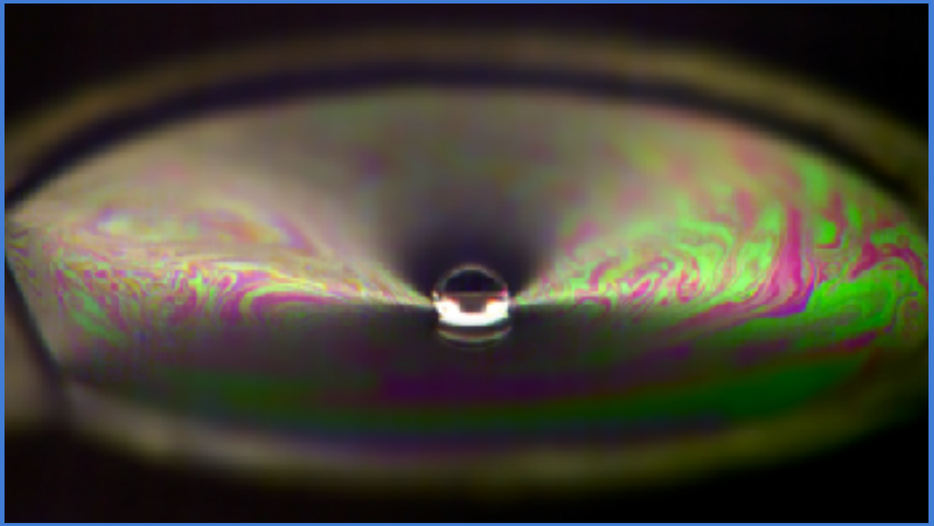
HQA Lecture 9 :

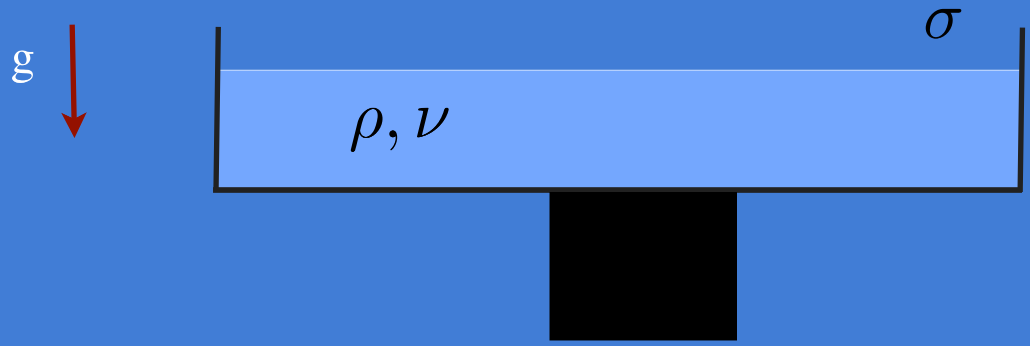
Faraday waves, Tibetan bowls and the fluid trampoline



Faraday waves

Faraday (1831)

- surface undulations with twice the forcing period, a parametric instability
- arise above a threshold that depends on fluid depth, viscosity, surface tension

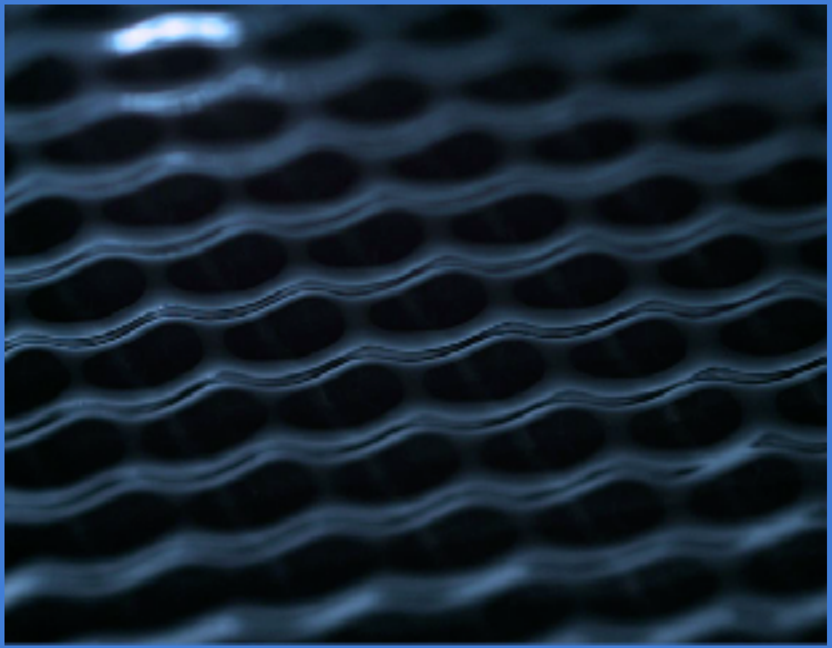


frequency f

amplitude A

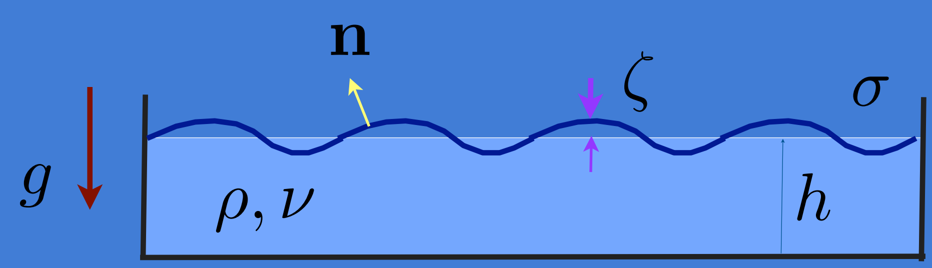
$$\Gamma > \Gamma_c \approx 4$$

Forcing parameter: $\Gamma = A(2\pi f)^2/g$



Unforced surface waves

$$\Gamma = 0$$



Dispersion relation

$$\omega^2 = \left(\frac{\sigma}{\rho}\, k^3 + g\, k\right) \tanh kh$$

- wave form depends on kh and $B_o = \dfrac{\rho g}{\sigma k^2} = \dfrac{\text{gravity}}{\text{capillarity}}$

Deep water: $\tanh kh \sim 1$

Gravity waves $(B_o \gg 1)$

$$\omega = \sqrt{gk}$$

Capillary waves $(B_o \ll 1)$

$$\omega = \left(\frac{\sigma}{\rho}\right)^{1/2} k^{3/2}$$

Shallow water: $\tanh kh \sim kh$

Gravity waves $(B_o \gg 1)$

$$\omega = \sqrt{gh}\; k$$

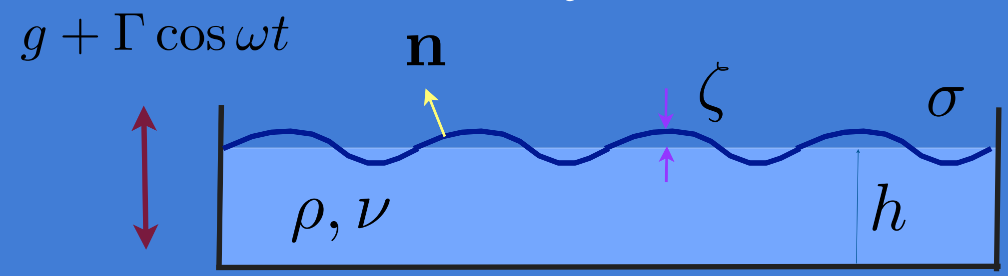
Capillary waves $(B_o \ll 1)$

$$\omega = \left(\frac{\sigma h}{\rho}\right)^{1/2} k^2$$

Benjamin & Ursell (1954)

Faraday instability

Consider an inviscid vibrating bath:



$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p - (g + \Gamma \cos \omega t) z \quad , \quad \nabla \cdot \mathbf{u} = 0$$

Conditions at surface: $z = h + \zeta(x, y, t)$

Bernoulli $$\frac{\partial \phi}{\partial t} + \frac{1}{2}\mathbf{u}^2 + \frac{\sigma}{\rho}\nabla \cdot \mathbf{n} + (g + \Gamma \cos \omega t) z = 0$$

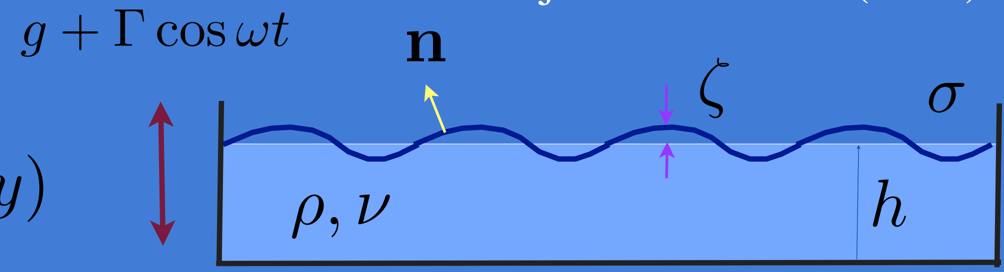
Kinematic $$\frac{D\zeta}{Dt} = u_z$$ where $\mathbf{u} = \nabla \phi$, ϕ is velocity potential

Linearize in ζ , ϕ . Expand ζ, ϕ in terms of eigenfunctions $S_m(x, y)$

s.t. $$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_m^2 \right) S_m(x, y) = 0$$ and k_m^2 are eigenvalues.

Benjamin & Ursell (1954)

Solution expansions:



$$\zeta(x,y,t) = \sum_{0}^{\infty} a_m(t)\ S_m(x,y)$$

Application of BCs, and linear independence of $S_m(x,y)$ require

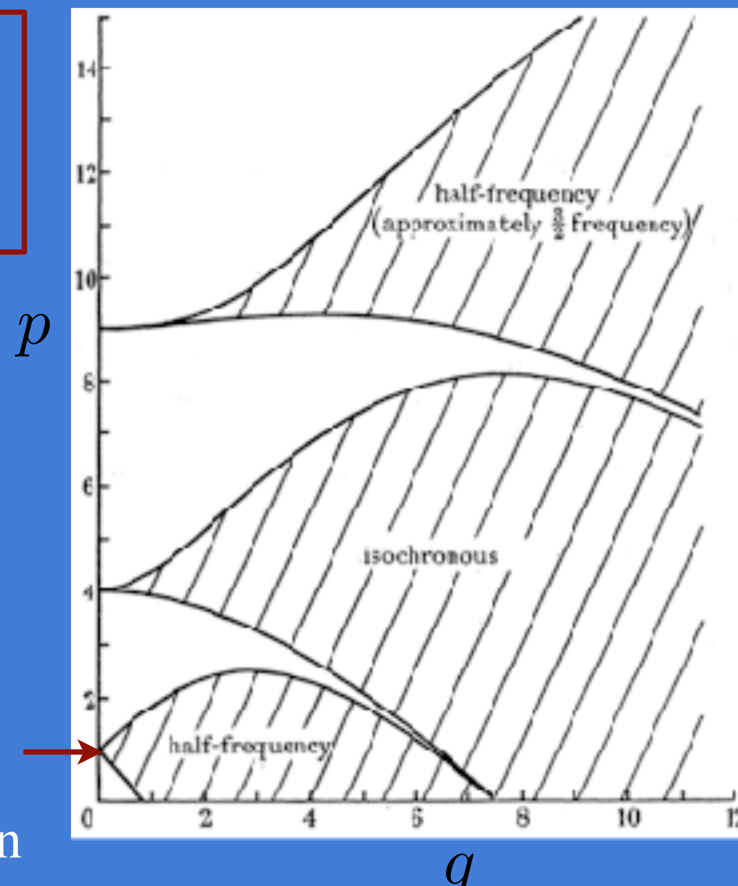
$$\frac{d^2 a_m}{dT^2} + [\ p_m + 2q_m \cos 2T\ ]\ a_m = 0$$

MATHIEU'S EQUATION

where $T = \omega t/2$

$$p_m = \frac{4k_m \tanh k_m h}{\omega^2}\left(g + \frac{k_m^2 \sigma}{\rho}\right)$$

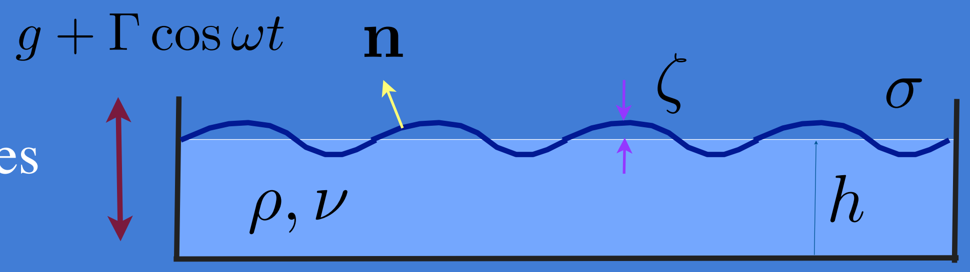
$$q_m = \frac{2k_m \Gamma \tanh k_m h}{\omega^2}$$



- an inviscid fluid is always unstable to vibration

The influence of viscosity

Kumar & Tuckerman (1994)



- stabilizes driven bath to Faraday waves

- prescribes critical acceleration required for instability

  e.g. deep water capillary waves, $\Gamma_c = 8 \left(\frac{\rho}{\sigma}\right)^{1/3} \nu \, \omega^{5/3}$ (Douady 1990)

  where $\omega = 2\omega_F =$ driving frequency

- wavelength of instability prescribed by forcing frequency

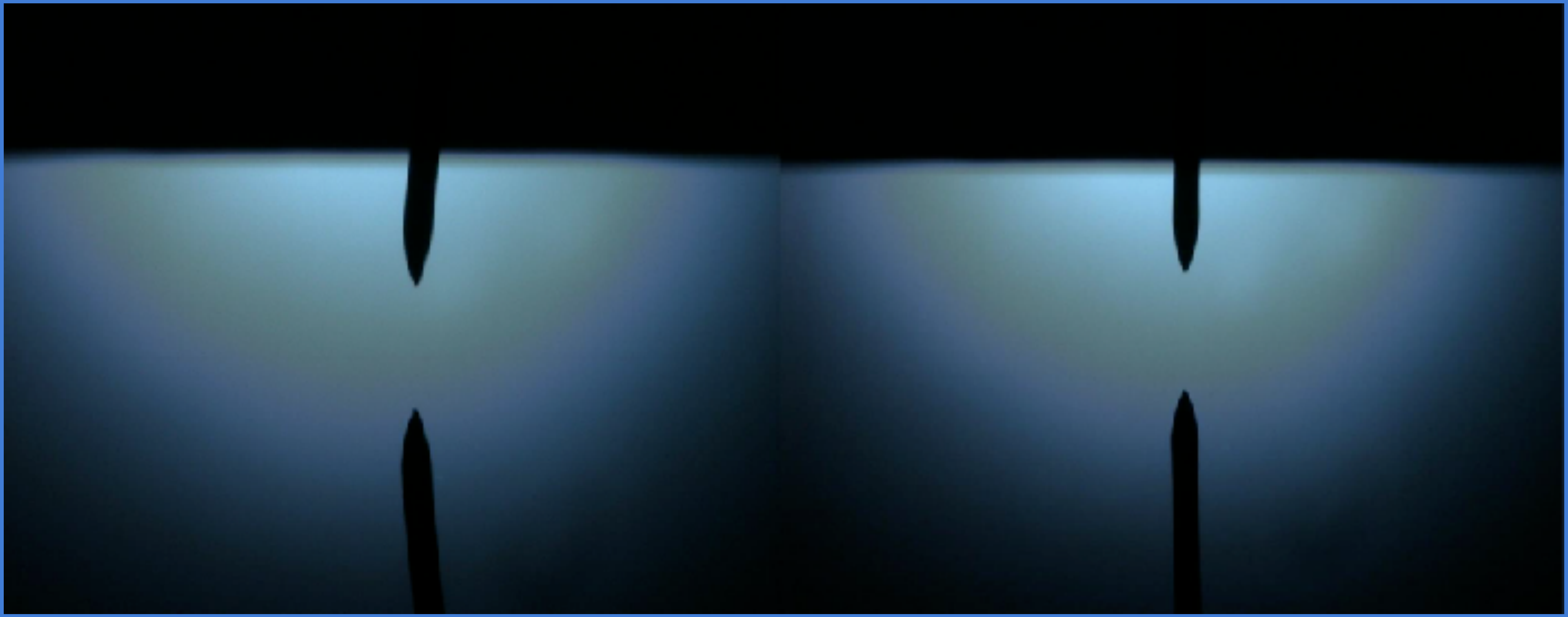
- if surface perturbed near onset, only the most unstable wavelength persists: other modes are damped by viscosity

- localized forcing near onset creates a monochromatic wave field

Disturbance of forced and unforced interfaces

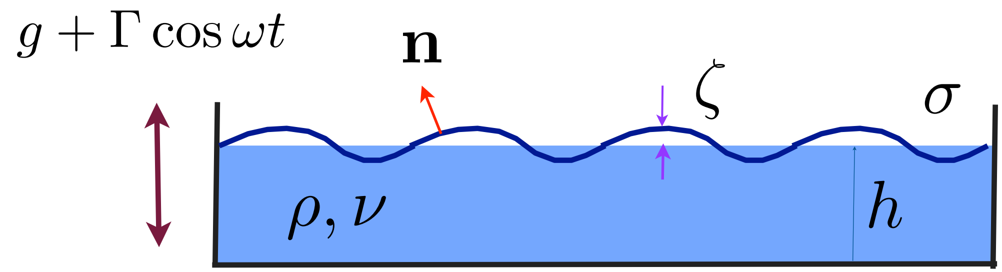
- withdraw millimetric needle from interface

No forcing | Faraday forcing



- waves quickly disperse
- field of Faraday waves persist

Faraday waves in PWH



- viscosity favors the subharmonic Faraday wave, since higher frequency waves suffer higher dissipation
- Faraday wavelength prescribed by standard dispersion relation with $\omega_F = \omega/2$

$$\omega_{\mathrm{F}}^2 = \left(\frac{\sigma}{\rho} k^3 + g\, k \right) \tanh kh$$

corresponds to wavelength accompanying walking droplets

E.g. for f = 80 Hz, λ_F = 4.75mm in deep water (h > 0.6 cm)

- above threshold, waves resisted by nonlinear effects, eventually break
- Faraday threshold, thus `memory', is depth dependent
- walkers bounce at $\omega_F = \omega/2$: bath as damped oscillator forced at resonance

Parametric Instability in Klein-Gordon Eqn?

$$\frac{1}{c^2}\Psi_{tt} - \nabla^2\Psi + \frac{m^2c^2}{\hbar^2}\Psi = 0$$

Seek modes:

$$\Psi(\mathbf{x},t) = e^{-i\mathbf{k}\cdot\mathbf{r}}\,\phi(t)$$

Force via mass oscillations:

$$mc^2 = m_0c^2\,(1+\epsilon\cos\omega t)$$

MATHIEU'S EQUATION

$$\phi_{TT} + [\,p + 2q\,\cos 2T\,]\,\phi = 0$$

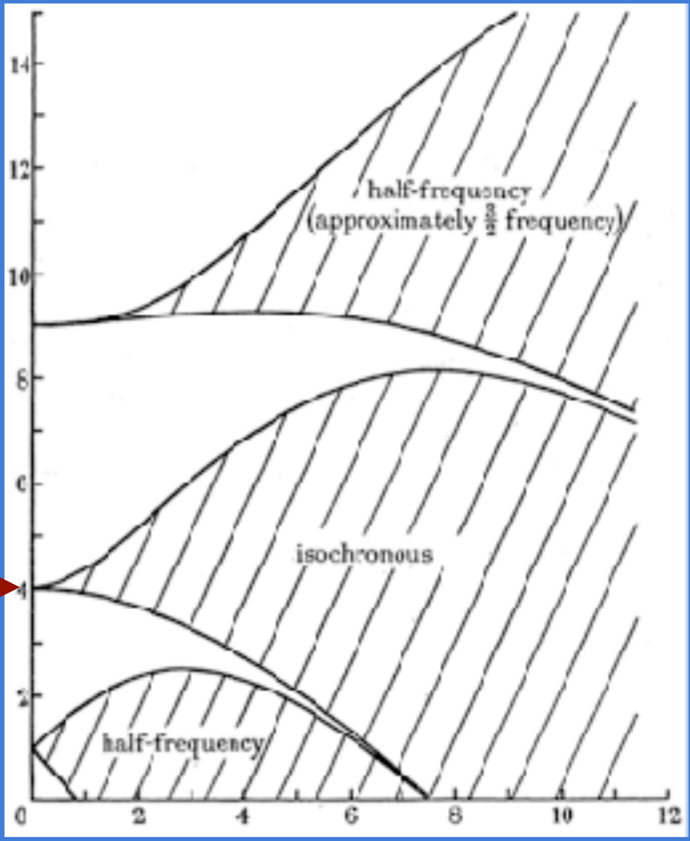
where $T = \omega t/2$, $\omega_c = \dfrac{m_o c^2}{\hbar}$

$$p = \frac{4}{\omega^2}(\omega_c^2 + c^2k^2)\,, \qquad q = 4\,\epsilon\,\frac{\omega_c^2}{\omega^2}$$

- any finite amplitude vibration ω will give rise to waves with a discrete set of frequencies:

$$\omega_q = \frac{n}{2}\,\omega \quad , \quad n = 1, 2, 3\ldots$$

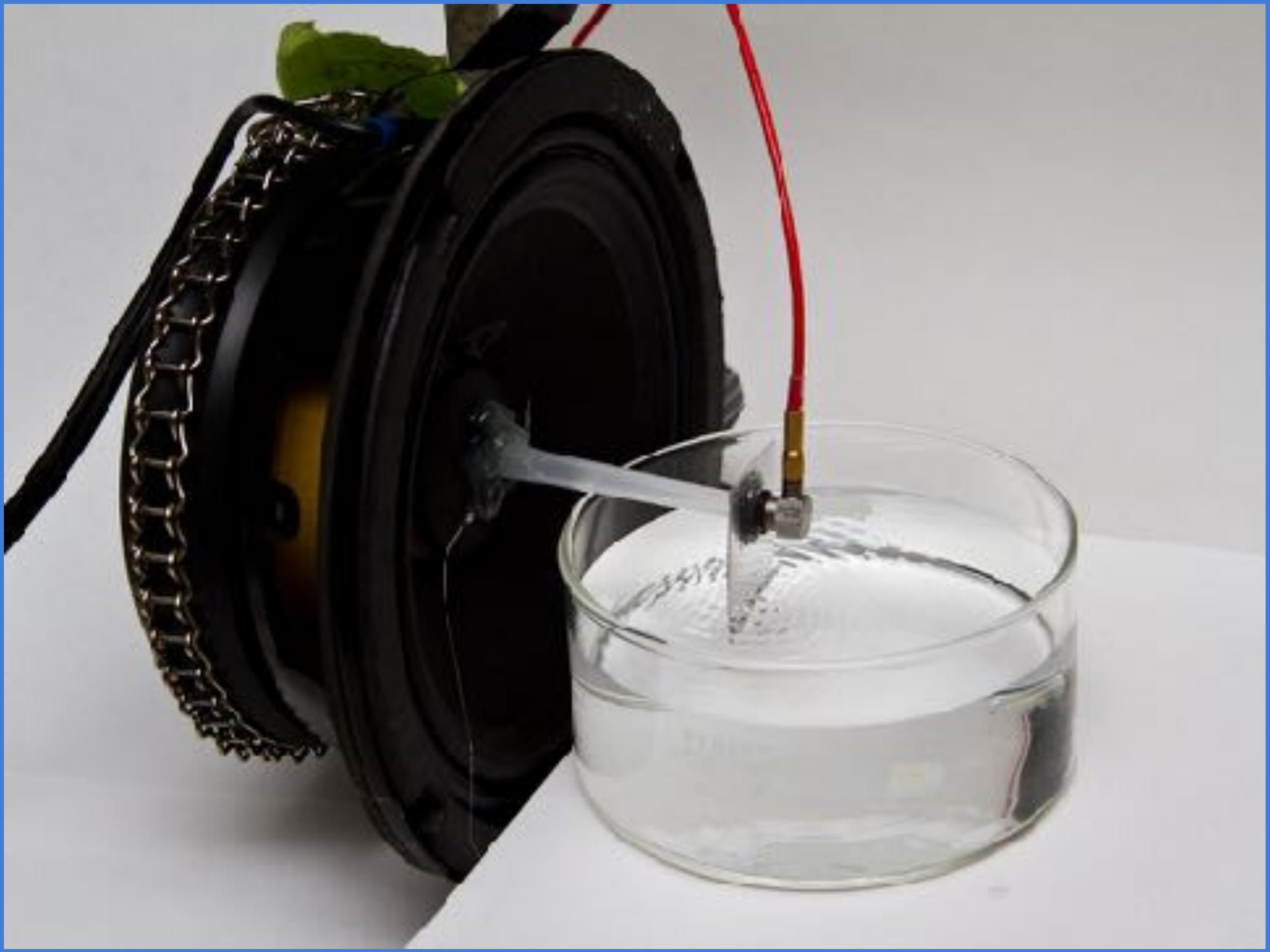
p



q

Faraday waves

- may also be generated by lateral boundary forcing



Faraday (1831)

Fig. 27.

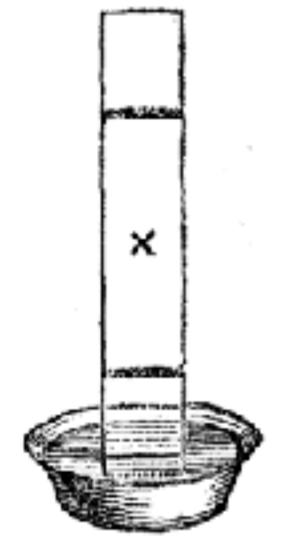
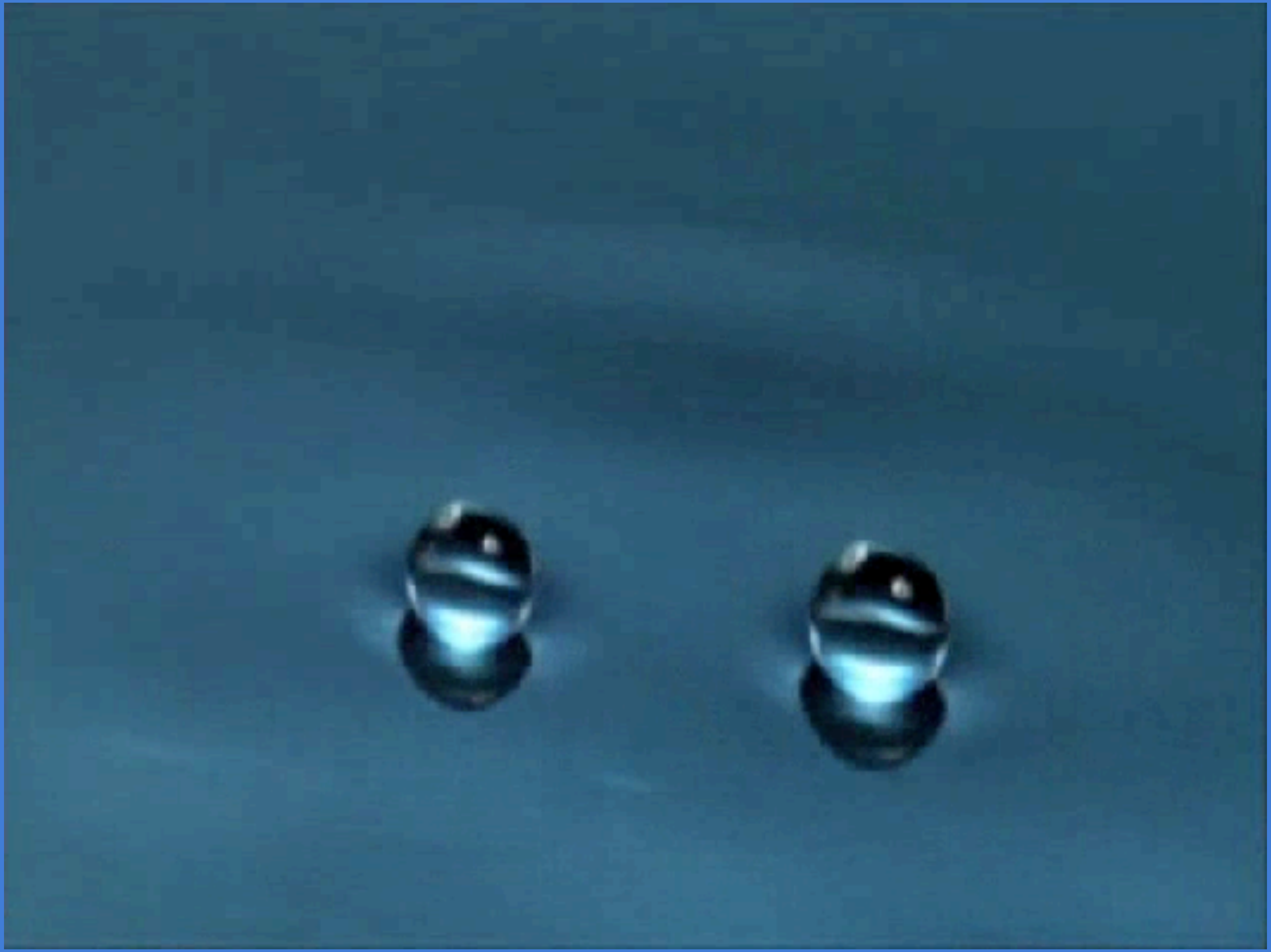


Fig. 28.

Non-coalescence on a vibrated fluid bath

30cS
Si oil

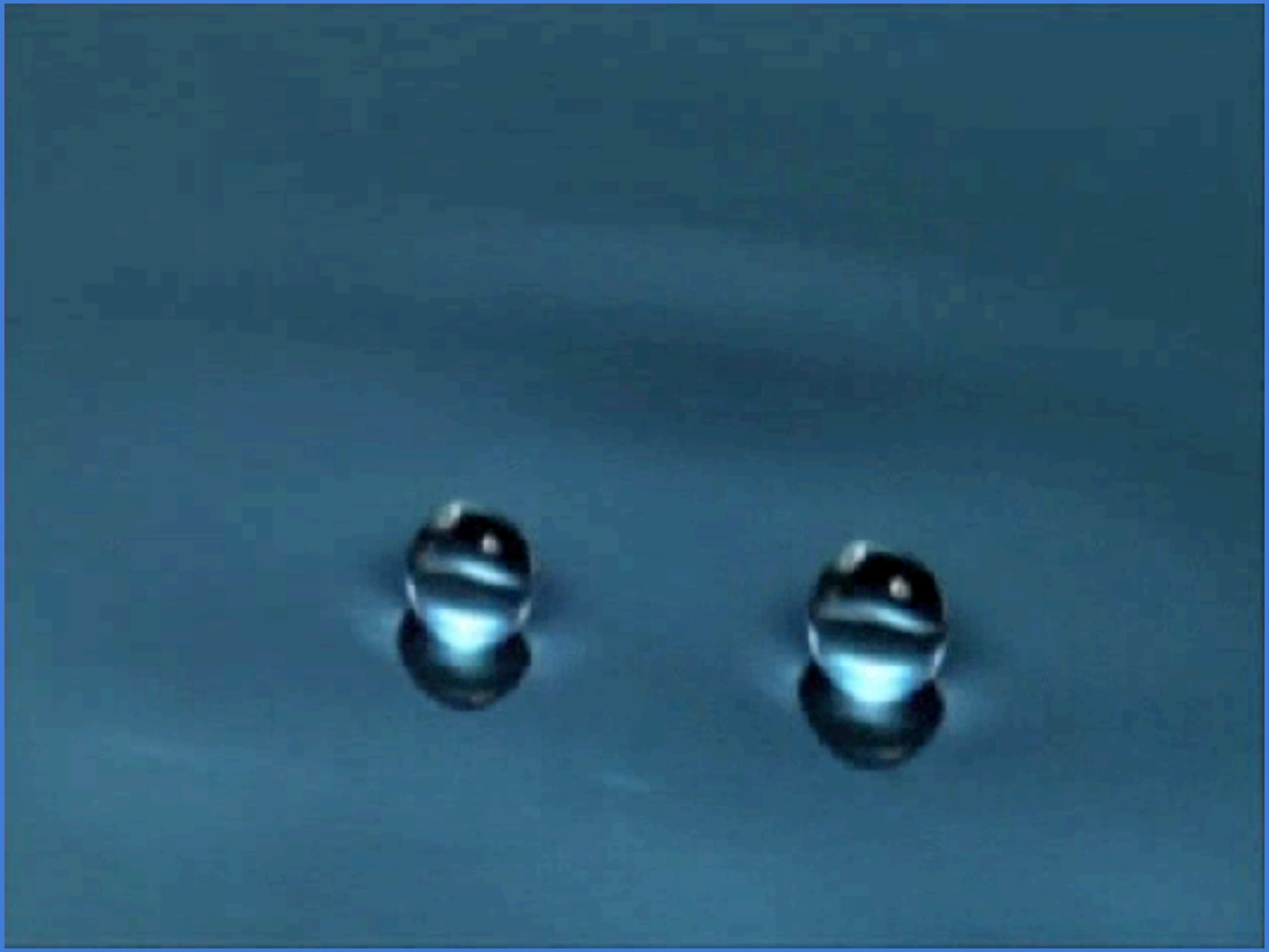


f ~ 30 Hz

Discovery Channel's `Time Warp'

Non-coalescence on a vibrated fluid bath

30cS
Si oil



f ~ 30 Hz

“Your experiments are proof that God exists.”

