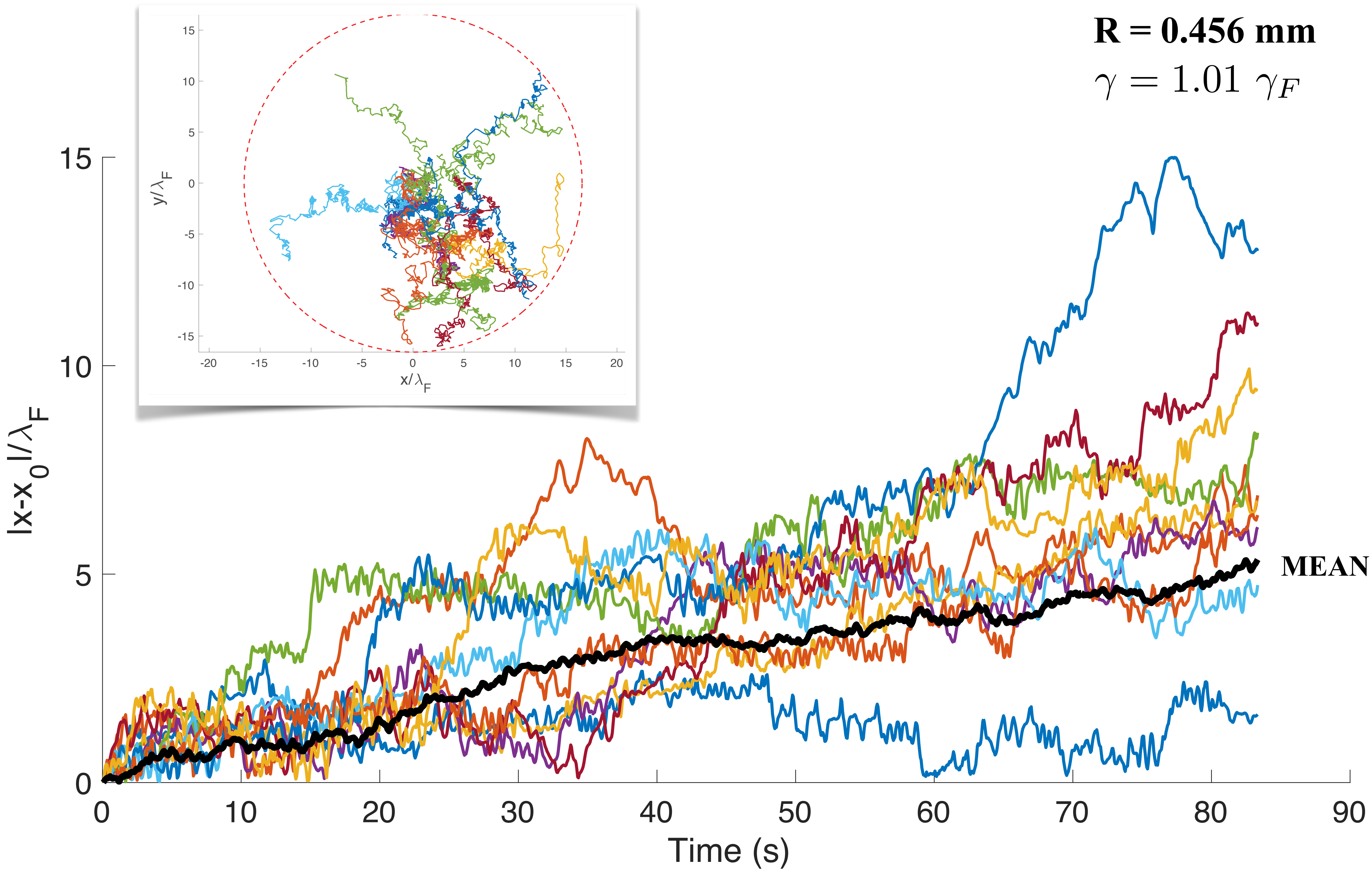
$\gamma = 1.01\ \gamma_F$



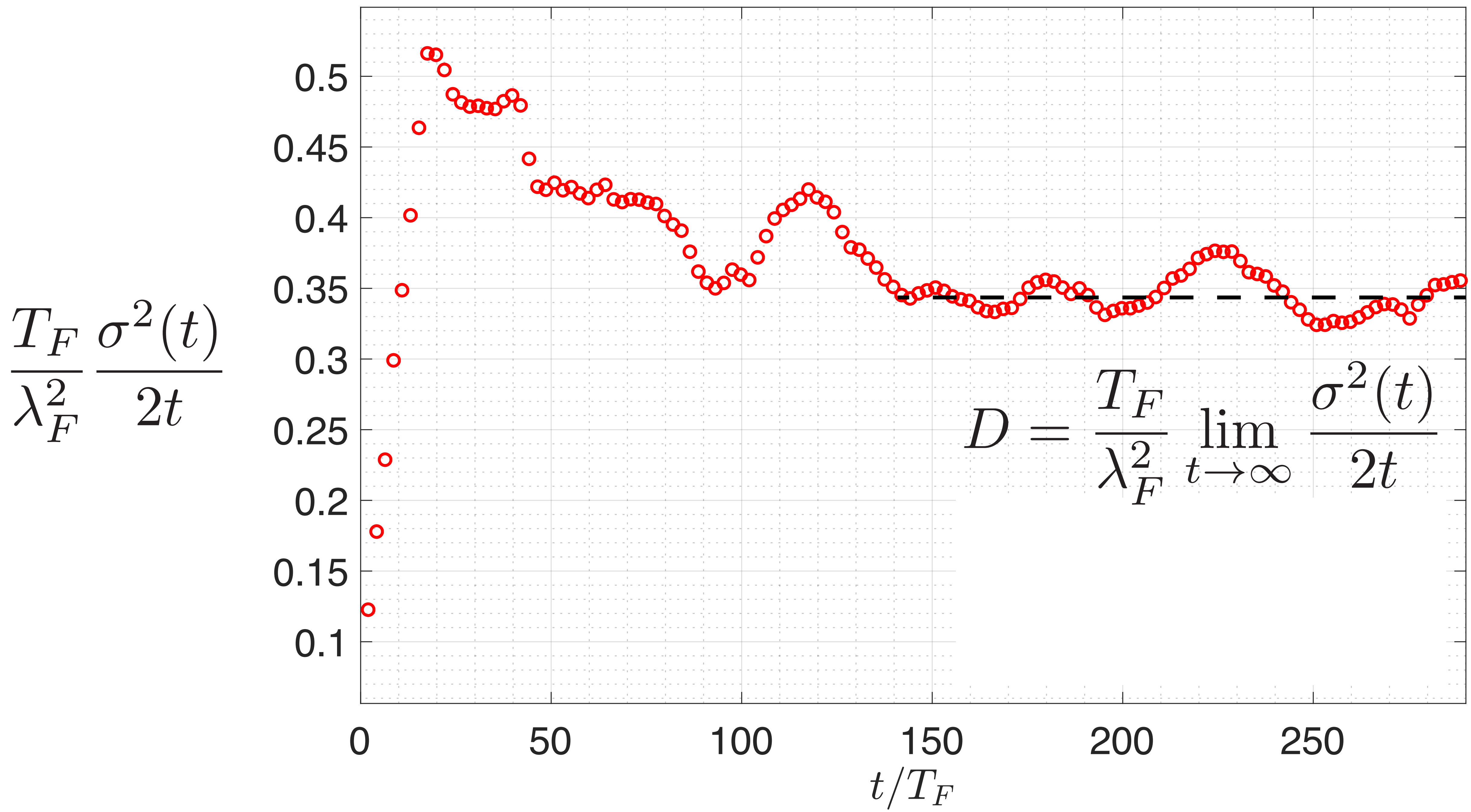
Diffusion in the chaotic regime

R = 0.456 mm

$\gamma = 1.01\ \gamma_F$

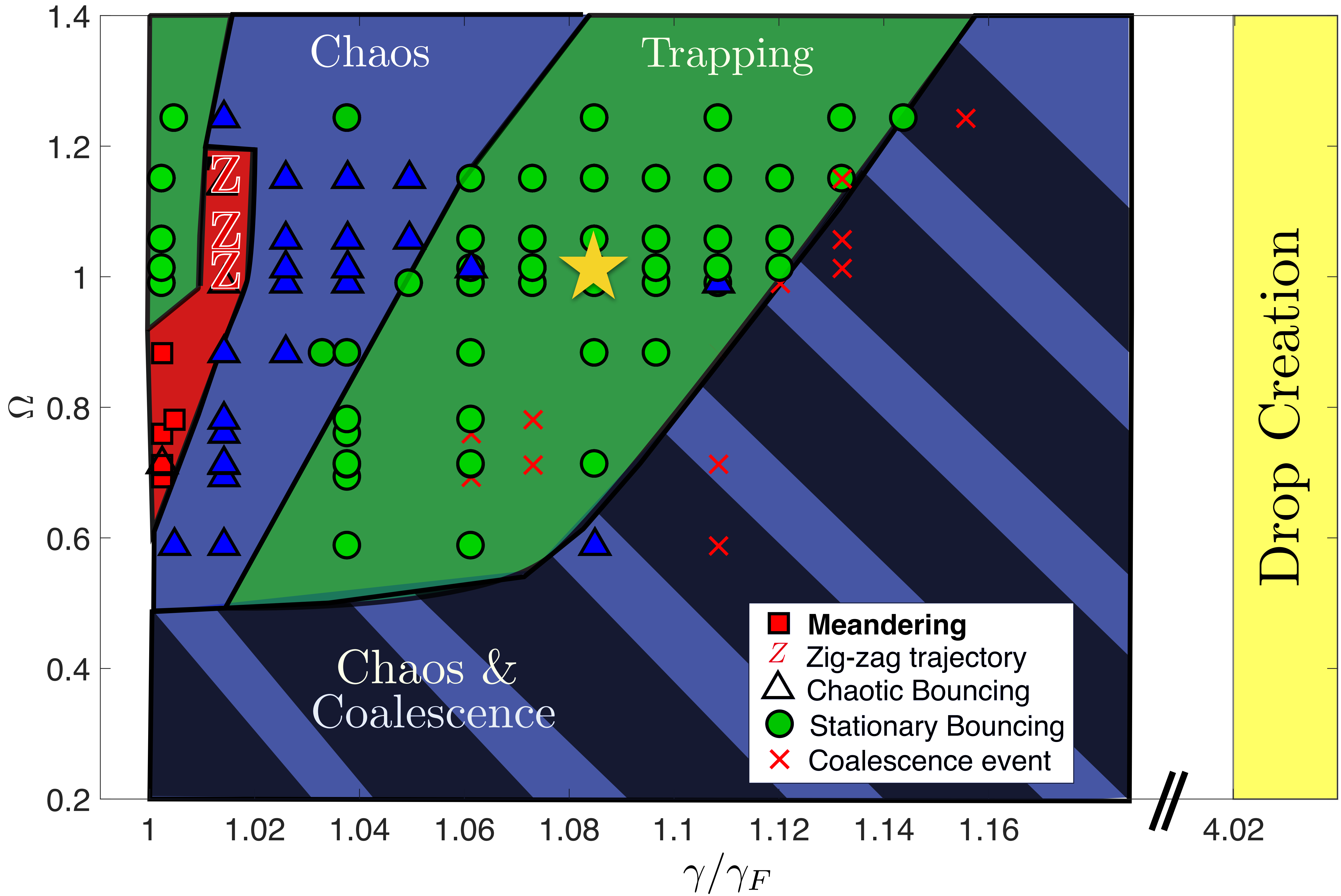


Classical diffusion in the chaotic regime

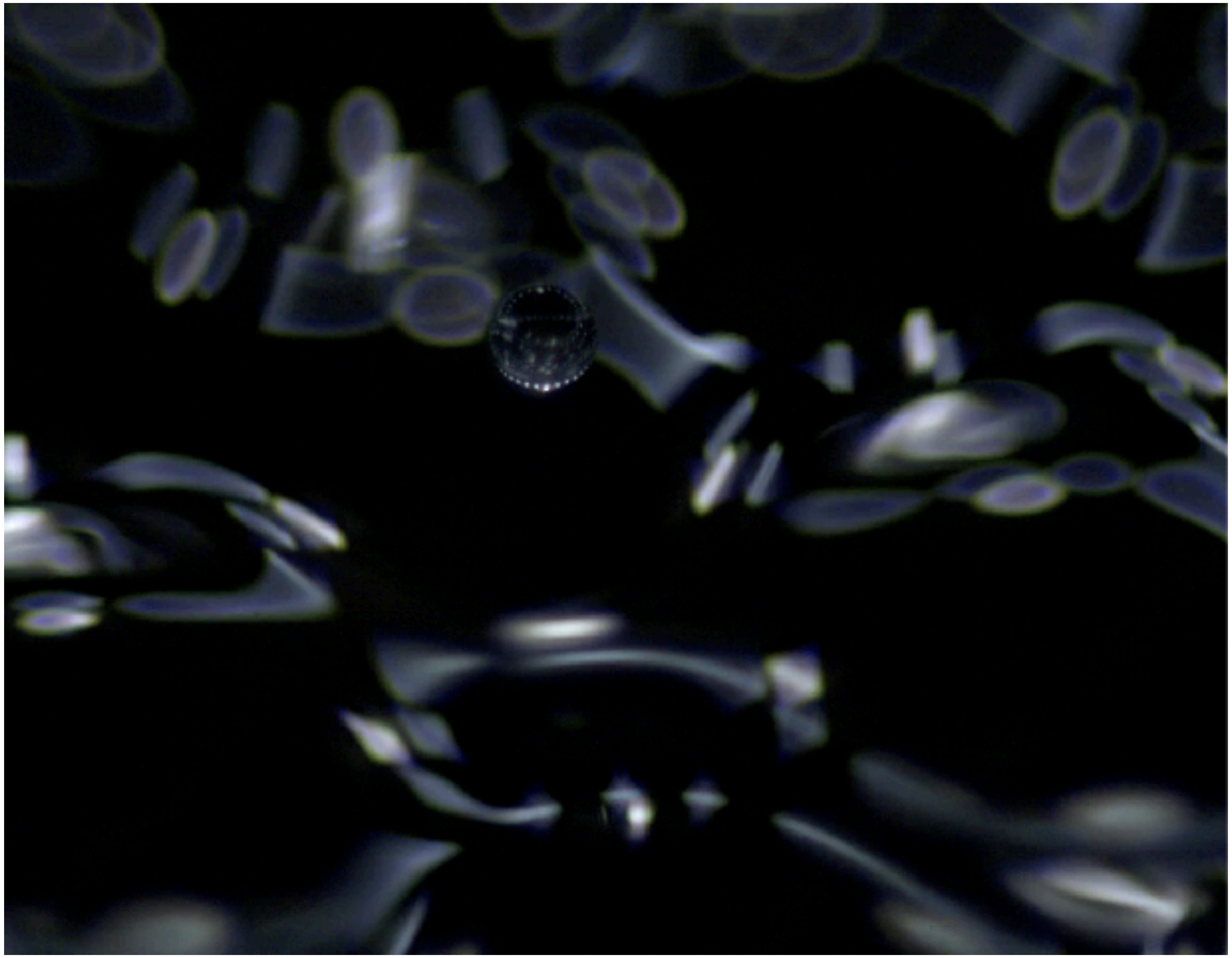


- **Brownian process**: mean-squared displacement scales with time t
- diffusivity D generally decreases with drop size, increases with memory

