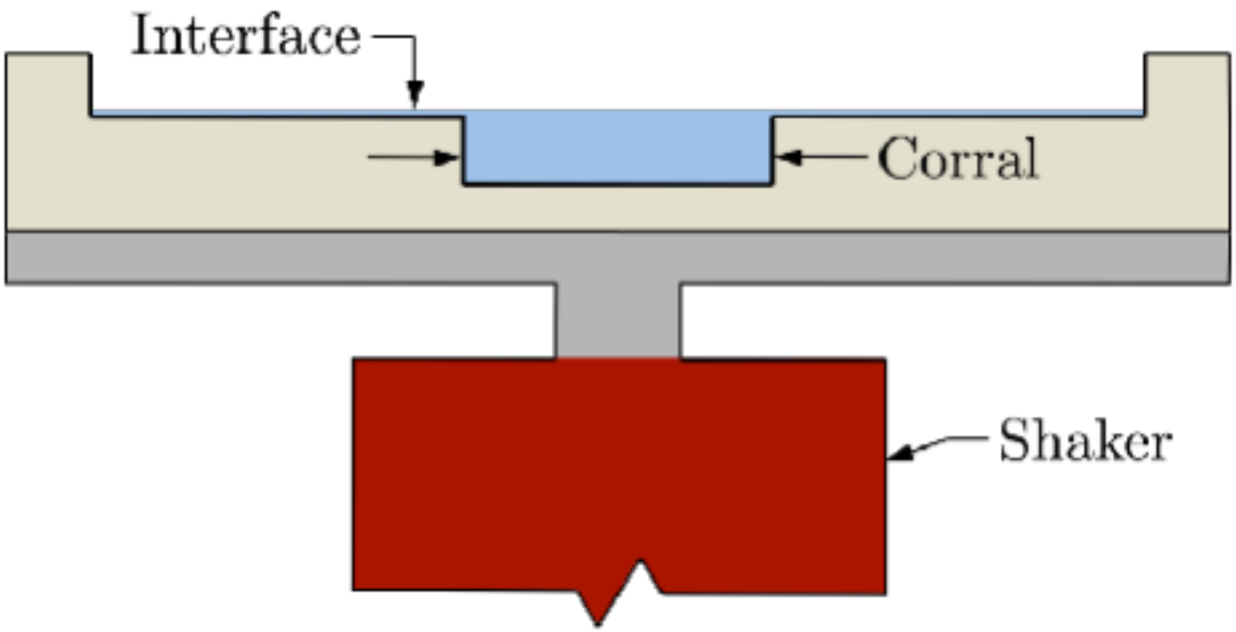
HQA Lecture 19

The hydrodynamic corral

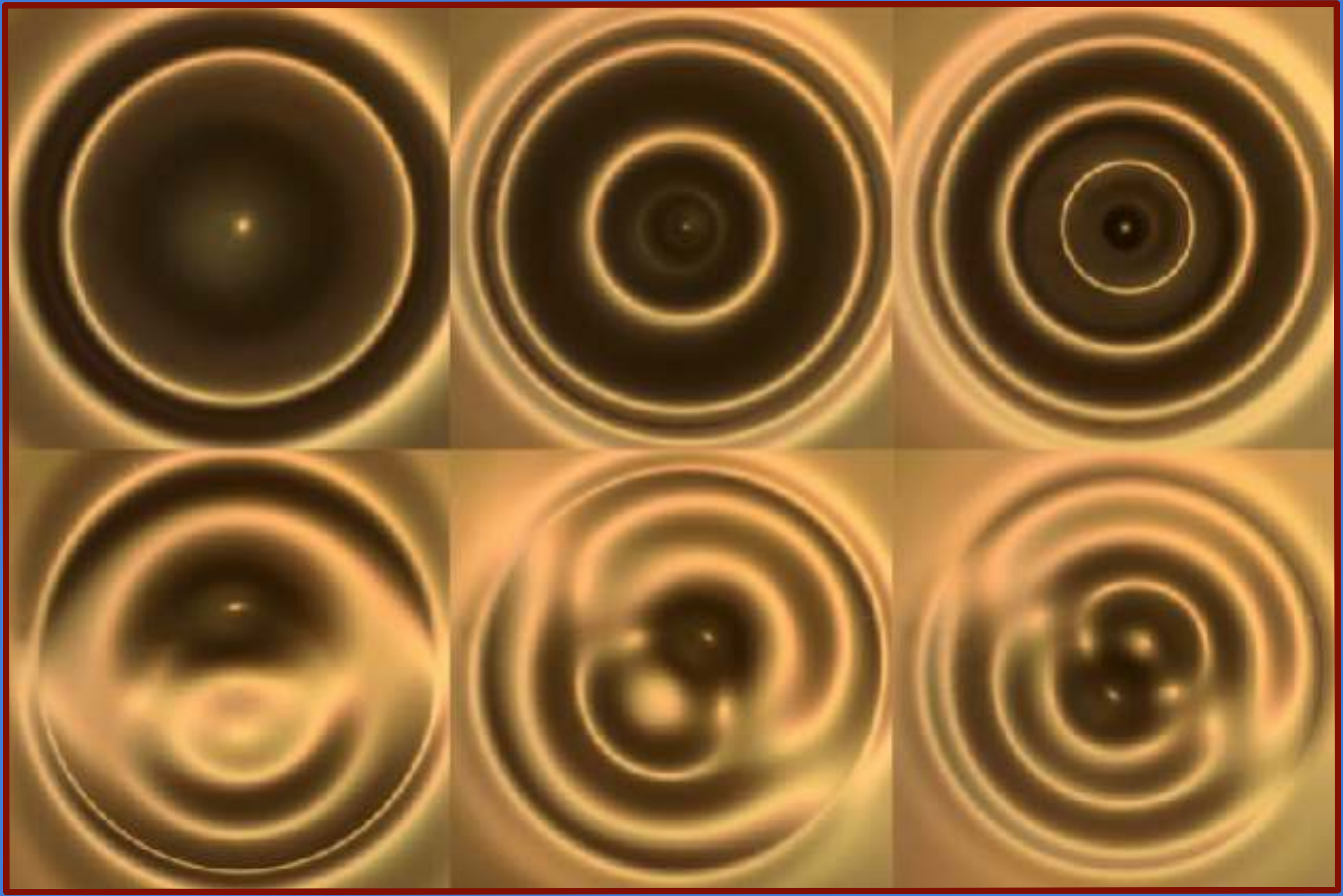
The fluid corral experiments

- corral depth of 6mm, radius 1-3cm; driving frequency 40-70Hz



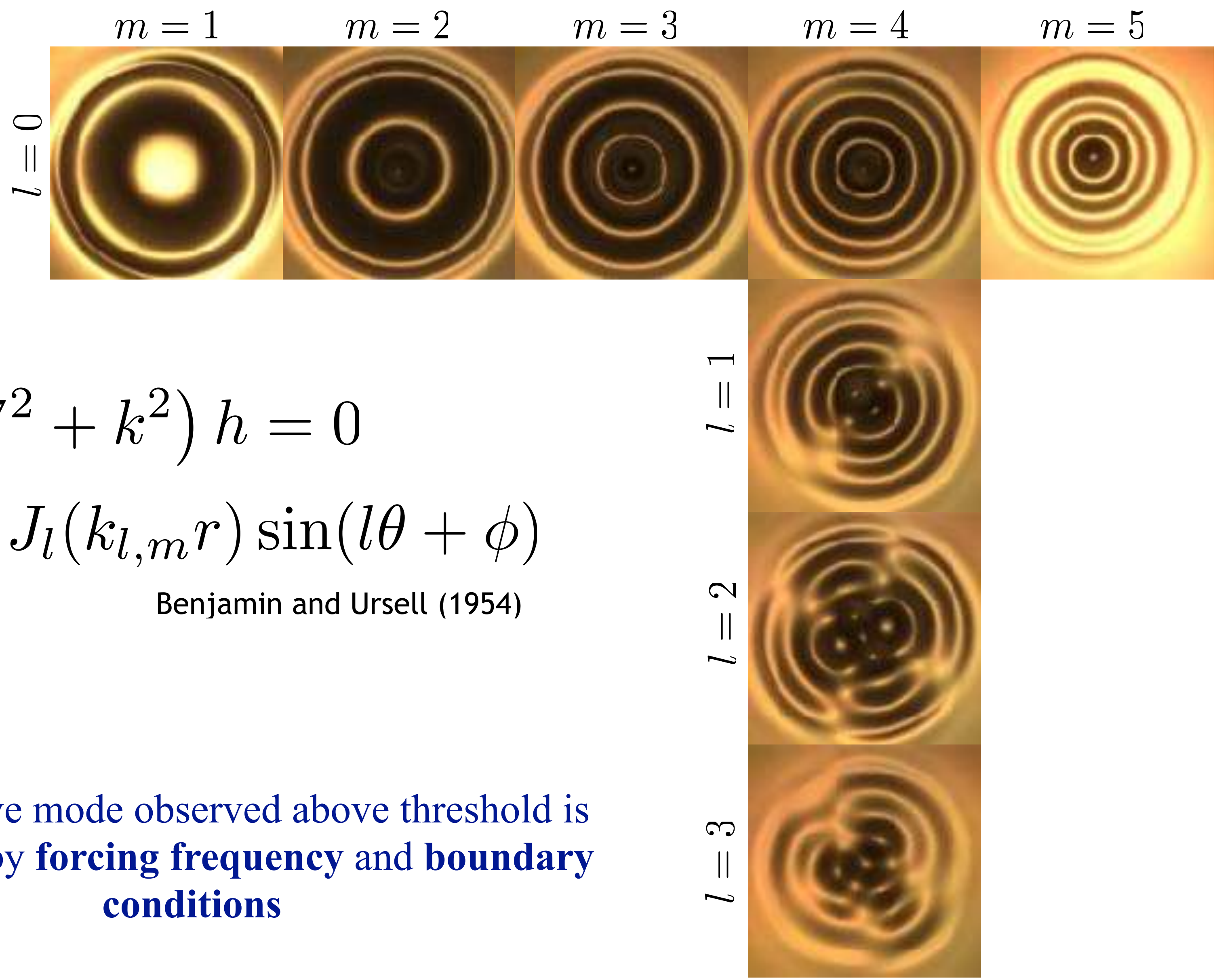
Droplet can only walk in the deep central region, thus remains confined within the “corral”

Faraday wave modes in a circular corral



- different modes selected at different driving frequencies

Circular Faraday modes



$$\left(\nabla^2 + k^2\right) h = 0$$

$$h_{l,m} = J_l(k_{l,m} r) \sin(l\theta + \phi)$$

Benjamin and Ursell (1954)

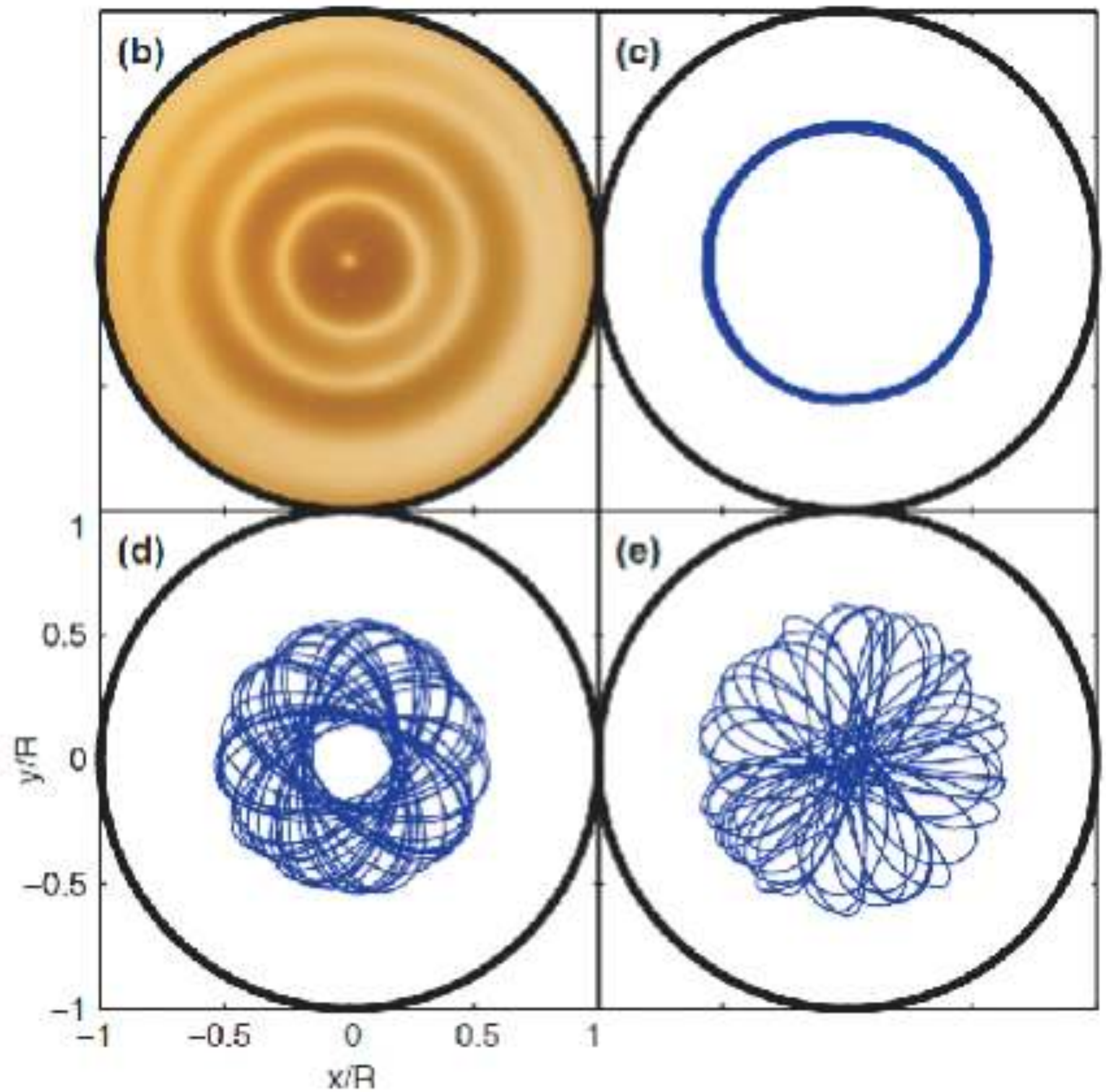
Faraday wave mode observed above threshold is determined by **forcing frequency** and **boundary conditions**

Small corrals

- walls play role of confining central force; periodic/quasiperiodic orbits prevalent

R = 10.1 mm

Wave mode



$\gamma/\gamma_F = 0.82$

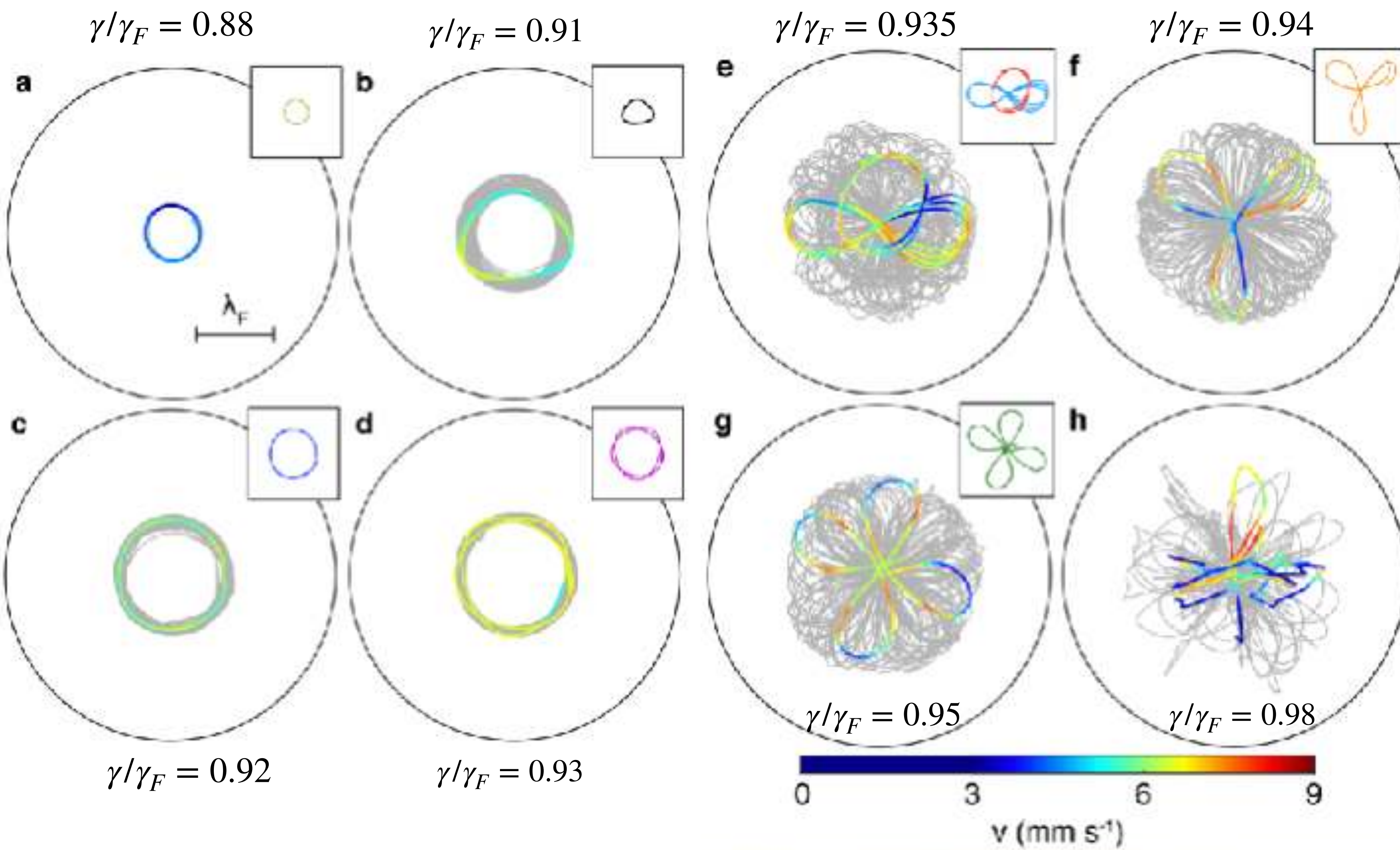
$\gamma/\gamma_F = 0.9$

$\gamma/\gamma_F = 0.94$

Small corrals

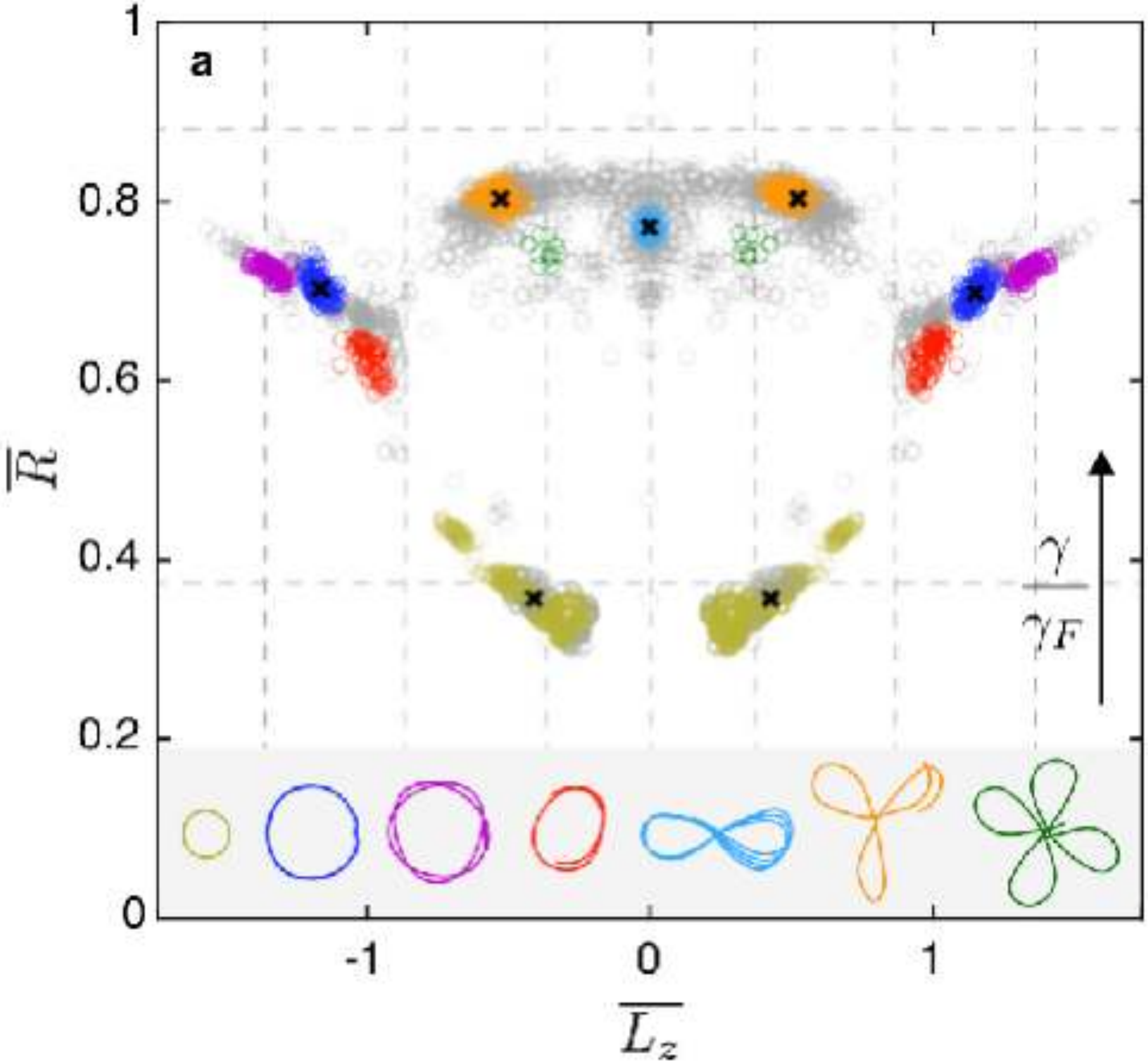
R = 10.1 mm

- walls play role of confining central force; periodic orbits prevalent



Small corrals

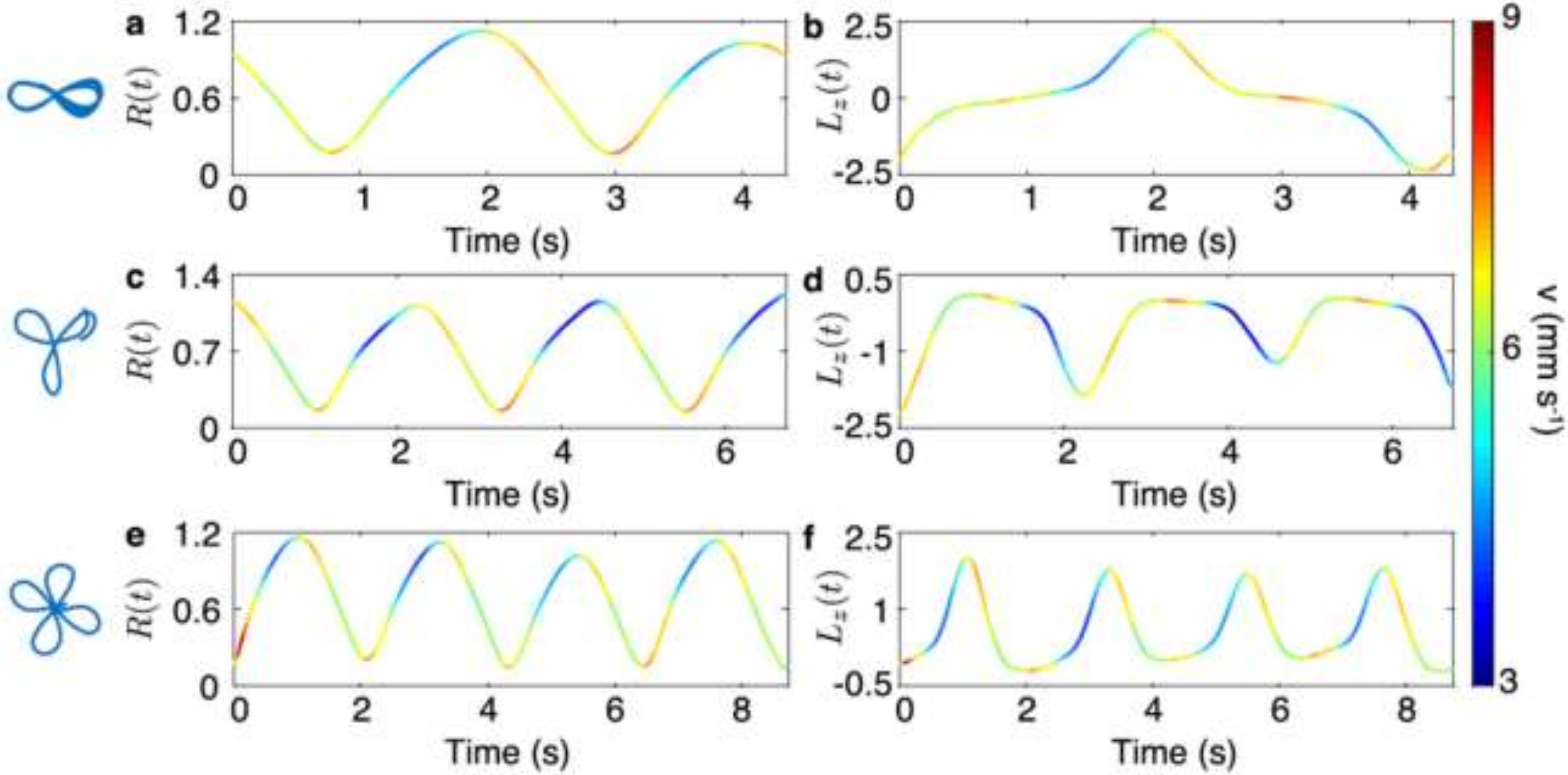
- walls play role of confining central force; periodic orbits prevalent



