HQA Lecture 18

I. Friedel oscillations

II. Spin lattices

III. Walkers on gentle slopes

Summary

- we now have the means to describe walkers interacting with boundaries

  1) Faria's strobe-based model for discrete steps in topography

  2) Nachbin's model for 1D motion with discrete steps in topography

- we here consider a new example of walker-topography interactions

  — will lead to Paradigm 2 for the emergence of quantum behavior

- we'll also consider the effects of slowly-varying topography

Interesting question

To what extent can we think of topographic anomalies as the generators of forces?

Friedel oscillations

- a walker interacts with a submerged well

SCIENCE ADVANCES | RESEARCH ARTICLE

PHYSICS

A hydrodynamic analog of Friedel oscillations

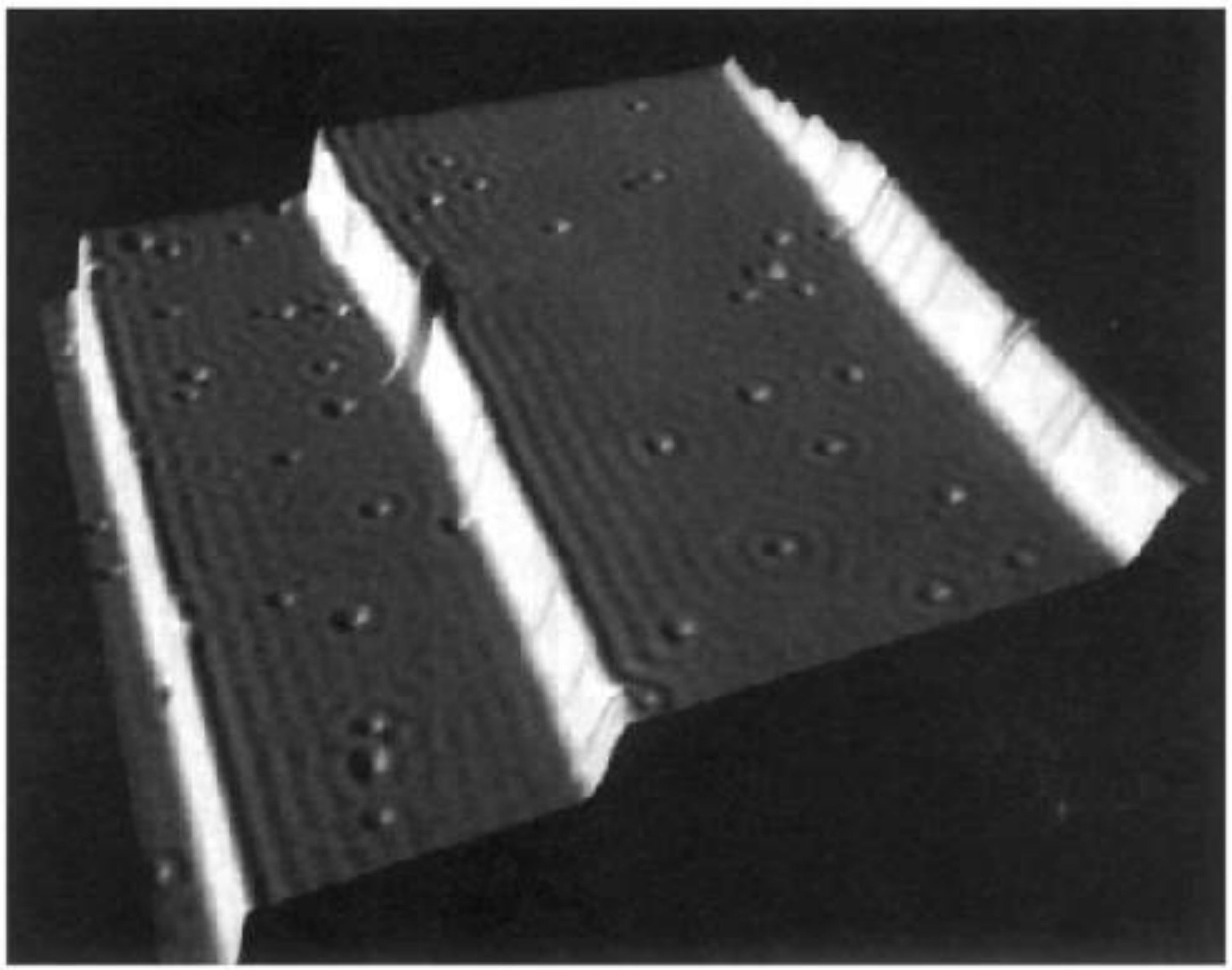
Pedro J. Sáenz[1,2]*, Tudor Cristea-Platon[2], John W. M. Bush[2]*

- yields the second paradigm for quantum behavior in HQAs

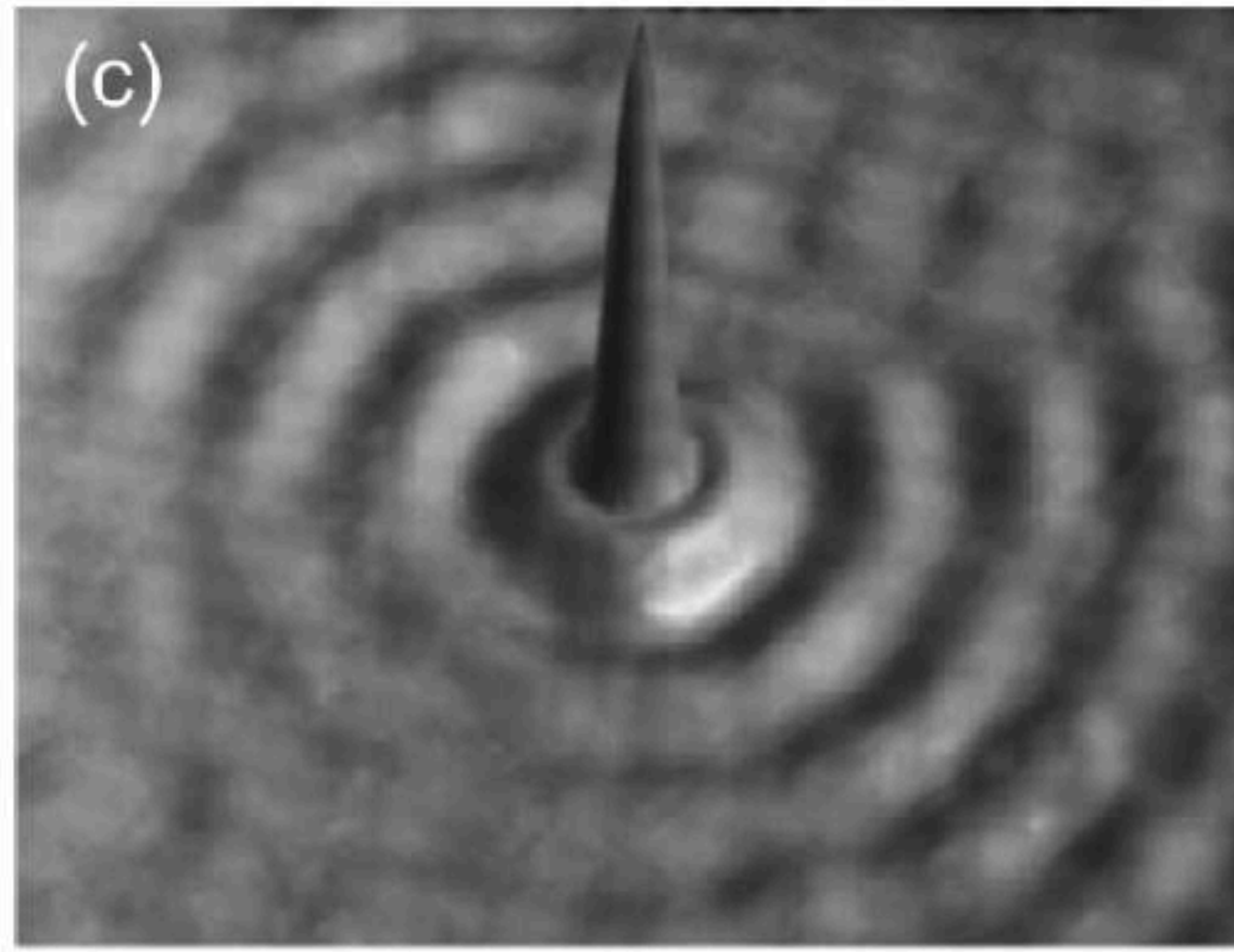
Friedel oscillations

- modulations of the probability density of the electron-sea on a metallic substrate due to the presence of a scattering impurity

Crommie *et al. (1993)*



Mallet *et al. (2016)*

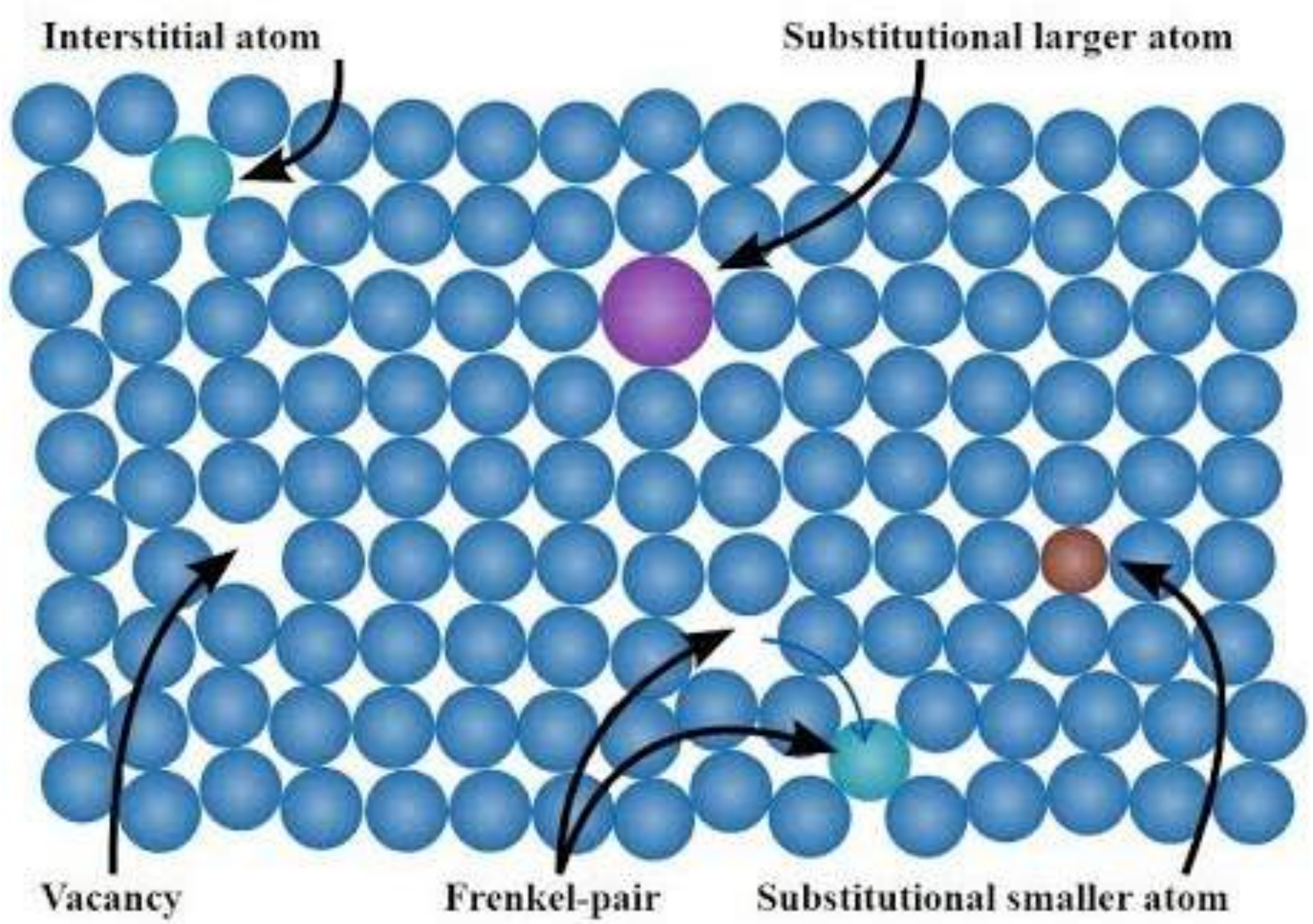


Unknown interaction mechanism

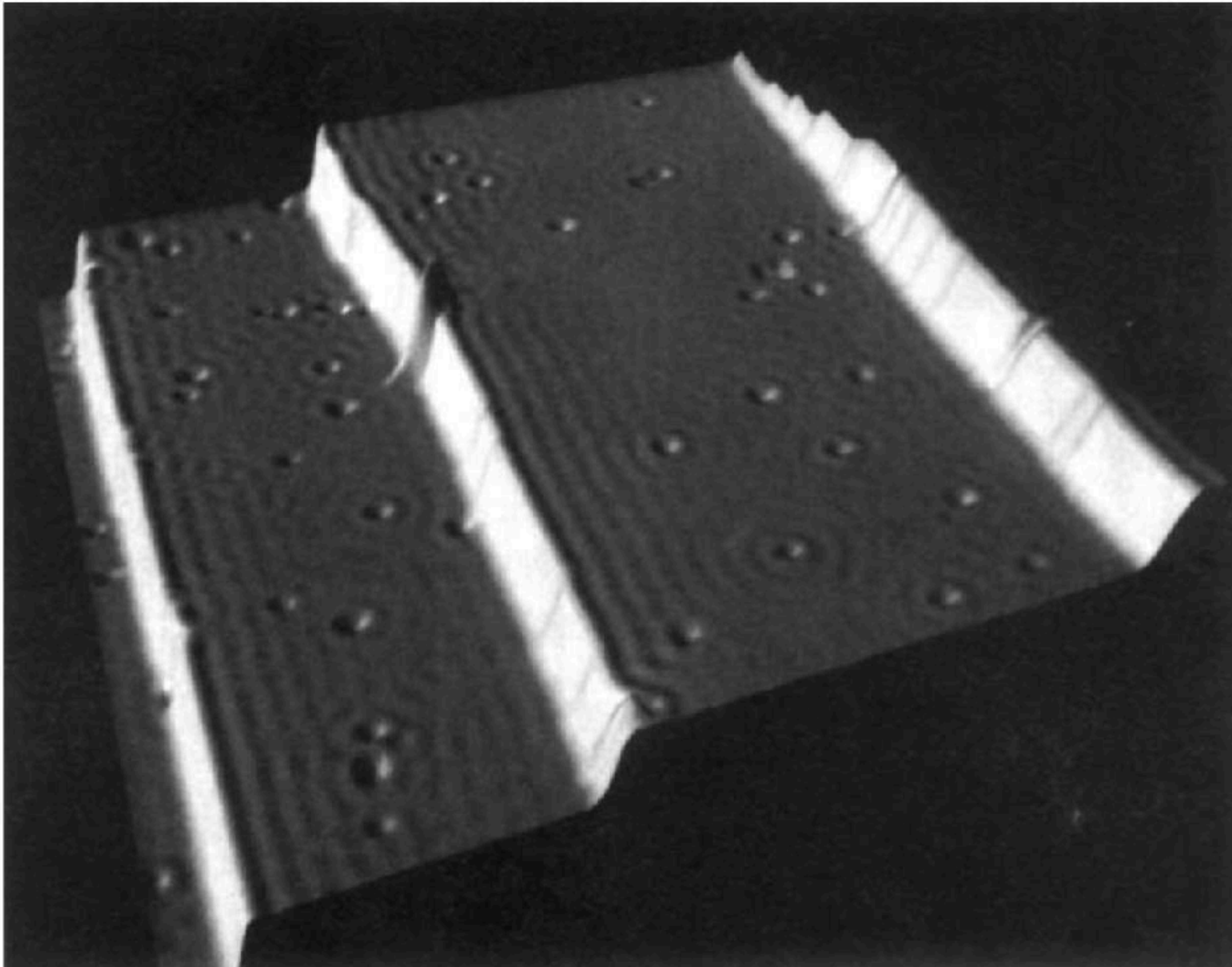
Modeled as localized scattering potentials

IMPURITIES IN CONDENSED MATTER

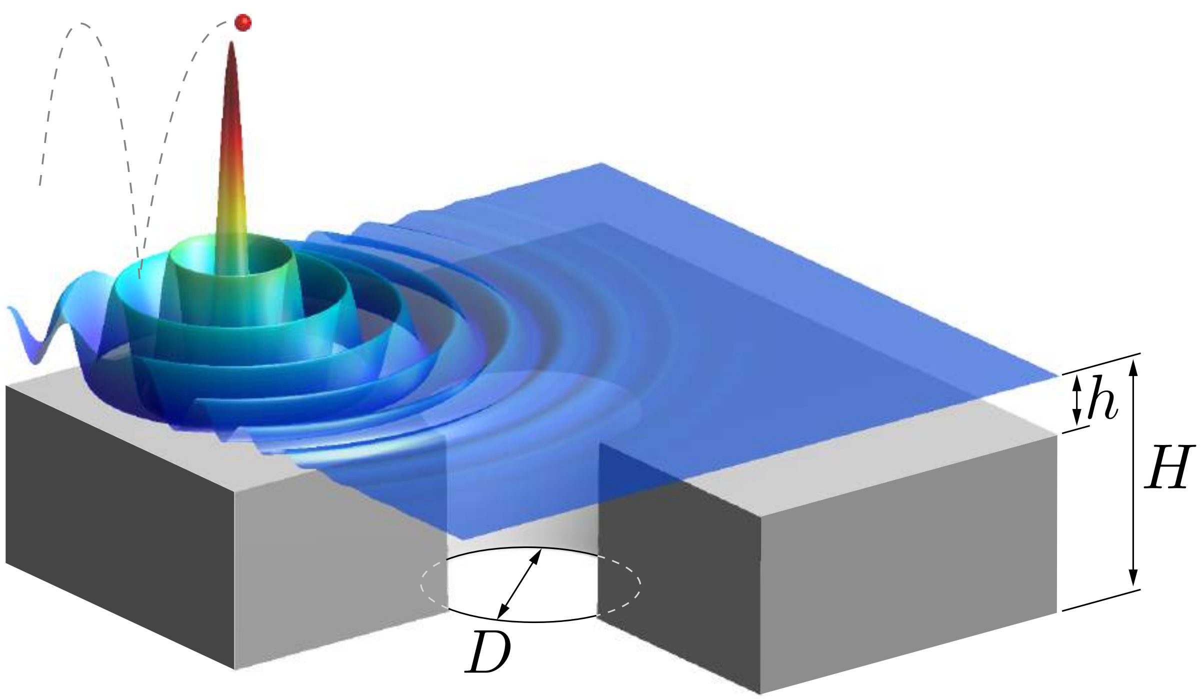
Localized irregularity in an otherwise homogeneous medium