Droplet behavior above threshold



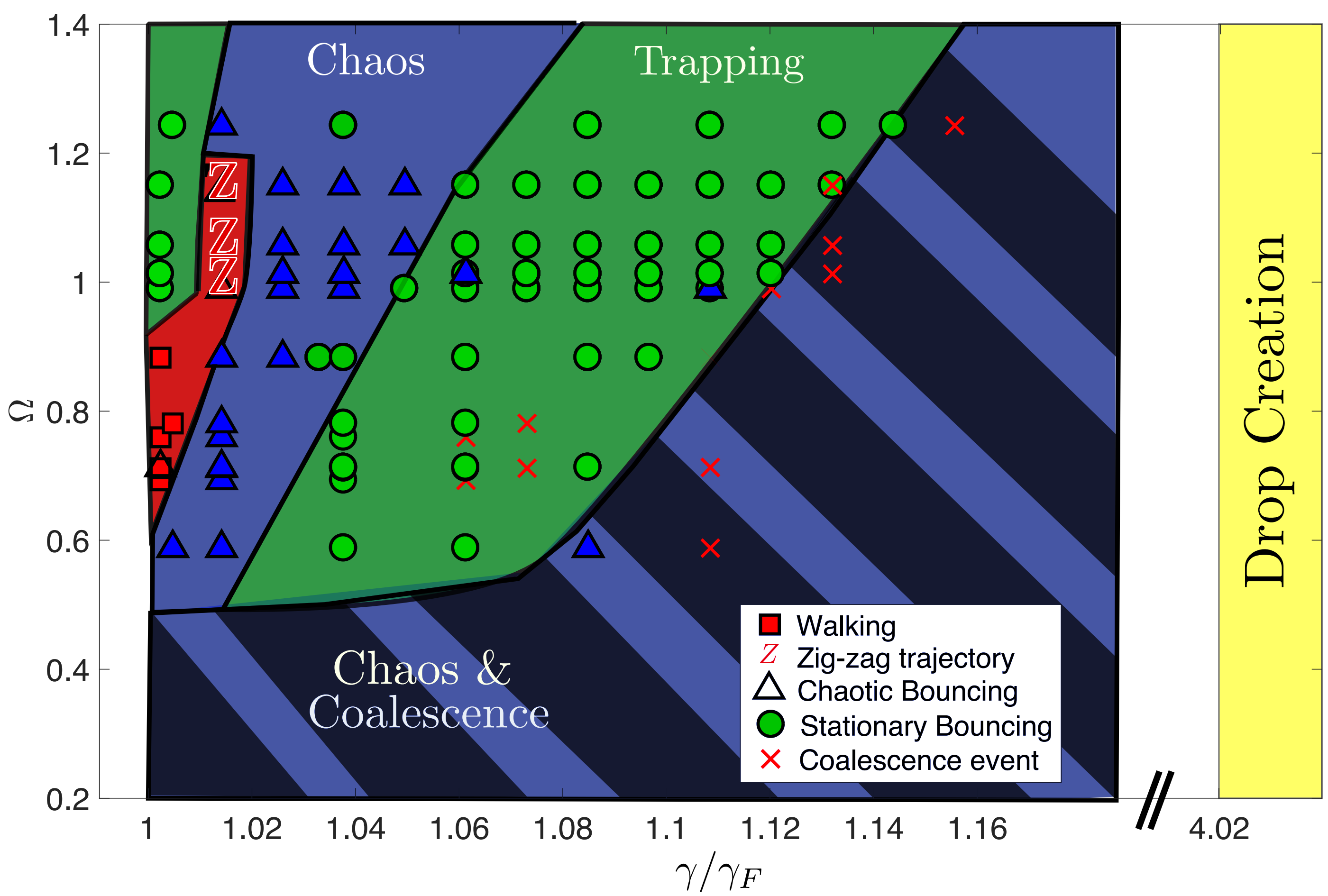
Wave-induced trapping



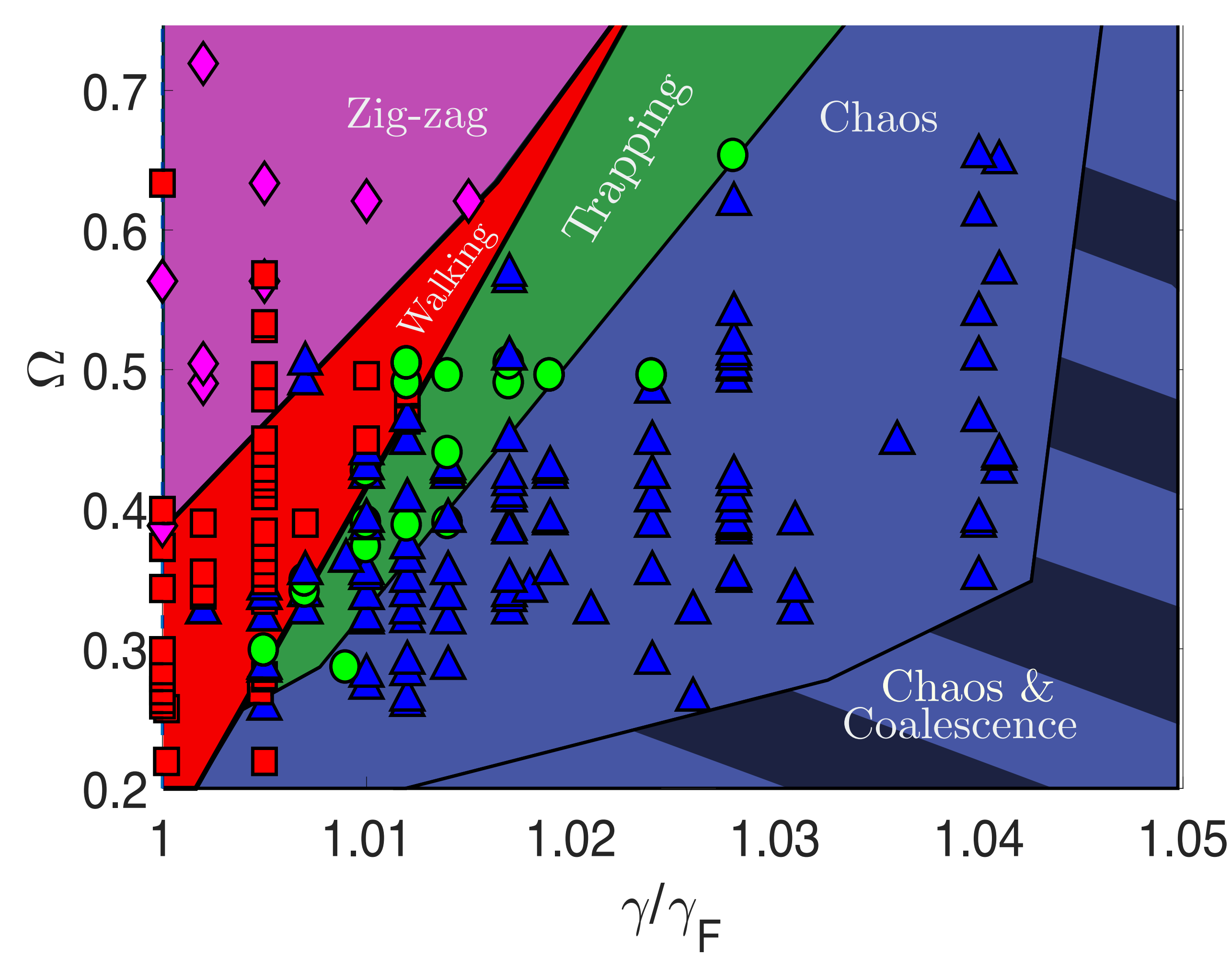
- droplet trapped by potential associated with Faraday wave field
- possible when bouncing period commensurate with Faraday period

Vary fluid, driving frequency

- qualitatively similar behavior: regions of meandering, chaos, trapping



20 cS, 80 Hz



50 cS, 50 Hz

Optical Talbot effect

PHYSICAL REVIEW FLUIDS **2**, 103602 (2017)

Hydrodynamic analog of particle trapping with the Talbot effect

N. Sungar*
Department of Physics, Cal Poly State University, San Luis Obispo, California 93407, USA

L. D. Tambasco, G. Pucci, P. J. Sáenz, and J. W. M. Bush
Department of Mathematics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
(Received 24 April 2017; published 11 October 2017)

RESEARCH ARTICLE | SEPTEMBER 11 2018

Faraday-Talbot effect: Alternating phase and circular arrays

Special Collection: Hydrodynamic Quantum Analogs

N. Sungar; J. P. Sharpe; J. J. Pilgram; J. Bernard; L. D. Tambasco

+ Author & Article Information

Chaos 28, 096101 (2018)

https://doi.org/10.1063/1.5031442 **Article history**

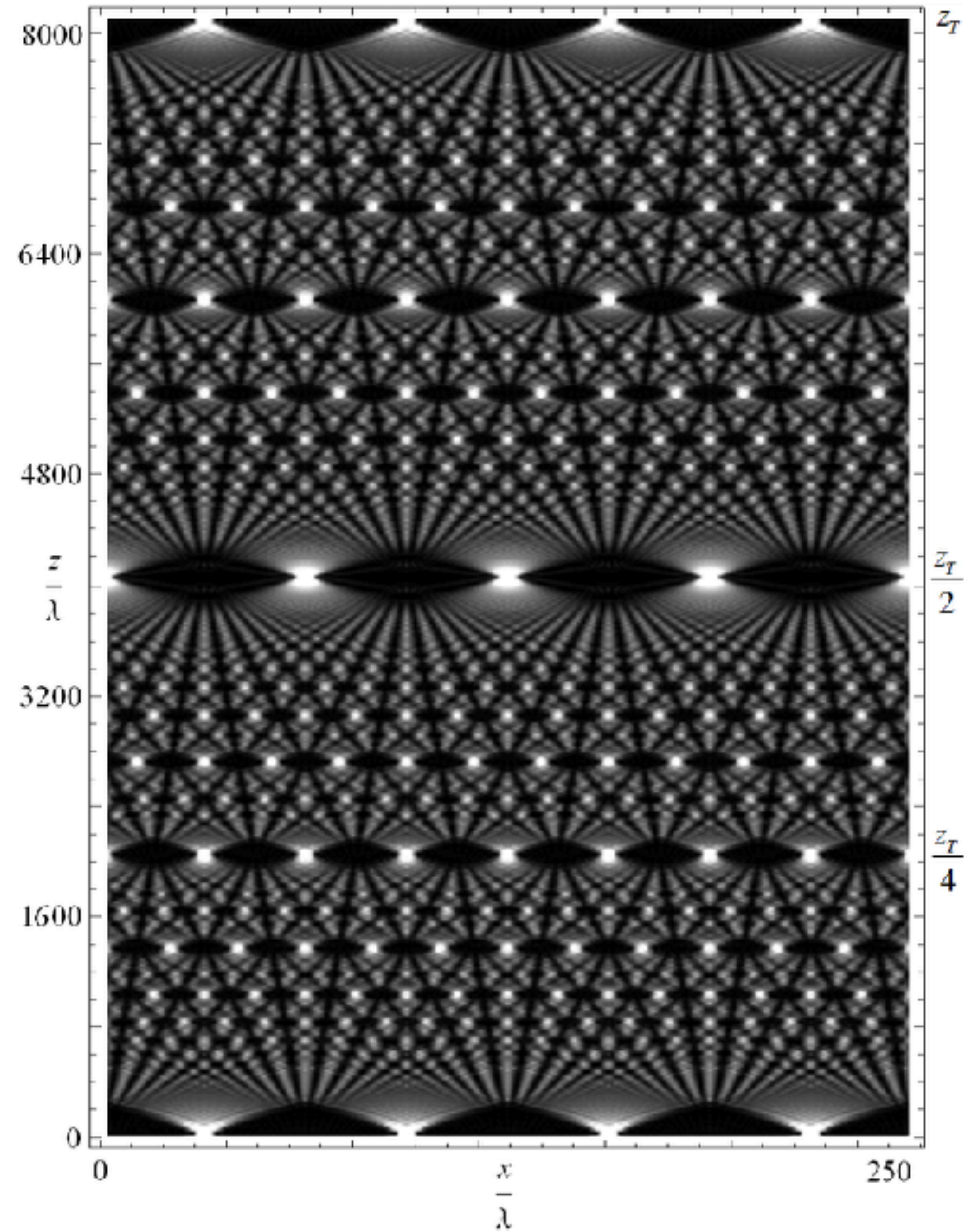
Optical Talbot effect

- plane wave of wavelength λ incident on an array of pillars with spacing d

$\Downarrow$

- image of pillar grating repeated at regular distances, the Talbot length:

$$z_T(\lambda) = \frac{\lambda}{2\left(1-\sqrt{1-(\frac{\lambda}{d})^2}\right)}$$