R = 10.1 mm

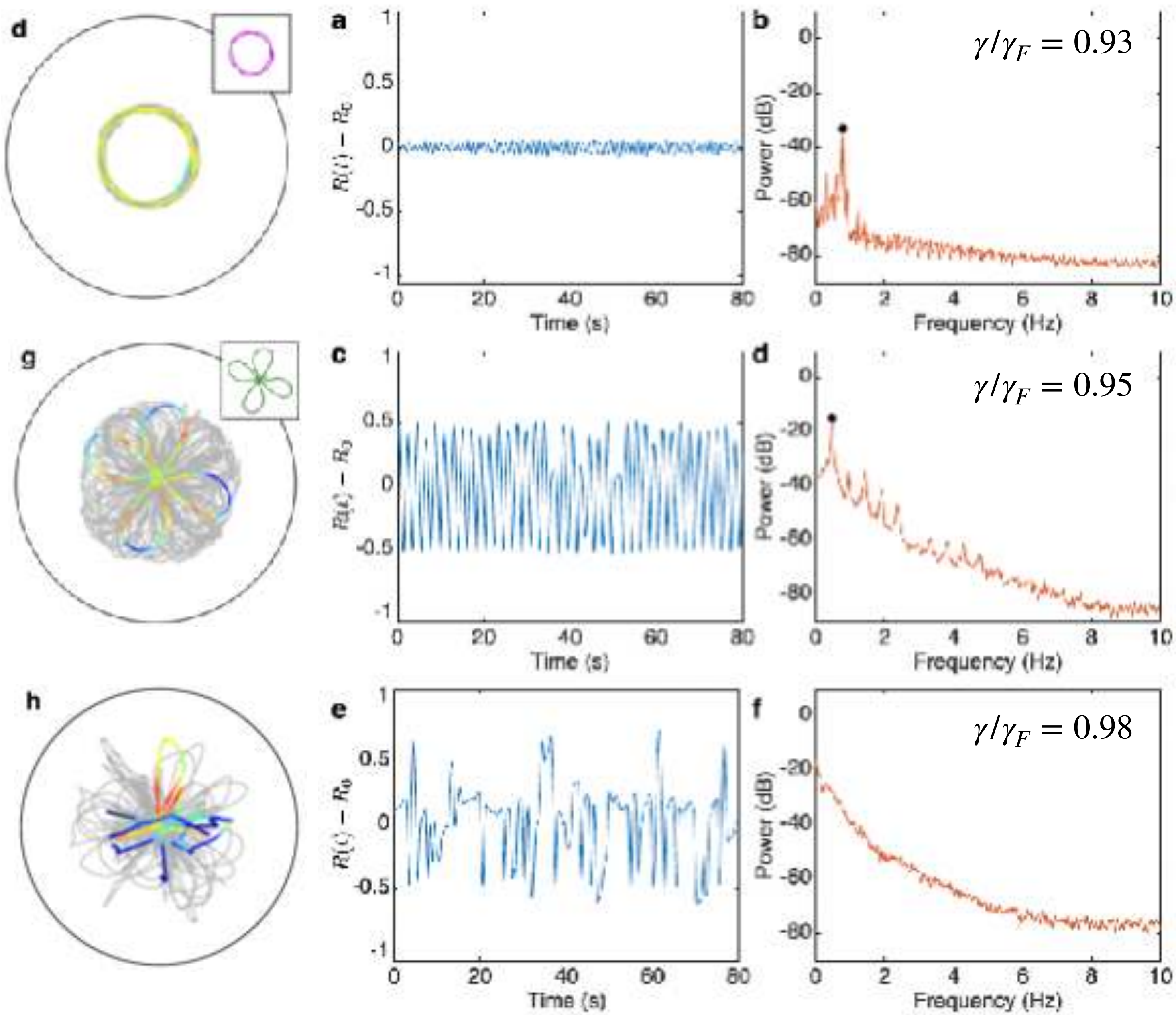
$\gamma/\gamma_F = 0.95$

Small corrals at high Me



- dynamical behavior reminiscent of that of a walker in a SHO

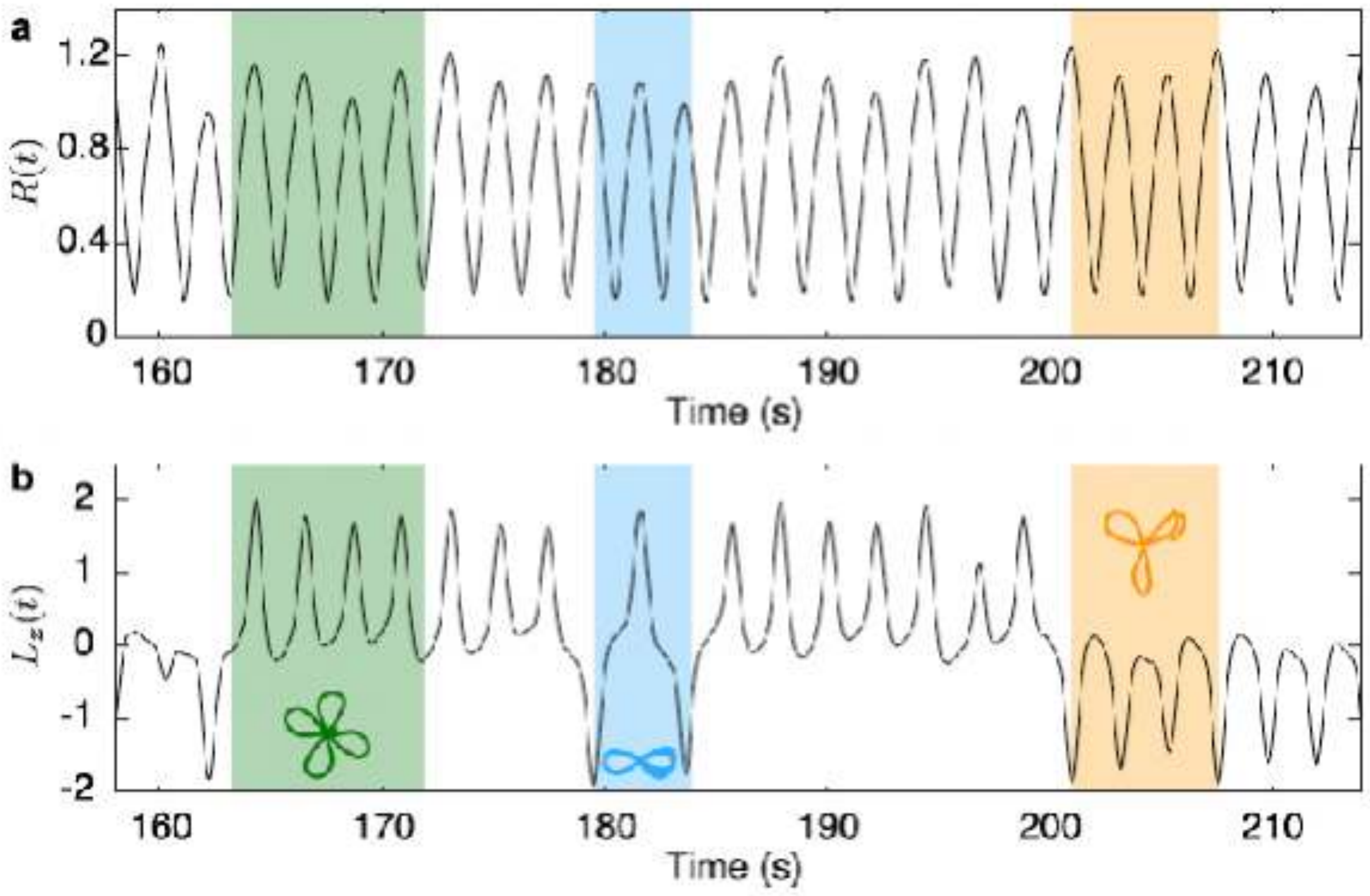
Small corrals at high Me



Small corrals at high Me

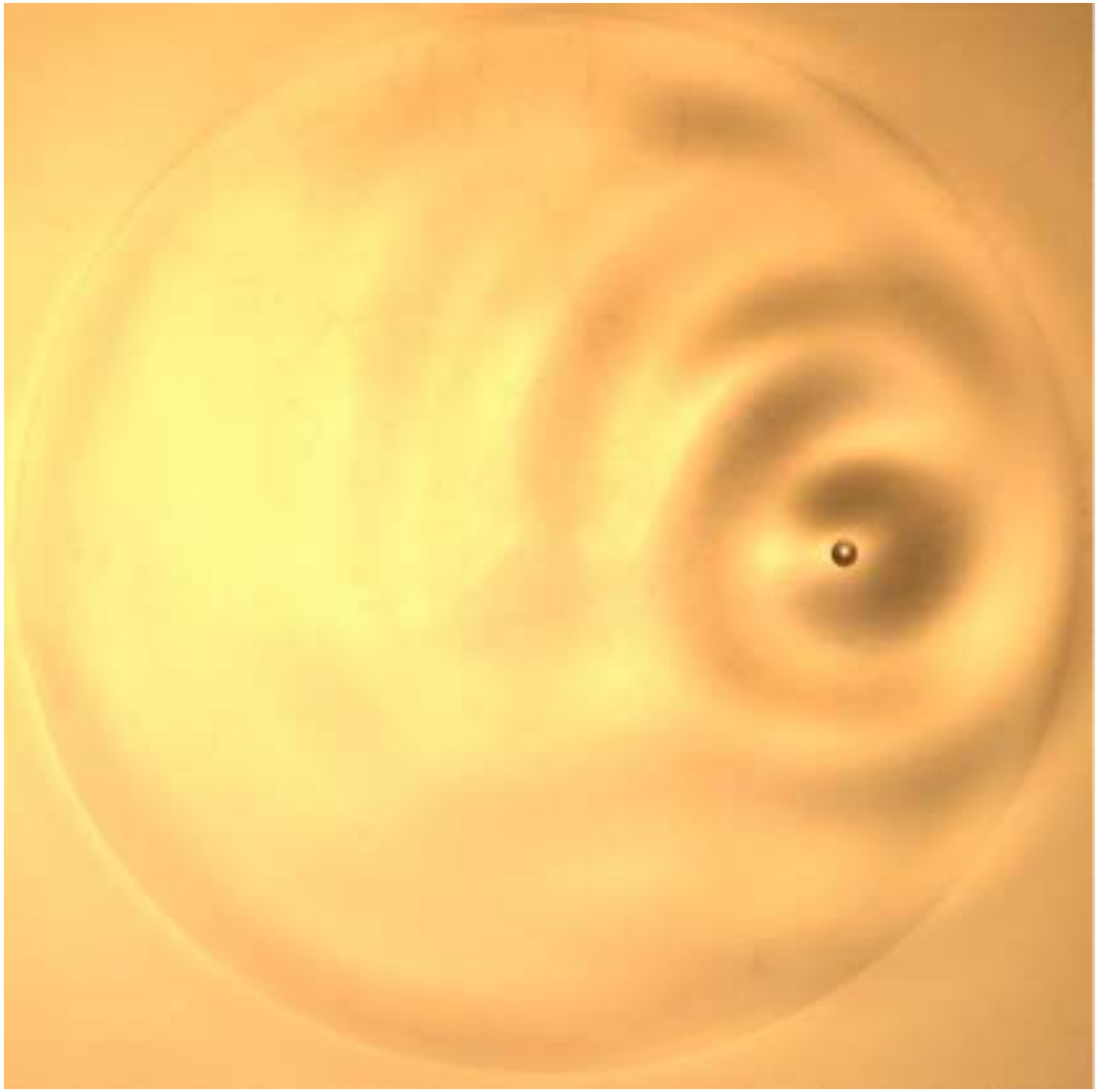
$\gamma/\gamma_F = 0.95$

- intermittent switching between a number of accessible periodic orbits



- statistical behavior reminiscent of that of a walker in a SHO

Walkers in a larger circular corral

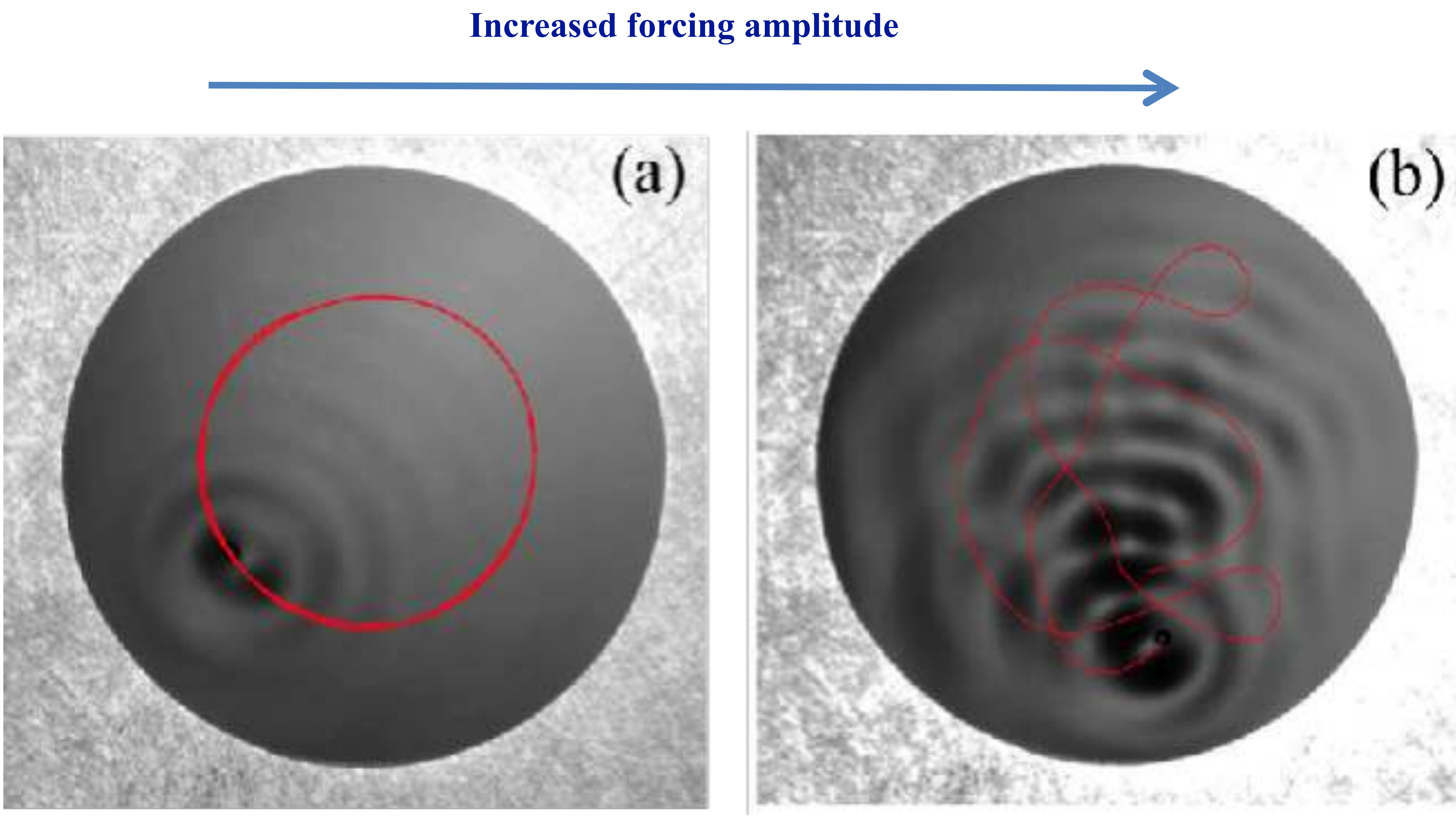


R = 20.2 mm

- the droplet excites and explores the resonant wave field of the cavity

Influence of memory

R = 20.2 mm



- walker motion becomes progressively more irregular with memory

A droplet walking in a circular corral

- the droplet generates and explores its wave field

A droplet surfing in a circular corral

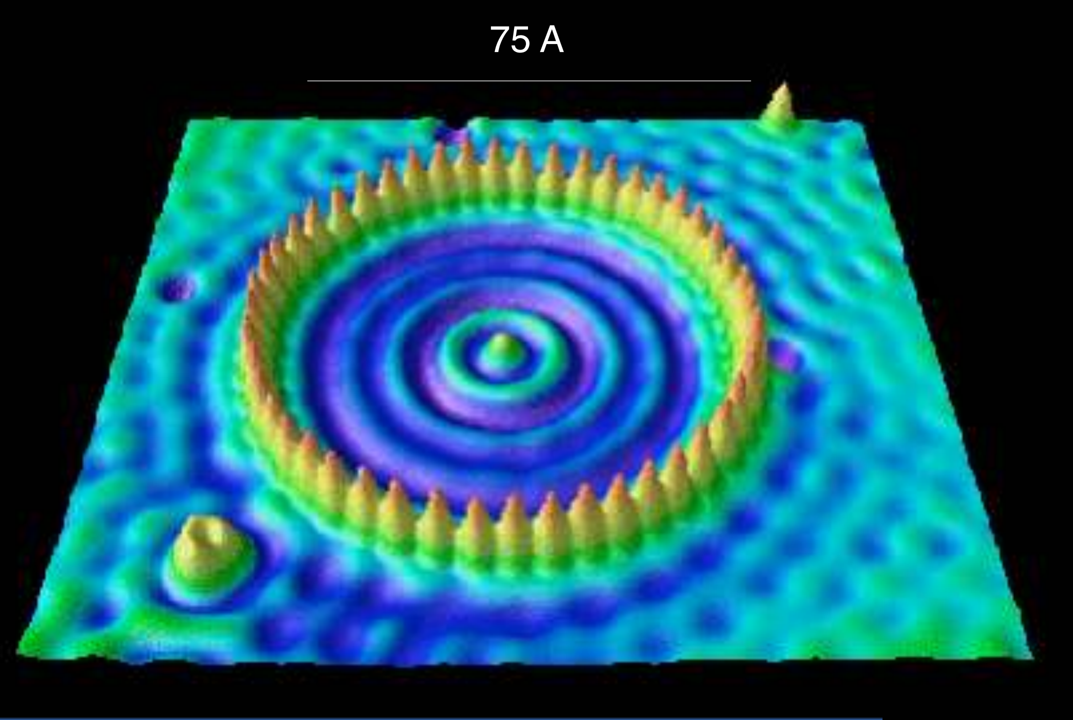
- strobe at the wave (and bouncing) frequency, 70 Hz
- fast bouncing dynamics filtered out

- drop appears to surf along the surface, guided by its pilot wave

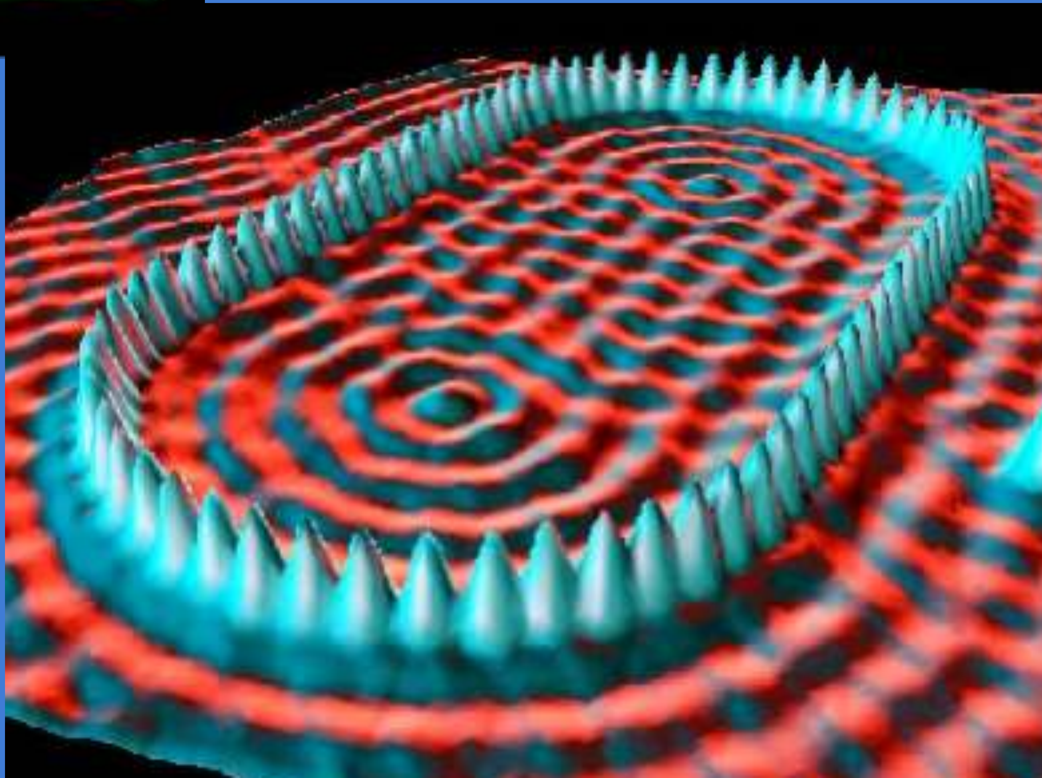
The quantum corral

Crommie, Lutz & Eigler (1993)

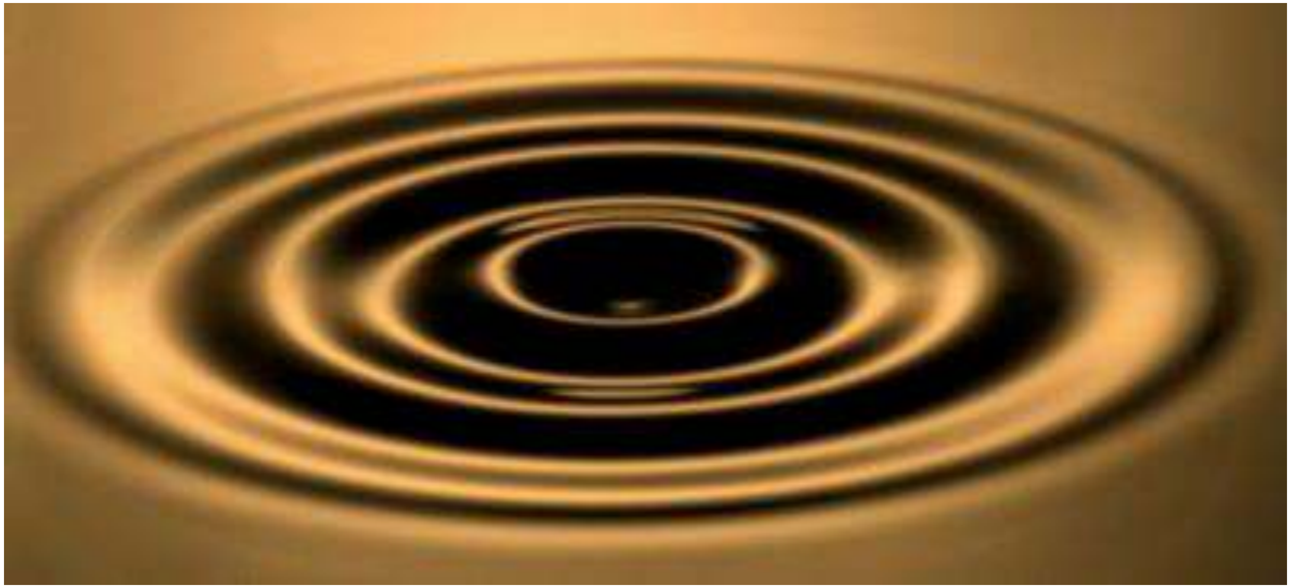
Fiete & Heller (2003)

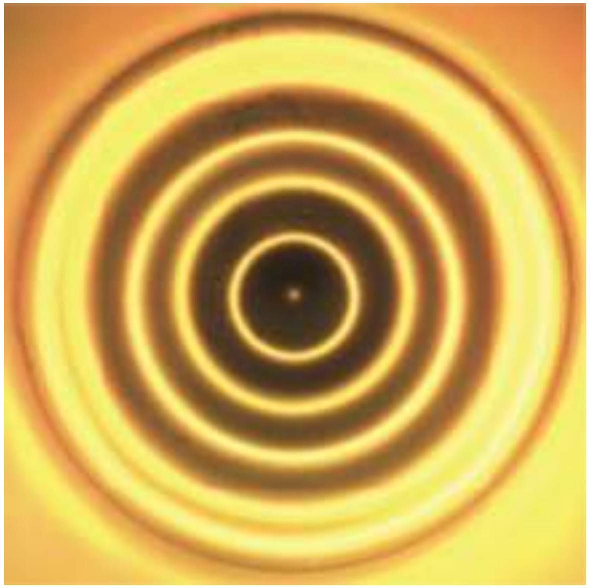


- de Broglie waves evident in the pdf of a sea of electrons trapped on a metal surface, excited by an SEM



Selected Faraday mode

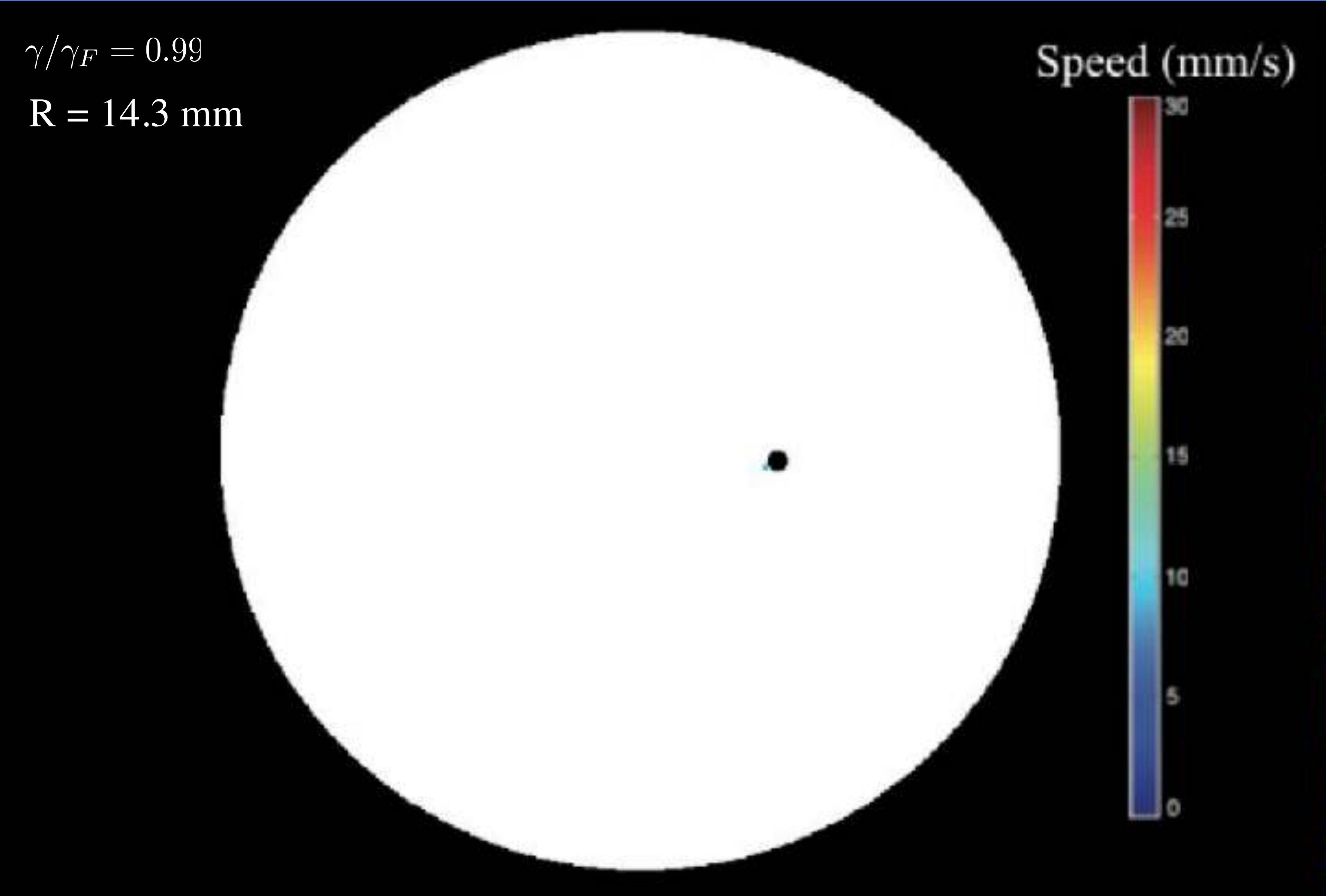




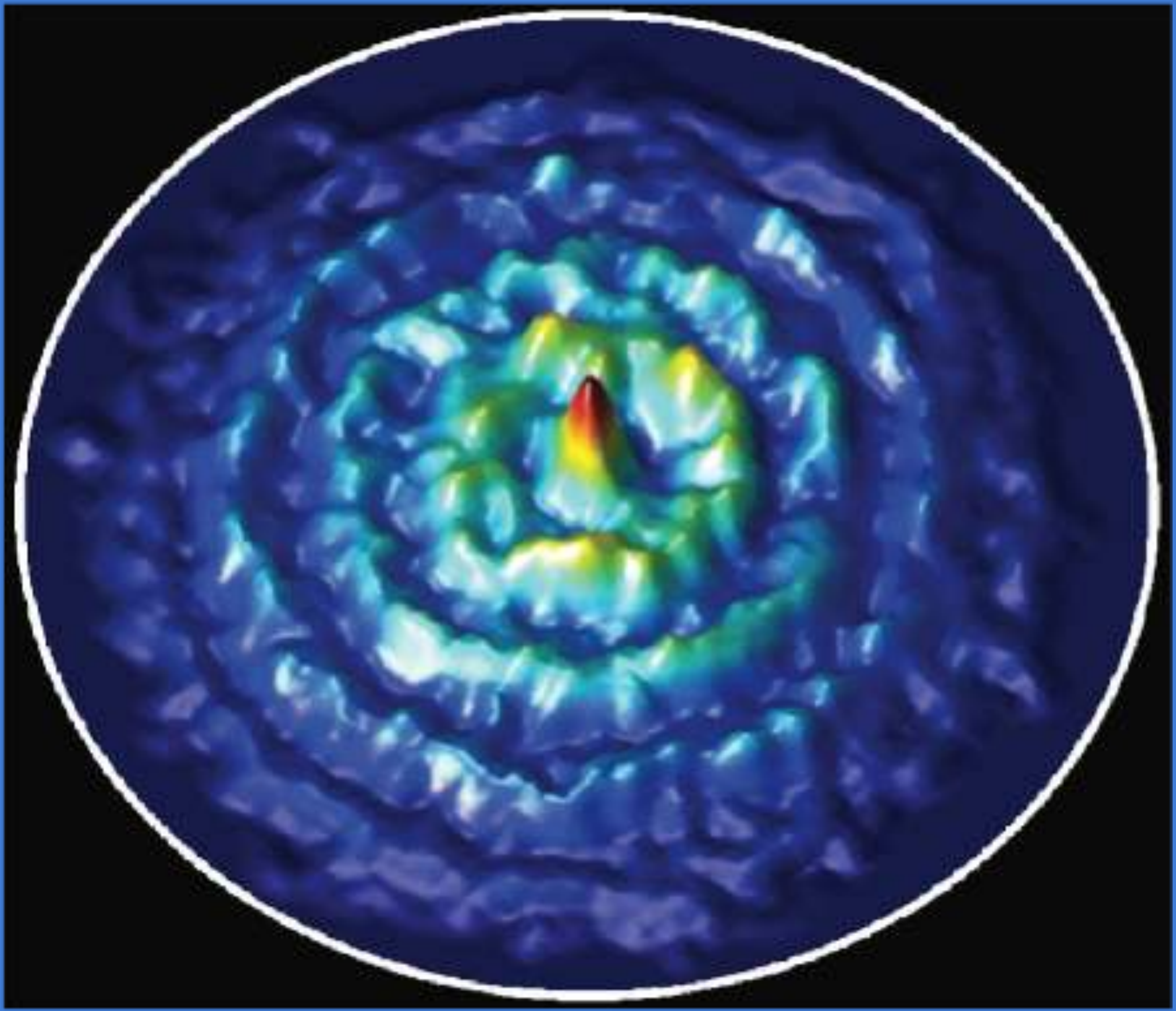
$m = 5, \ l = 0$

Most unstable mode above the Faraday threshold at $f = 70 \text{ Hz}$

Droplet walking in a circular corral

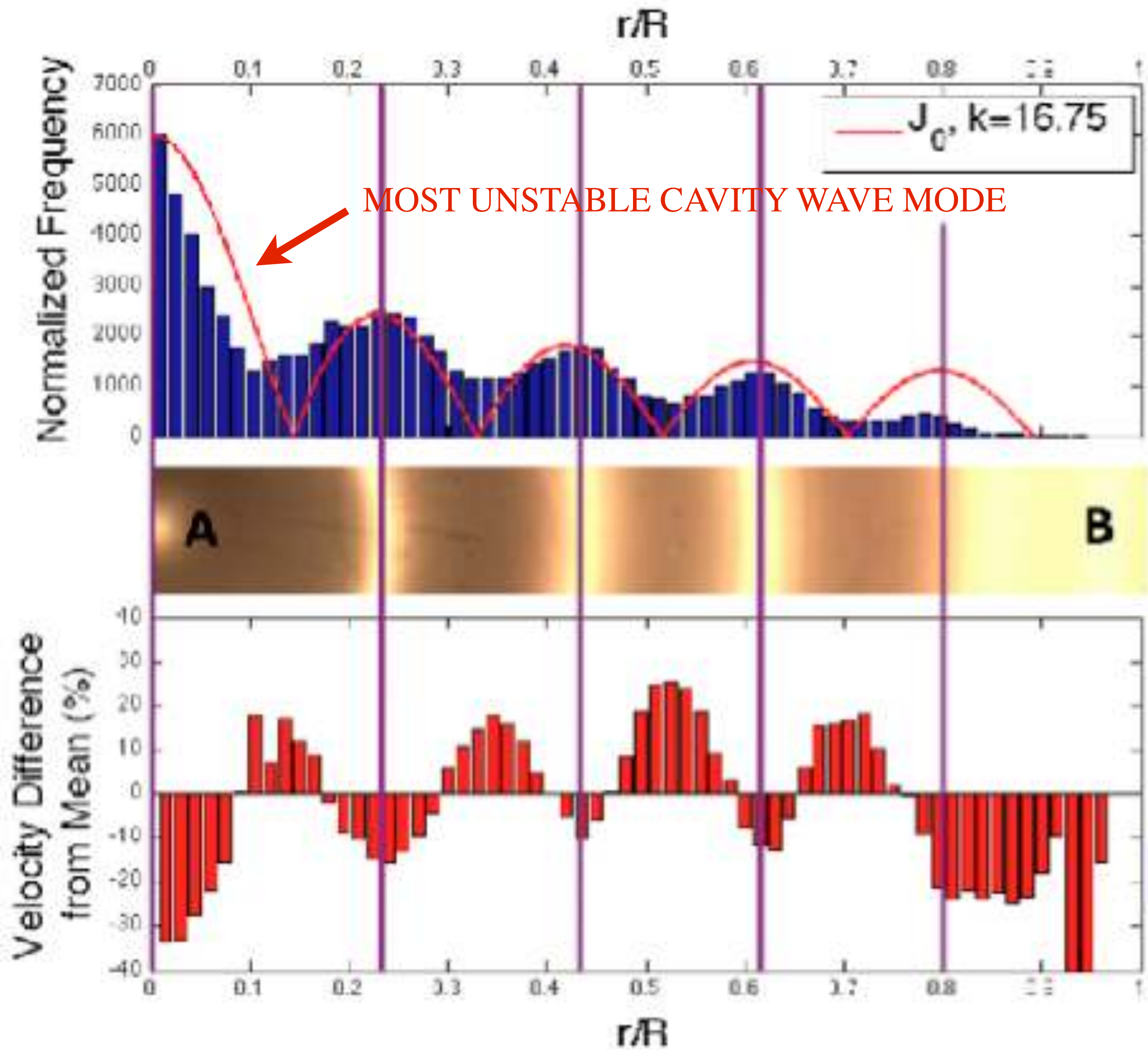


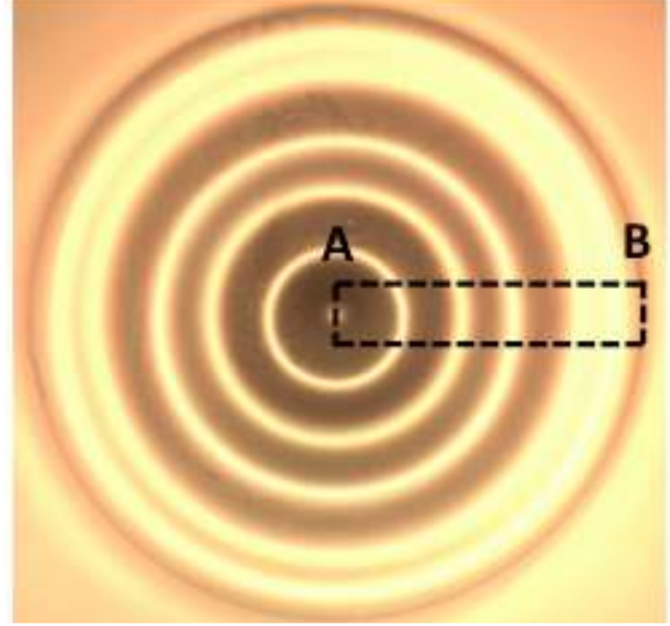
Probability density function

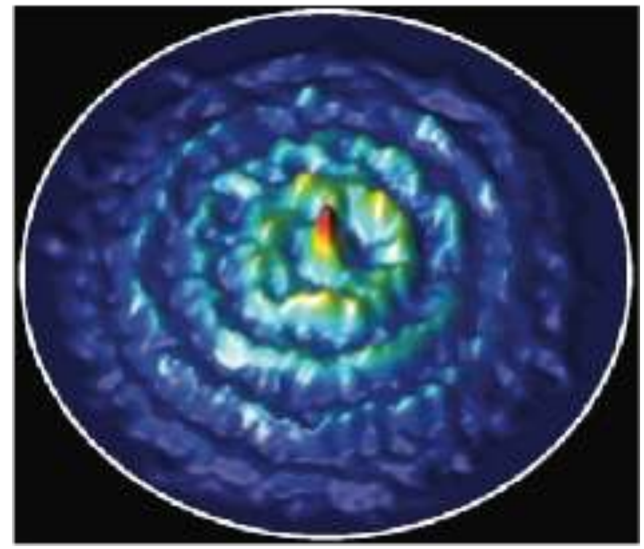


Harris, Moukhtar, Fort, Couder & Bush (PRE, 2013)

- **coherent, wave-like statistics emerge from chaotic pilot-wave dynamics**
- **emergent statistics not inconsistent with the notion of particle trajectories**







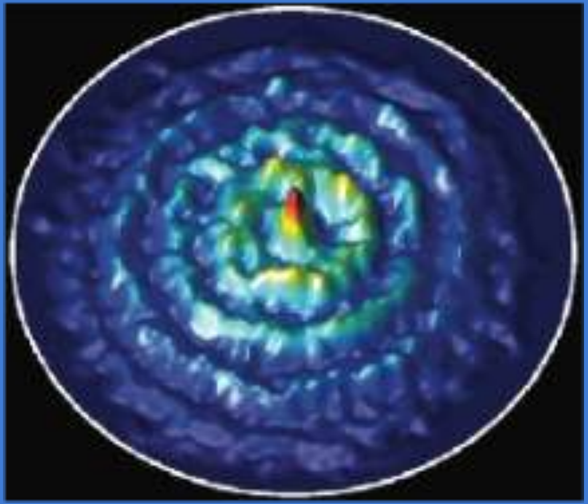
- max in surface perturbation amplitude correspond to peaks in pdf
- pdf prescribed by amplitude of the most unstable resonant wave mode of the cavity

Emerging physical picture: 3 time scales

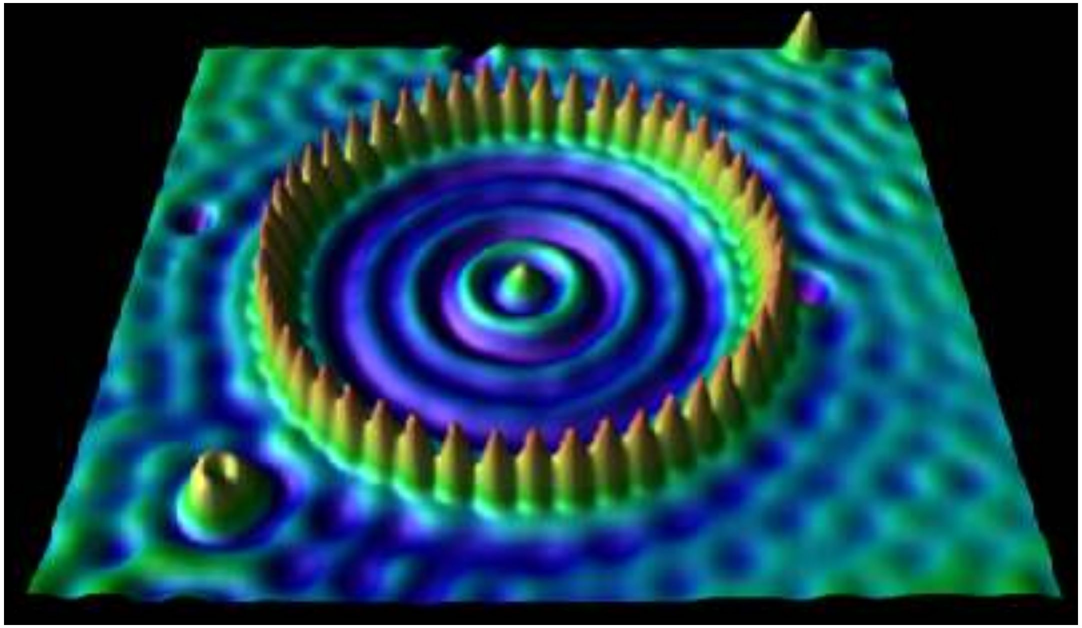
- **fast** dynamics: bouncing at resonance creates monochromatic wave field

- **intermediate** (strobed) pilot-wave dynamics: droplet rides its instantaneous guiding wave

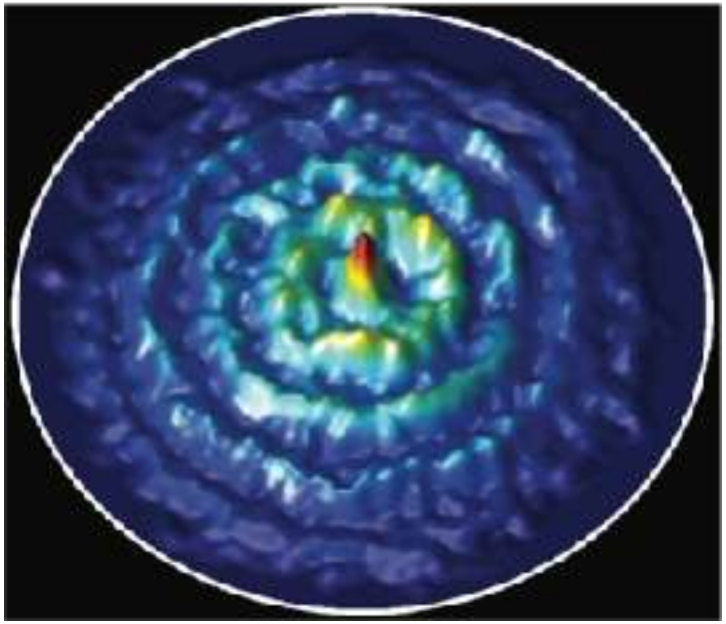
- **long-term statistical** behaviour described by Faraday wave modes



Analogy with quantum corrals



Crommie, Lutz, & Eigler, *Science* (1993)



Harris *et al.*, *PRE*

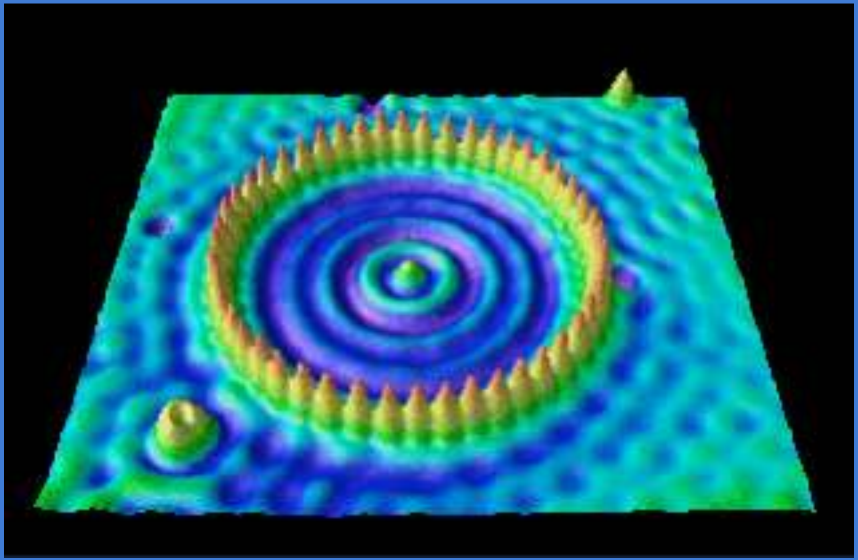
- in quantum corral, electron statistics prescribed by the solution to the time-independent Schrodinger equation in circular geometry with **de Broglie** wavelength
- in fluid corral, walker statistics are defined by the solution to wave equation in circular geometry with **Faraday** wavelength

- statistics prescribed by Born rule in QM, not in the hydrodynamic system

Quantum particles

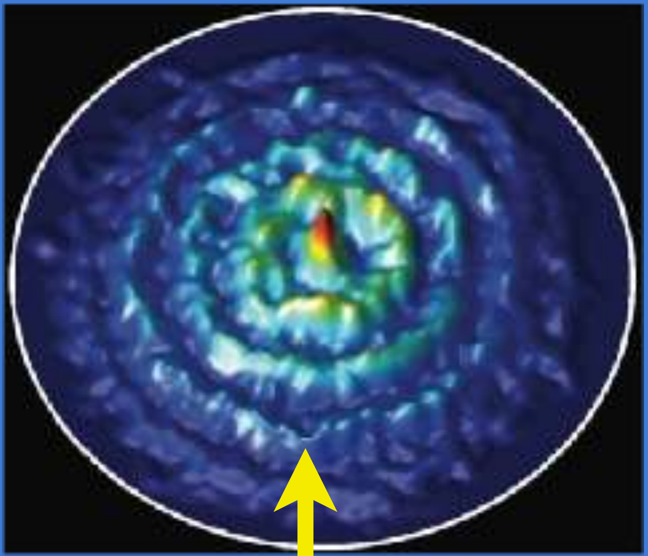
Bouncing droplets

Statistical description



Fiete & Heller (2003)