Impurities play a critical role on the **mobility of charge carriers**



WALKER-WELL INTERACTION

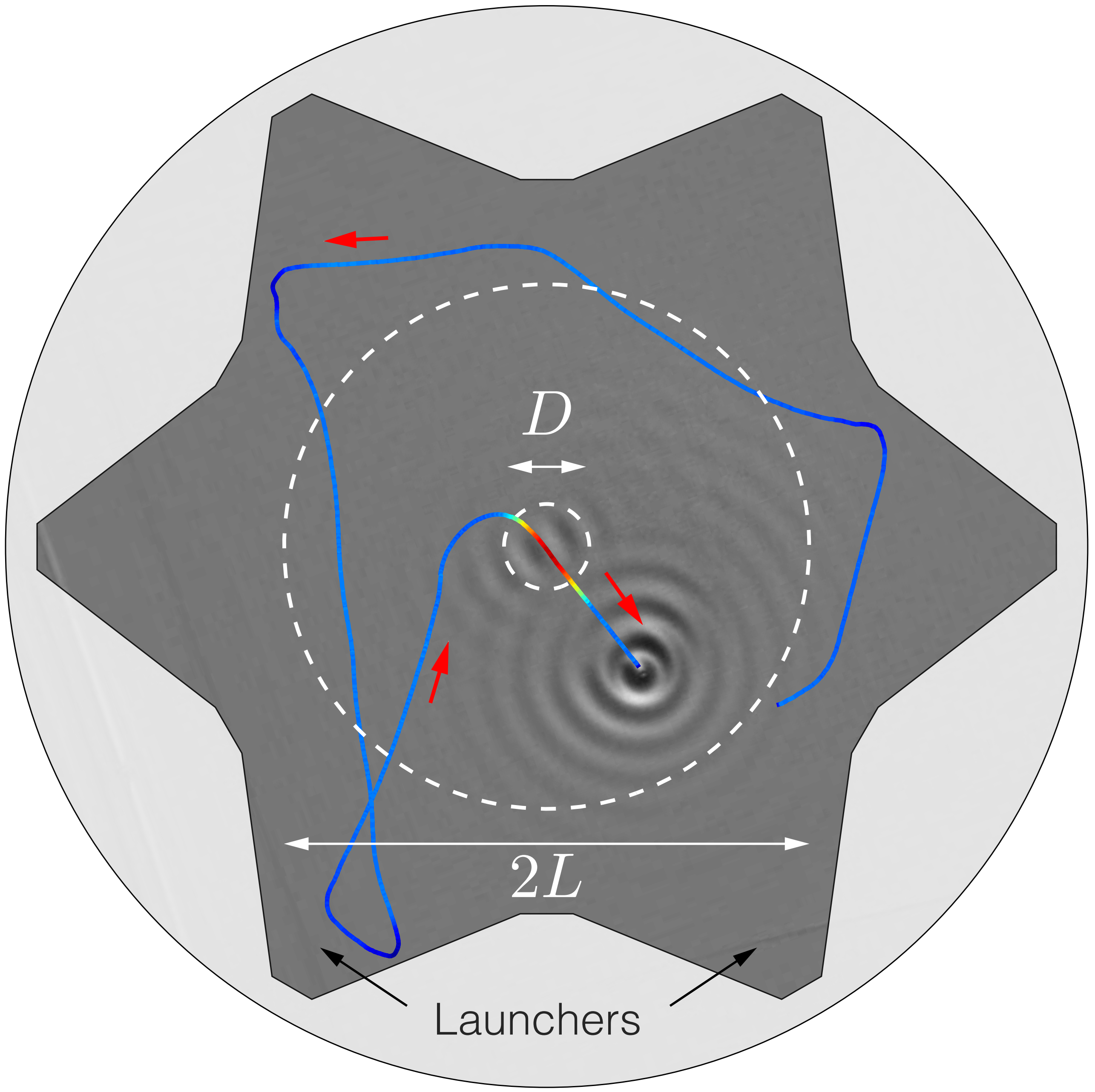


Well

Region of high excitability

$$\gamma_F^H < \gamma_F < \gamma_F^h$$

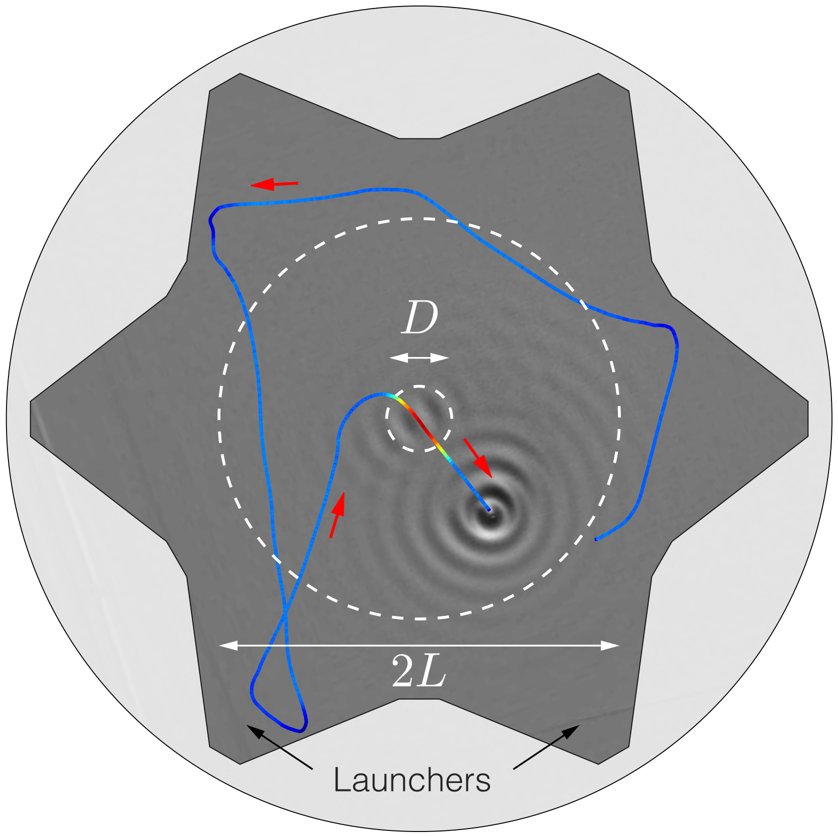
Experimental Setup



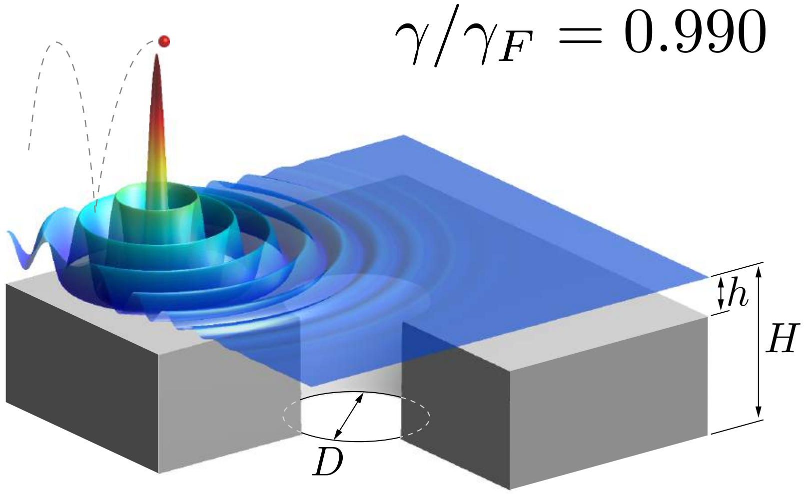
$$\gamma/\gamma_F = 0.990$$

WALKER-WELL INTERACTION

Experimental Setup



$$\gamma/\gamma_F = 0.990$$



Well

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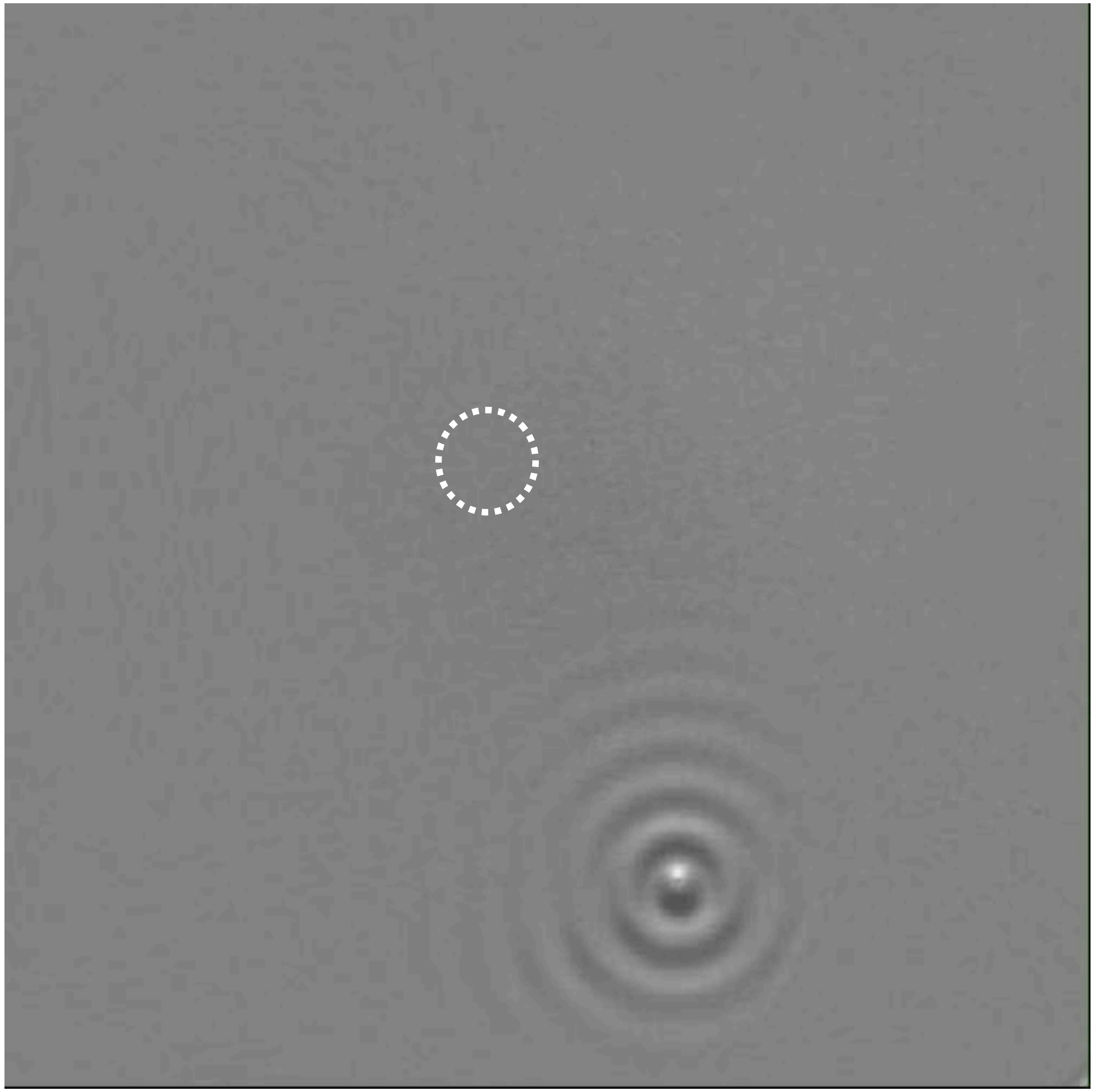
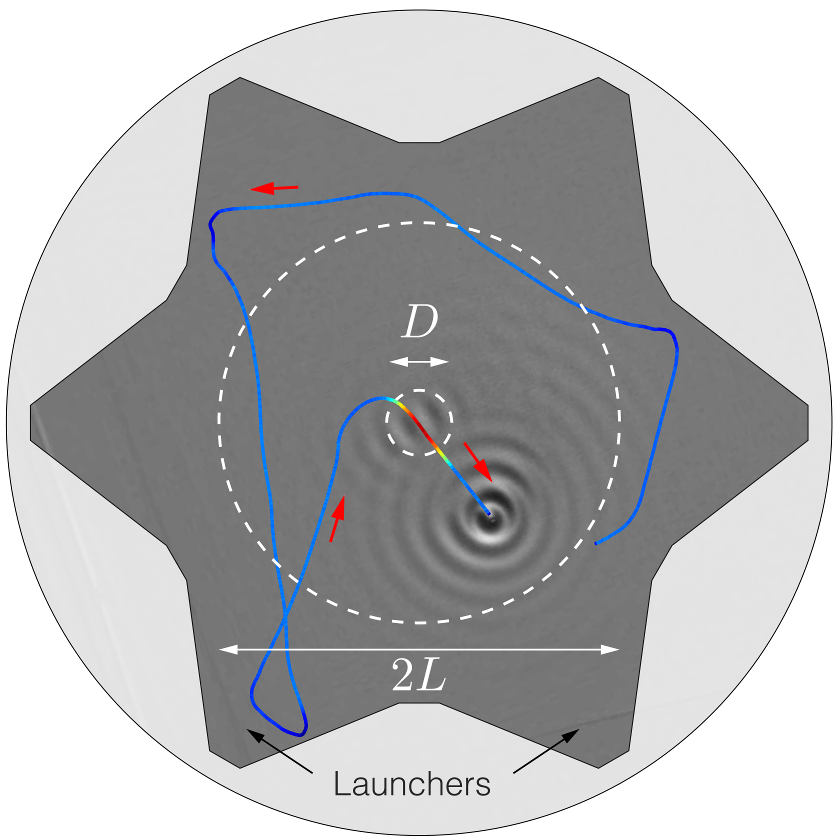


Drop drawn in along an Archimedean spiral

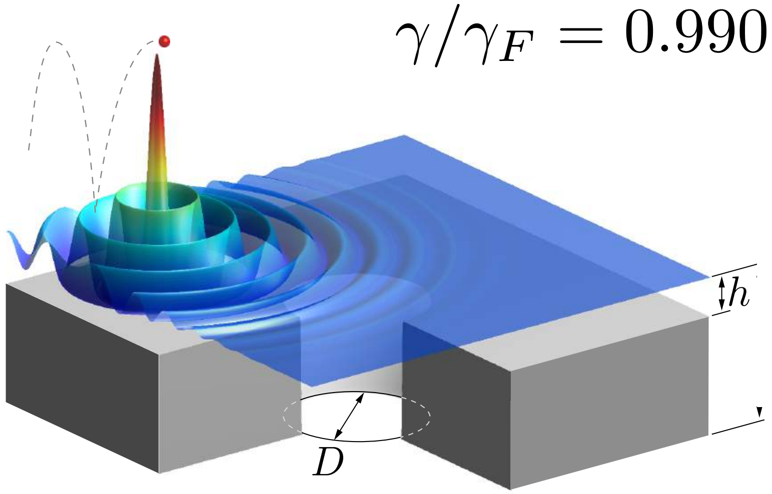
Speed modulations induced by interaction with waves generated above the well