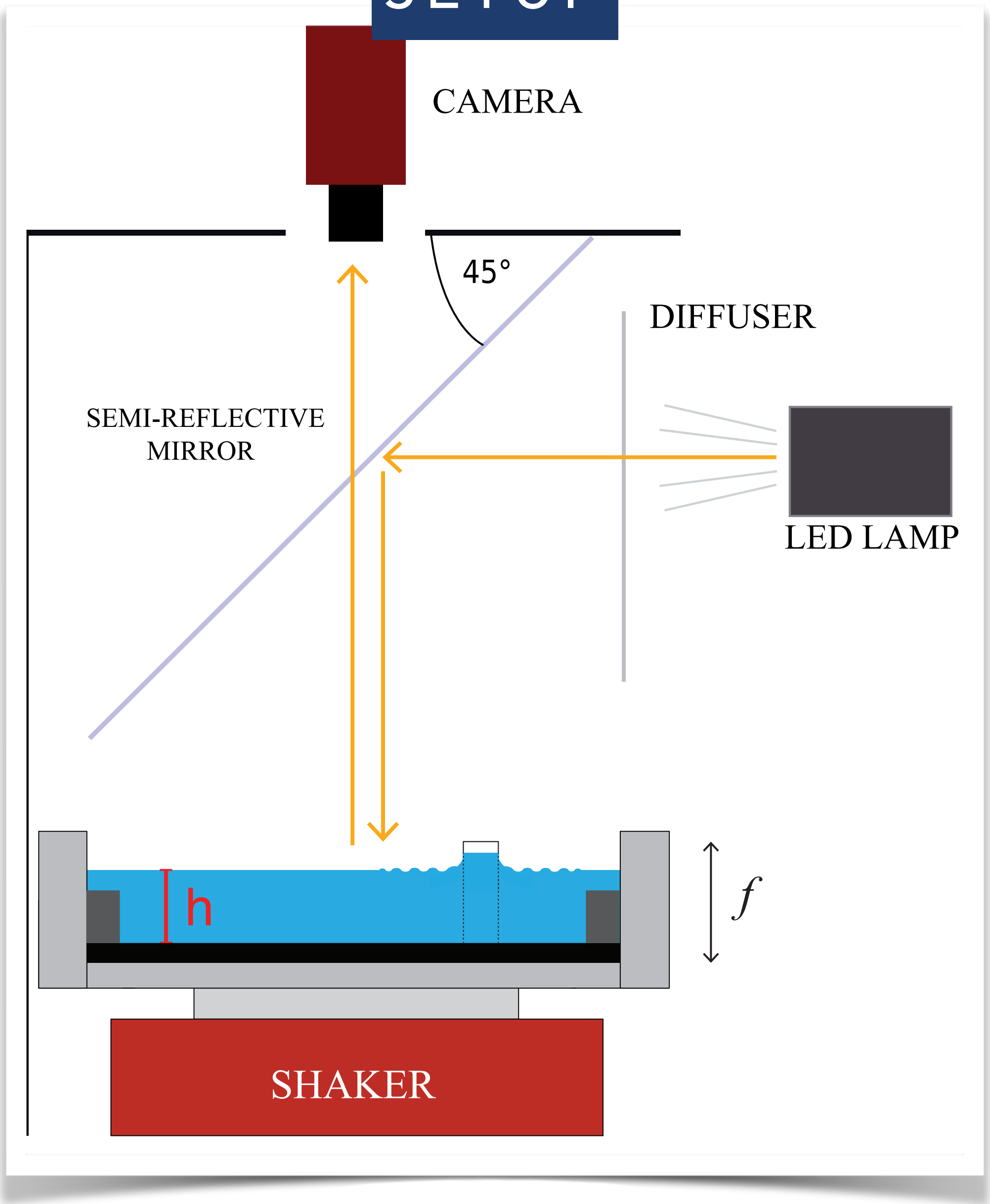


THE FARADAY-TALBOT EFFECT

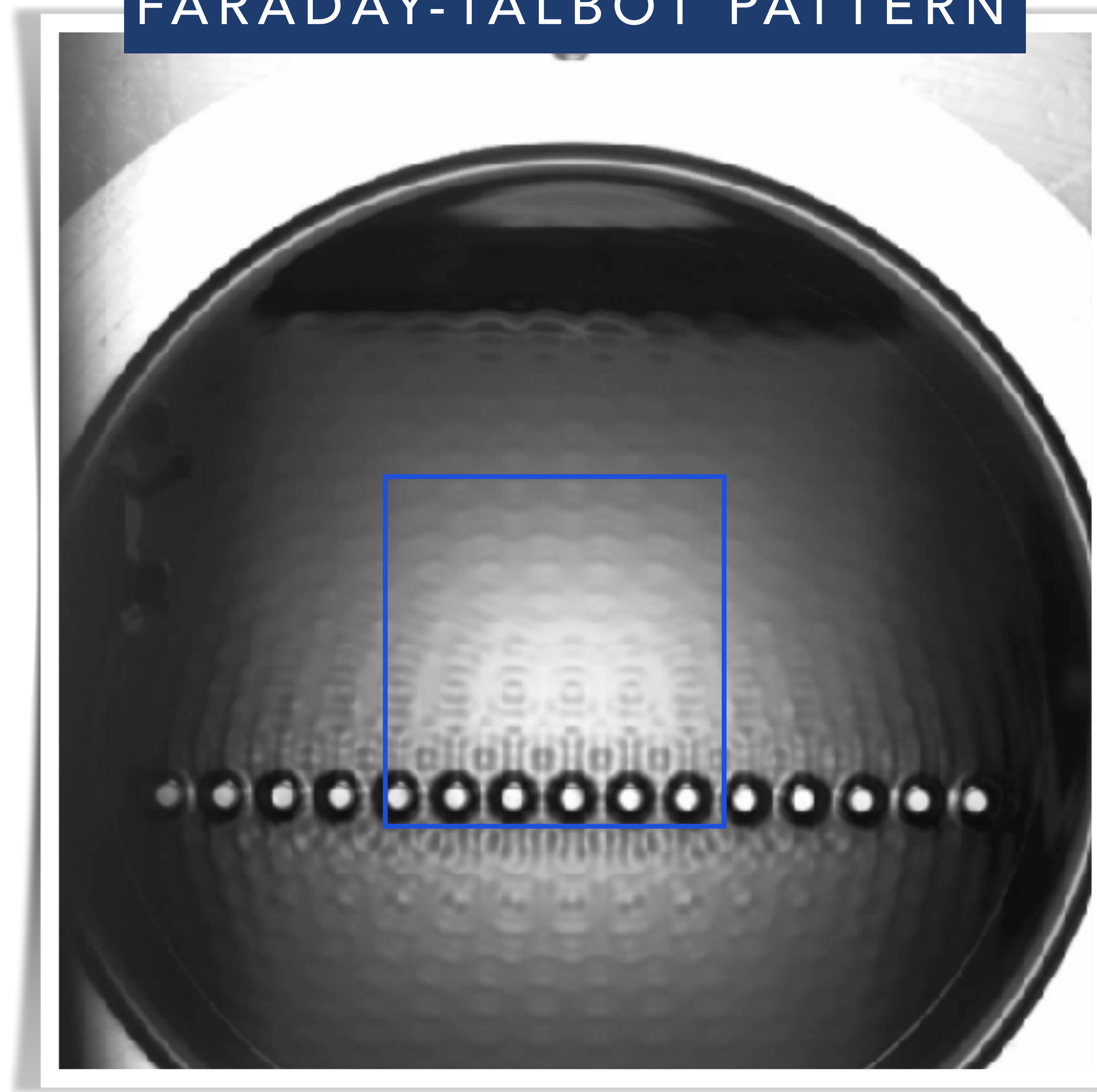
LINEAR ARRAY: SELF-IMAGES

$\gamma / \gamma_F = 1.007$

SETUP



FARADAY-TALBOT PATTERN



$f = 80$ Hz

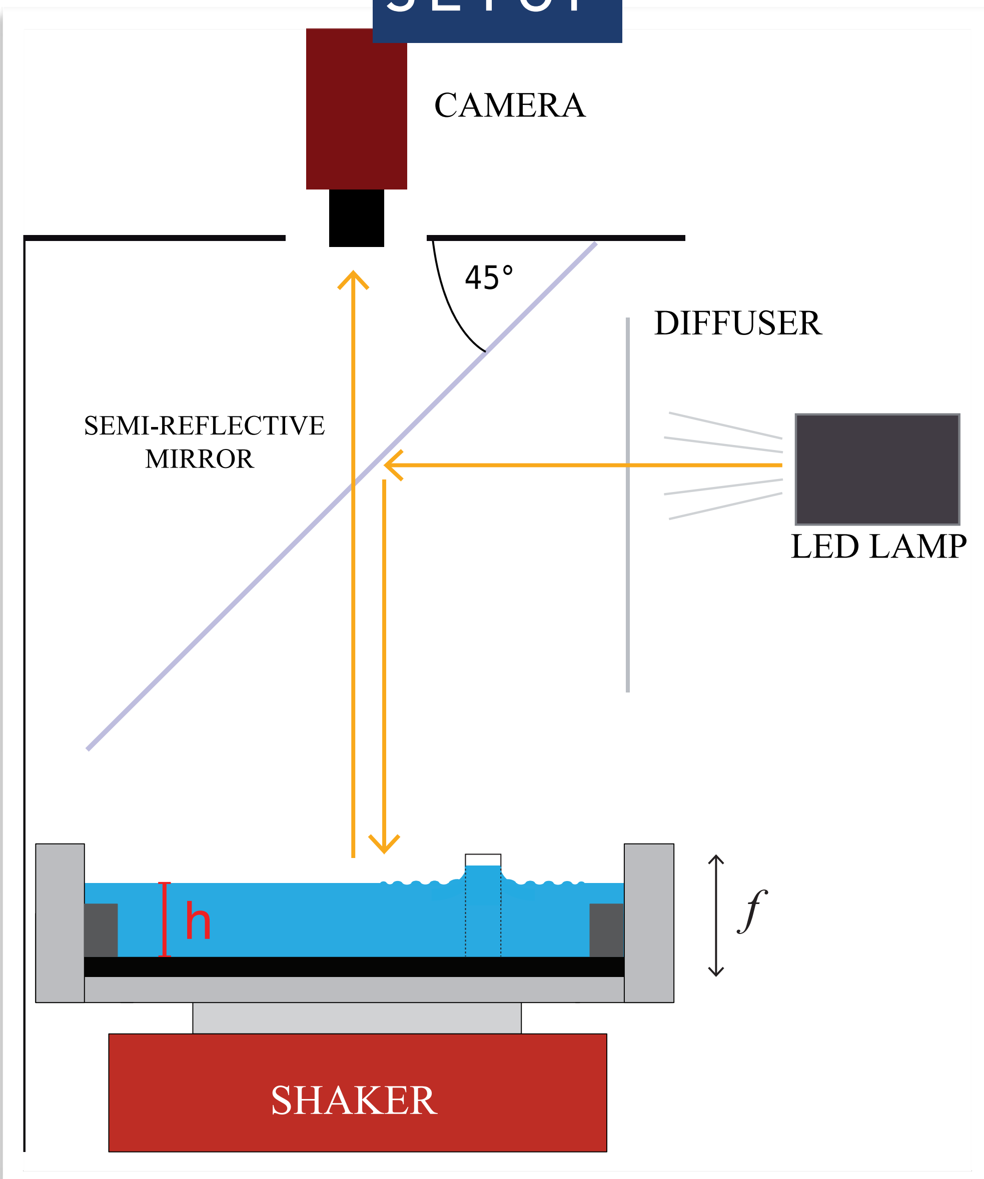
$d = 2\lambda_F = 9.5$ mm

THE FARADAY-TALBOT EFFECT

LINEAR ARRAY: SELF-IMAGES

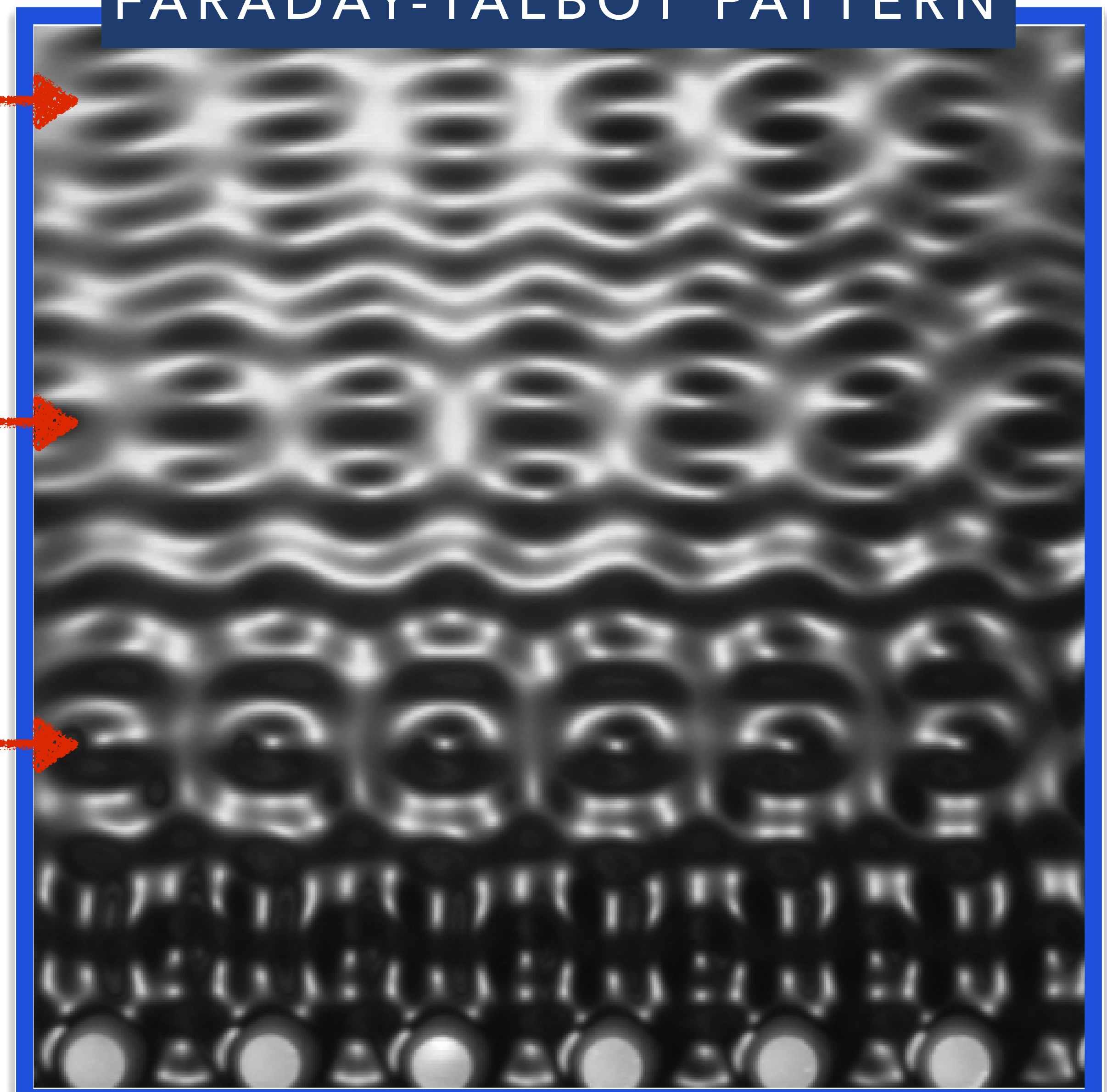
$\gamma/\gamma_F = 1.007$

SETUP



SELF-IMAGES

FARADAY-TALBOT PATTERN



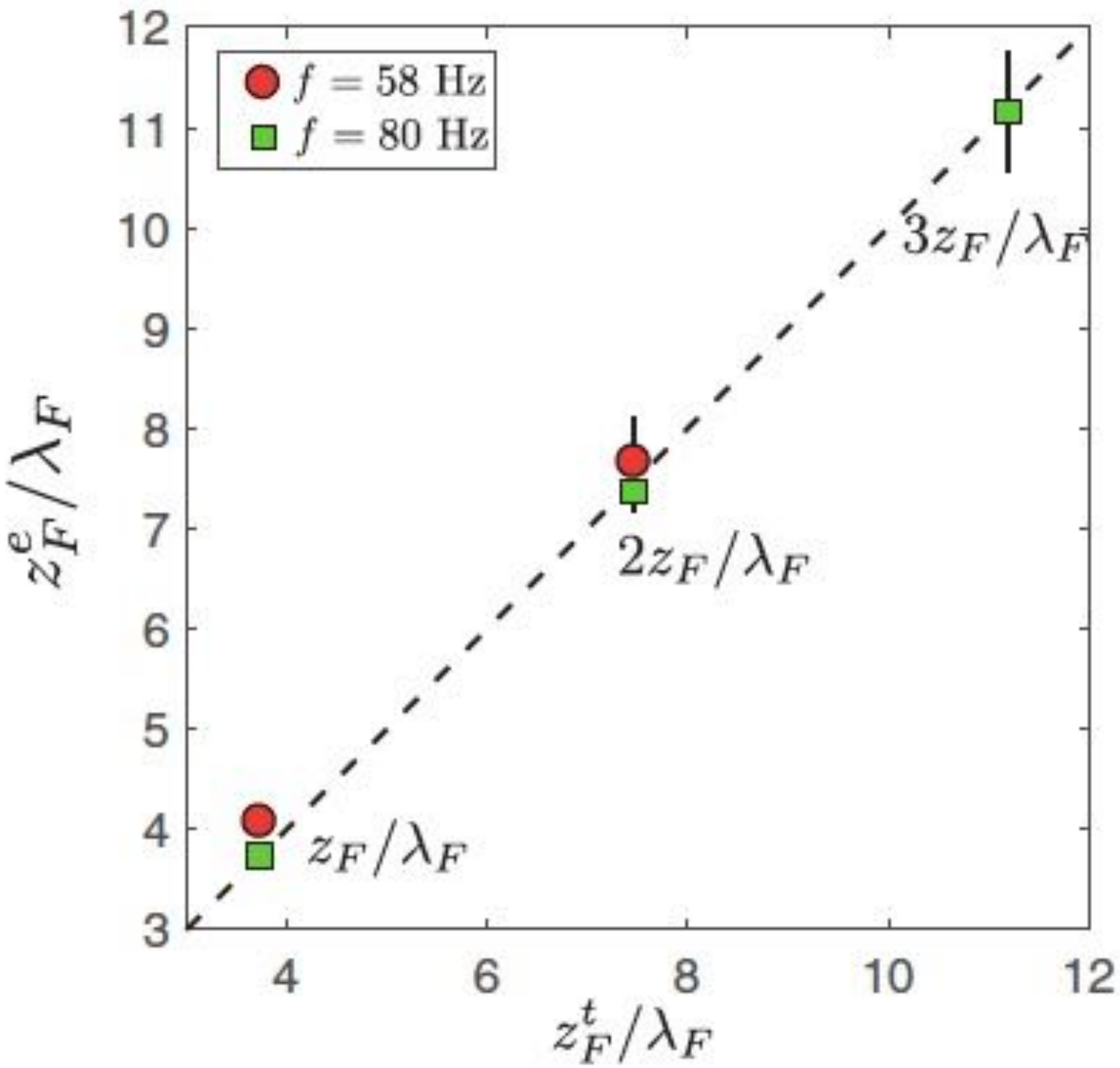
$f = 80$ Hz

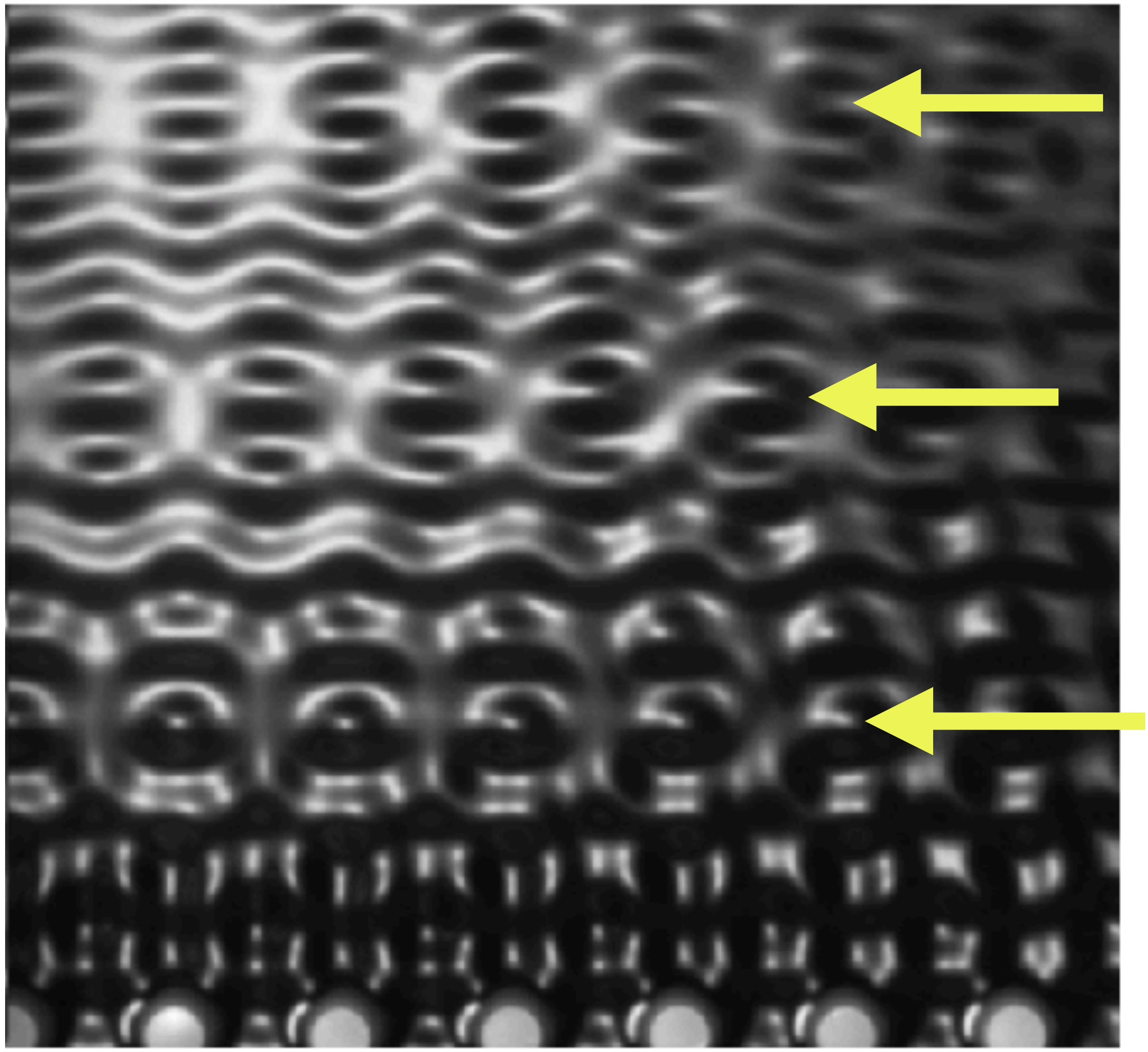
$d = 2\lambda_F = 9.5$ mm

Hydrodynamic Talbot effect

- observed just above the Faraday threshold: $\gamma/\gamma_F = 1.007$
- sloshing ridges between pillars with spacing d generate images at the Faraday-Talbot length:

