HQA Lecture 8: Water waves

Why can one only surf near the beach?

For a localized source, what relates the frequency to the wavelength?

Big splash

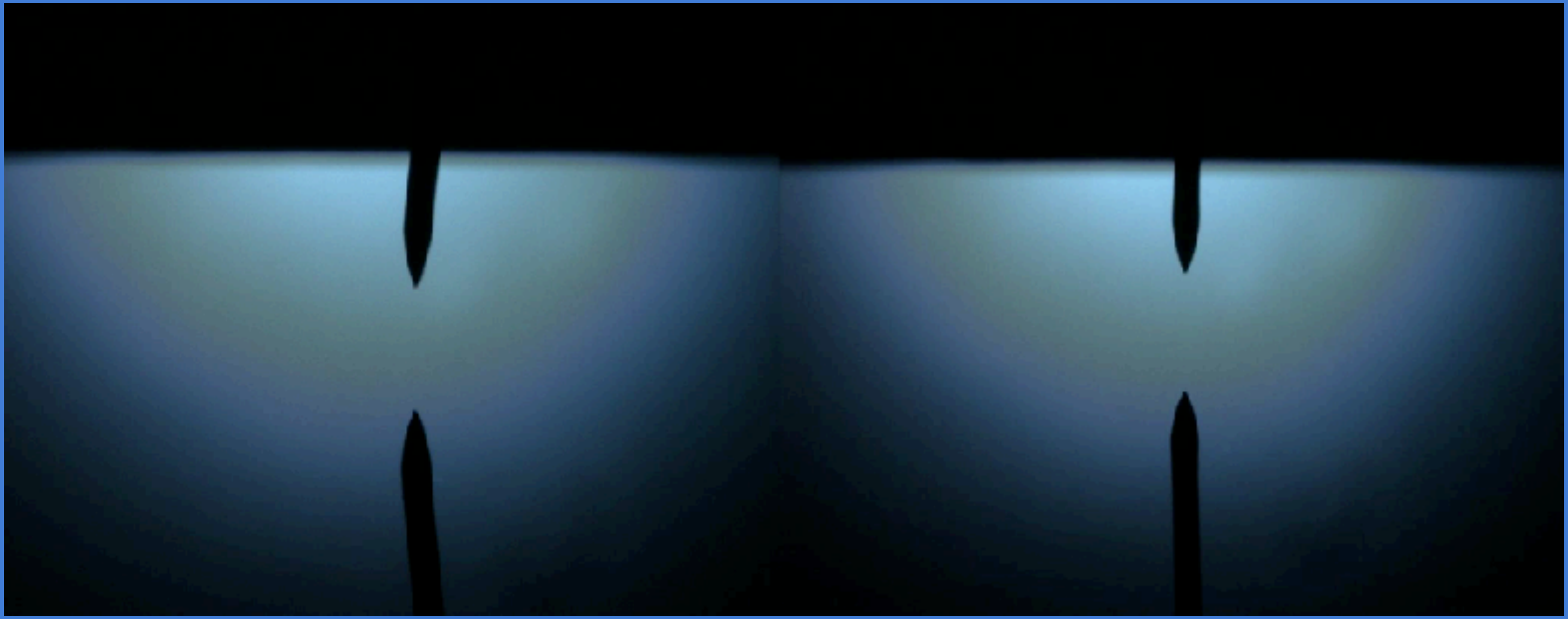
Small splash

Disturbance of forced and unforced interfaces

- withdraw millimetric needle from interface

No forcing

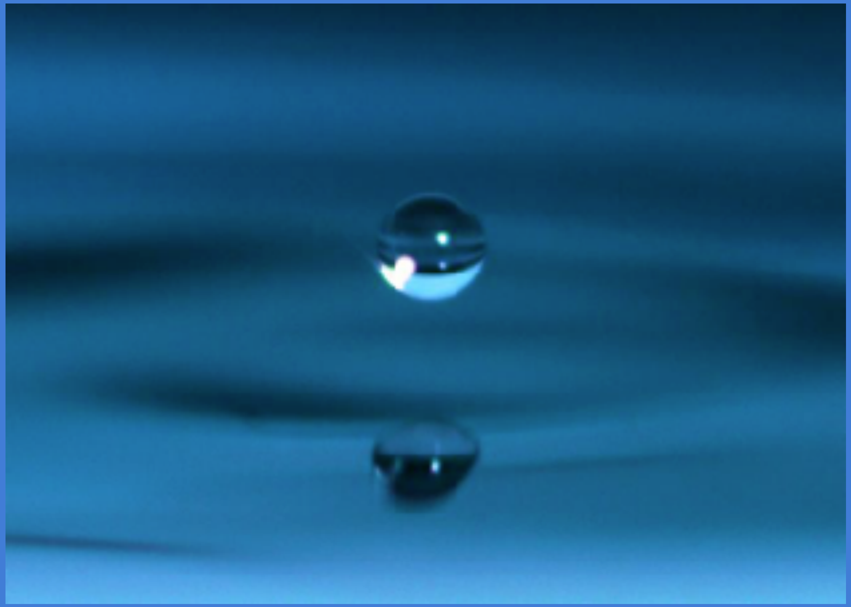
Faraday forcing



- waves quickly disperse
- field of Faraday waves persist

Why are these wave forms so different?

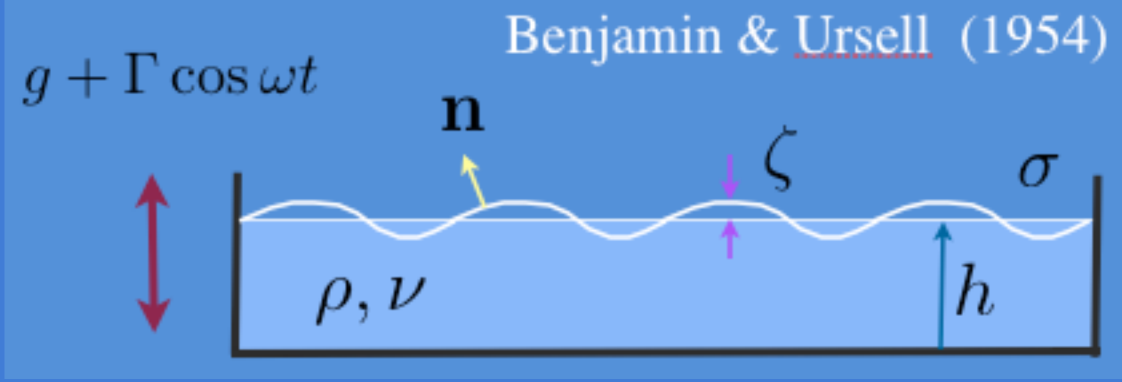
The wave fields of bouncing and walking droplets



- drops initially bounce at the forcing period: waves quickly dissipated
- walking requires period doubling, so that drop bouncing period becomes commensurate with the Faraday wave frequency
- fluid becomes a damped oscillator forced at resonance by the drop

Unforced surface waves

$$\Gamma = 0$$



Dispersion relation

$$\omega^2 = \left(\frac{\sigma}{\rho} k^3 + g\, k \right) \tanh kh$$

- wave form depends on kh and $B_o = \dfrac{\rho g}{\sigma k^2} = \dfrac{\text{gravity}}{\text{capillarity}}$

Deep water: $\tanh kh \sim 1$

Gravity waves $(B_o \gg 1)$

$$\omega = \sqrt{gk}$$

Capillary waves $(B_o \ll 1)$

$$\omega = \left(\frac{\sigma}{\rho} \right)^{1/2} k^{3/2}$$

Shallow water: $\tanh kh \sim kh$

Gravity waves $(B_o \gg 1)$

$$\omega = \sqrt{gh}\; k$$

Capillary waves $(B_o \ll 1)$

$$\omega = \left(\frac{\sigma h}{\rho} \right)^{1/2} k^2$$

Big splash

Small splash