WALKER-WELL INTERACTION

Experimental Setup



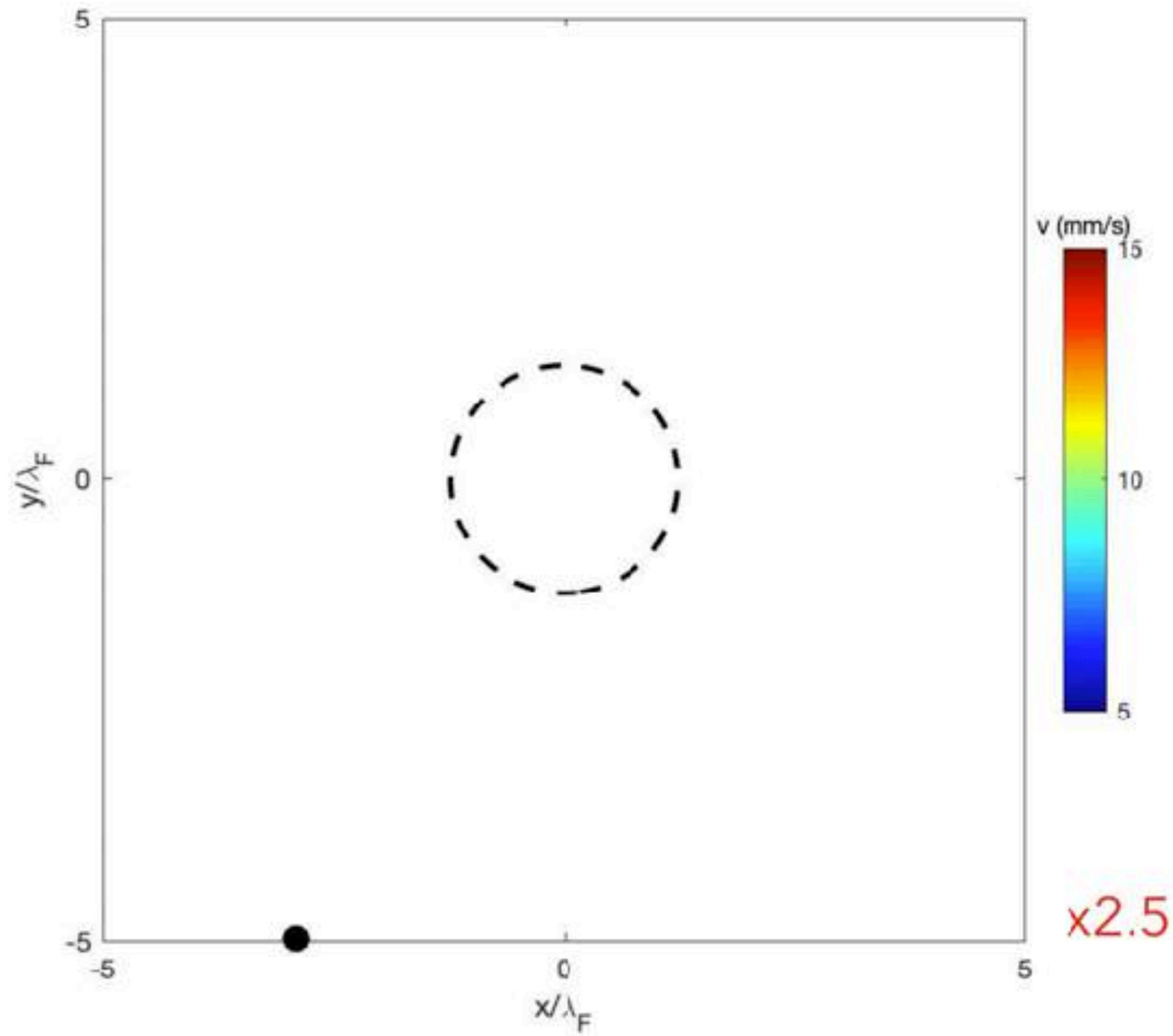
$$\gamma/\gamma_F = 0.990$$



Well

Region of high excitability

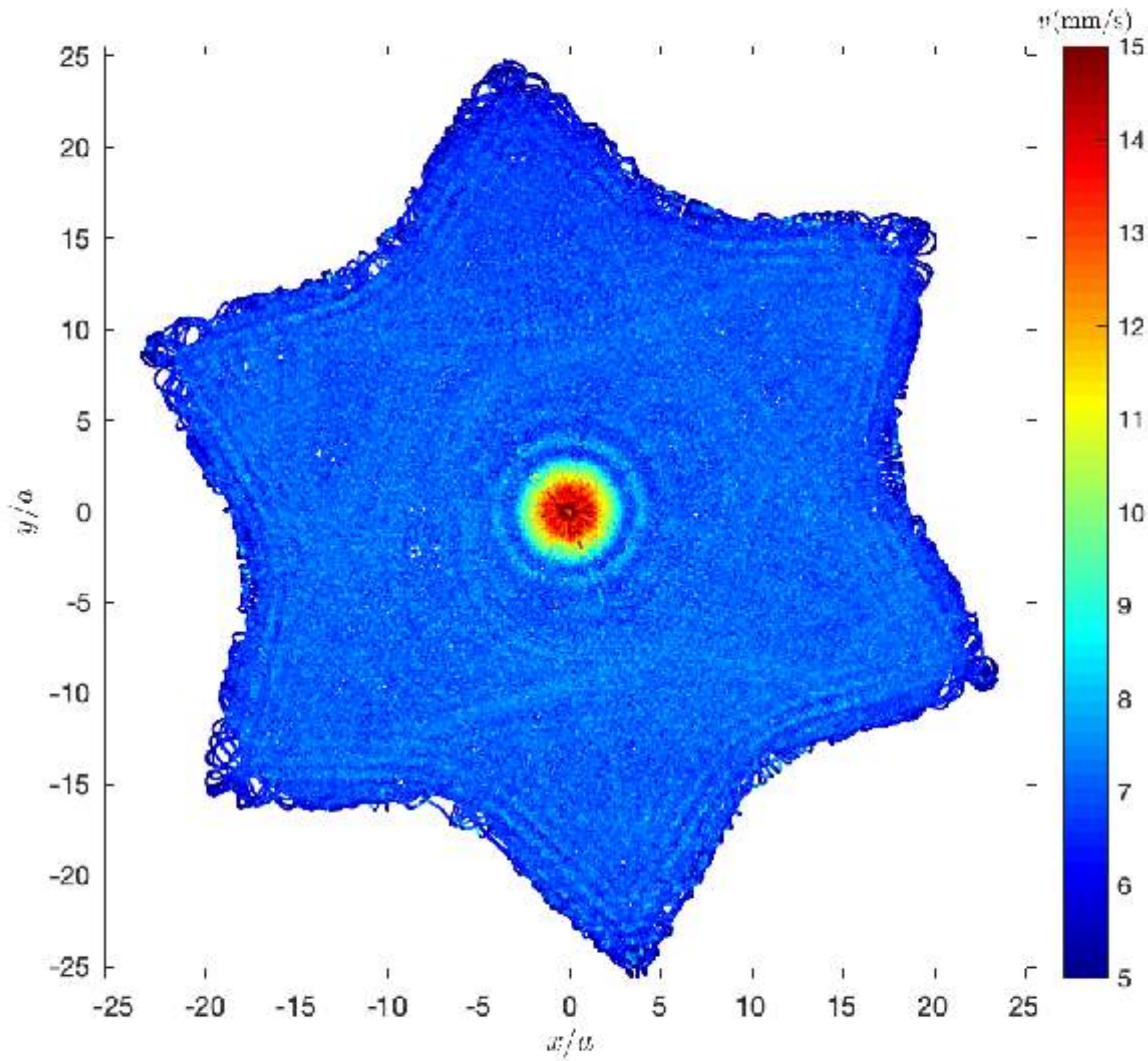
$$\gamma_F^H < \gamma_F < \gamma_F^h$$

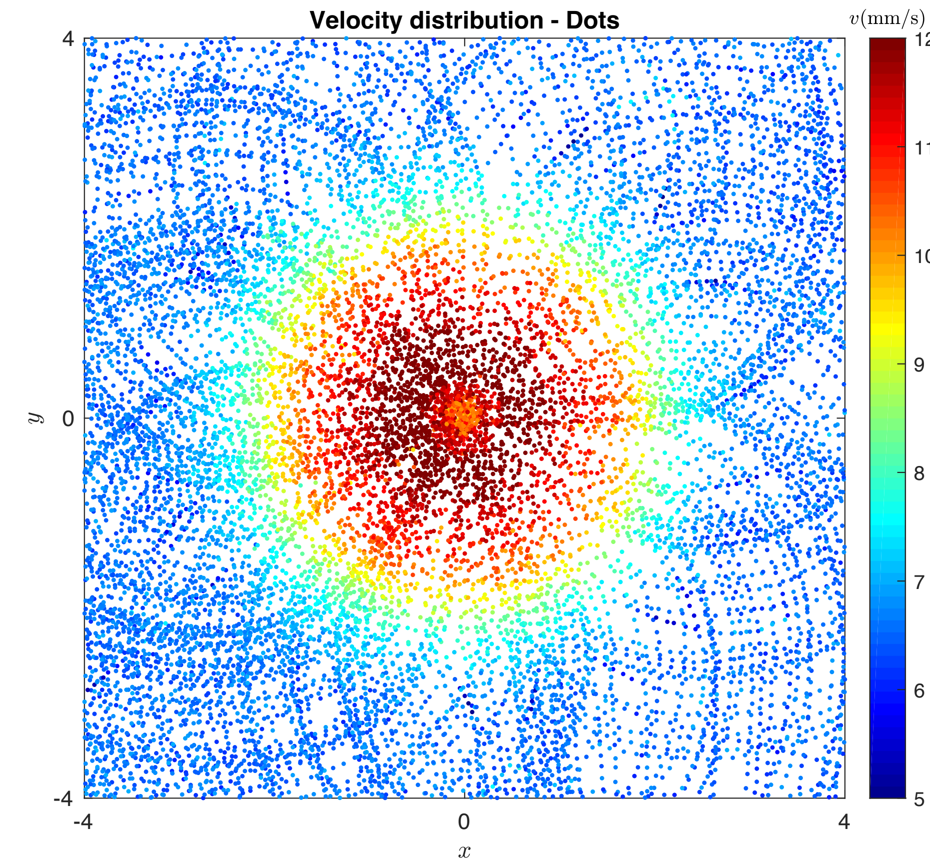


Drop drawn in along an Archimedean spiral

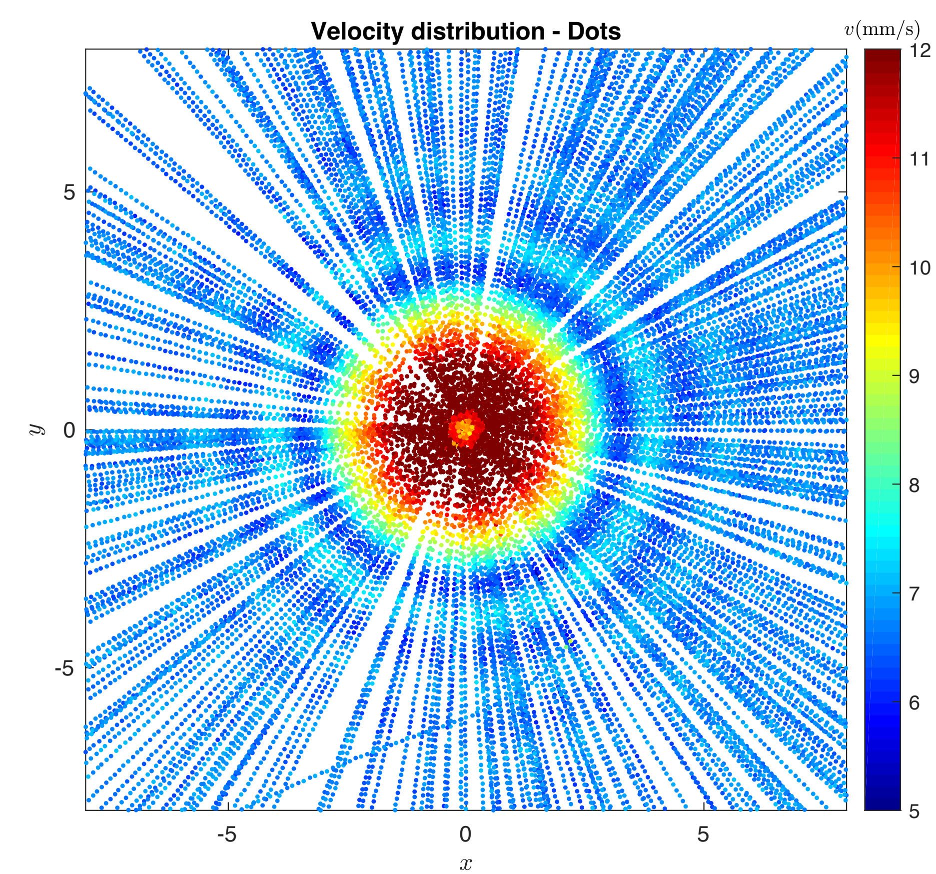
Speed modulations induced by interaction with waves generated above the well

Droplet speed map



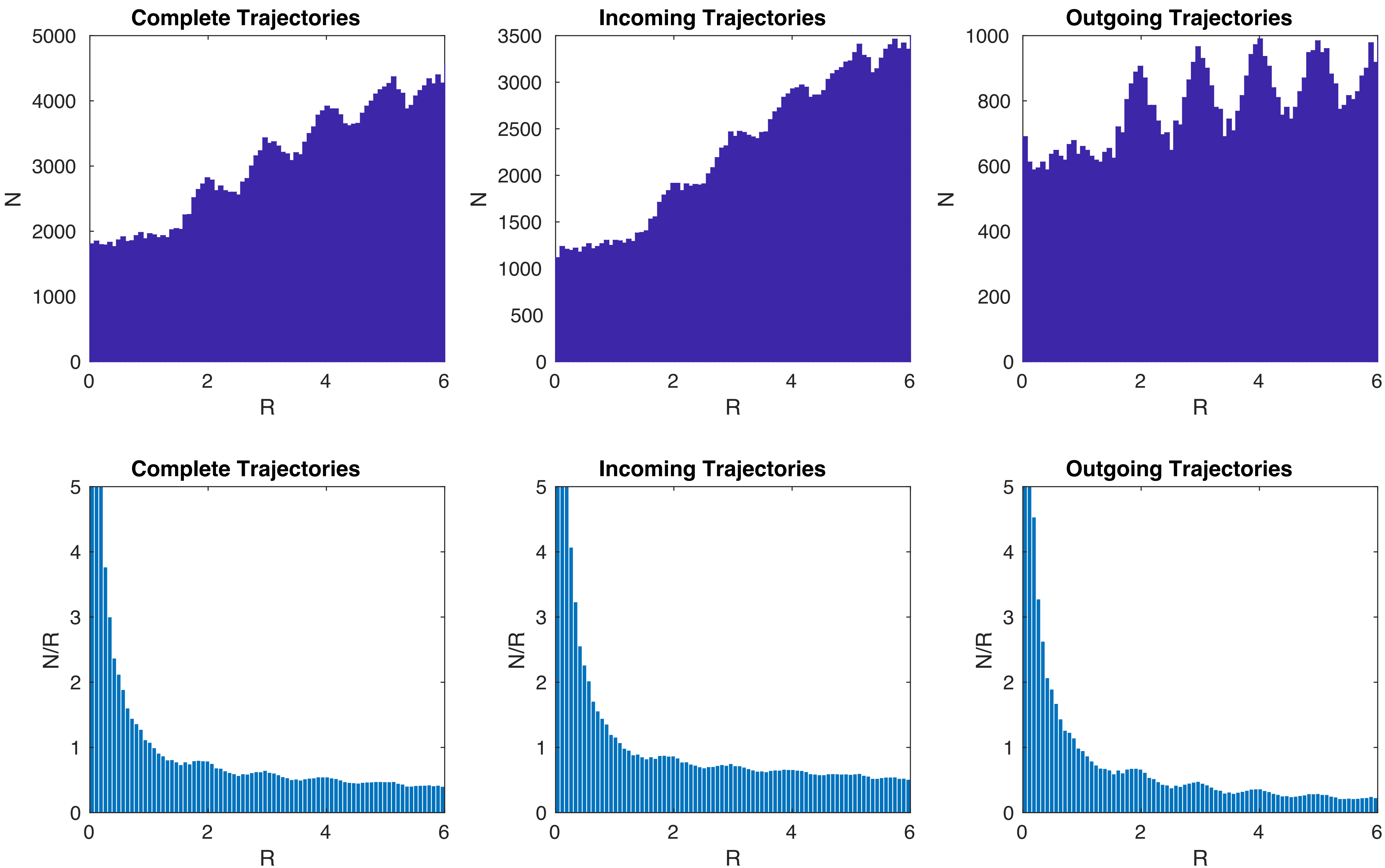


Incoming trajectories



Outgoing trajectories

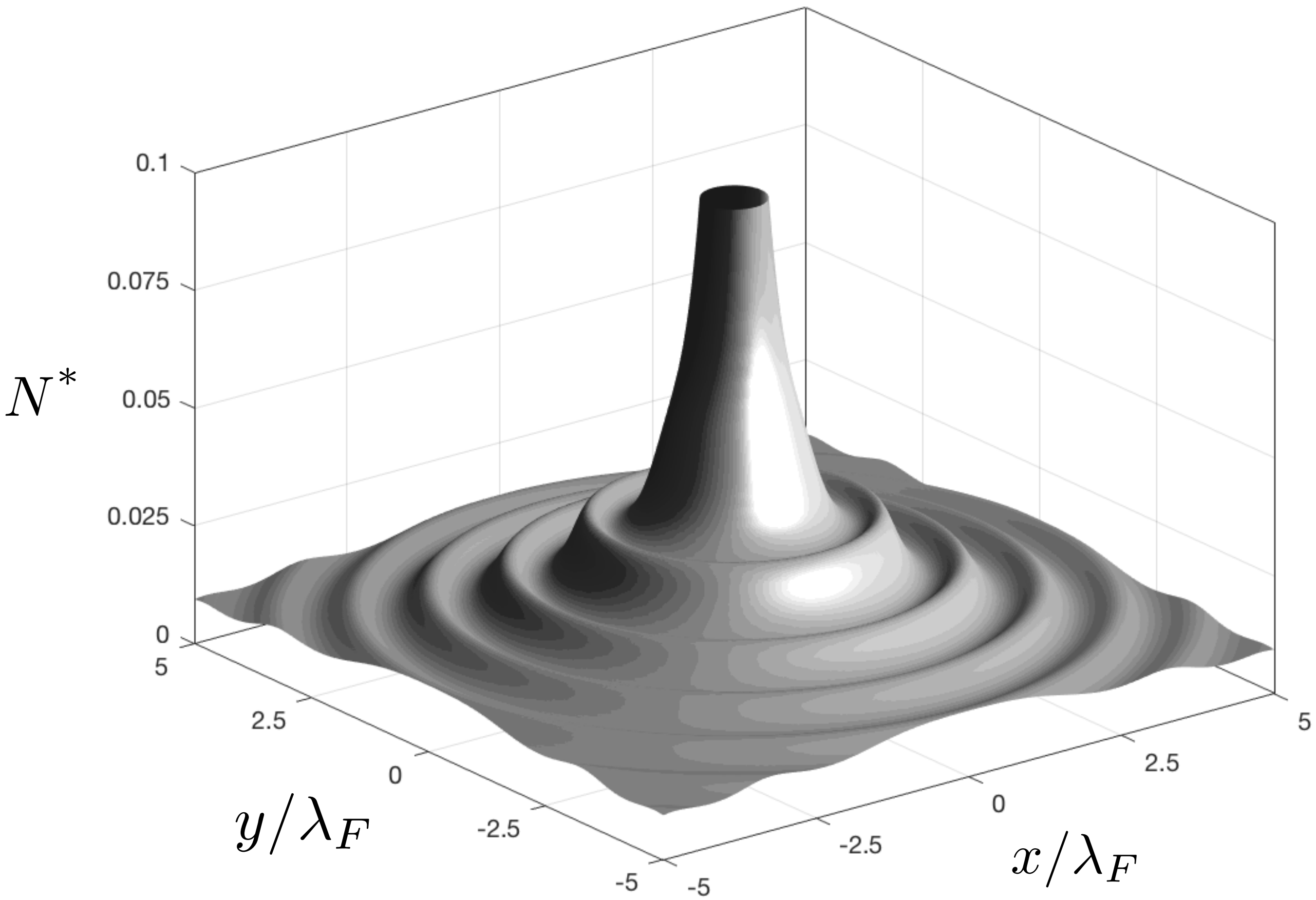
Statistics

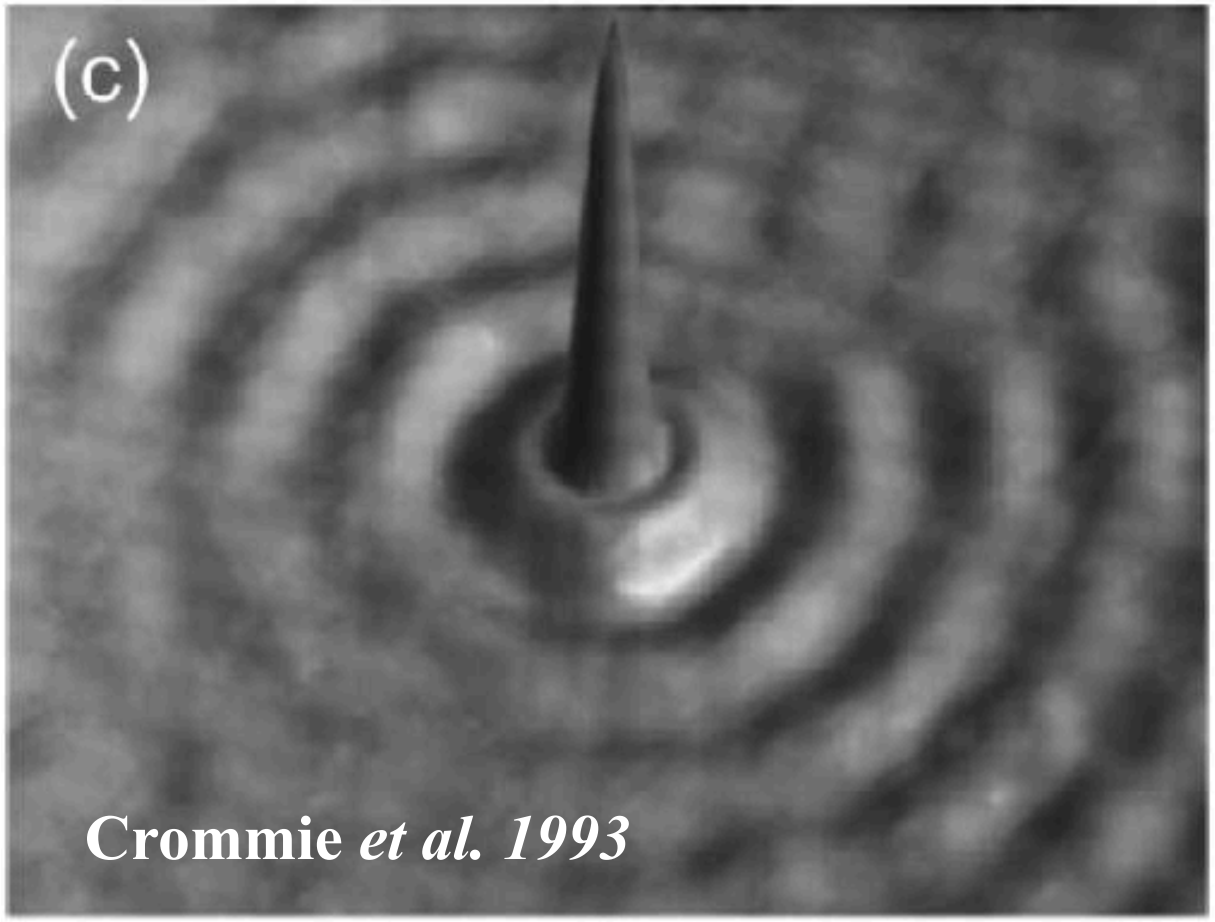

Complete Trajectories
Incoming Trajectories
Outgoing Trajectories
N
R
Complete Trajectories
Incoming Trajectories
Outgoing Trajectories
N/R
R