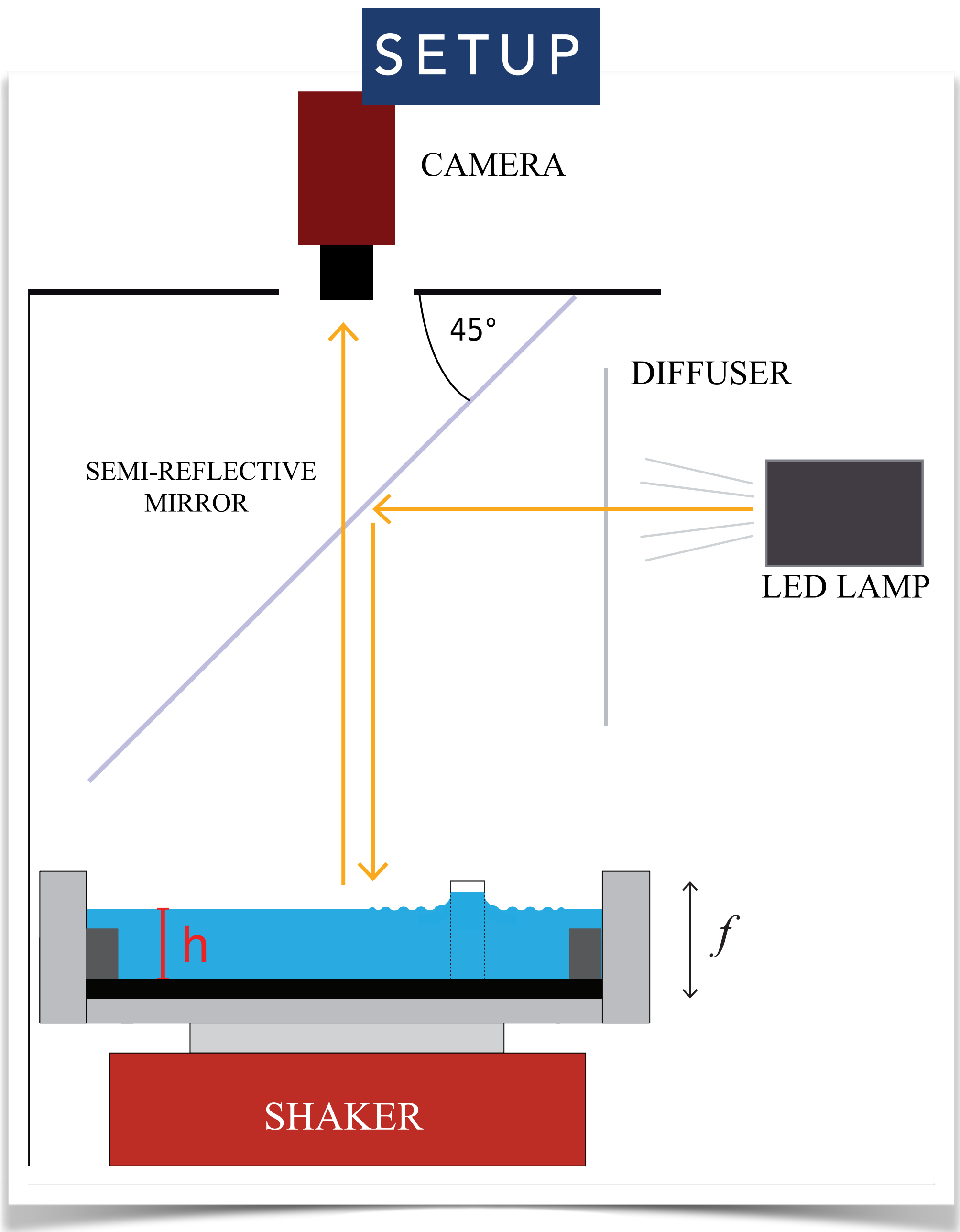
$$z_T(\lambda_F) = \frac{\lambda_F}{2\left(1 - \sqrt{1 - (\frac{\lambda_F}{d})^2}\right)}$$

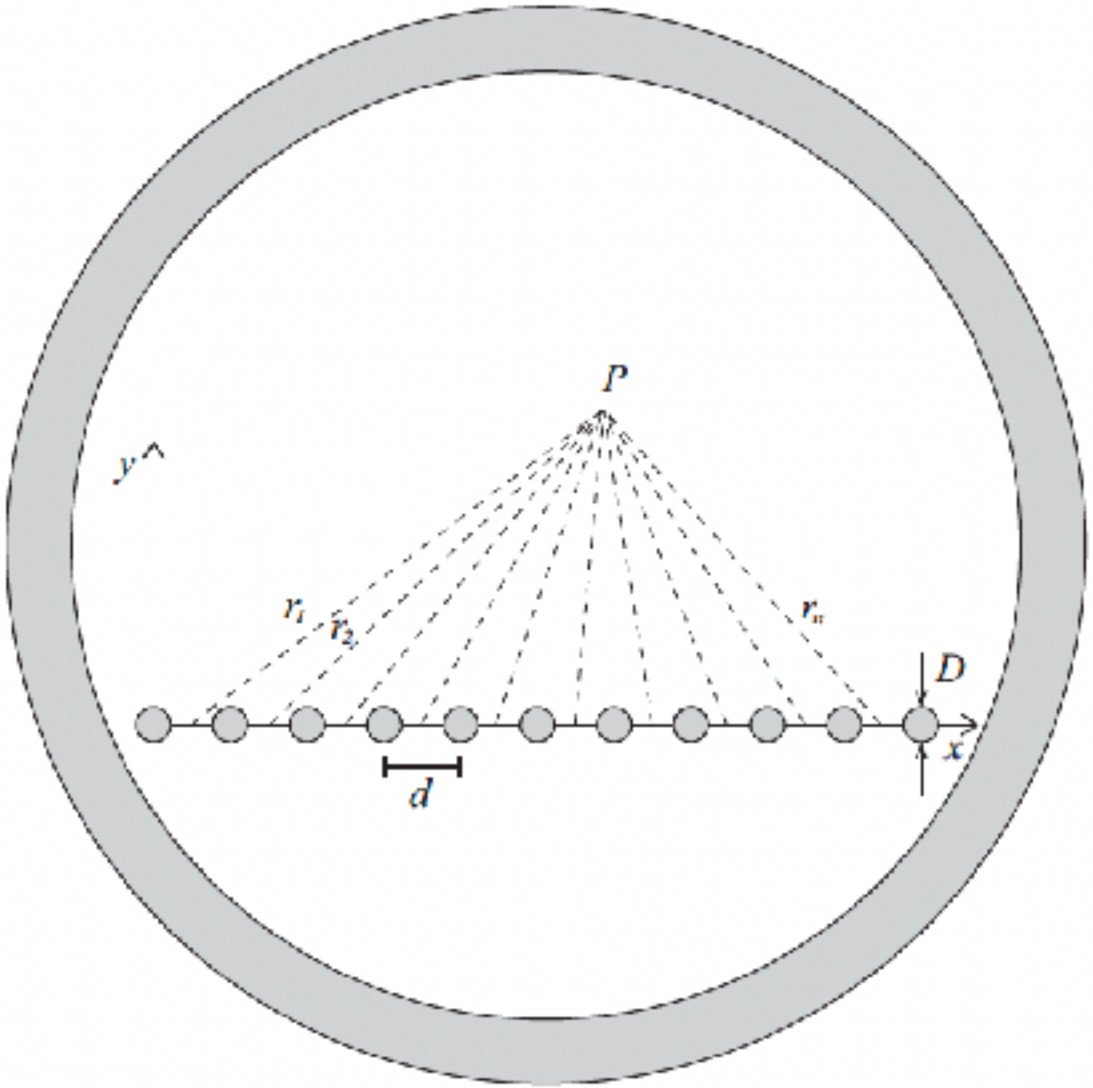




THE FARADAY-TALBOT EFFECT

LINEAR ARRAY: SELF-IMAGES





$f = 80$ Hz

$d = 2\lambda_F = 9.5$ mm

Approximate point source $J_0(kr)e^{-i\omega t}$

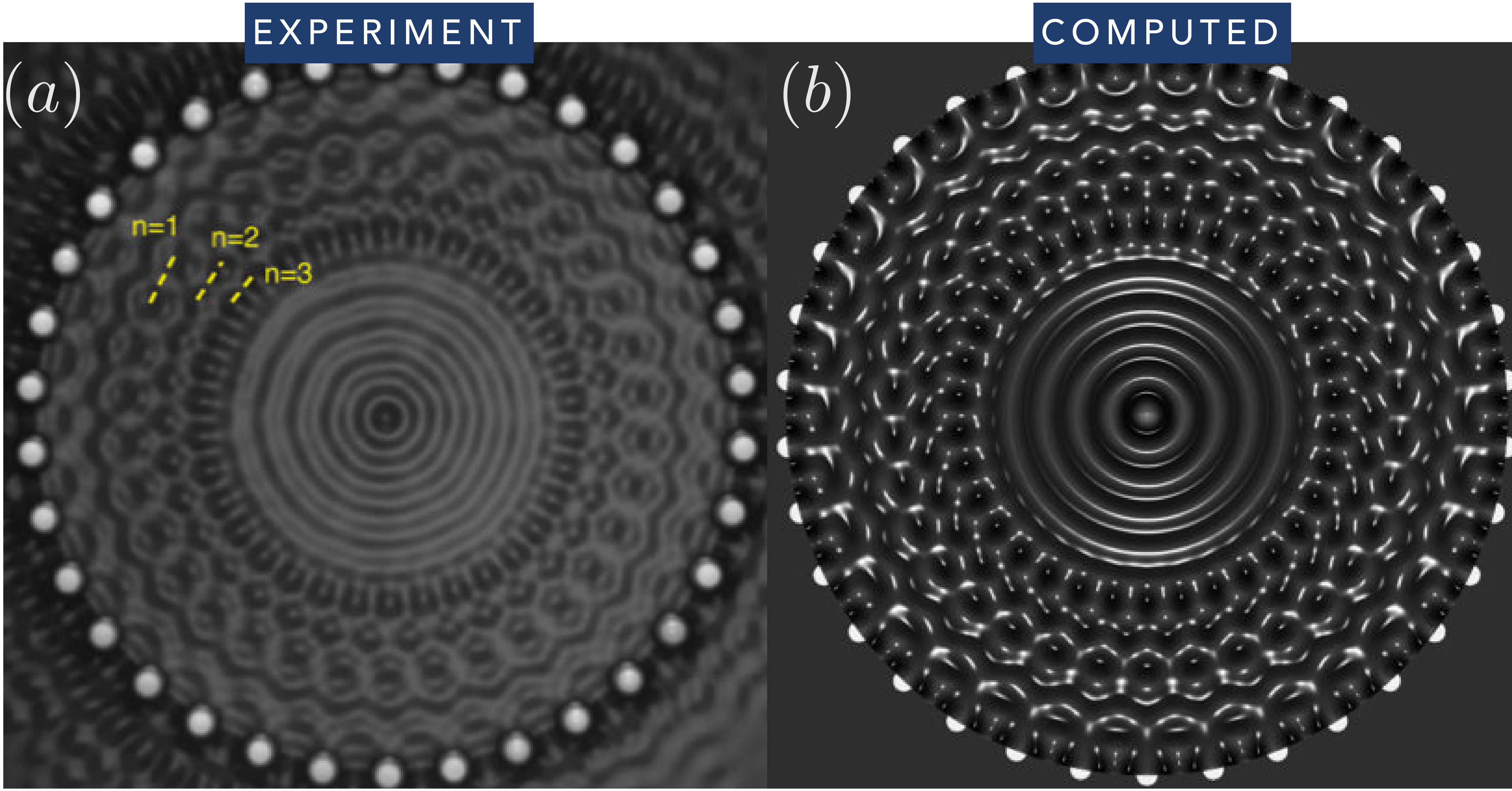
using $J_0(x) \sim \cos(x - \pi/4)/\sqrt{\pi x/2}$

Sum sources between pillars: $u(x,y,t) = A_F \sum_{n=1}^{N-1} \frac{\cos(k_F r_n - \omega_F t)}{\sqrt{k_F r_n}}$

where A_F is the wave amplitude, $r_n = \sqrt{y^2 + (x - (n - 1/2)d)^2}$, $\omega_F = \pi f$.

THE FARADAY-TALBOT EFFECT

CIRCULAR ARRAYS



$f = 80$ Hz

$d = 2\lambda_F$

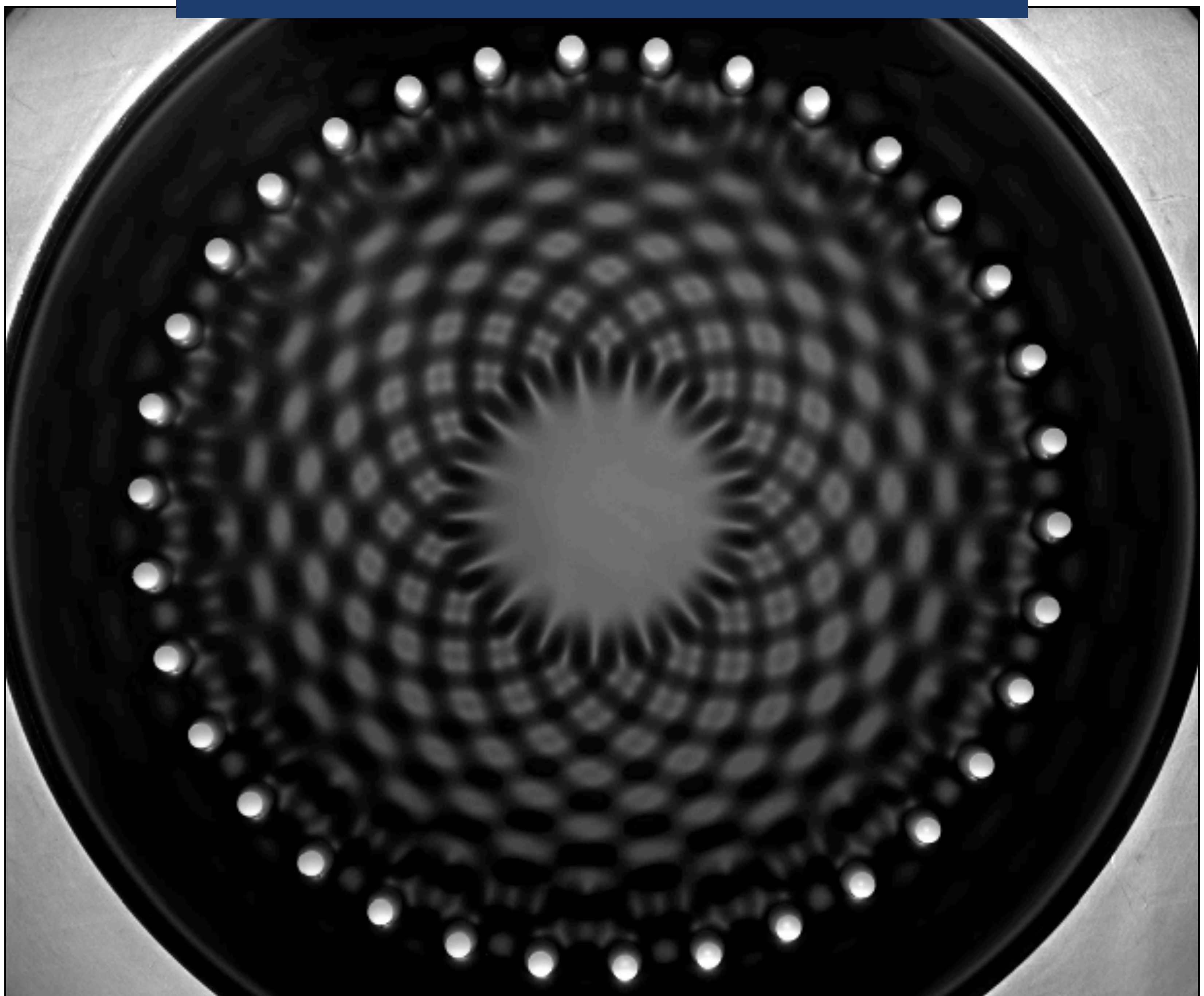
$R = 51.5$ mm

Surface deflection: $$u(x,y,t) = A_F \sum_{n=1}^{N-1} \frac{\cos(k_F r_n - \omega_F t)}{\sqrt{k_F r_n}}$$

THE FARADAY-TALBOT EFFECT

CIRCULAR ARRAYS

STABLE PATTERN - EXPERIMENT



$f = 55$ Hz

$d = 11$ mm

$\lambda_F = 6.40$ mm

$R = 59.5$ mm

Talbot trapping


Large-scale optical traps on a chip for optical sorting

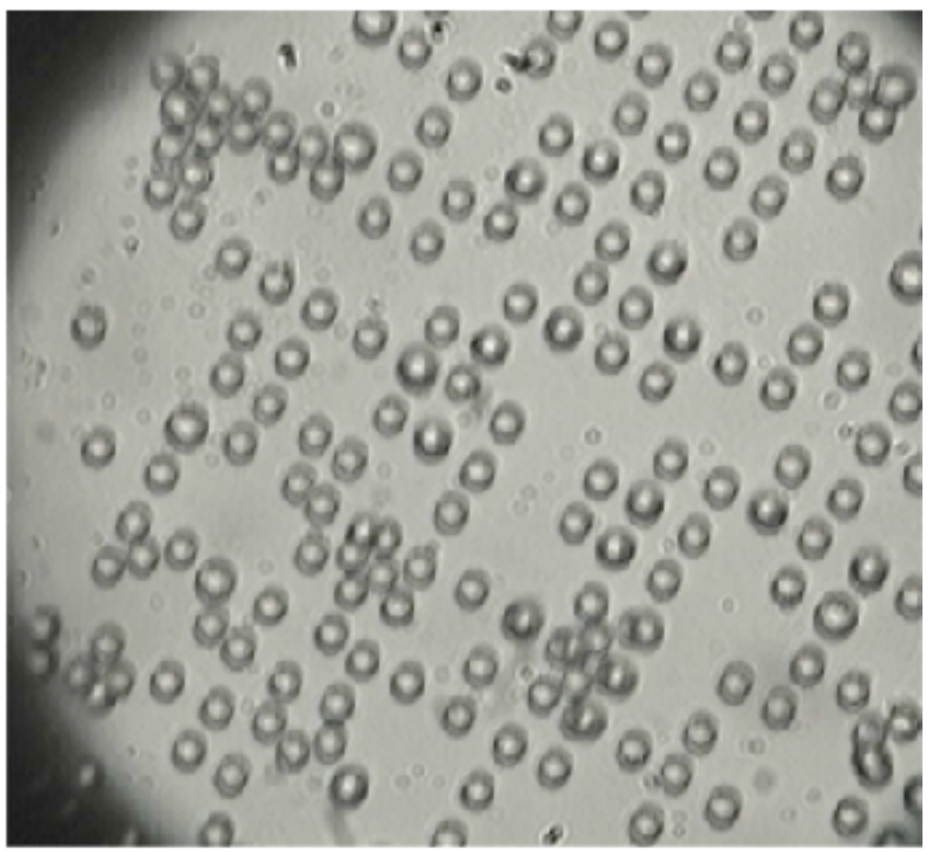
Y. Y. Sun
College of Engineering, Nanyang Technological University, Nanyang Avenue, Singapore 639798, Singapore