- Rosie Warburton

Sound Body Wholistic Health Center

"I have seen exactly what you describe - in my Tibetan singing bowls."
- Rosie Warburton

The Tibetan singing bowl

- produced by Himalayan fire cults as early as 500 BC
- composed of an 11 metal alloy, plus traces of meteorite
- used in shamanic rituals and religious ceremonies for:

  healing, exorcism, shamanic journeying, meditation, chakra adjustment, and...

... levitation.

The Tibetan Singing Bowl

"Here, amongst the waning, be, in this realm of decline, be a sounding glass, shattering itself in its sound. Be - and be aware the same, of the conditions of not being - the infinite reason of your deep-rooted vibration, that you perform it to the fullest, this one time.

- Rilke, Sonette to Orpheus

The history of this bowl

- 1600s: hand made by Himalayan fire cults for shamanic rituals
- 1950: taken to Tibet by those fleeing Chinese invasion
- 1980: imported to Texas by a collector

- 2000: purchased by Rosie
- 2010: sent to MIT

1950

Singing bowls

Tibetan

Chinese

Singing bowls

Tibetan

Chinese

French

Singing bowls

Tibetan

Chinese

French

American

Vibrational modes

Rossing (1990)

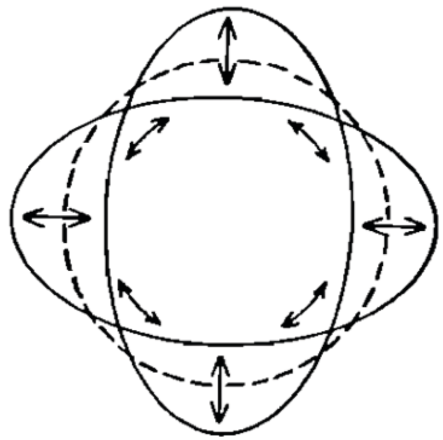
Deformation modes

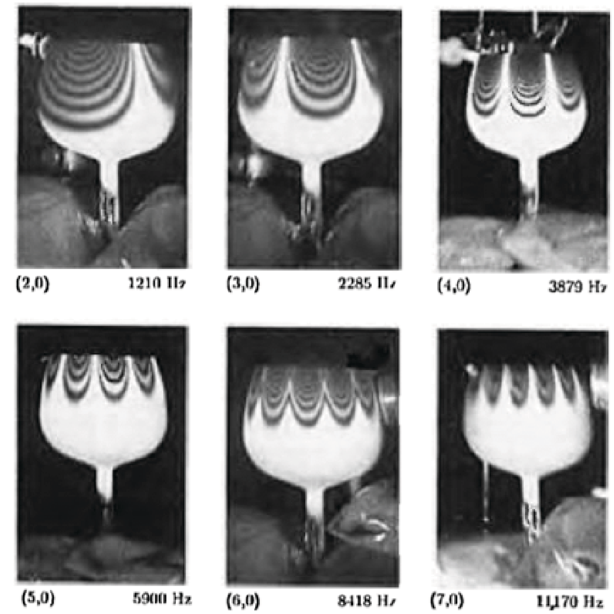
(n, m)

nodal meridions

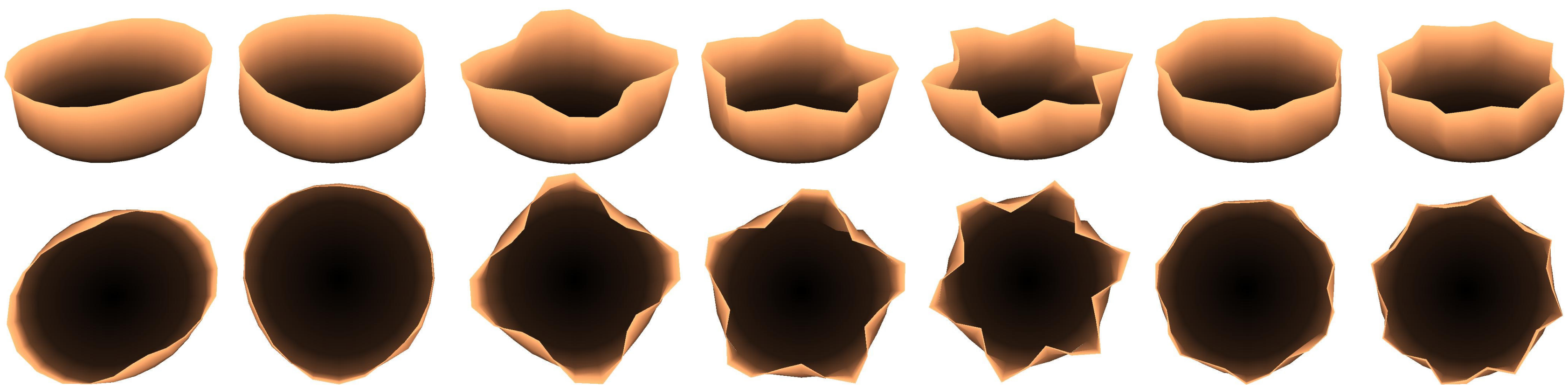
nodal parallels

(2,0)





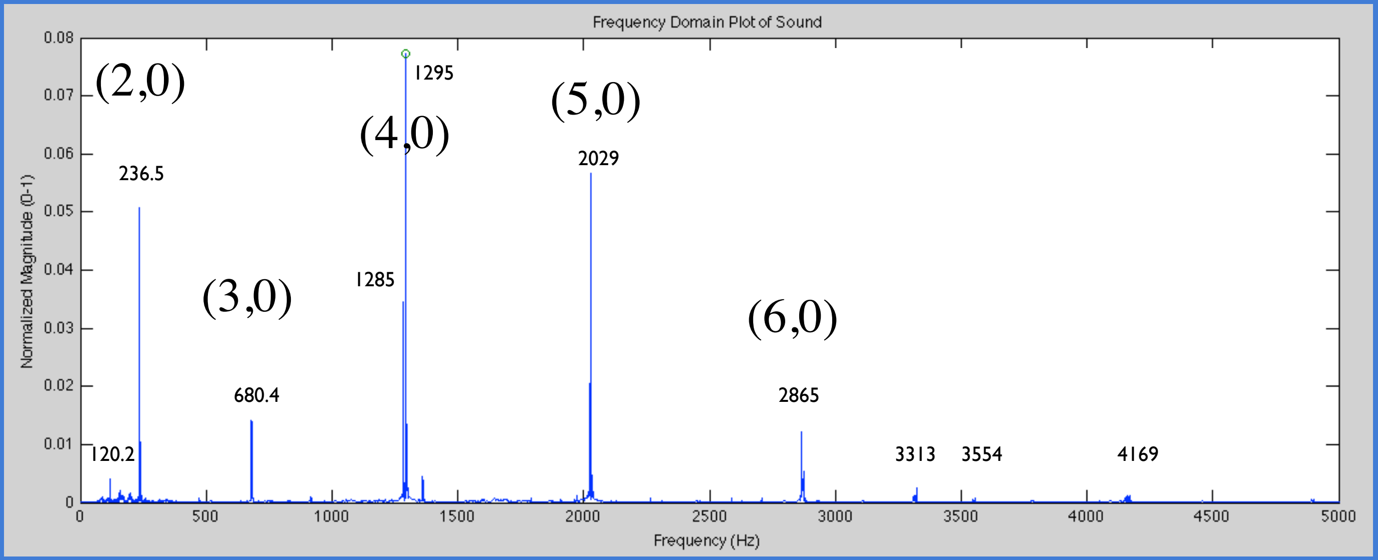
(2,0) (3,0) (4,0) (5,0) (6,0) (7,0) (8,0)



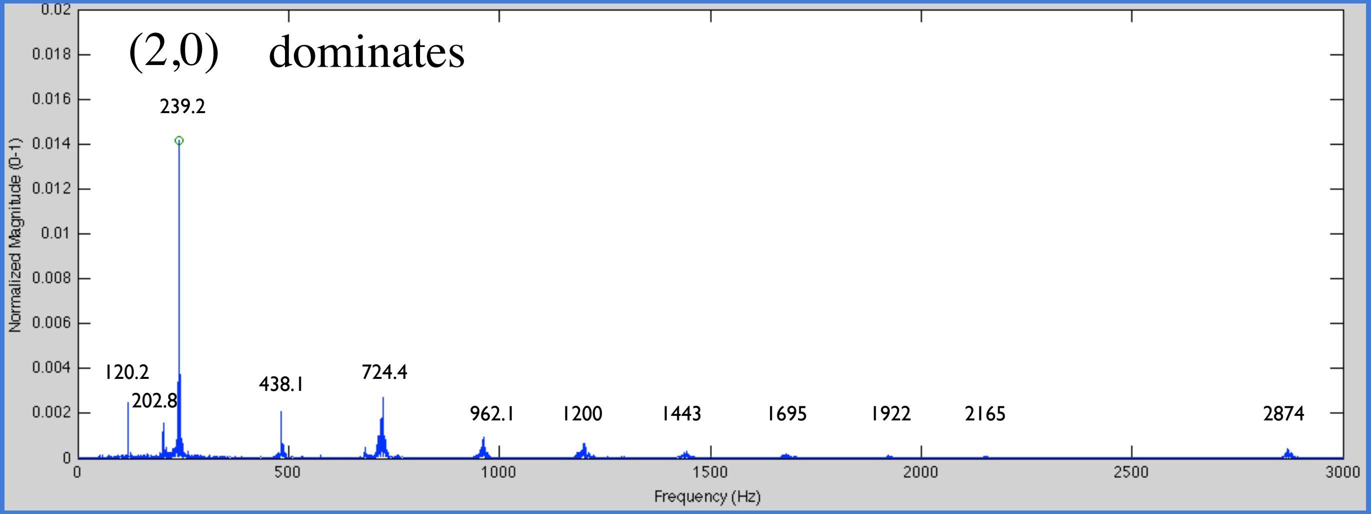
Ignacio et al. (2006)

Acoustics of the singing bowl

Striking



Rubbing



Acoustics of a struck bowl

Bending wave speed

$$V_b = \left(\frac{\pi V_L f e}{\sqrt{3}}\right)^{1/2} \quad \text{where} \quad V_L = \sqrt{E/\rho_s}$$

Bending wave period

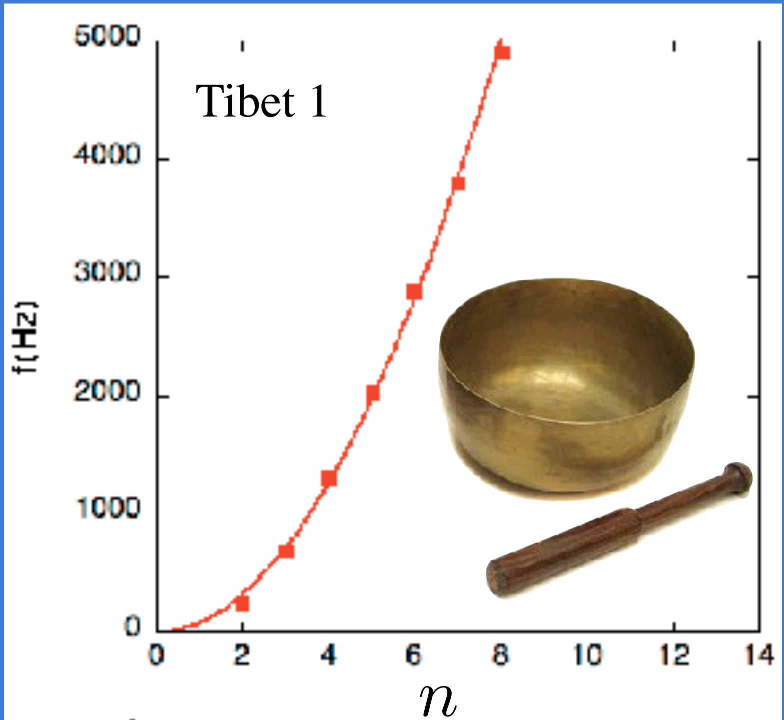
$$T = \frac{1}{f} = \frac{2\pi r}{V_b} \longrightarrow \boxed{f \propto \frac{1}{r^2}}$$

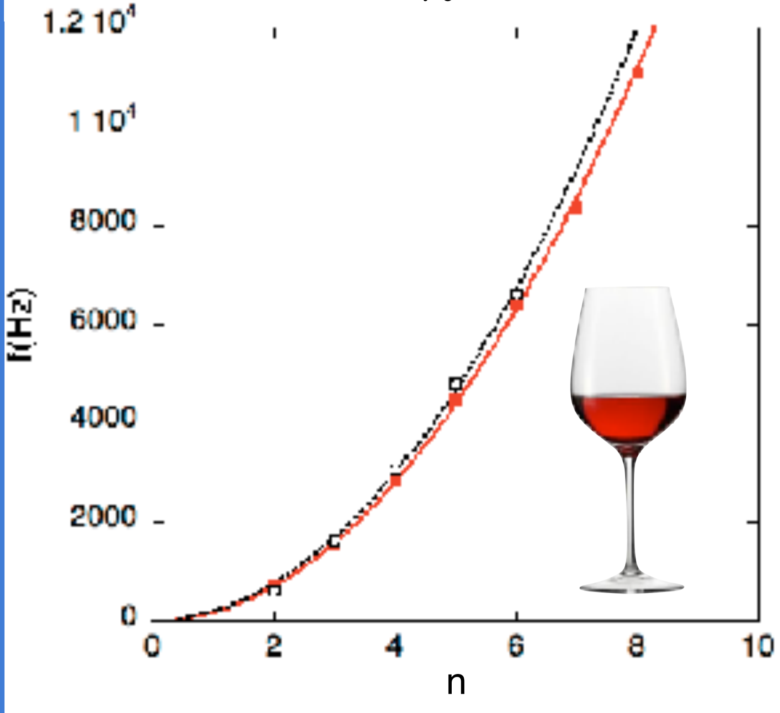
Modal dependence

$$\lambda = \frac{V_b}{f} \propto \frac{1}{\sqrt{f}}$$

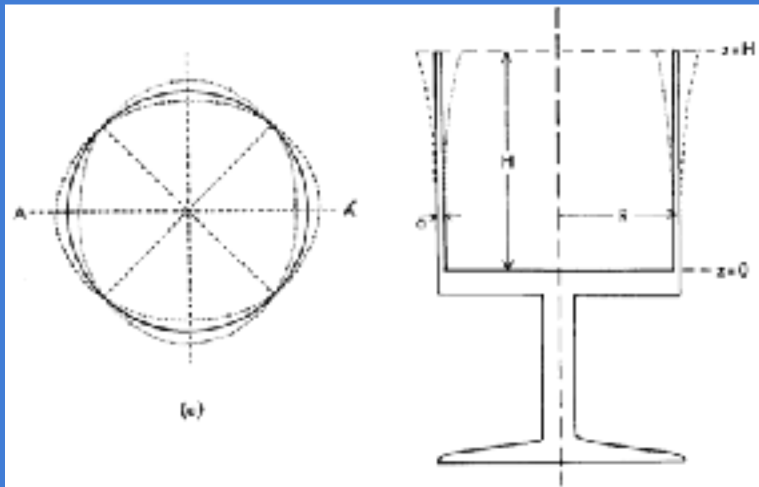
$$2\pi r = n\lambda_{(n,0)}$$

$$\longrightarrow \boxed{f_{(n,0)} \propto n^2}$$





The acoustics of an empty wine glass (A.P. French, 1982)



Deformation mode (2,0)

Displacement: $\Delta(t) = \Delta_0 \cos \omega t$

System energy:

$$E = A\left(\frac{d\Delta}{dt}\right)^2 + B\Delta^2$$

KINETIC POTENTIAL

Frequency: $\omega^2 = B/A$

Vibrational frequency

$$\omega_0 = \frac{1}{2\pi}\left(\frac{3Y}{5\rho_s}\right)\frac{a}{R^2}\left[1+\frac{4}{3}\left(\frac{R}{H}\right)^4\right]^{1/2}$$

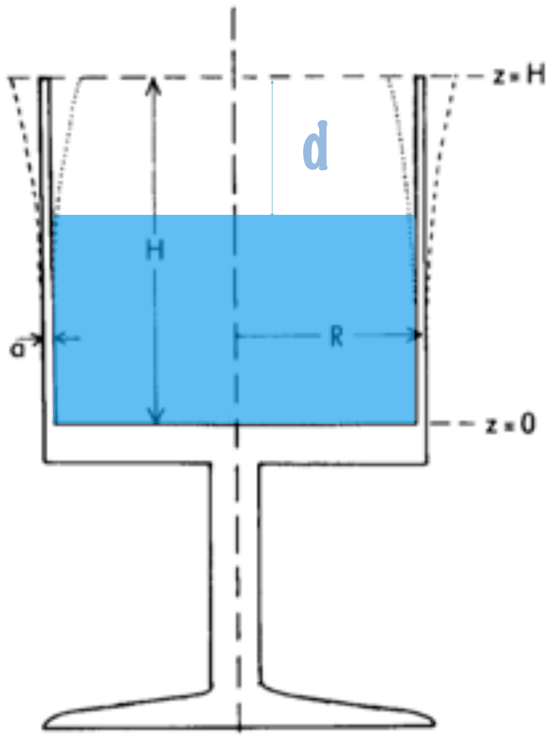
French (1982), Apfel (1985)

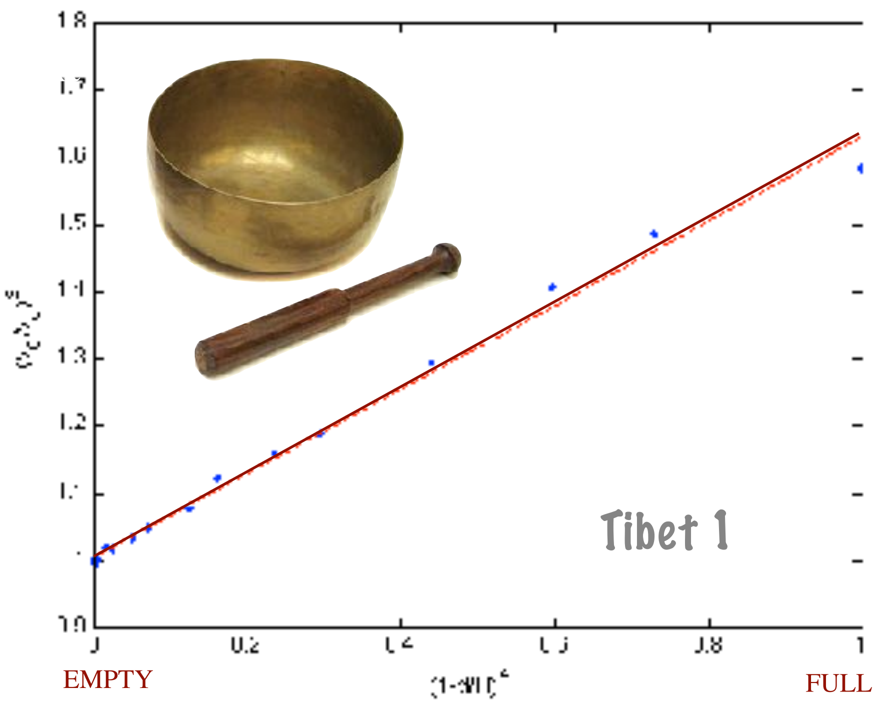
The glass half full

- consider the additional kinetic energy of the fluid

$$\left(\frac{\omega_0}{\omega}\right)^2 \sim 1 + \frac{\alpha}{5}\frac{\rho_\ell R}{\rho_s a}\left(1 - \frac{d}{H}\right)^4$$

- frequency decreases with increasing depth





The glass harp

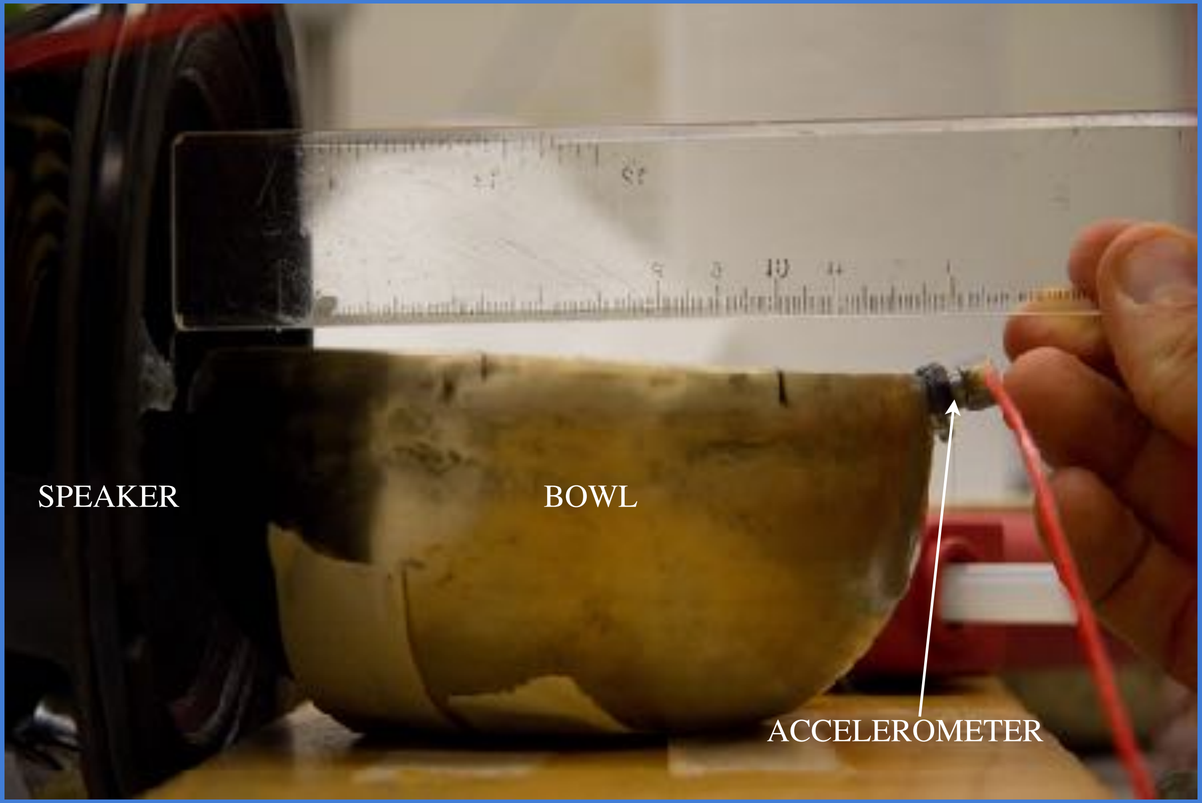
Bach's Toccata and Fugue in D minor
Robert Tiso

Experiments: the hydrodynamics of the Tibetan singing bowl

- measure natural frequencies of bowl following strike
- force with a loud speaker at these natural frequencies
- measure frequency f and amplitude Δ of wall motion via accelerometer, strain gauge
- observe progression of flows as forcing acceleration Γ is increased

Dimensionless acceleration:

$$\Gamma = \frac{4\pi^2 f^2 \Delta}{g}$$



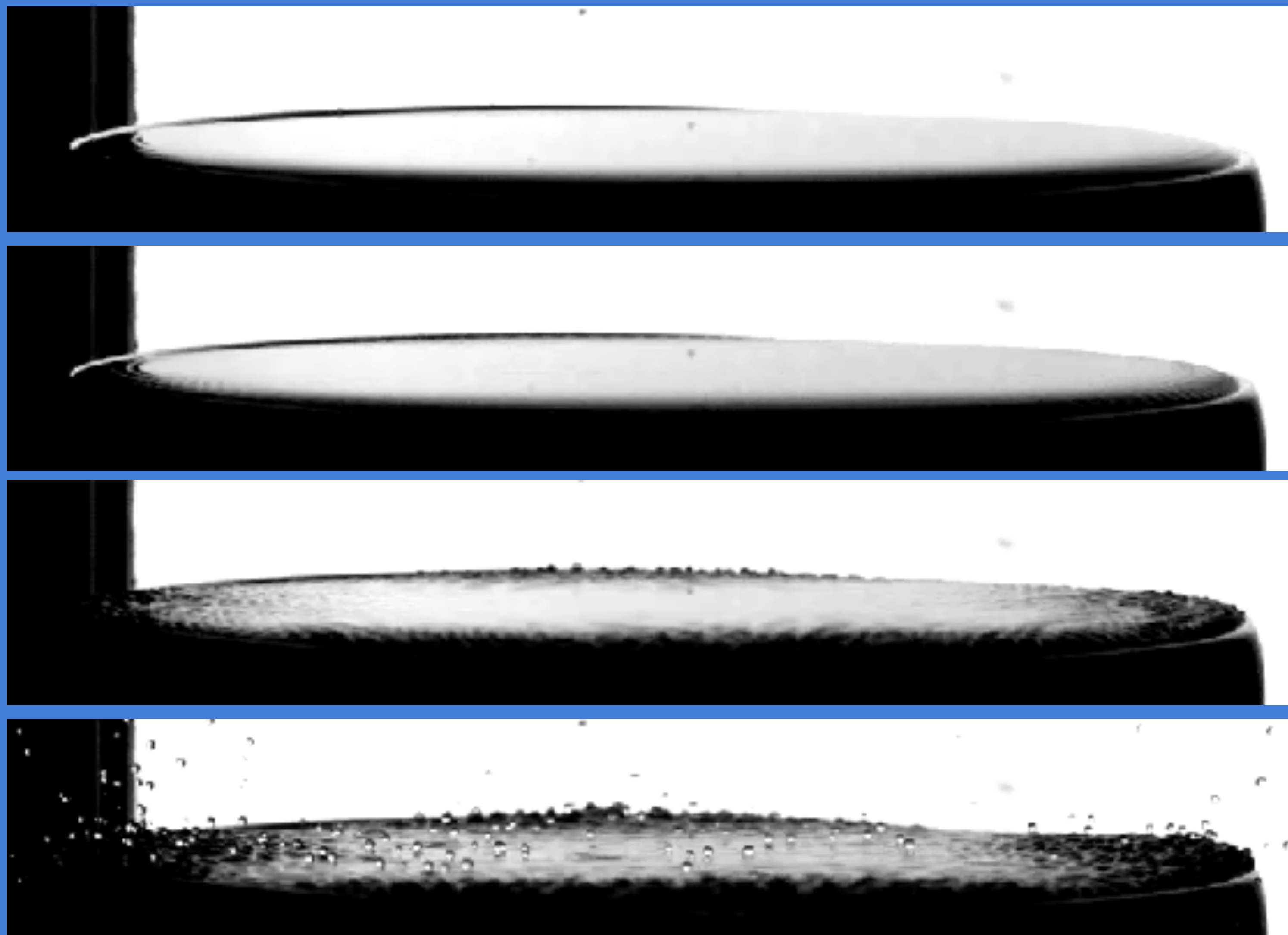
$$\Gamma = \frac{4\pi^2 f^2 \Delta}{g}$$