A hydrodynamic analog of Friedel oscillations

Sáenz, Cristea-Platon & Bush (Sci. Advances, 2020)

- arises due to wave-induced speed modulations in outgoing trajectory





Friedel-like oscillations are *not* inconsistent with the notion of particle trajectories

Simulations via Faria…

J. Fluid Mech. (2017), vol. 811, pp. 51–66. © Cambridge University Press 2016 51
doi:10.1017/jfm.2016.750

A model for Faraday pilot waves over variable topography

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$$\phi_t = -g(t)\eta + \frac{\sigma}{\rho}\Delta_\perp \eta + 2\nu^* \Delta_\perp \phi - \frac{1}{\rho} P_D(\boldsymbol{x} - \boldsymbol{x}_p(t), t), \tag{2.13}$$

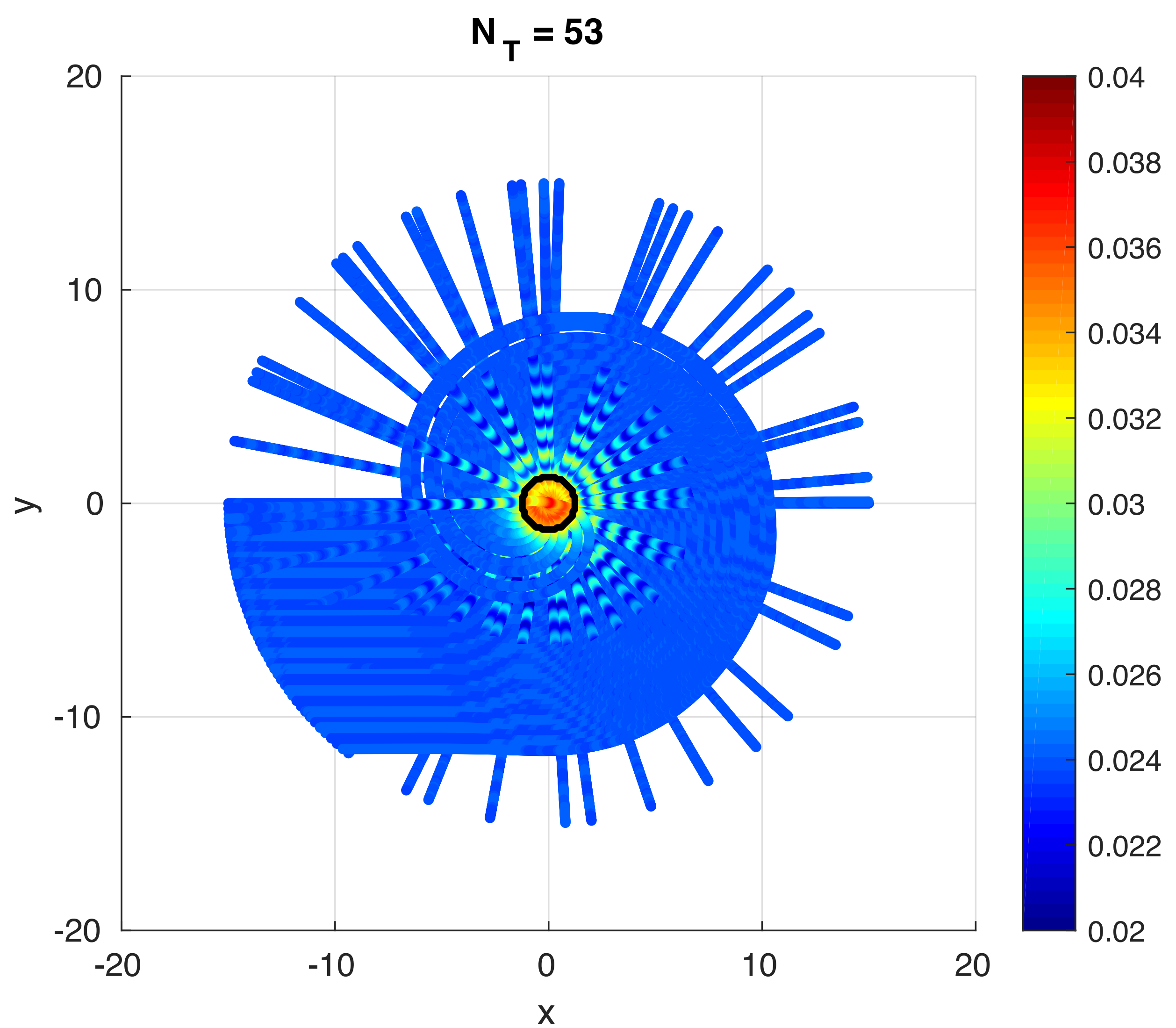
$$\eta_t = -\boldsymbol{\nabla}_\perp \cdot (b \boldsymbol{\nabla}_\perp \phi) + 2\nu^* \Delta_\perp \eta. \tag{2.14}$$

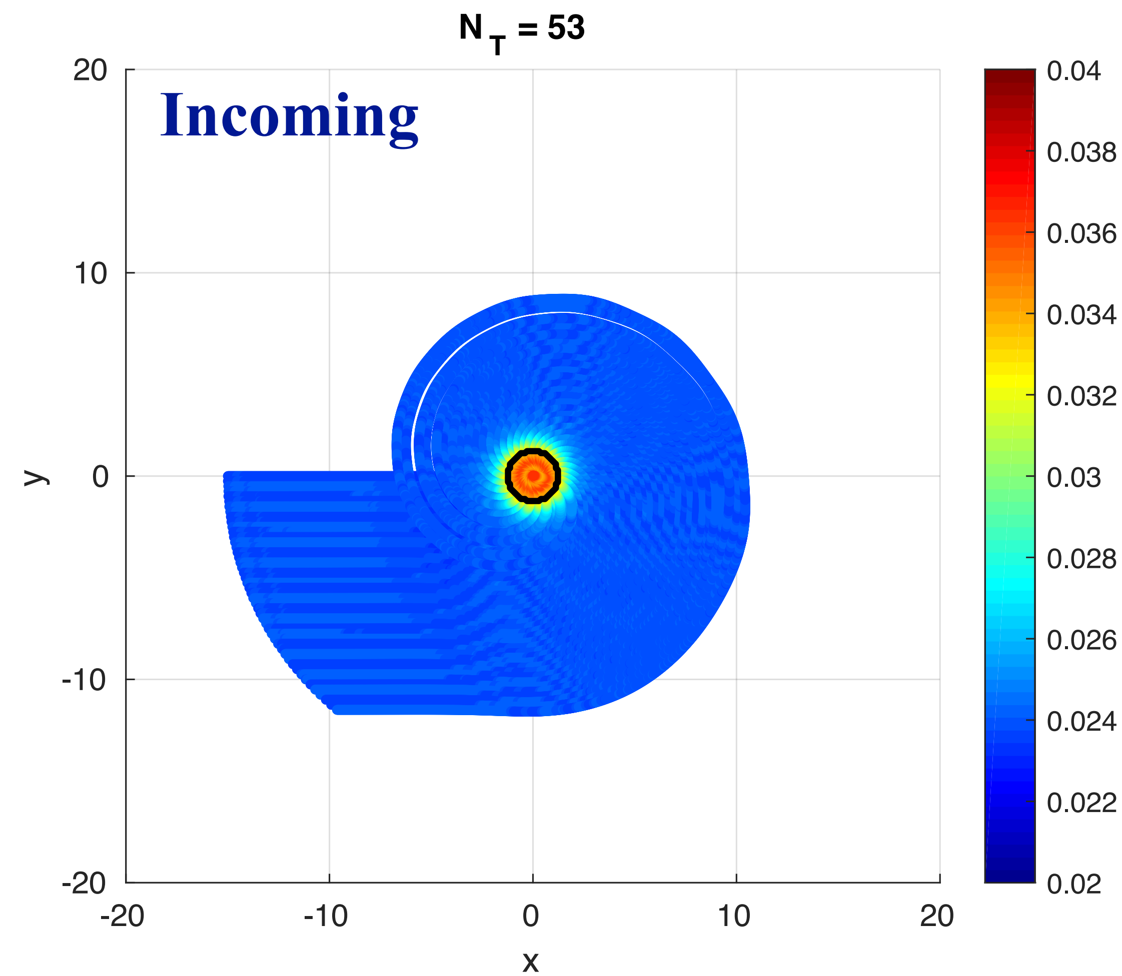
$$m \frac{\mathrm{d}^2 \boldsymbol{x}_p}{\mathrm{d}t^2} + \left(c_4 \sqrt{\frac{\rho R_0}{\sigma}} F(t) + 6\pi R_0 \mu_{air} \right) \frac{\mathrm{d}\boldsymbol{x}_p}{\mathrm{d}t} = -F(t) \boldsymbol{\nabla} \eta |_{\boldsymbol{x}=\boldsymbol{x}_p}. \tag{2.15}$$

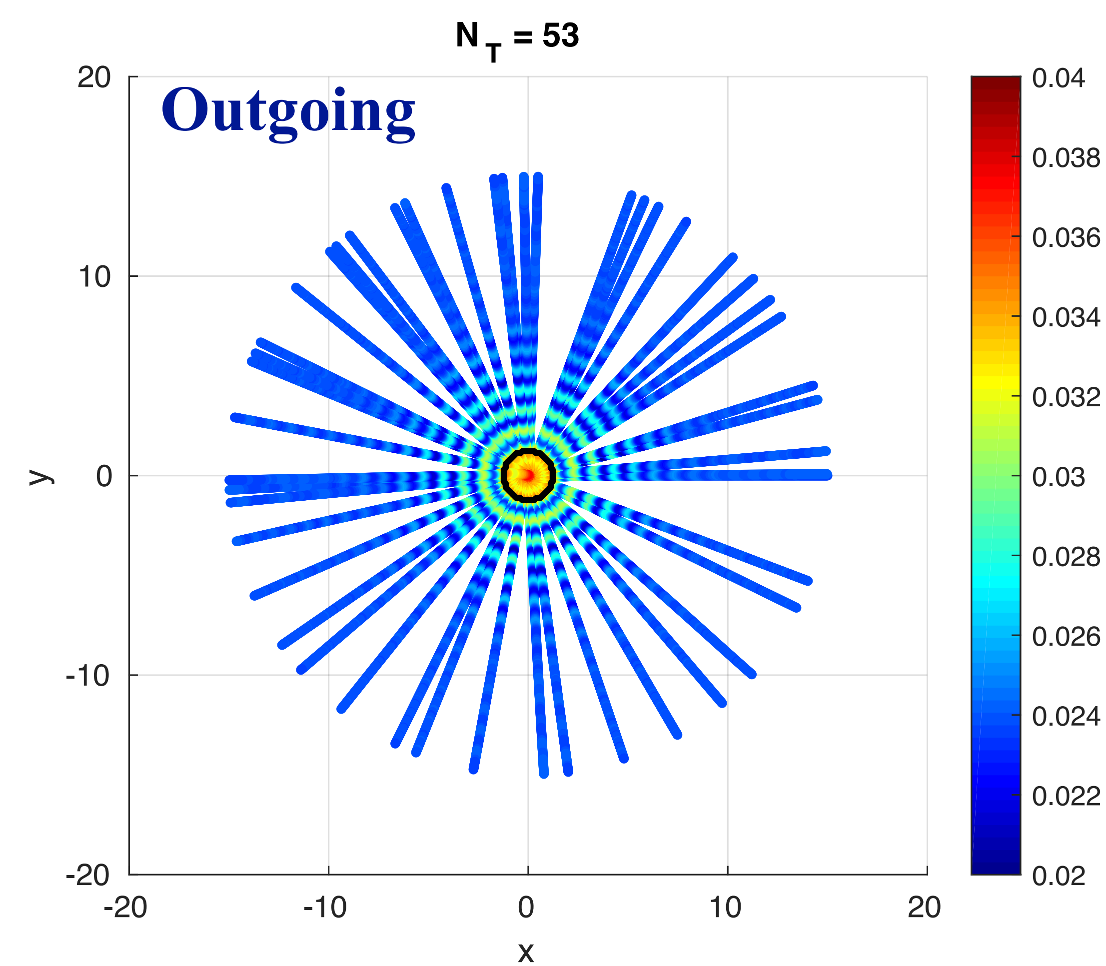
$$b(\boldsymbol{x}) = \begin{cases} \tanh(k_{F_0} h_0)/k_{F_0} & \text{for } \boldsymbol{x} \in \mathcal{D}, \\ \tanh(k_{F_1} h_1)/k_{F_1} & \text{for } \boldsymbol{x} \notin \mathcal{D}. \end{cases} \tag{2.10}$$

$$P_D = \frac{F(t)}{\lambda_F^2} \delta \left(\frac{\boldsymbol{x} - \boldsymbol{x}_p}{\lambda_F} \right). \tag{3.9}$$

