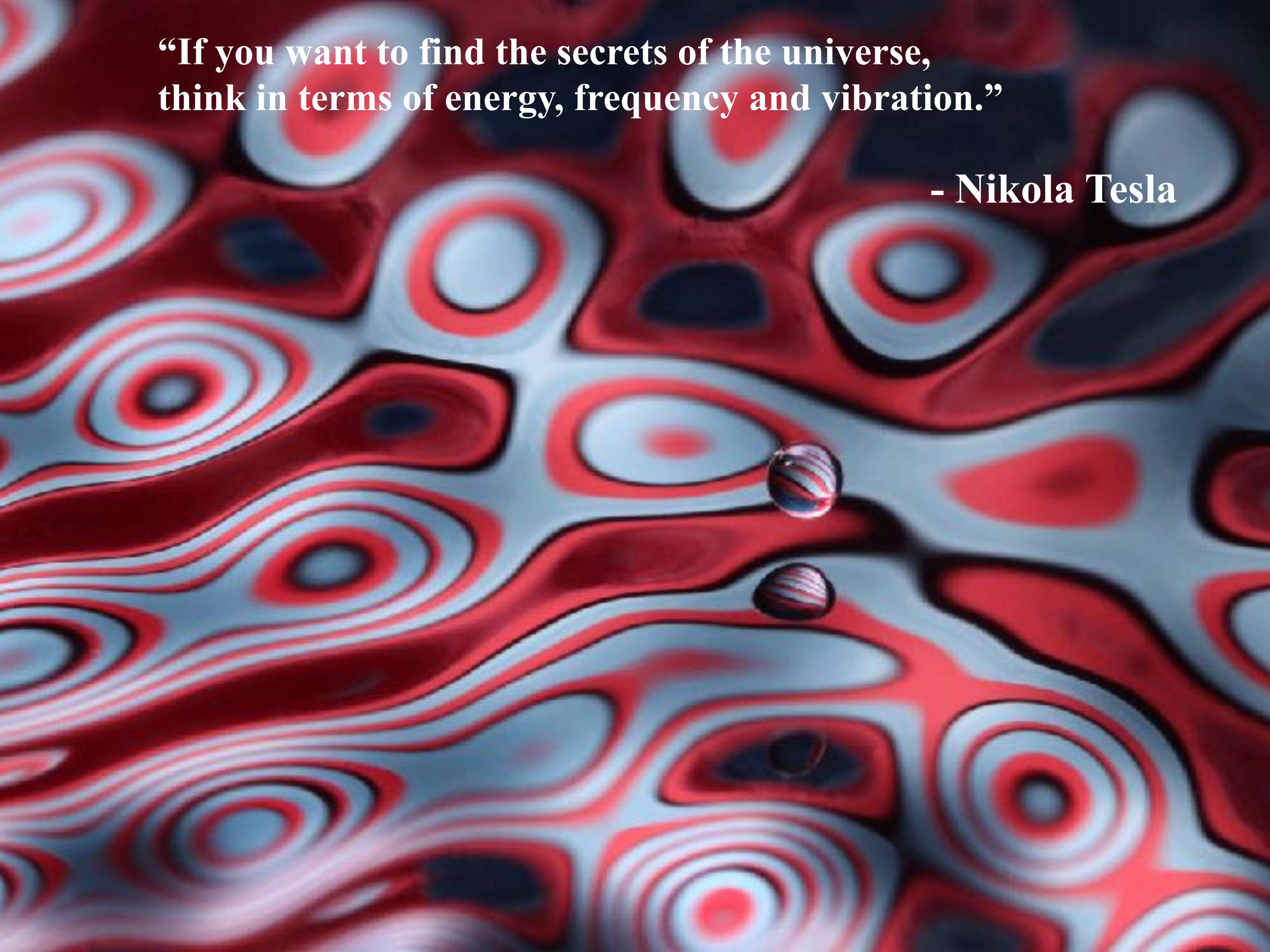
18.S996 Hydrodynamic quantum analogs

Lecture 11: Finale: the theory of walking droplets


"If you want to find the secrets of the universe,
think in terms of energy, frequency and vibration."
- Nikola Tesla