Statistical description



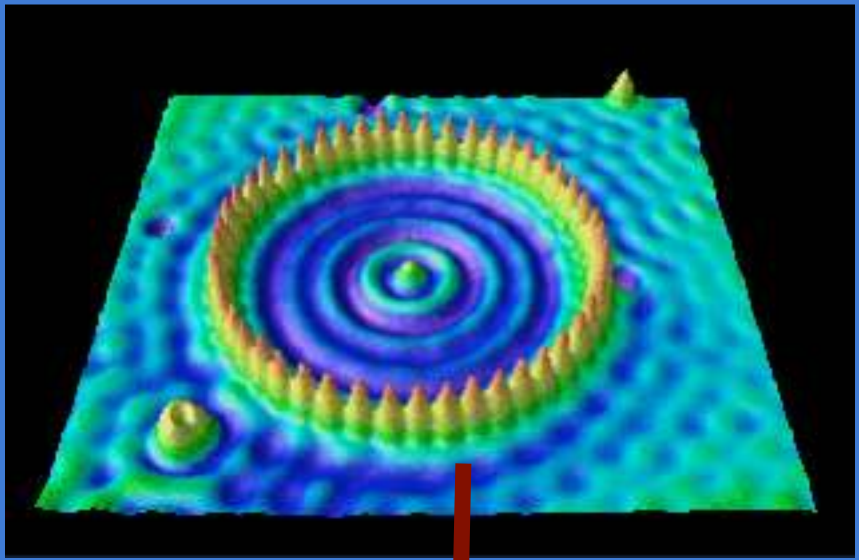
Harris et al. (2012)

Underlying dynamics

An Exposed Variable Theory

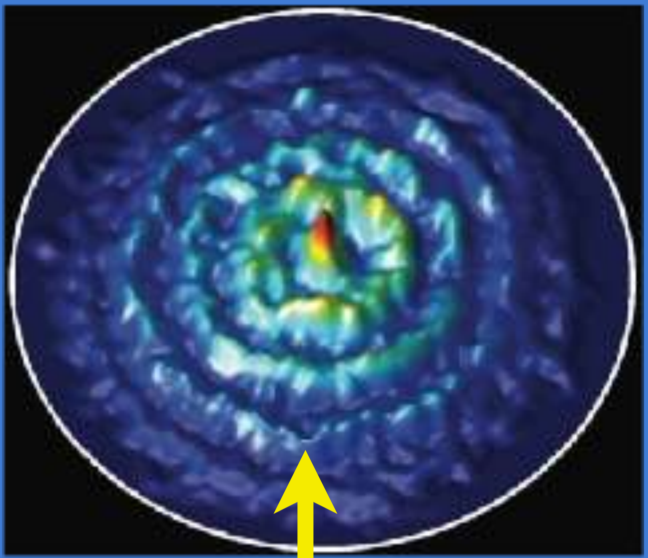
Quantum particles | Bouncing droplets

Statistical description



Fiete & Heller (2003)

Statistical description



Harris et al. (2012)

Underlying dynamics

Hidden Variable Theory

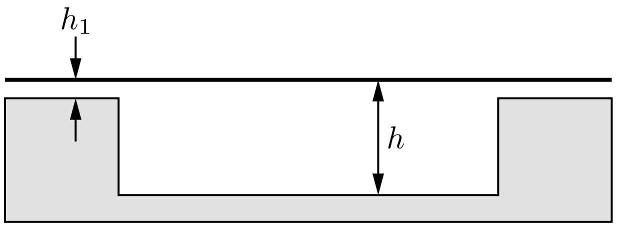
Underlying dynamics

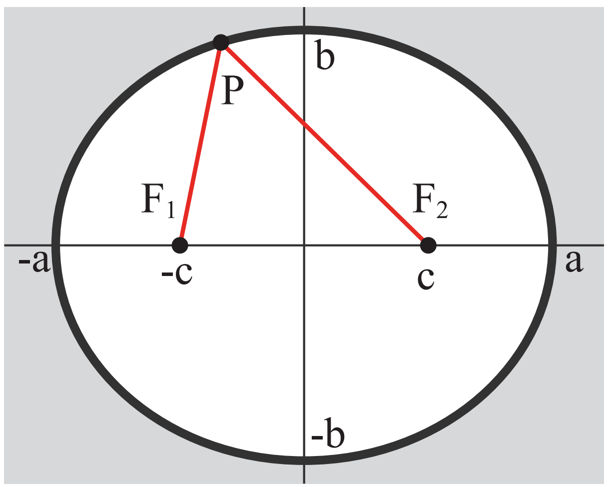
An Exposed Variable Theory

Elliptical corral

Faraday Waves: Oscillation of an elliptic membrane

E. Mathieu, "Le mouvement vibratoire d'une membrane de forme elliptique," J. Math. Pures Appl. 13, 137–203 (1868).





Path length: $P_L = 2a = 28.5\,\mathrm{mm}$

Eccentricity: $e = \frac{c}{a} = 0.5$

Two-dimensional Helmholtz equation:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + k^2 U = 0$$

Transform to elliptic coordinates and seek solutions of the form:

$$U(\xi, \eta) = R(\xi)\Phi(\eta)$$

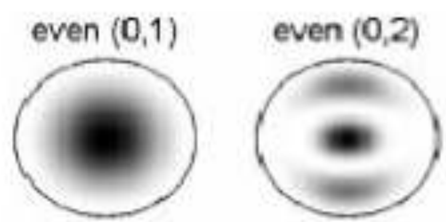
Angular and radial Mathieu equations:

$$\frac{d^2\Phi}{d\eta^2} + (a - 2q\cos 2\eta)\Phi = 0$$

$$\frac{d^2 R}{d\xi^2} - (a - 2q\cosh 2\xi)R = 0$$

$$q = \frac{f^2 k^2}{4}$$

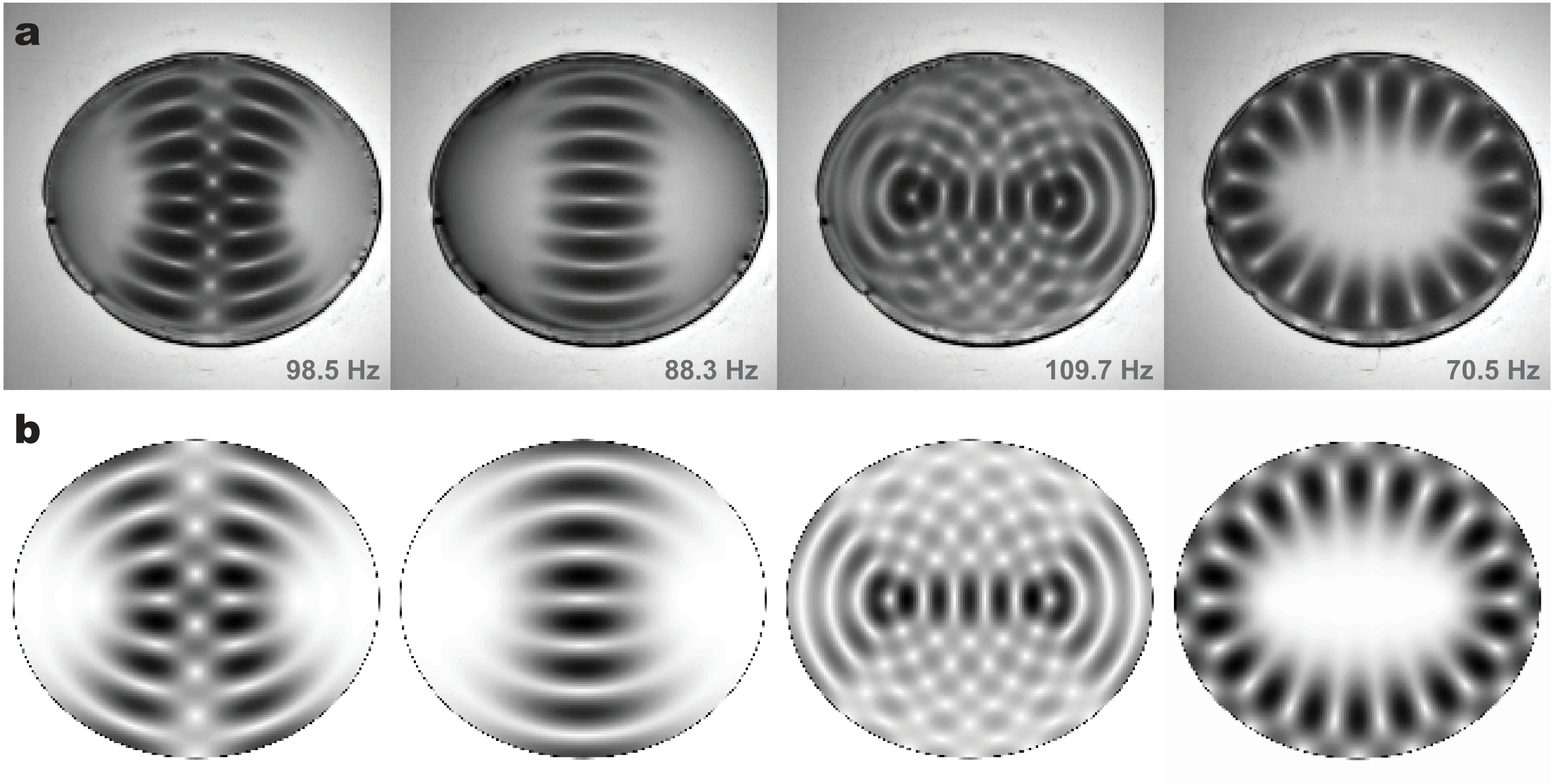
Dirichlet BC



Even solutions

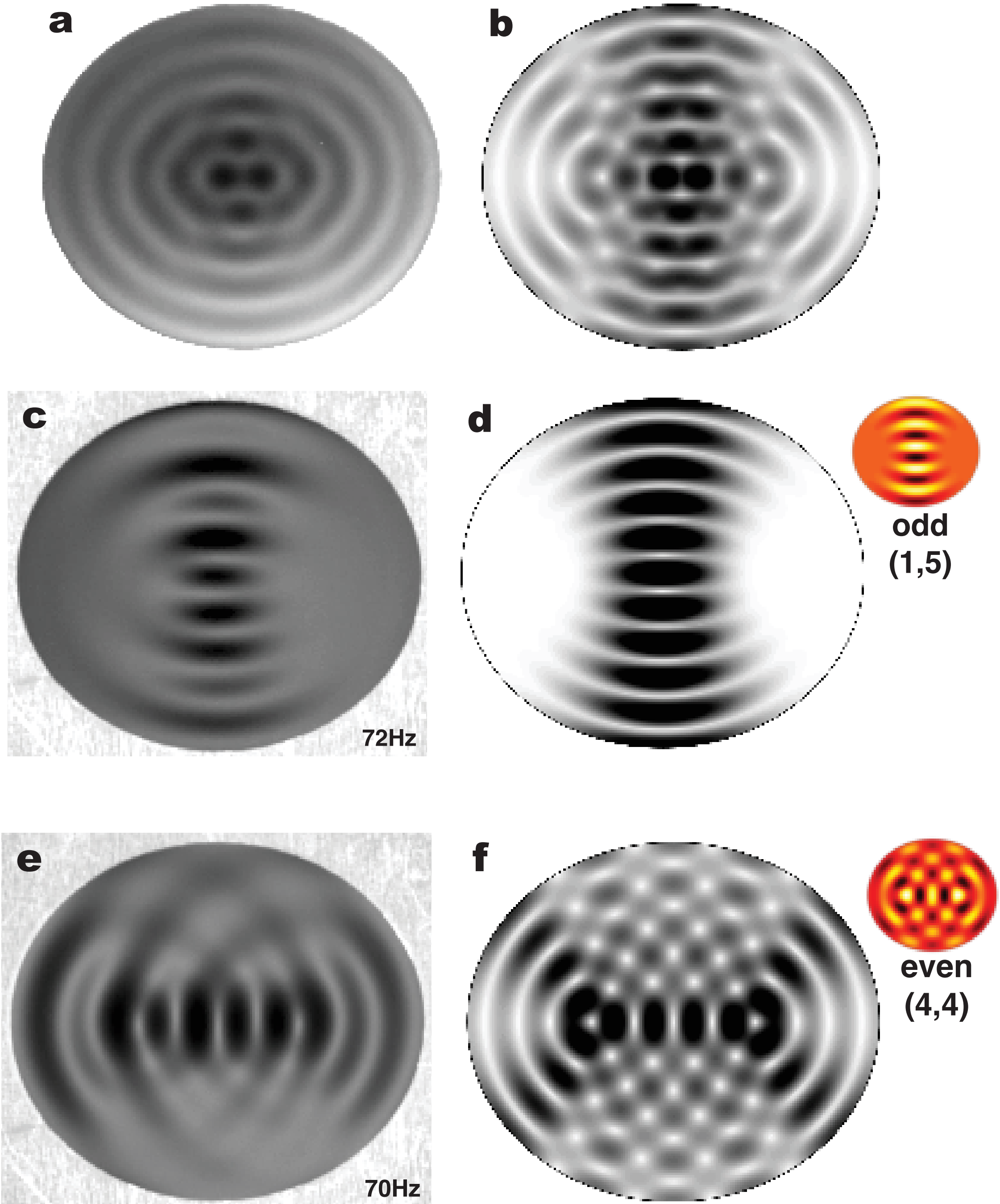
Waves modes

Experiments

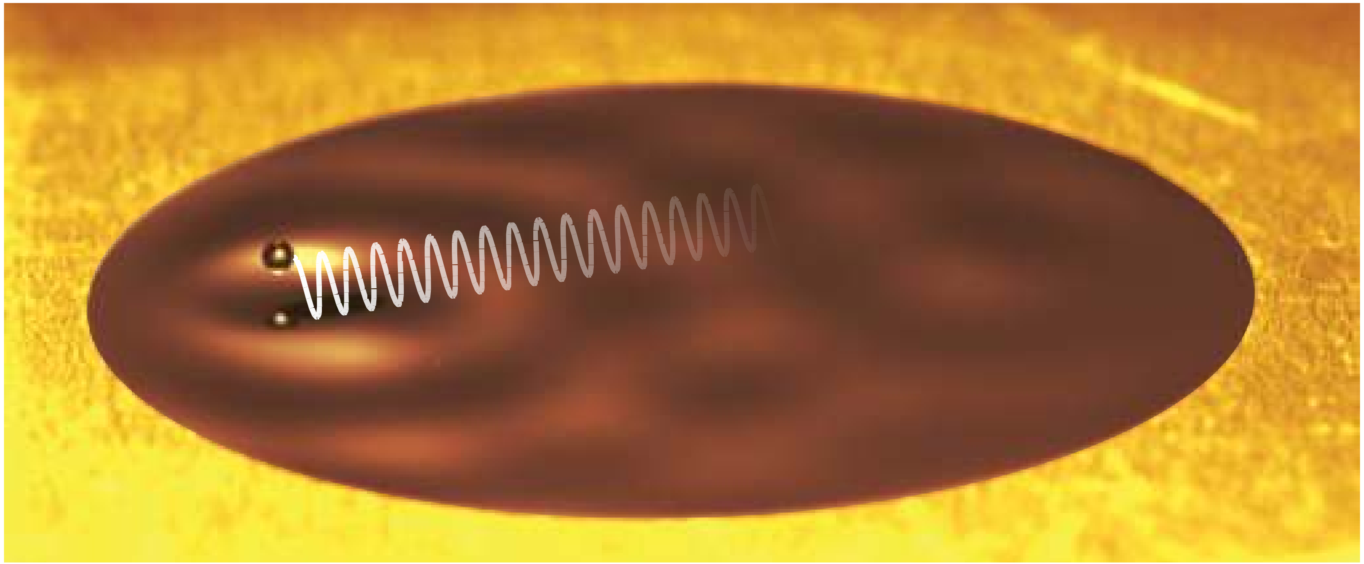


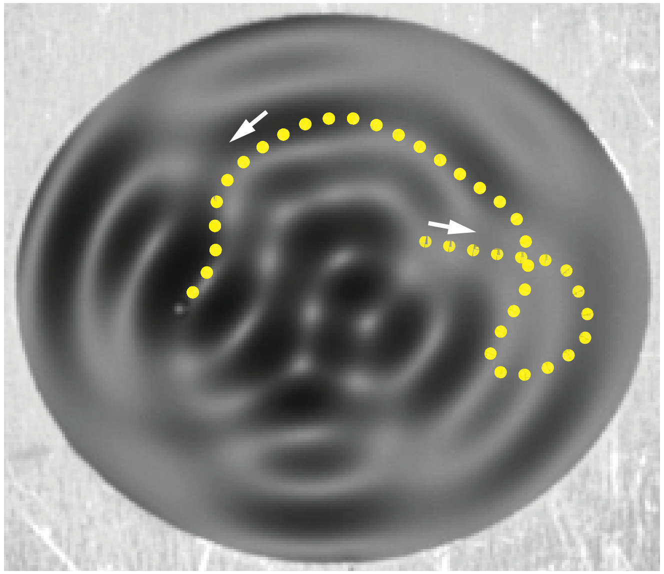
Theory

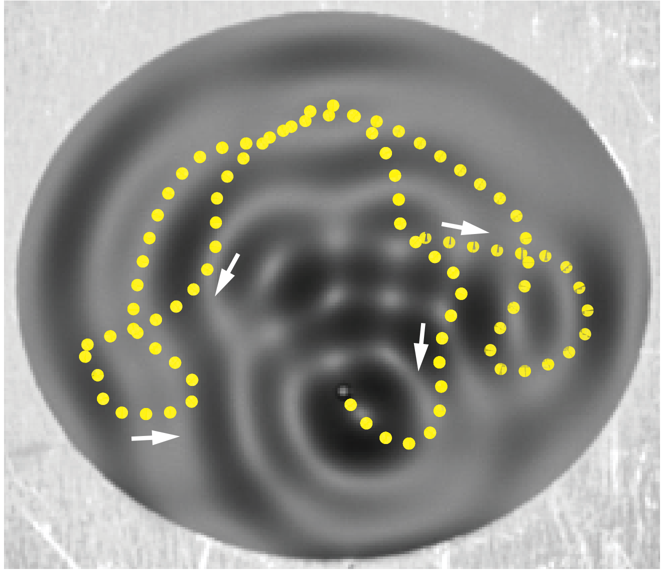
Waves modes



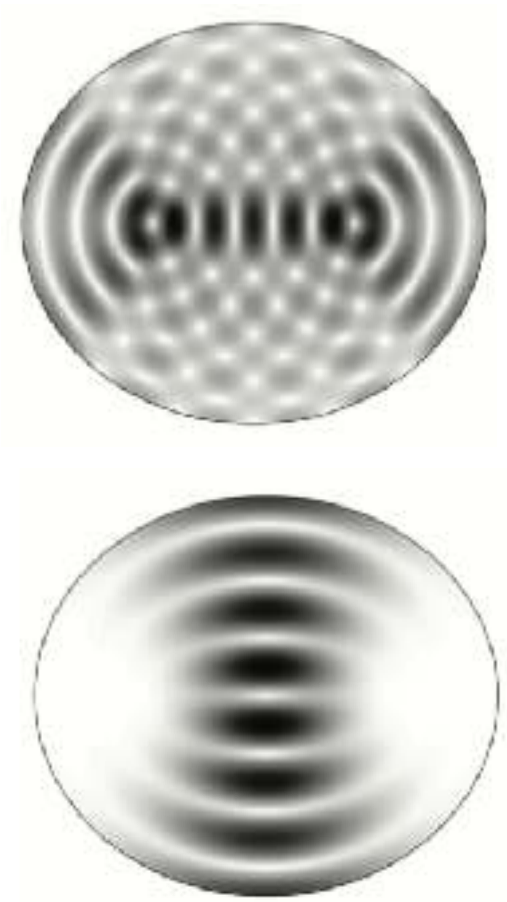
Wave forms arising at high Me

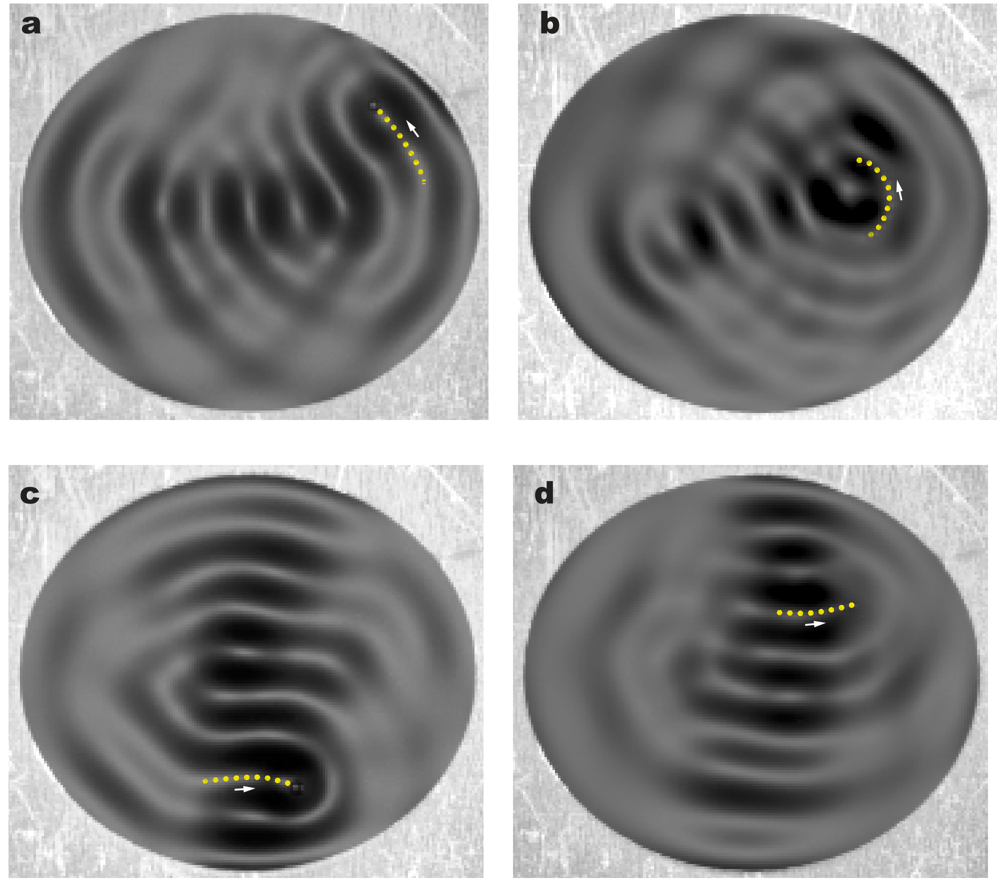






Instantaneous pilot-wave field

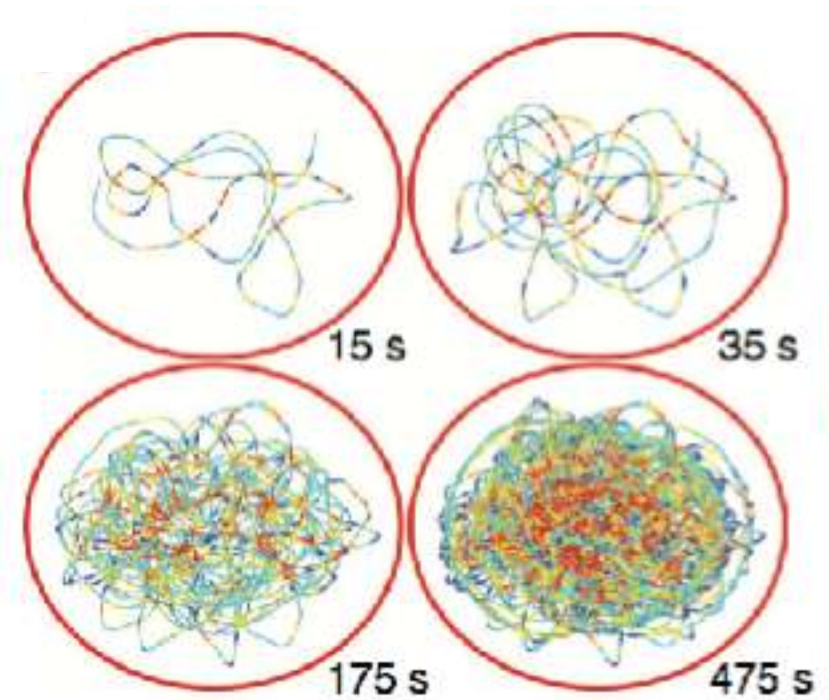




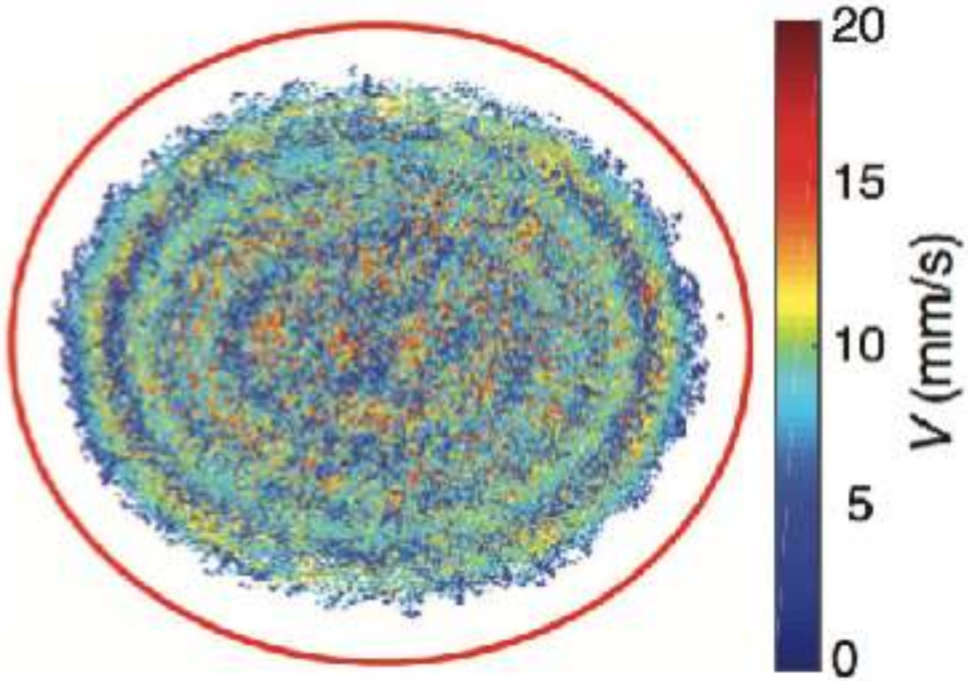
- at any given instant, the pilot-wave differs from the most unstable cavity modes
- but vestiges of the cavity modes may be apparent

The elliptical corral

Trajectories



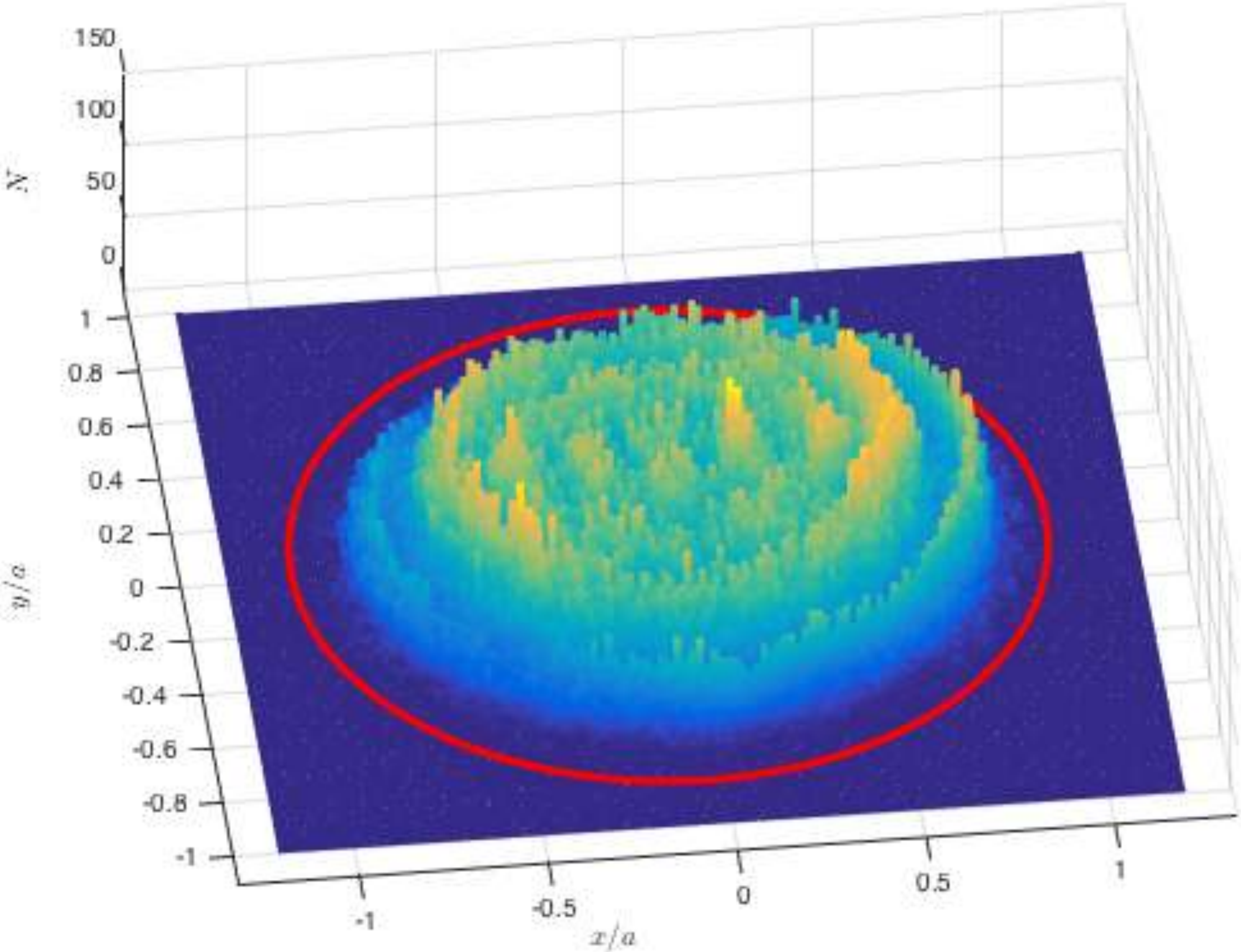
Mean speed



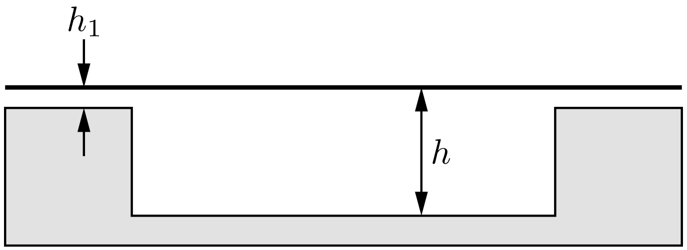
- correlation between position and speed, as in the circular corral

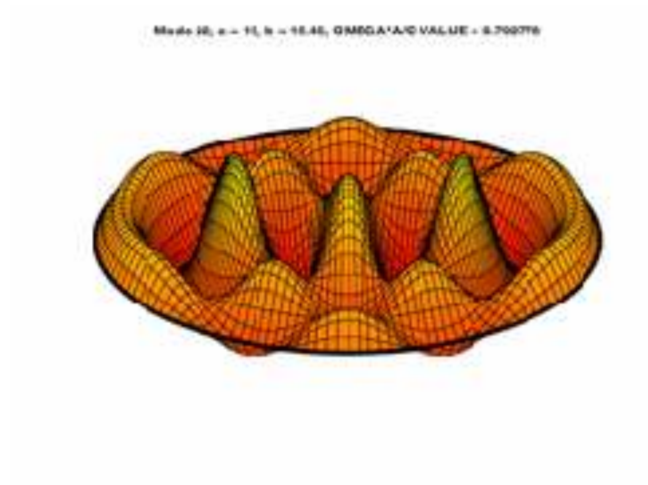
- Sáenz, Cristea-Platon & Bush, Nat. Phys. (2018)

Elliptical corral - Probability density function



Elliptical corral - Probability density function



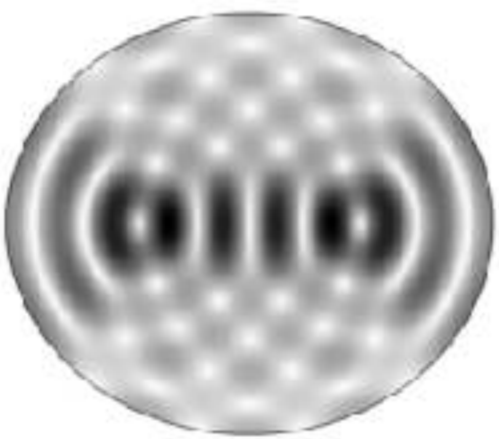


$f = 72\,\mathrm{Hz}$ $\qquad \gamma/\gamma_F = 99.8\%$

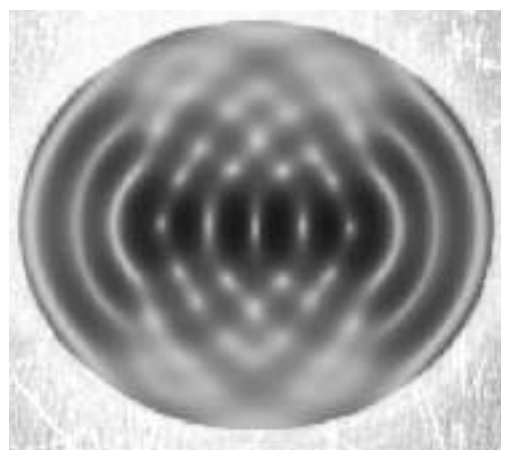
$h = 6\,\mathrm{mm}$ $\qquad D = 0.696\,\mathrm{mm}$

$h_1 = 0.1\,\mathrm{mm}$

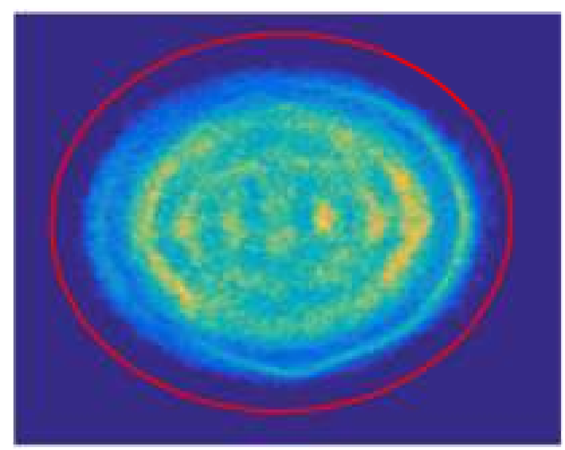
$3\,\mathrm{h} \times 20\,\mathrm{fps} \rightarrow 36,000\,\mathrm{points}$



Theoretical eigenmode



Faraday waves

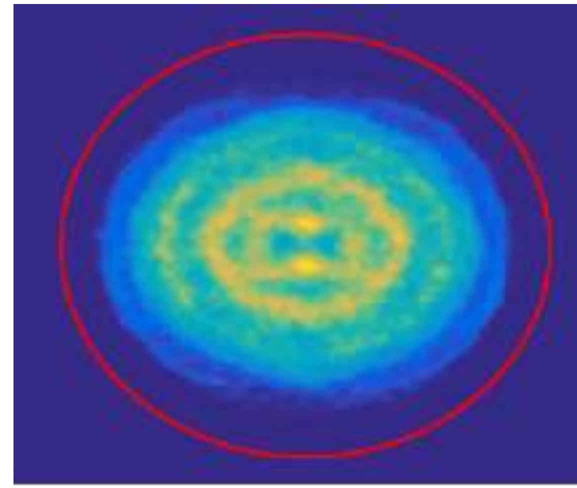


Walker's histogram

Mode superposition

$f = 72\,\mathrm{Hz}$

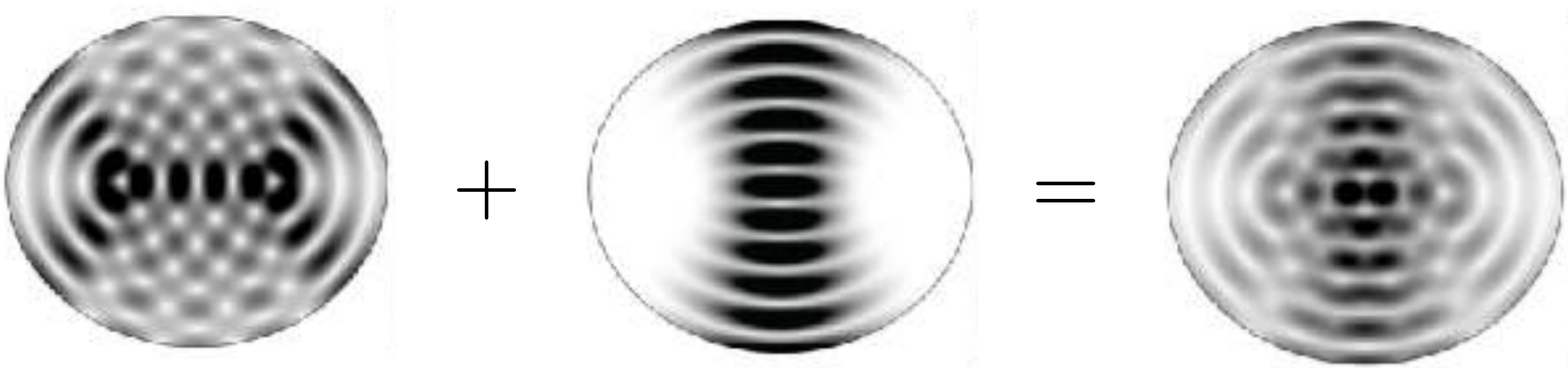
Particle's histogram



- emergent statistics do *not* correspond to the most unstable Faraday mode at 72 Hz
- drop introduces a second mode that is the most unstable Faraday mode at 70 Hz