Increasing vibration amplitude

Tibet 3

$f = 279\,\mathrm{Hz}$

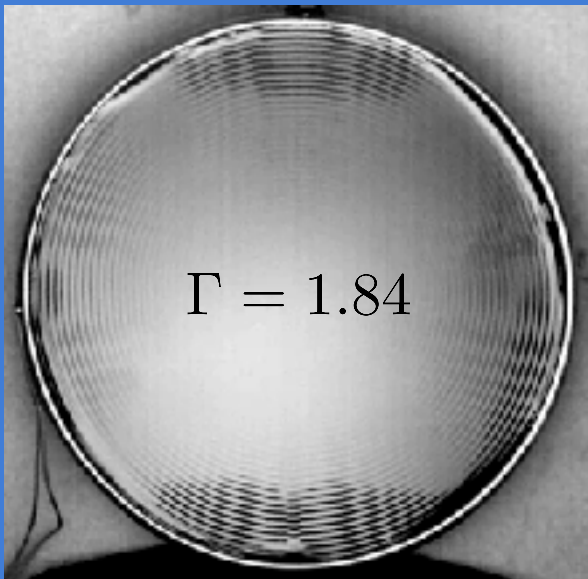
$\Gamma \uparrow$

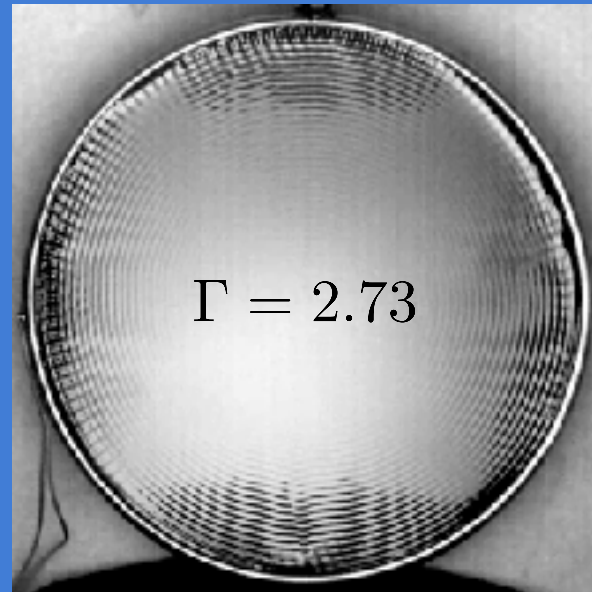


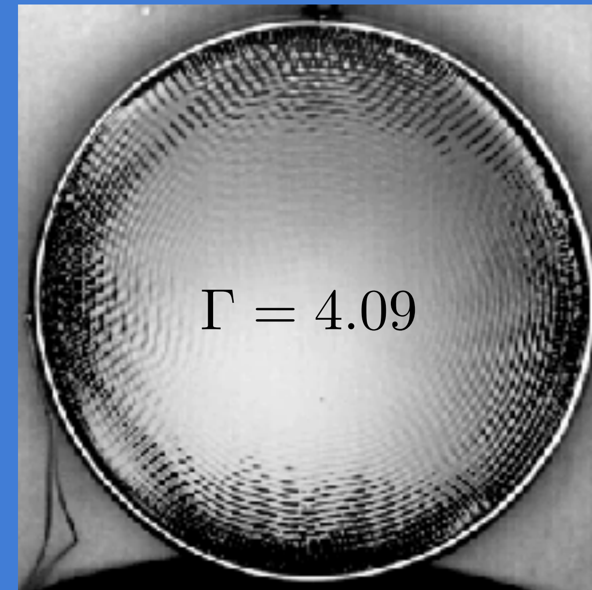
Increasing vibration amplitude

Tibet 1, water

$f = 187.5 \text{ Hz}$

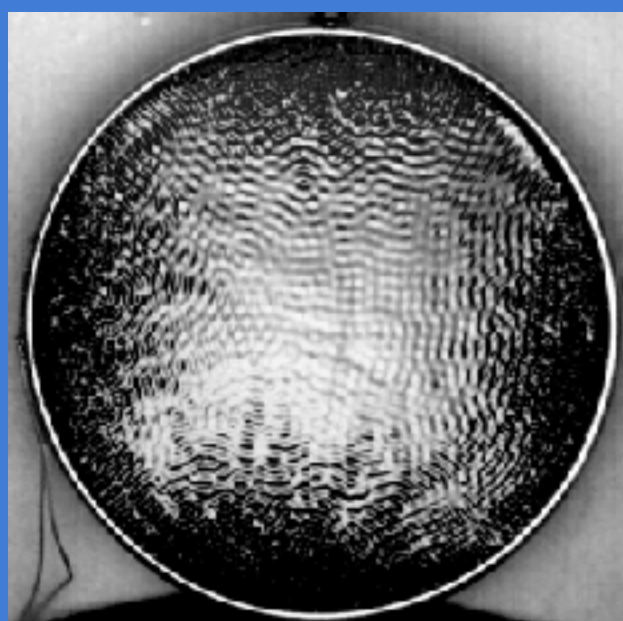


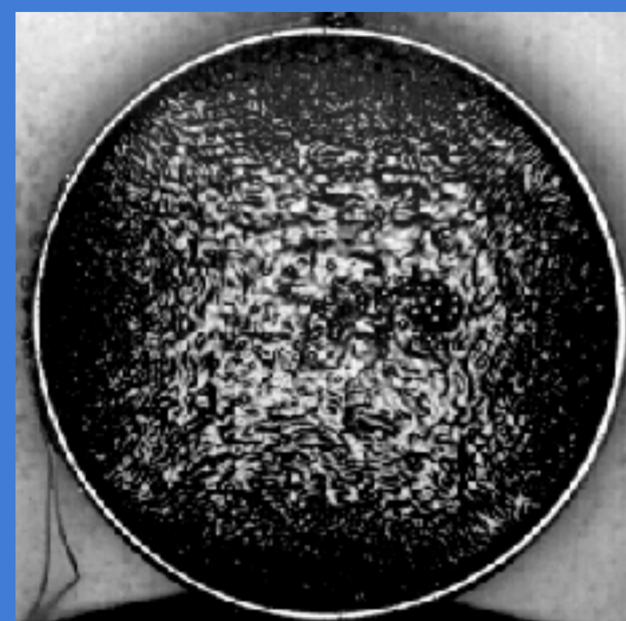




$$\Gamma = \frac{4\pi^2 f^2 \Delta}{g}$$

$\Gamma = 10.15$



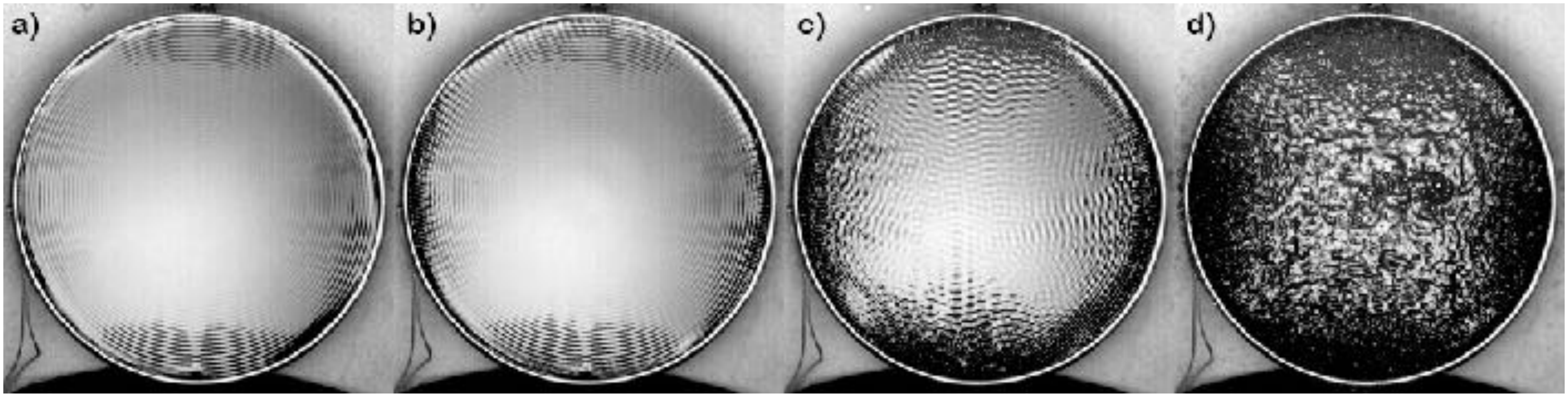


$\Gamma = 16.25$

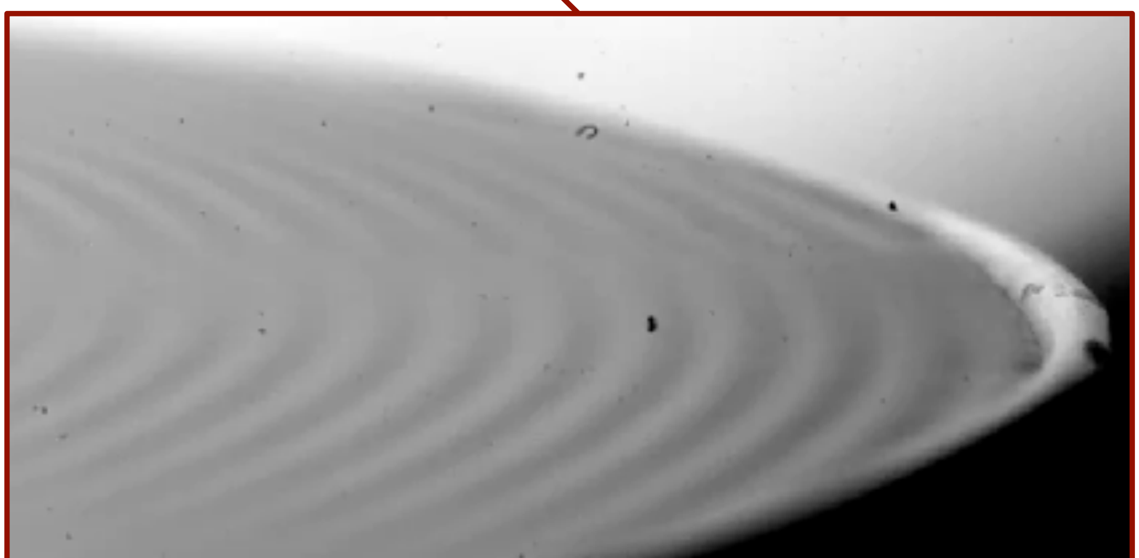
The influence of increasing amplitude

Tibet 1, water, $f = 187.5$ Hz

Faraday waves | Faraday waves | Faraday waves | Drop ejection



$\Gamma = 1.84$ | $\Gamma = 2.73$ | $\Gamma = 10.15$ | $\Gamma = 16.25$



$$\omega^2 = [gk + \frac{\sigma}{\rho}k^3] \tanh hk$$

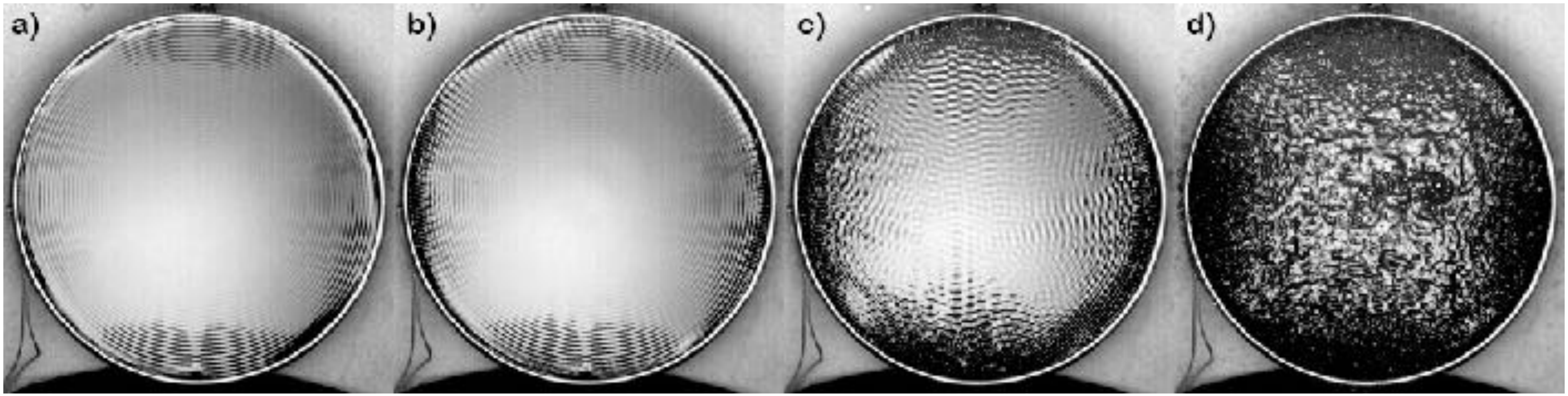
$\lambda \sim 0.28$ cm

$v \sim 39.2$ cm/s

The influence of increasing amplitude

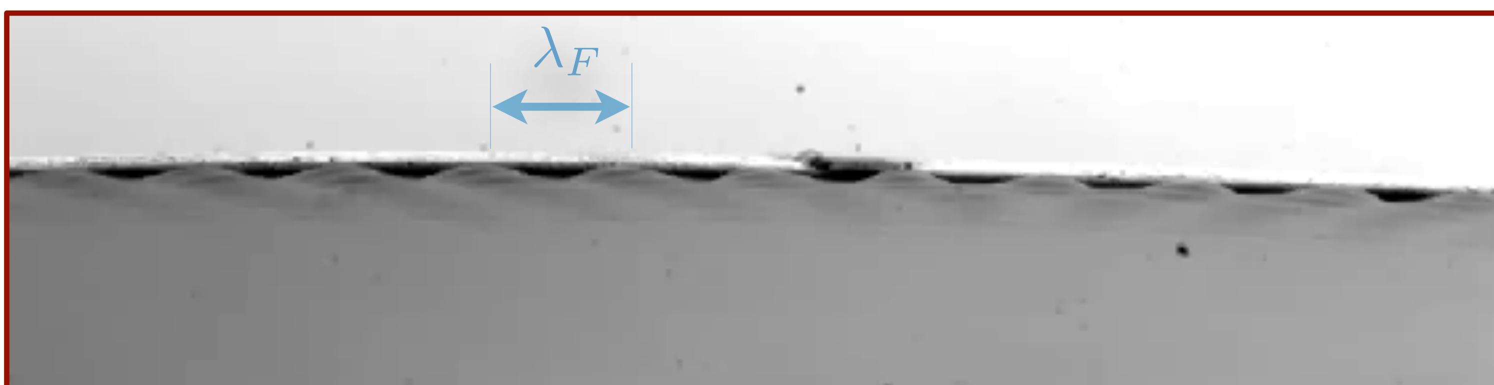
Tibet 1, water, $f = 187.5$ Hz

Faraday waves | Faraday waves | Faraday waves | Drop ejection



$\Gamma = 1.84$ | $\Gamma = 2.73$ | $\Gamma = 10.15$ | $\Gamma = 16.25$

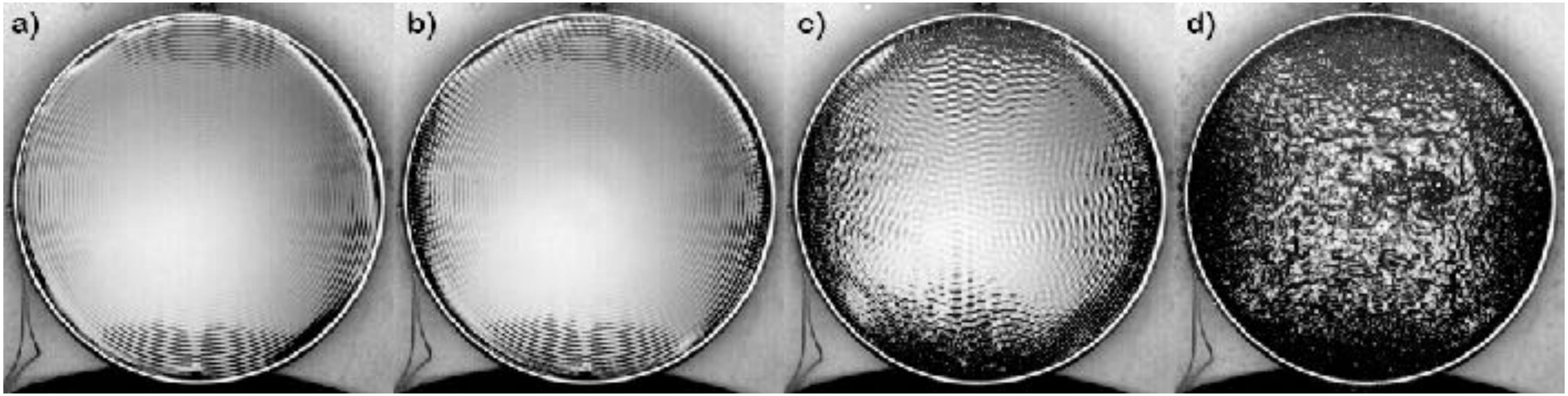
$\lambda_F \sim 0.36$ cm



The influence of increasing amplitude

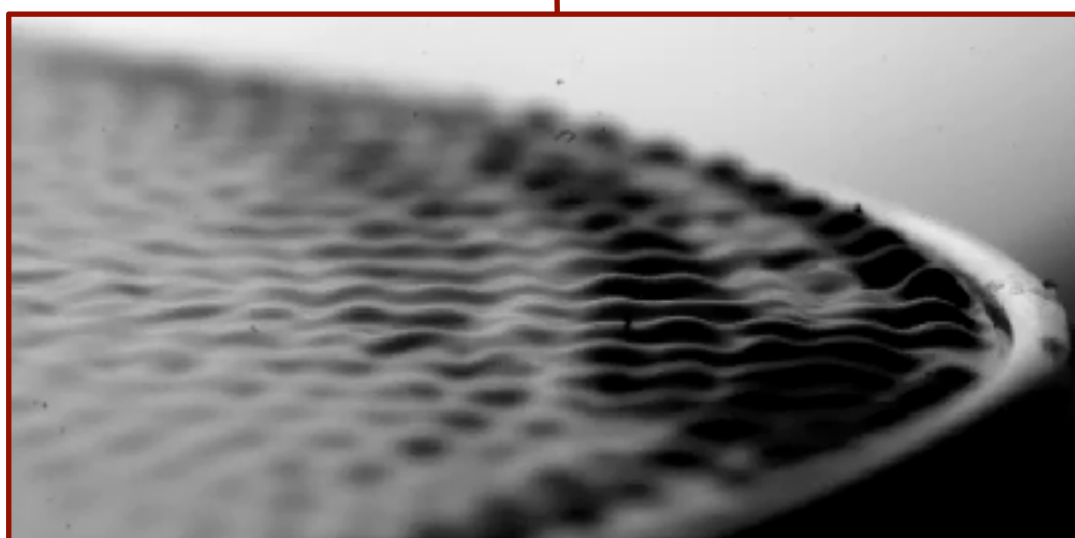
Tibet 1, water, $f = 187.5$ Hz

Faraday waves | Faraday waves | Faraday waves | Drop ejection



$\Gamma = 1.84$ | $\Gamma = 2.73$ | $\Gamma = 10.15$ | $\Gamma = 16.25$

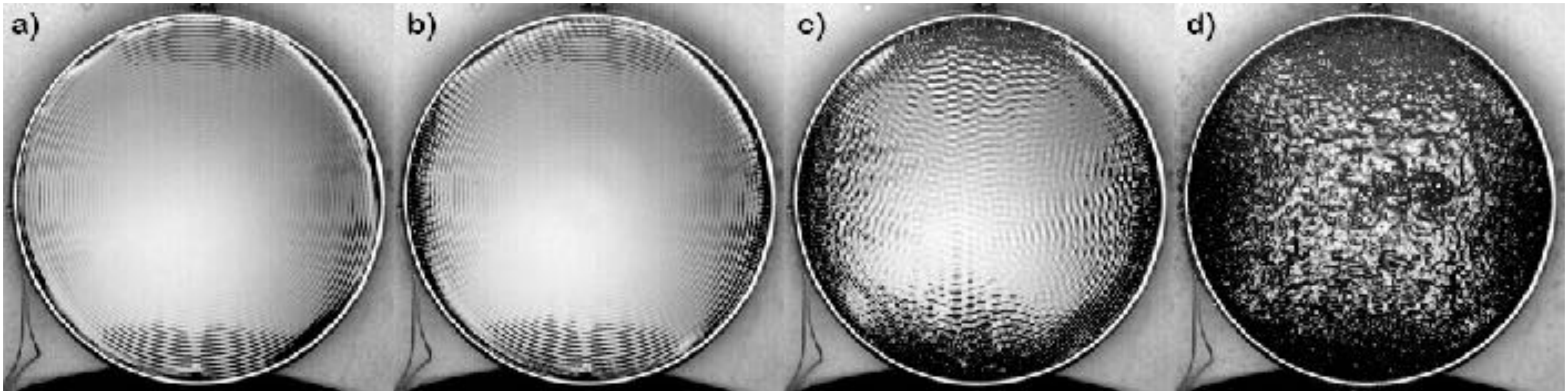
$\lambda_F \sim 0.36$ cm



The influence of increasing amplitude

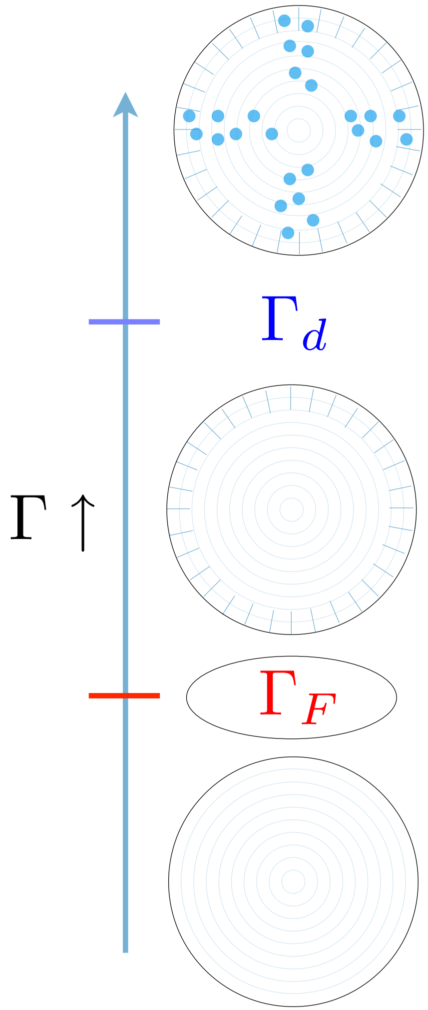
Tibet 1, water, $f = 187.5$ Hz

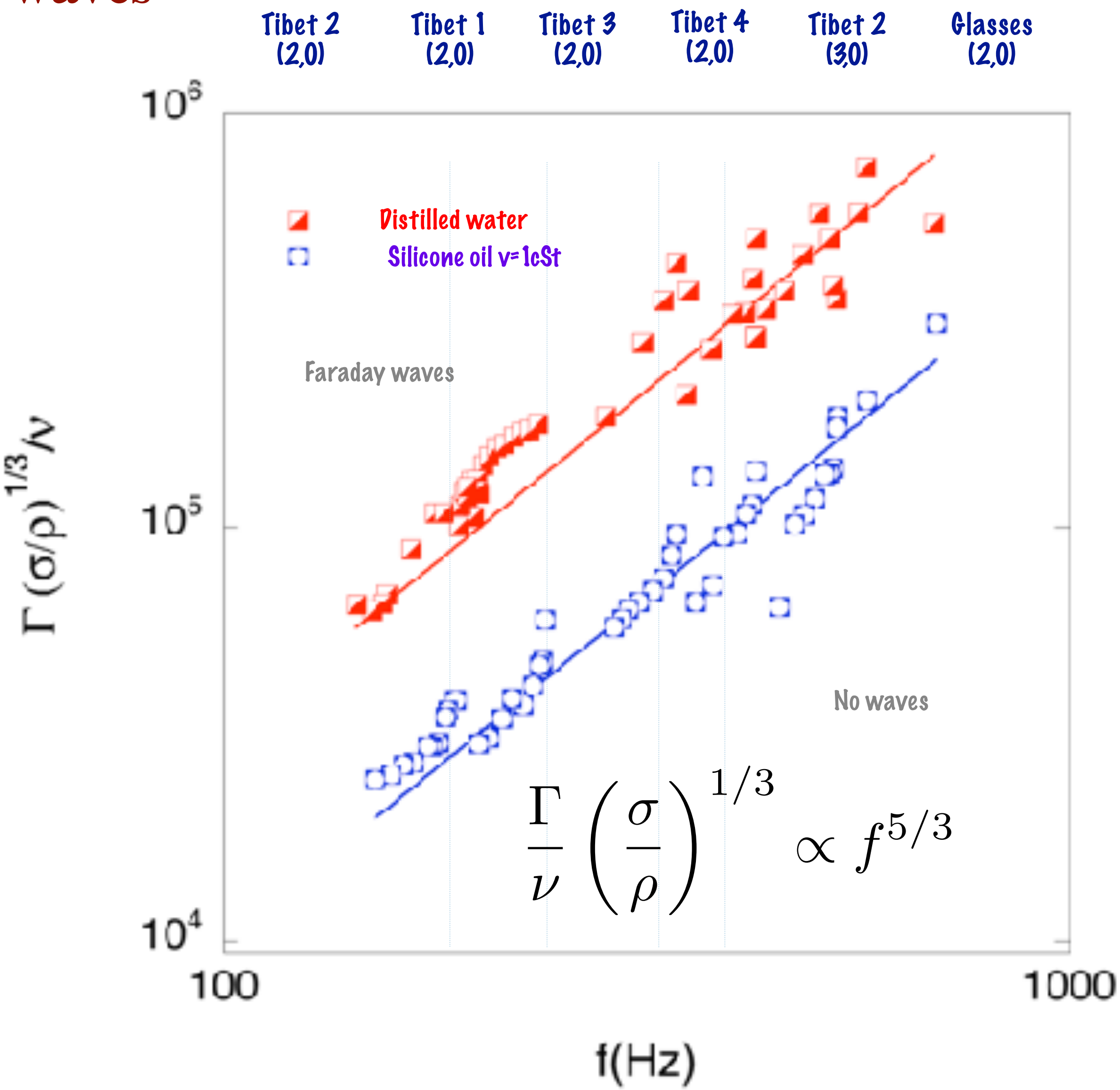
Faraday waves	Faraday waves	Faraday waves	Drop ejection



$\Gamma = 1.84$	$\Gamma = 2.73$	$\Gamma = 10.15$	$\Gamma = 16.25$

Thresholds for Faraday waves





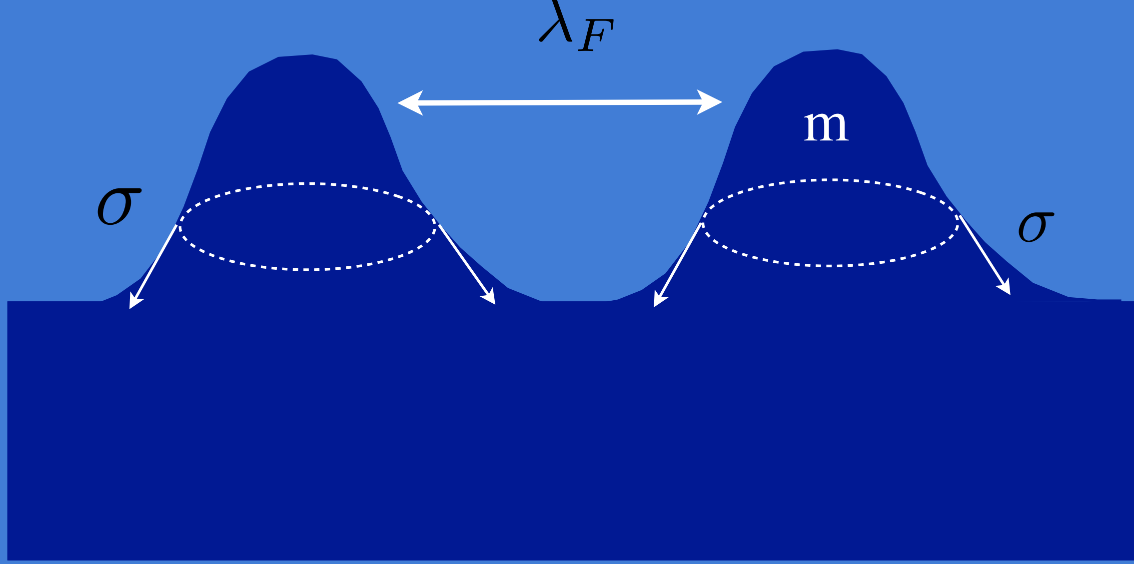
- consistent with $\Gamma_F = 2^{4/3}(\rho/\sigma)^{1/3}\nu\ \omega_0^{5/3}$ Kumar & Tuckerman (1994)

Droplet generation via vibration

- at sufficiently high forcing accelerations, Faraday waves break

Interfacial fracture

$$g + \Gamma \cos \omega t$$



Fracture criterion

$$m\,\Gamma_c \;\geq\; \pi\;\sigma\;\lambda_F \;\longrightarrow\; \Gamma_c \;\geq\; \frac{\sigma}{\rho}\,\frac{1}{\lambda_F^2}$$

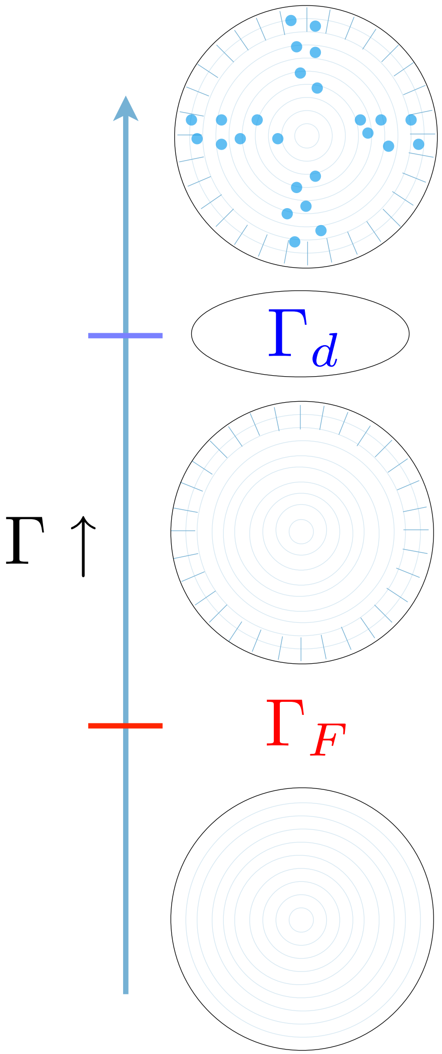
Use dispersion relation for deep water capillary waves:

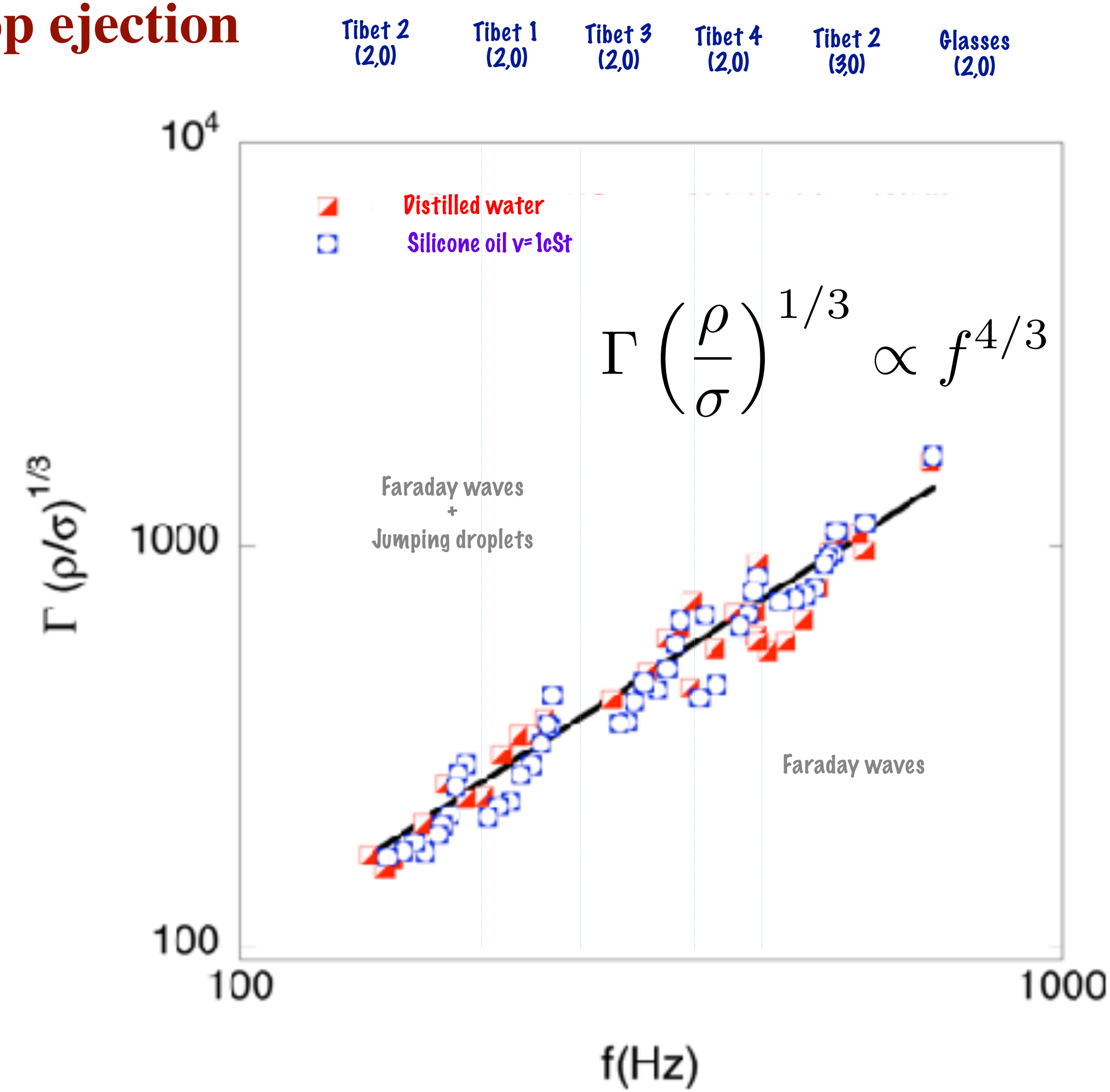
$$\omega = \left(\frac{\sigma}{\rho}\right)^{1/2} k^{3/2} \qquad \text{i.e.} \qquad \lambda_F \sim \frac{\sigma}{\rho}\,\omega^{4/3}$$

to predict

$$\Gamma_c \;\geq\; \left(\frac{\sigma}{\rho}\right)^{1/3} \omega^{4/3}$$

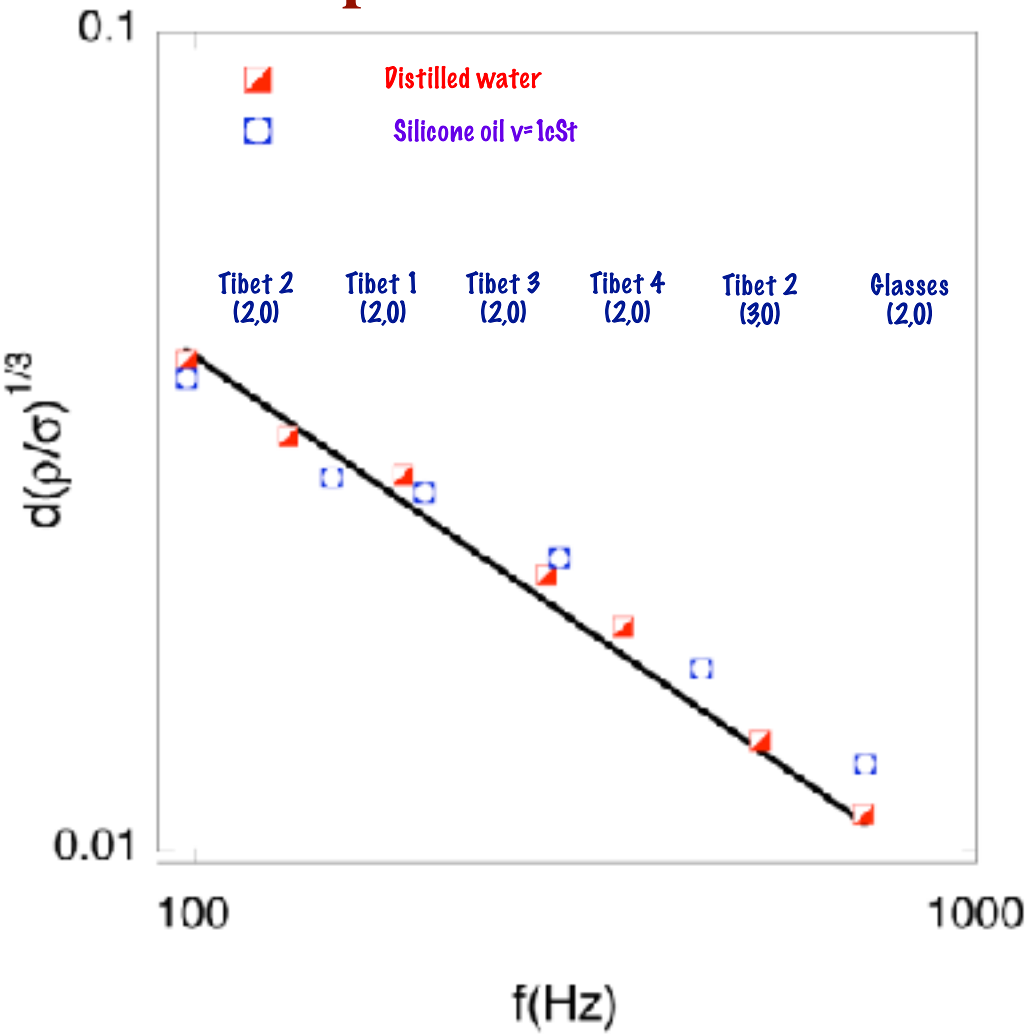
Thresholds for drop ejection





- consistent with $\Gamma_d \sim 0.26(\sigma/\rho)^{1/3}\omega^{4/3}$ Goodrich et al. (1996, 97, 99)

Droplet size distribution



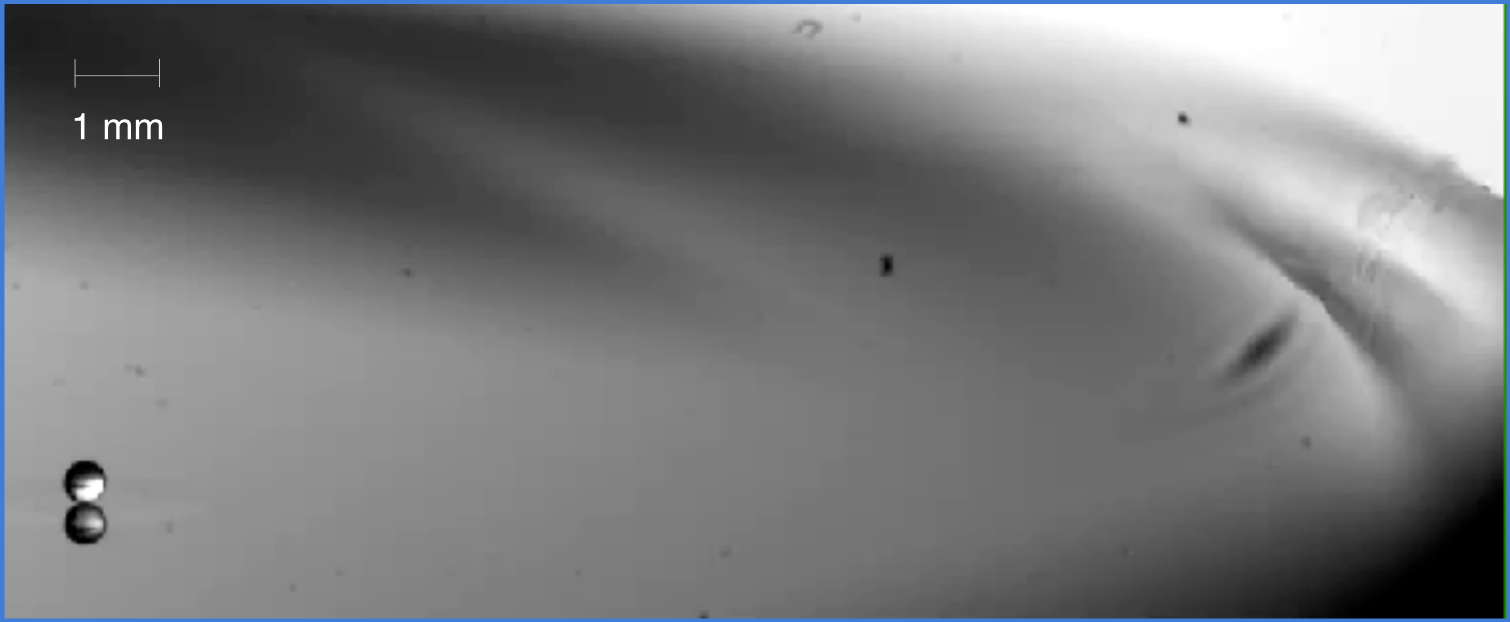
- droplet size consistent with Faraday wavelengths:

$$d_m = B(\sigma/\rho)^{1/3}\omega^{-2/3}$$

Puthenveethil et al. (2009)

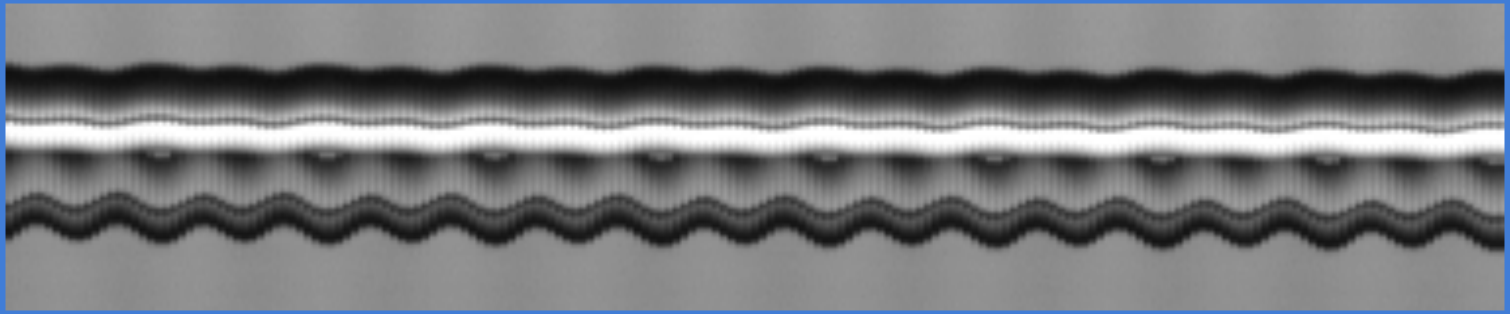
What about levitation?

Levitating drops in the Tibetan singing bowl



$f = 188$ Hz

$\nu = 10$ cSt

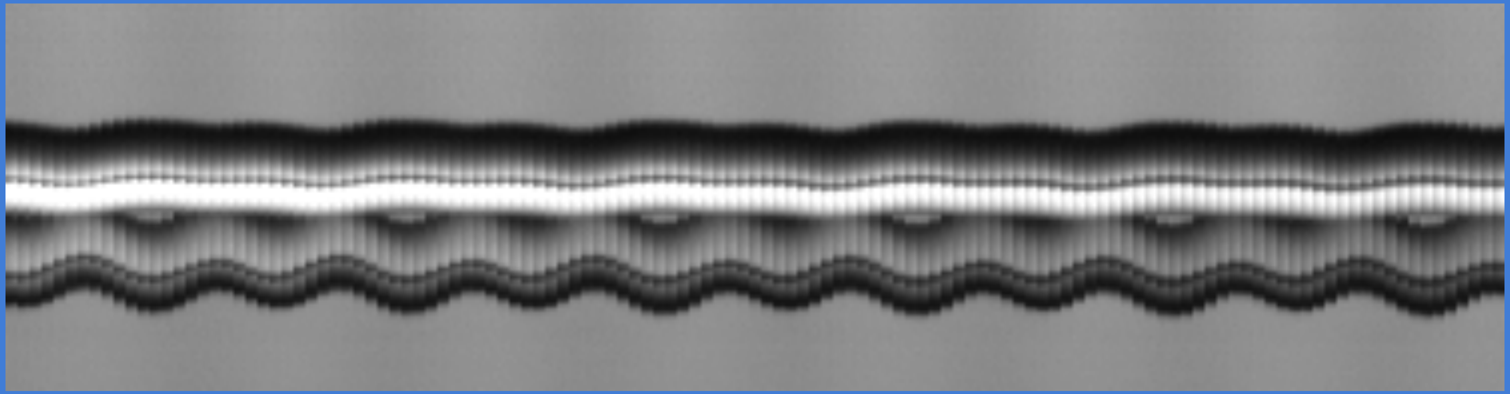


time

Levitating drops in the Tibetan singing bowl

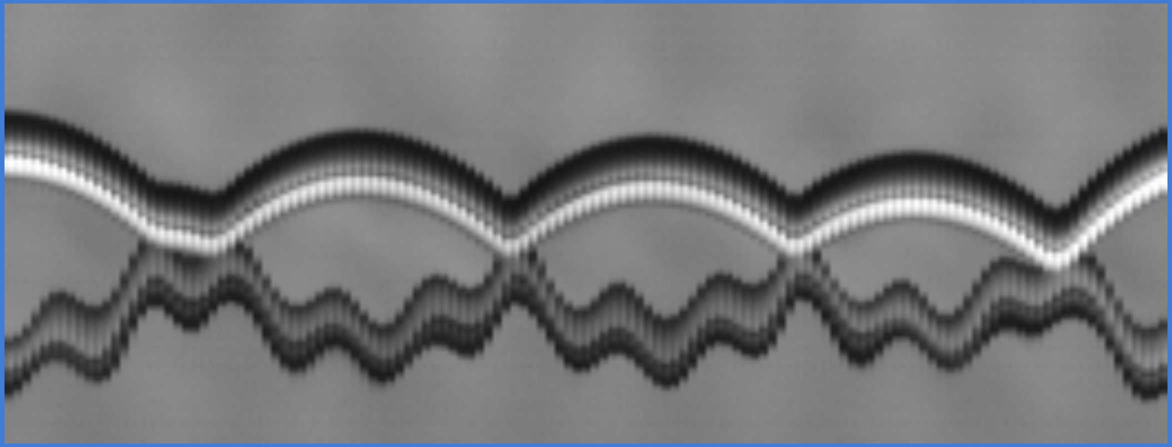
$f = 188$ Hz

$\nu = 10$ cSt



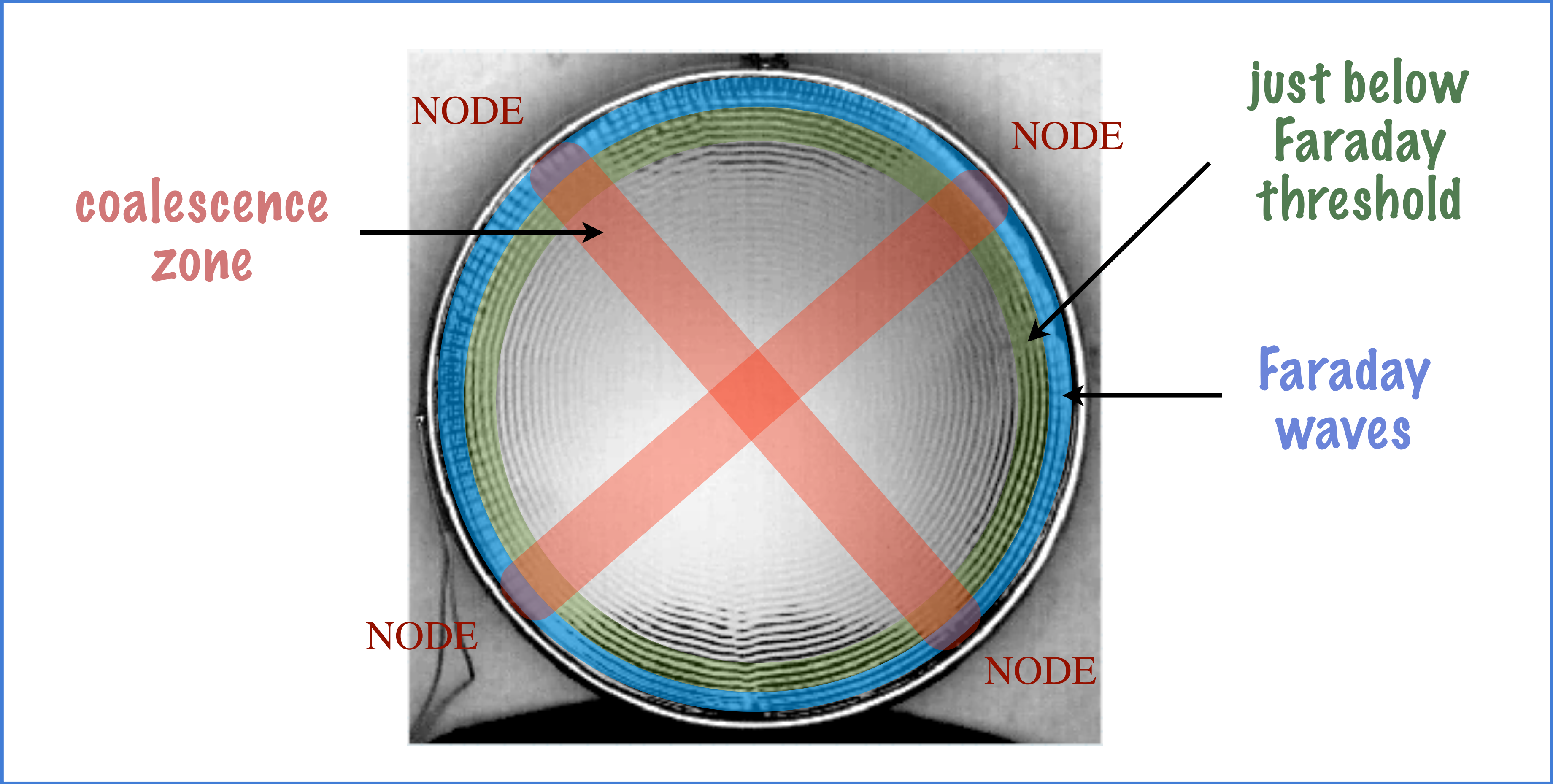
Levitating drops in the Tibetan singing bowl

$$f = 188 \text{ Hz}$$
$$\nu = 10 \text{ cSt}$$



Can droplets walk in the singing bowl?

- amenability to walking depends on position
- region just below Faraday threshold limited to 4 circular arcs



Conclusions

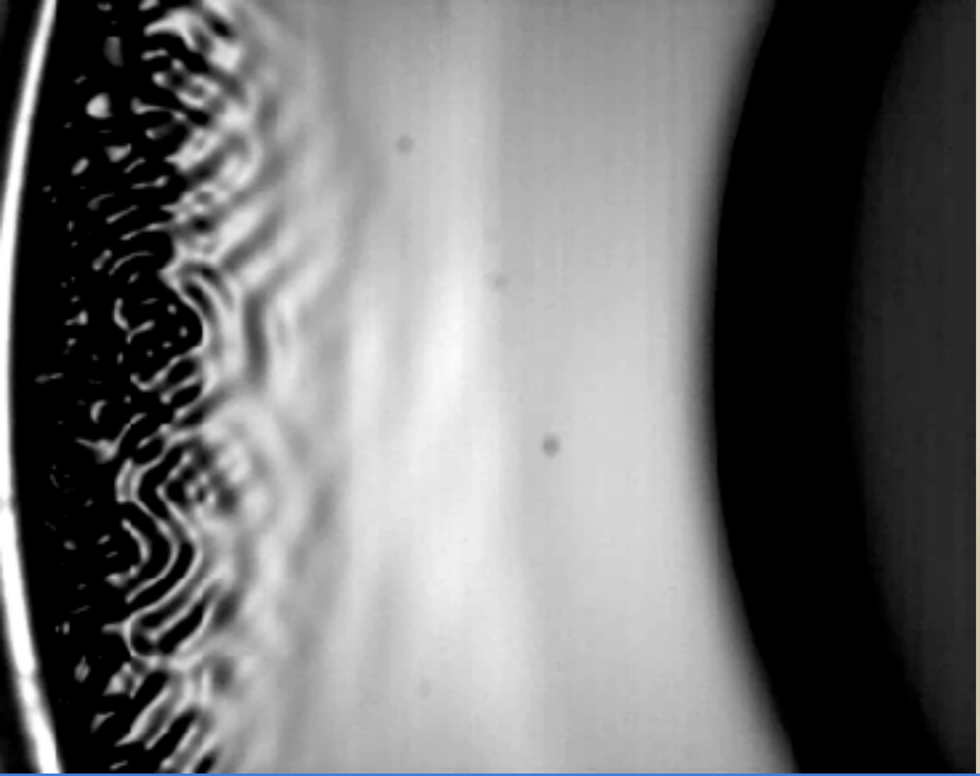
- have characterized the acoustics, hydrodynamics of the Tibetan singing bowl

- vibrational modes excite Faraday waves at its edge

- Faraday waves break, releasing droplets onto the surface

- droplets may bounce (but not walk) on the field of Faraday waves

- Tibetan singing bowls: good for levitation, bad for quantum analogs

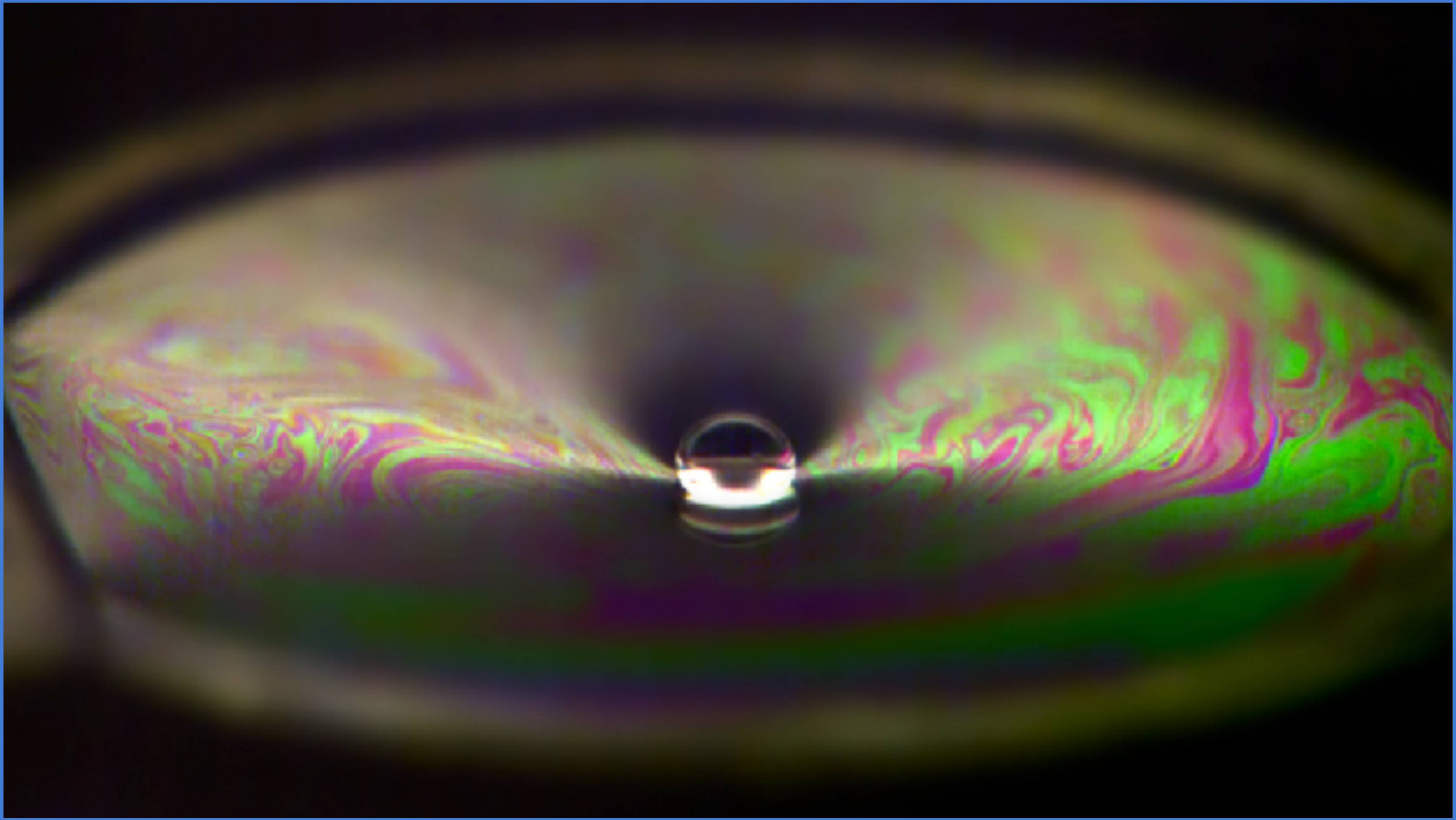
` And he showed Pierre a globe, a living wavering ball of no dimensions, its surface consisting of drops tightly packed together. The drops moved and shifted, now merging from several into one, now dividing from one into many. Each drop strove to spread and take up the most space, but the others, striving to do the same, pressed against it, sometimes destroying, sometimes merging.”

“This is life”, said the old teacher. “In the center is God, and each drop strives to expand in order to reflect Him in the greatest measure. It grows, merges and shrinks, is obliterated on the surface, vanishes into the depths, then resurfaces.” ’

- Tolstoy, War and Peace

Half time

Part II The fluid trampoline: droplets bouncing on a soap film



John W. M. Bush
Department of Mathematics, MIT

Tristan Gilet
University of Liege

Inspiration: walking droplets on a vibrating bath

- Couder's wave-particle duality on the macroscopic scale
- modeling difficulties: must describe flow in drop, bath and intervening air layer

Video courtesy of Suzie Protiere

A simple variant

We here examine drops on a soap film, for which bouncing states can be characterized exactly.

This will turn out to be the simplest fluid mechanical chaotic oscillator yet explored.

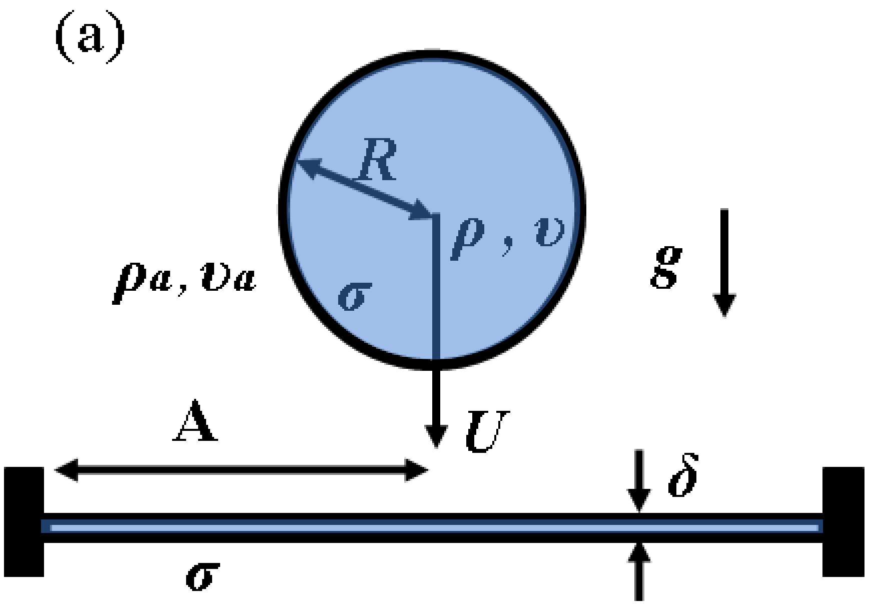
A brief (and woefully incomplete) history of Chaos:

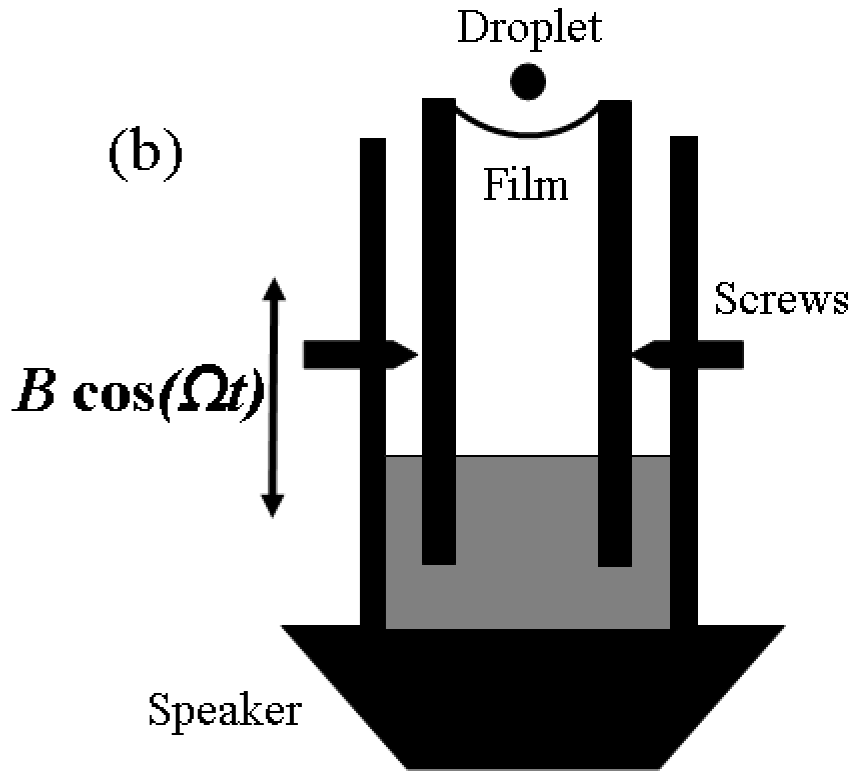
- Henri Poincaré (1903): discovered chaos in exploring celestial mechanics
- Ed Lorenz (EAPS, MIT): *Deterministic non-periodic flow* (1963)
- the Howard-Malkus water wheel, a mechanical analogue of the Lorenz system, developed in Applied Math Lab
- Feigenbaum (1978) predicted chaos in 1D iterative maps, transition to chaos via period-doubling cascades (governed by Feigenbaum numbers)
- period-doubling transitions to chaos reported in various systems:
  - thermal convection (Libchaber 1980, Gollub & Swinney 1981)
  - the dripping faucet (Shaw, 1981)
  - an elastic ball bouncing on an oscillating substrate (Tufillaro et al. 1992)

... and now.... ... the fluid trampoline .....