… performed by Pedro Sáenz

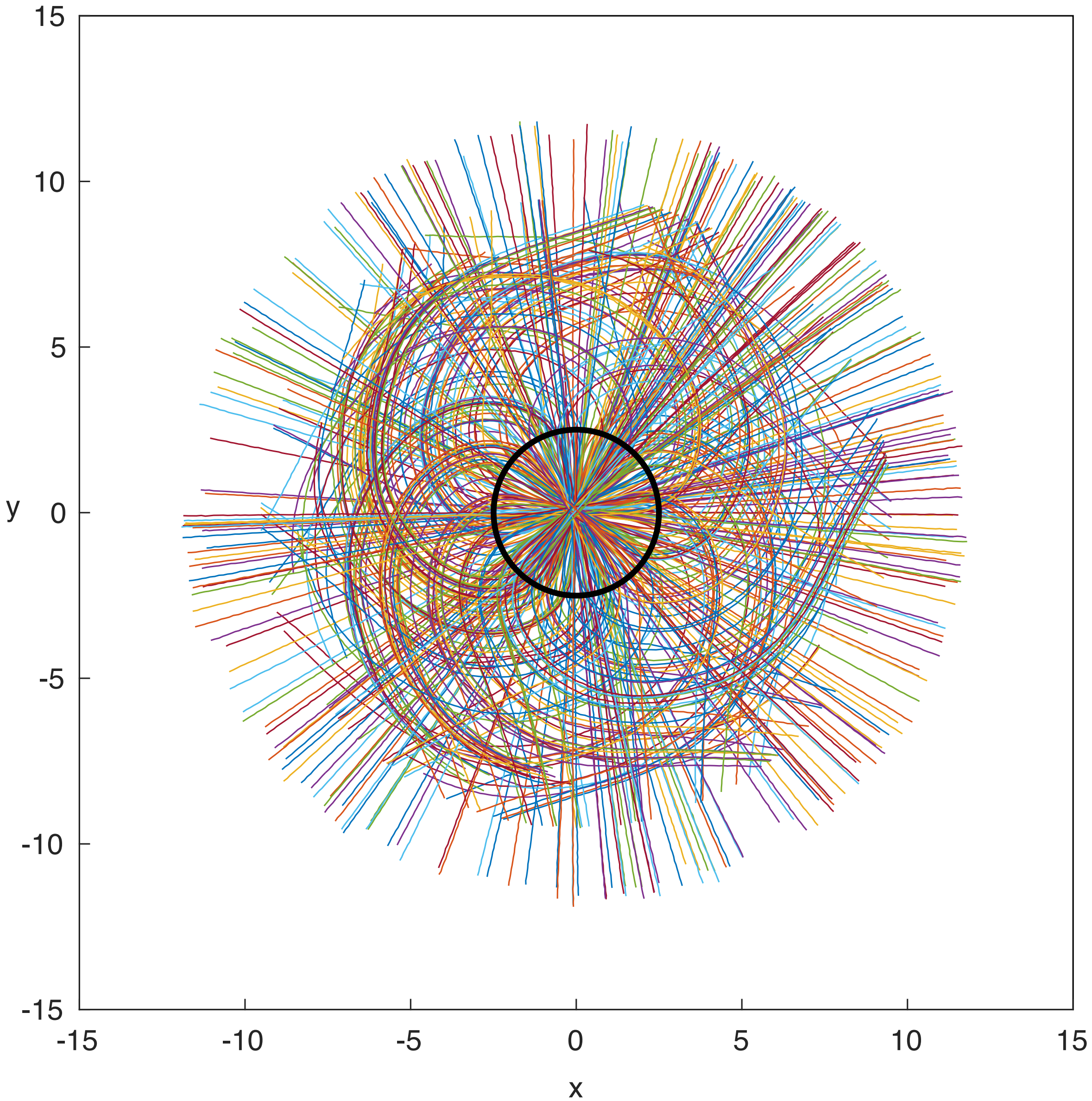


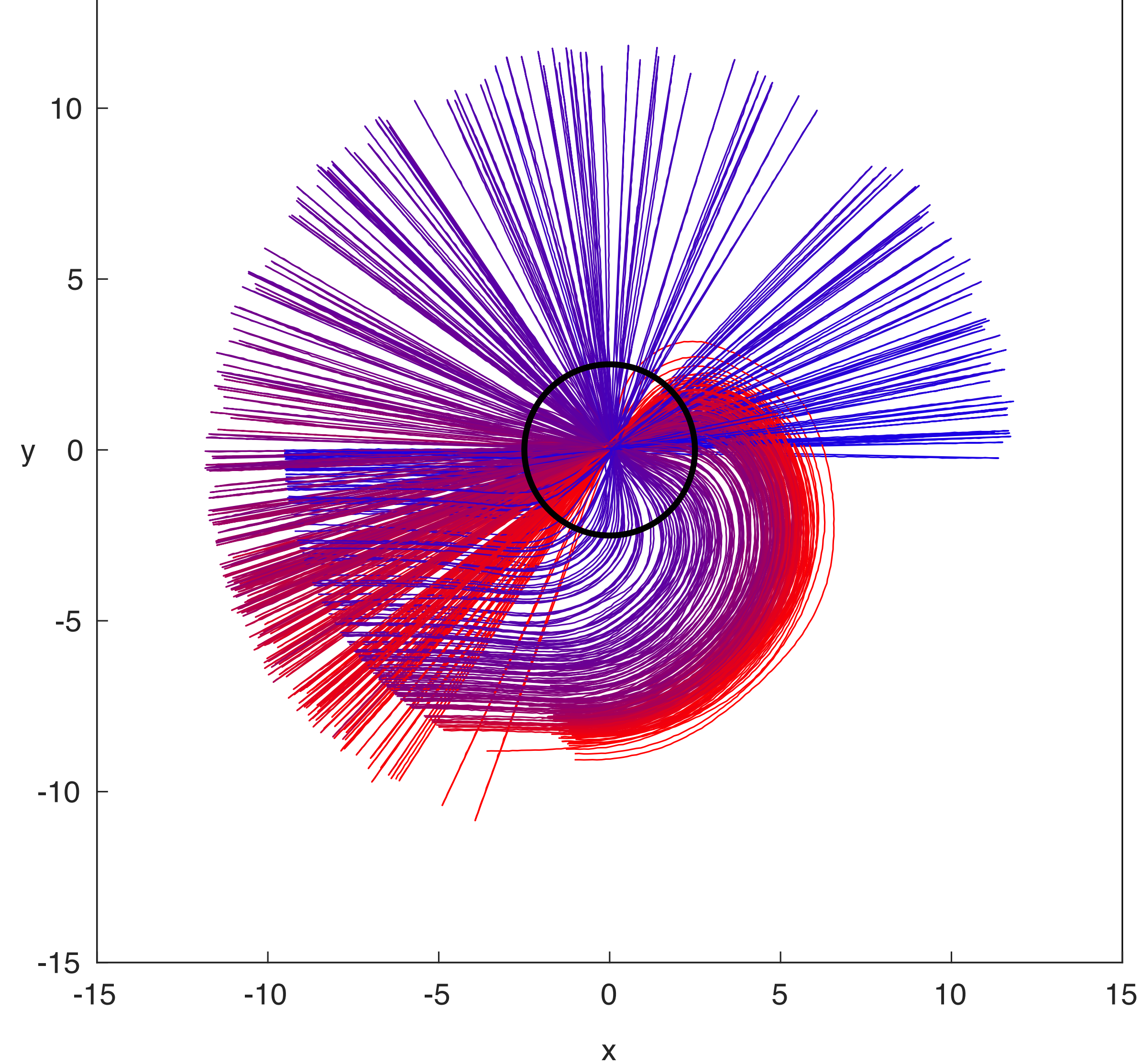


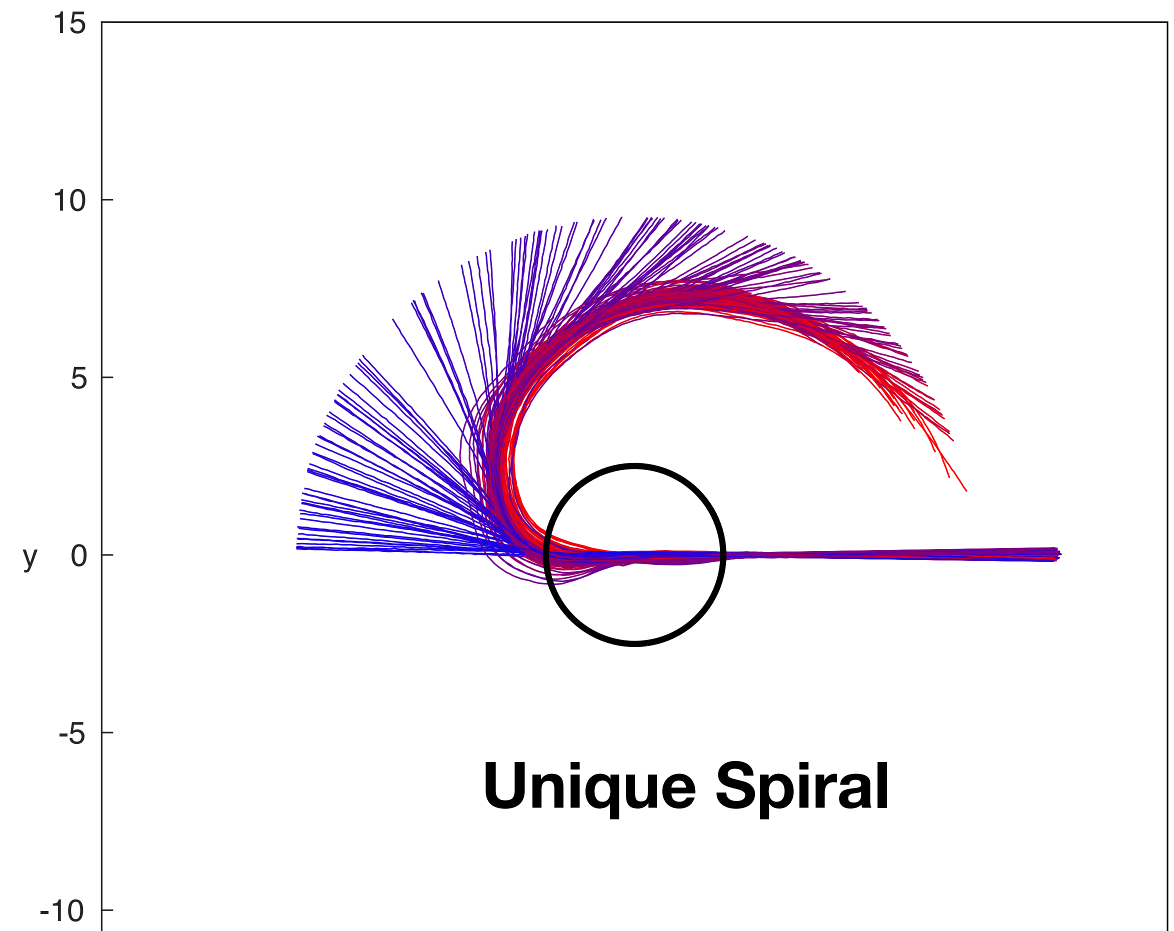


Droplet Speed Map

Experimental spirals

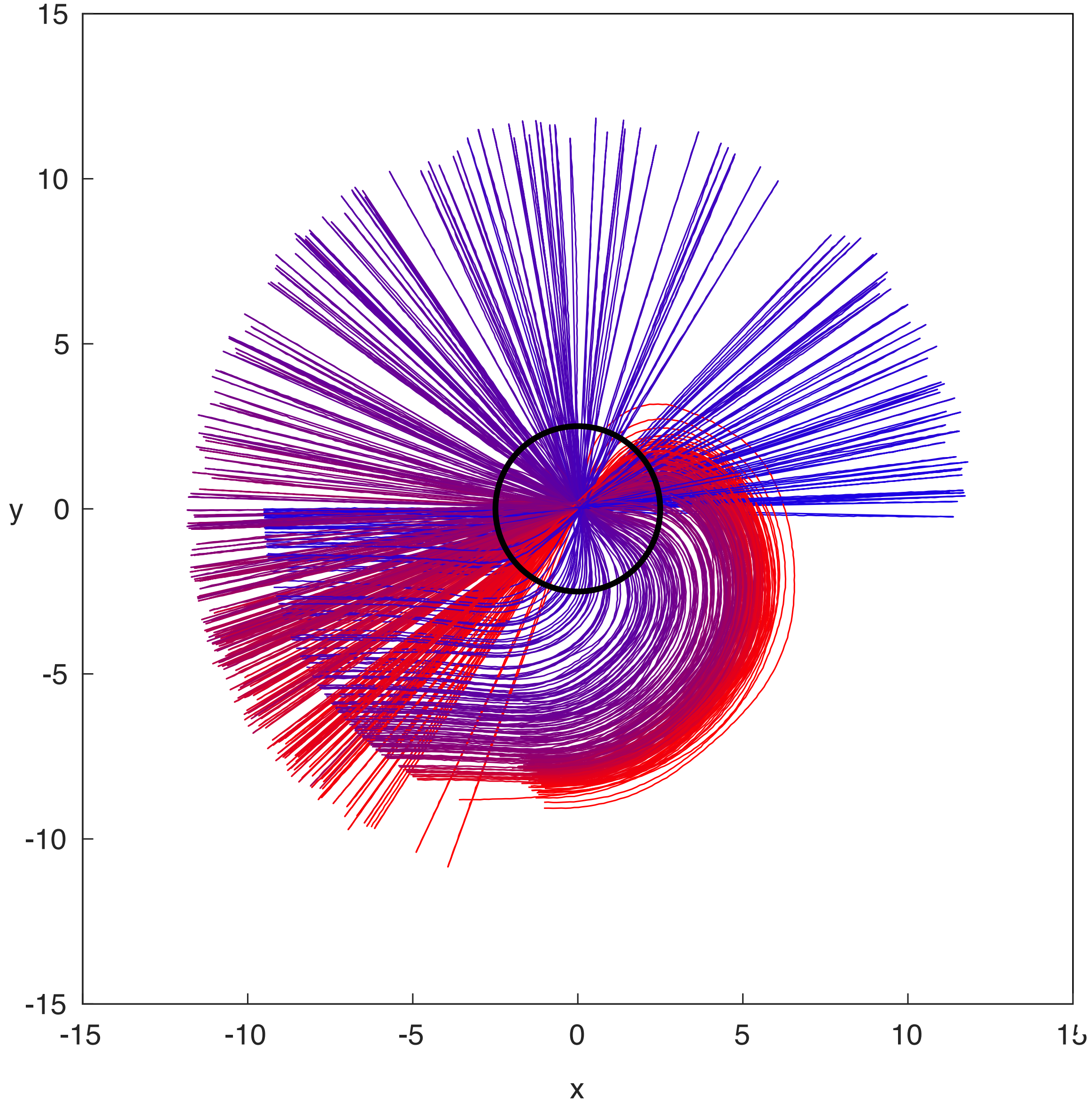


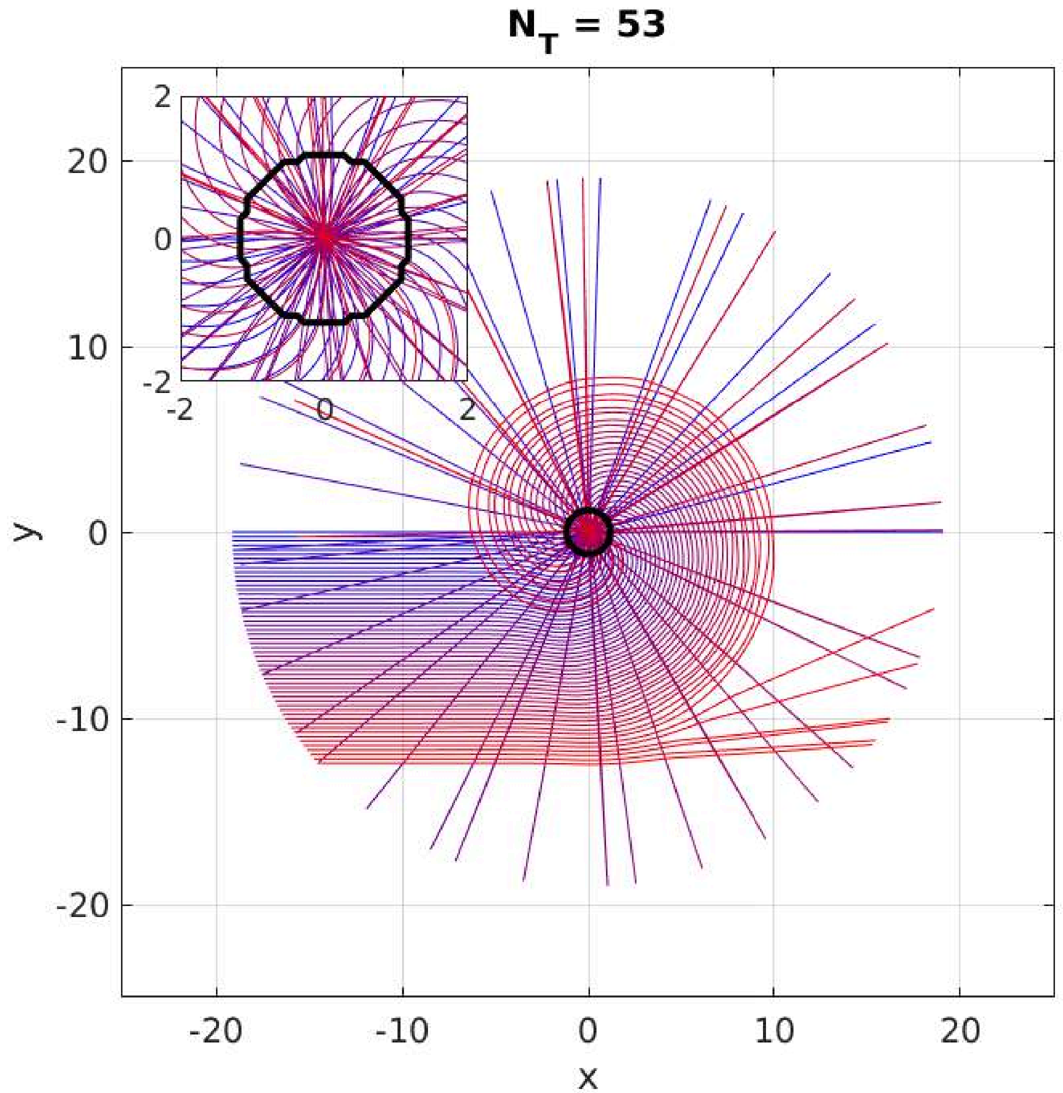


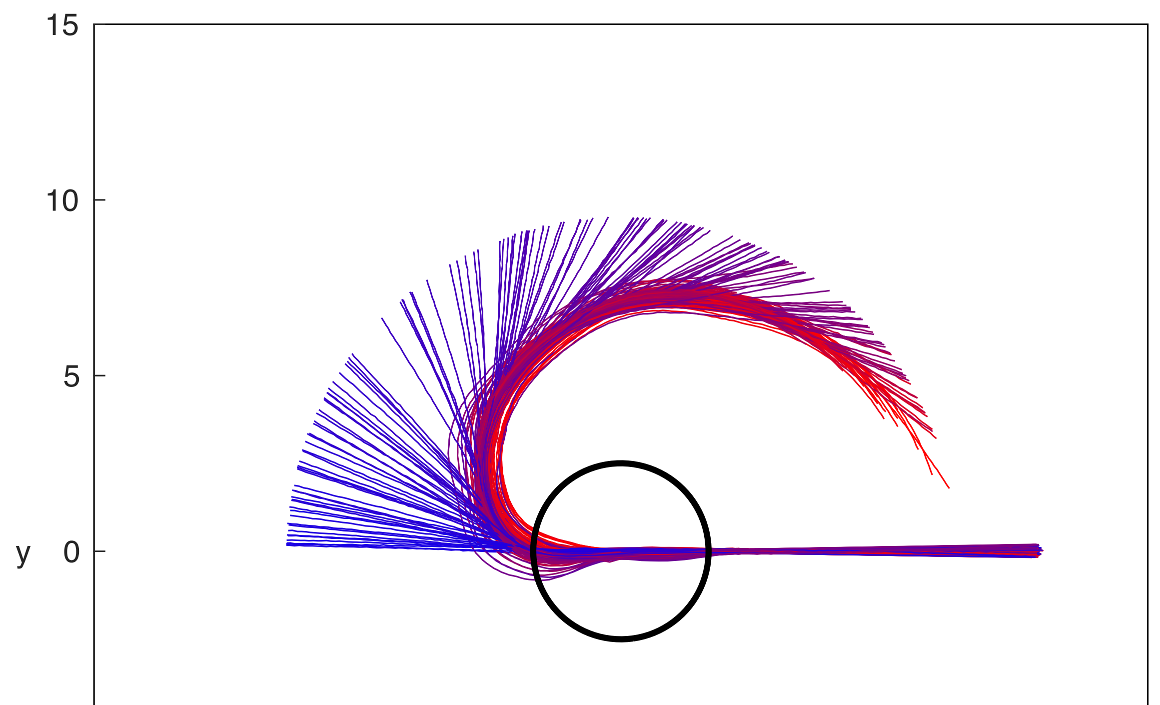


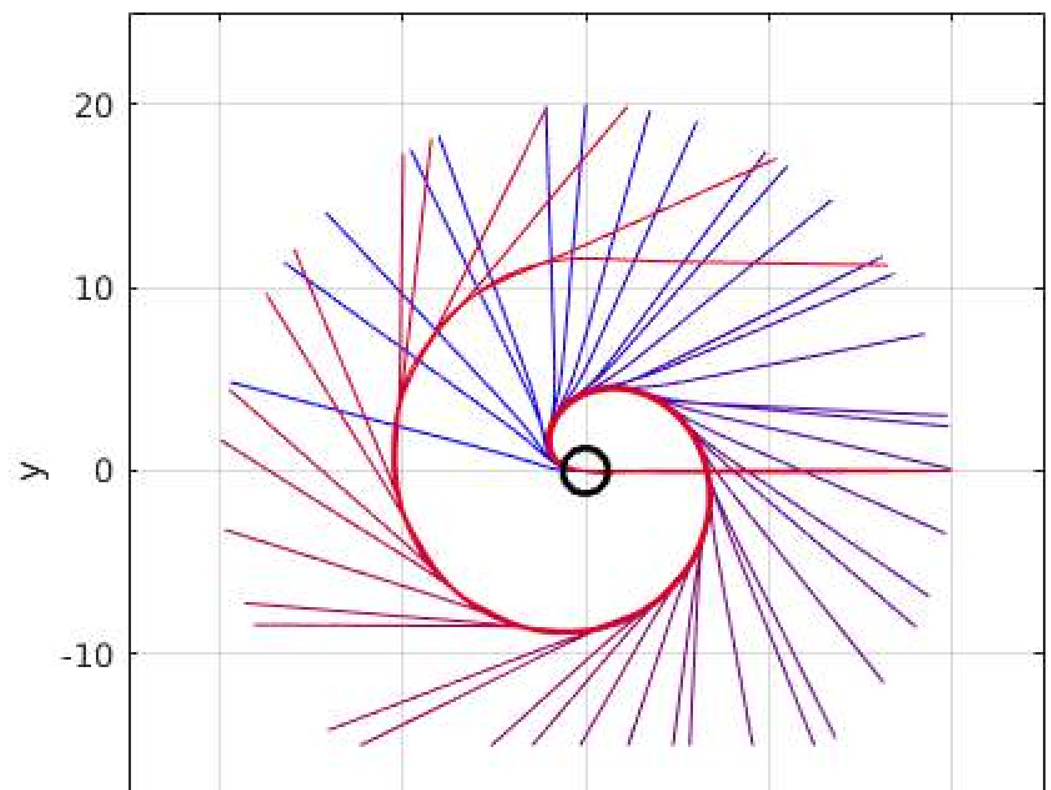
Experiments

Simulations





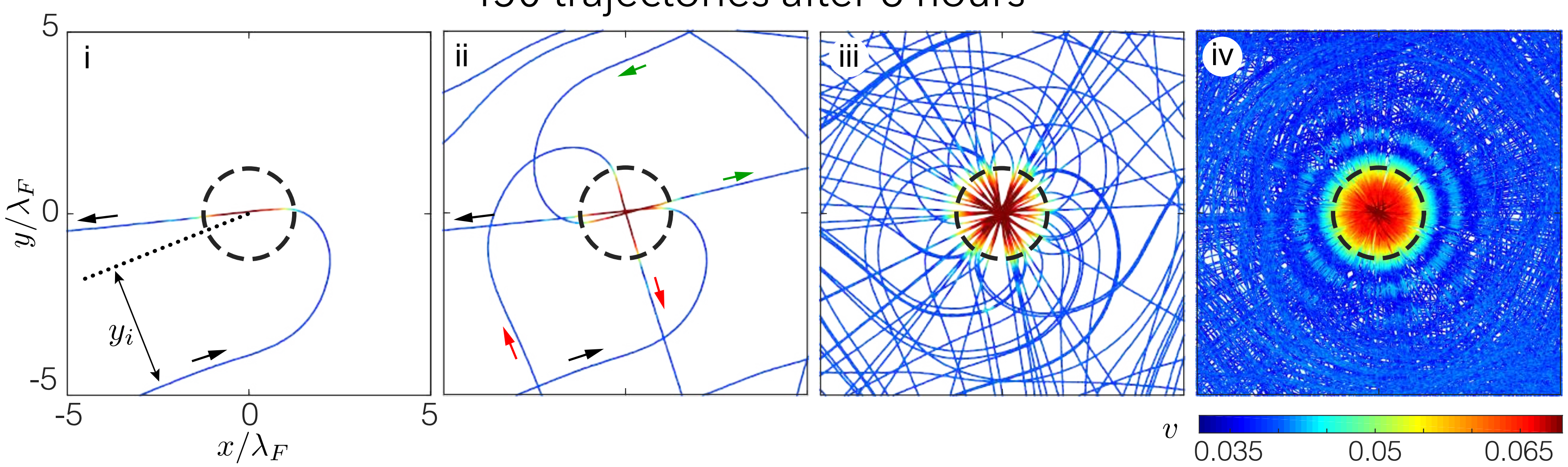