Mode superposition

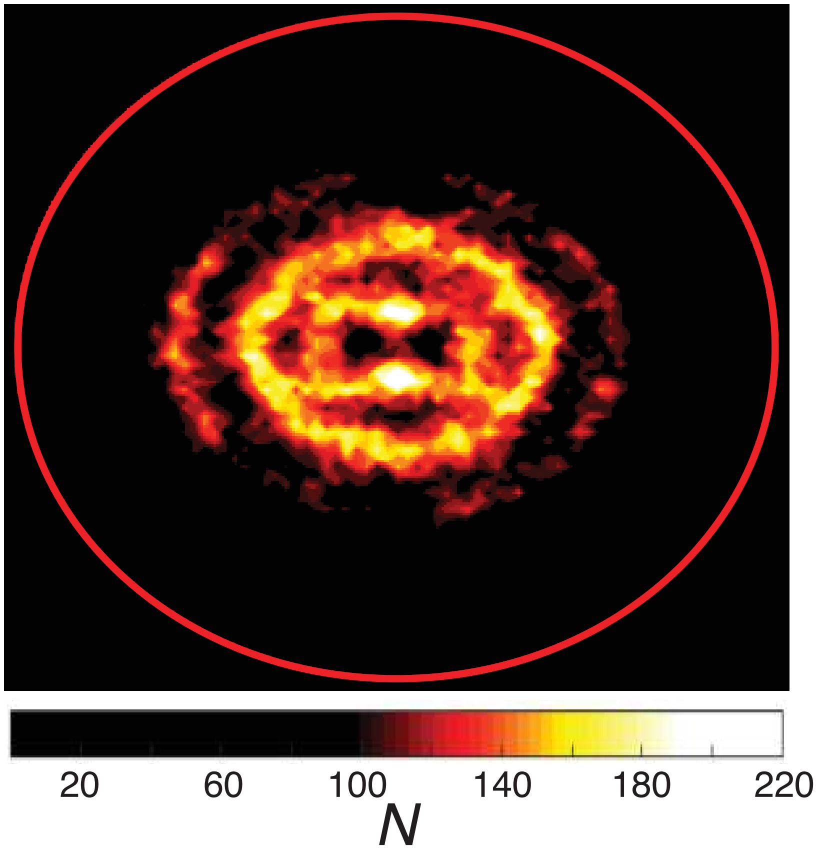
$f = 70\ Hz$ $\qquad f = 72\,\mathrm{Hz}$



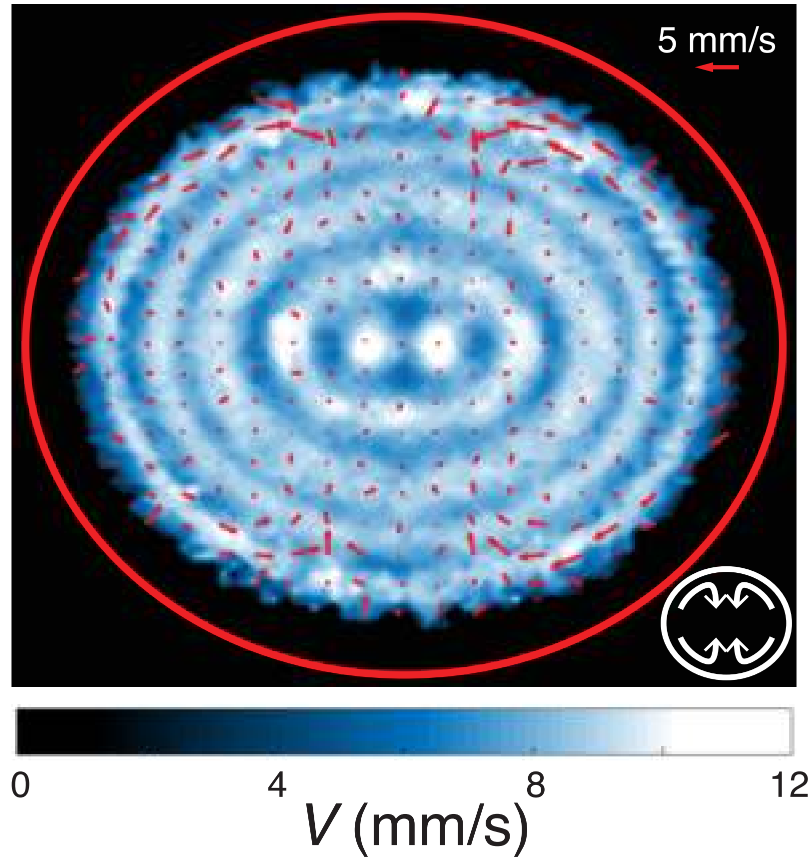
A superposition of statistical states

A new diagnostic: mean velocity

Position histogram (pdf)



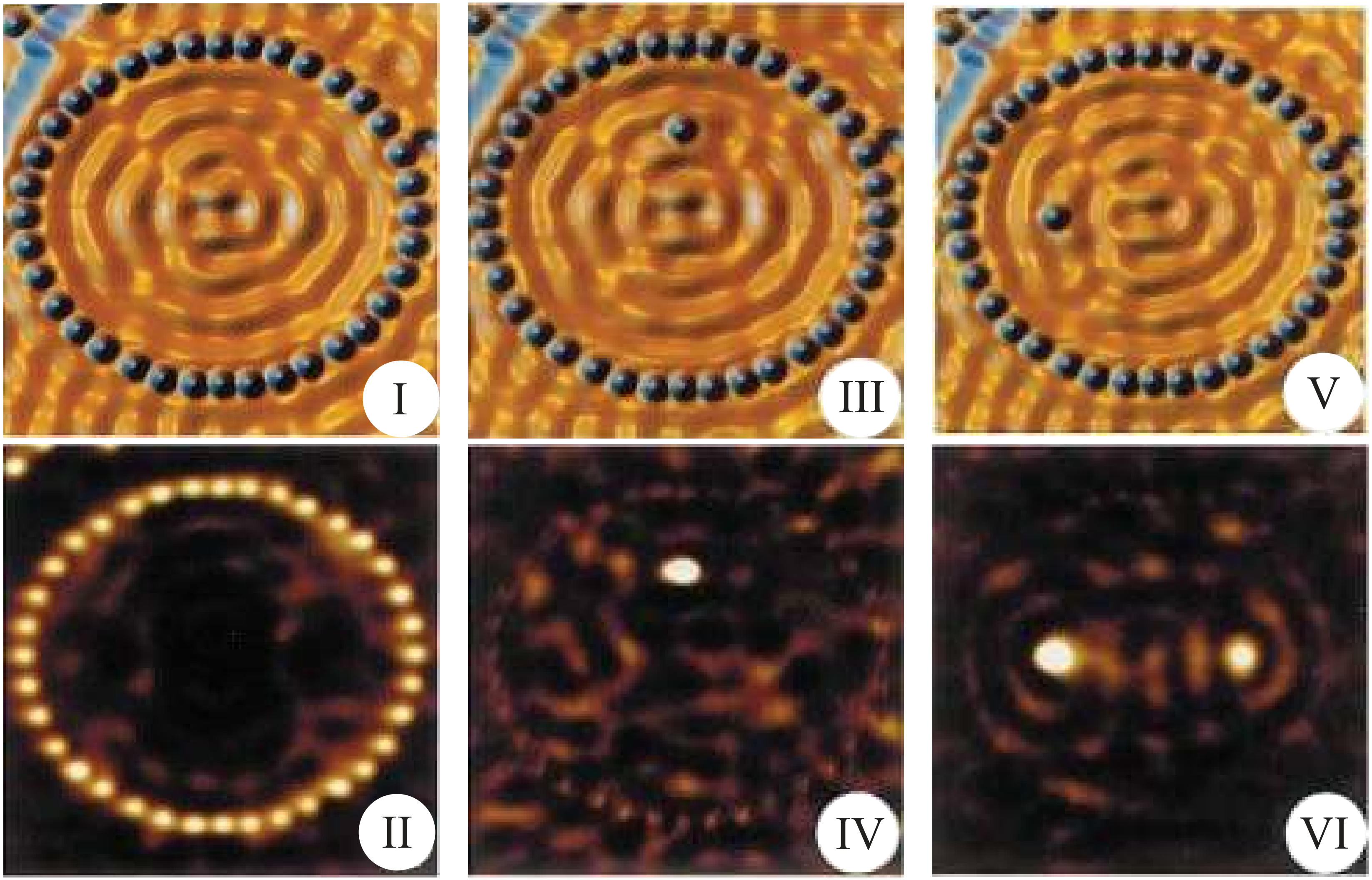
Mean speed and velocity



- while the mean velocity is zero in a circular corral, a quadrupolar flow emerges in the ellipse
- relation to Bohmian mechanics?

Fiete & Heller (2002)

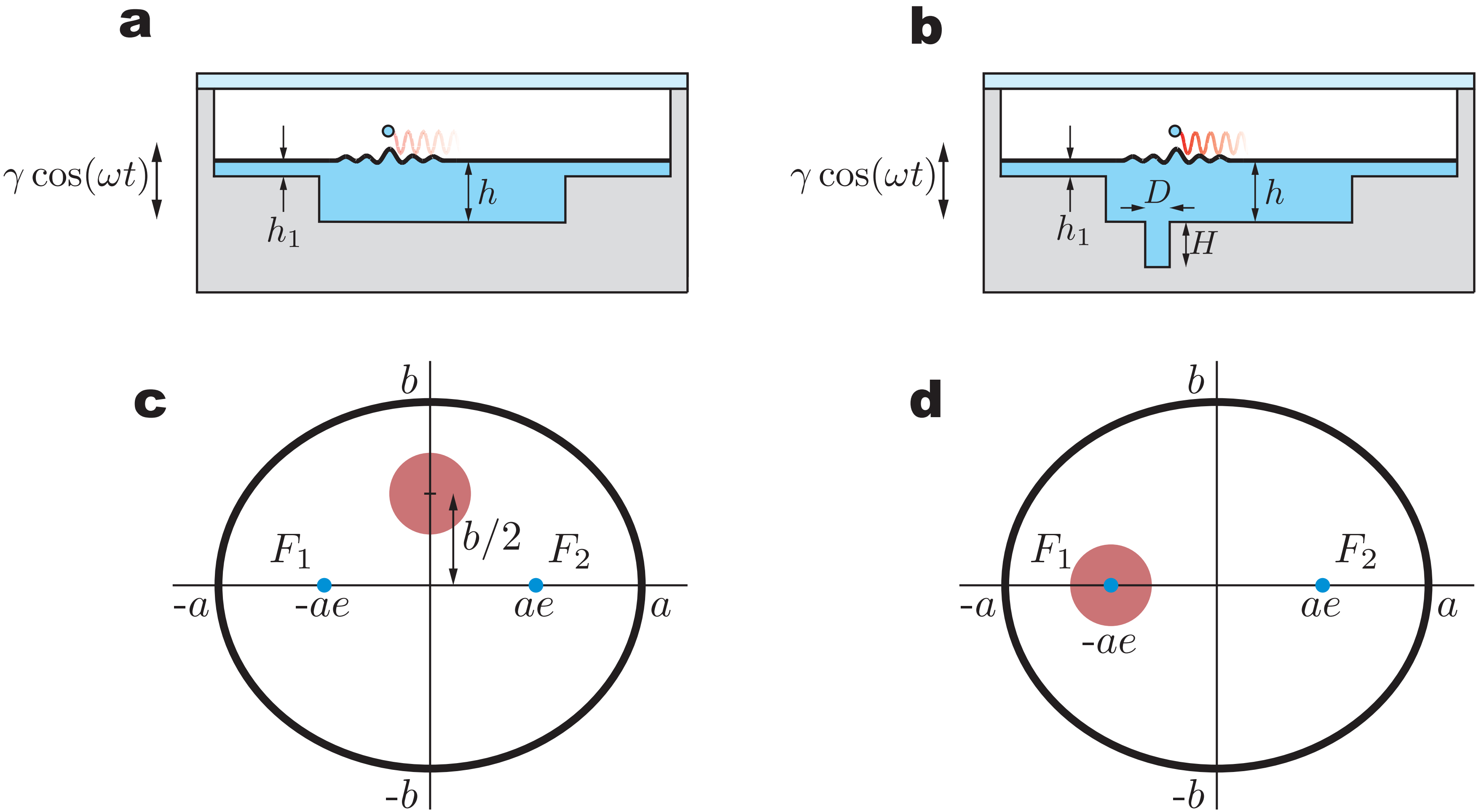
Projection effects: the `quantum mirage'



- placing an impurity at one focus results in a `mirage' at the other focus
- effect pronounced in differential conductivity, as depends strongly on pdf

Topographic control of walker statistics

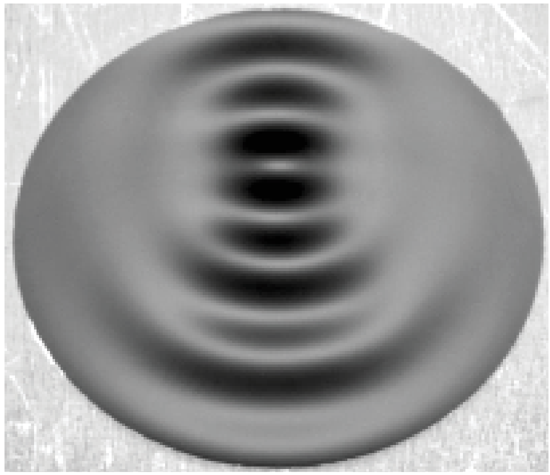
- in shallow-water limit, h = 1.7mm, the walker feels the bottom topography



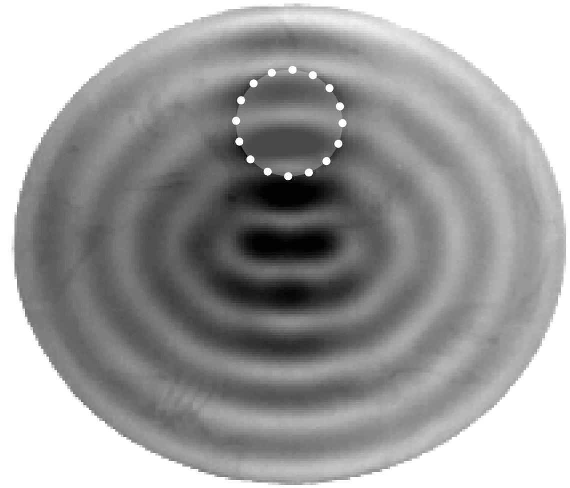
- the walker is generally attracted to the well
- arbitrary placement of well simply disrupts coherence of pdf

Well at midpoint of semi-minor axis

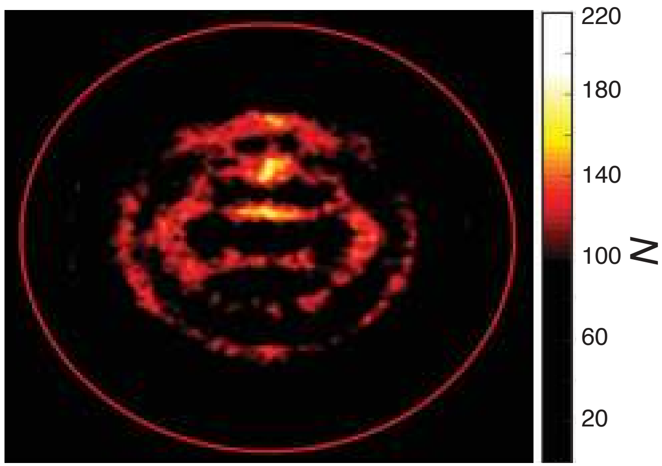
Cavity mode



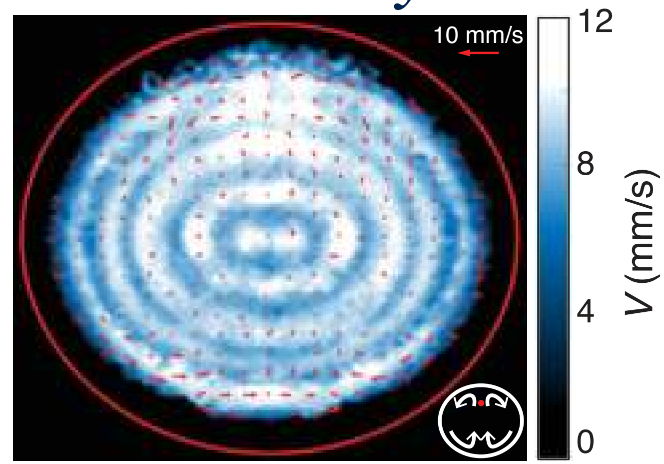
Mean wave field



PDF



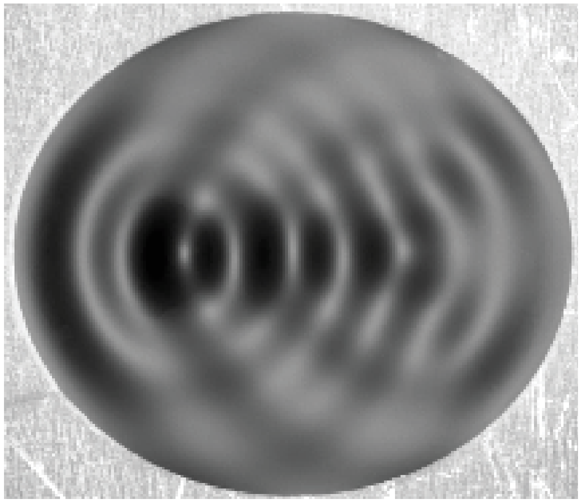
Mean velocity



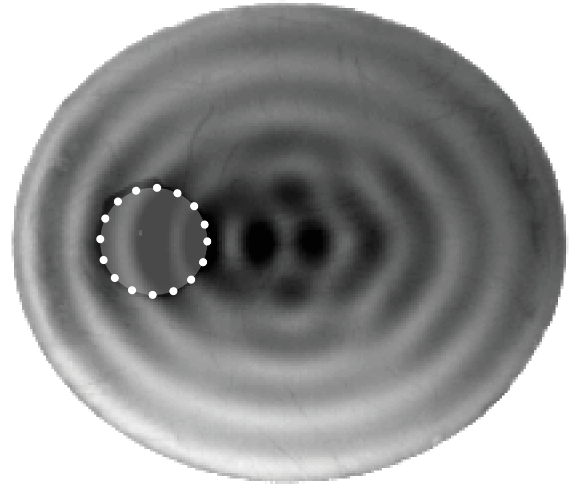
- well is not projected: acts to disrupt *pdf*, which is relatively incoherent

Well at left focus

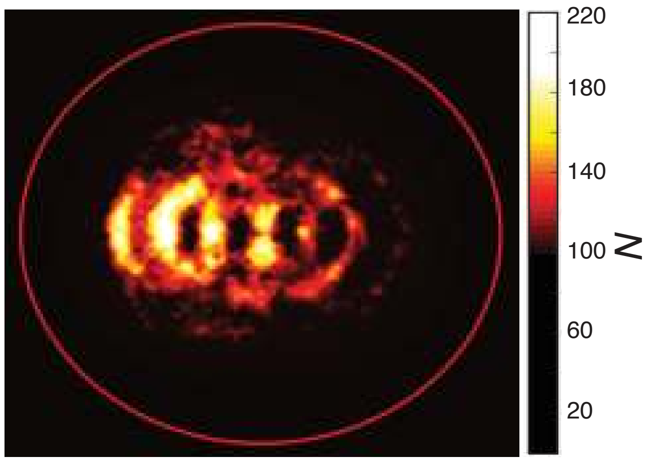
Cavity mode



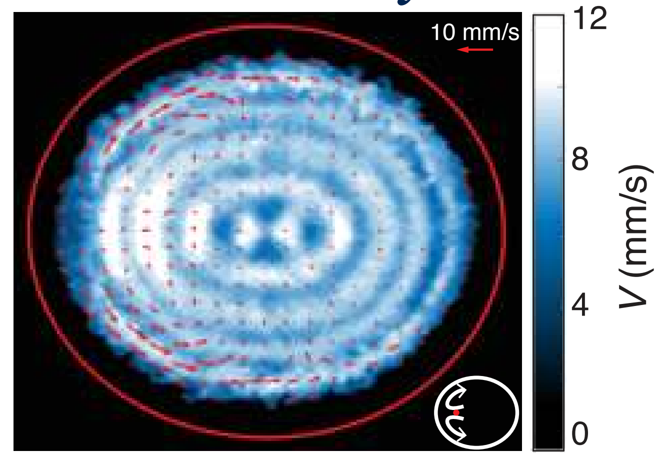
Mean wave field



PDF

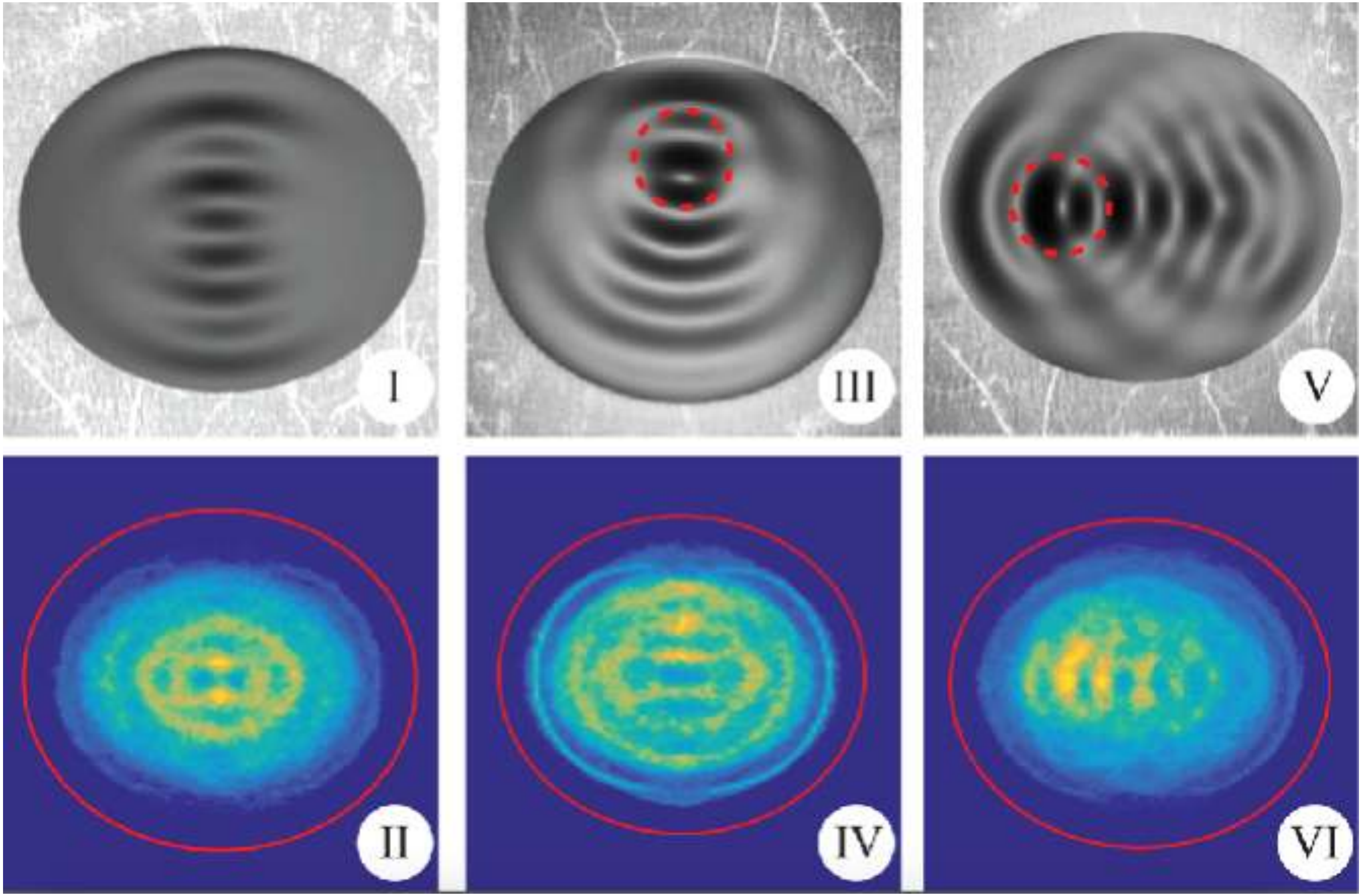


Mean velocity



- by preferentially selecting (4,4) mode, well is projected towards empty focus
- effect on pdf ***more*** pronounced than in quantum corral (Eric Heller)

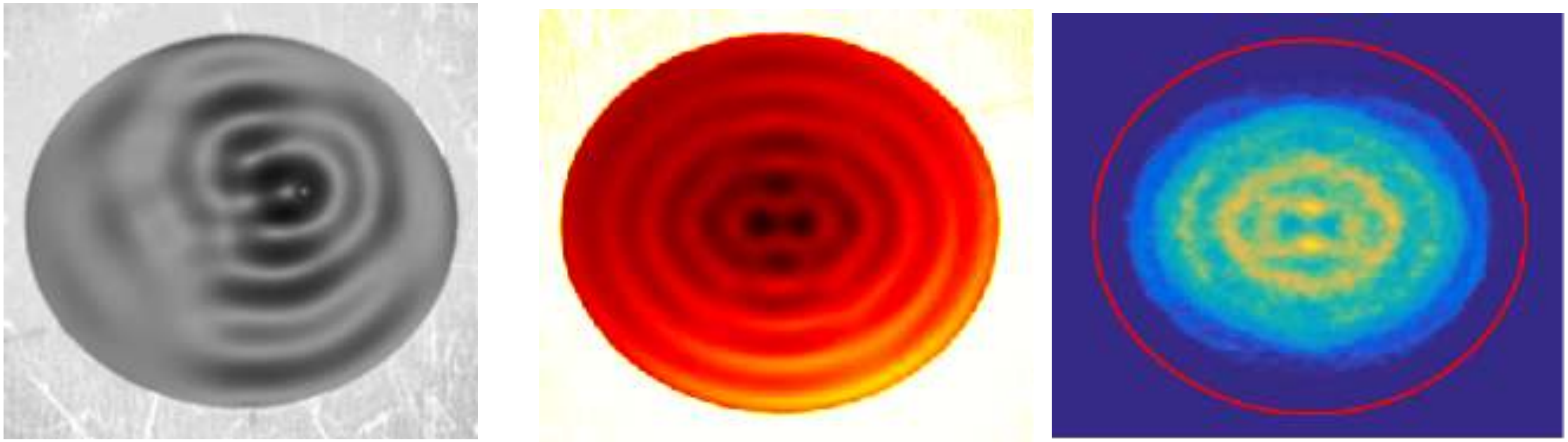
An analog of the `quantum mirage'



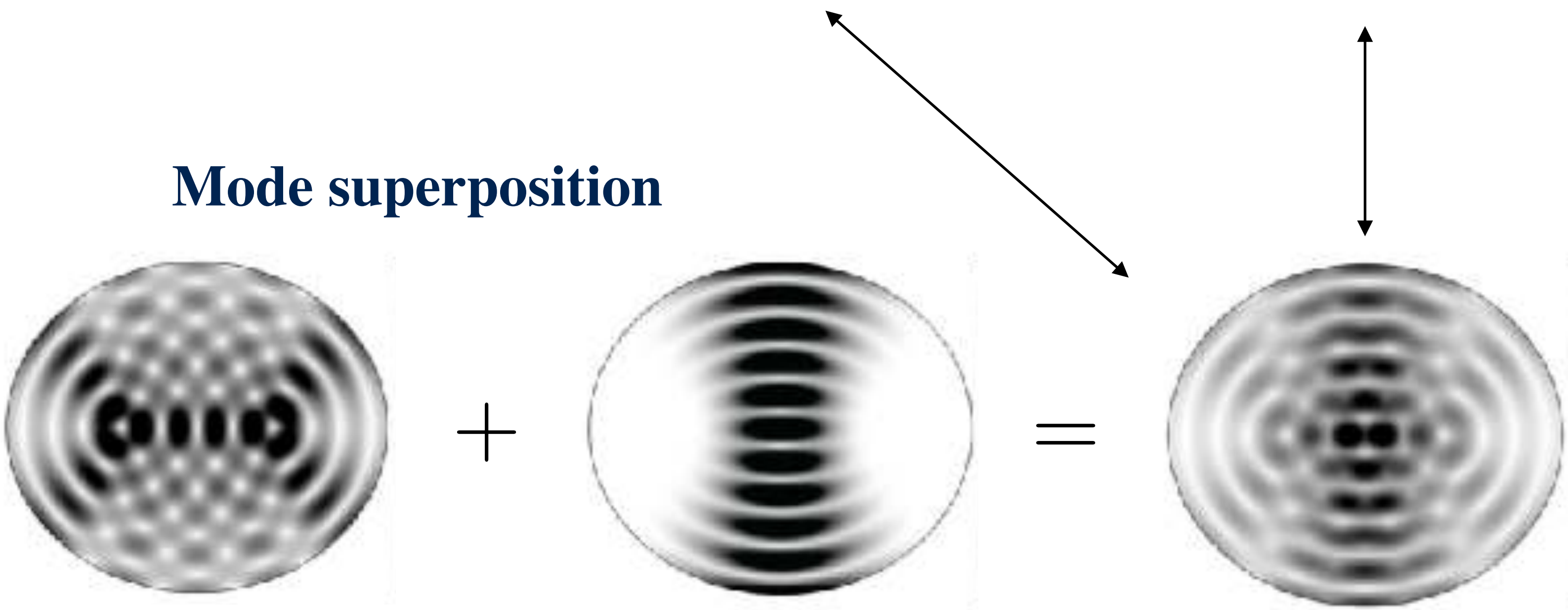
- a hole at one focus induces a mirage at the other by favoring one cavity mode

A striking equivalence

Instantaneous wave | Average wave | Particle's histogram



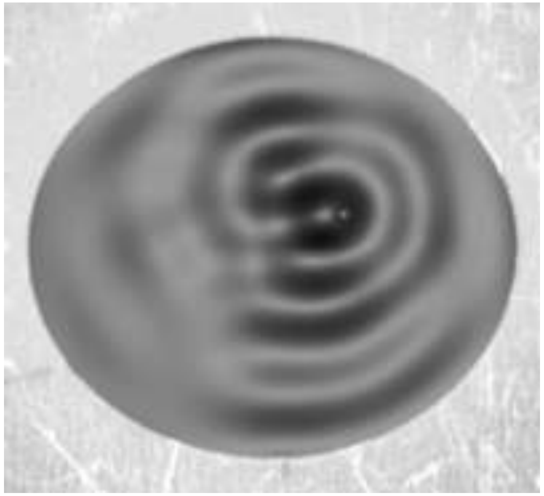
Mode superposition



A superposition of statistical states

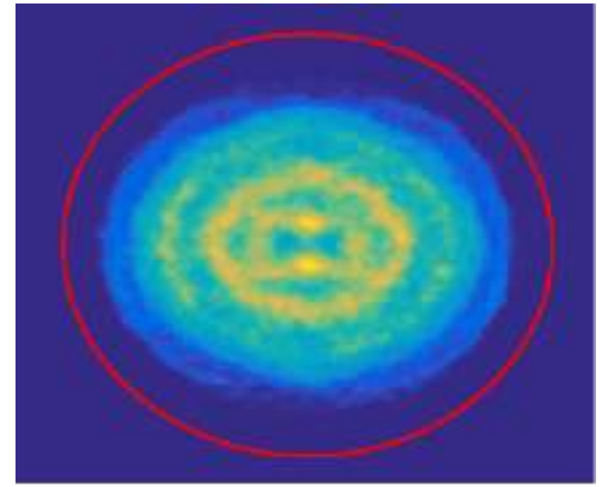
The mean pilot-wave field

Instantaneous wave



Average wave $\bar{\eta}(\mathbf{x})$

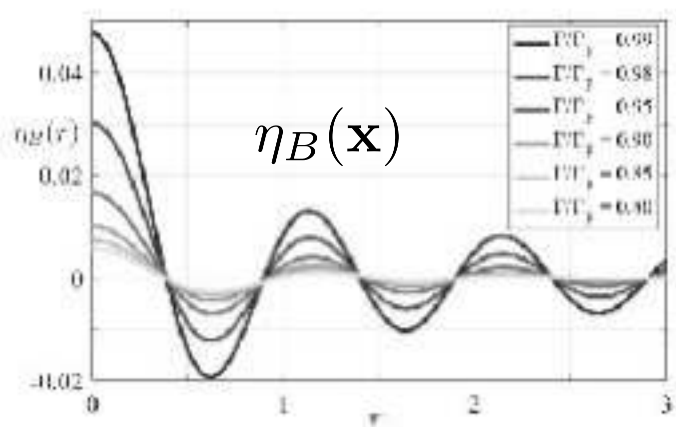
Particle's histogram $\mu(\mathbf{x})$



Theorem *(Durey, Milewski & Bush 2018)*

$$\bar{\eta}(\mathbf{x}) = \int_{\mathbf{R}^2} \eta_{\mathbf{B}}(\mathbf{x}-\mathbf{y})\ \mu(\mathbf{y})\ \mathbf{dy} = (\eta_{\mathbf{B}} * \mu)(\mathbf{x})$$

FOR PERIODIC OR CHAOTIC TRAJECTORIES

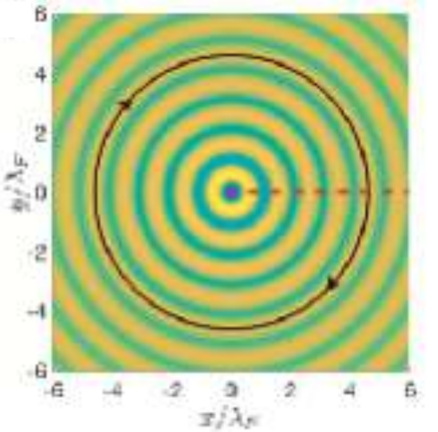


- the average wave field, $\bar{\eta}(\mathbf{x})$, corresponds to the convolution of the *pdf,* $\mu(\mathbf{x})$, and the wave field of a stationary bouncing droplet, $\eta_B(\mathbf{x})$
- result deduced from the stroboscopic assumption, which breaks down at high Me

The mean pilot-wave field of a circular orbit

We consider a drop in a circular orbit of radius r_0 with constant speed. Its mean wavefield may be computed analytically in polar coordinates

$$\eta_B = A_B J_0(|\mathbf{x}|) \quad \text{and} \quad \mu(\mathbf{y}) = \delta(|\mathbf{y}|)/(2\pi r_0)$$



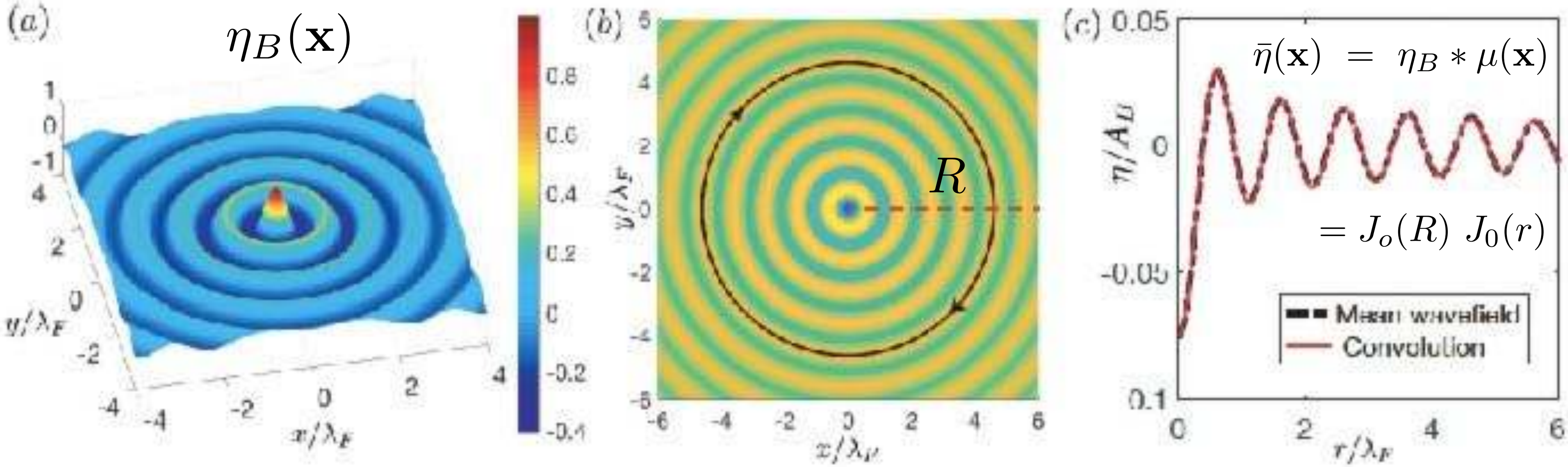
$$\bar{\eta}(\mathbf{x}) = \int_{\mathcal{R}^2} \eta_B(\mathbf{x}-\mathbf{y})\mu(\mathbf{y})\,\mathrm{d}\mathbf{y}$$

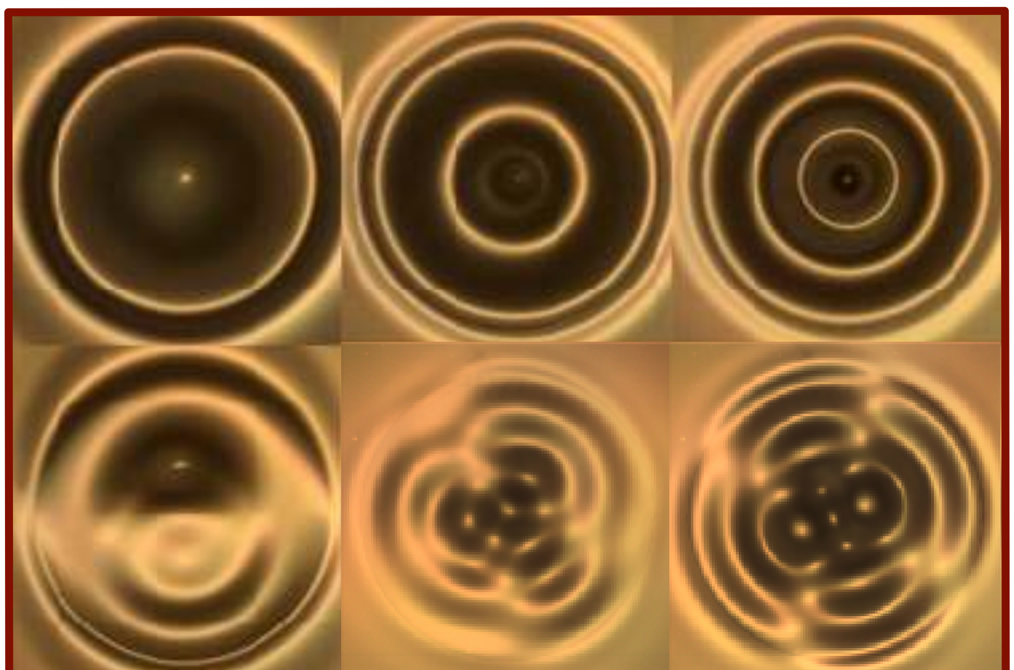
$$= A_B \int_{\mathcal{R}^2} \frac{J_0(\sqrt{r^2+\rho^2-2r\rho\cos(\theta)})\delta(\rho-r_0)}{2\pi r_0}\rho\,\mathrm{d}\rho\,\mathrm{d}\theta$$

$$\boxed{\bar{\eta}(r) = A_B J_0(r_0) J_0(r)}$$

The mean wavefield has the form of a Bessel function centered on the orbital center, and an amplitude prescribed by the orbital radius.