Molacek: model summary

Vertical dynamics

$$m_D \ddot{Z} = -m_D g \left(1 + \Gamma \sin \omega t\right) \quad \text{when} \quad Z \geq 0$$

$$m_D \ddot{Z} \left(1 + \frac{c_3}{Q^2(Z)}\right) + \frac{\mu R_0 c_2}{Q(Z)} \dot{Z} + \frac{3\sigma Z}{2Q(Z)} = -m_D g \left(1 + \Gamma \sin \omega t\right) \quad \text{otherwise}$$

Standing wave evolution

$$h(X, \tau) = \sum_{n=1}^{N} h_0(X, X_n, \tau, \tau_n)$$

$$h_0(X, X_n; \tau, \tau_n) \approx \frac{4\sqrt{2\pi}}{3} \frac{k_C^2 k_F \mathrm{Oh}_e^{1/2}}{3k_F^2 + \mathrm{Bo}} \left[\int F(u) \sin \frac{\Omega u}{2} \mathrm{d}u\right] \frac{H(\tau)}{\sqrt{\tau - \tau_n}} \exp\left\{(\Gamma/\Gamma_F - 1) \frac{\tau - \tau_n}{\tau_d}\right\} J_0\left(k_C(X - X_n)\right)$$

$$X_n = \int_{\tau_c} F(u) X(u) \mathrm{d}u \Big/ \int_{\tau_c} F(u) \mathrm{d}u \quad , \quad \tau_n = \int_{\tau_c} F(u) u \mathrm{d}u \Big/ \int_{\tau_c} F(u) \mathrm{d}u$$

Horizontal dynamics

$$m_D \ddot{X} + \left[C F_N \sqrt{\rho R_0 / \sigma} + 6\pi \mu_a R_0\right] \dot{X} = -\frac{\partial h}{\partial X} F_N$$

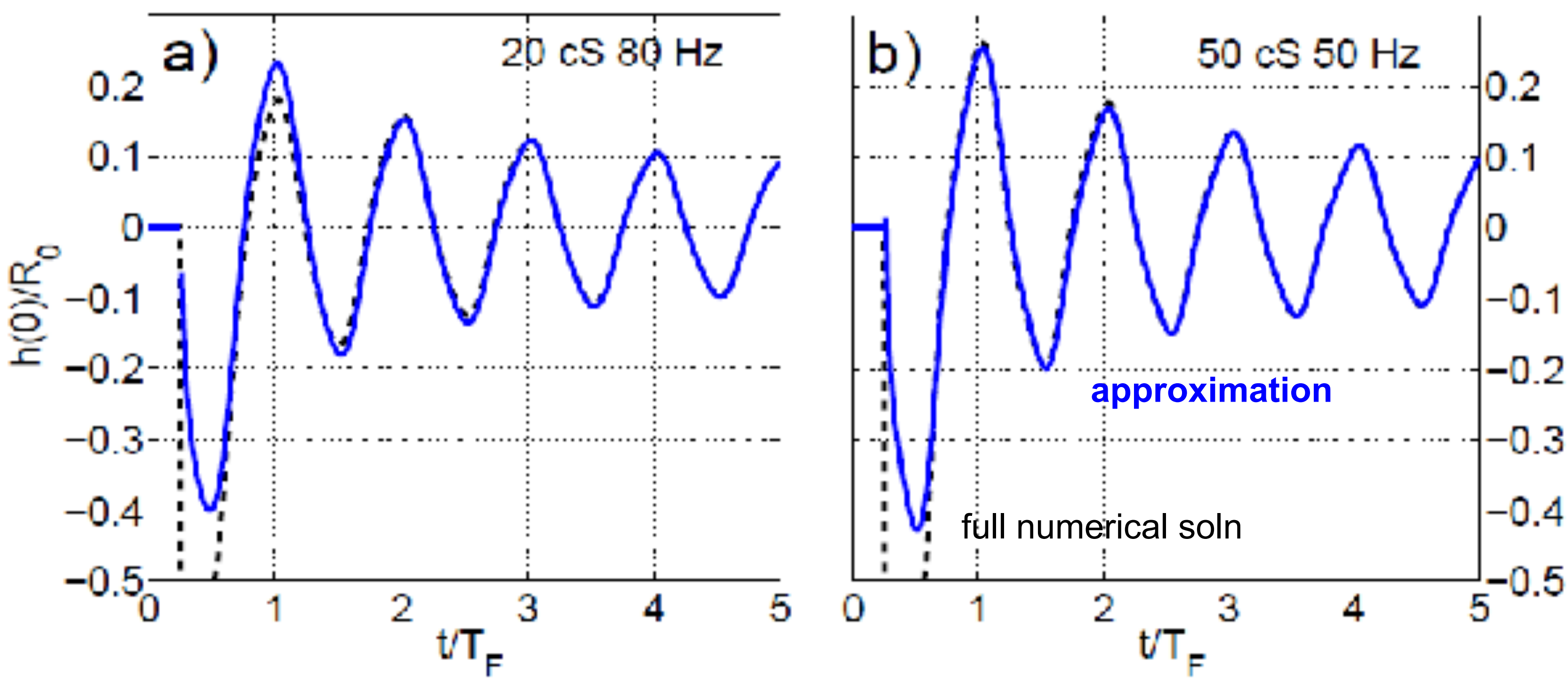
contact drag — air drag — sloped interface