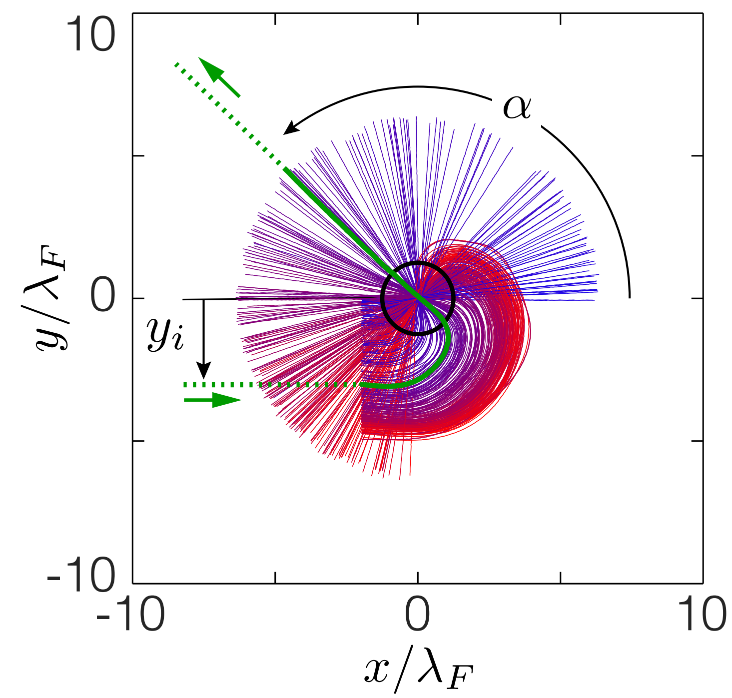
Droplet trajectories

Experiments

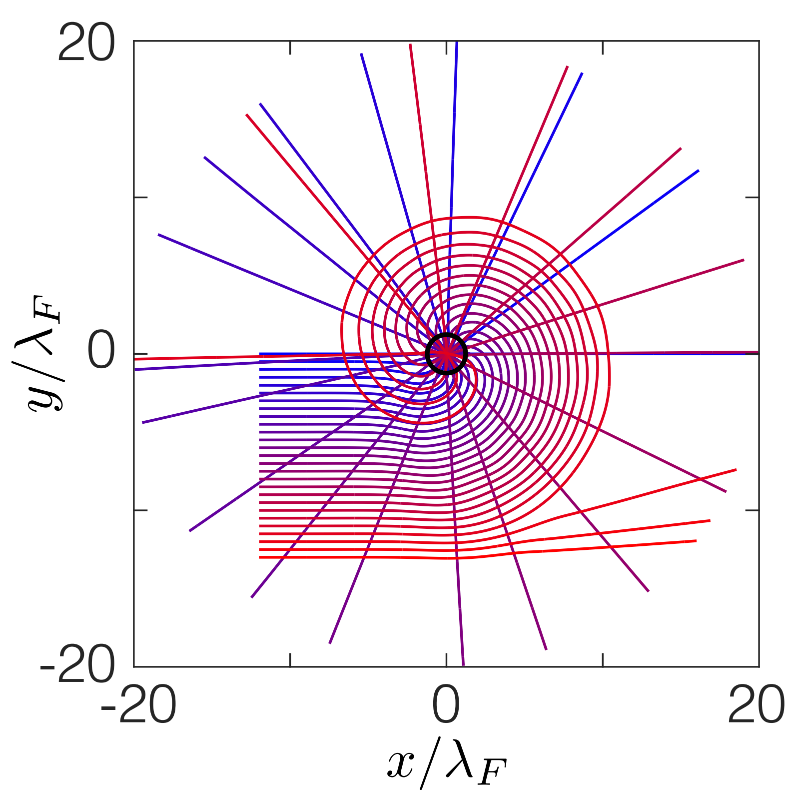
~450 trajectories after 6 hours



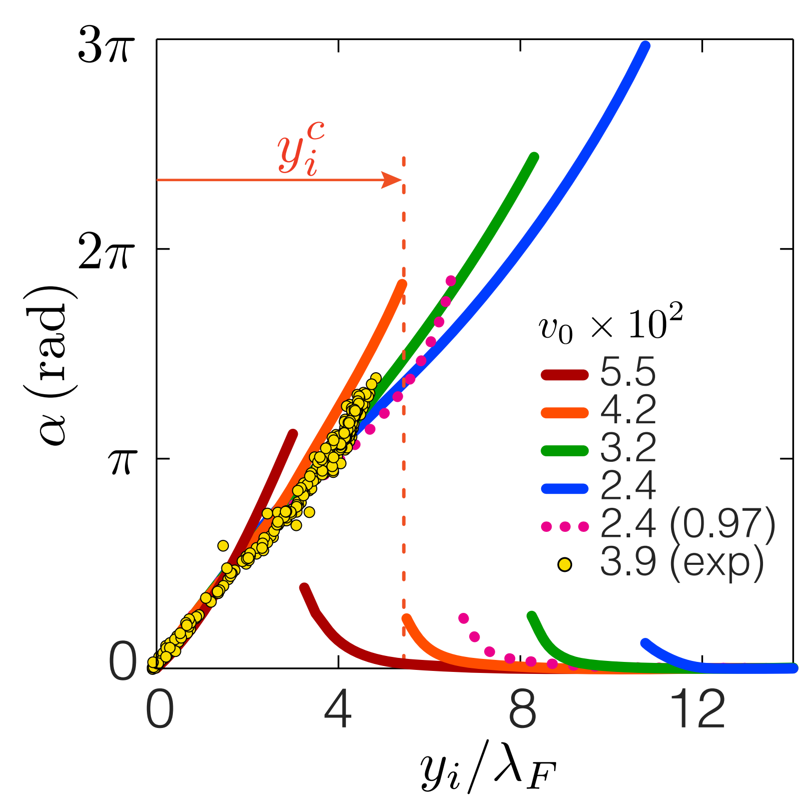
Ordered by impact parameter



Simulations

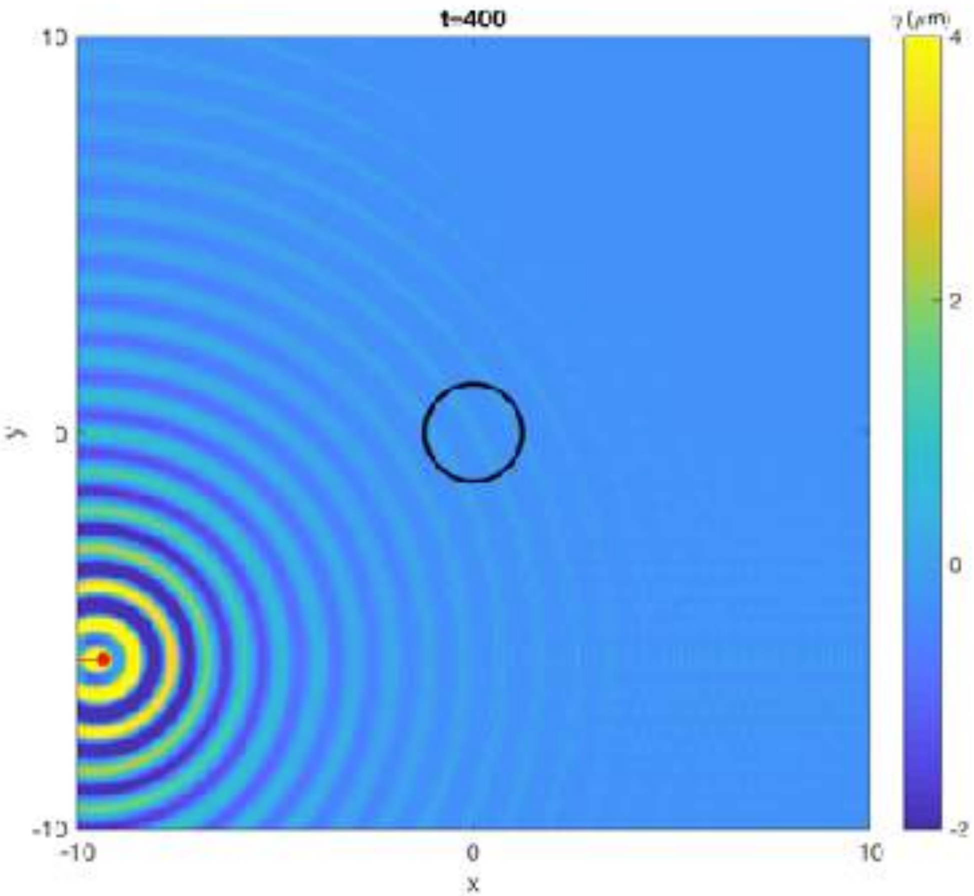


Scattering Angle

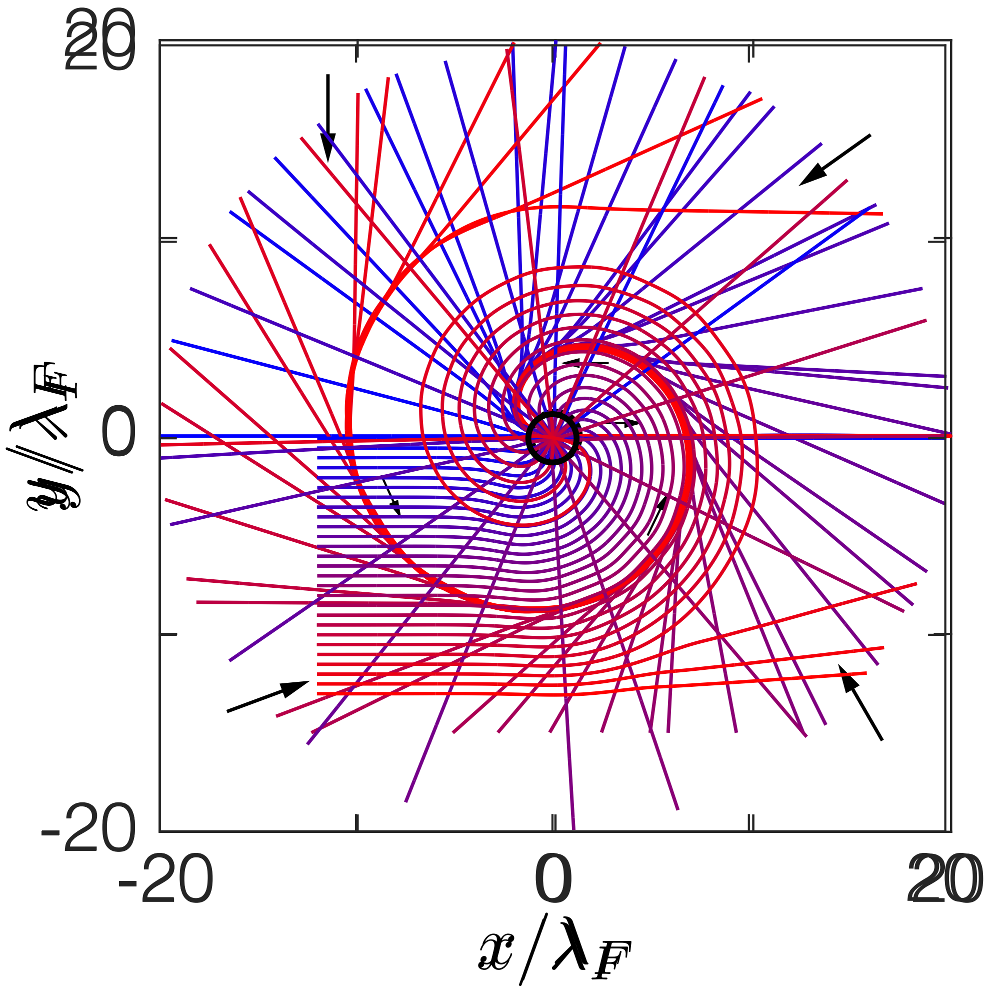


- slower drops spend more time on spiral
- tethering length decreases with memory