The convolution result: relates waveforms to trajectories

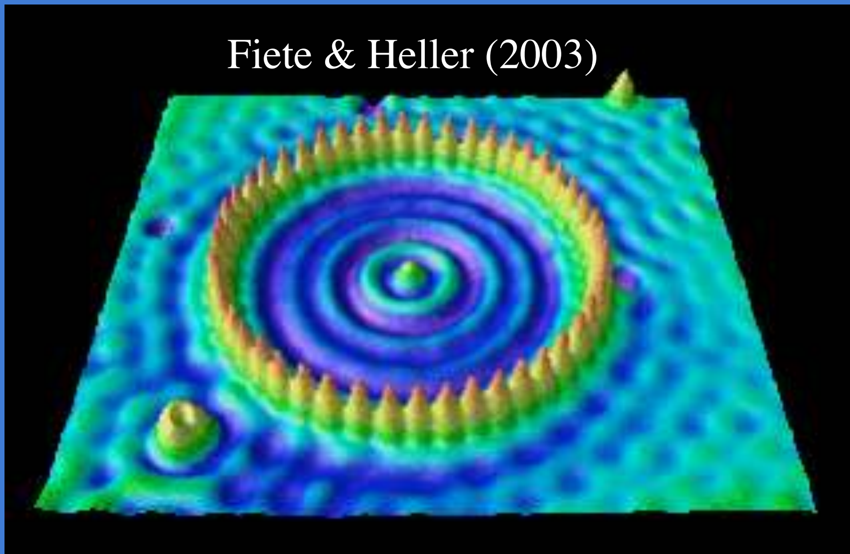




- prevalence of orbits with radii at zeros of $J_0(r)$ suggests dominance of wave energy

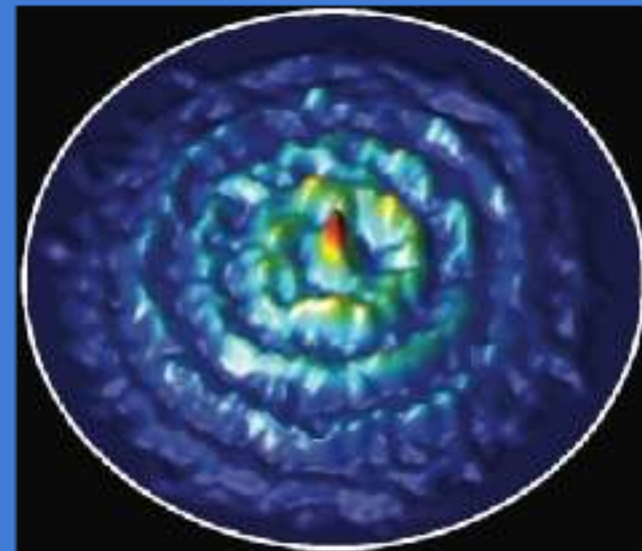
- suggests that wave modes have corresponding periodic trajectories

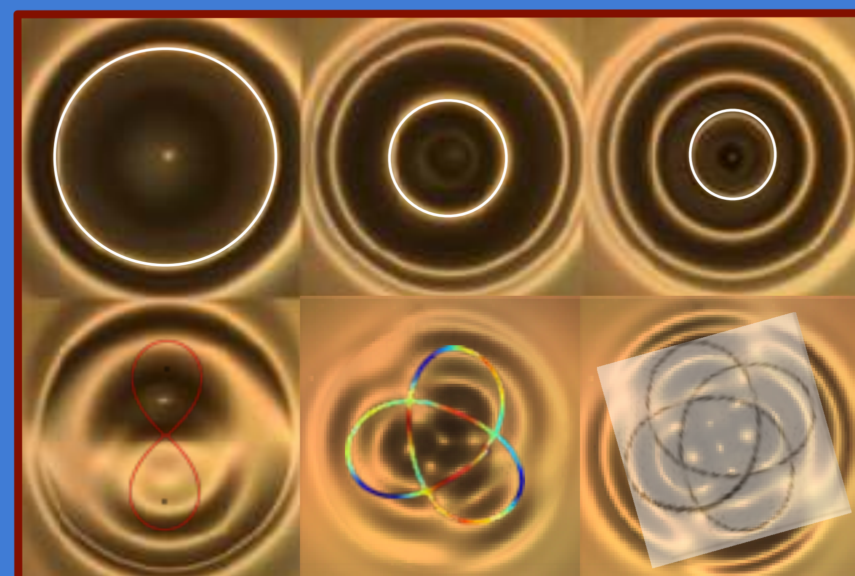
Quantum particles



- superposition of wave modes

Bouncing droplets



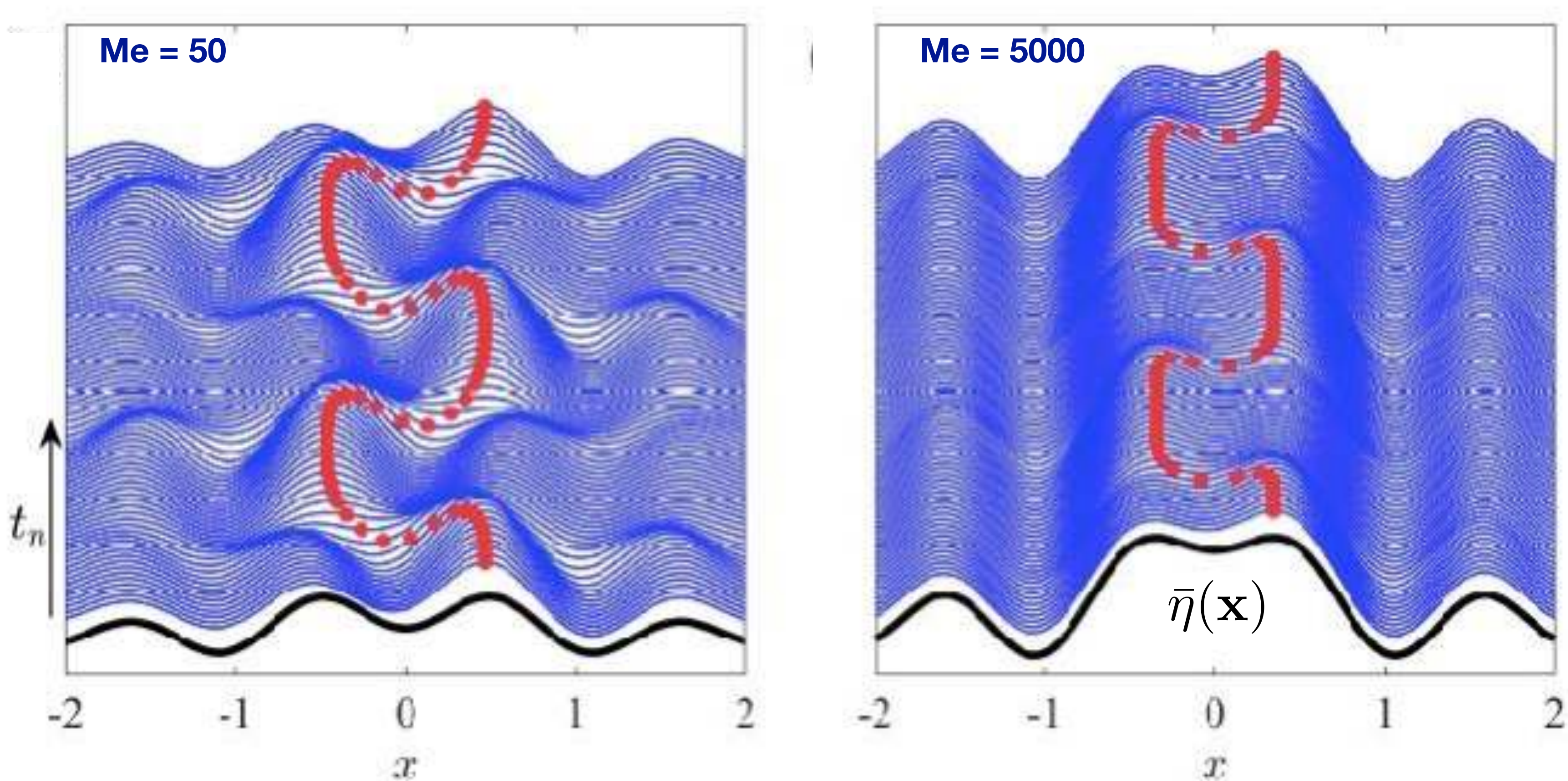


- superposition of subtrajectoriess

Observation (from both experiments and simulations)

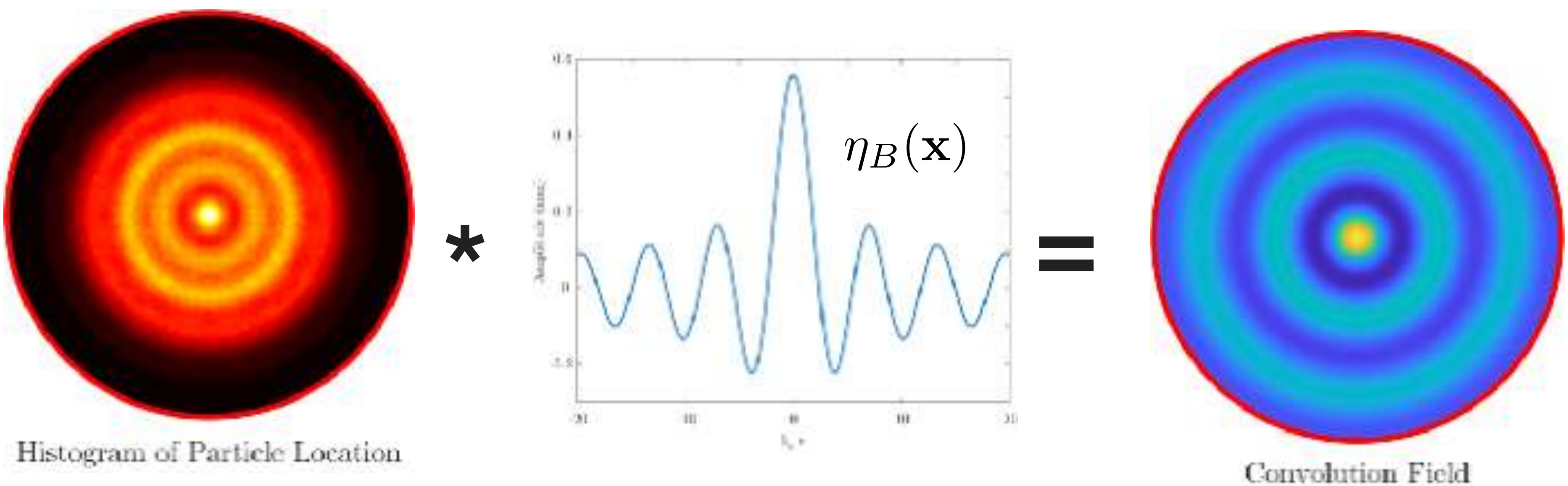
- when walker motion is confined by boundaries or applied forces, the instantaneous pilot wave approaches the mean wave field at high Me

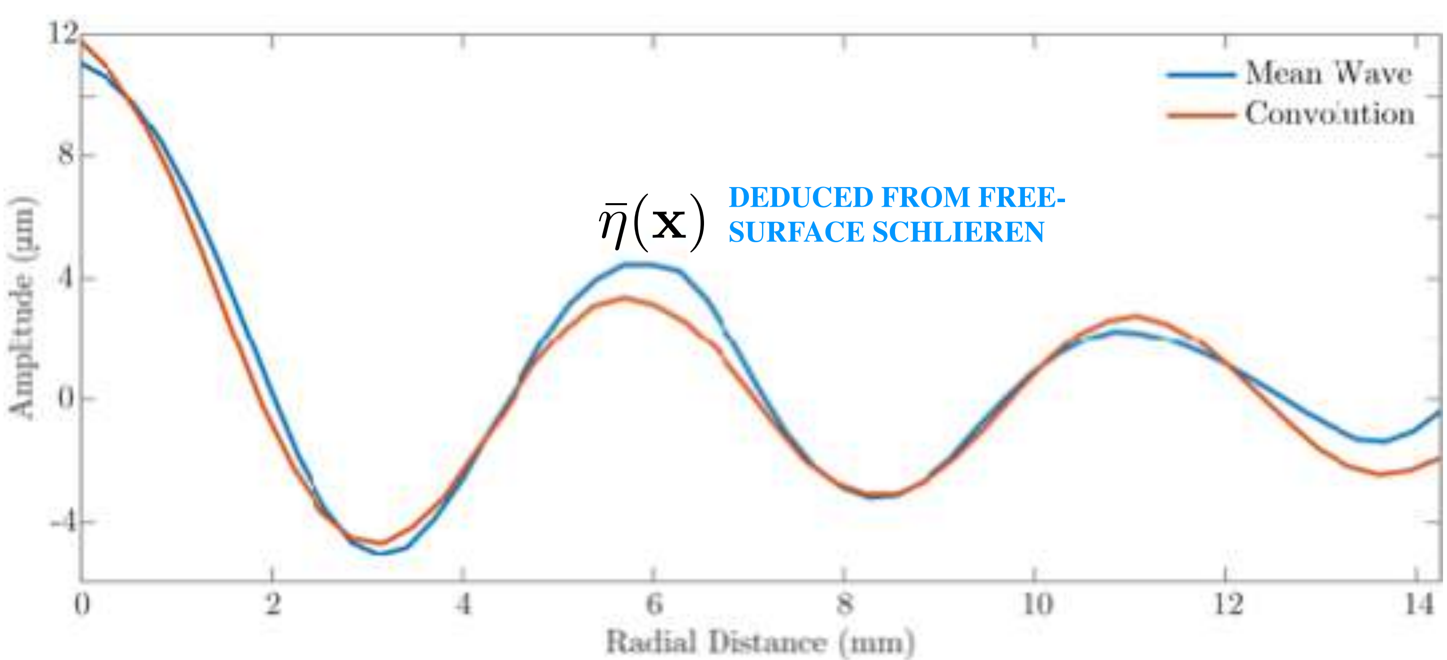
e.g. simulated 1D pilot-wave dynamics in a simple harmonic potential



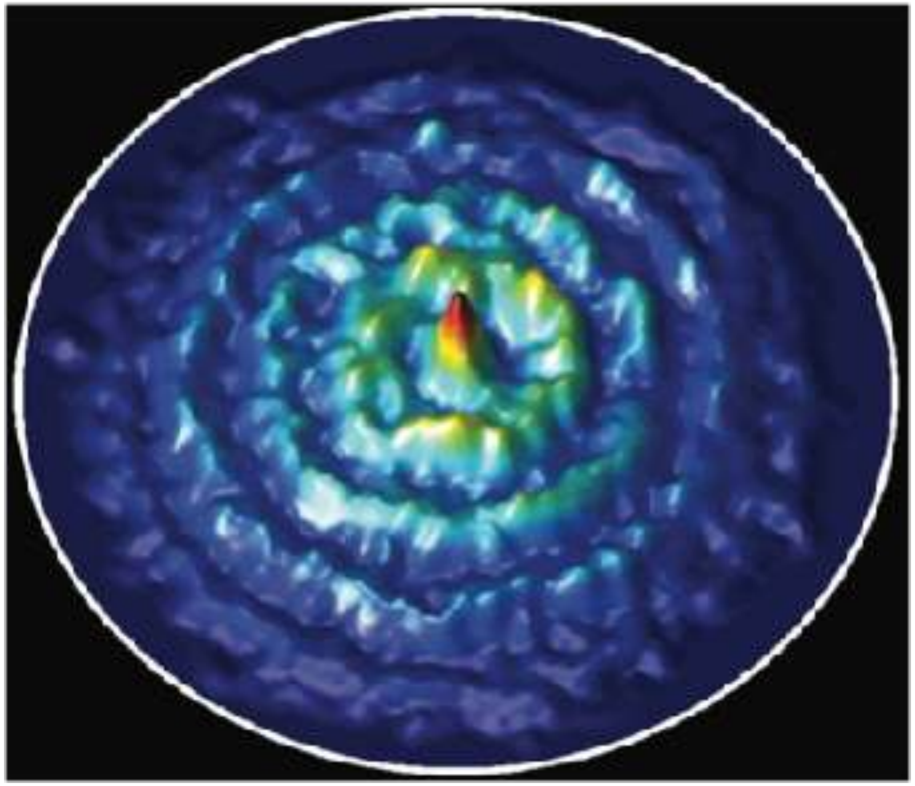
Durey, Milewski & Bush (2018)

The mean pilot-wave field



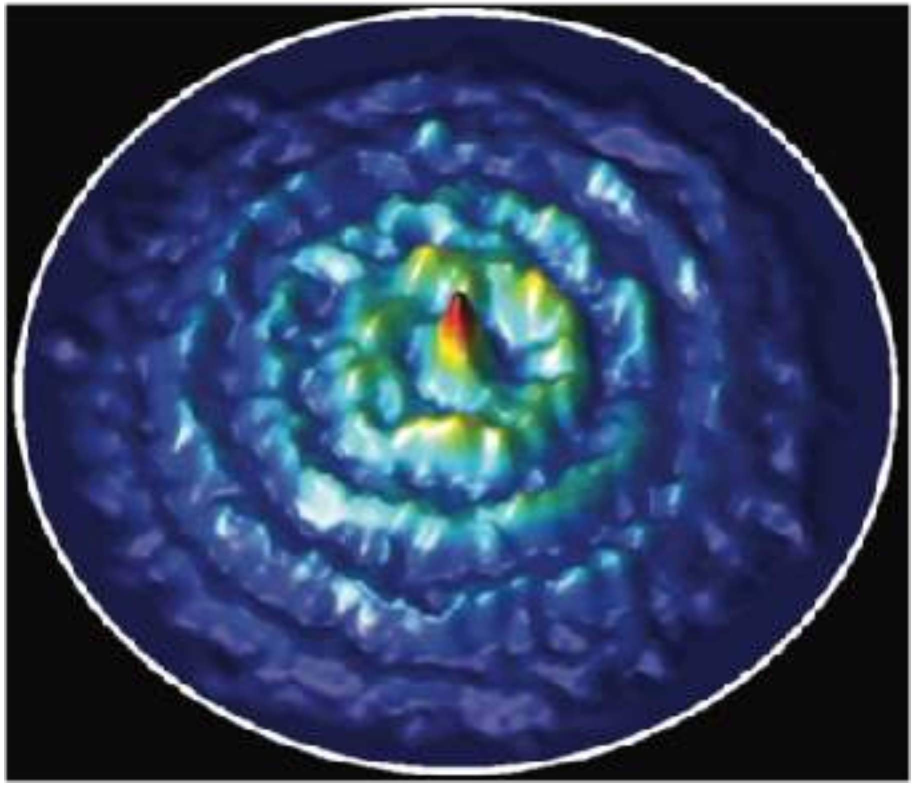


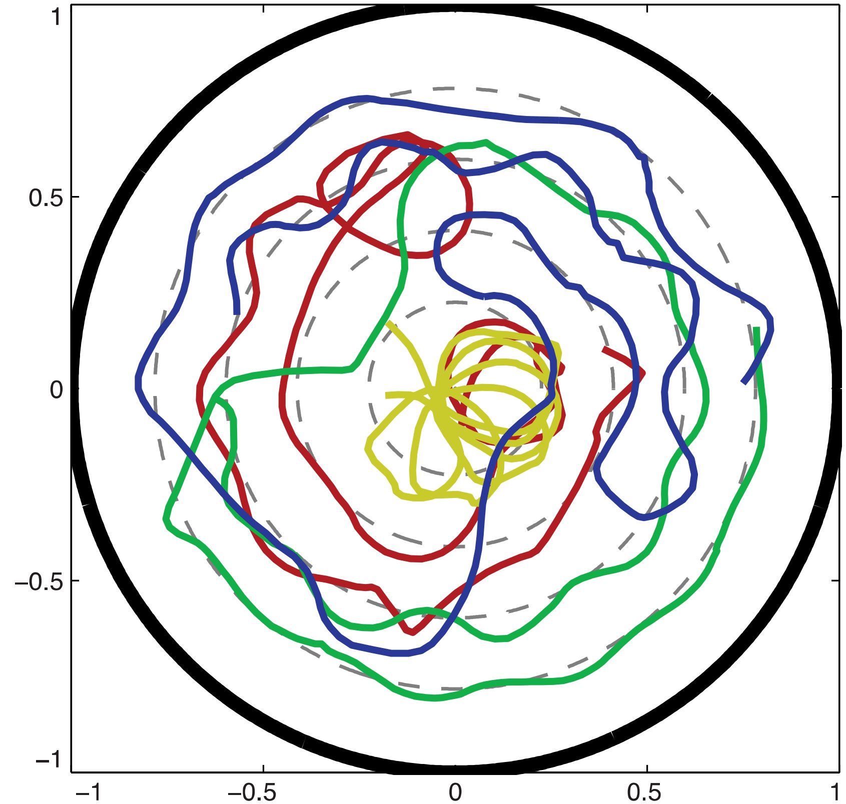
Mechanism for the coherent emergent statistics at hight Me?



- two possible mechanisms have been proposed
- based on the 2 existing HQA paradigms
- their shortcomings have prompted the development of Paradigm III

Paradigm I: orbital suggested by dynamics

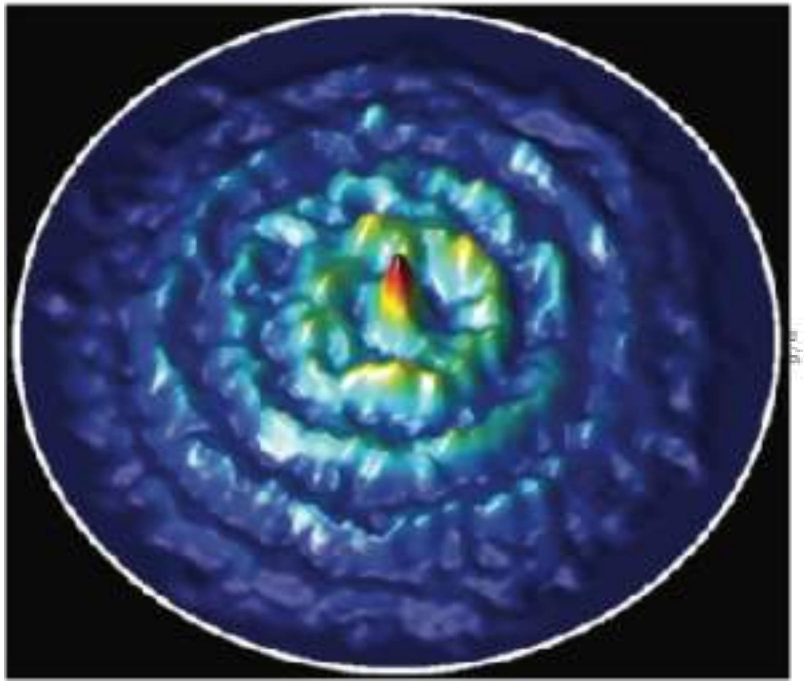


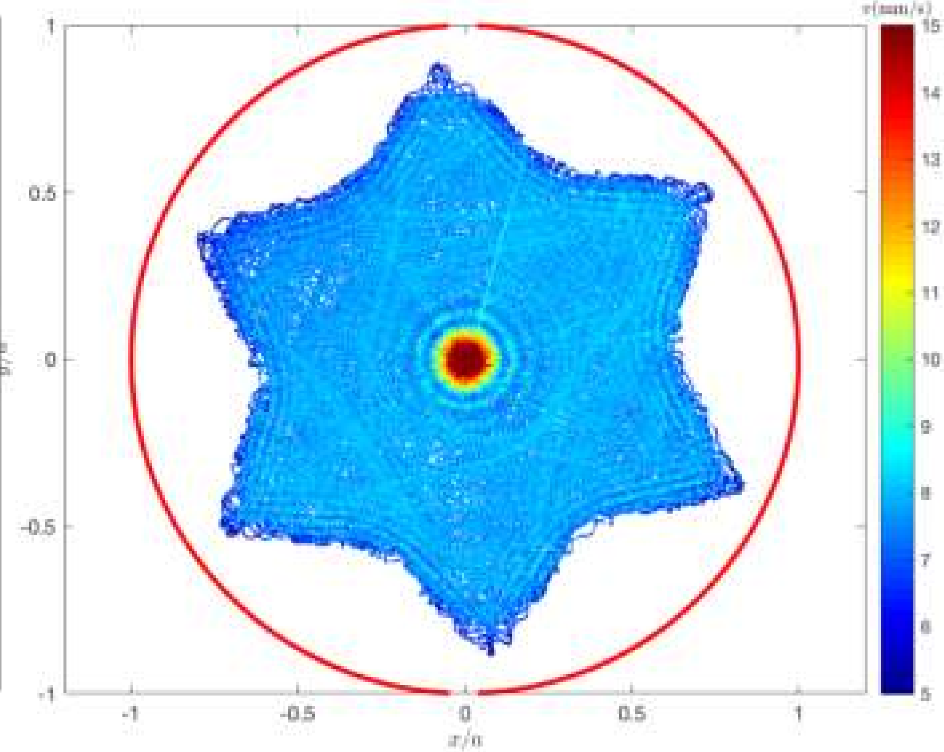


- at low memory, circular orbits along extrema of cavity mode are stable
- at higher memory, these orbits destabilize, yield to chaotic pilot-wave dynamics
- intermittent switching between periodic states results in multimodal statistics

Harris et al. (2013)

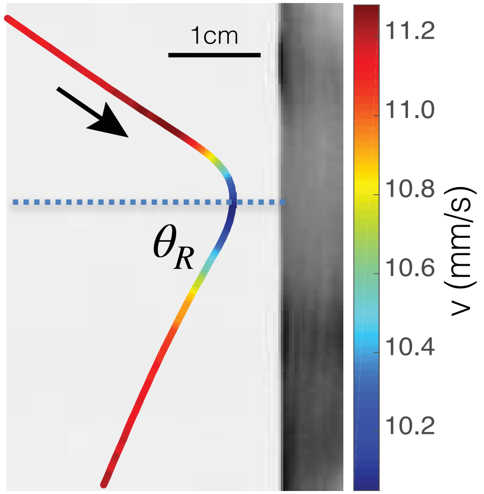
Paradigm II: Friedel oscillations from the outer boundaries



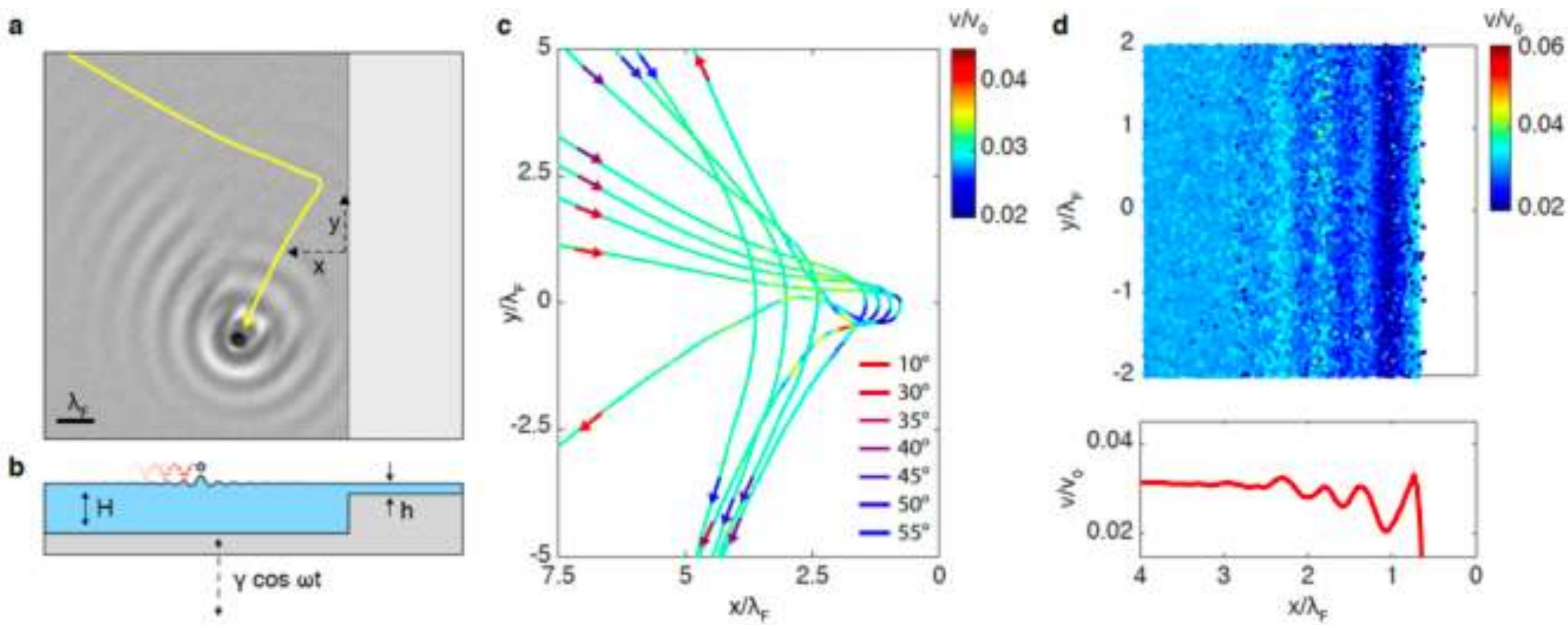


- in-line oscillations with λ_F excited at corral's edge
- preferred reflection angle of $\theta_R = 60^0$ gives rise to statistical signature with wavelength

$$\lambda_F \cos \pi/3 = \lambda_F/2$$



Paradigm II: Friedel oscillations from the outer boundaries



- in-line oscillations with λ_F excited at corral's edge
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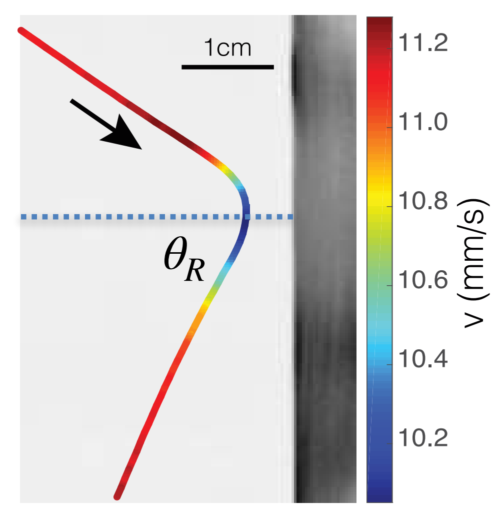


Trajectory Equation in the high memory limit

$$m\,\ddot{\mathbf{x}}_{\mathbf{p}} \;=\; -D\,\dot{\mathbf{x}}_{\mathbf{p}} \;+\; \nabla\eta(\mathbf{x},t) \;-\nabla V$$

- decompose wave force into mean and perturbation components

$$\nabla\eta(\mathbf{x},t) \;=\; \nabla\bar{\eta}(\mathbf{x}) \;+\; \nabla\eta^*(\mathbf{x},t)$$

SMALL

- view the perturbation pilot-wave field as a stochastic forcing

Langevin-type Equation

STOCHASTIC TERM

$$m\ddot{\mathbf{x}}_{\mathbf{p}} \;=\; -D\dot{\mathbf{x}}_{\mathbf{p}} \;+\; \nabla\bar{\eta}(\mathbf{x}) \;+\; \nabla\eta^*(\mathbf{x},t) - \nabla V$$

- mean wave field plays the role of an imposed potential, and is related to the pdf through the convolution relation $\bar{\eta}(\mathbf{x}) \;=\; \eta_B * \mu(\mathbf{x})$
- ***the statistics appear to be driving the dynamics…***

What relation does this physical picture have to quantum mechanics?