I. Stationary film

II. Driven film



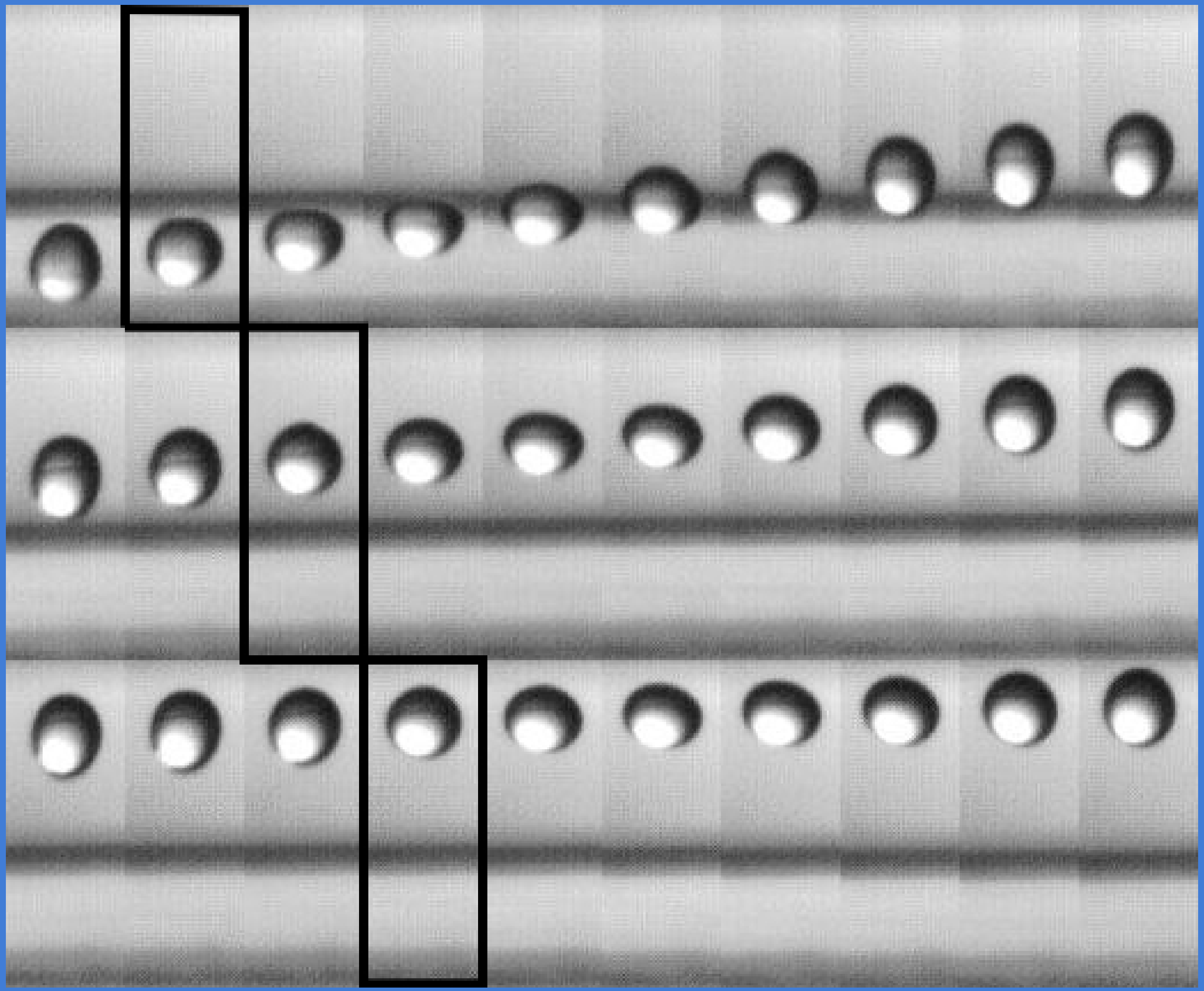


Key parameter: $$We = \frac{\rho U^2 R}{\sigma}$$

Acceleration: $$\Gamma = B\Omega^2$$

- fluid is glycerine, water, Dove: viscosity 2 cS, surface tension 22 dynes/cm
- drop radius: R = 0.8 mm; frame radius A = 8 mm

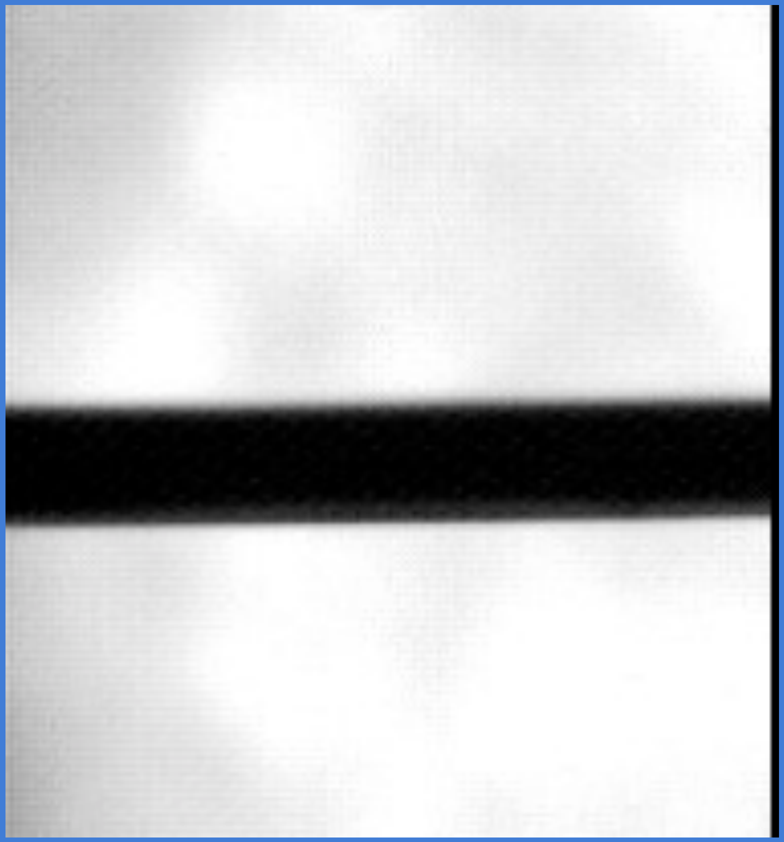
Inference of surface tension from drop oscillations:



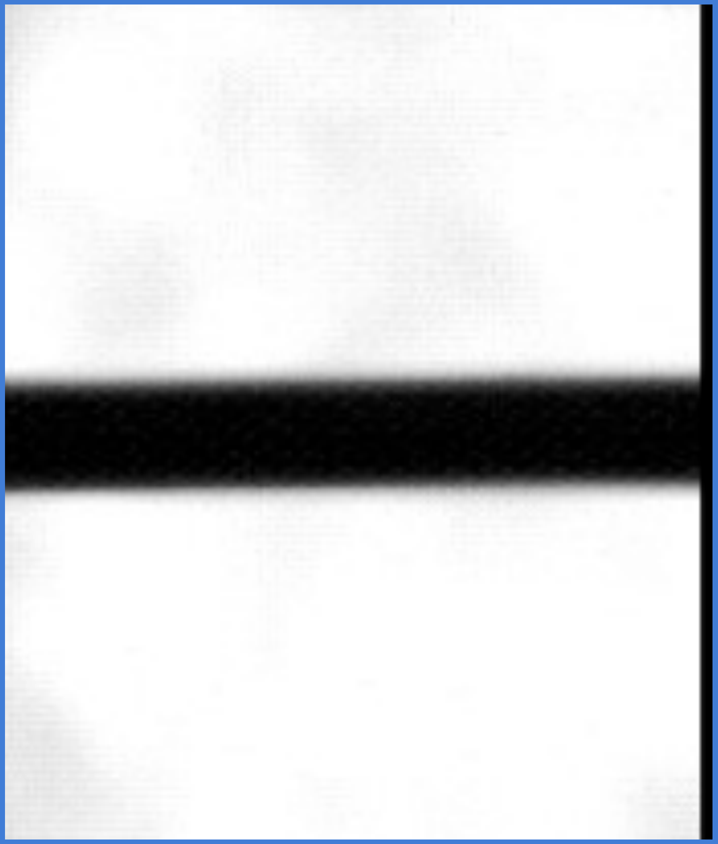
Surface tension: $\sigma = \dfrac{3\pi m}{8T^2}$ = 22 dynes/cm

Impact

- falling water droplets strike a horizontal soap film



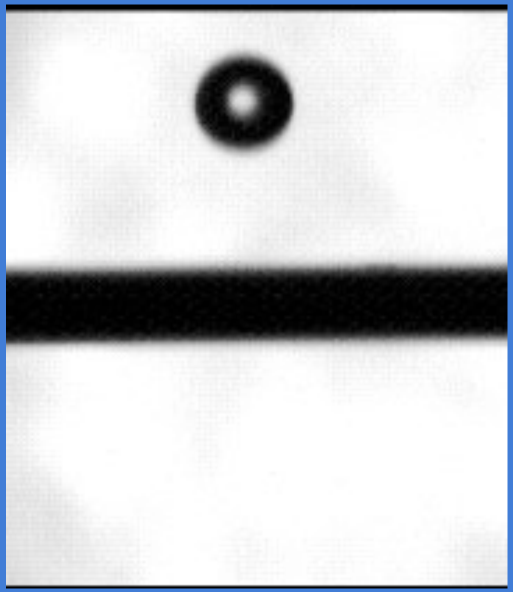
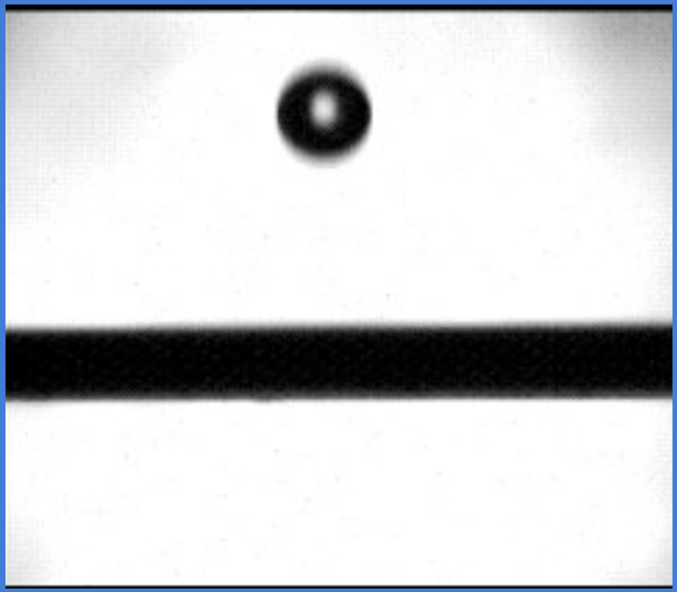
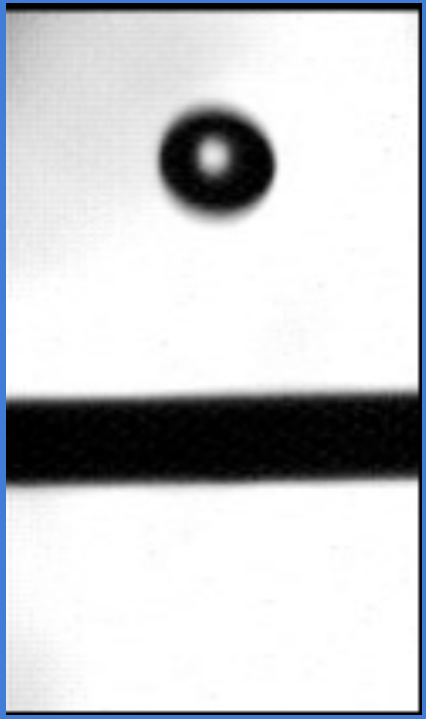
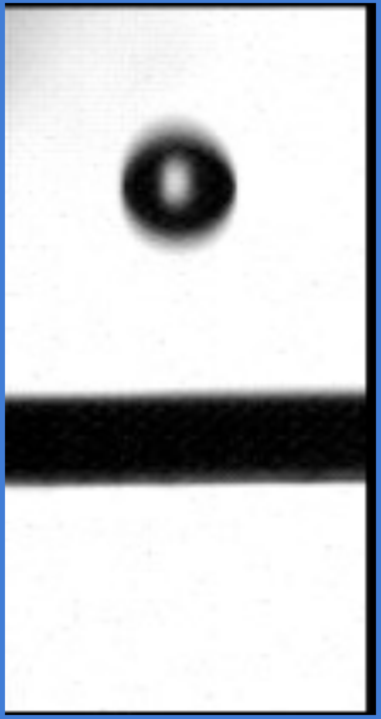
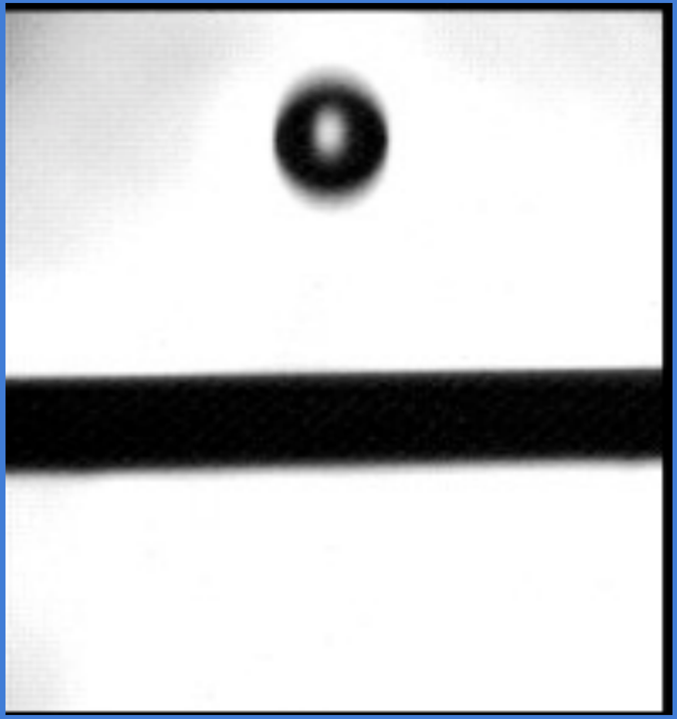
BOUNCES

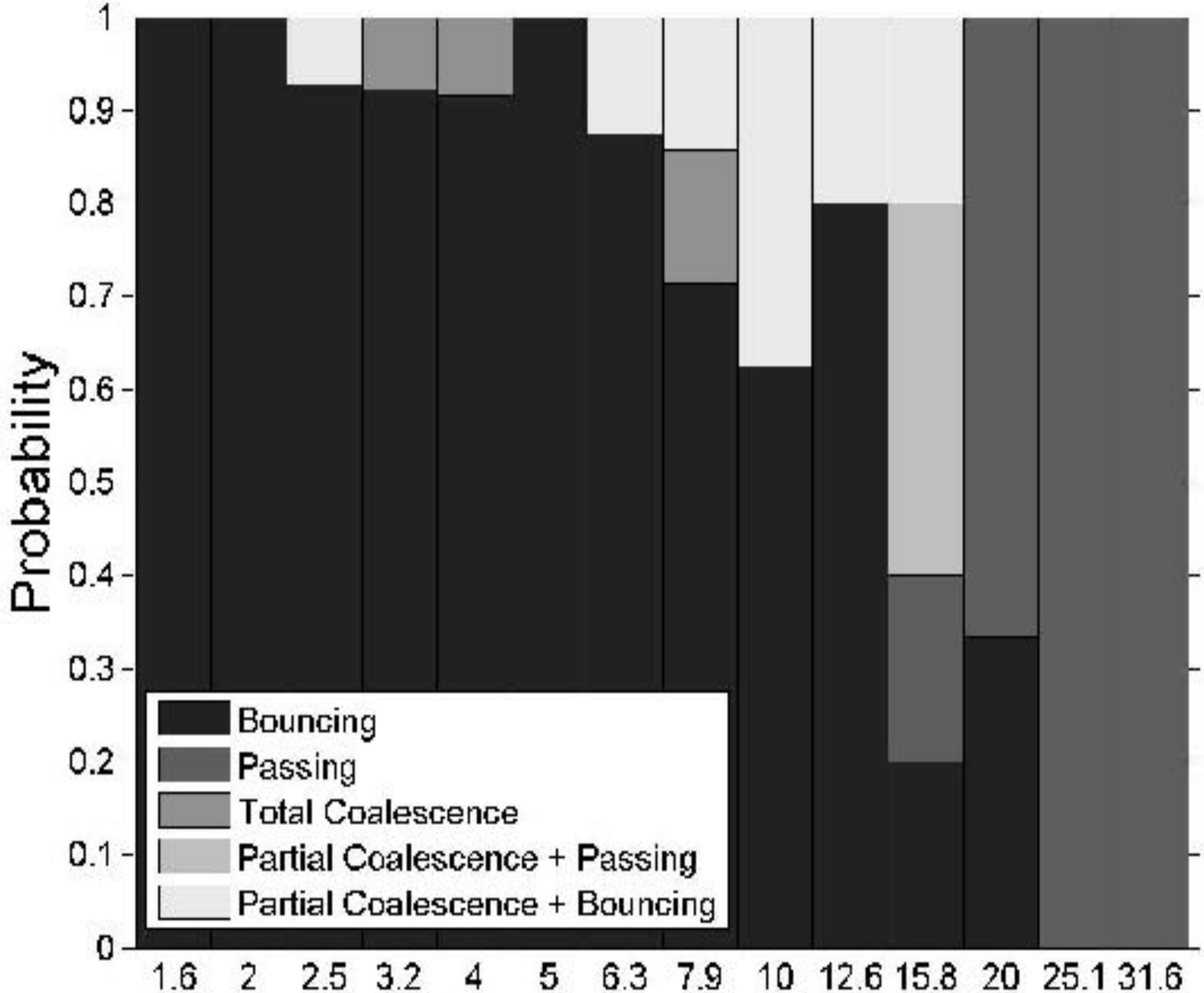


PASSES THROUGH

Other possibilities:

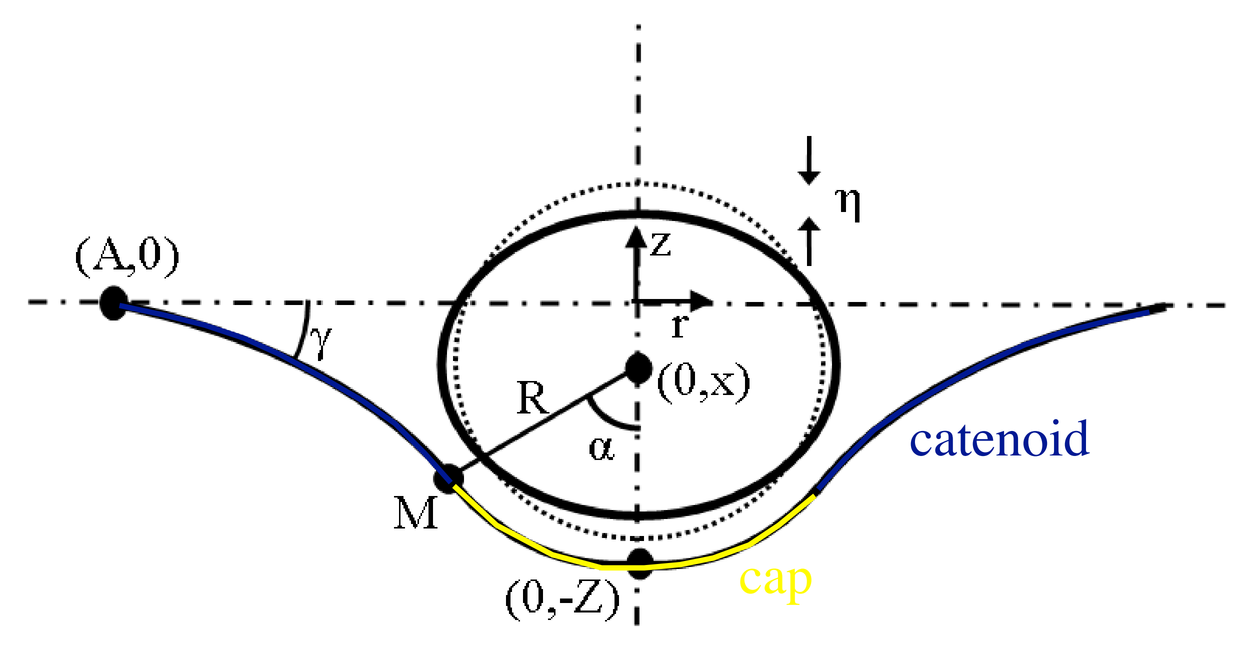
partial coalescence
in various guises



$$We = \frac{\rho U^2 R}{\sigma}$$

Impact model



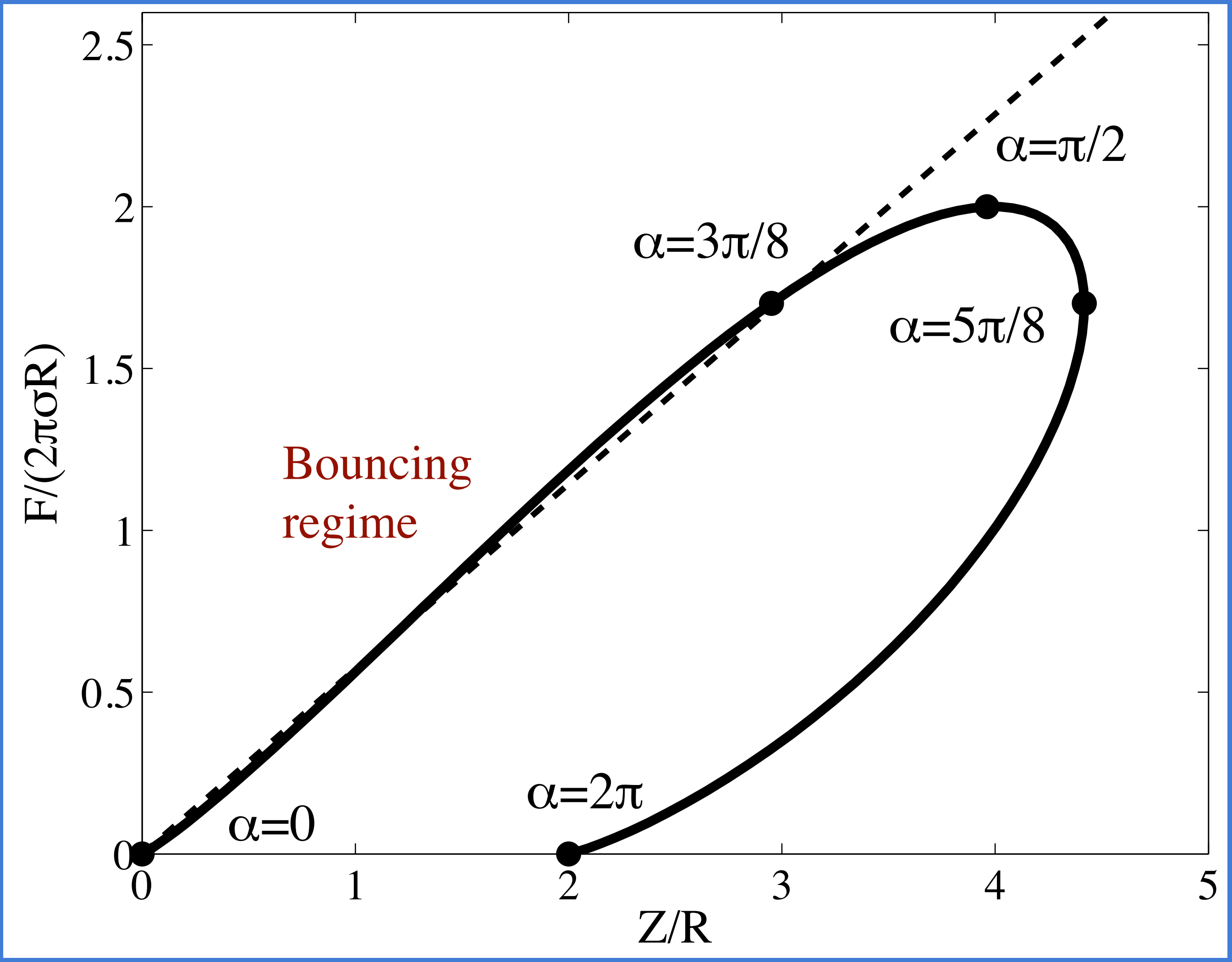
- air layer communicates stress from film to drop
- can deduce force on drop from film shape

Film shape

- quasistatic: valid since impact speed (~20 cm/s) much less than capillary wave speed on soap film (~3m/s)

→ spherical cap & catenoid

Force-displacement curve



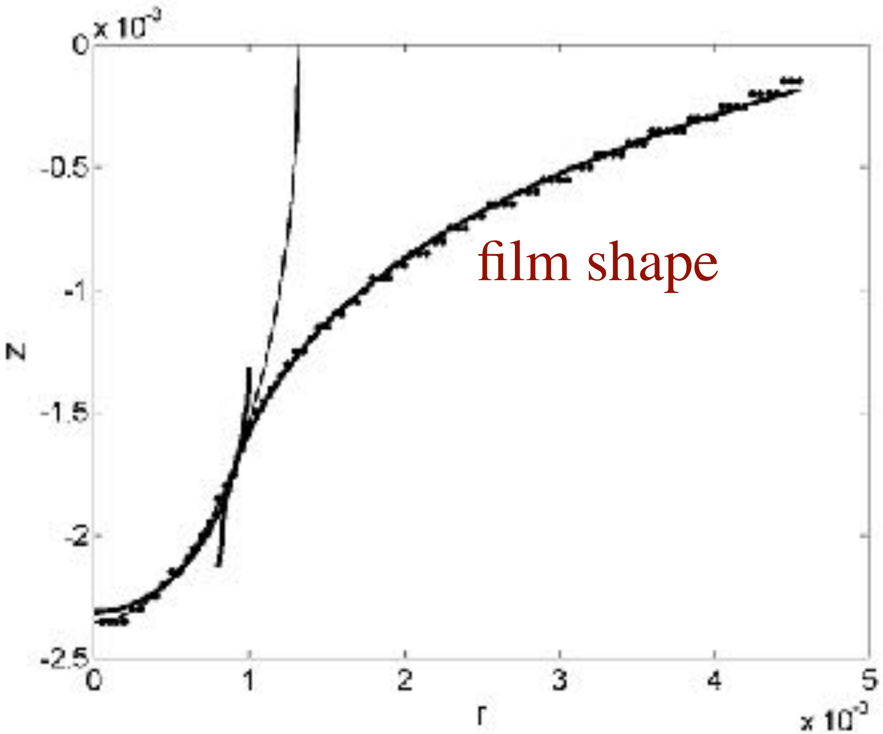
In bouncing regime, film acts like linear spring with spring constant:

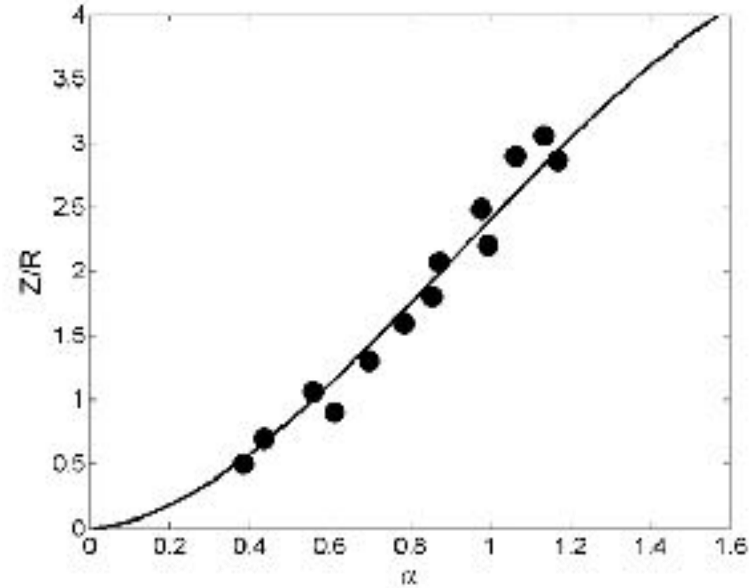
$$k = \frac{8\pi}{7}\sigma$$

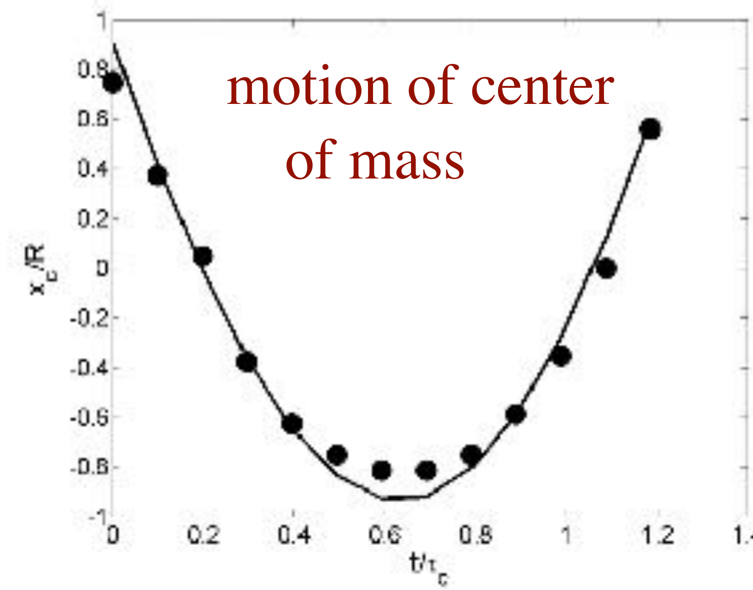
for $\beta = \frac{\text{film radius}}{\text{drop radius}} = 10$

Soap film model

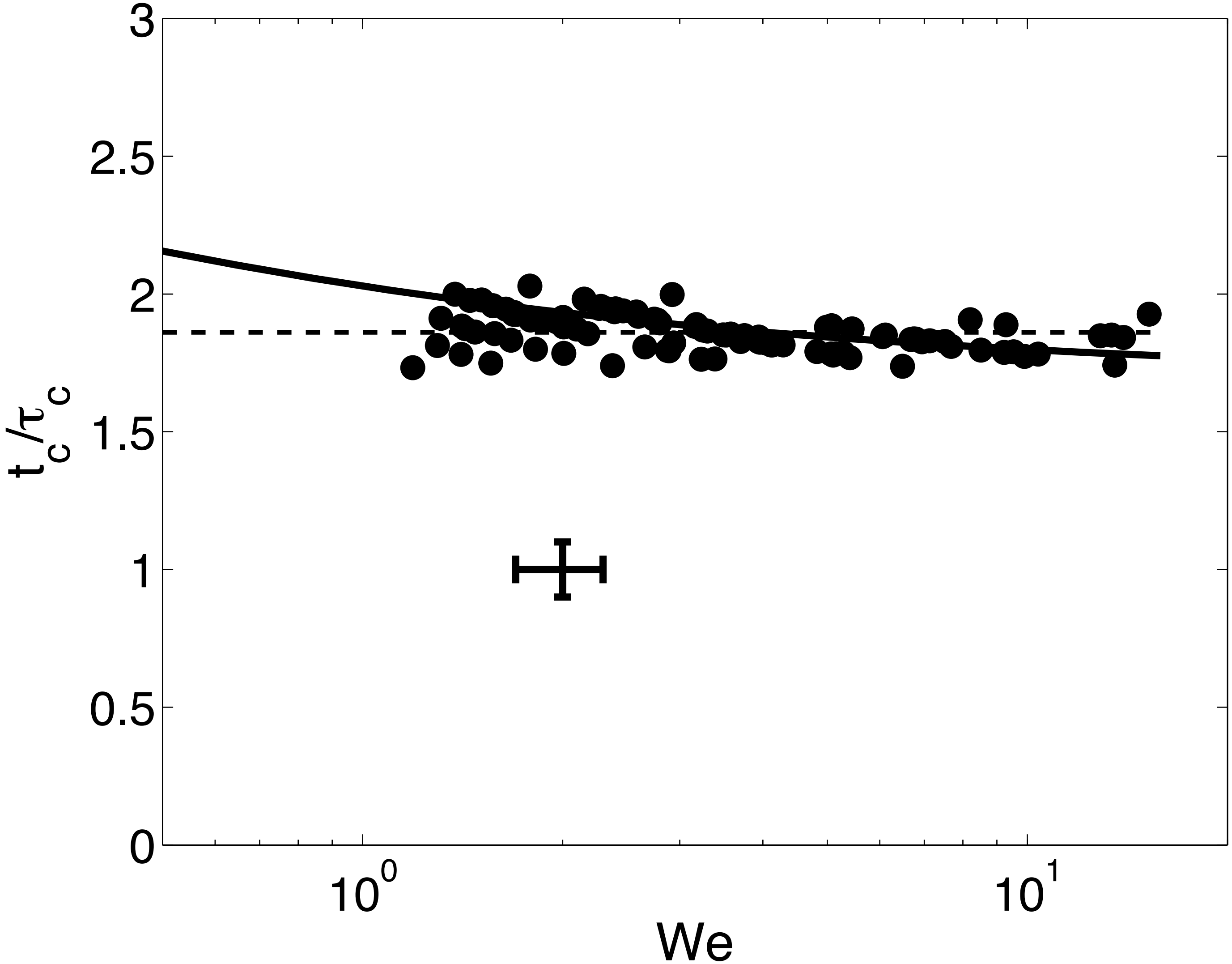
- experiments versus theory







Contact time

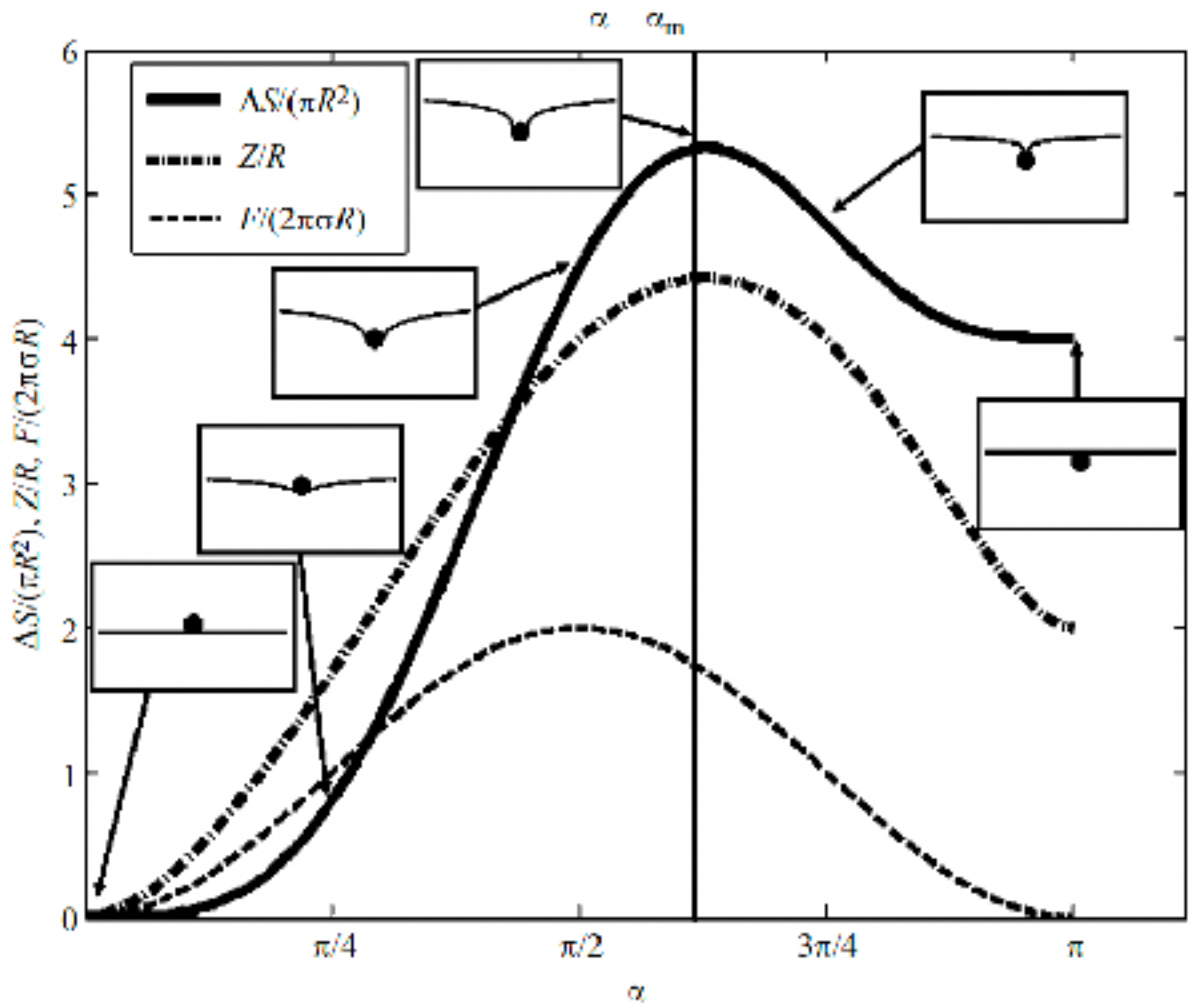


Linear spring: $t_c = \sqrt{m/k}$ where $k = \frac{8\pi}{7}\sigma$

Breakthrough criterion

Drop kinetic energy > Surface energy at breakthrough

$$\frac{2\pi}{3}\rho R^3 U^2 > 2\sigma \Delta S_m(\beta) \quad \rightarrow \quad \boxed{We^* = \frac{3\Delta S_m(\beta)}{\pi R^2}}$$