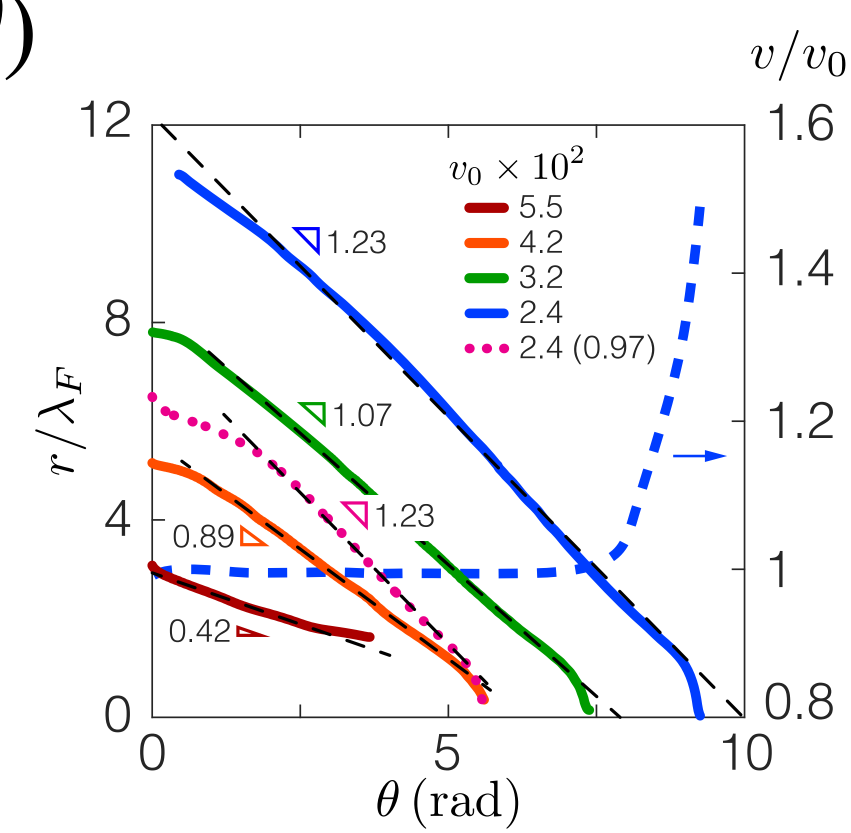
Infer effective force from trajectory



Unique spiral



$r(\theta)$



Archimedean Spiral: $r(\theta) = a + b\theta$

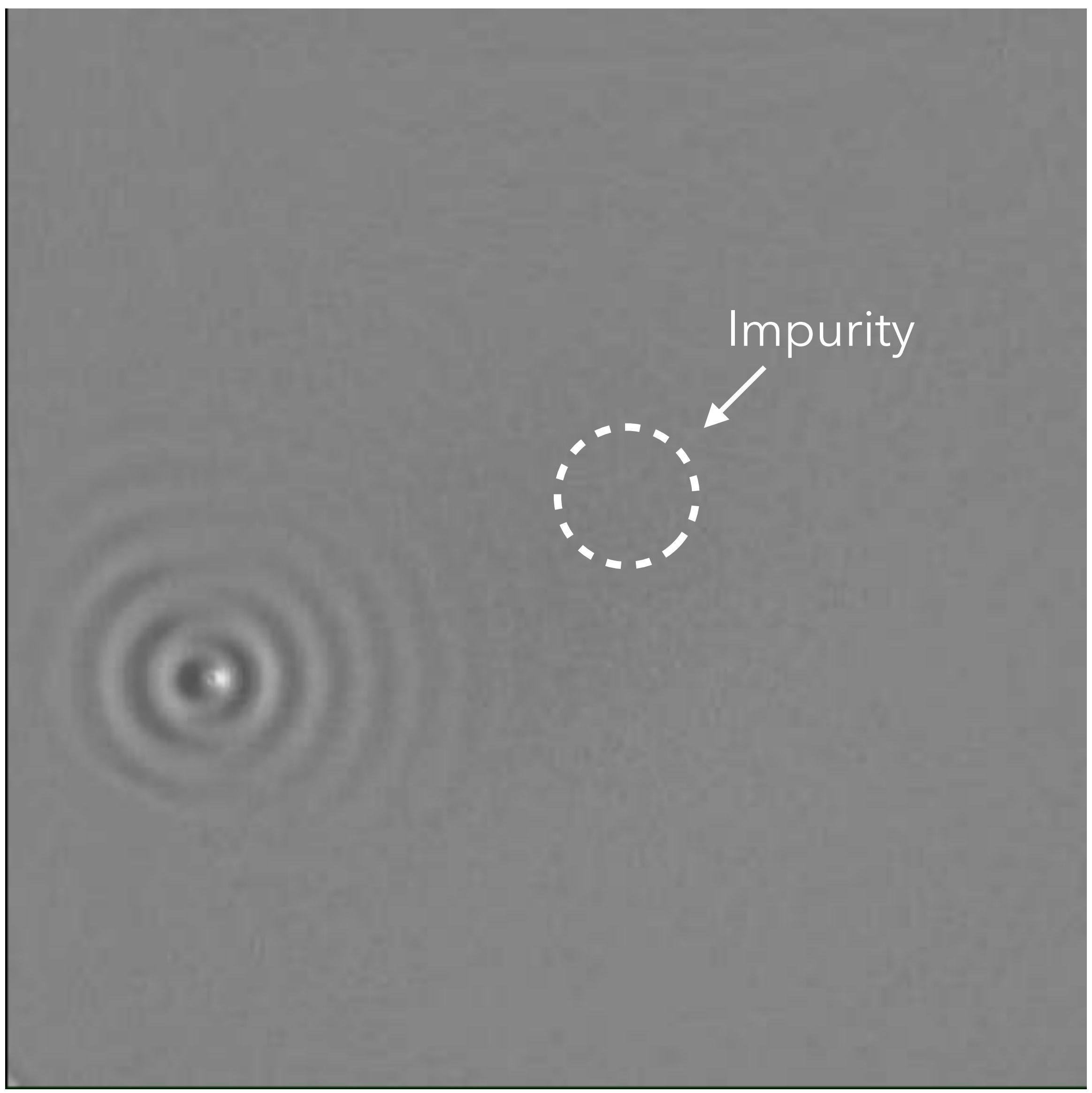
Constant speed: $v^2 = \dot{r}^2 + (r\dot{\theta})^2$

Equation of Motion: $m\dfrac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{F}_w$

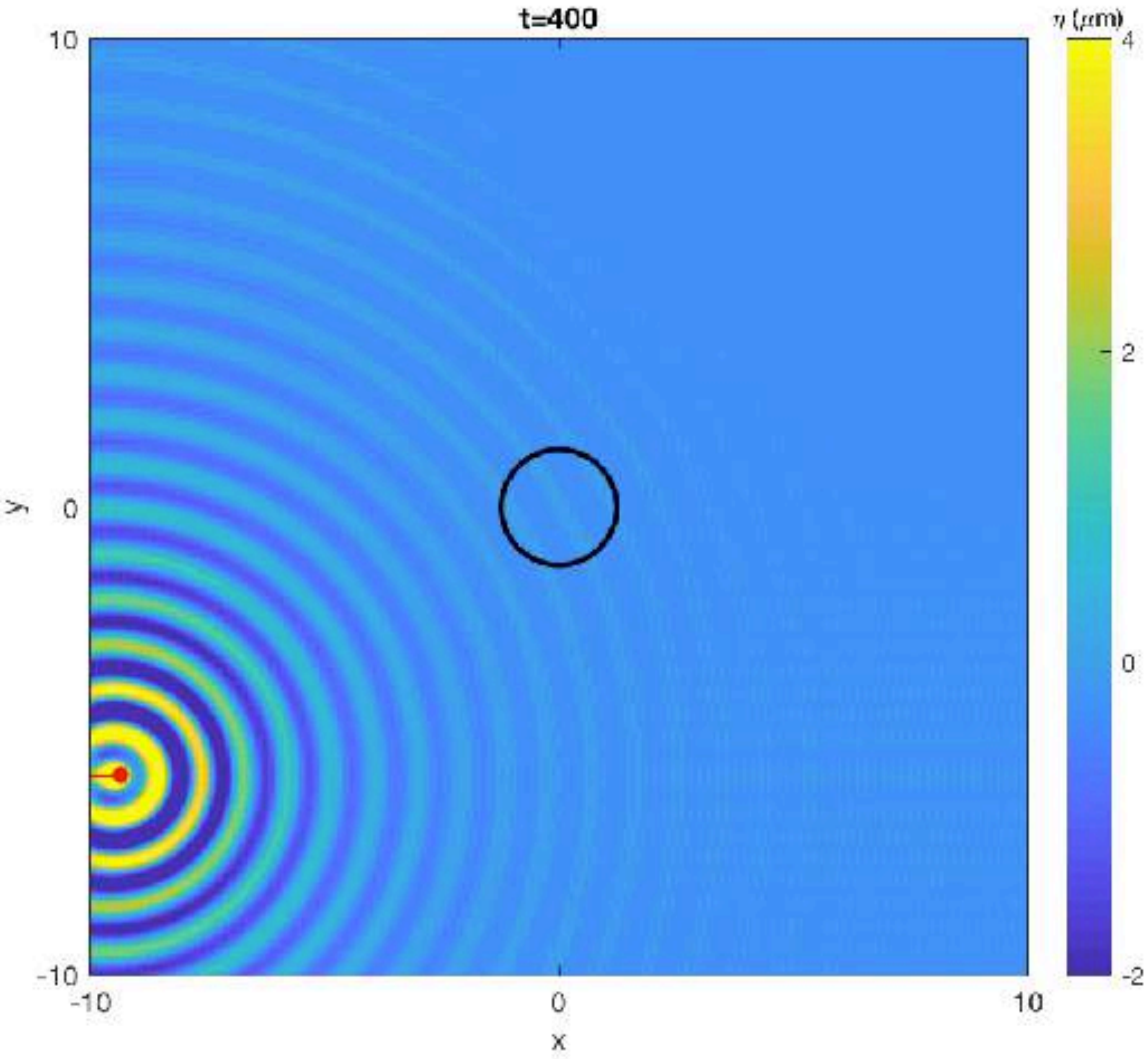
Effective force: $\mathbf{F}_w = m\left(1 + \dfrac{\dot{r}^2}{v^2}\right)(\dot{\boldsymbol{\Theta}} \times \mathbf{v})$

Wave fields

Experiment



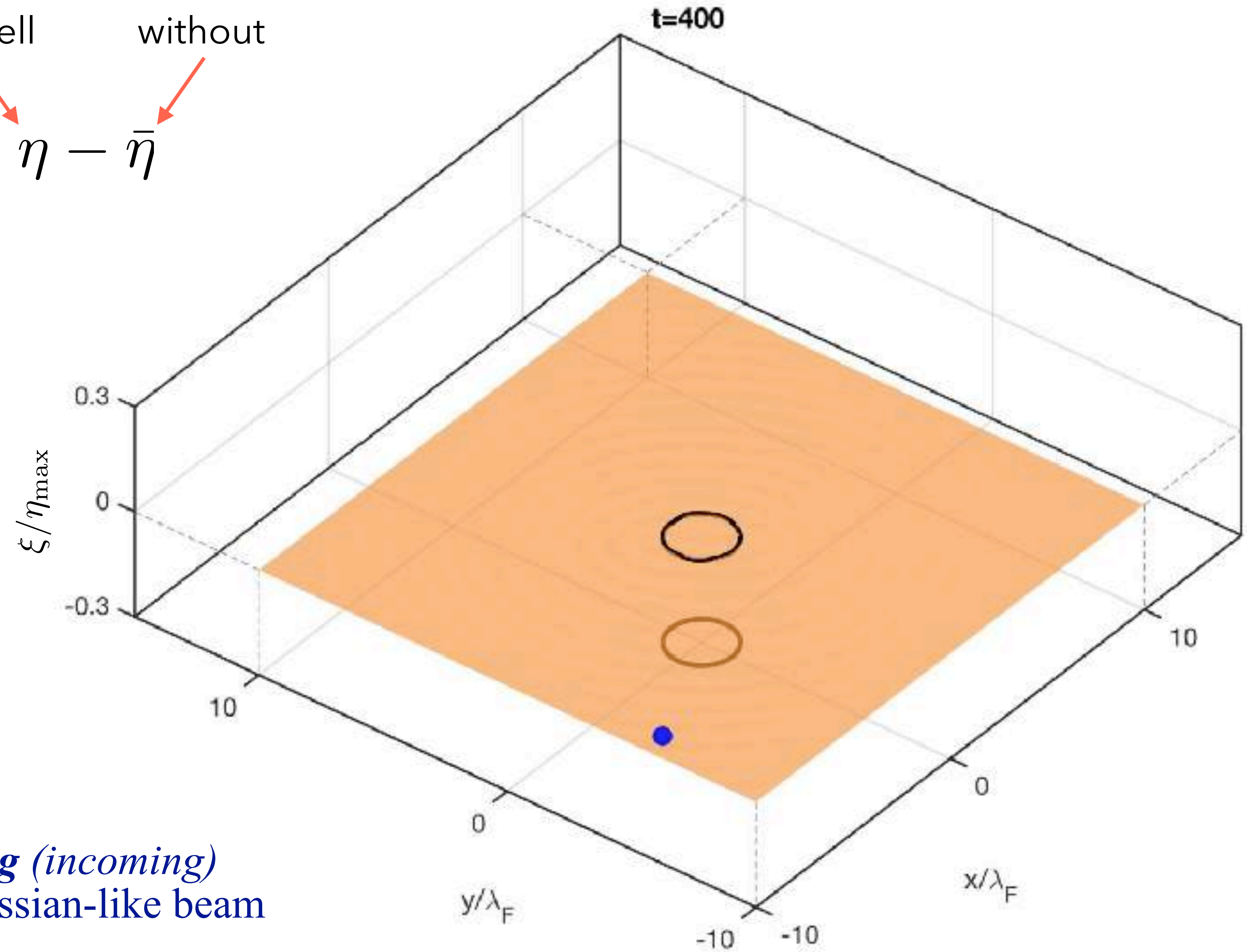
Simulations



Anomalous wave field

with well → η, without → $\bar{\eta}$

$$\xi = \eta - \bar{\eta}$$

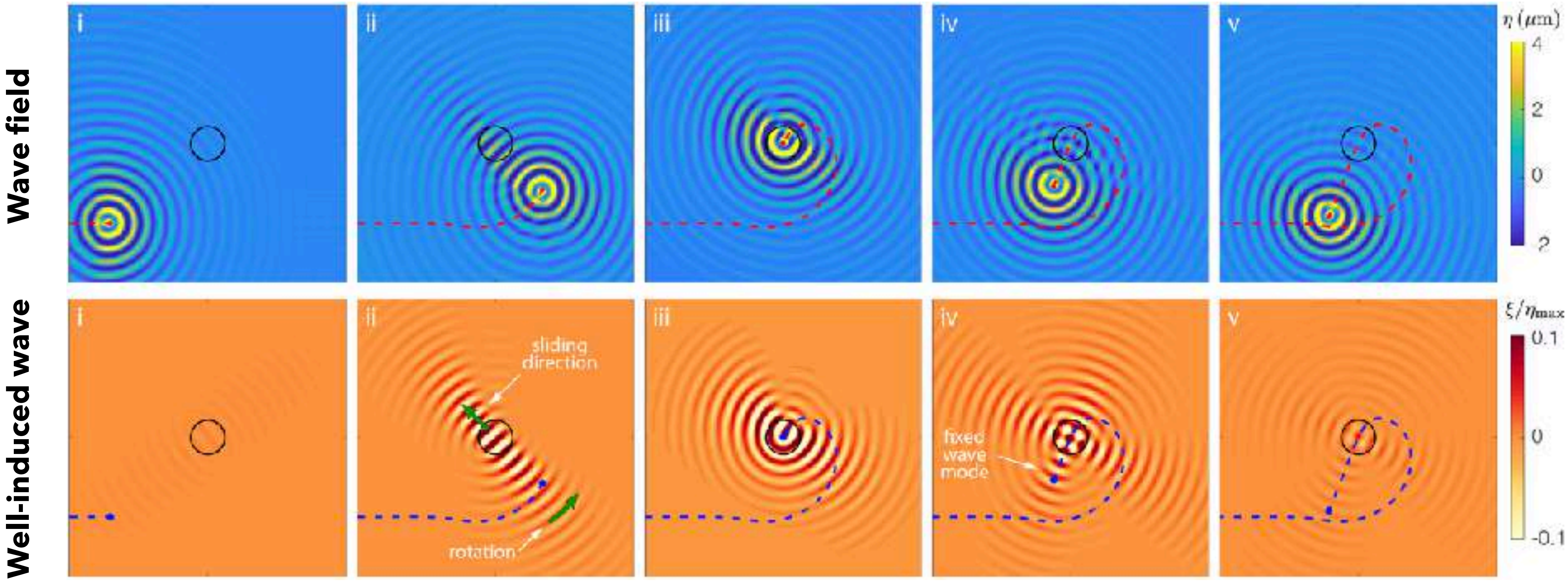


Scattering *(incoming)*
Sliding Gaussian-like beam

$$\mathbf{F}_w = m\left(1 + \frac{\dot{r}^2}{v^2}\right)(\dot{\boldsymbol{\Theta}} \times \mathbf{v})$$

Wavelike statistics *(outgoing)*
Resonant waves centered at well

Mechanism



Sliding beam attracts drop | Resonant excitation | Drop 'rides' fixed mode | Fixed mode decays slowly

Wave decay

$$\sim e^{-t/T_M}$$

$$T_M = \frac{T_d}{1 - \gamma/\gamma_F}$$

Effect of the well

Enhanced memory time $T_M^h \sim 0.6\,\mathrm{s} \rightarrow T_M^H \sim 2.5\,\mathrm{s}$