X.-C. Yuan[a)]
College of Engineering, Nanyang Technological University, Nanyang Avenue, Singapore 639798, Singapore and Institute of Optoelectronics, Key Laboratory of Optoelectronic Devices and Systems of Chinese Education Ministry, Shenzhen University, 518060 Shenzhen, China

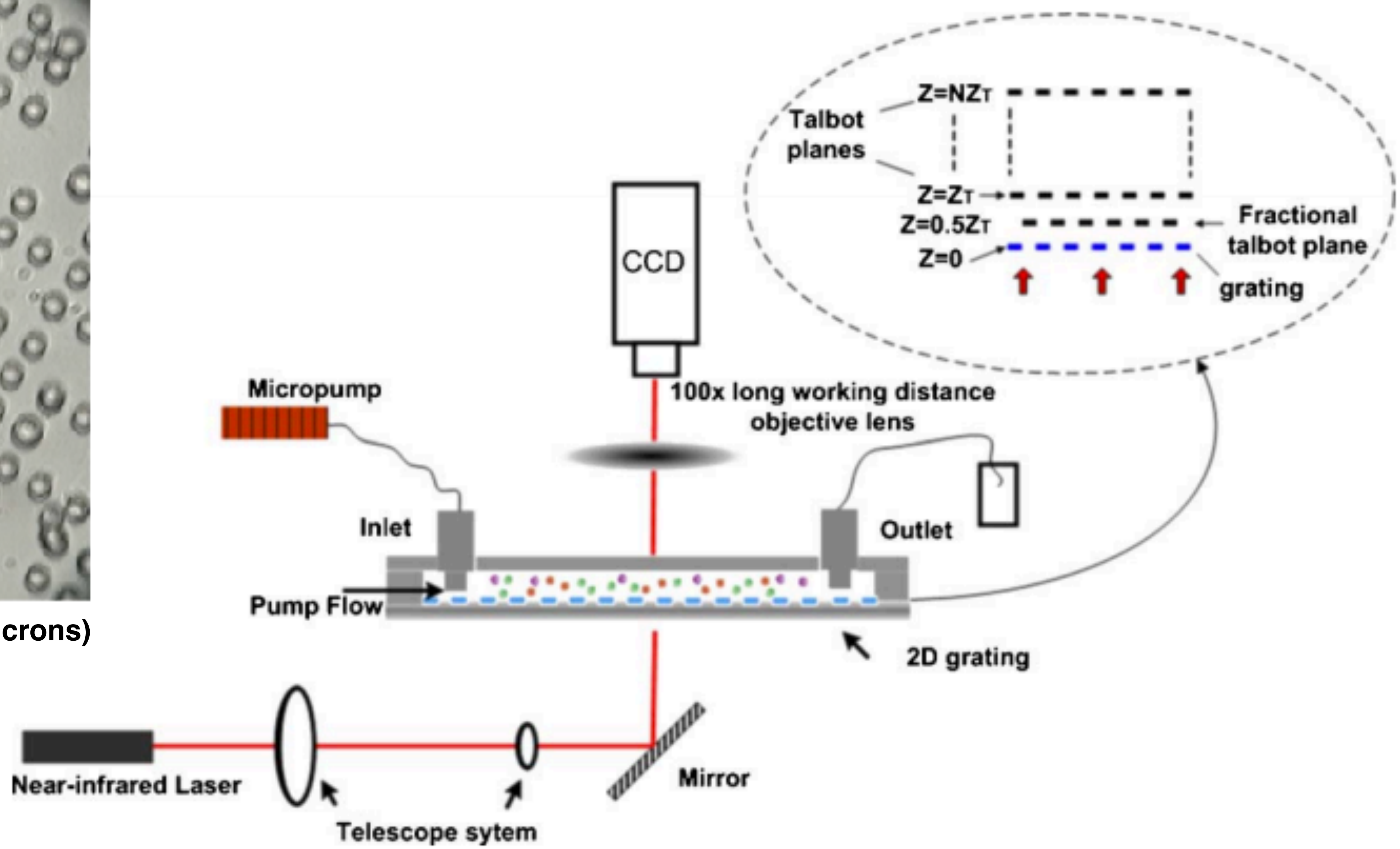
L. S. Ong and J. Bu
College of Engineering, Nanyang Technological University, Nanyang Avenue, Singapore 639798, Singapore

S. W. Zhu and R. Liu
Tianjin Union Medical Centre, Tianjin 300121, People's Republic of China

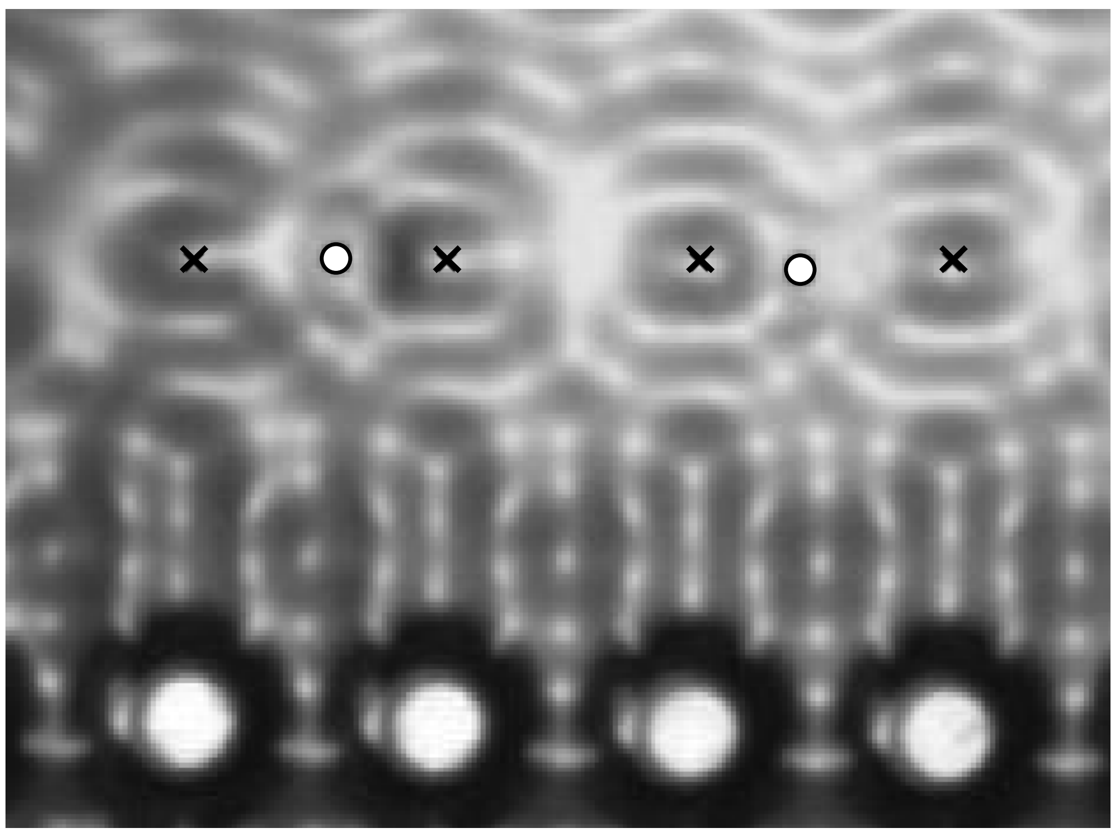
Talbot effect can trap polymer particles in image planes



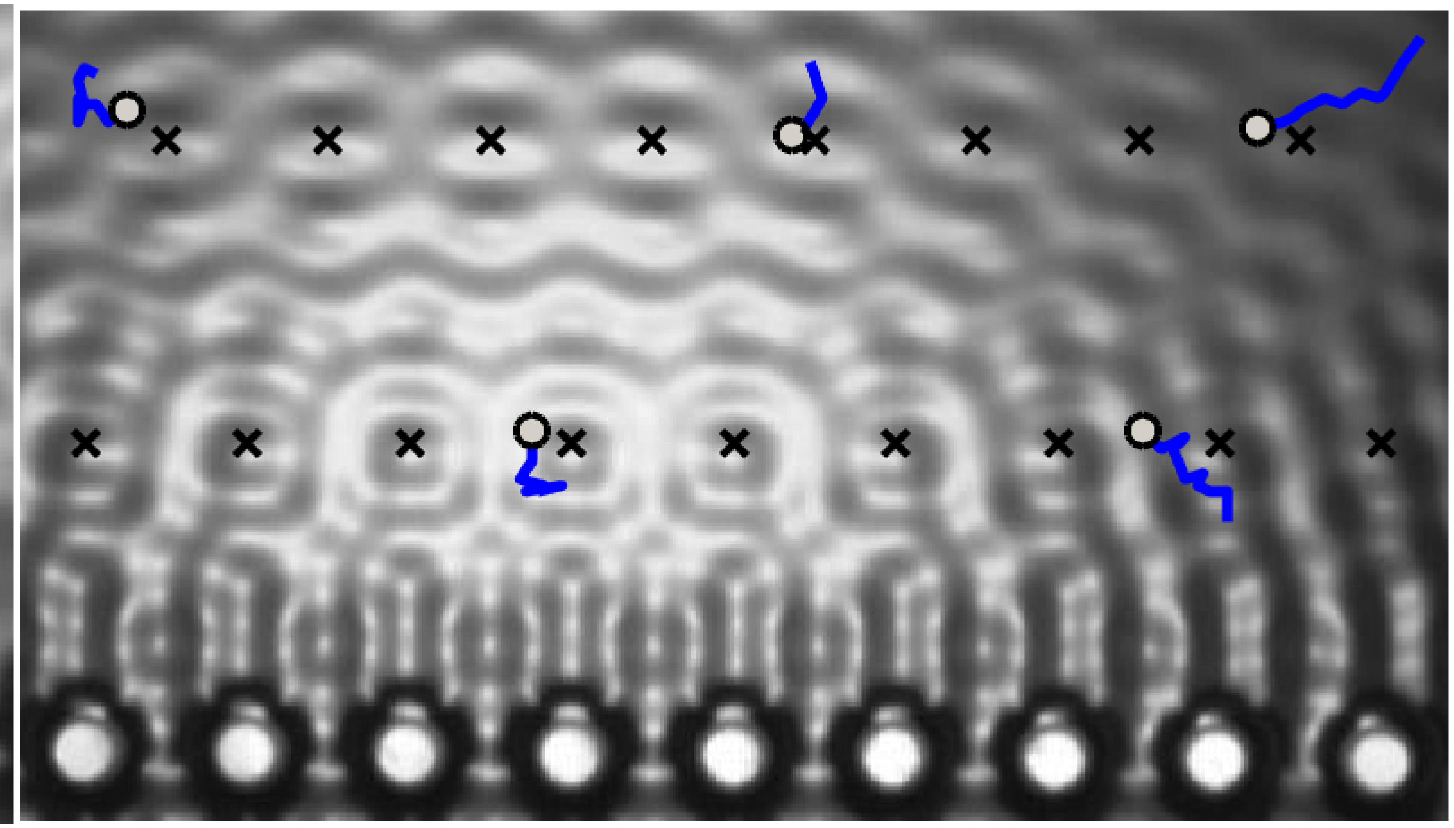
Array of trapped polymer particles (3.1 microns)



Faraday-Talbot trap for bouncers

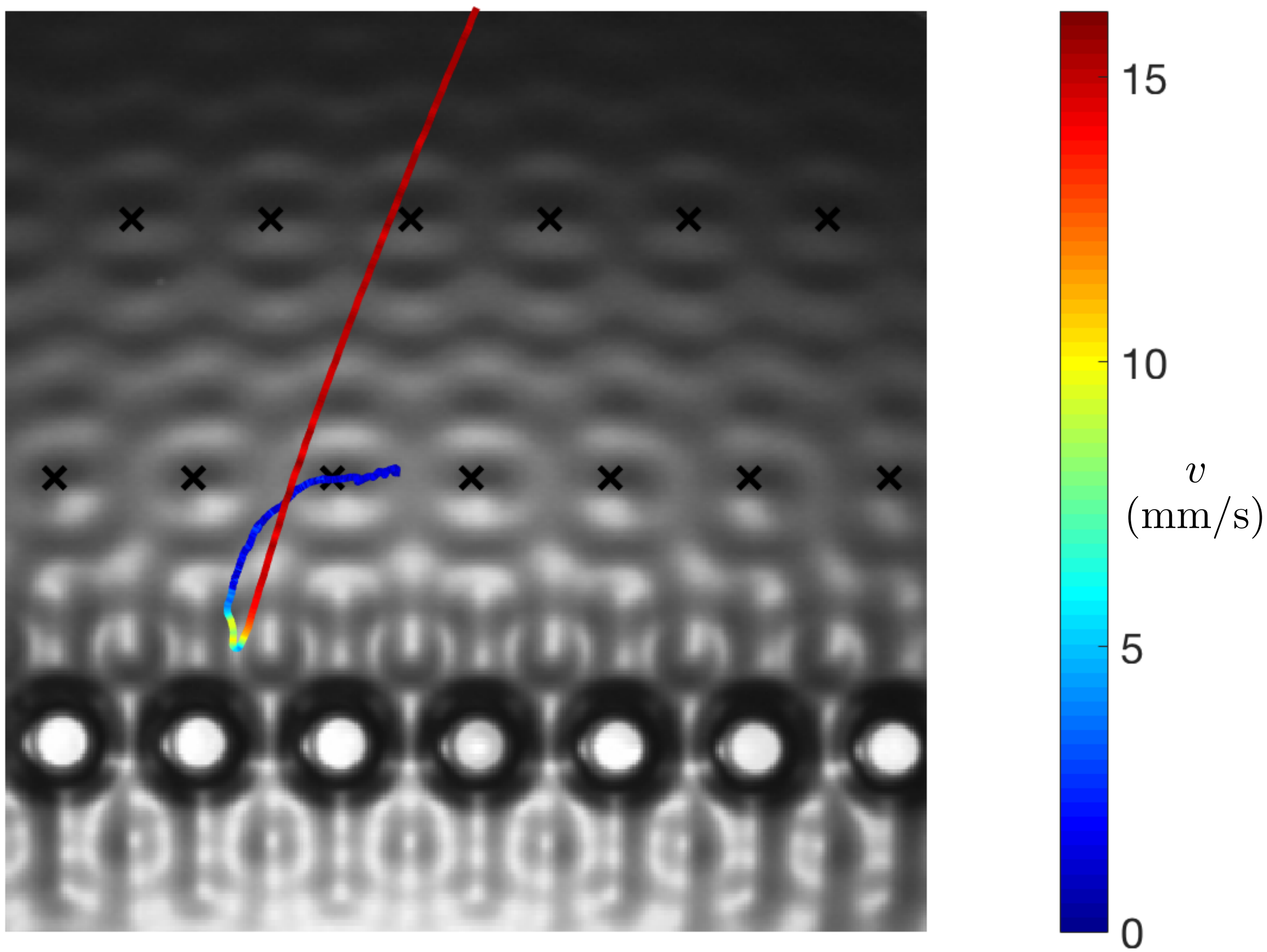


Pair of bouncers trapped between Talbot images



Array of bouncers drifting to row of images.

Trapping of a fast walker above the Faraday threshold



- a hydrodynamic analog of particle trapping with the Talbot effect
- trapping accompanied by disruption of vertical dynamics: evidence of ponderomotive effects?