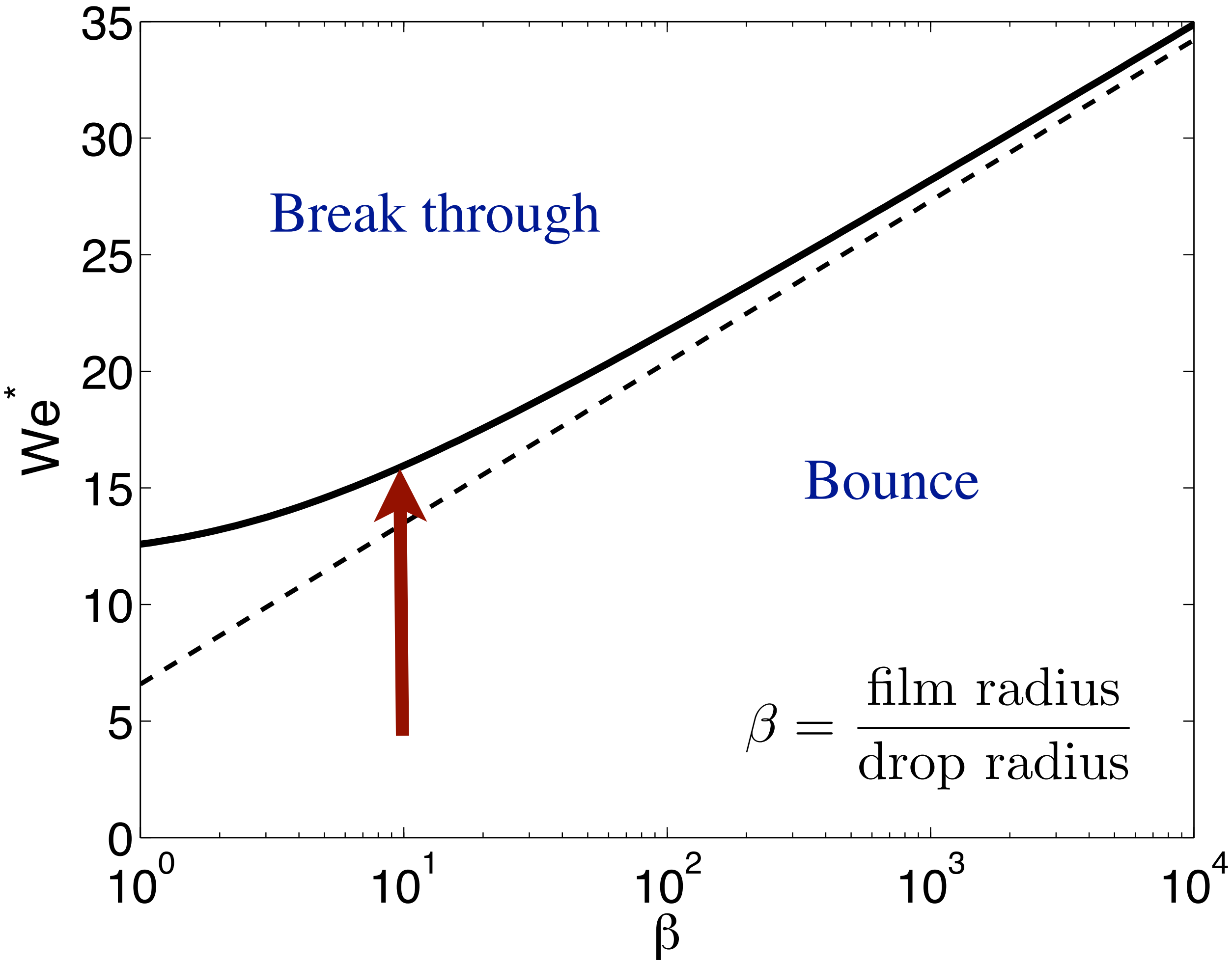
$$\beta = \frac{\text{film radius}}{\text{drop radius}} = 10$$

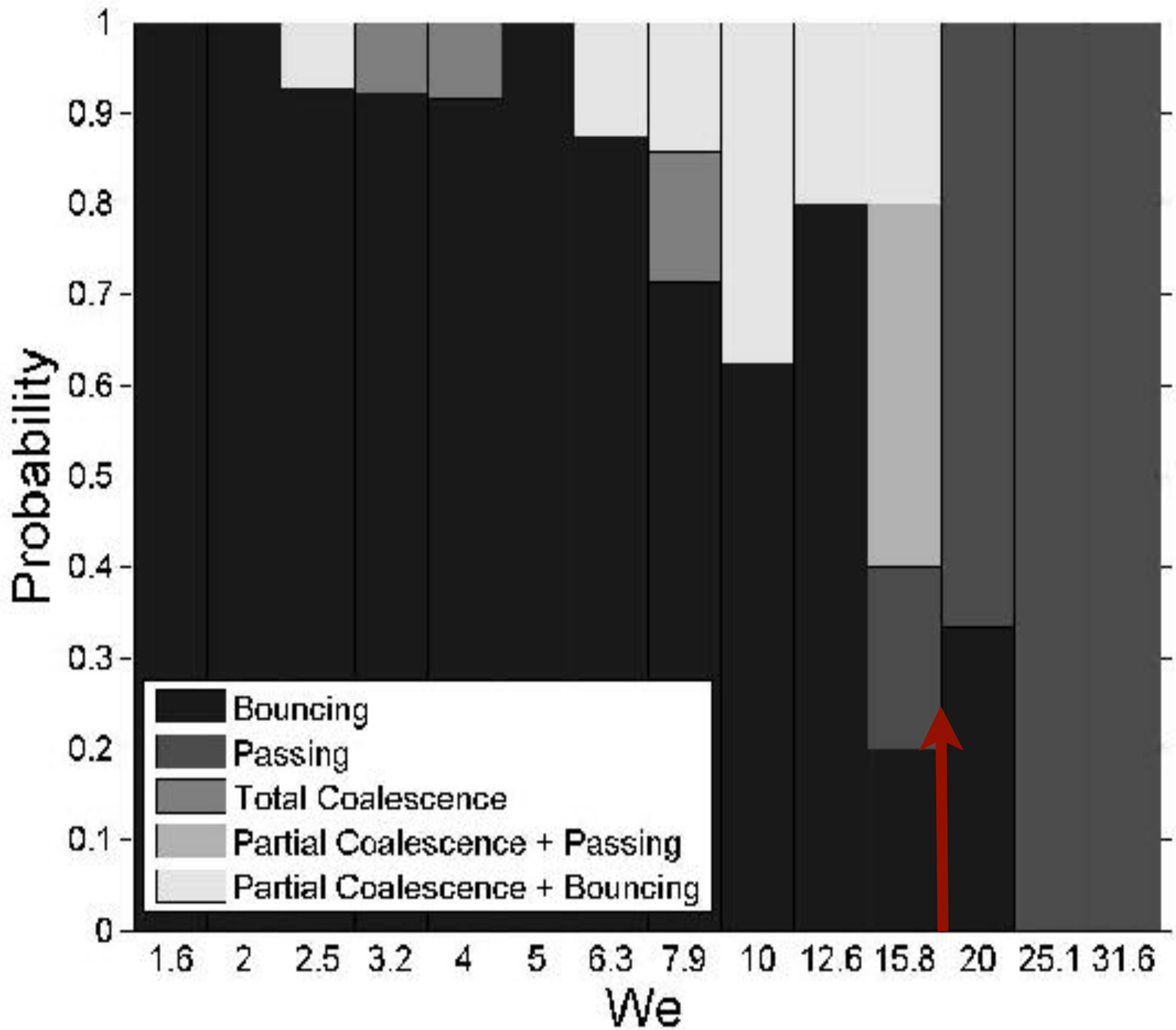
Breakthrough criterion

Drop kinetic energy > Surface energy at breakthrough

$$\frac{2\pi}{3}\rho R^3 U^2 > 2\sigma \Delta S_m(\beta) \quad \rightarrow \quad We^* = \frac{3\Delta S_m(\beta)}{\pi R^2}$$

$$We = \frac{\rho U^2 R}{\sigma}$$

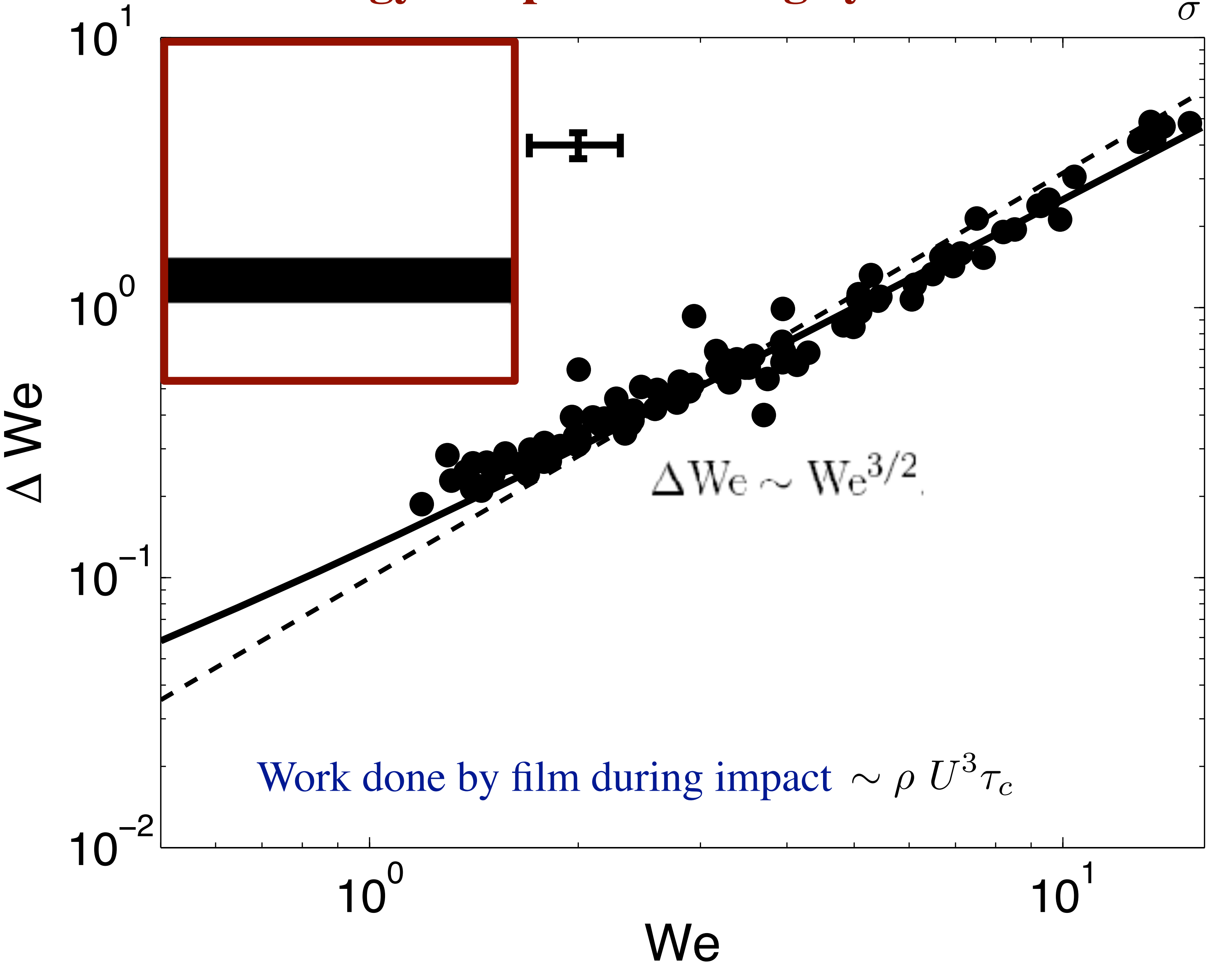




Probability
We
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1.6
2
2.5
3.2
4
5
6.3
7.9
10
12.6
15.8
20
25.1
31.6
Bouncing
Passing
Total Coalescence
Partial Coalescence + Passing
Partial Coalescence + Bouncing

Energy lost per bouncing cycle

$$We = \frac{\rho U^2 R}{\sigma}$$



Equation of motion for impact on a stationary film

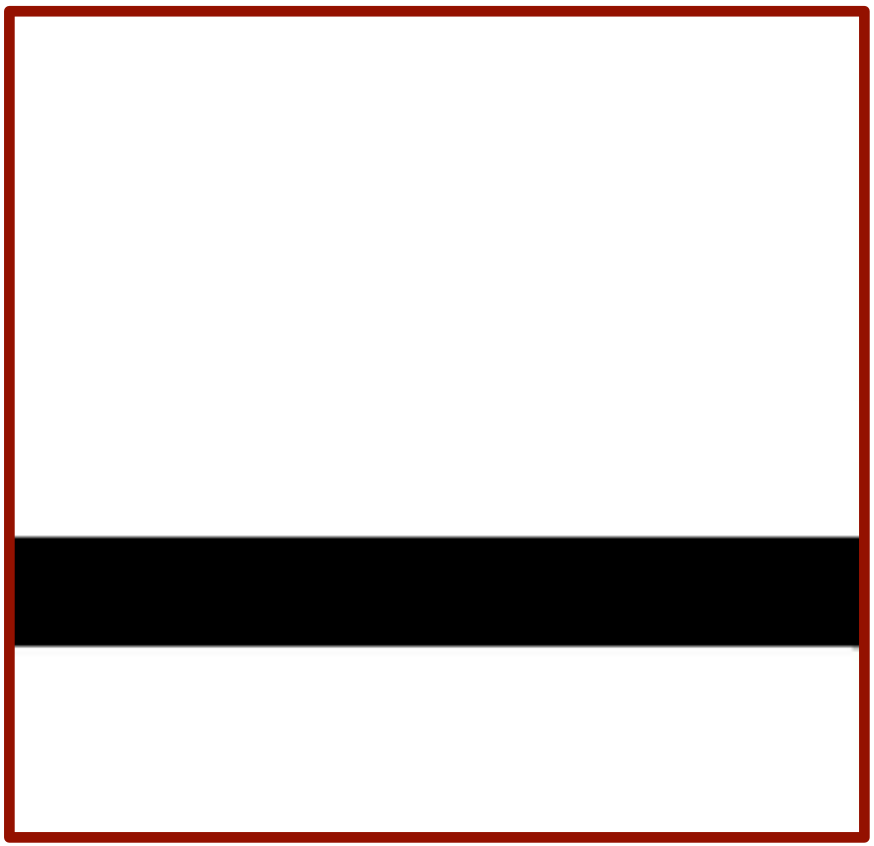
$$m\ddot{Z} = mg - kZH(Z) - DH(Z)\dot{Z}|\dot{Z}|$$

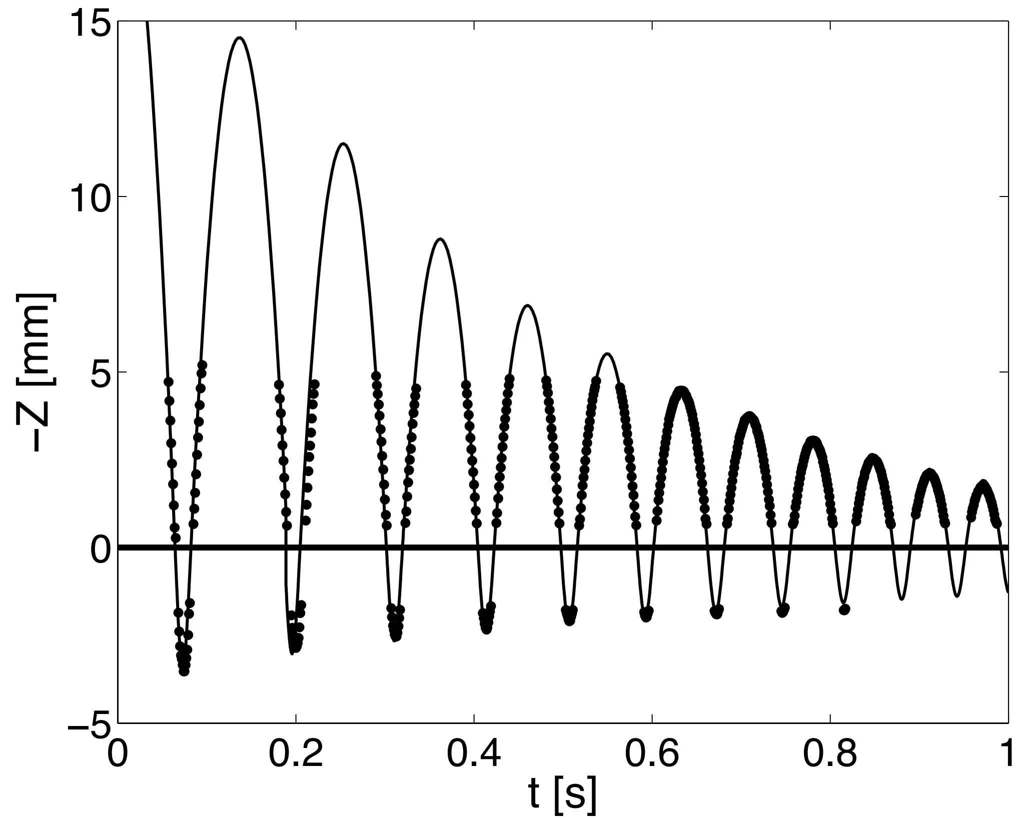
gravity spring dissipation

- spring acts only during impact with constant $k = \frac{8\pi}{7}\sigma$
- form of dissipation suggested by experiment: $\Delta We \sim We^{3/2}$
- dissipation constant prescribed by experiment $D = 8 \times 10^{-5}$ kg/m

Bouncing on a stationary film

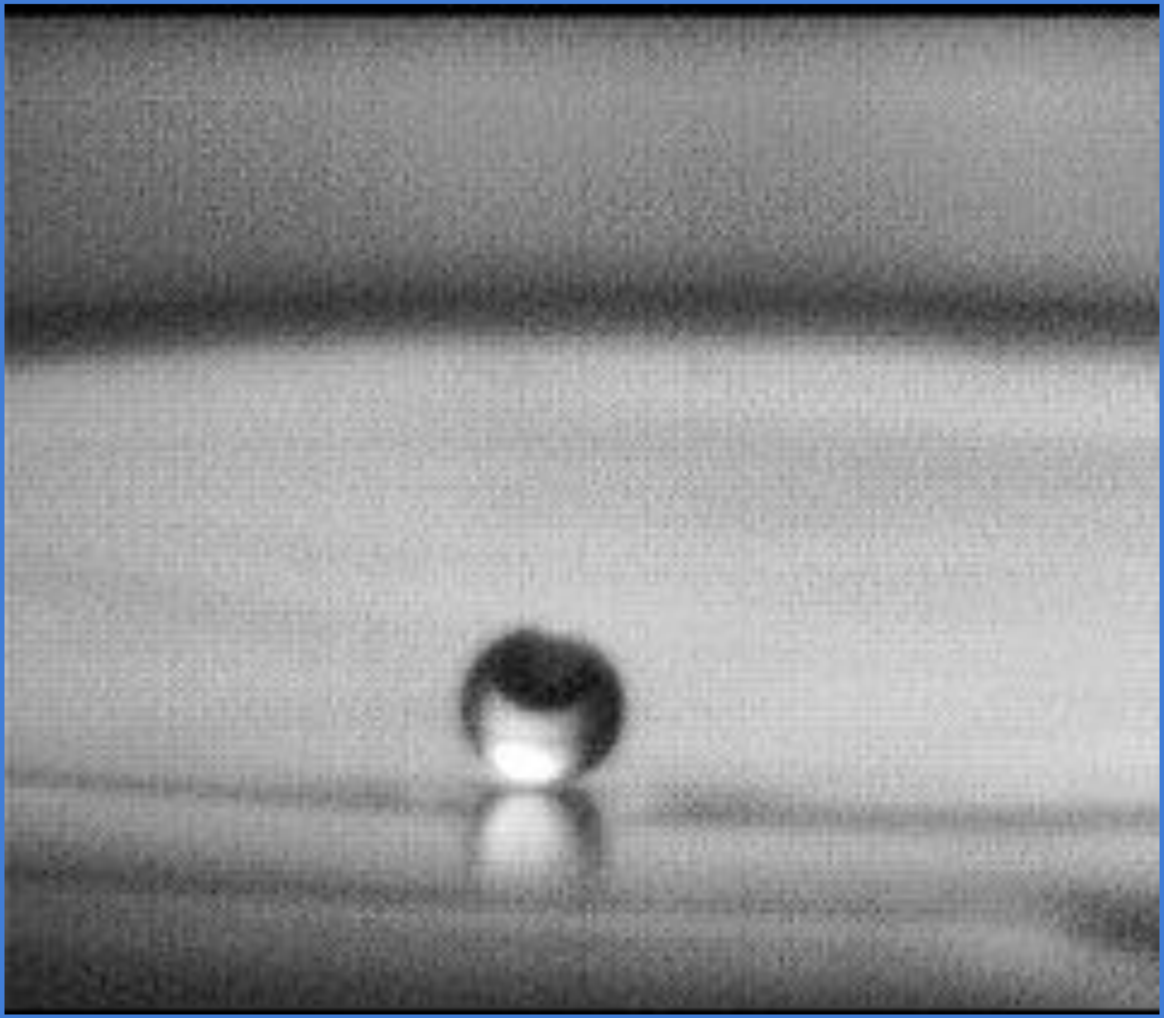
$$m\ddot{Z} = mg - kZH(Z) - DH(Z)\dot{Z}|\dot{Z}|$$





- KE lost with each successive impact

DROPS BOUNCING ON A DRIVEN FILM

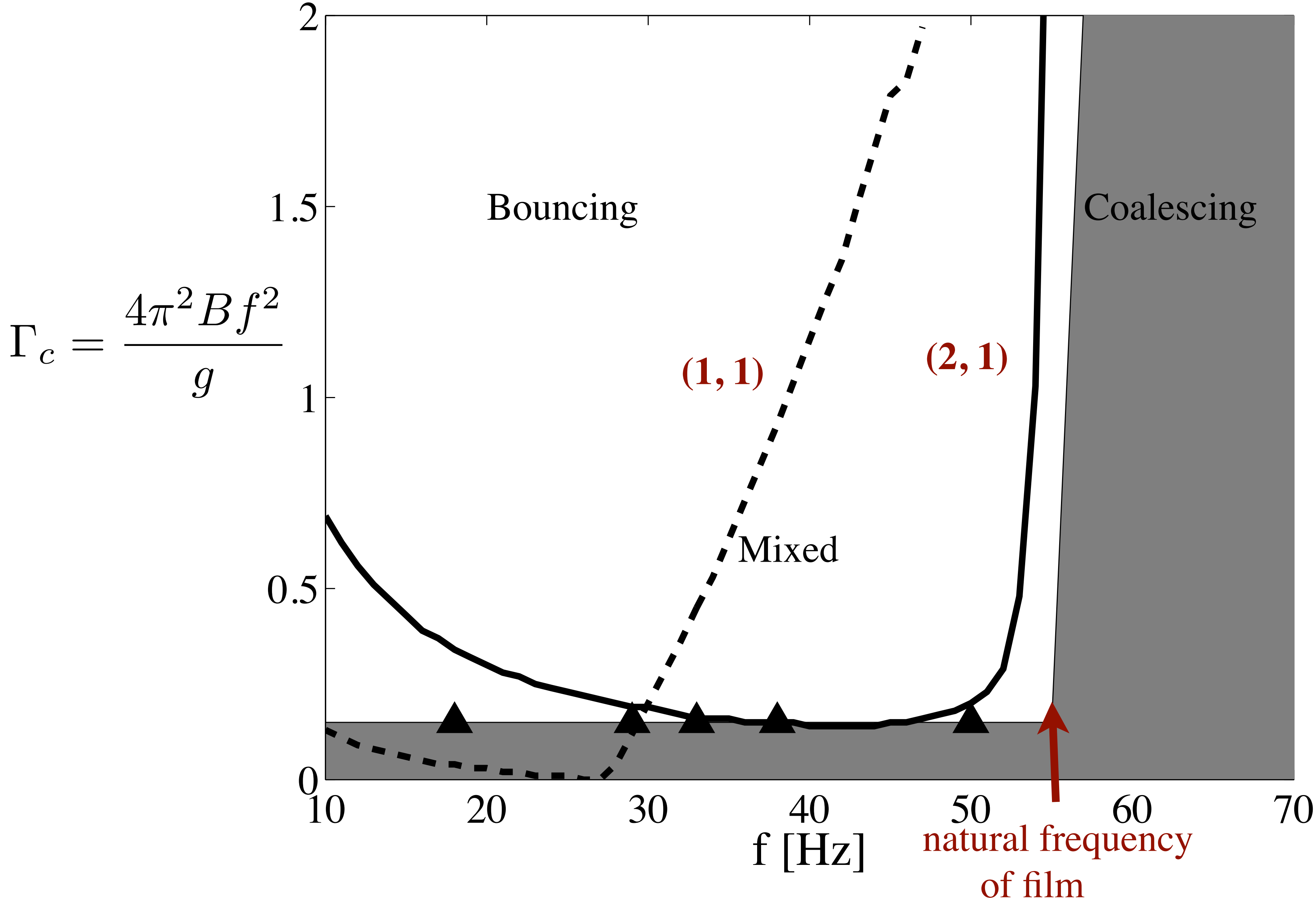


frequency f

amplitude B

- drops may be sustained indefinitely on the film, bounce periodically or chaotically

Bouncing criterion



The zoology of the bouncing states: nomenclature

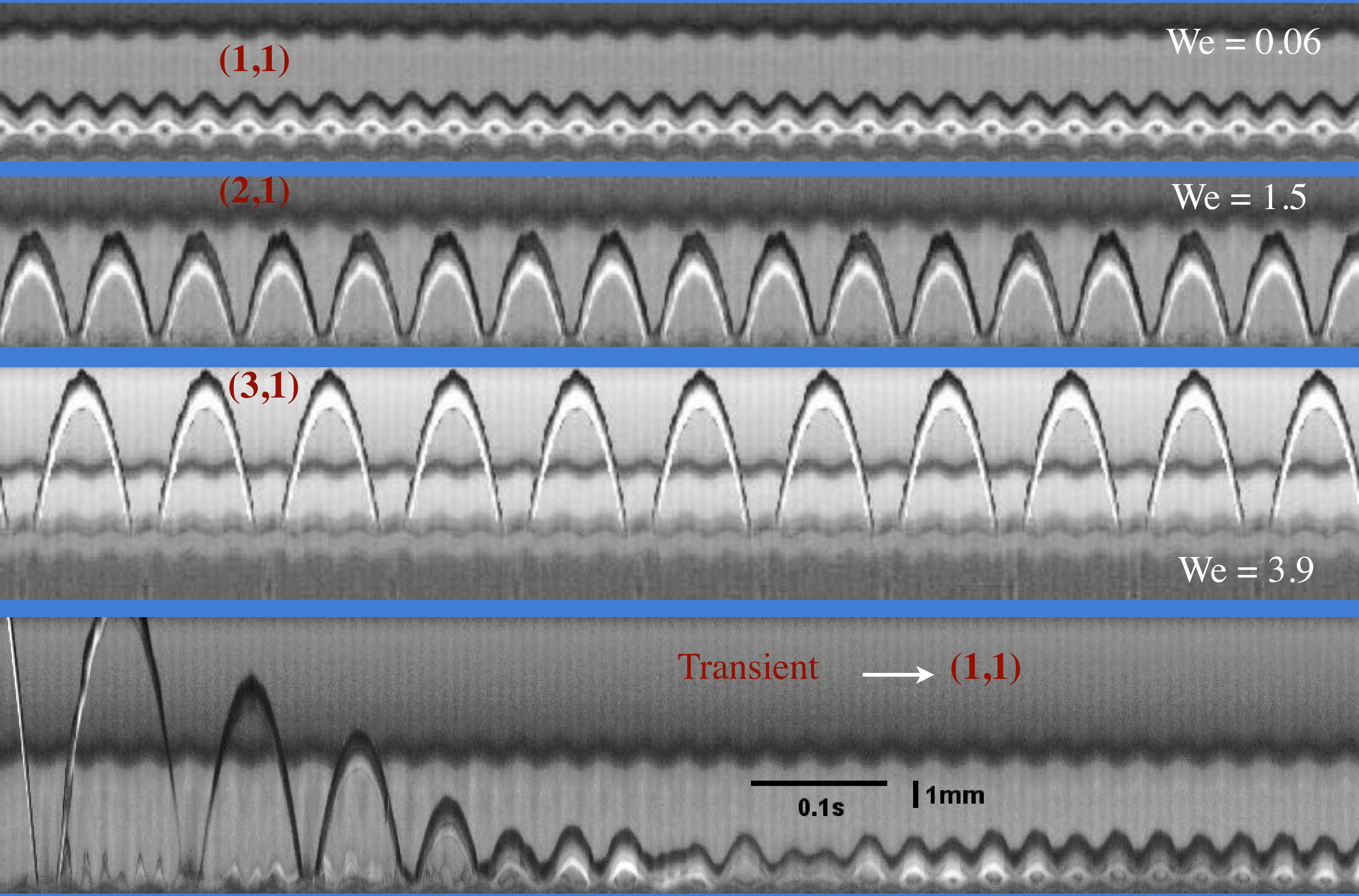
A bouncing state (m, n) bounces n times in m forcing periods.