Standing wave field

- standing wave created by a single drop impact:

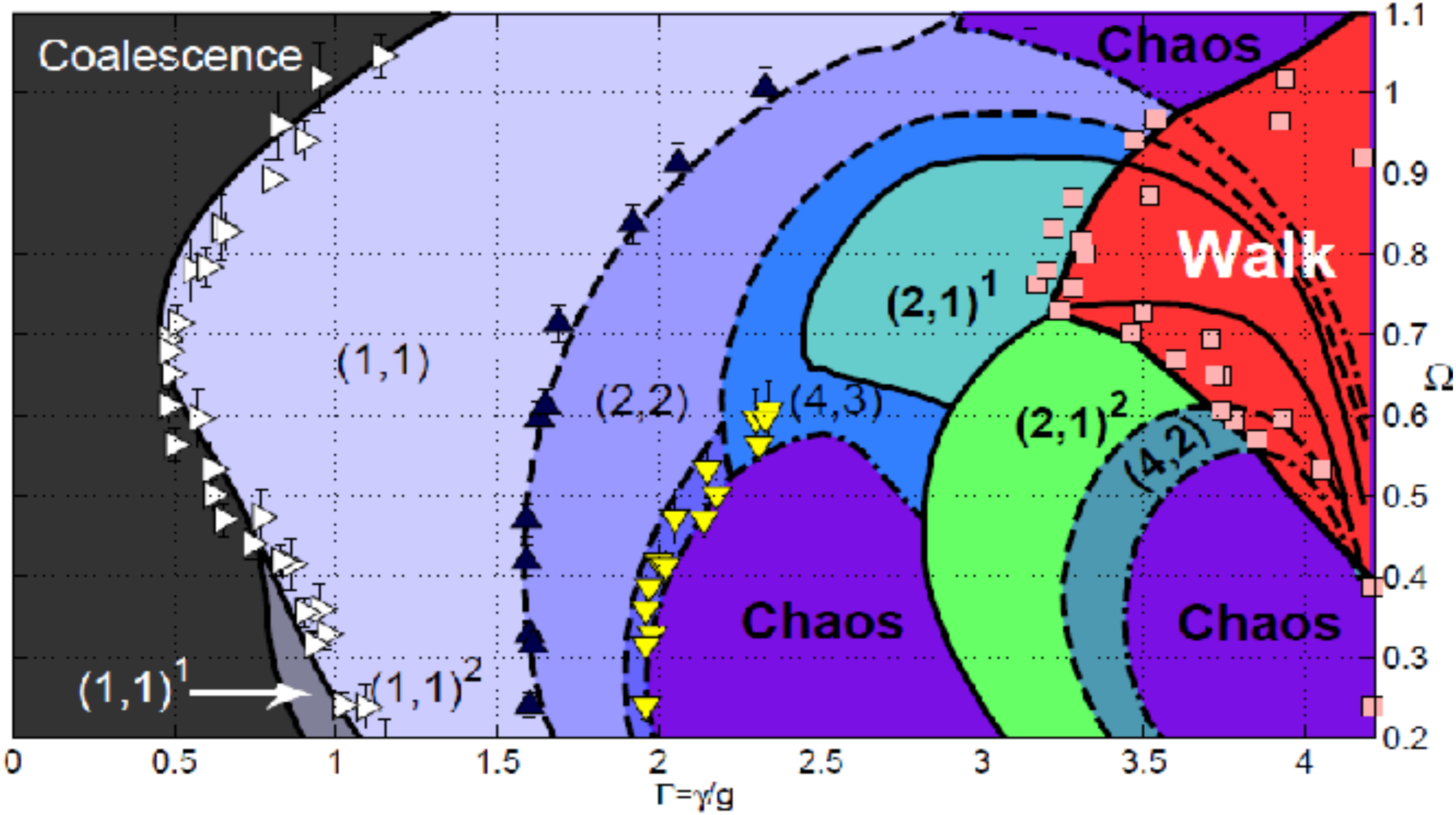
$$h(r,\tau) \approx \frac{4\sqrt{2\pi}}{3\sqrt{\tau}} \frac{k_C^2 k_F \Omega h_e^{1/2}}{3k_F^2 + \mathrm{Bo}} \left[\int F(u) \sin\frac{\Omega u}{2} \mathrm{d}u\right] \cos\frac{\Omega \tau}{2} \exp\left\{\left(\frac{\Gamma}{\Gamma_F} - 1\right)\frac{\tau}{\tau_d}\right\} J_0(k_C r) \,.$$