Hydrodynamic analog of superradiance

PHYSICAL REVIEW LETTERS **130**, 064002 (2023)

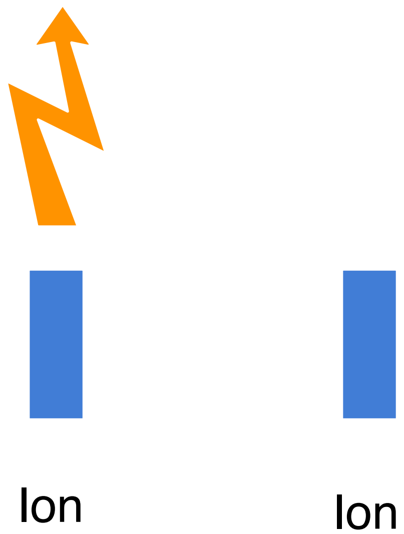
Featured in Physics

Superradiant Droplet Emission from Parametrically Excited Cavities

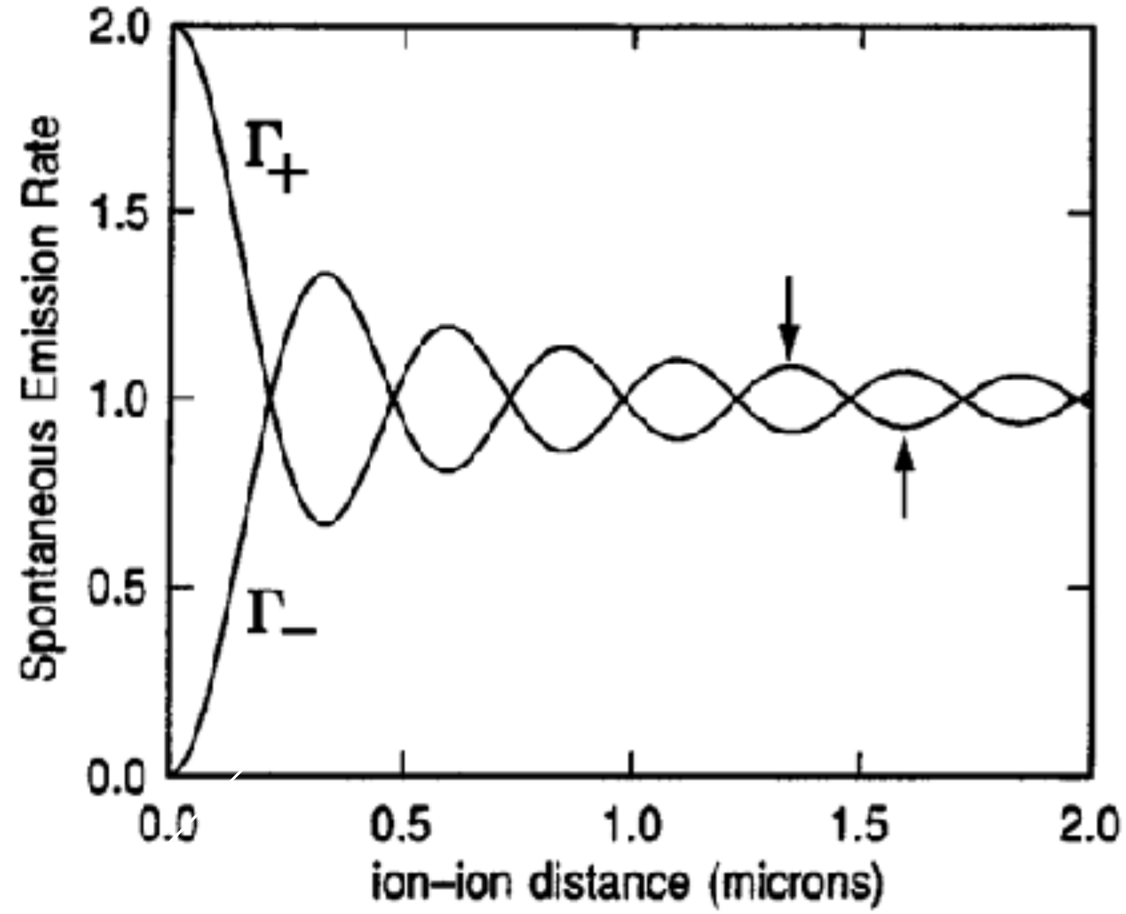
Valeri Frumkin* and John W. M. Bush†
Department of Mathematics, Massachusetts Institute of Technology

Konstantinos Papatryfonos*
Department of Mathematics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
and Gulliver UMR CNRS 7083, ESPCI Paris, Université PSL, 75005 Paris, France

Quantum superradiance and subradiance

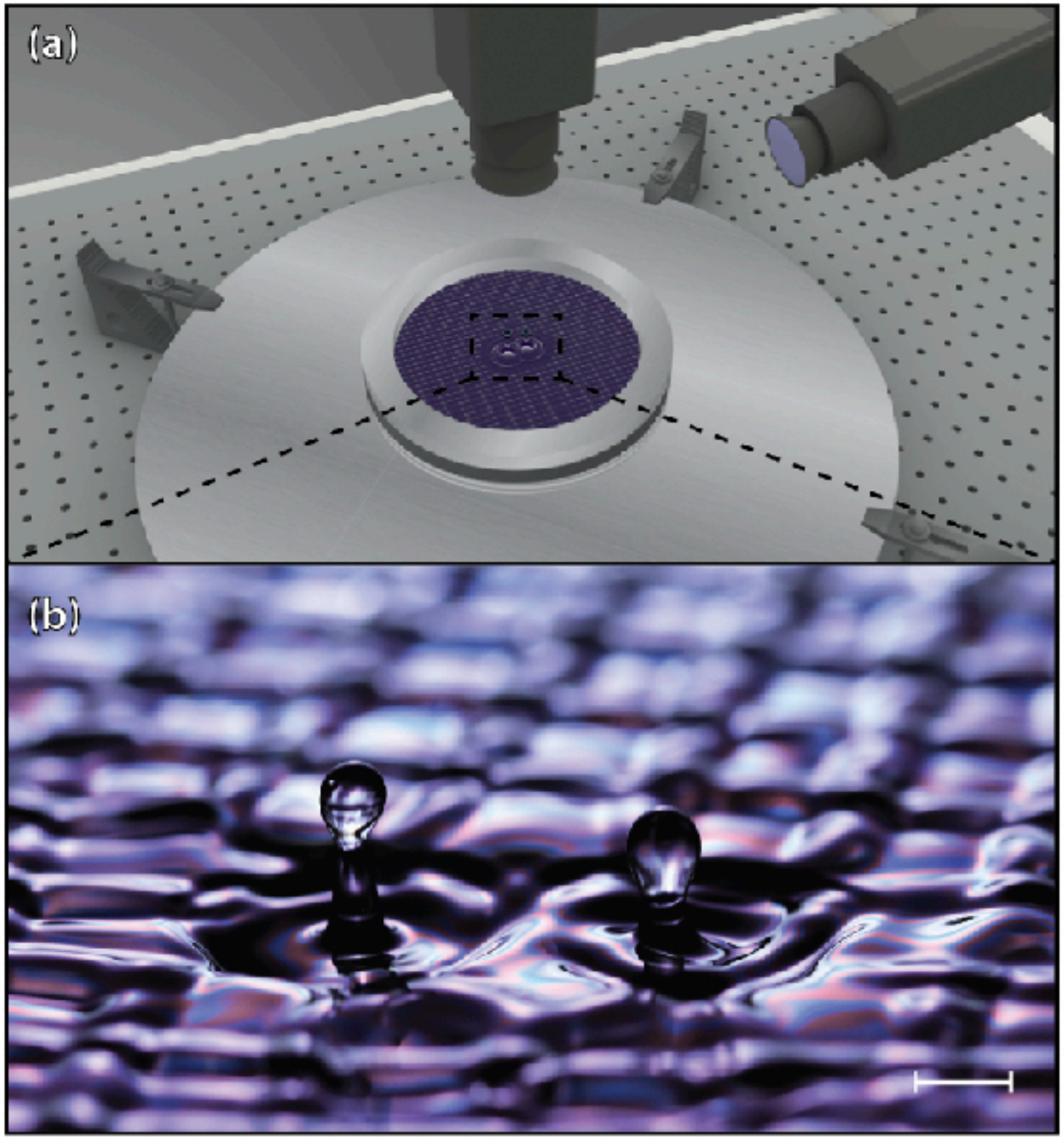


- *Dicke (1954); DeVoe & Brewer (1996)*



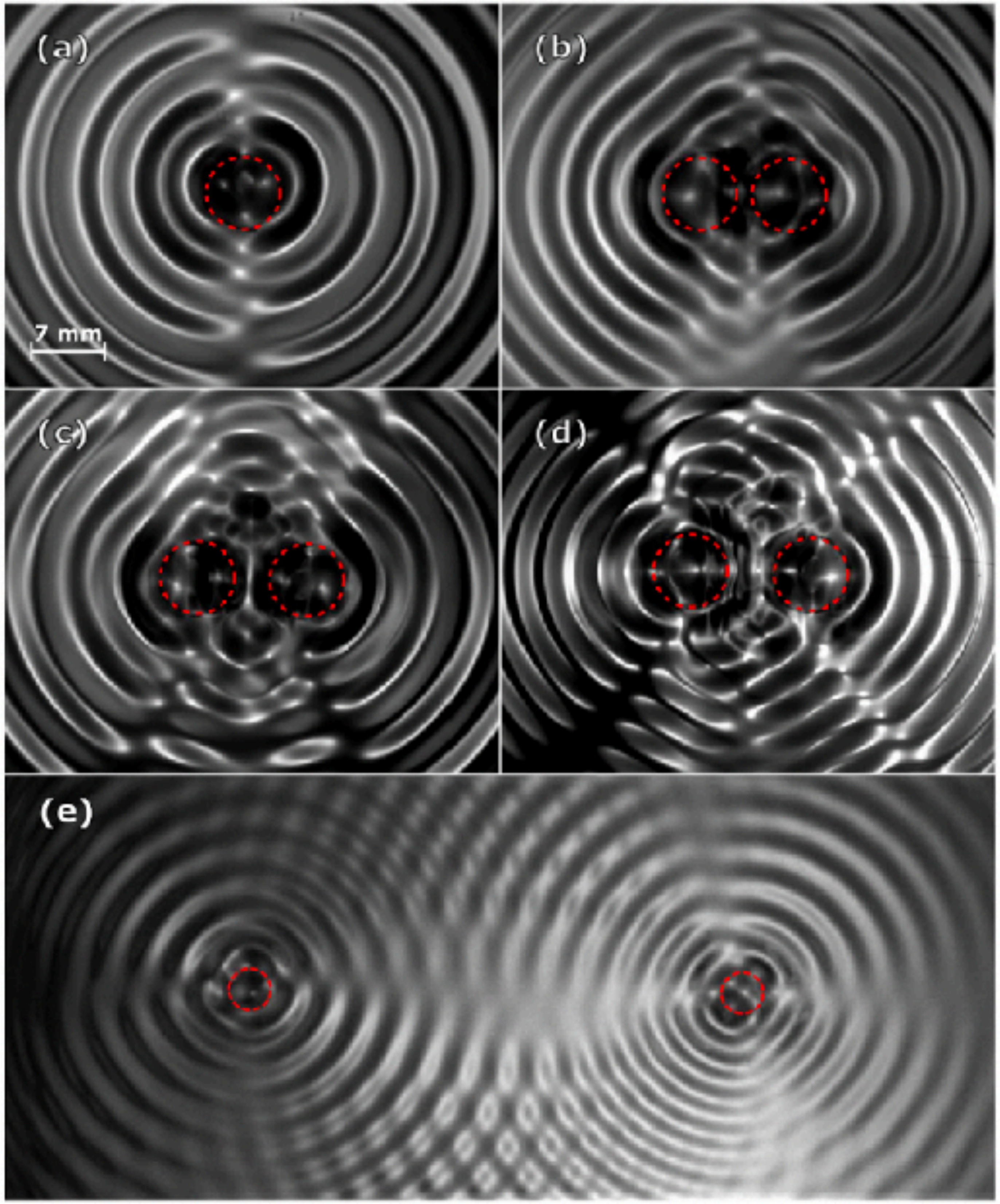
- the emission rate of photons from a pair of ions is enhanced or diminished relative to that of two isolated ions
- anomalous emission rate varies sinusoidally with distance between ions
- the mechanism responsible for the anomalous emission is unclear
- the phenomenon is often taken to be a manifestation of quantum entanglement

Hydrodynamic analog of superradiance

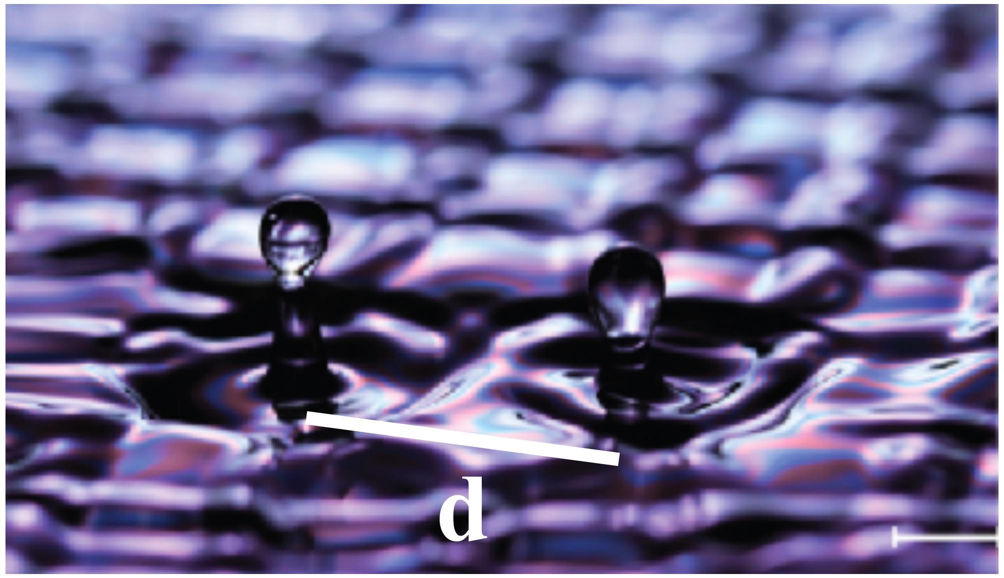


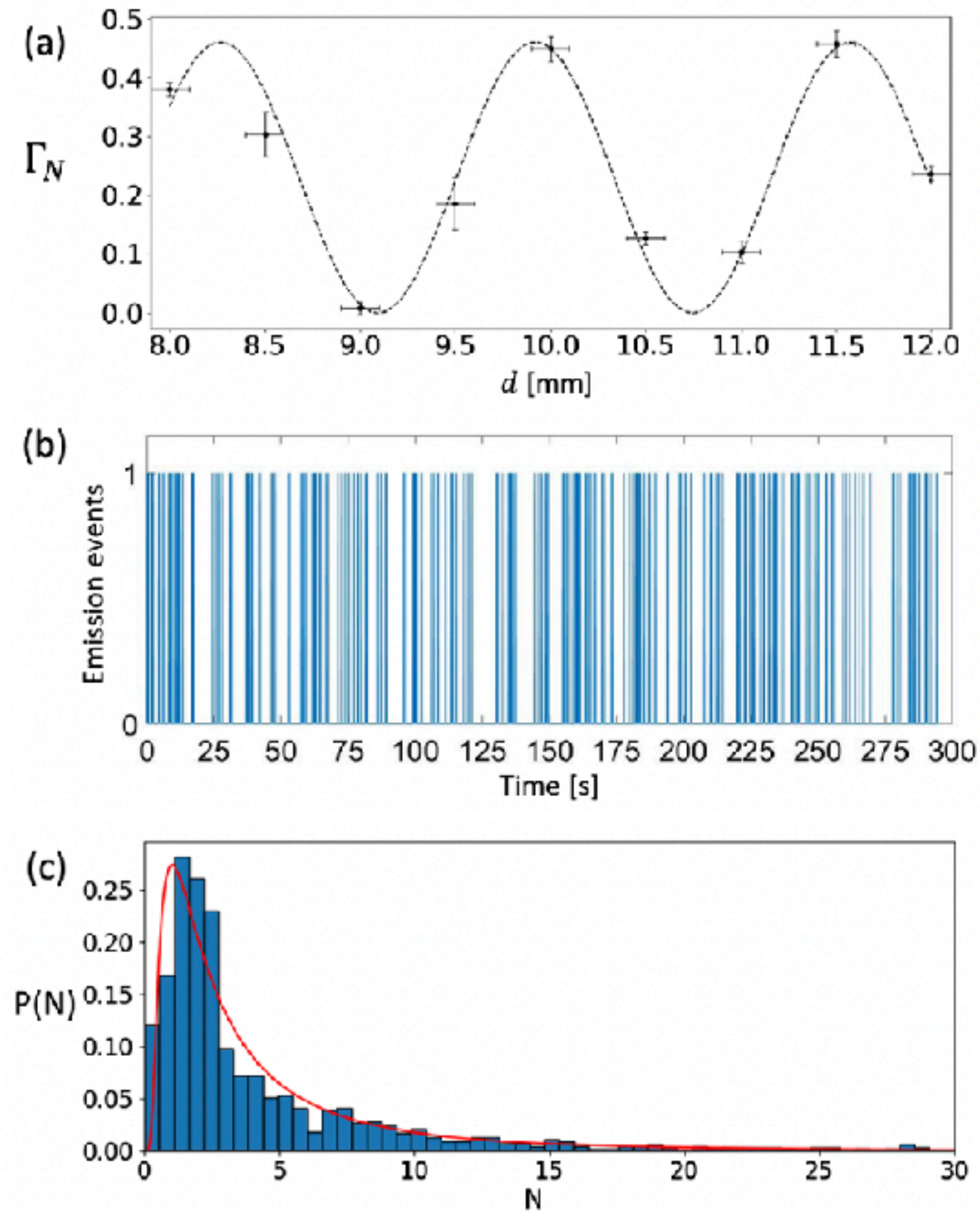
- consider droplet emission from a pair of deep regions (cavities) via interfacial fracture of a vibrating bath

Hydrodynamic analog of superradiance



Hydrodynamic analog of superradiance





- emission rate enhanced by presence of neighboring cavity, varies sinusoidally with distance between cavities
- superradiant droplet emission may be rationalized in terms of wave-mediated interactions between cavities
- the first HQA involving particle creation via interfacial fracture

Conclusions

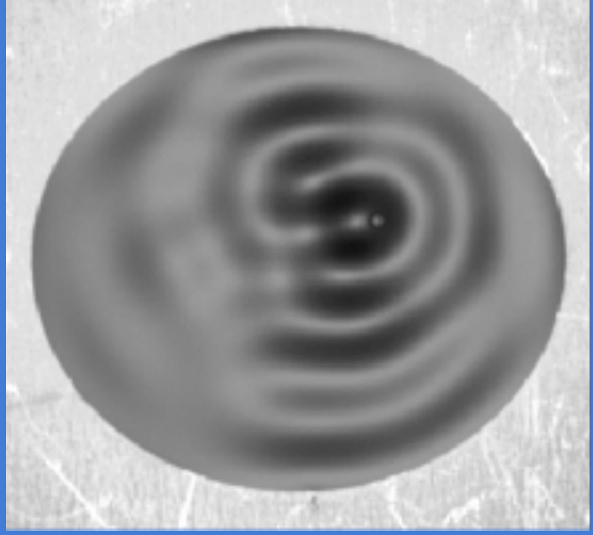
Crossing the threshold

- allows for tuning of relative magnitudes of self-generated and ambient waves
- allows for a number of new droplet behaviors: meandering, zigzagging, trapping, Brownian motion
- allows for further hydrodynamic analogs of EM systems (e.g. Talbot trapping)

Questions raised

- is there a parameter regime in which the diffusion is anomalous, quantum-like?
- what new quantum/EM analogs might be achievable above threshold?
- how can we model such effects theoretically?