The (Old) Hydrodynamic Interpretation of Quantum Mechanics

Schrodinger:

$$i\hbar \, \Psi_t = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V \, \Psi$$

Madelung transformation (1928):

$$\Psi = \sqrt{\rho} \, e^{iS/\hbar}$$

Continuity:

$$\frac{D\rho}{Dt} + \rho \, \nabla \cdot \mathbf{u} = 0$$

Quantum Hamilton-Jacobi:

$$\frac{\partial S}{\partial t} + \frac{1}{2} \mathbf{u}^2 - \frac{\hbar^2}{2m^2} \frac{1}{\sqrt{\rho}} \nabla^2 \sqrt{\rho} + \frac{V}{m} = 0$$

QUANTUM POTENTIAL Q

where $\rho = |\Psi|^2$ is the probability density, S is the action,

$\mathbf{u} = \nabla S/m$ is the quantum velocity of probability,

$\mathbf{j} = \rho \, \mathbf{u}$ is the quantum probability flux.

Bohmian Mechanics (1952)

- equate quantum velocity of probability $\mathbf{u}$ and particle velocity $\dot{\mathbf{x}}_p$
- solve Schrodinger's equation for Ψ , from which Q is computed
- solve trajectory equation

David Bohm

$$m\,\ddot{\mathbf{x}}_p = -\nabla Q - \nabla V$$

Problems

- quantum potential is nonlocal, imposed by fiat
- requires $|\Psi|^2$ distribution as initial conditions in order for results to be equivalent to those of standard QM (Keller 1966) ; e.g. in corral

Extensions (Bohm & Vigier 1954)

- invoke a stochastic forcing $\nabla\Phi_S$ from a `sub quantum realm':

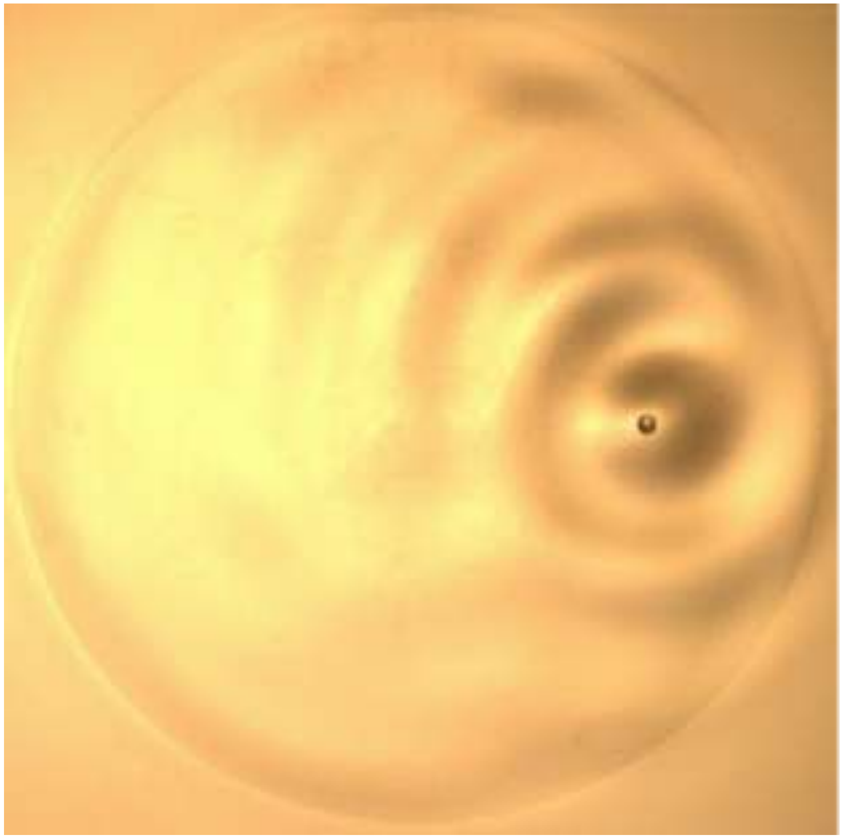
$$m\,\ddot{\mathbf{x}}_p = -\nabla Q - \nabla V + \nabla\Phi_S$$

- particles jostle about $\mathbf{u}$ like Brownian motion of gas molecules about streamlines

	Bohmian mechanics	Walkers
WAVELENGTH	λ_B	λ_F
GUIDANCE	$m\,\ddot{\mathbf{x}}_p \;=\; -\nabla Q \;-\; \nabla V \;+\; \nabla\Phi_S$	$m\,\ddot{\mathbf{x}}_\mathbf{p} \;=\; -D\,\dot{\mathbf{x}}_\mathbf{p} \;+\; \nabla\eta(\mathbf{x},t) \;-\; \nabla V$
WAVE POTENTIAL	$\mathbf{Q} = -\frac{\hbar^2}{m^2}\frac{1}{\sqrt{\rho}}\nabla^2\sqrt{\rho}$ QUANTUM POTENTIAL	$\bar{\eta}(\mathbf{x}) \;=\; \eta_B * \mu(\mathbf{x})$ MEAN WAVE FIELD
STOCHASTIC FORCING	$\nabla\Phi_S$ **ARBITRARY,** ***ad hoc***	$-\nabla\eta^*(\mathbf{x},t)$ PERTURBATION WAVE FIELD
WAVE ORIGIN	NONE	PARTICLE VIBRATION

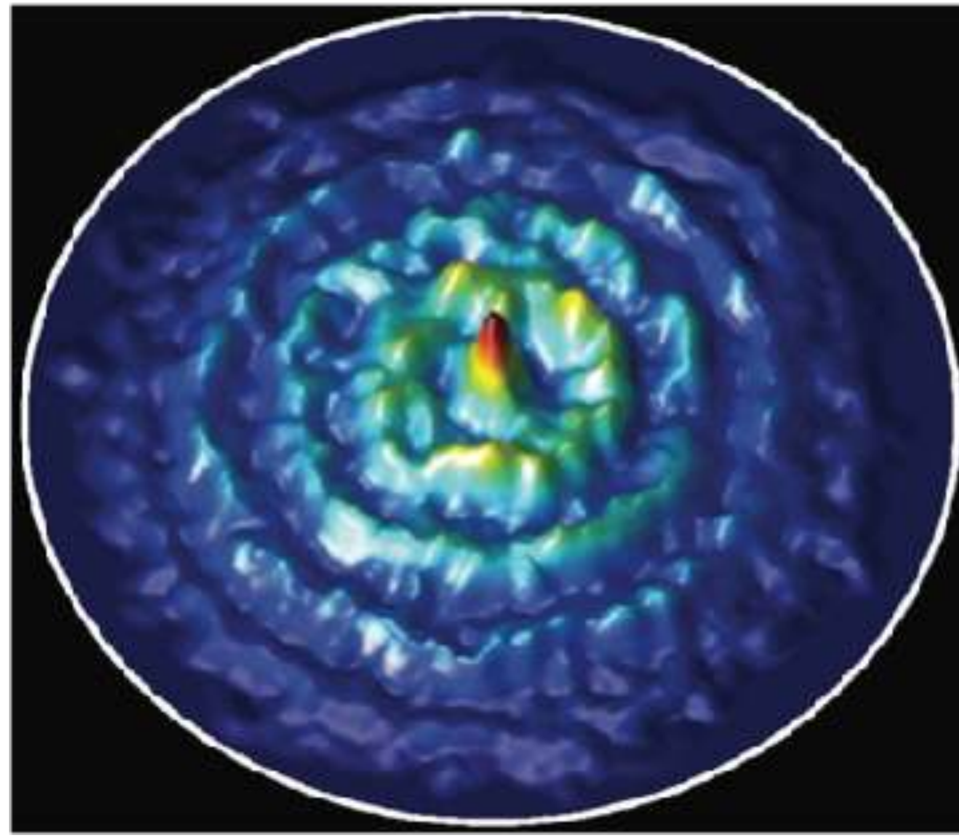
This is NOT Bohmian mechanics!

Time-dependent pilot-wave



Particle's histogram

`Bohmian' pilot wave = time-averaged pilot wave

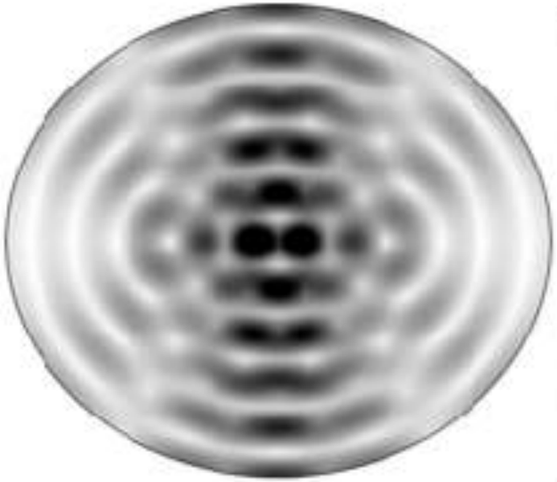


- Bohm: predictions are consistent if you choose ICs appropriately
- Joe Keller: "A self-consistent theory need not impose prescribed ICs."
- Bohm: ``You can never have a pure cavity mode".

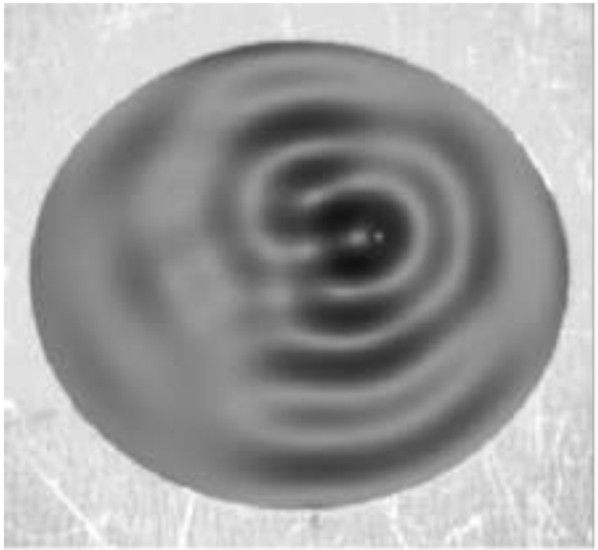
This is NOT Bohmian mechanics, but…

- can we formulate a Bohmian mechanics to describe the mean velocity?

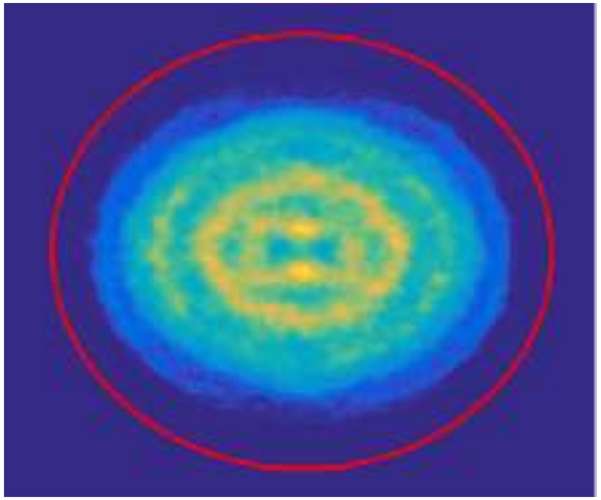
Cavity mode



Time-dependent pilot-wave



`Bohmian' pilot wave = time-averaged pilot wave

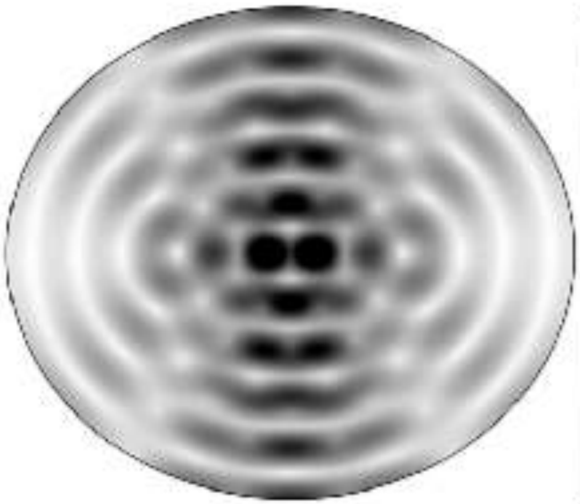


Particle's histogram

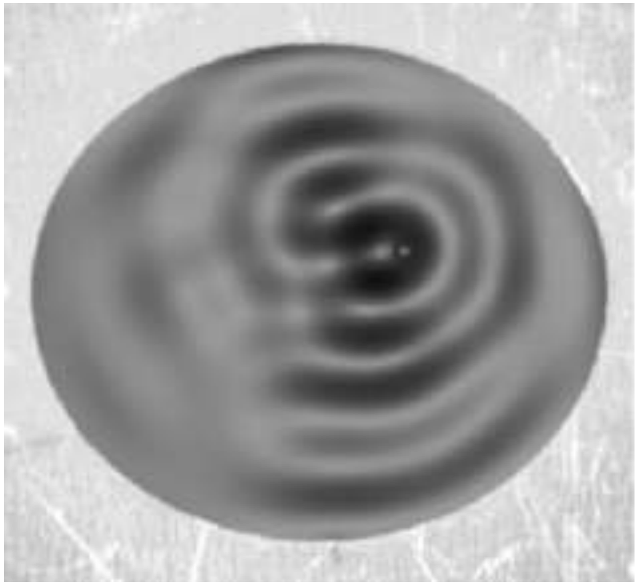
Walkers suggest a means of revising Bohmian mechanics

- consider high-frequency particle vibrations as the source of pilot wave
- the mean-pilot-wave field is related to the emergent statistical waveform (analog of Q)
- the instantaneous pilot wave differs owing to the disturbance induced by particle

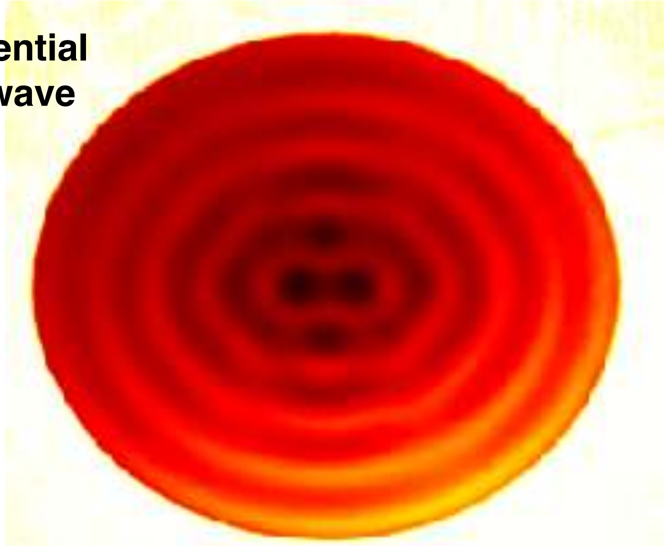
Cavity mode



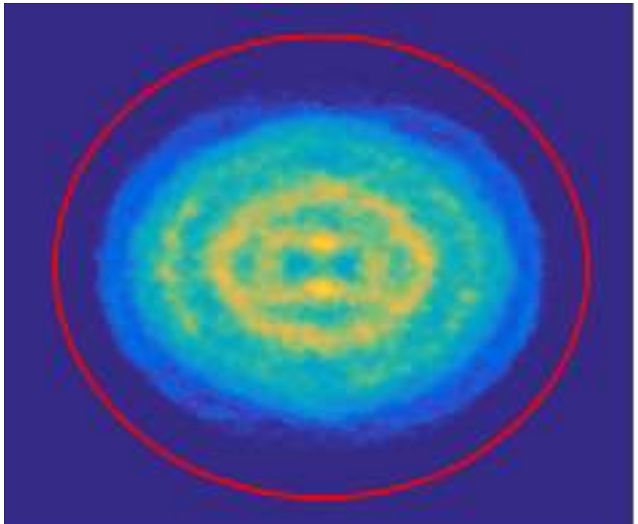
Instantaneous pilot wave



`Bohmian' potential = mean pilot wave



Particle histogram



de Broglie's pilot-wave theory: The double-wave solution

" A freely moving body follows a trajectory that is orthogonal to the surfaces of an associated wave guide".

- **Louis de Broglie (1892-1987)**

- Ψ is the probability wave, as prescribed by standard quantum theory
- $\Psi^{dB} = |\Psi^{dB}|\, e^{i\phi/\hbar}$ is a real physical wave responsible for guiding the particle
- wave generated by internal particle vibration (*Zitterbewegung*) at the Compton frequency: $\omega_c = \dfrac{m_0 c^2}{\hbar}$

- a solution of Klein-Gordon equation triggered by oscillations in rest mass
- particle pushed perpendicular to surfaces of constant phase:

  $\mathbf{p} = m\dot{\mathbf{x}}_\mathbf{p} = \nabla\phi = \hbar\mathbf{k}$ for a monochromatic wave $\Psi^{dB} = |\Psi^{dB}| e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$
- **Harmony of Phases**: the particle oscillates in resonance with its guiding wave
- Ψ^{dB} *differs from* Ψ *owing to nonlinearity in the vicinity of the particle*

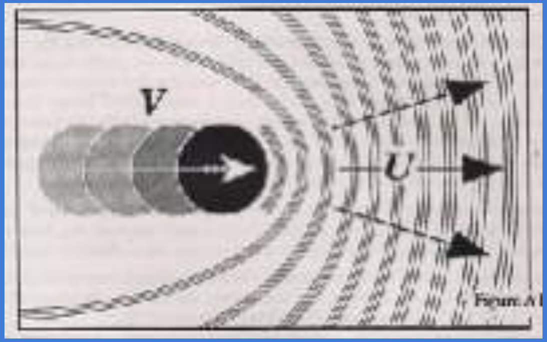
de Broglie's pilot-wave theory

- fast dynamics: mass oscillations at

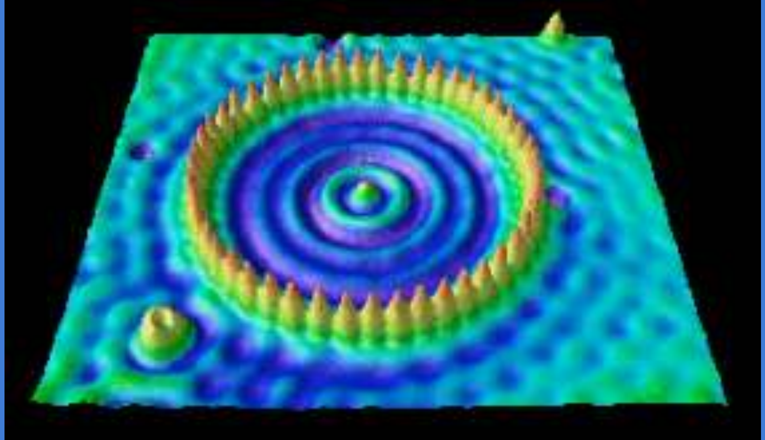
$$\omega_c = \frac{m_0 c^2}{\hbar}$$ create wave field centered on particle

- intermediate pilot-wave dynamics: particle rides its guiding wave field

$$\mathbf{p} = \hbar\, \mathbf{k}$$



- long-term statistical behaviour described by wave function of standard quantum theory



Fiete & Heller (2003)

Subsequent work on corrals

— unpublished experiments

Later work … Wave mode analysis

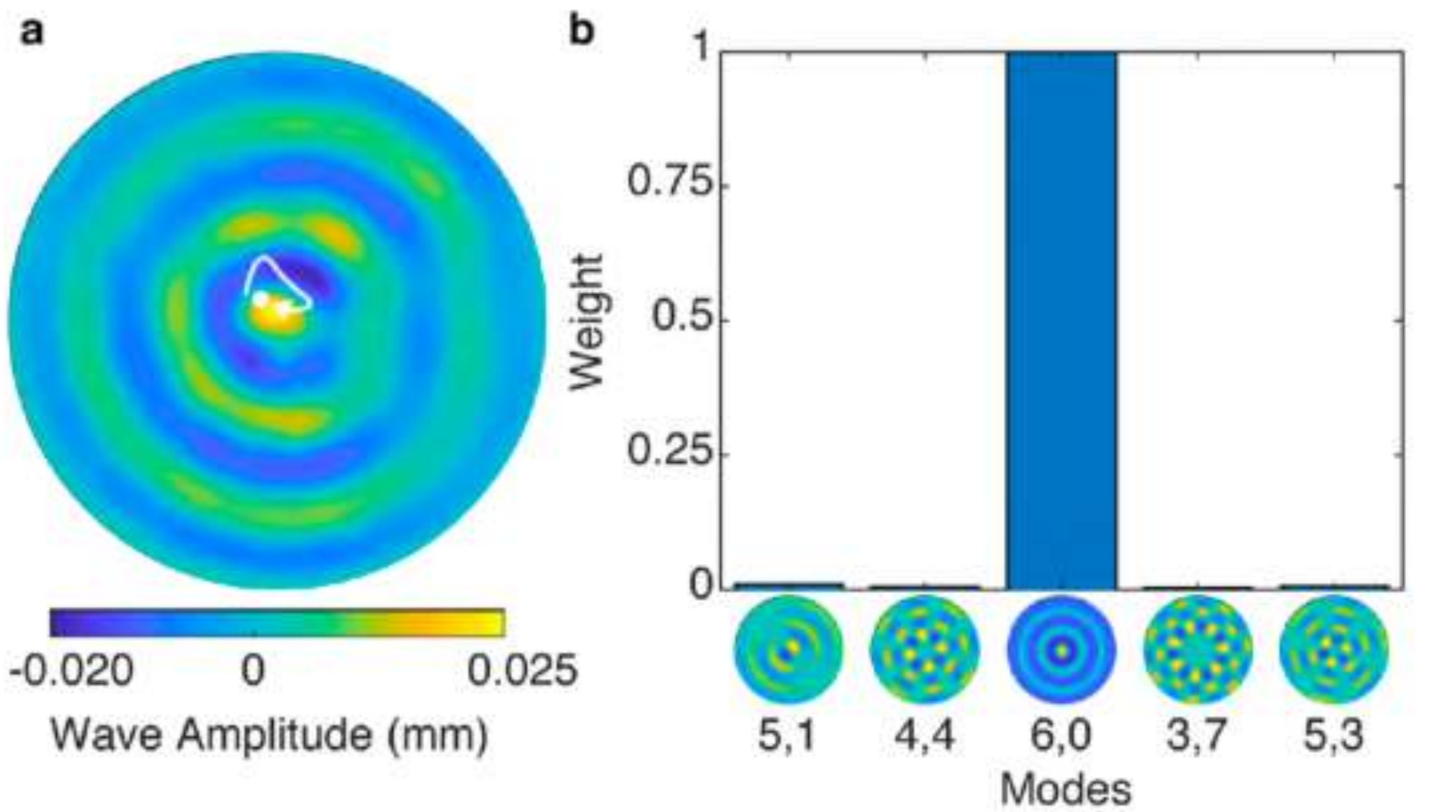


Figure A-26: Azimuthally symmetric instantaneous wave field when the droplet is exploring the circular corral of diameter $D = 28.5$ mm, vibrated vertically at $f = 70$ Hz. **a** Instantaneous wave field displaying the dominantly described by one azimuthally symmetric mode. Overlaid in white is an example of the walker trajectory (of duration ~ 1 s). **b** The weights of the five most prominent modes present in the reconstruction of the mean wave field.

Wave mode analysis

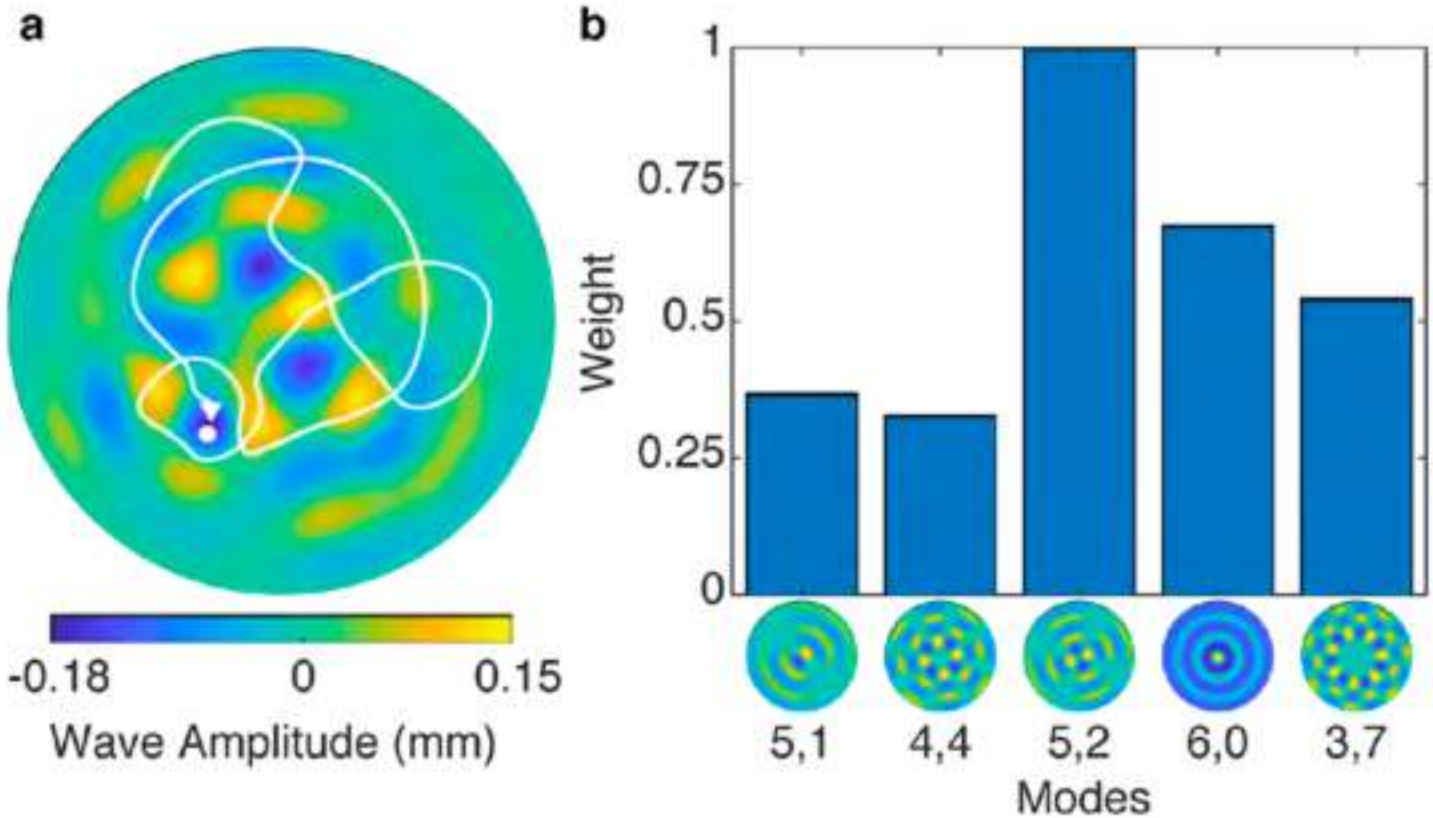


Figure A-25: Instantaneous wave field when the droplet is exploring the circular corral of diameter $D = 28.5$ mm, vibrated vertically at $f = 70$ Hz. **a** An arbitrary selected instantaneous wave field. Overlaid in white is an example of the walker trajectory (of duration ~ 10 s). **b** The weights of the five most dominant modes present in the reconstruction of an arbitrary selected instantaneous wave field.

Wave mode analysis

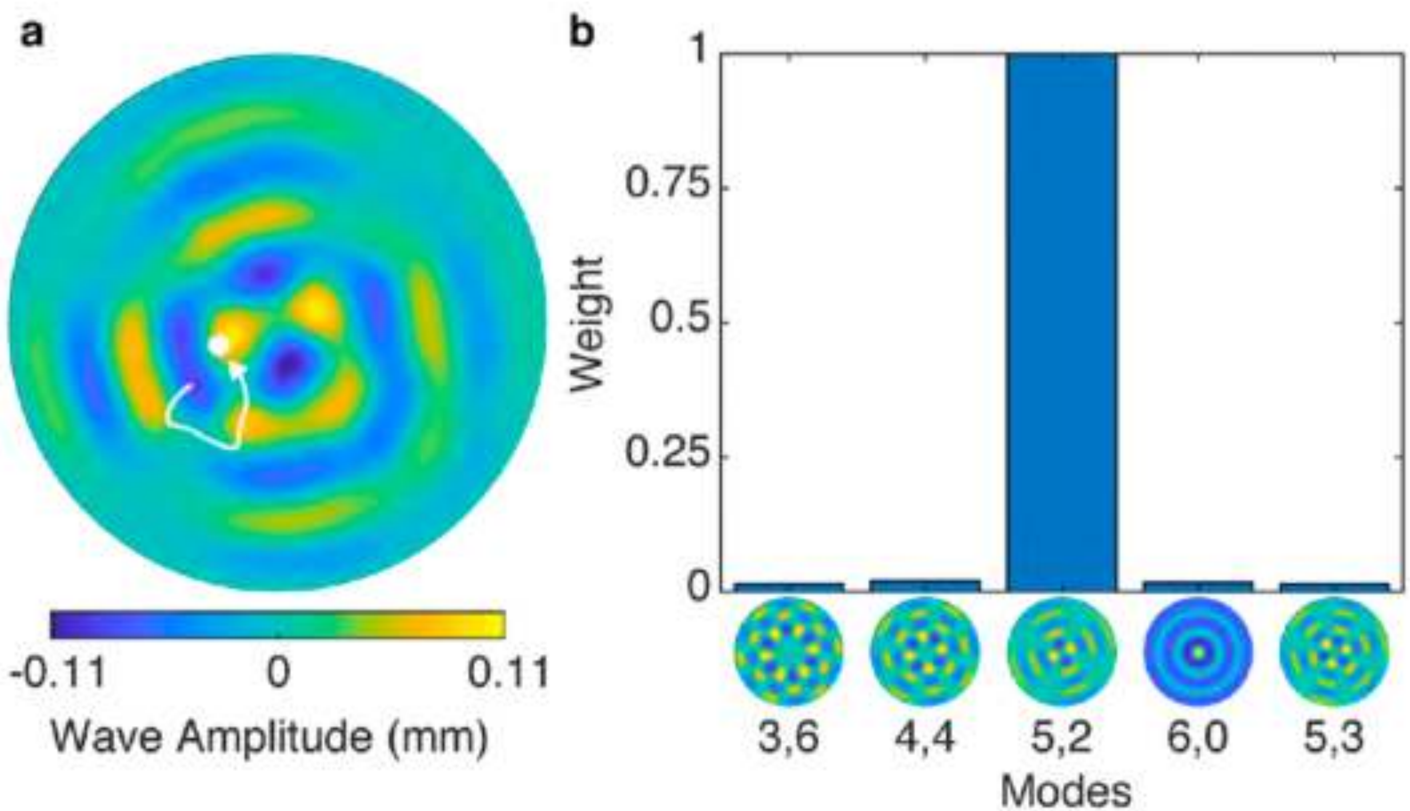
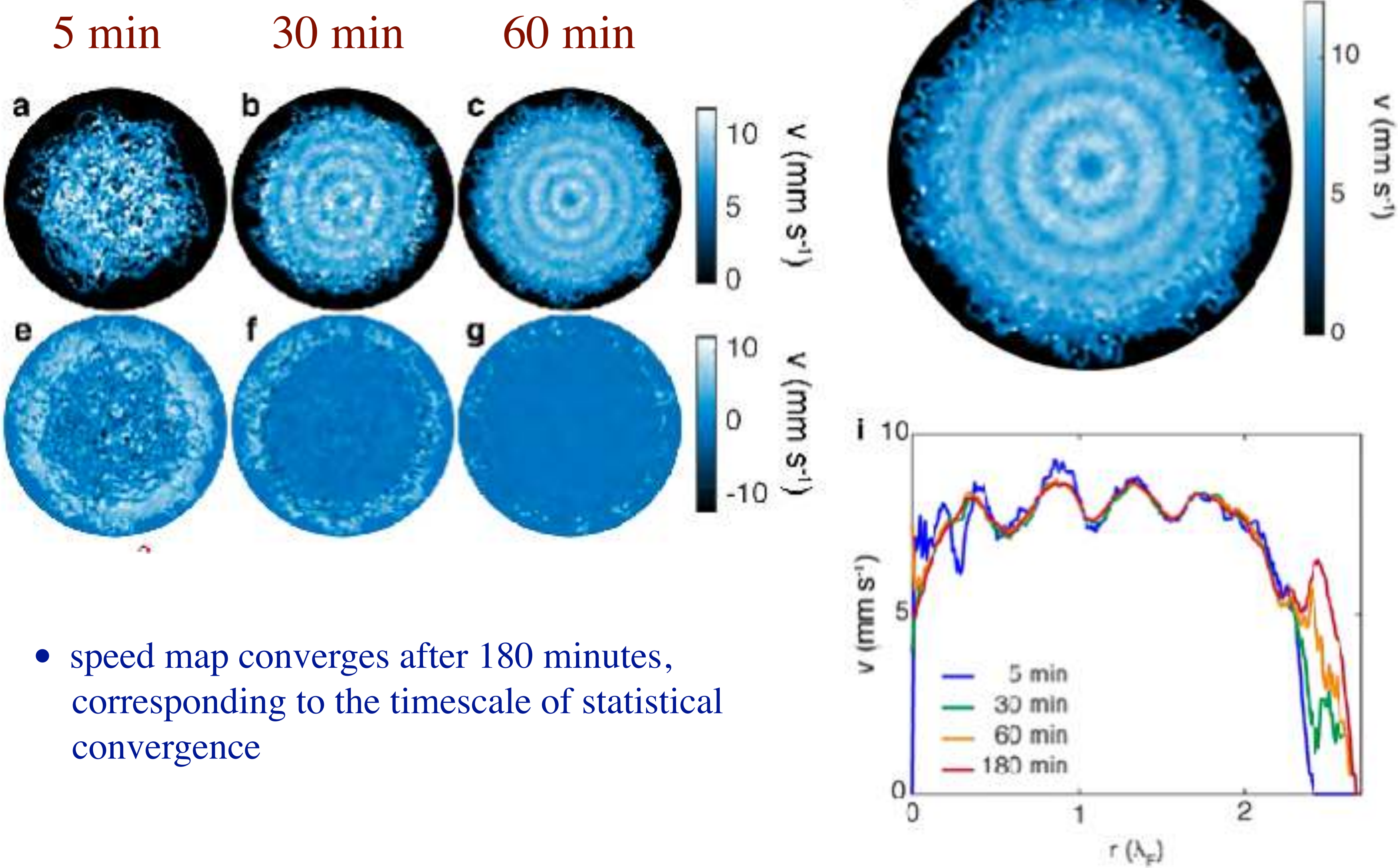


Figure A-27: Non-azimuthally symmetric instantaneous wave field when the droplet is exploring the circular corral of diameter $D = 28.5$ mm, vibrated vertically at $f = 70$ Hz. **a** Instantaneous wave field displaying the dominantly described by one non-azimuthally symmetric mode. Overlaid in white is an example of the walker trajectory (of duration ~ 1 s). **b** The weights of the five most prominent modes present in the reconstruction of the mean wave field.