- most unstable mode $k_C \approx k_F$ where $k_F^3 + \mathrm{Bo} \cdot k_F = \frac{1}{4}\Omega^2$

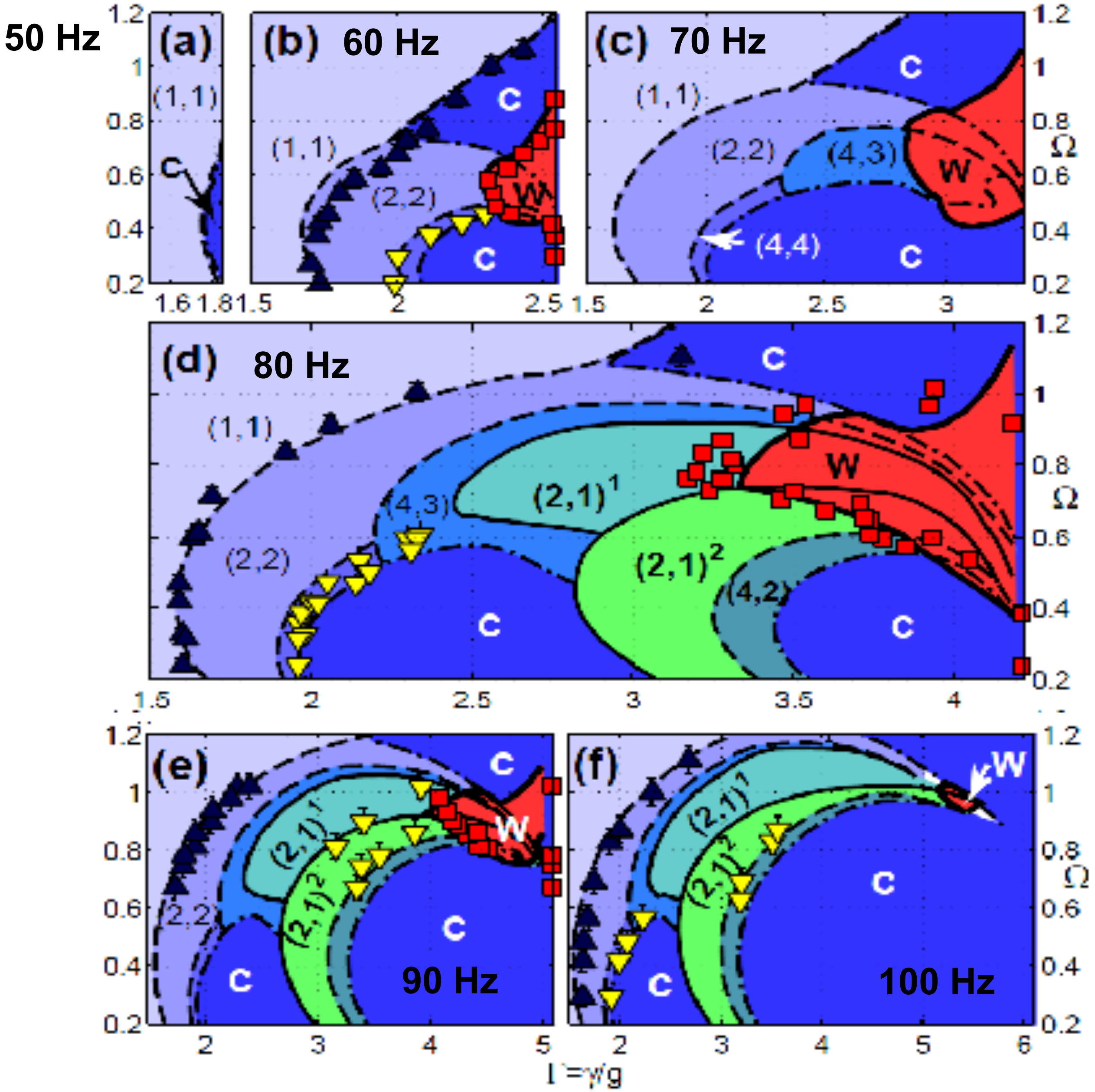


Predicted regime diagrams

20 cS, 80 Hz