`Closed' pilot-wave systems

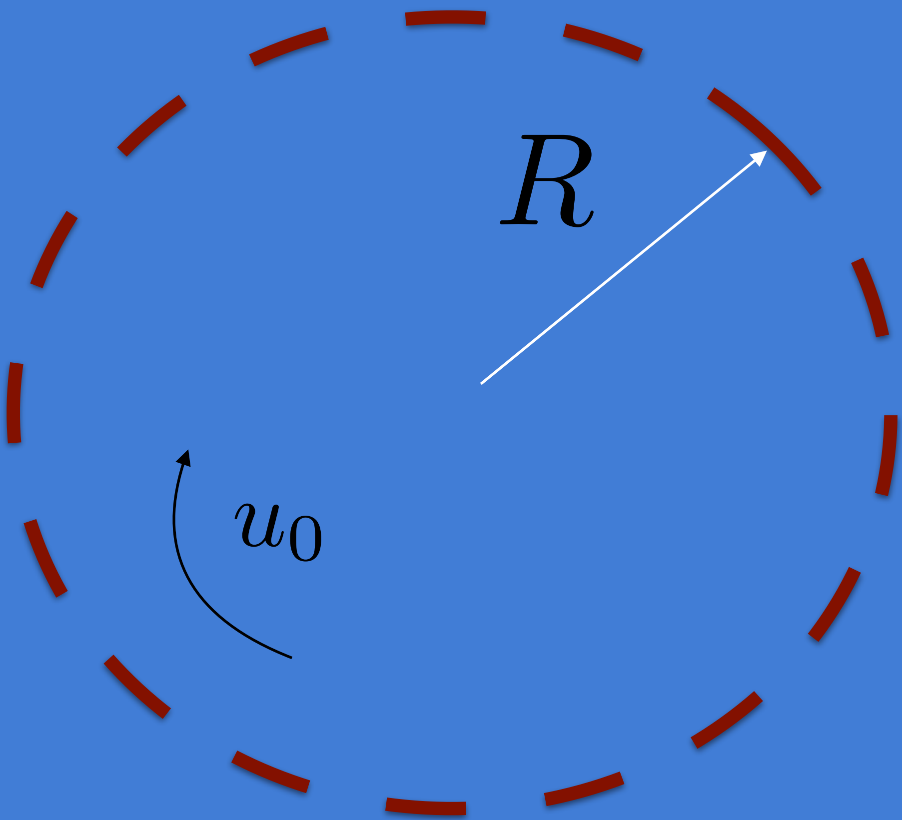


- walker motion confined by either boundaries or applied force.

Requirement for quantum-like behavior:

$$T_M > u_0/R$$

MEMORY TIME — CROSSING/ORBITAL TIME



→ waves persist beyond characteristic crossing/orbital time

→ system is effectively `***closed***' and `***above threshold***'

How do we model walkers in this regime?

Droplet tunneling

Eddi et al. (2009)

- probability of tunneling decreases with wall width and distance from threshold
- tunneling requires proximity to Faraday threshold, pronounced waves

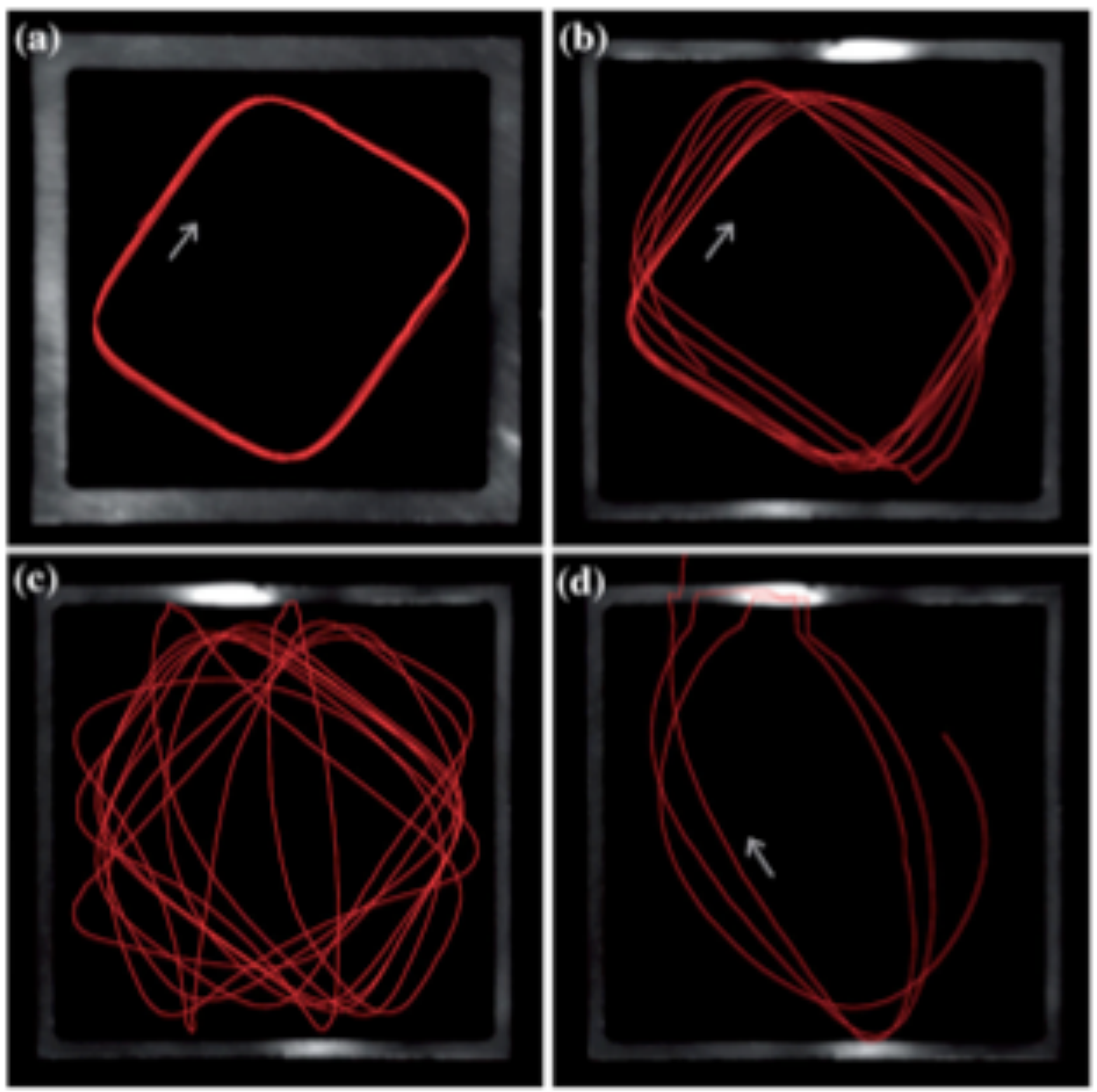
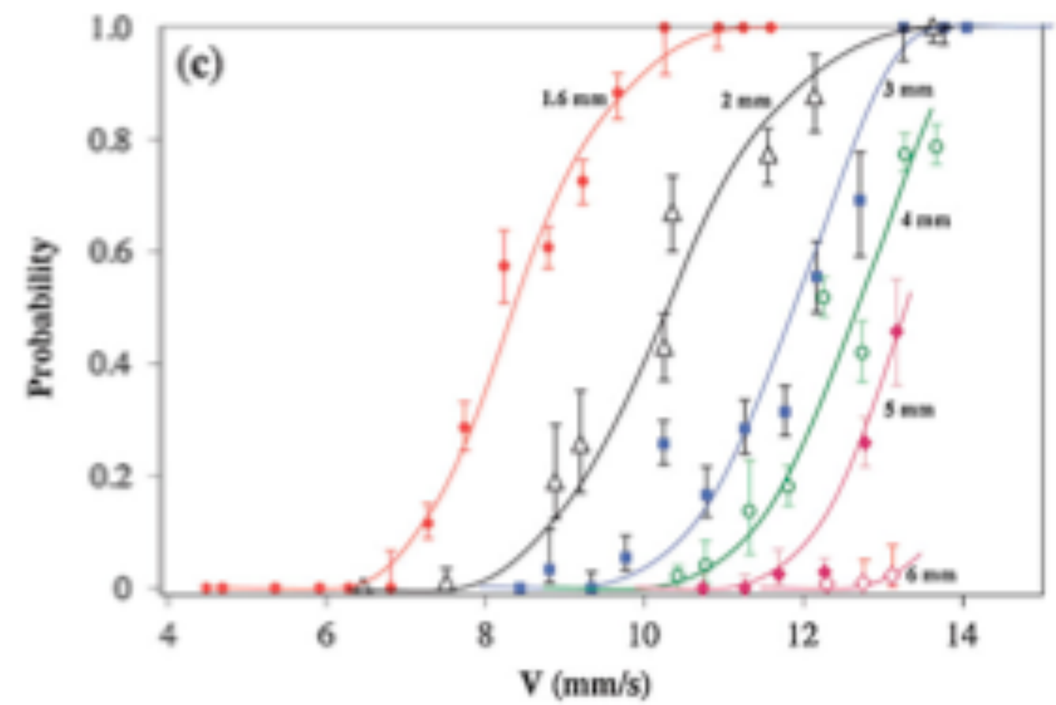
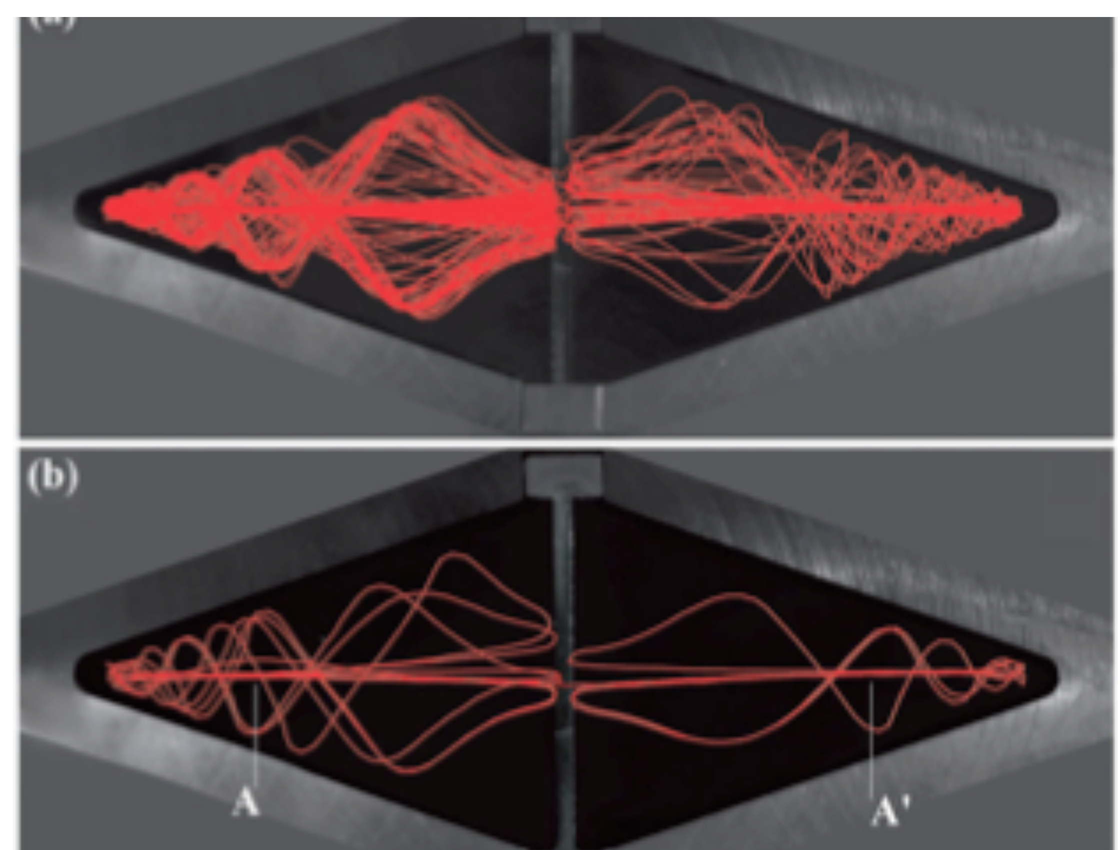
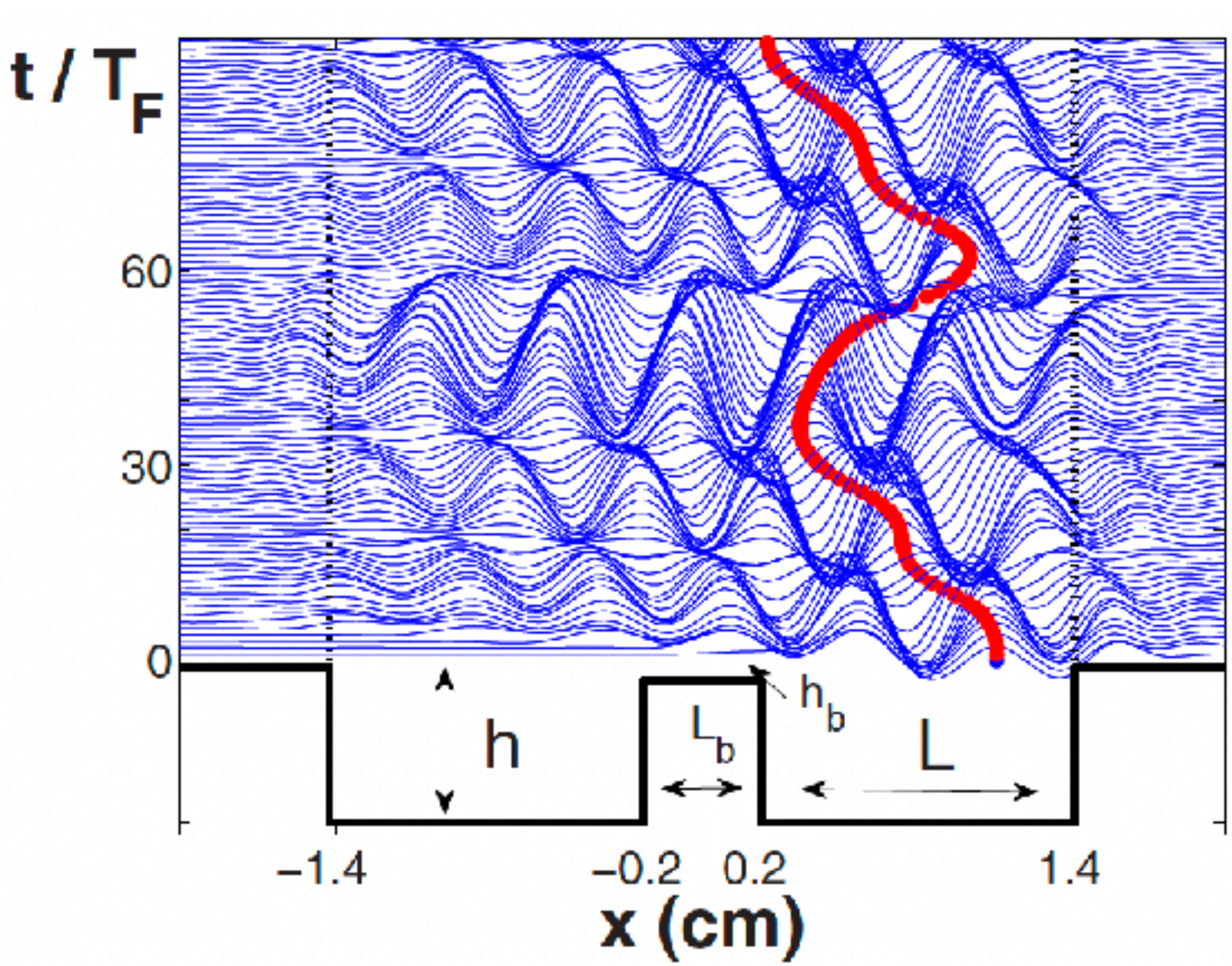