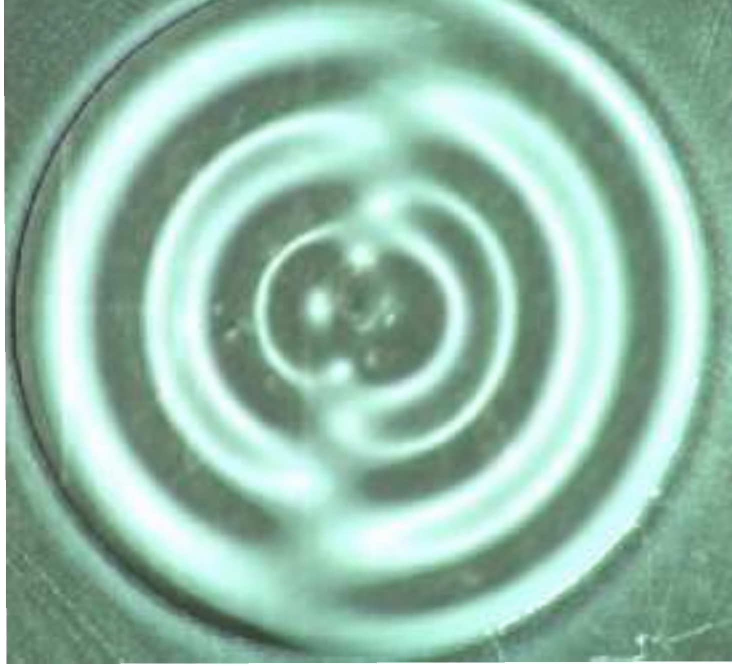
Average speed map

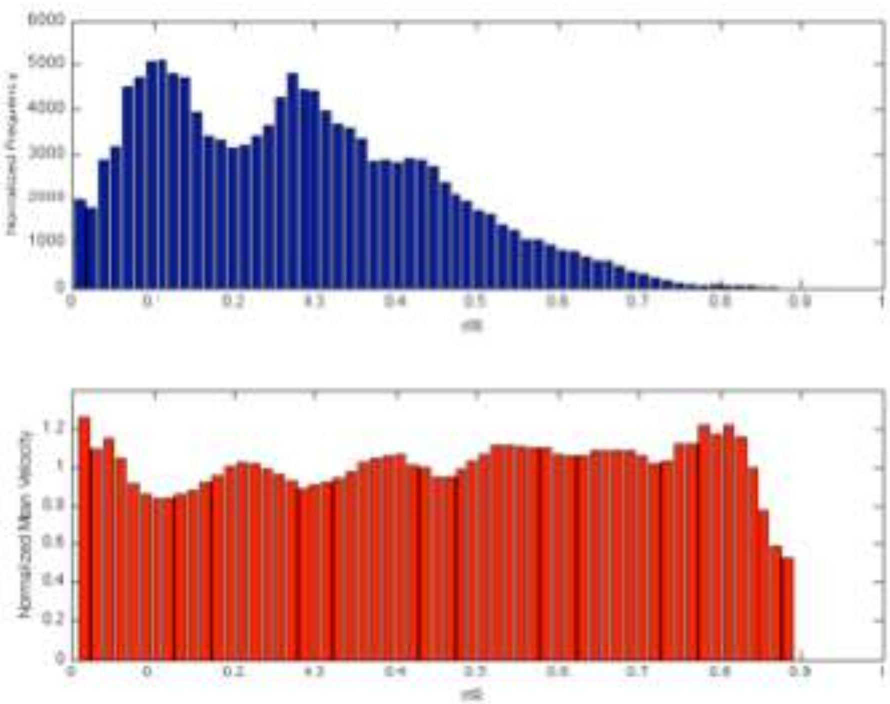


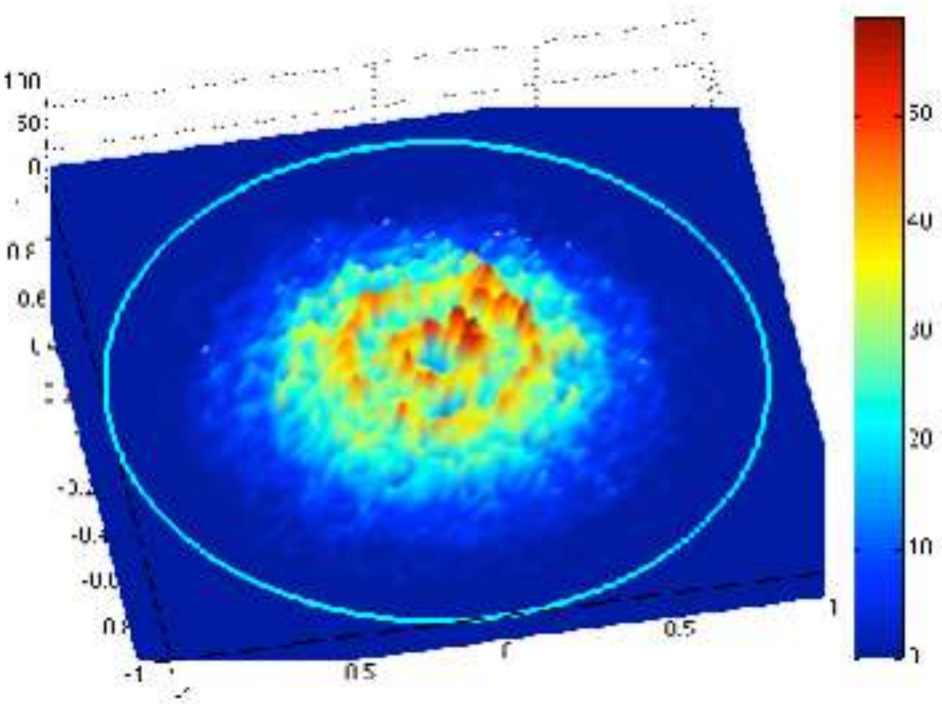
- speed map converges after 180 minutes, corresponding to the timescale of statistical convergence

Non-axisymmetric modes?

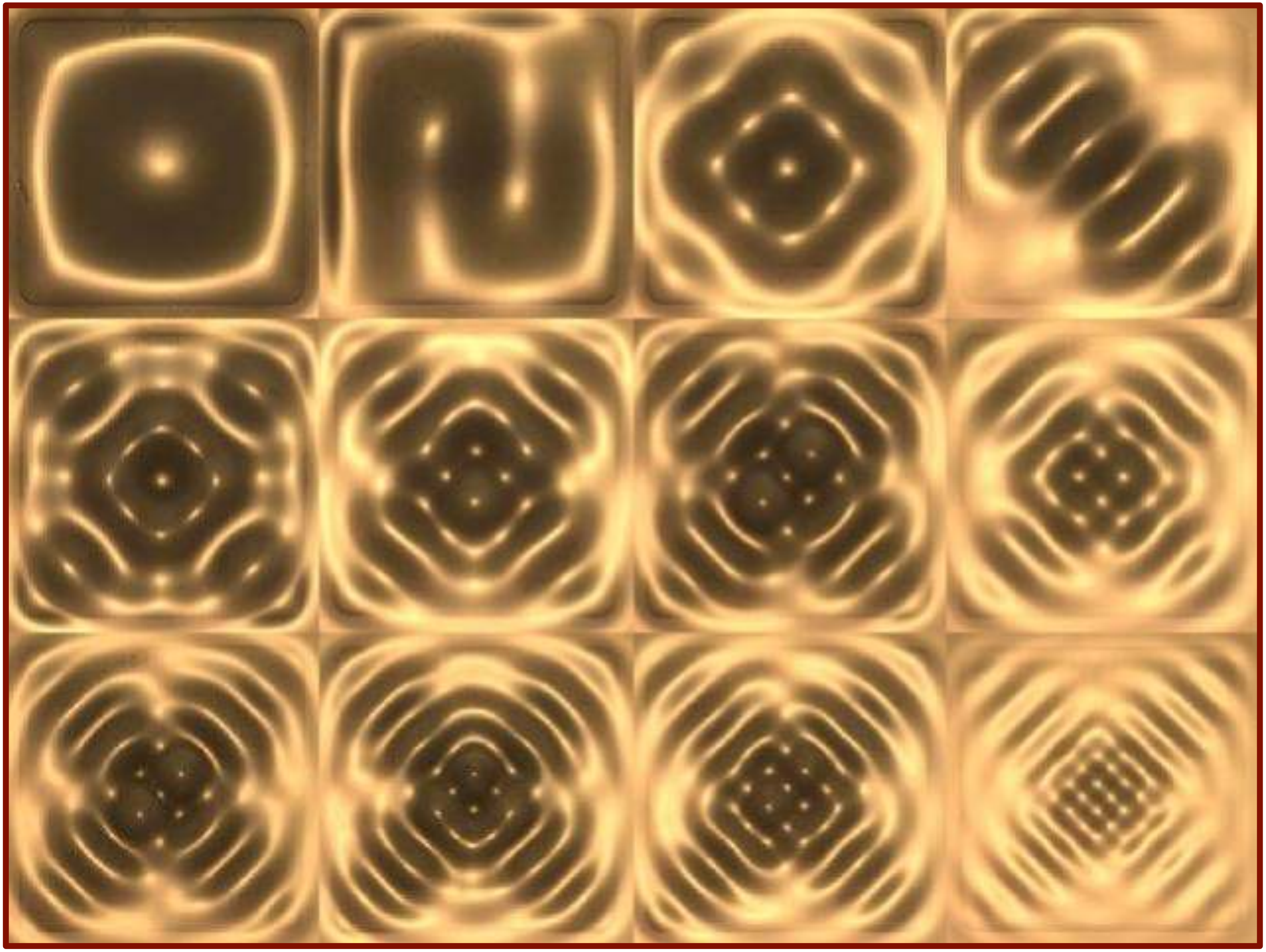
- non-axisymmetry lost in long-term statistics, since waves mostly generated by the walker
- observed pdf represents azimuthally averaged wave amplitude
- or is the minimum on axis retained?



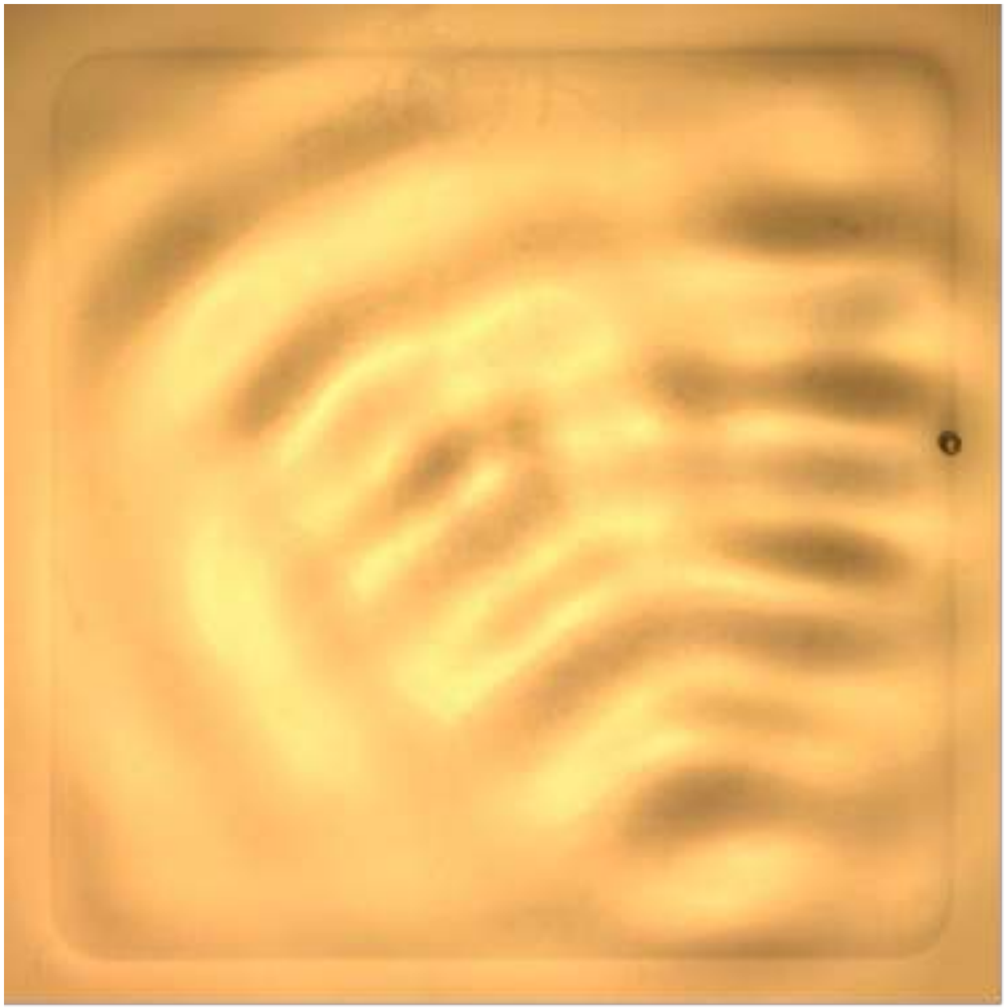


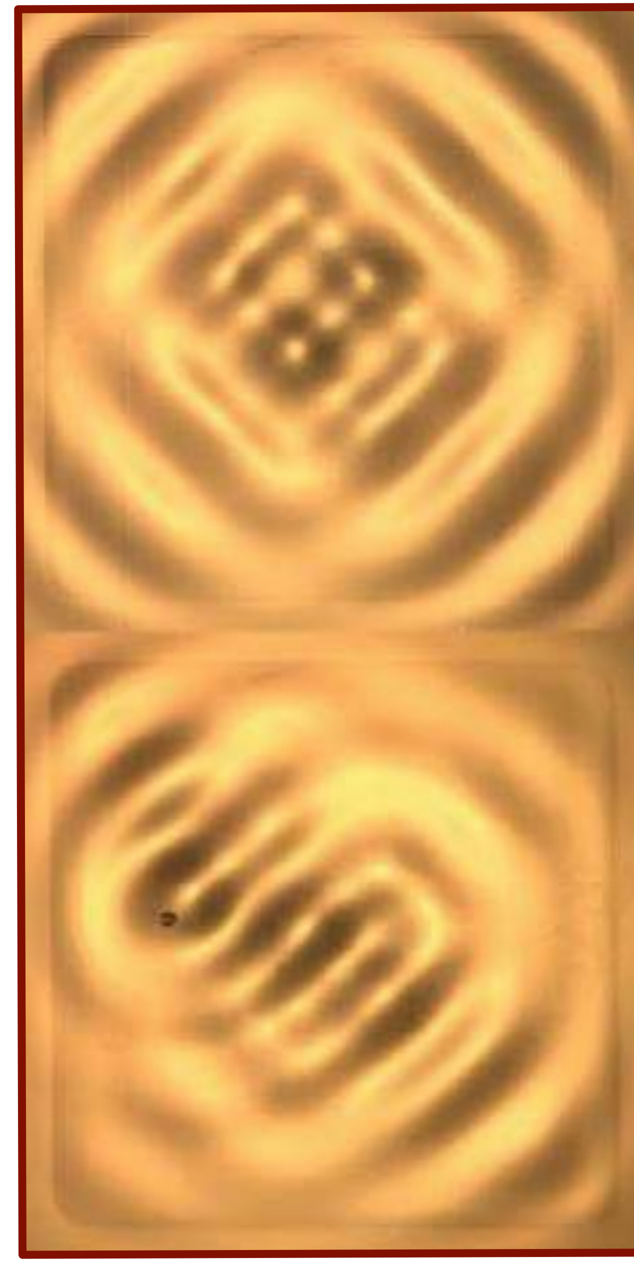


Modes of the square corral



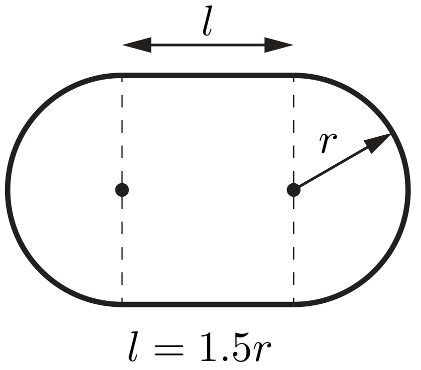
Walker in a square corral



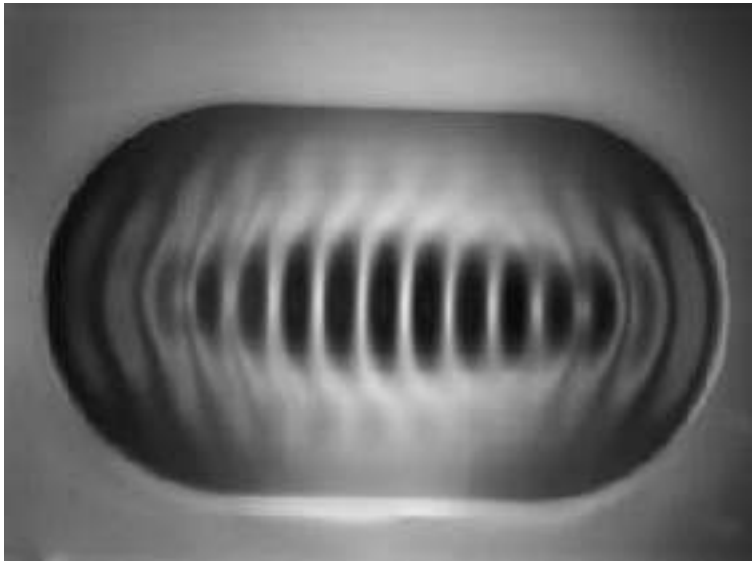


- the walker excites and explores the resonant wave field of the cavity

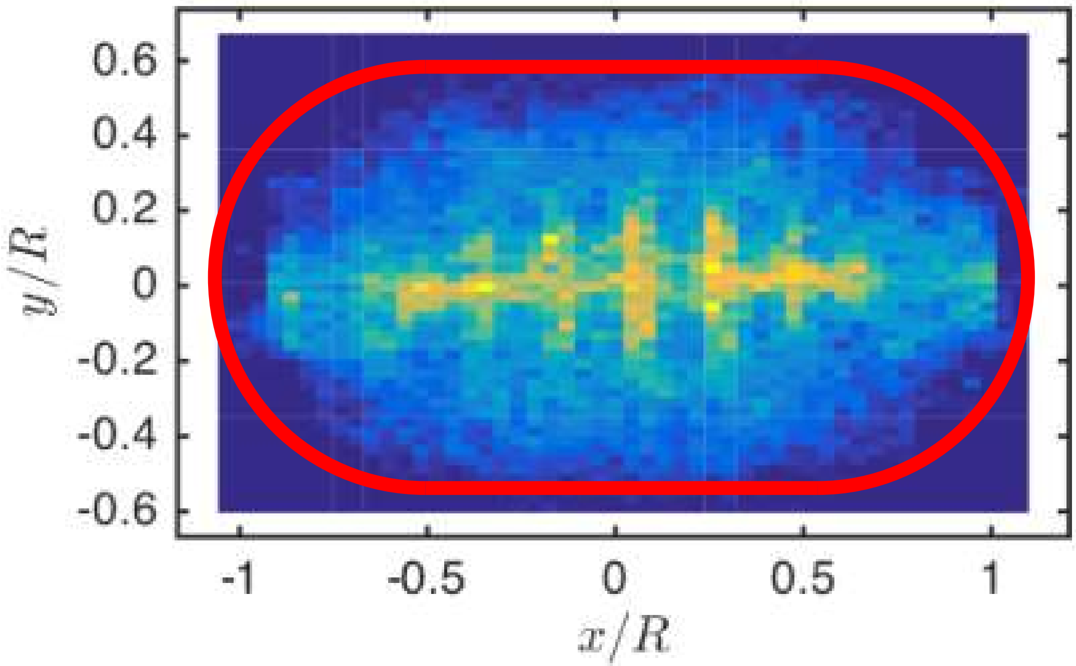
Stadium corral



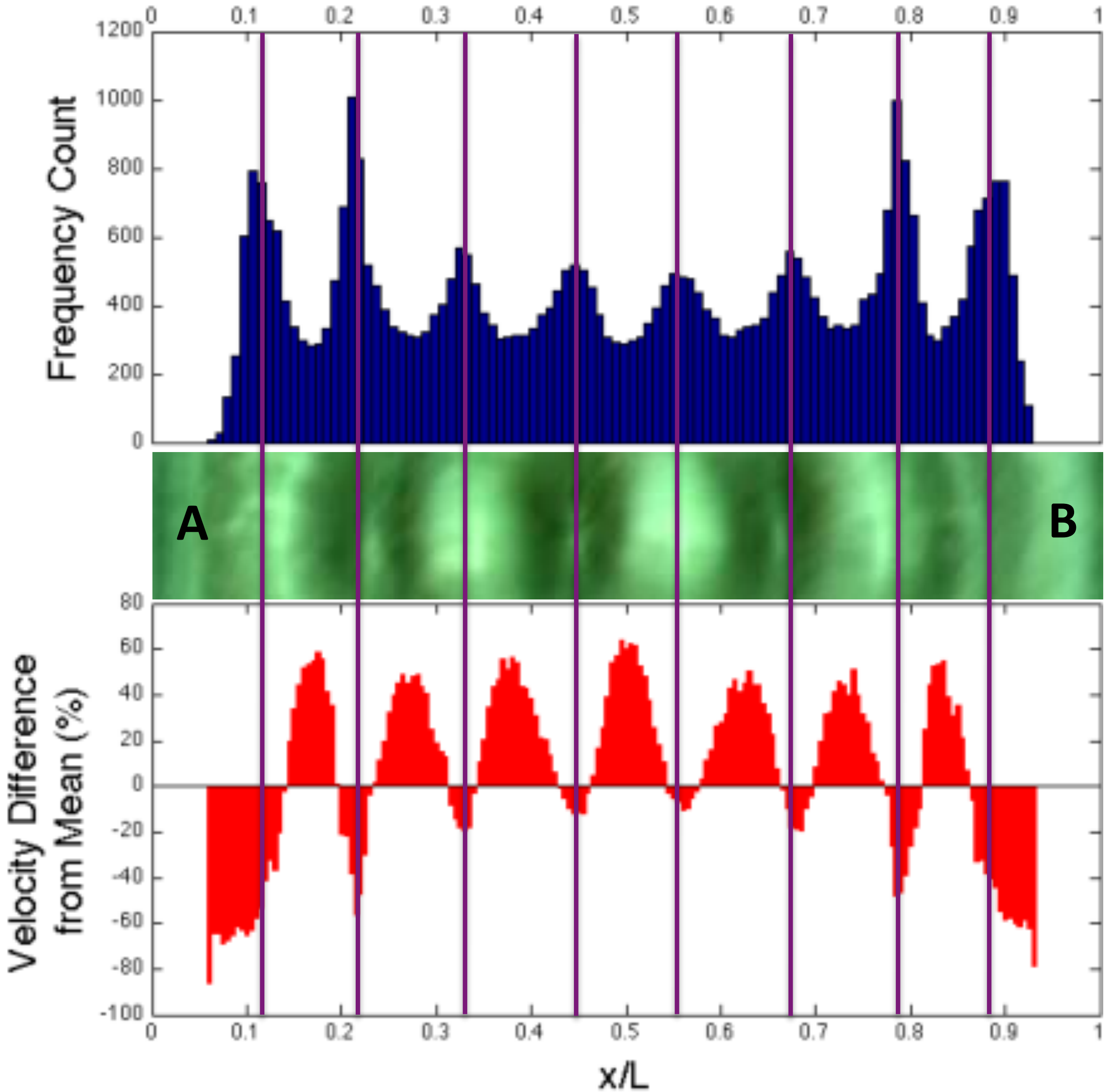
Faraday waves

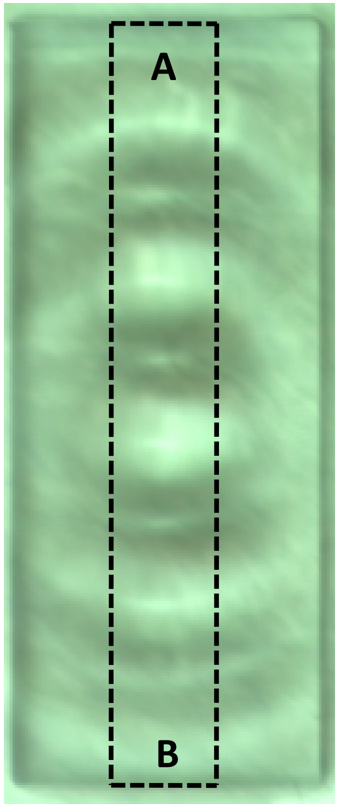


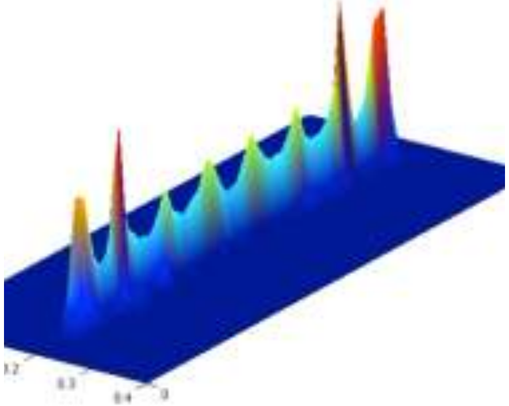
Walker histogram

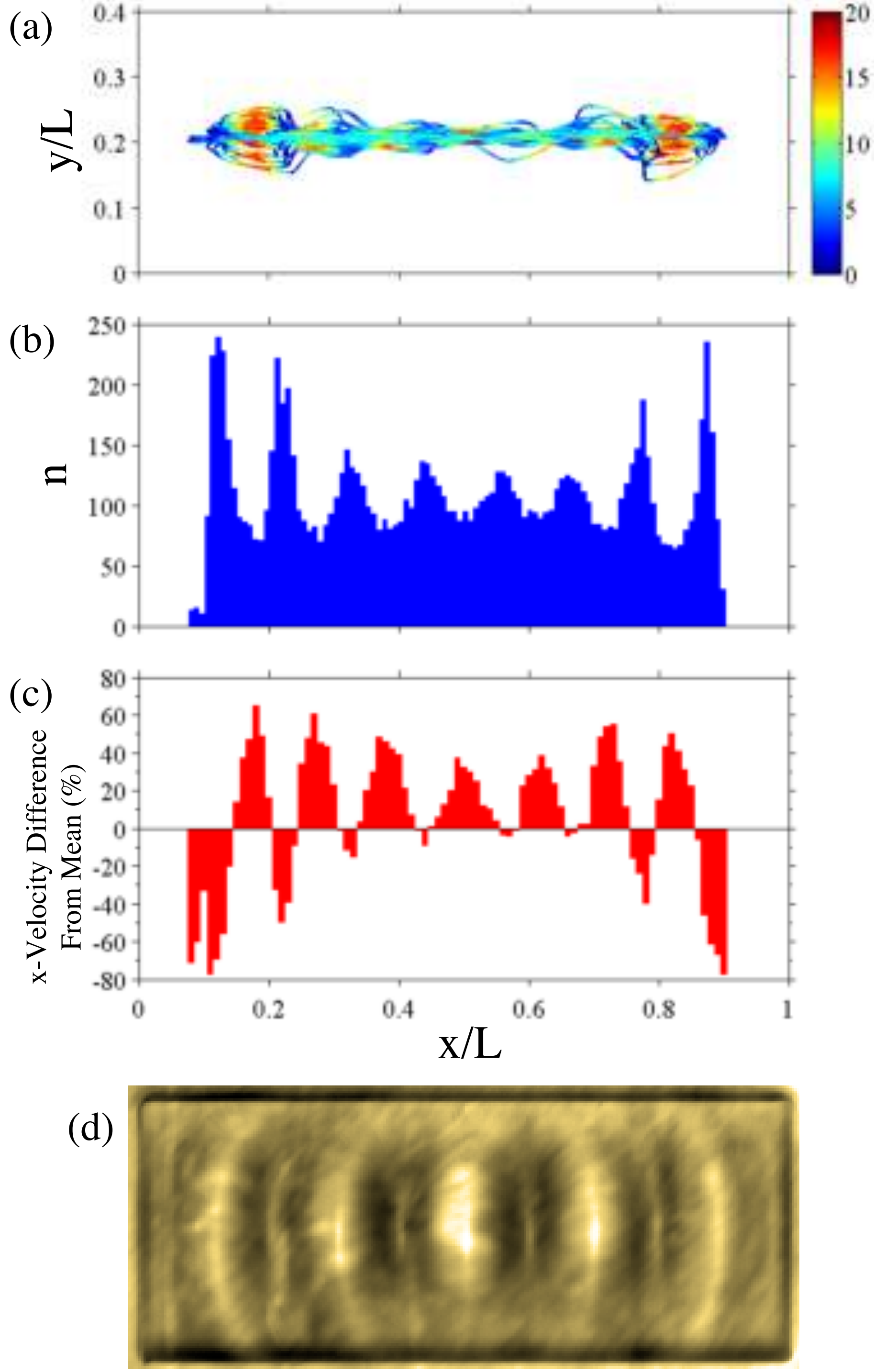


A droplet walking in a narrow rectangular corral



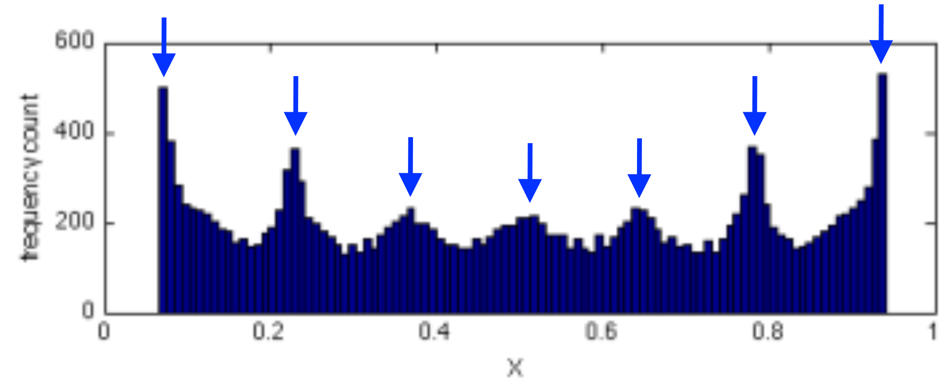




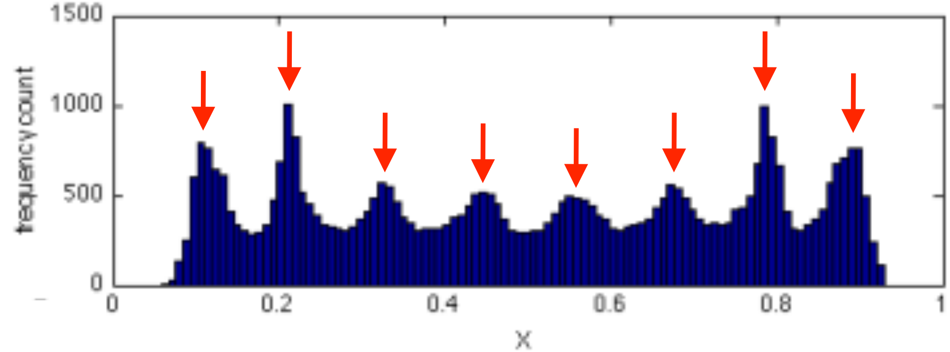


(a)
y/L
0.4
0.3
0.2
0.1
0
20
15
10
5
0
(b)
n
250
200
150
100
50
0
(c)
x-Velocity Difference
From Mean (%)
80
60
40
20
0
-20
-40
-60
-80
0
0.2
0.4
0.6
0.8
1
x/L
(d)