Larger amplitude $\max(\xi/\eta_{\max}) \sim 30\%$

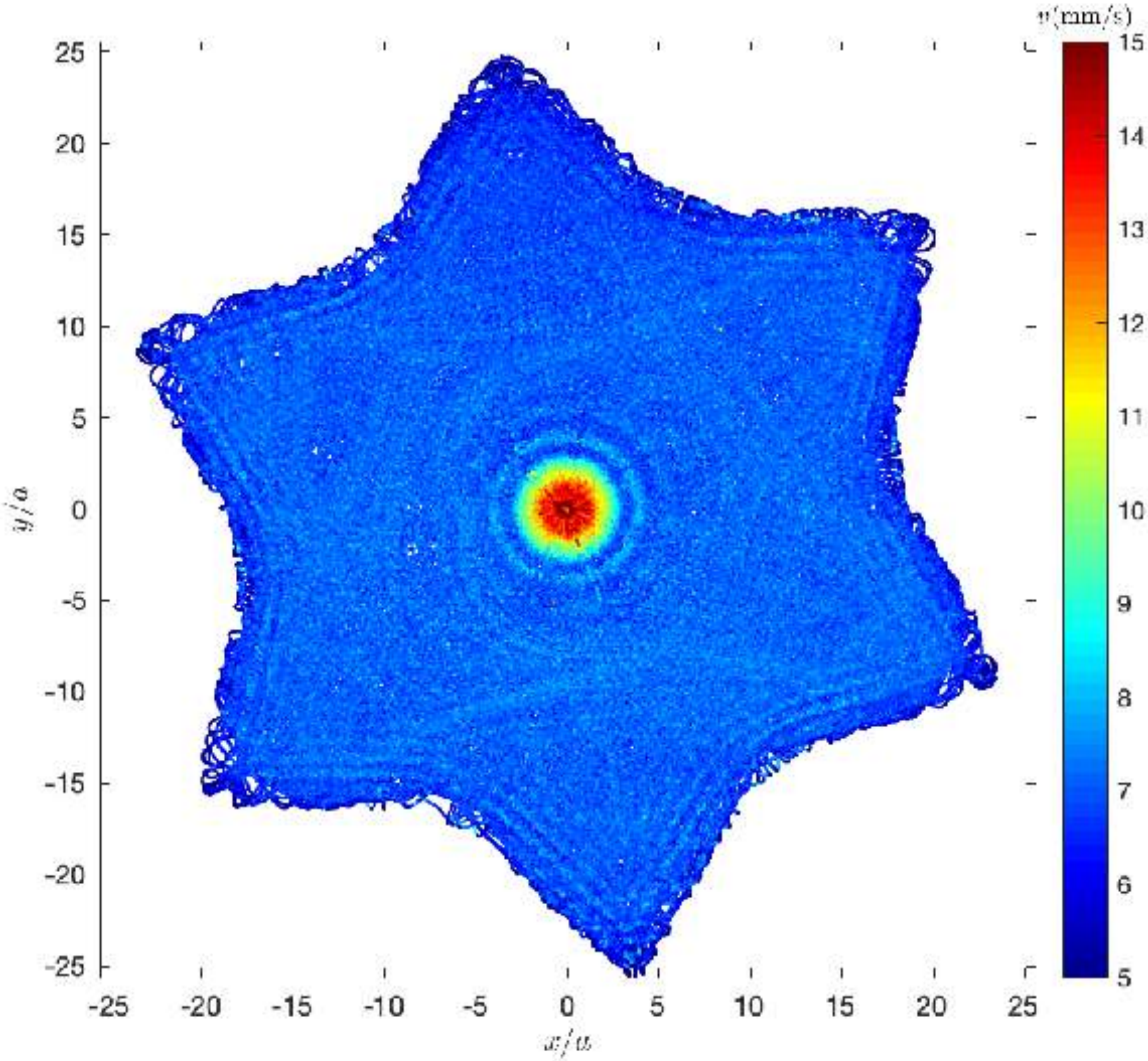
Crossing time

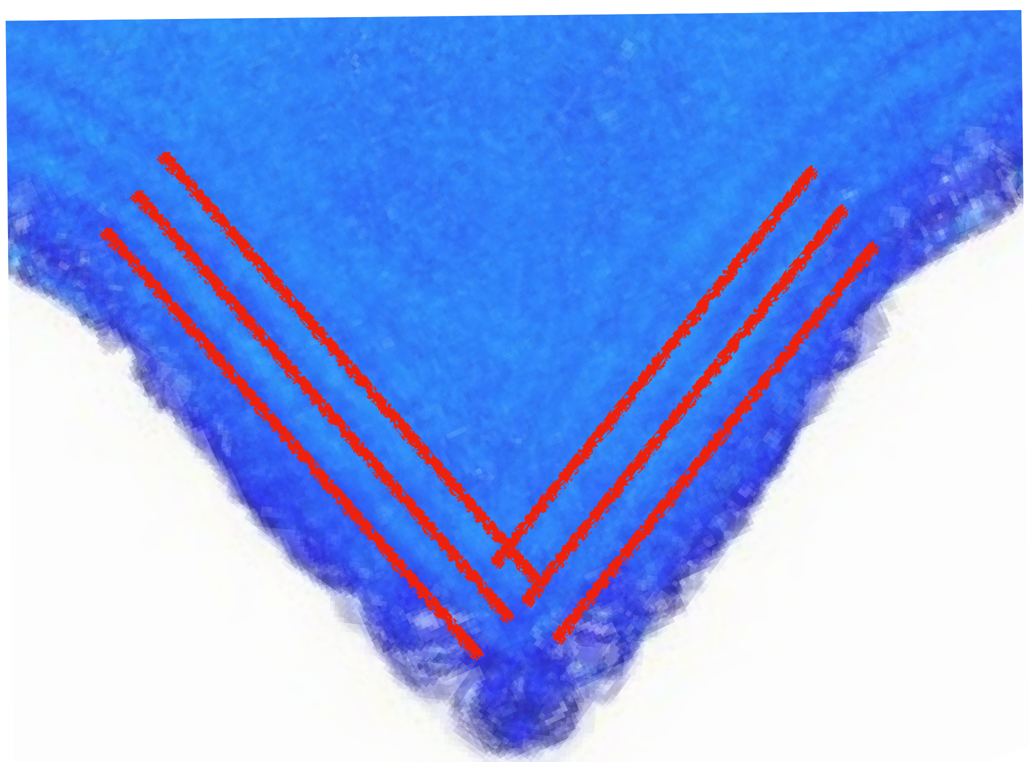
$$\lambda_F/v_0 \sim 0.7\,\mathrm{s}$$

$$T_M > \lambda_F/v_0$$

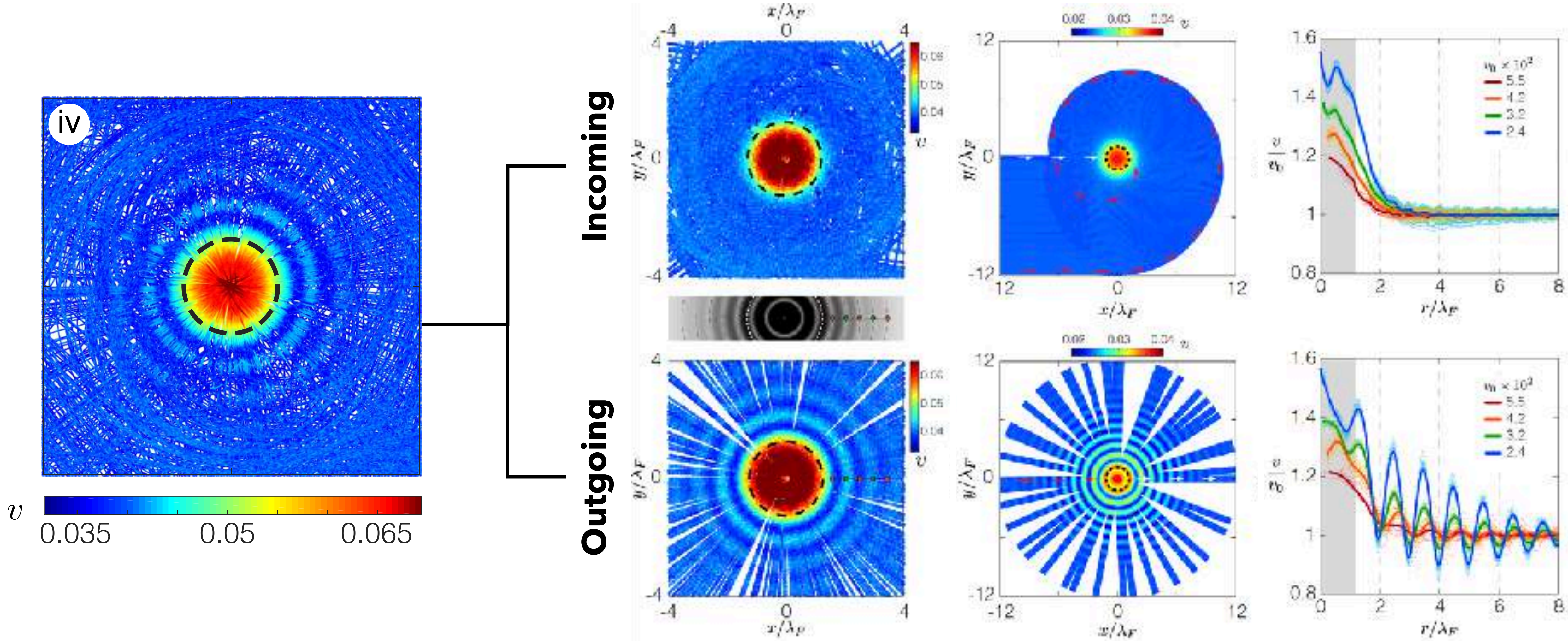
Detailed wave-mediated interaction mechanism

Friedel from the border





Summary

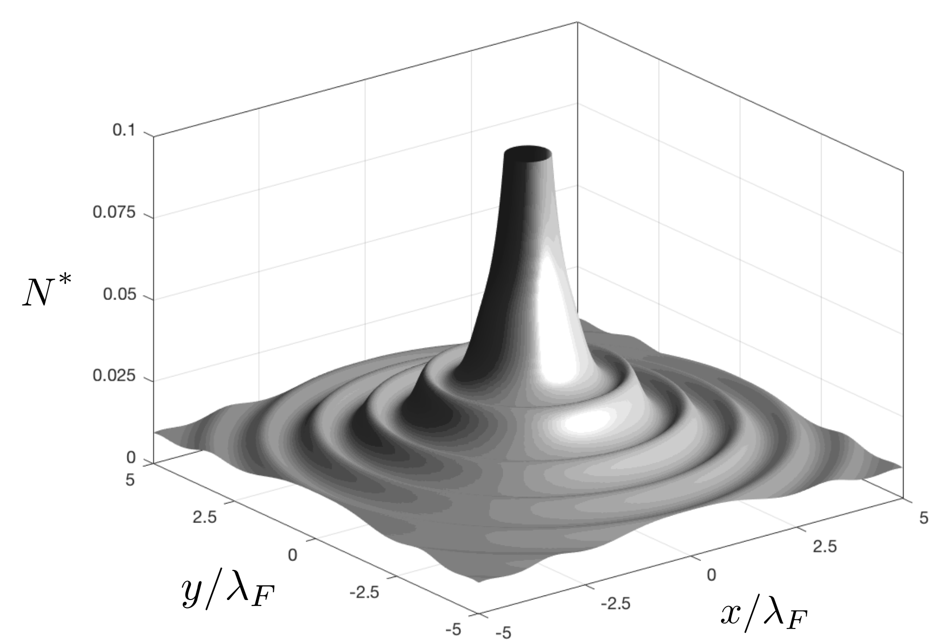


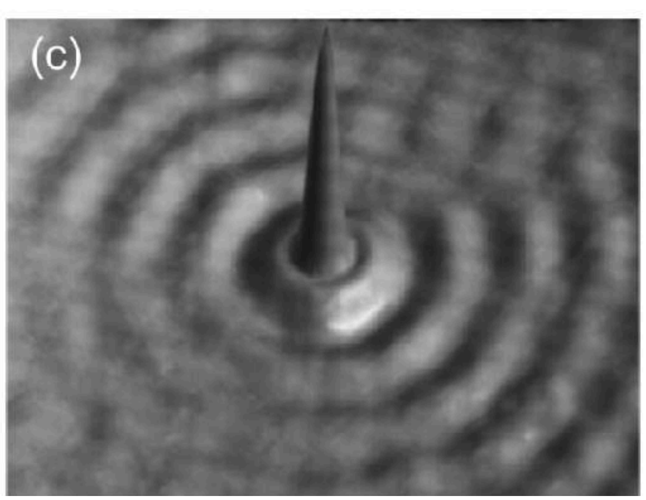
Experiments

Simulations

$v(r)$

Localized wavelike statistics from speed modulation in outgoing trajectories

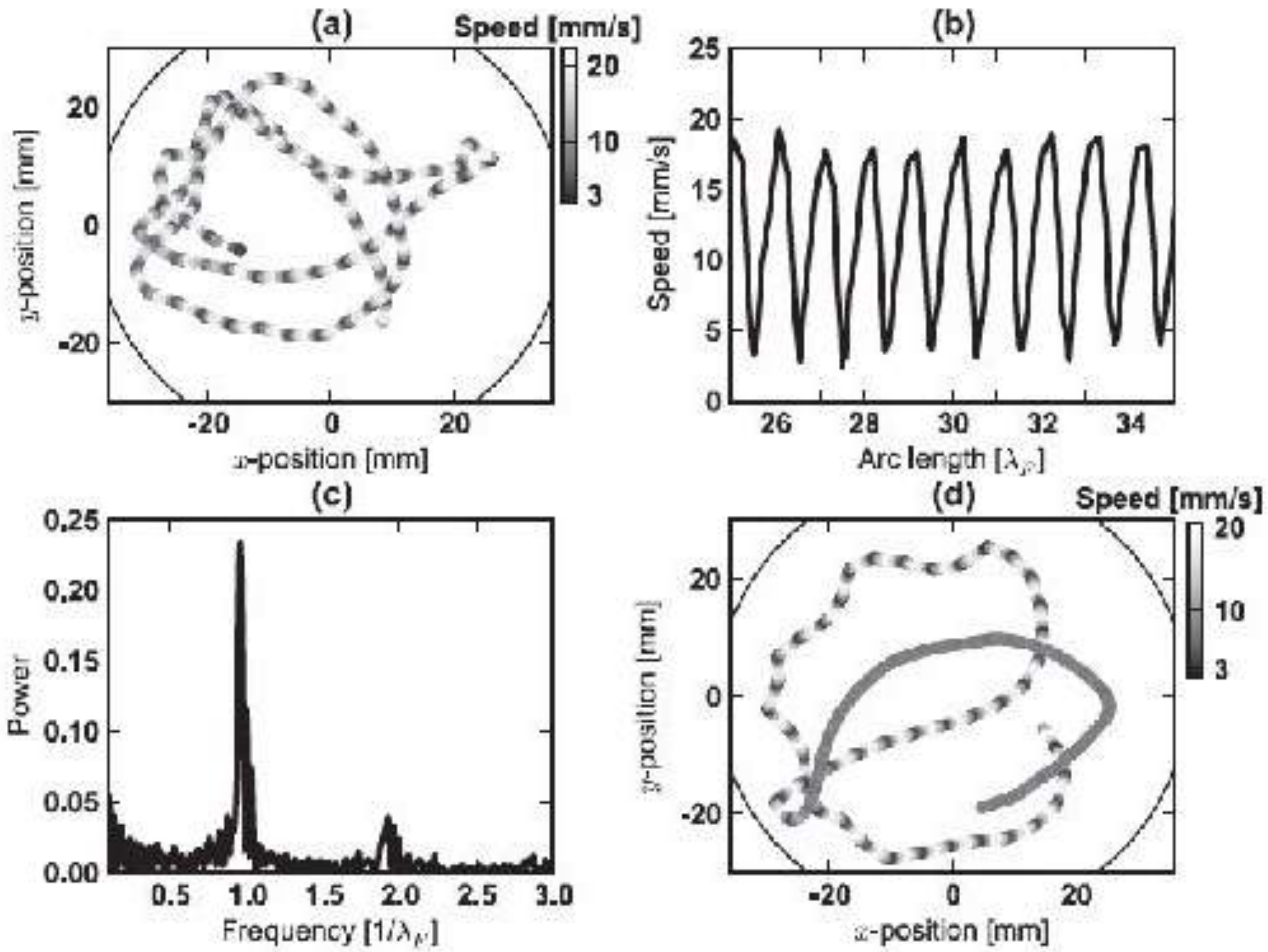




Friedel Oscillations

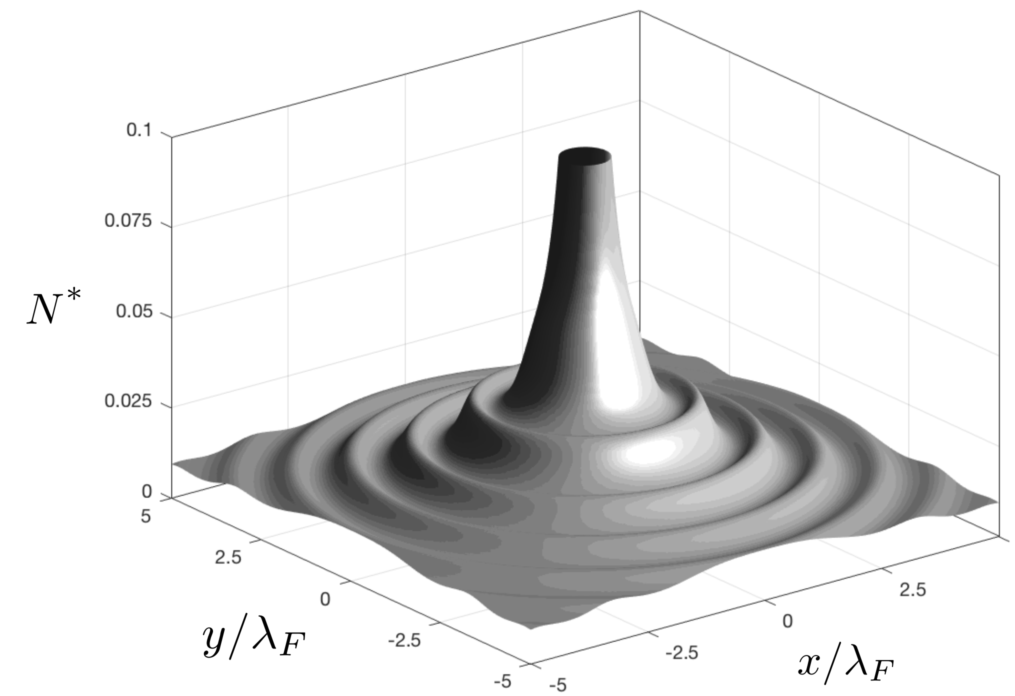
HQA Paradigm II : in-line oscillations

I. FREE WALKER

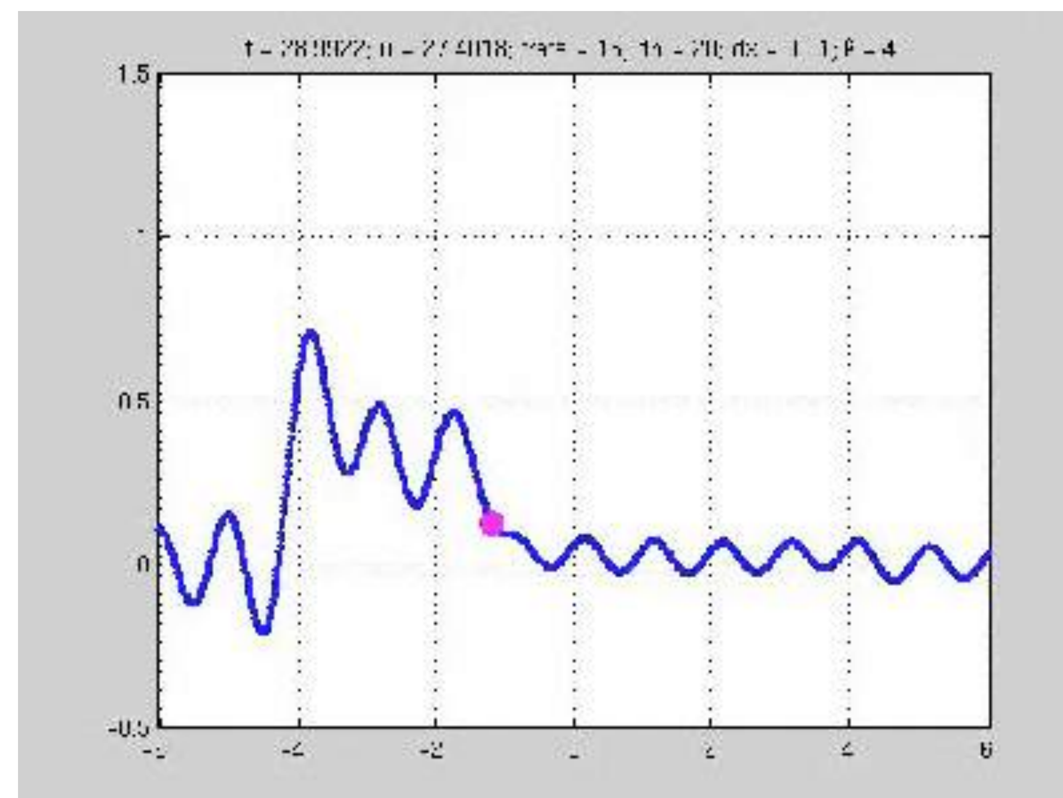


Wind-Willassen et al. (2013), Bacot et al. (2019)

II. FRIEDEL OSCILLATIONS



III. 1D SPRING FORCE



- correlation between speed and position lead to a statistical signature with the pilot wavelength

- provides mechanism for emergent statistics in Friedel oscillations and 1D SHO, other systems where topography provides geometric anchor