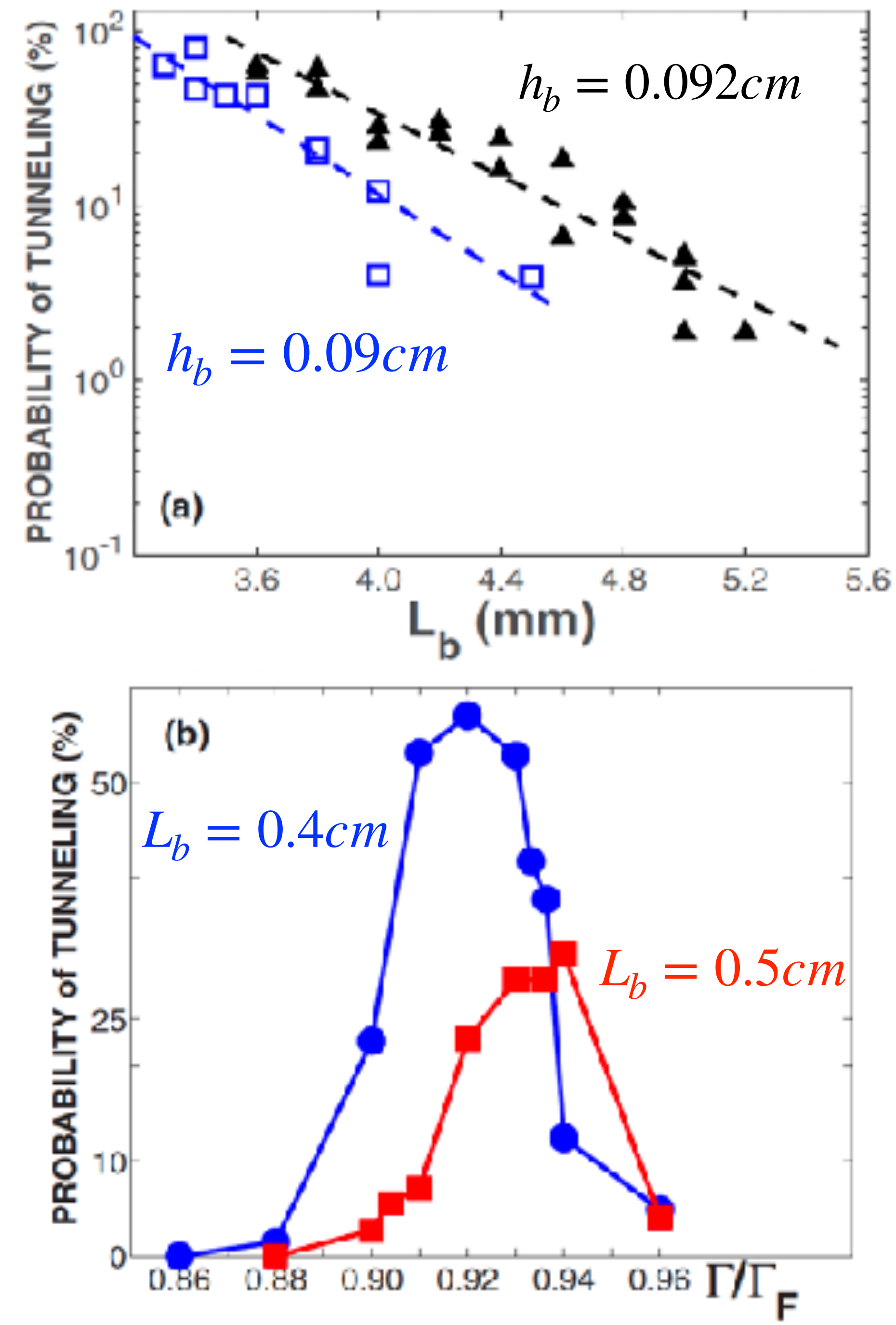
FIG. 4 (color online). The recorded trajectories of the walker inside the square trap of side $L = 55$ mm. In (a) $e = 4.5$ mm and $V = 9.95$ mm/s. In (b) $e = 2.5$ mm and $V = 9$ mm/s. The probability of escape P is of the order of 1%. In (c) $e = 2.5$ mm and $V = 11.8$ mm/s. $P \simeq 10\%$. In (d) $e = 2.5$ mm and $V = 13.2$ mm/s. $P \simeq 30\%$.





Tunneling: Numerics with André's model





- drop-wave-barrier interaction is chaotic, leading to lack of predictability
- tunneling probability decreases exponentially with barrier width, as in QM

Tunneling

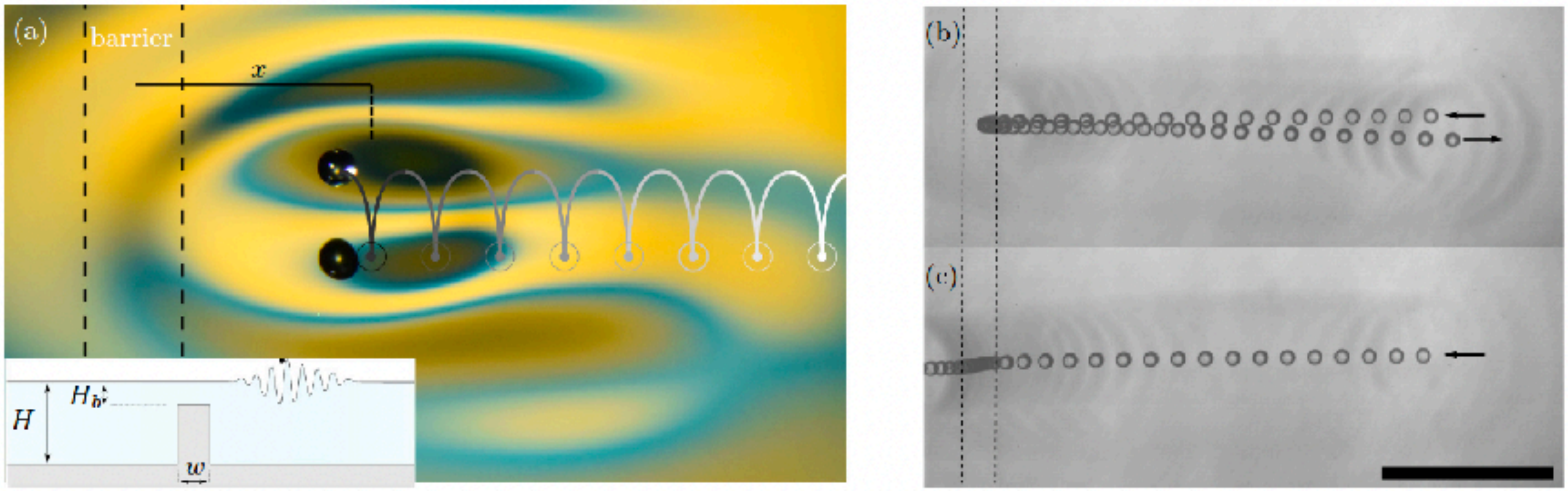
PHYSICAL REVIEW E **102**, 013104 (2020)

Predictability in a hydrodynamic pilot-wave system: Resolution of walker tunneling

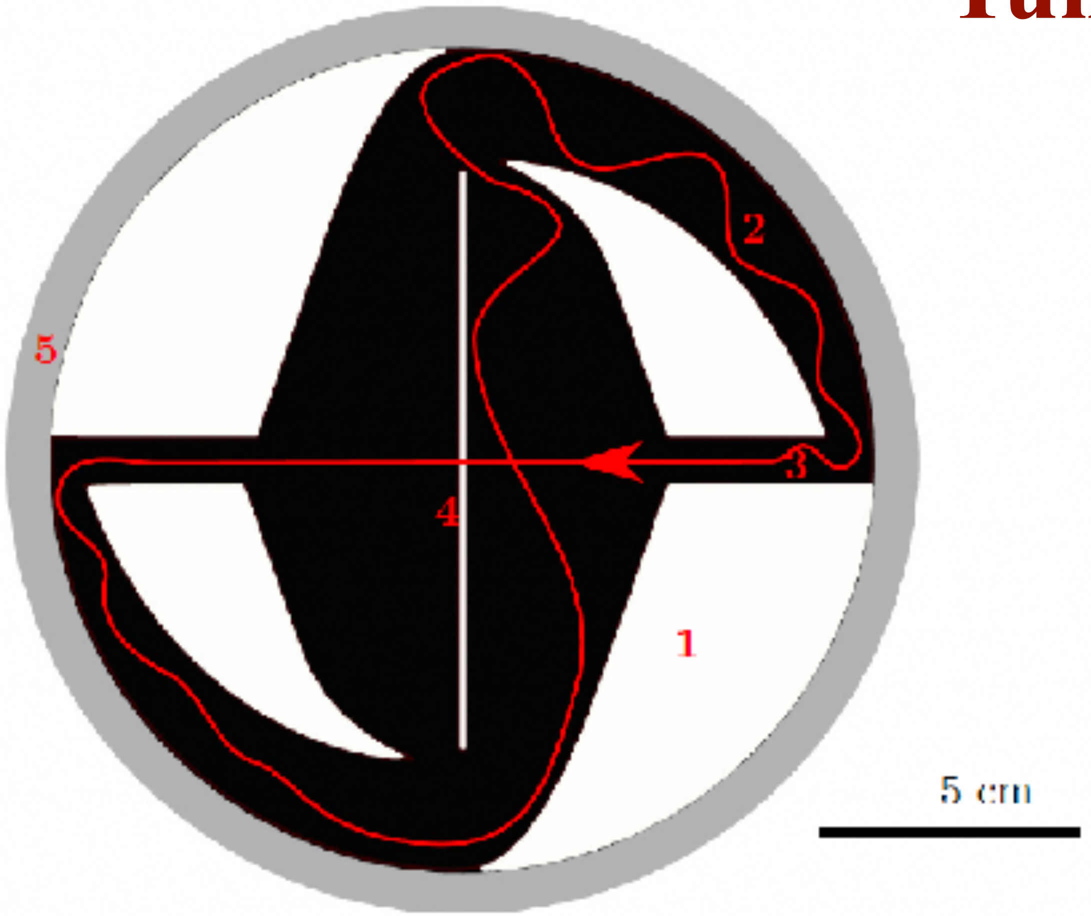
Loïc Tadrist* and Tristan Gilet
Microfluidics Lab, Aerospace and Mechanical Engineering, University of Liege, Allée de la découverte 9, 4000 Liège, Belgium

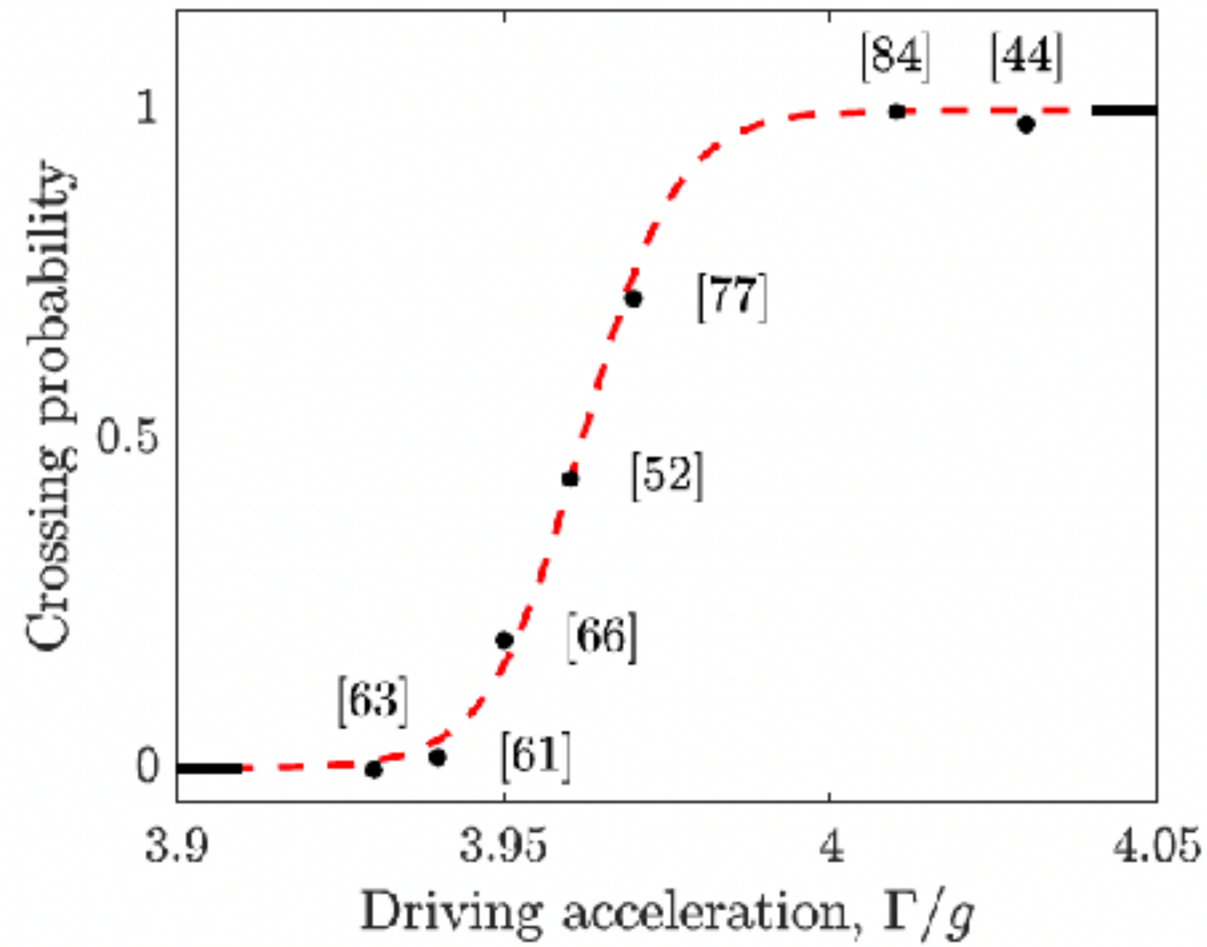
Peter Schlagheck
IPNAS, CESAM research unit, University of Liege, Allée du 6 Août 15, 4000 Liège, Belgium

John W. M. Bush
Department of Mathematics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

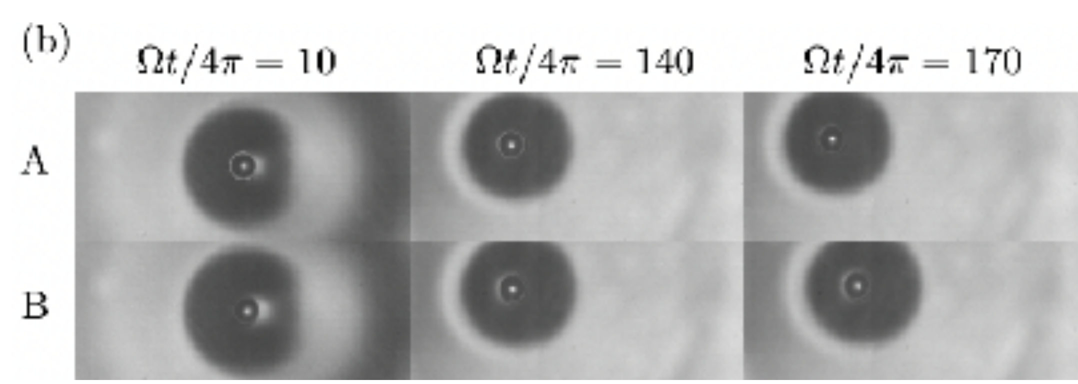


Tunneling





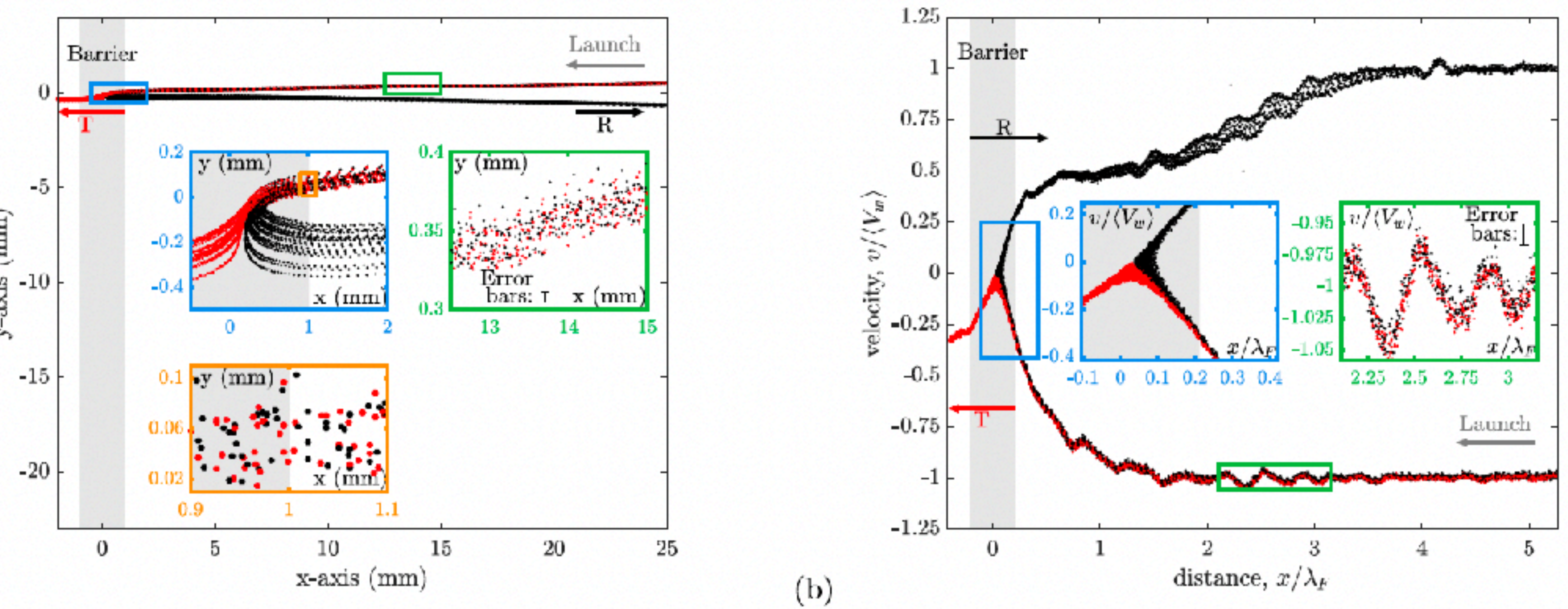
- robust probabilistic behavior as reported in previous studies



Question

- can resolution of the fast timescale render the theory deterministic?
- characterized the footprints of the walker

Tunneling



- tunneling is not rendered predictable through resolution of the fast timescale
- drop-wave-barrier interaction is chaotic, leading to lack of predictability