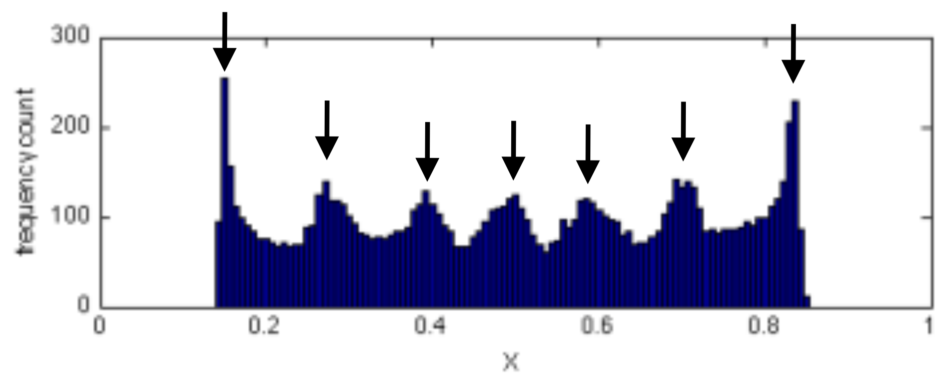
$\bar{v} = 9.63\ mm/s$



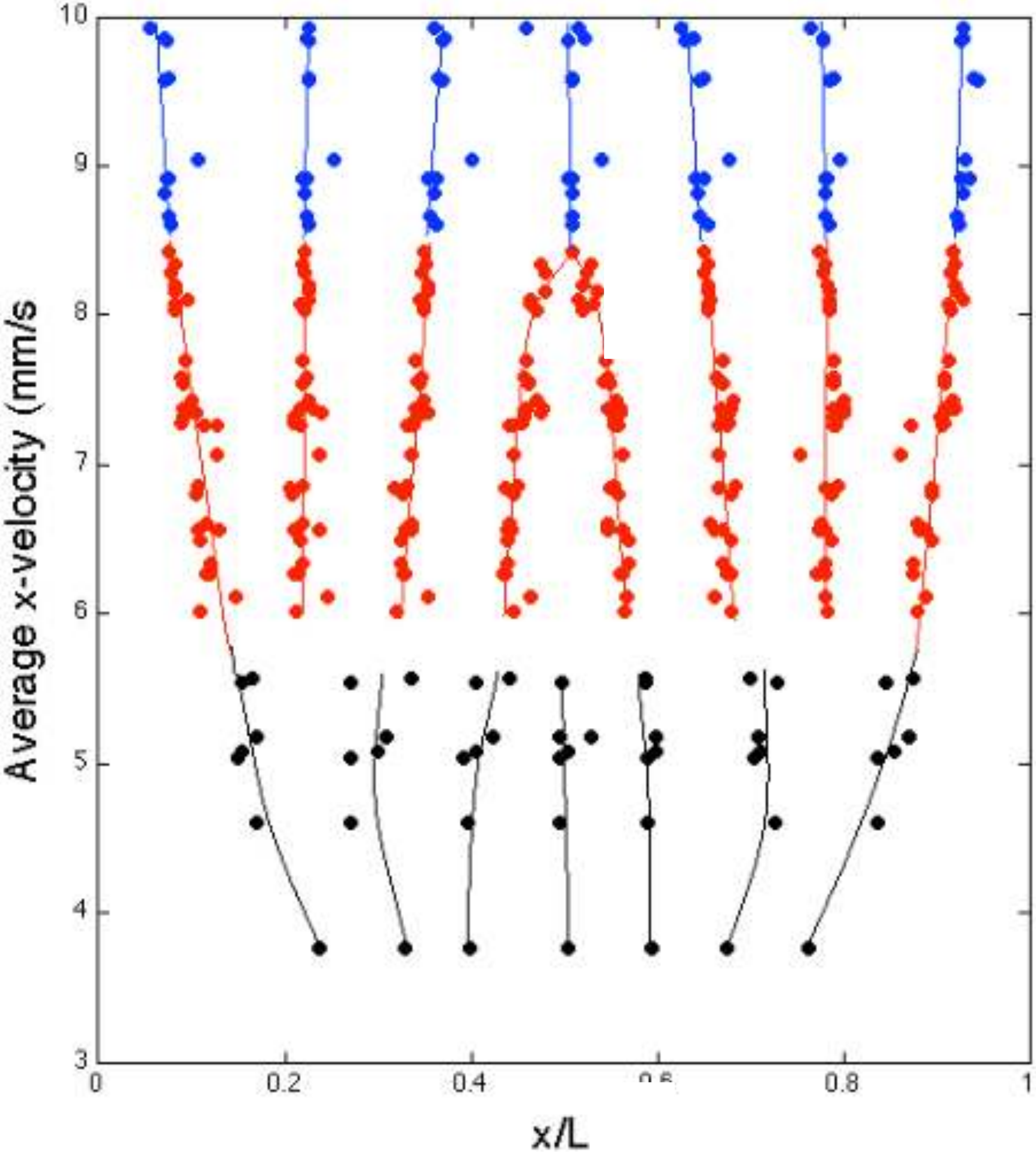
$\bar{v} = 6.75\ mm/s$

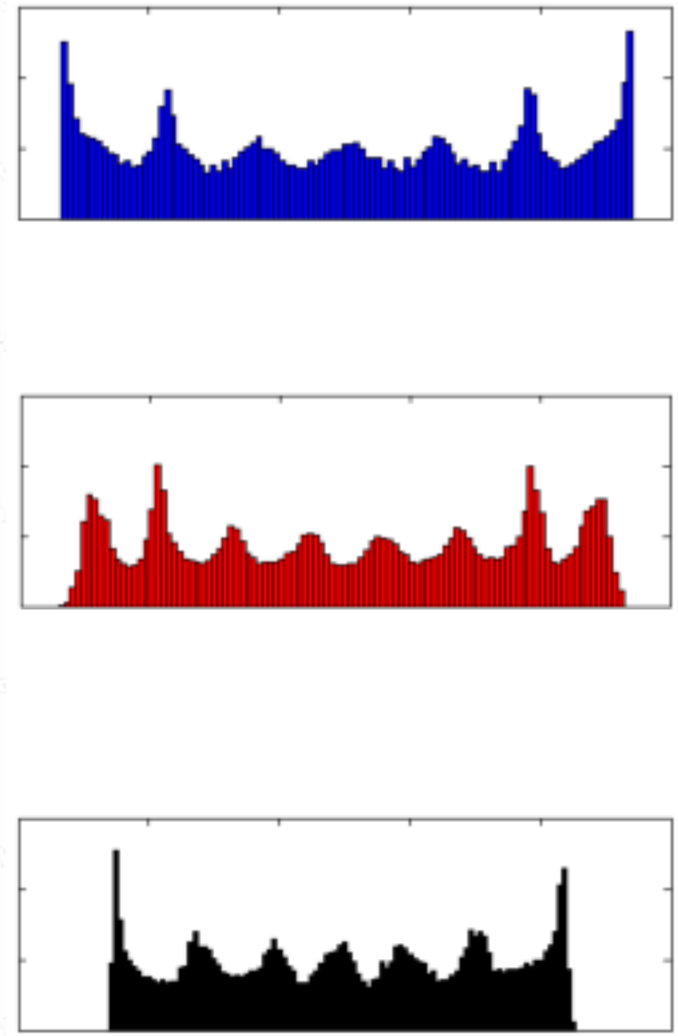


$\bar{v} = 5.23\ mm/s$

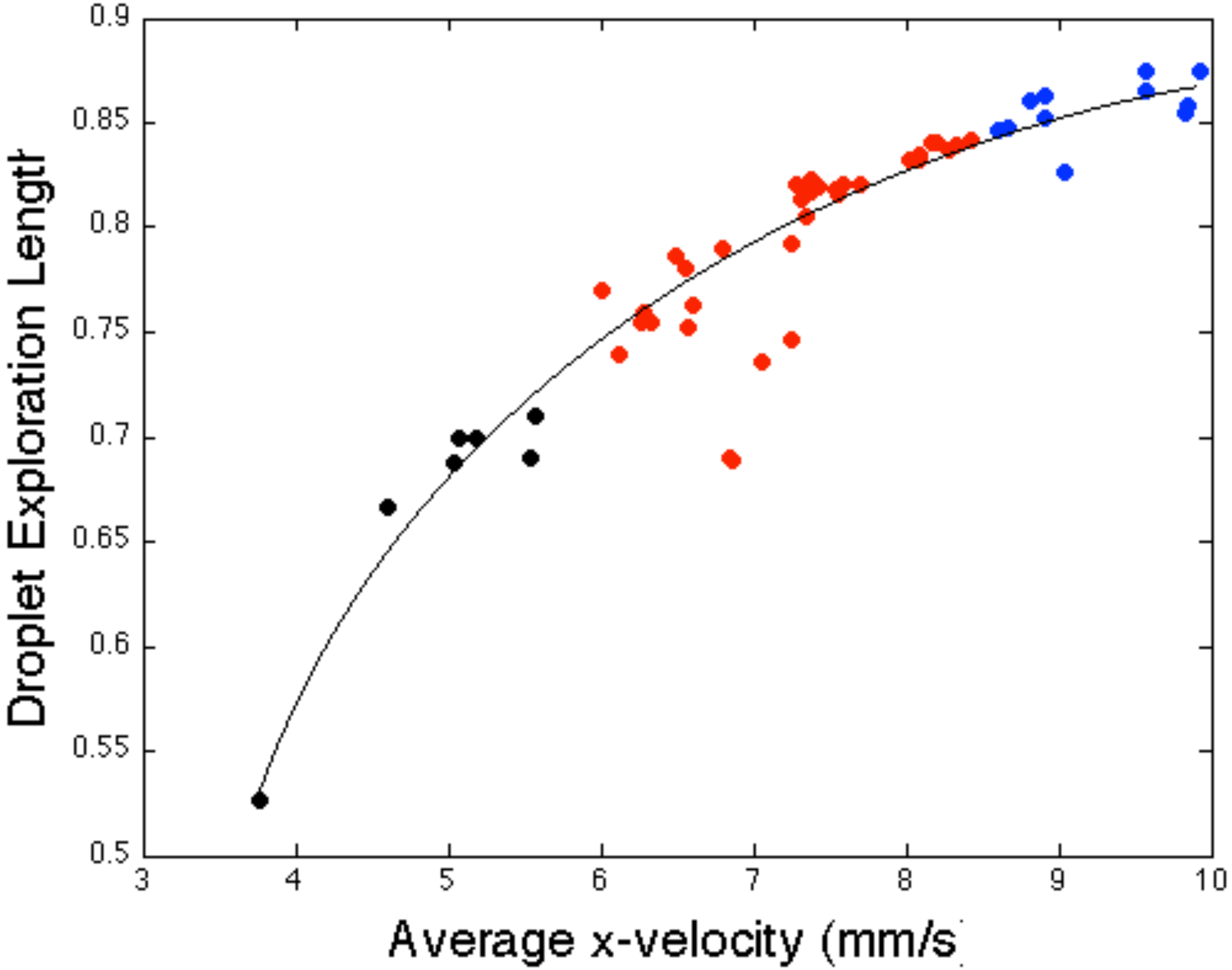


52 Experiments, ~11,000 Images Each

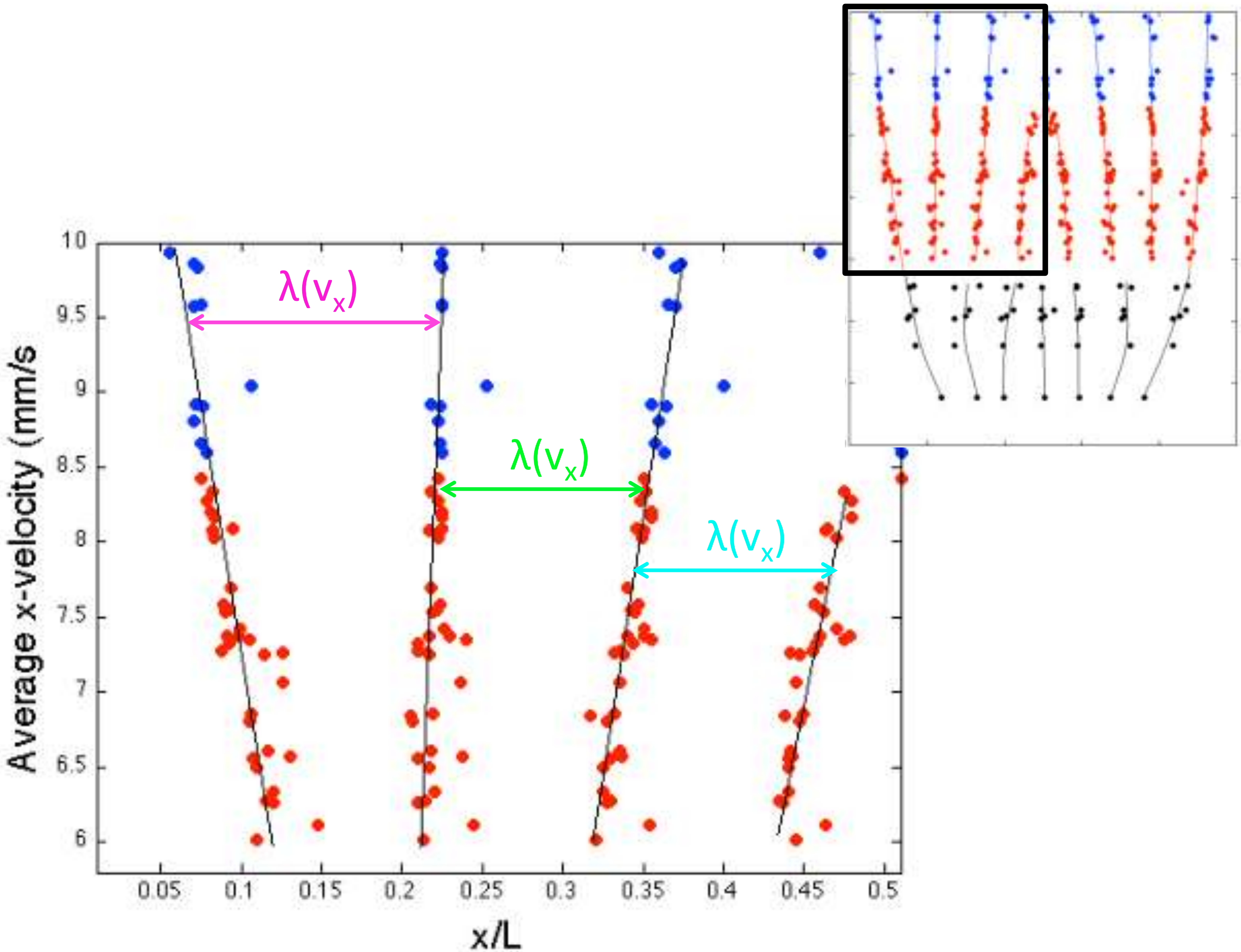




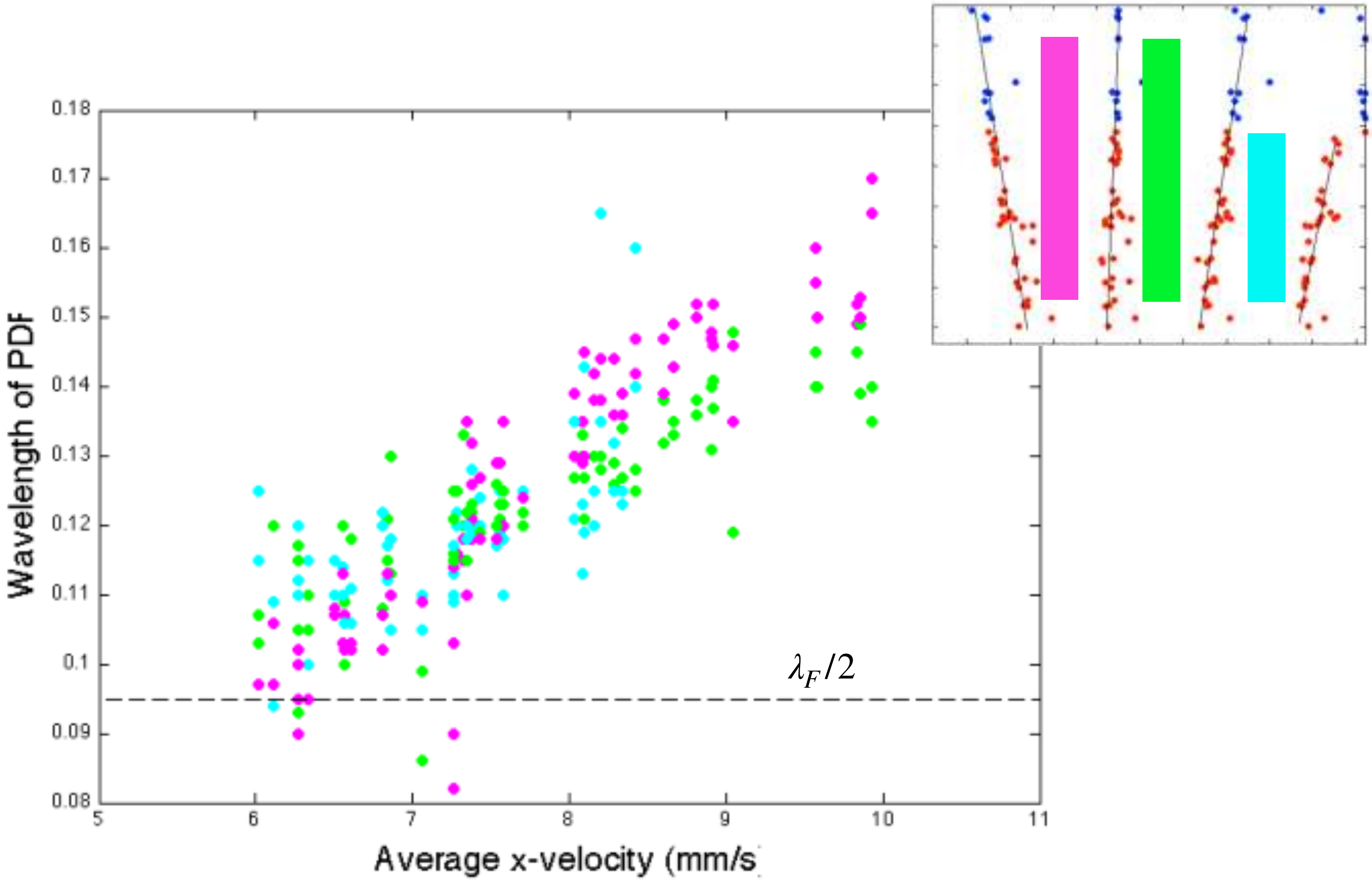
pdf



Observation 1: Exploration Length Increases with Speed



Average x-velocity (mm/s
x/L
λ(v_x)
λ(v_x)
λ(v_x)
10
9.5
9
8.5
8
7.5
7
6.5
6
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5



<u>Observation 2</u>: Wavelength Increases with Speed

Theoretical modeling of the circular corral

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Faraday pilot-wave dynamics in a circular corral

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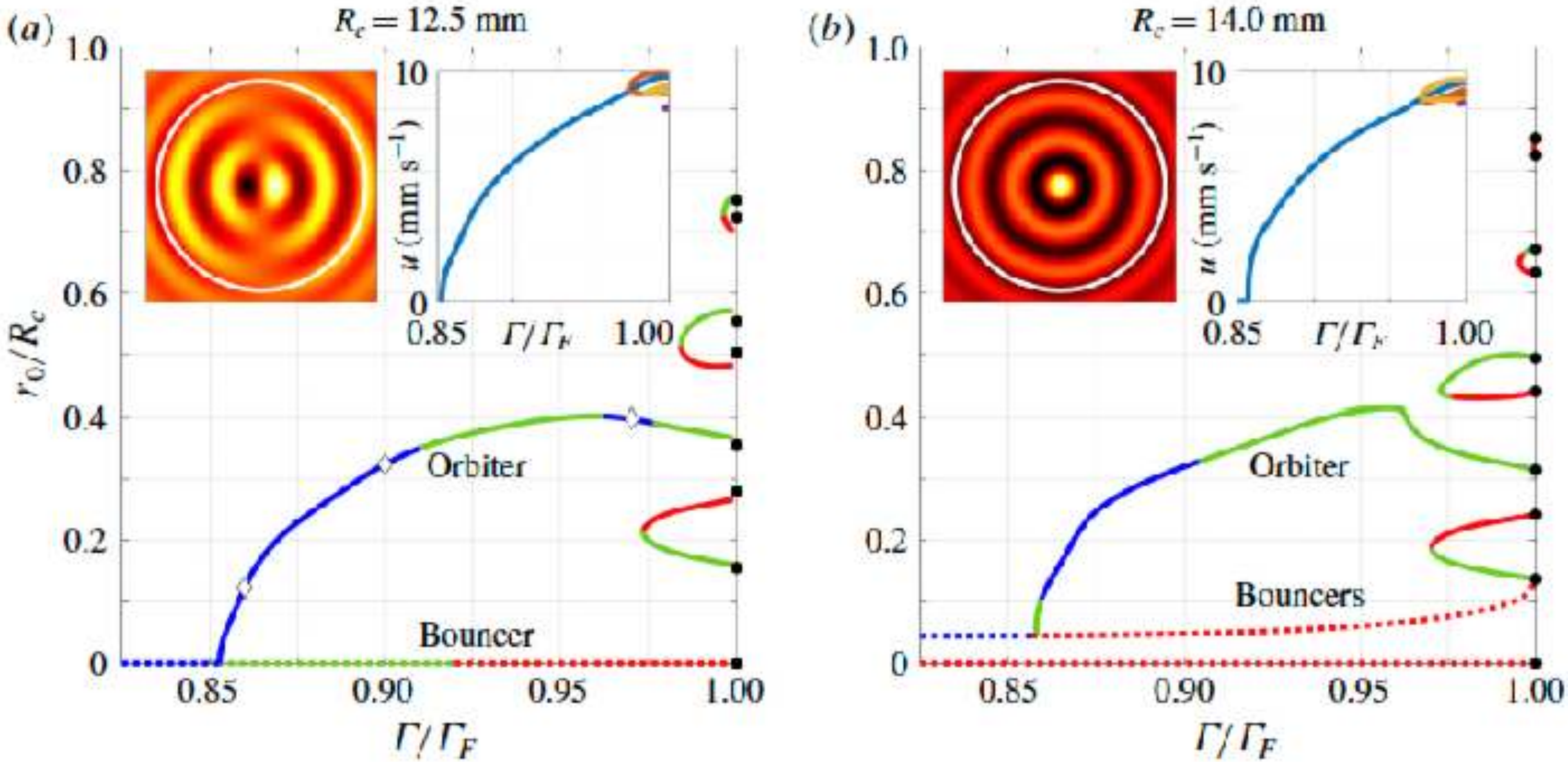
[3]Key Laboratory for Mechanics in Fluid Solid Coupling Systems, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, PR China

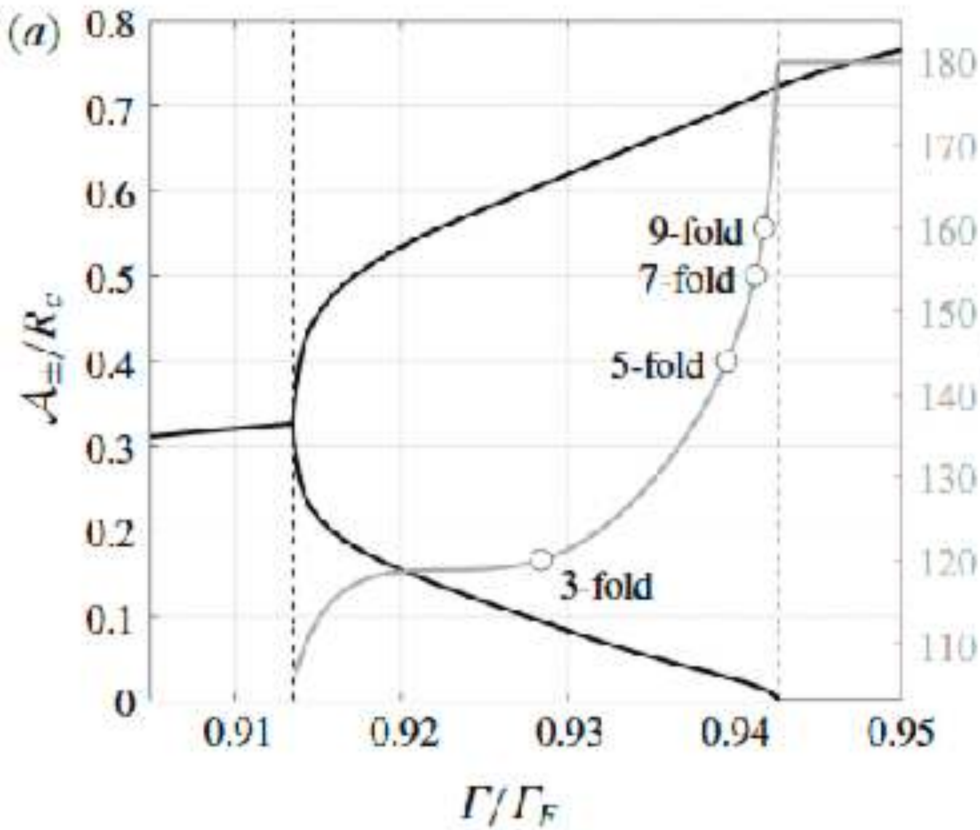
- strobe-based model correctly captures low Me behavior
- fails to predict emergent statistical behavior arising at high Me

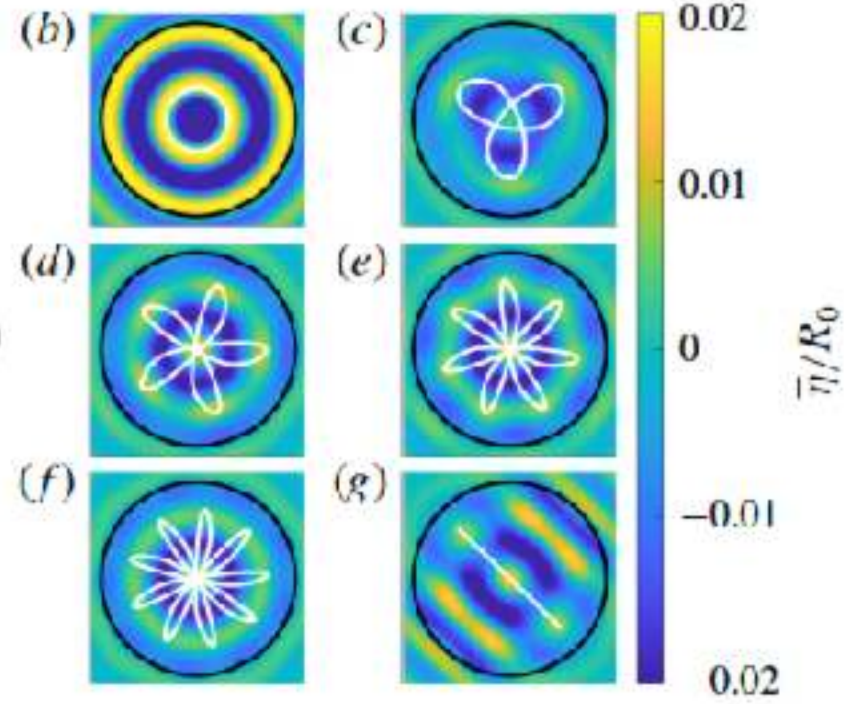
Stability of circular orbits



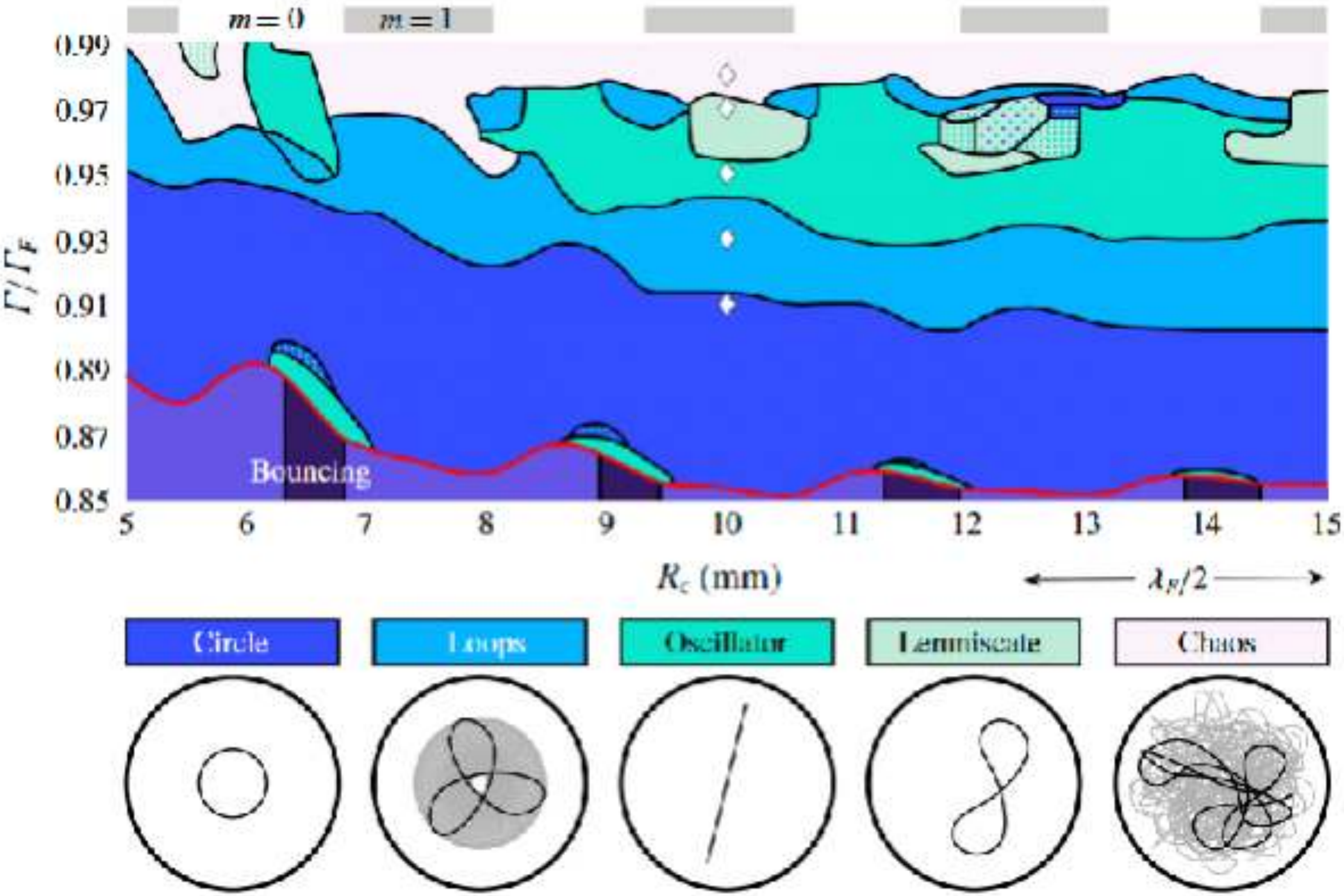
Evolution of periodic orbits

R = 10 mm



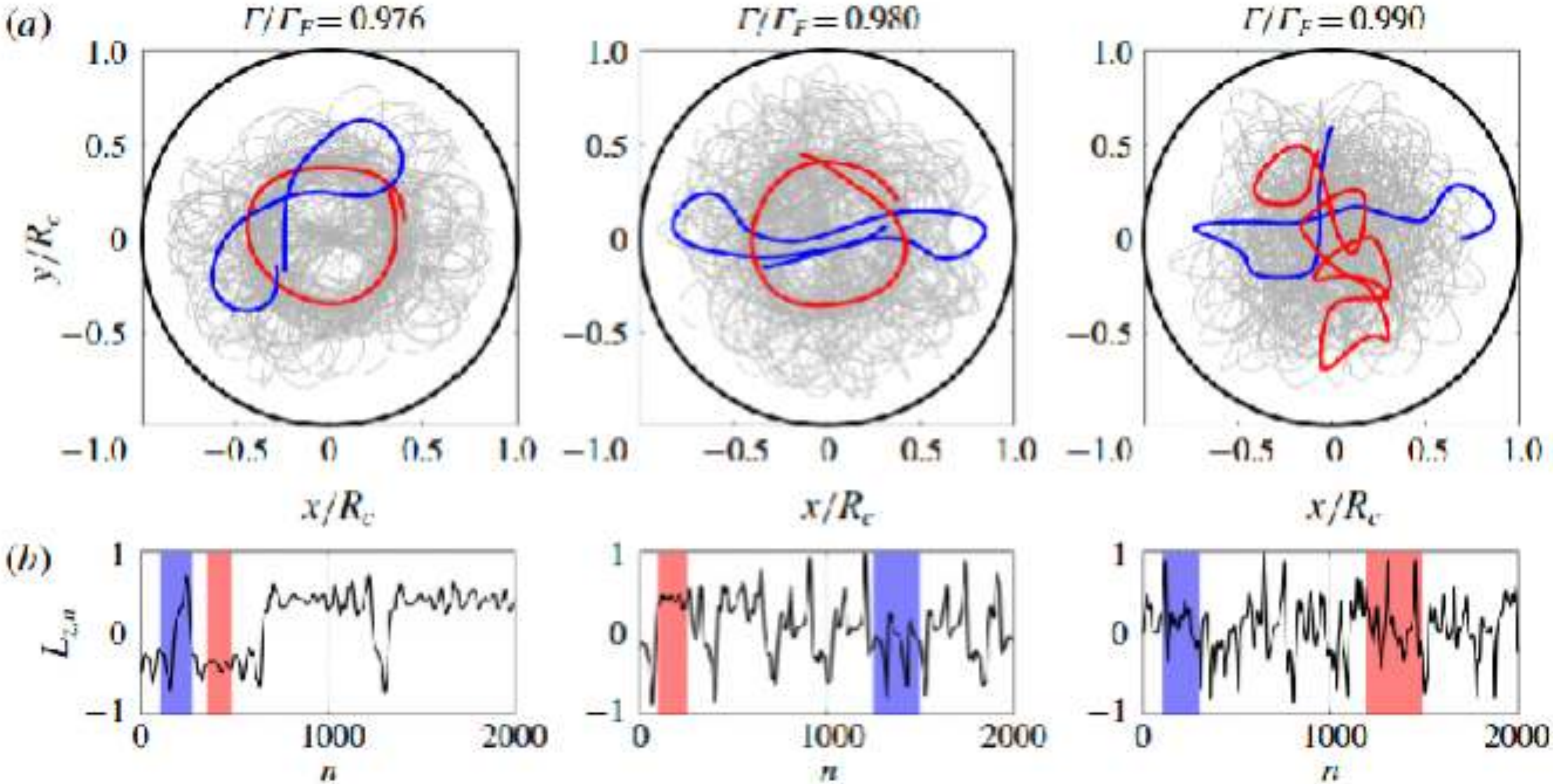


Regime diagram: dependence on corral radius, memory



Regime diagram: circular corral experiments

R = 12.125 mm



Theoretical modeling of the circular corral

PHYSICAL REVIEW E **93**, 042202 (2016)

Quantumlike statistics of deterministic wave-particle interactions in a circular cavity

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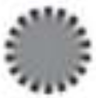
Microfluidics Lab, Department of Aerospace and Mechanics, University of Liège, B-4000 Liège, Belgium
(Received 21 January 2016; published 5 April 2016)

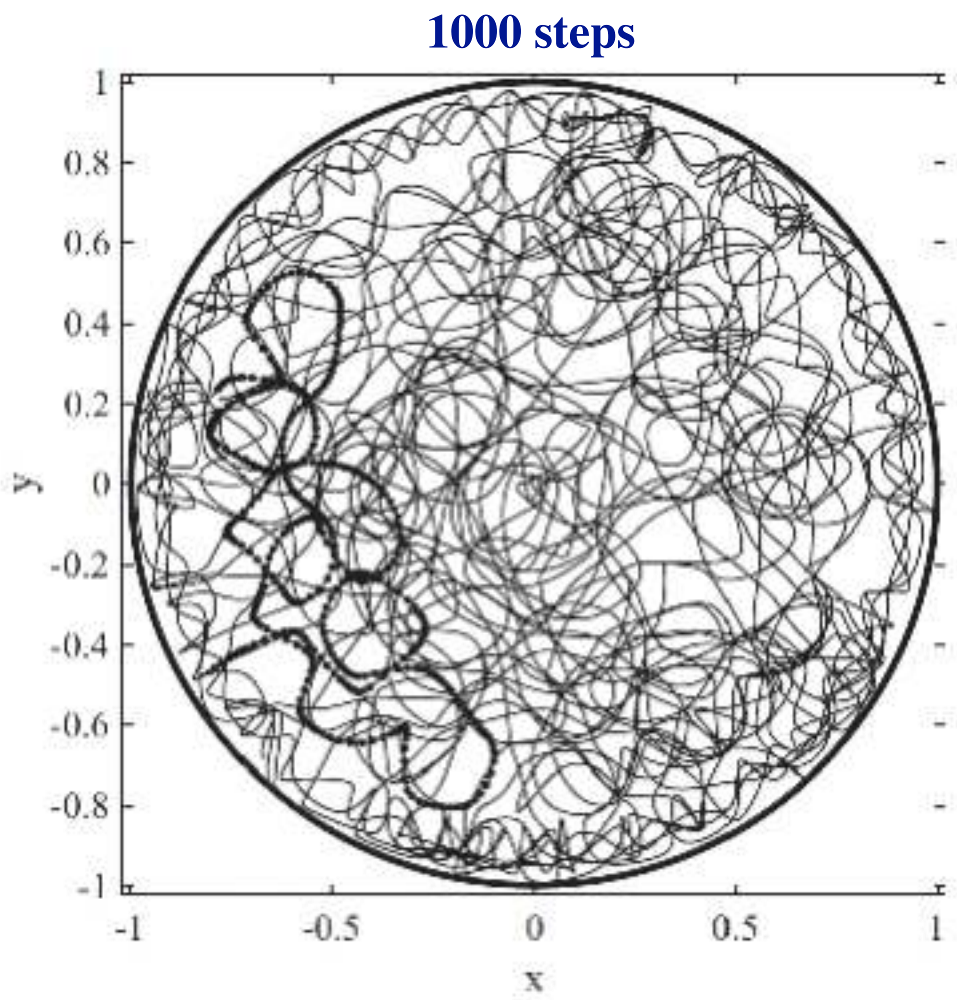
- neglect inertia: first order dynamics
- decompose pilot-wave field into cavity modes

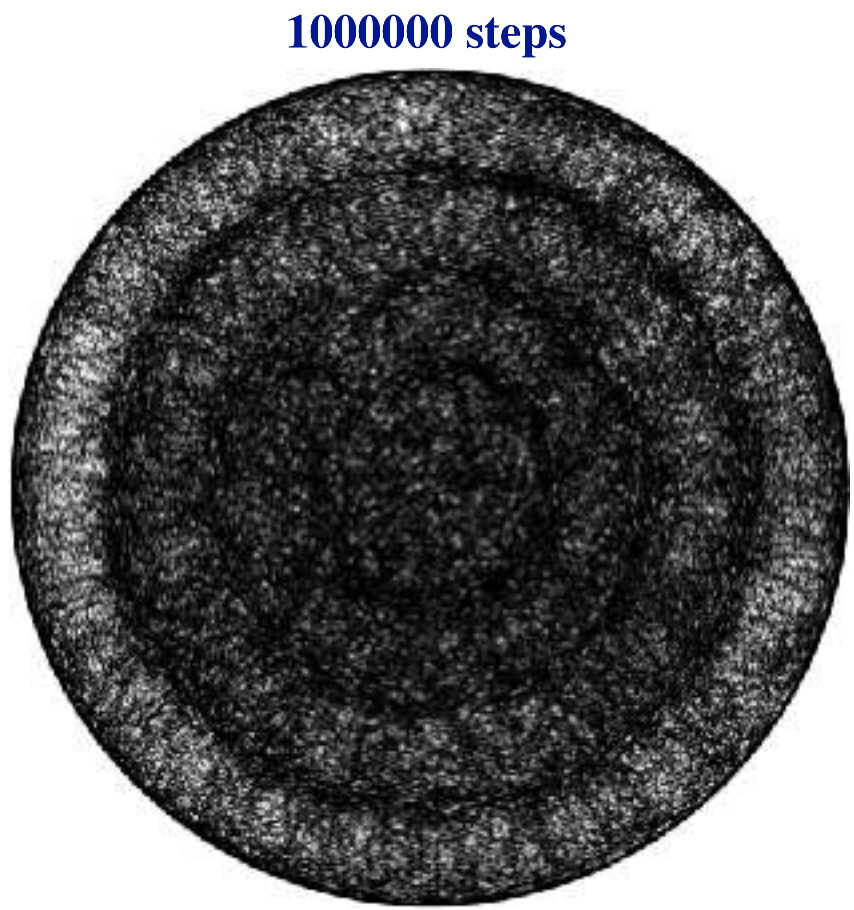
$$\mathbf{X}_{n+1} - \mathbf{X}_n = -\delta \sum_k W_{k,n} \nabla \Phi_k]_{\mathbf{X}_n}$$

TABLE I. Twelve dominant Neumann eigenmodes for a cavity of radius 14.3 mm filled with 20 cS oil and forced at 83 Hz.

Mode	k,ℓ	Λ_F	μ	M	Mode	k,ℓ	Λ_F	μ	M
	2,6	0.31	0.999	738		9,3	0.33	0.983	60
	0,7	0.32	0.998	460		18,1	0.34	0.974	39
	4,5	0.34	0.993	141		10,3	0.32	0.968	32
	13,2	0.33	0.990	99		12,2	0.34	0.962	26
	17,1	0.33	0.989	90		6,4	0.34	0.951	20
	7,4	0.32	0.987	77		5,5	0.32	0.933	15

Theoretical modeling of the circular corral





- only model to capture emergent statistical behavior in the circular corral

Summary of theoretical modeling

- stroboscopic models of Faria, Durey capture low Me behavior
- these models fail to capture chaotic dynamics arising at high Me
- they also fail to capture the coherent statistics arising at high Me
- only Gilet's model successfully models emergent high Me statistical behavior

Why?

- stroboscopic models neglect non-resonant effects
- neglect of drop inertia makes drop more skittish, as do non resonance effects

What's next?

HQA Paradigm 3

Conclusions

- in the high-memory limit, the mean-pilot-wave field plays the role of the quantum potential in Bohmian mechanics

    - *the statistics appear to influence the dynamics*

- the instantaneous wave field differs from the mean in a manner that depends on the system memory

    - *one expects the relative magnitudes of the mean and perturbation fields to be an important parameter*

- the hydrodynamic pilot-wave system suggests a means of resolving difficulties of Bohmian mechanics, by enriching the dynamics a la de Broglie

Future directions

- consider relaxation to statistical steady state from an ensemble of ICs
- consideration of non-resonant effects will lead us to **Paradigm III** in HQA