(1,1) (2,1) (3,1)

i.e. one period of the trajectory corresponds to m forcing periods and n bounces of the droplet

(3,3)

Multiple simple periodic modes at $f = 33\ Hz\ ,\ \Gamma = 0.6$



Equation of motion of a droplet on a driven film

$$m\ddot{Z} = mg - kZH(-Z) - DH(Z)\dot{Z}|\dot{Z}| - mg\Gamma\cos(\Omega t + \phi)$$

gravity spring dissipation forcing

- spring acts only during impact with constant $k = \frac{24\pi}{25}\sigma$
- form of dissipation suggested by experiment: $\Delta We \sim We^{3/2}$
- dissipation constant prescribed by experiment $D = 8 \times 10^{-5}$ kg/m
- 2nd order equation rendered non-autonomous by the forcing

Equation of motion

$$m\ddot{Z} = mg - kZH(-Z) - DH(Z)\dot{Z}|\dot{Z}| - mg\Gamma\cos(\Omega t + \phi)$$

Introduce nondimensional variables

$$y = \frac{-kZ}{mg}; \quad \tau = \sqrt{\frac{k}{m}}t; \quad V^2 = \frac{kU^2}{mg^2}; \quad \Psi = \frac{Dg}{k}; \quad \omega = \Omega\sqrt{\frac{m}{k}}$$

Nondimensional governing equation

$$\ddot{y} + H(-y)y + 1 = -H(-y)\Psi|\dot{y}|\dot{y} + \Gamma\cos(\omega\tau + \phi)$$

- solve subject to initial conditions $y(0) = 0$, $\dot{y}(0) = -V$ at impact
- recast as a system of three first order autonomous equations

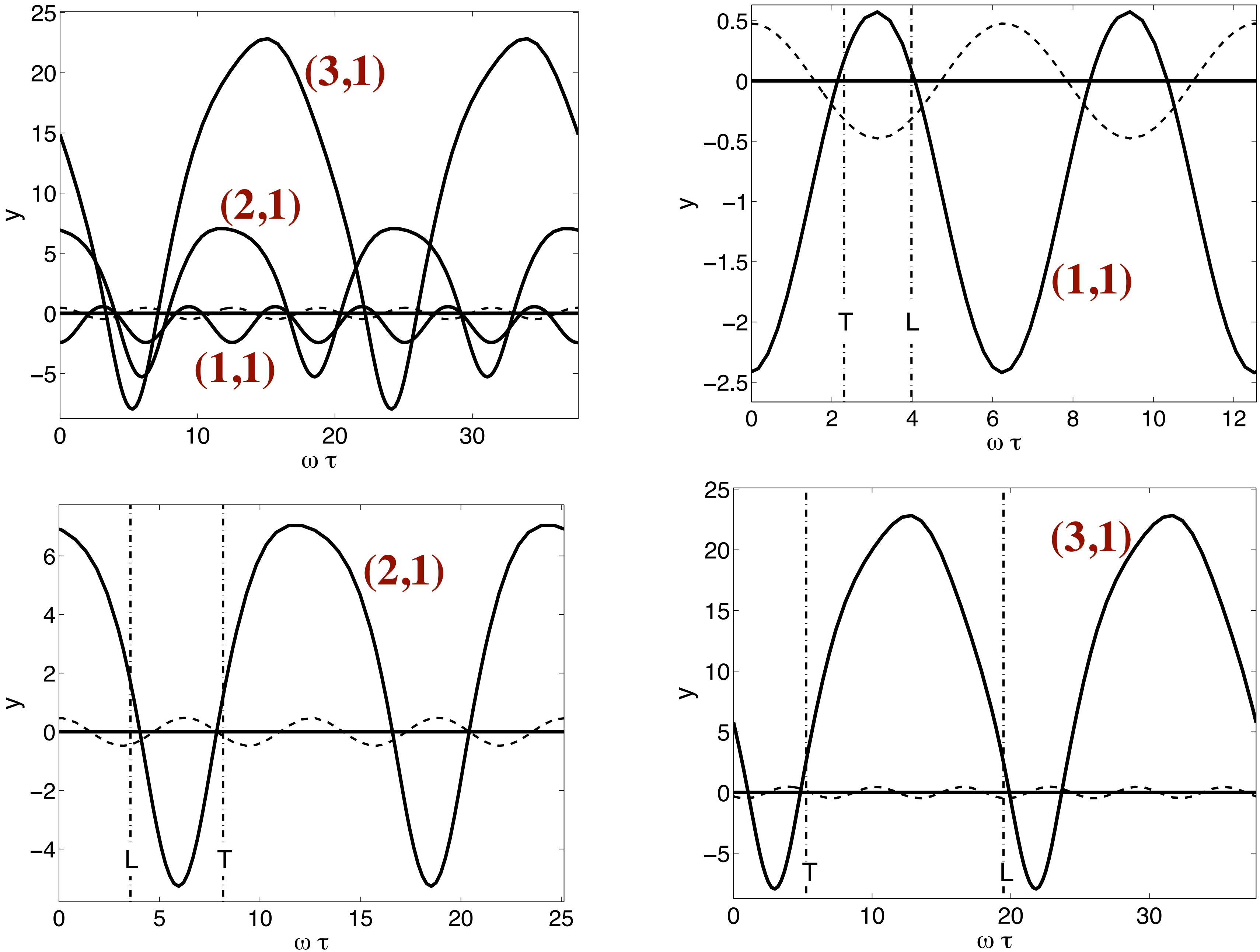
Dynamical system

Choose variables $y(\tau), \dot{y}(\tau)$ and $\theta(\tau) = \mathrm{mod}(\omega\tau + \phi, 2\pi)$

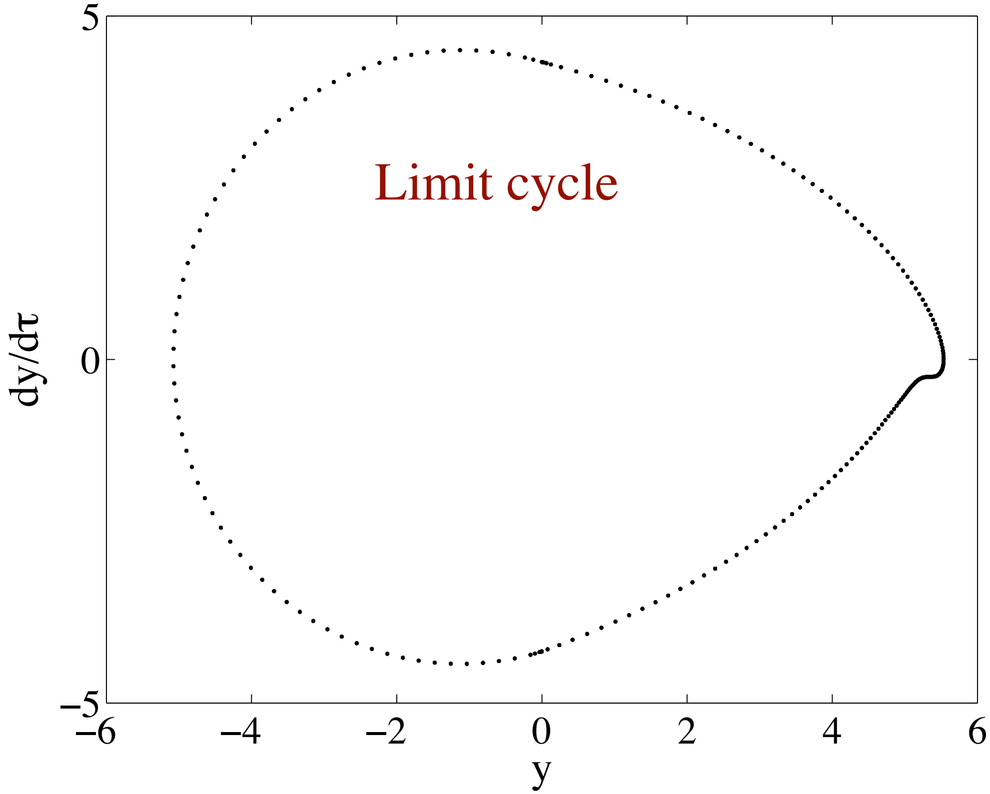
$$
\begin{aligned}
\frac{dy}{d\tau} &= \dot{y} \\
\frac{d\dot{y}}{d\tau} &= -1 - H(-y)[y + \Psi|\dot{y}|\dot{y}] + \Gamma\cos\theta \\
\frac{d\theta}{d\tau} &= \omega
\end{aligned}
$$

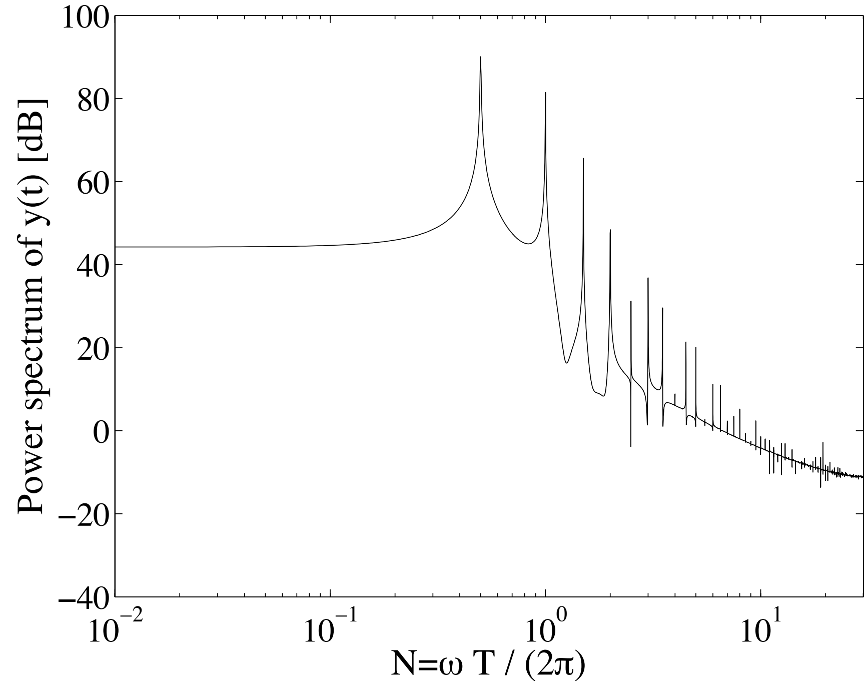
- similar to the Duffing equations, as describe 2 classic chaotic oscillators
    - the inelastic bouncing ball (Mehta & Luck 1990)
    - the parametrically forced pendulum (McLaughlin 1981)
- integrate system with a variety of initial conditions: $(y, \dot{y}, \theta) = (0, -V, \phi)$

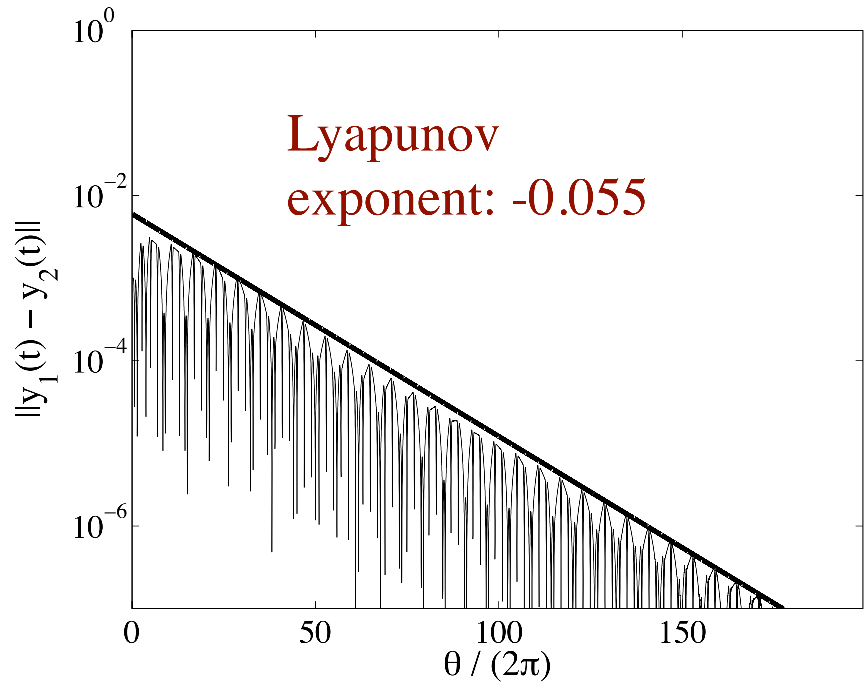
Multiple modes: permissible by virtue of different impact phases



Periodic mode (2,1) at $\Gamma = 1$







A 2D iterative map

- solutions may be displayed on a Poincare section made at impact:

$$(y, \dot{y}, \theta) = (0, -V, \phi)$$

- system integrated numerically from one impact to the next for various initial conditions (V, ϕ)

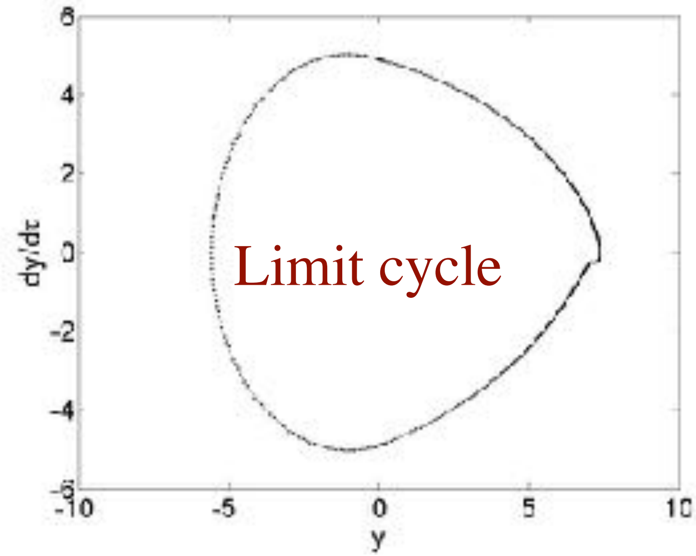
- define a 2D iterative map:

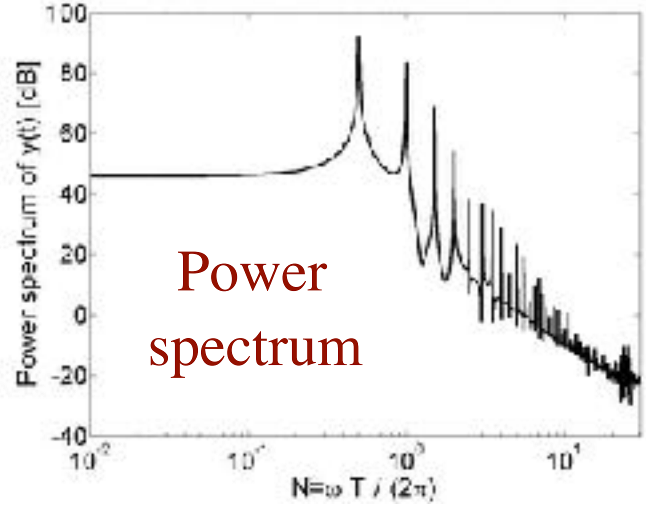
$$V_{i+1} = f(V_i, \phi_i)$$
$$\phi_{i+1} = g(V_i, \phi_i)$$

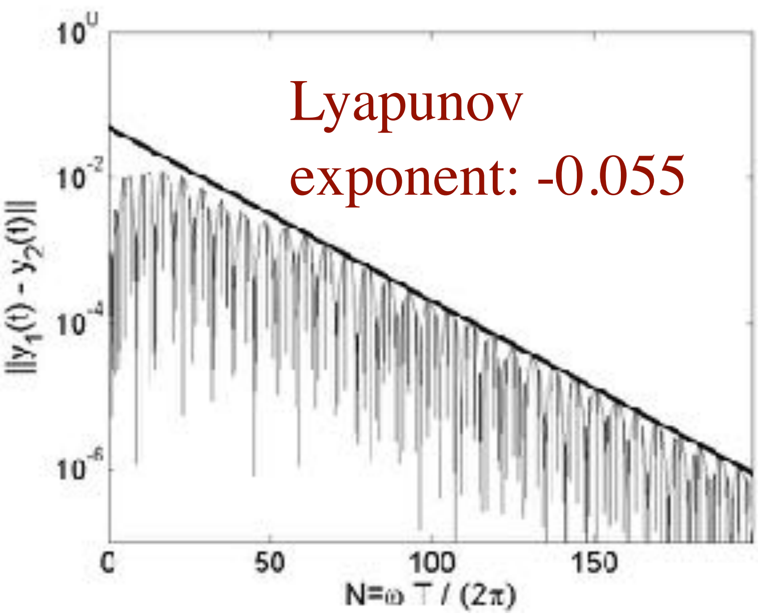
- net energy gained by the drop during the i th bounce depends on (V_i, ϕ):

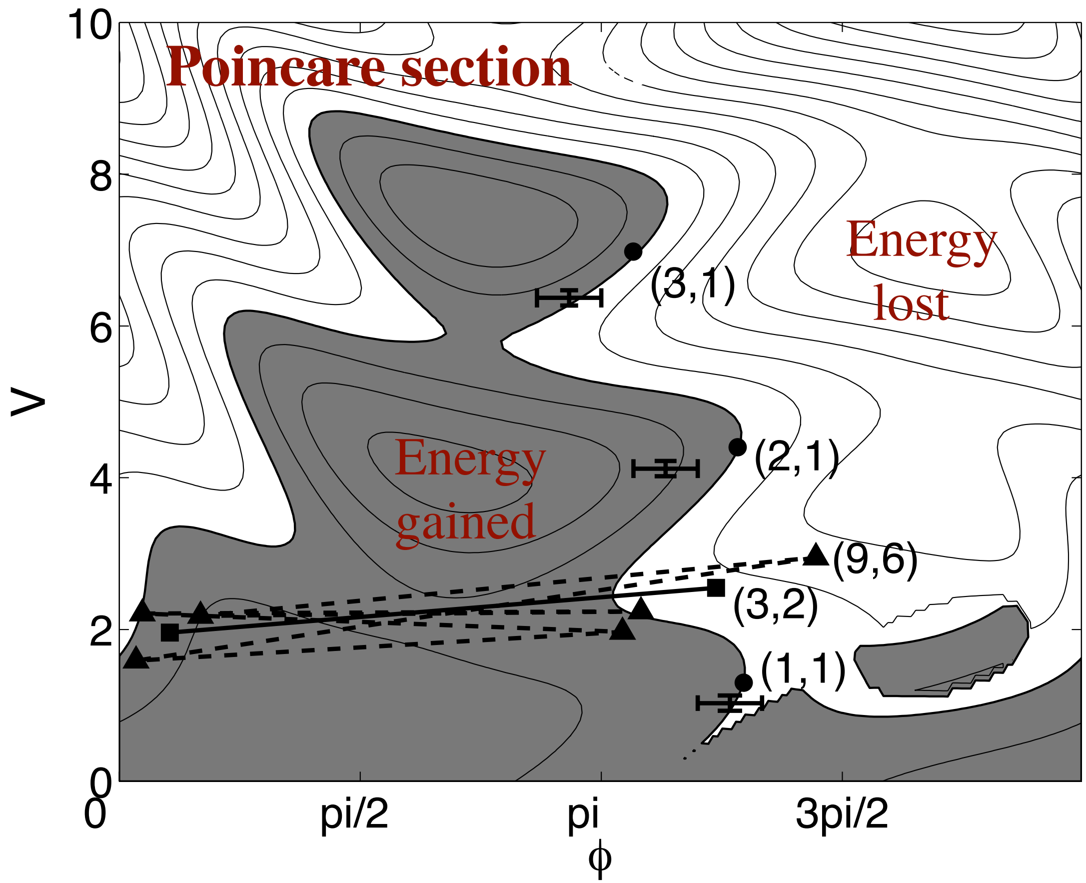
$$\Delta E = (V_{i+1}^2 - V_i^2)/2$$

Periodic mode (2,1) at $\Gamma = 1, \omega = 1.1$



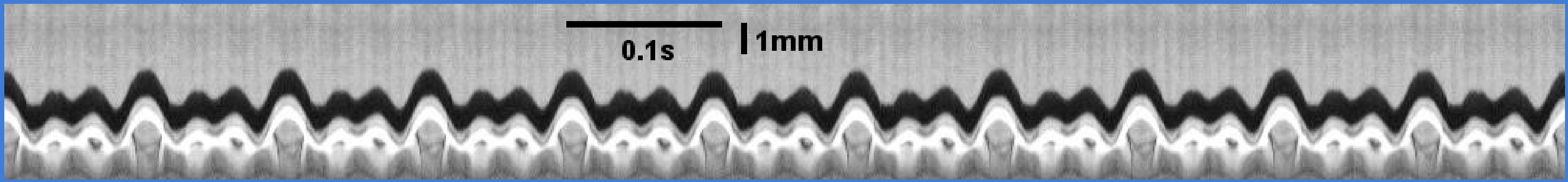




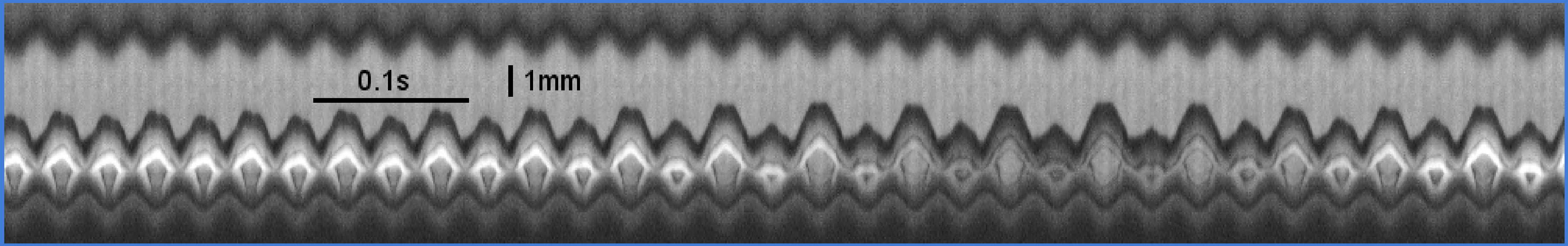


Complex periodic and aperiodic modes

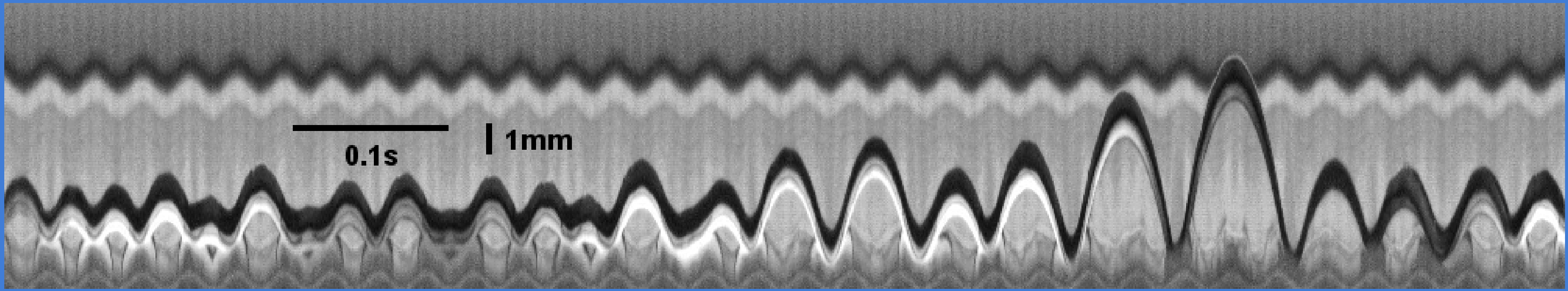
Recall: a state (m, n) bounces n times in m forcing periods



Periodic mode (3,3) at $f = 33\ Hz\ ,\ \Gamma = 0.7\ g$

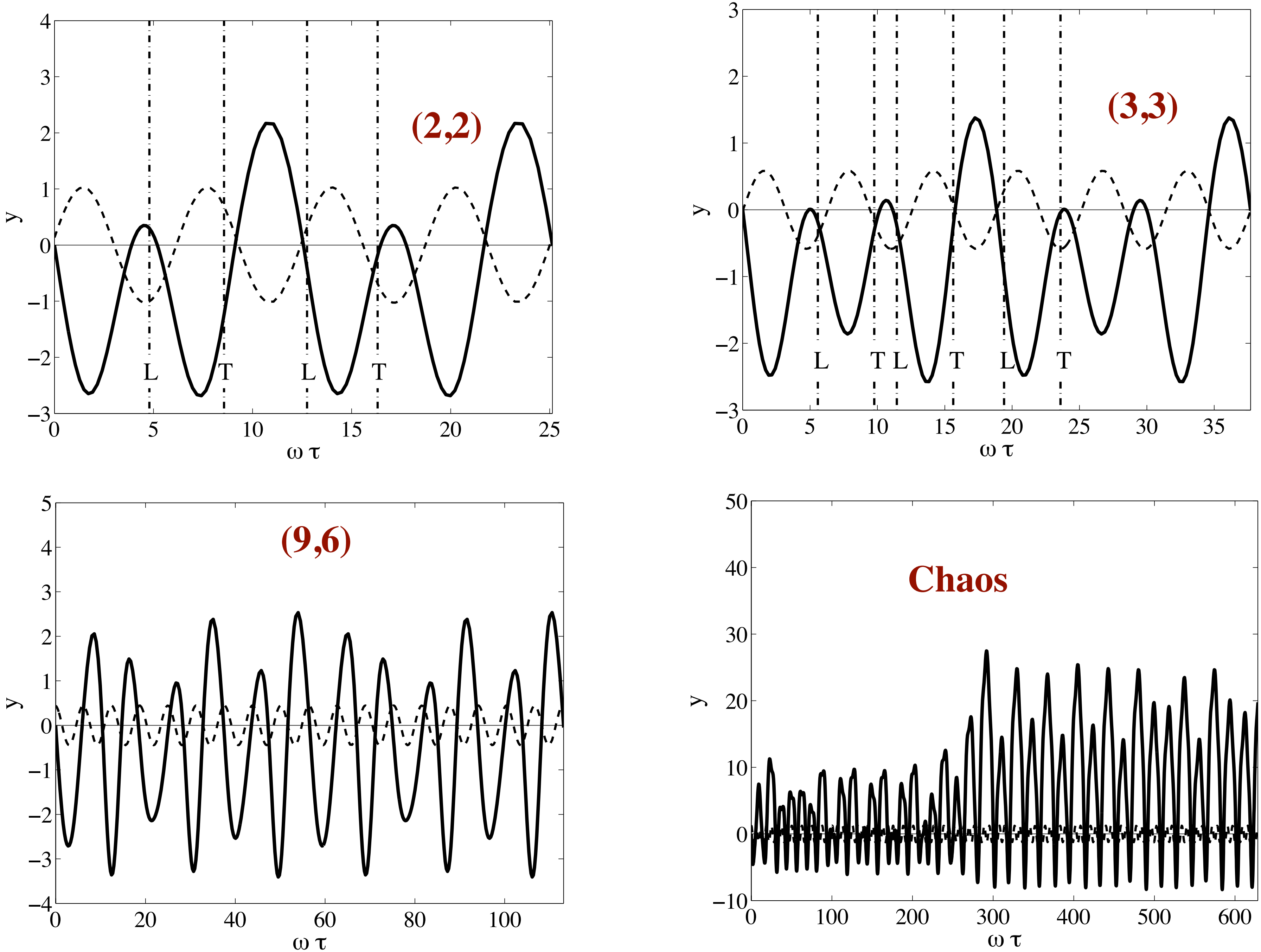


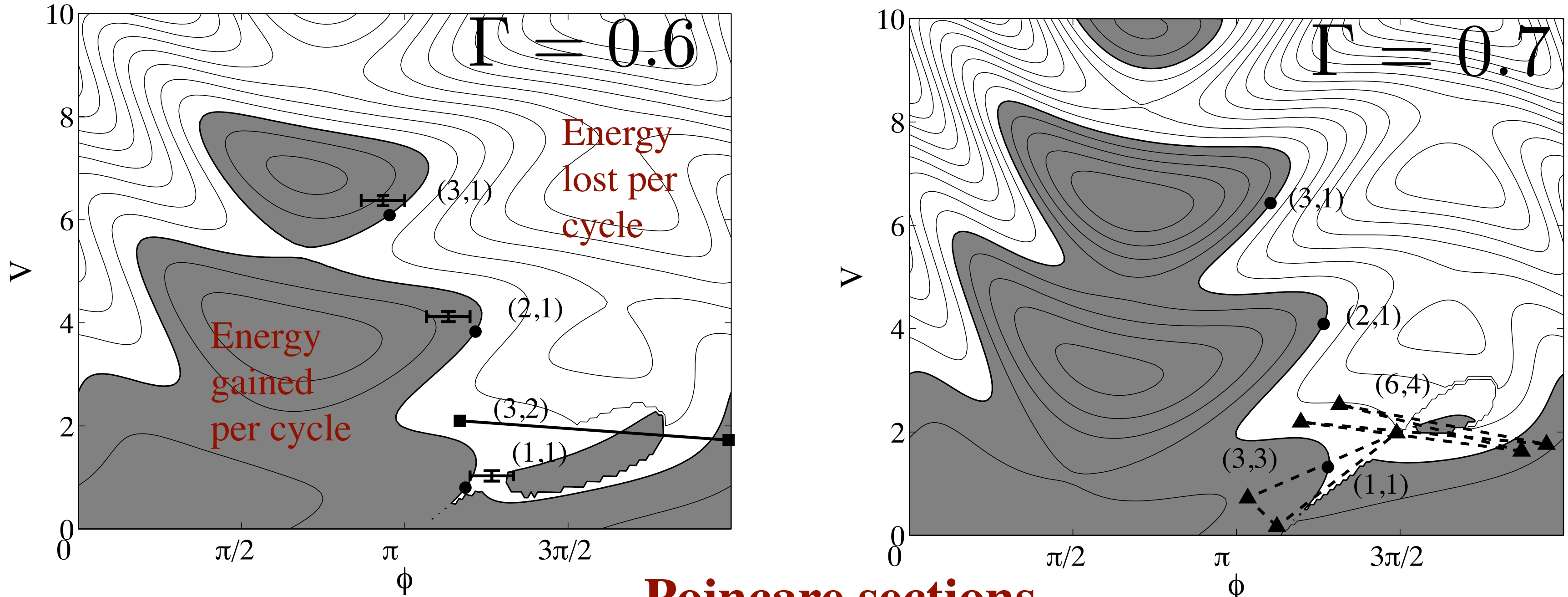
Period-doubling transition from mode (1,1) to (2,2): $f = 33\ Hz\ ,\ \Gamma = 1.1\ g$



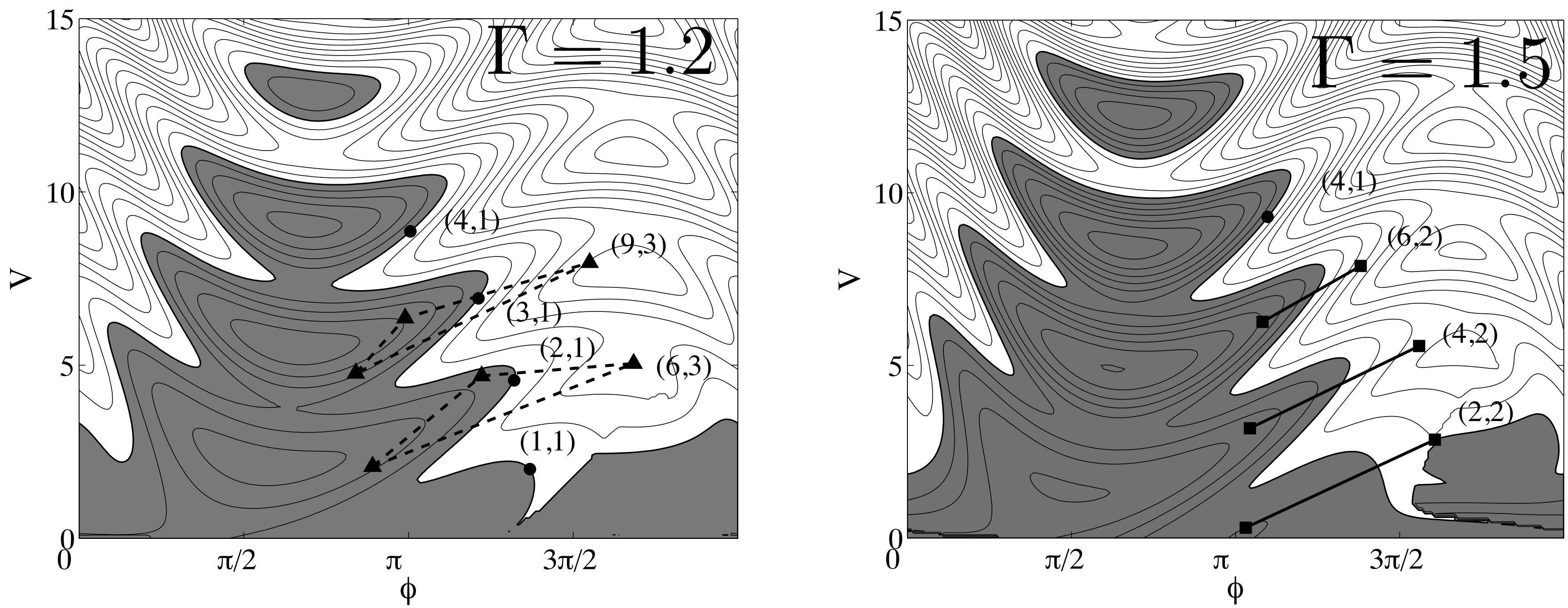
Chaotic solution at $f = 33\ Hz\ ,\ \Gamma = 1.2\ g$

Complex periodic modes



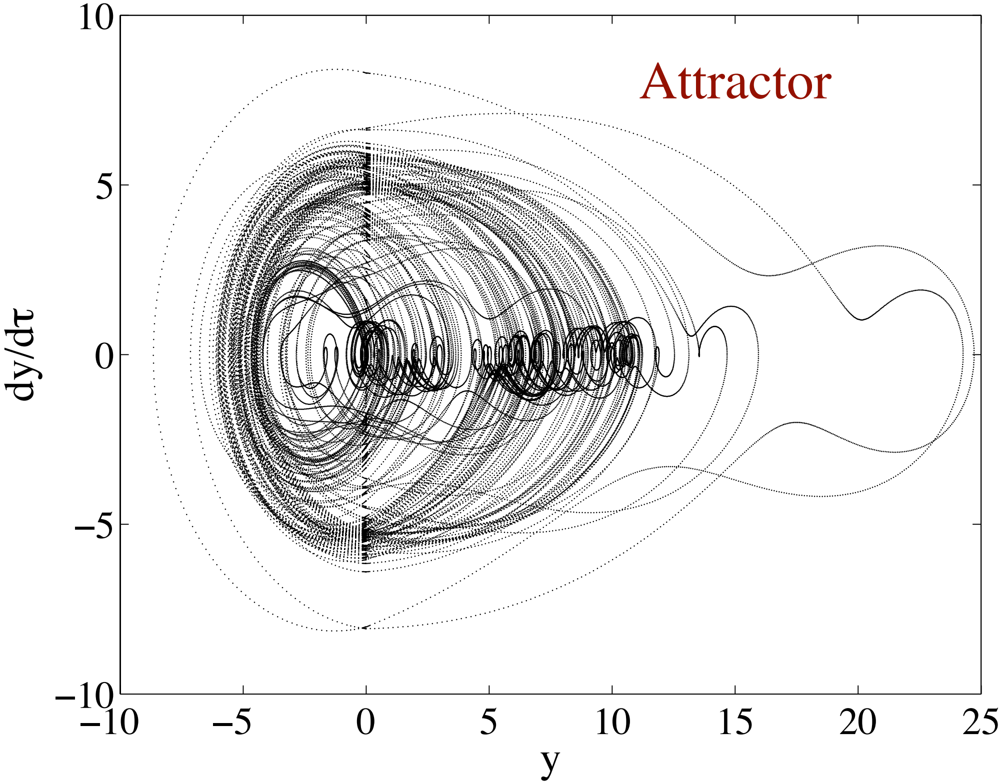


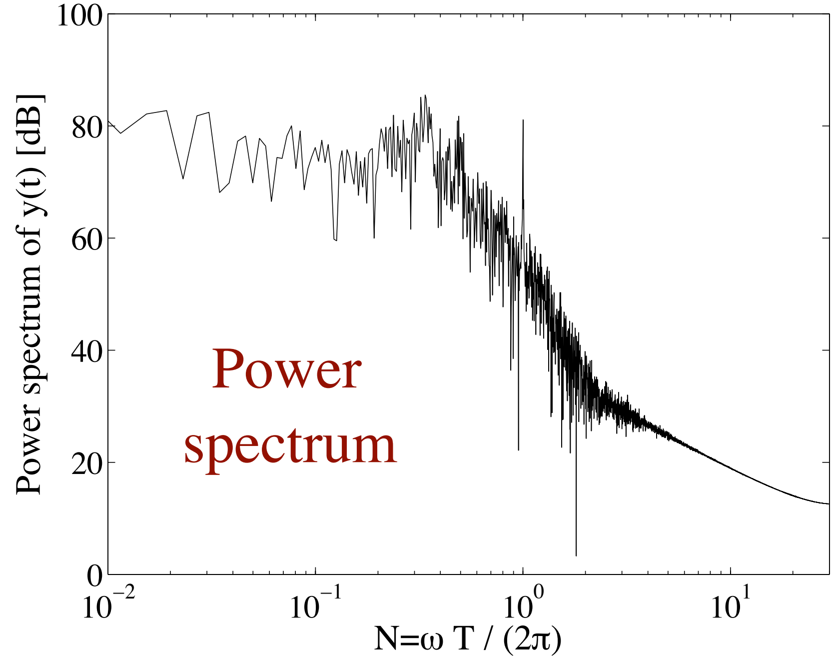
Poincare sections

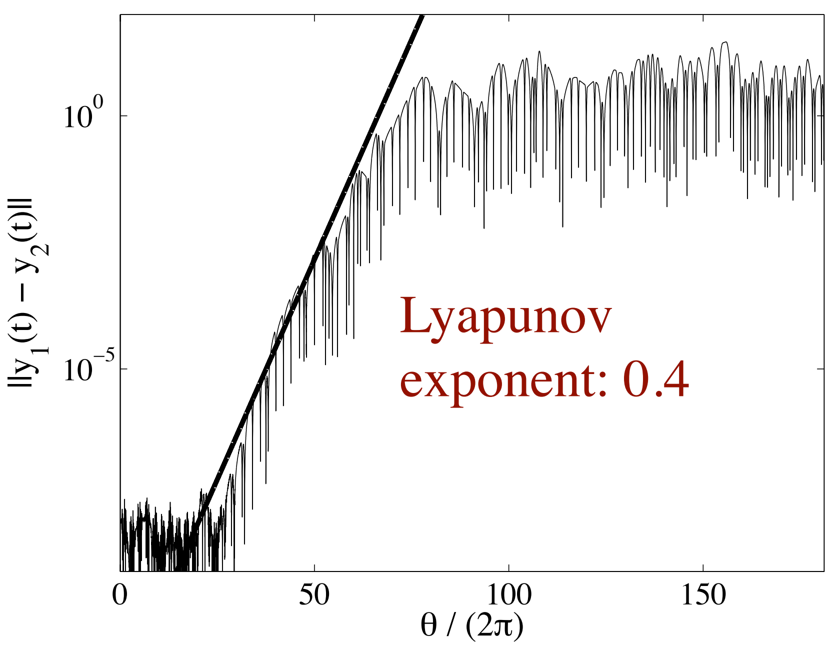


CHAOS at $\Gamma = 1.82$

$\omega = 1.1$

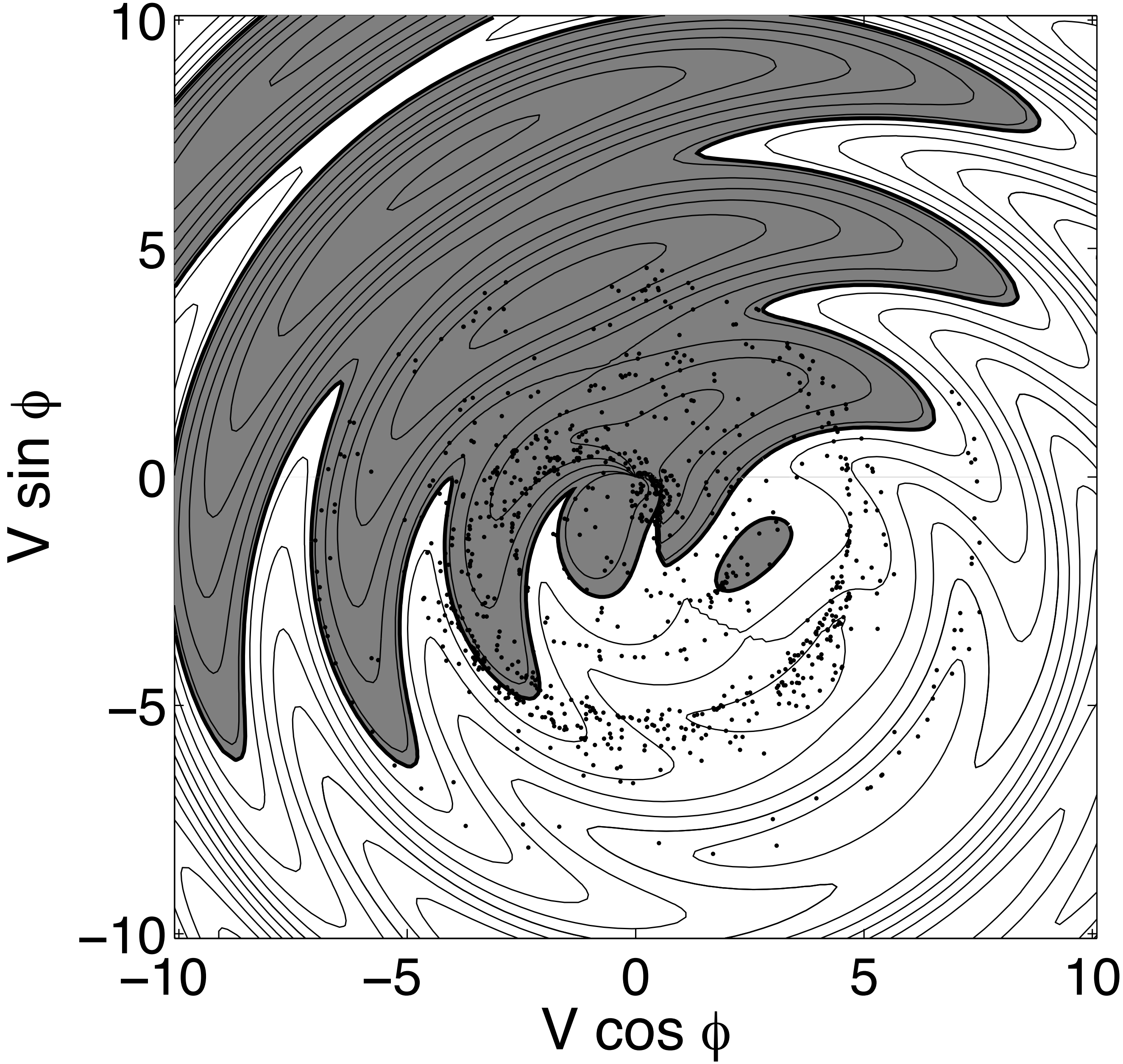






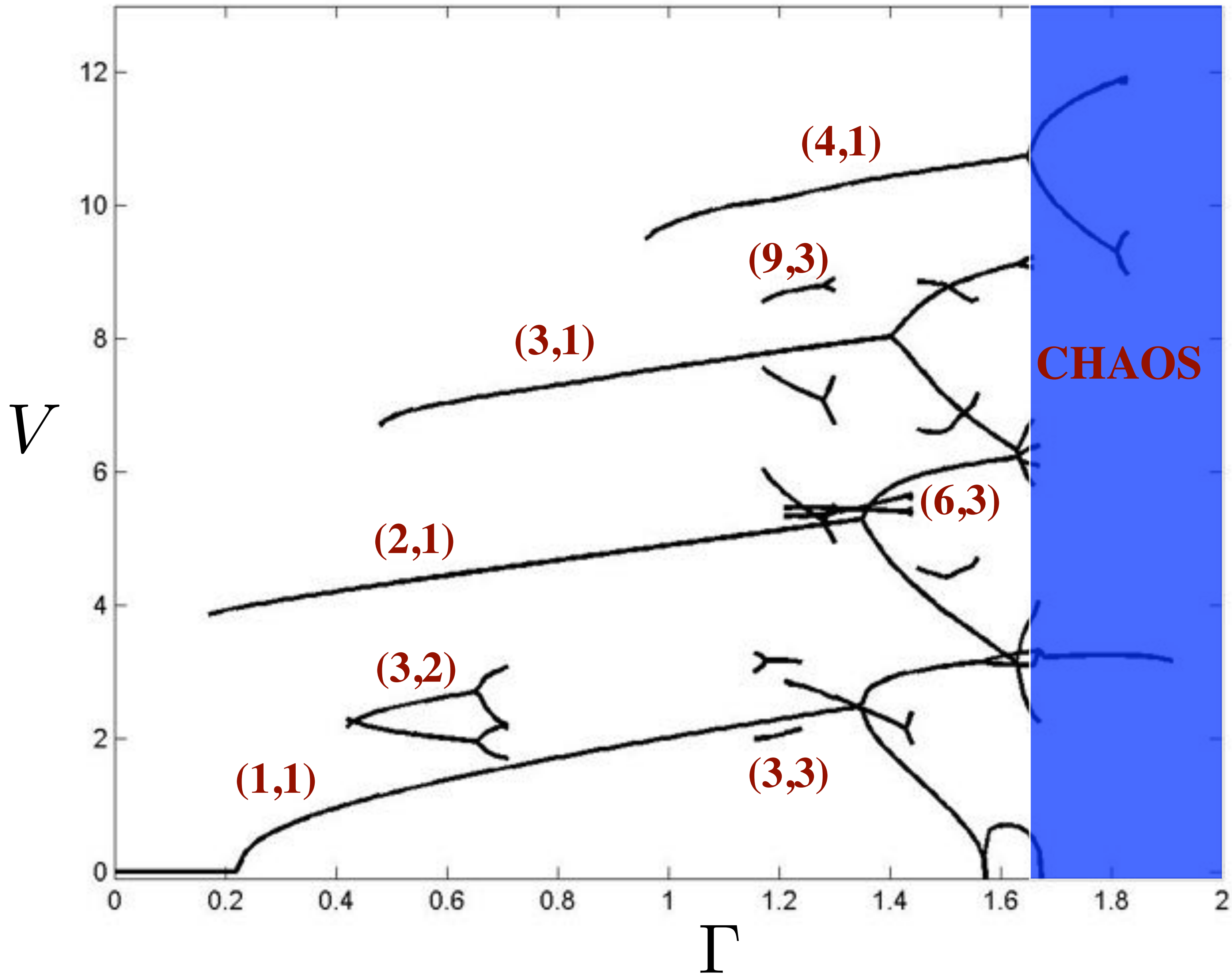
Poincare section of a chaotic solution

$\Gamma = 1.82$
$\omega = 1.1$

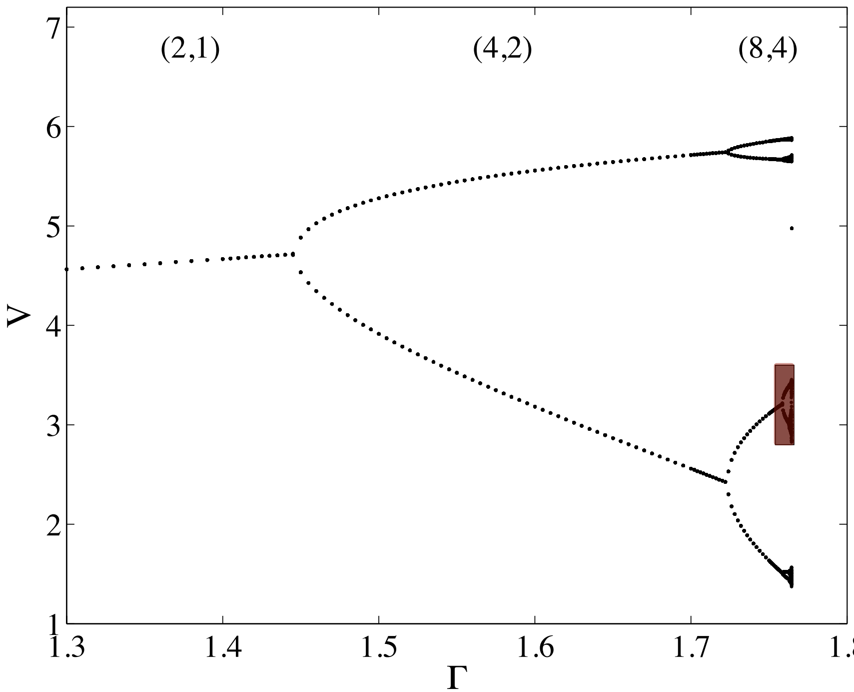


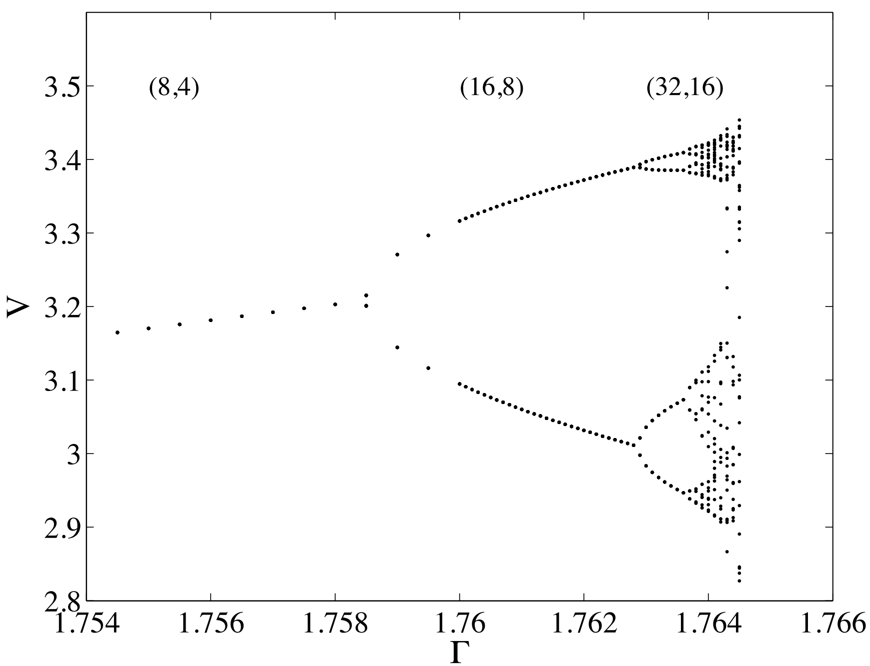
The bifurcation diagram

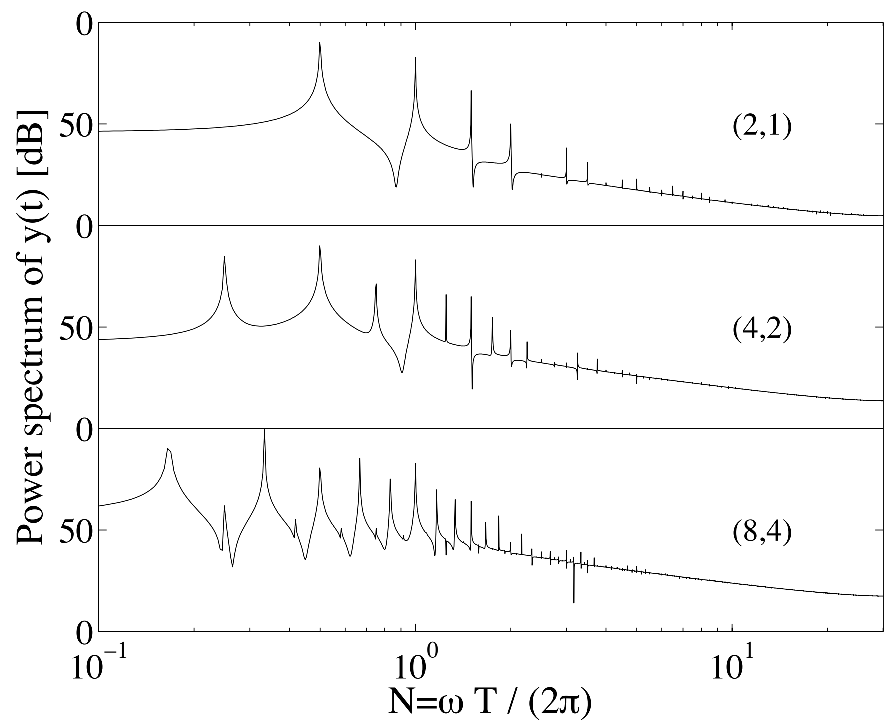
Bifurcation diagram at $\omega = 1.1$



Period-doubling transition to Chaos



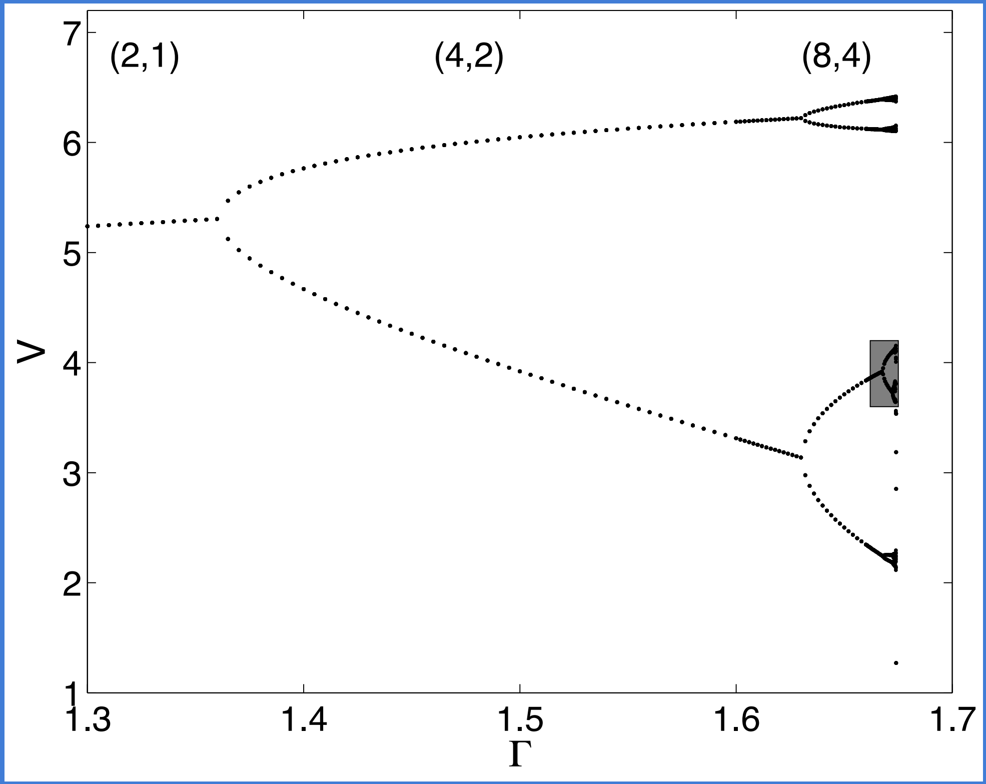




Feigenbaum numbers

- period doubling at Γ values: 1.361, 1.631, 1.6679, 1.67244, 1.67319, 1.673349, 1.673380,...

$$\delta_i = \frac{\Gamma_{i+1} - \Gamma_i}{\Gamma_{i+2} - \Gamma_{i+1}}$$



- the first terms of the δ suite:

  7.3, 8.1, 6.0, 4.7, 5.1, ...

- converging to the Feigenbaum number, 4.6692... ?
- ... and the Golden Mean?

CONCLUSIONS

- have deduced break-through criterion for drop striking a soap film
- in the bouncing regime, the soap film behaves like a linear spring
- simple We-dependence of coefficient of restitution allows for accurate model of dissipation
- bouncing dynamics described (nearly!) exactly by a 2nd order ODE
- simple model captures simple periodic modes quantitatively, complex periodic and chaotic modes qualitatively
- bifurcation diagram reveals period-doubling transitions to Chaos

BIG PICTURE

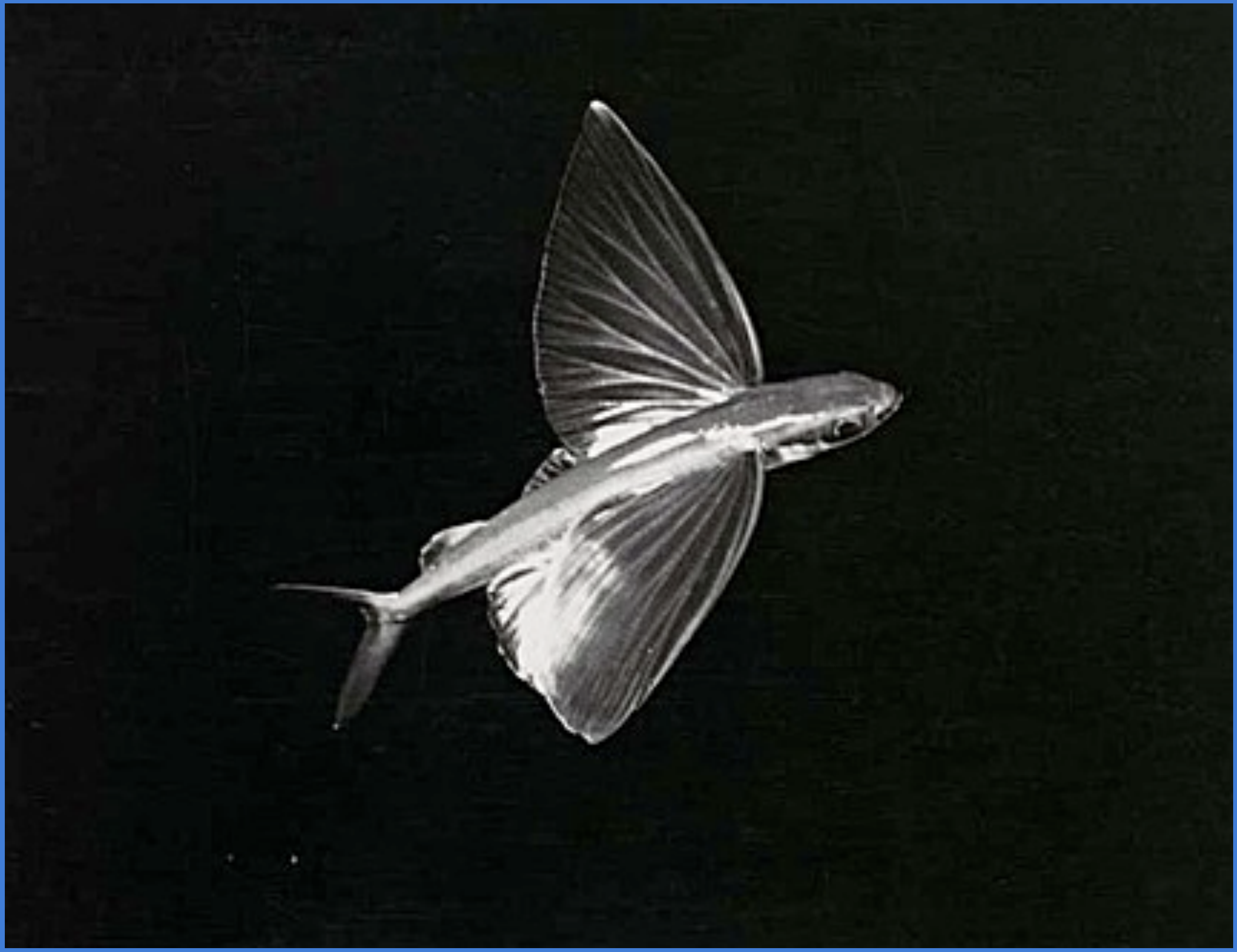
- **among the simplest chaotic fluid systems yet explored**
- first step towards modeling bouncing drops

Biological application ?

Postulate: every problem you work on has a biological application

Even here?

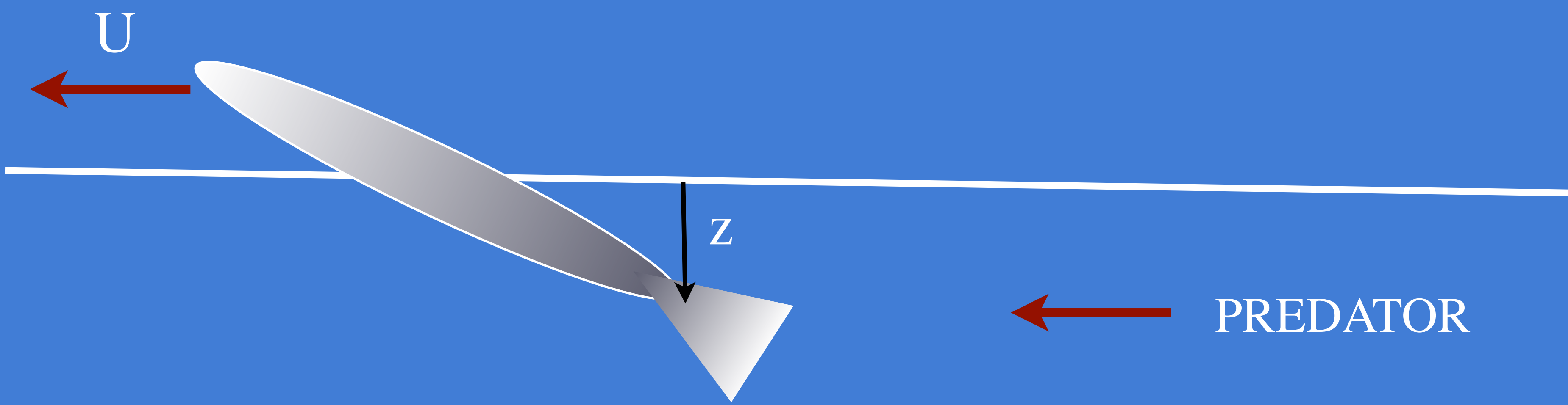
Biological application



Doc Edgerton’s “Flying fish”

On the escape strategies of flying fish

Lift force: $F = kz\ H(z)$ where $k \sim \rho\, U^2 w$



- if it swims through a periodic wave field at uniform speed, its dynamics will be precisely analogous to that of a drop on a driven film

- a chaotic trajectory would surely assist its escape