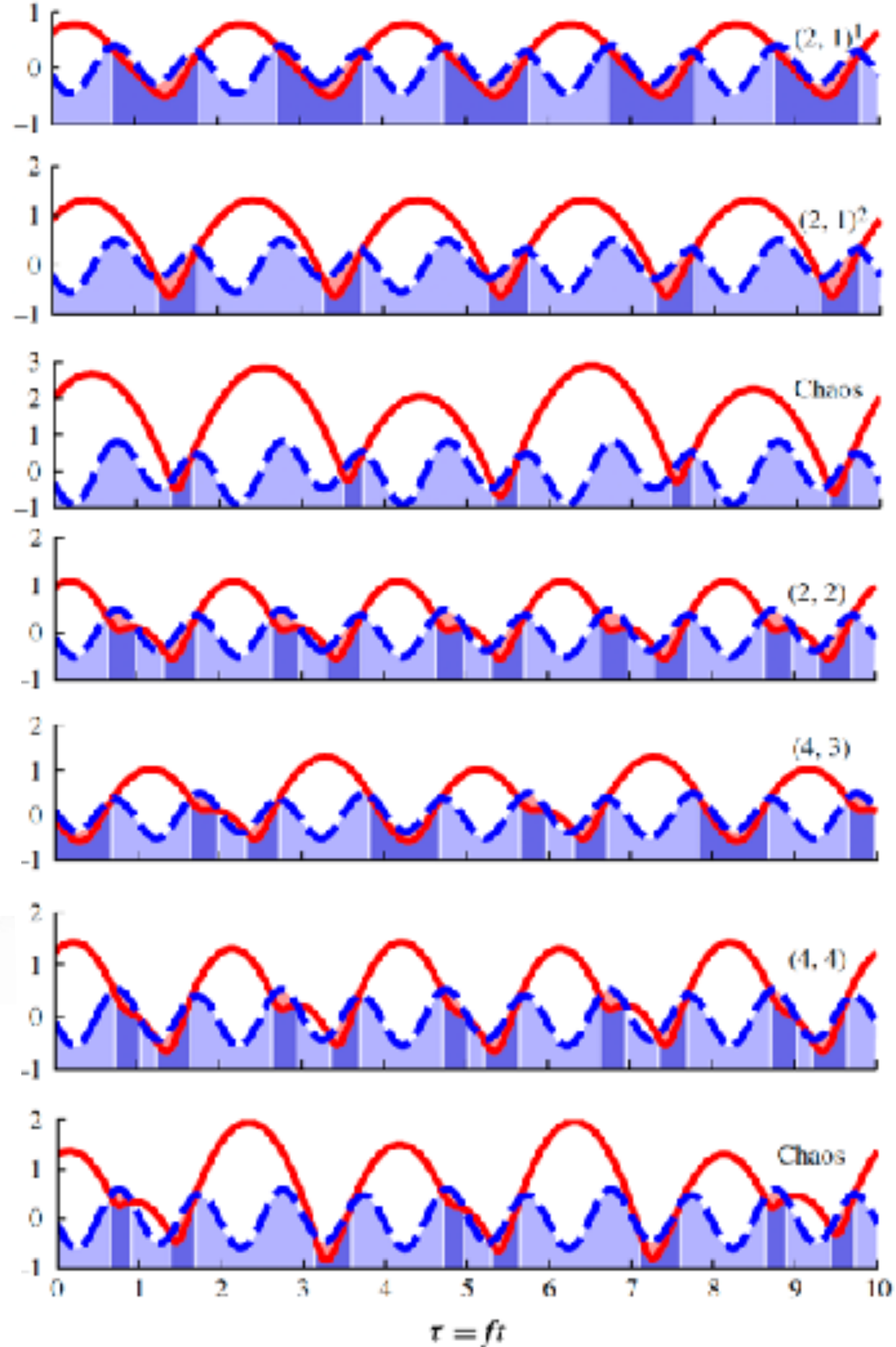
Predicted regime diagrams for 20 cS



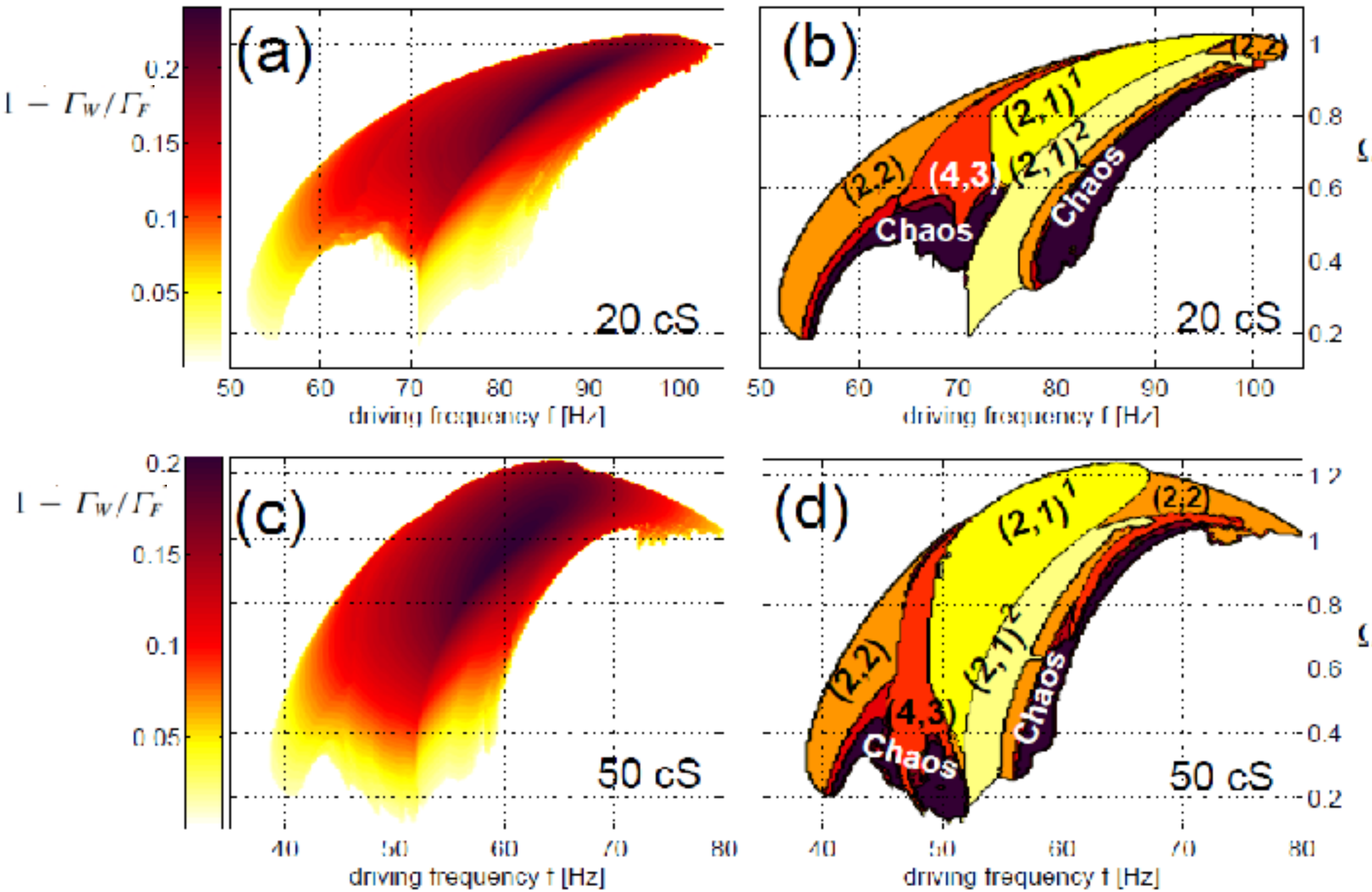
Walking modes

arising at various f for 20 cS

$$Z = z/R_0$$



The walking regime



PHYSICS OF FLUIDS **25**, 082002 (2013)

Exotic states of bouncing and walking droplets

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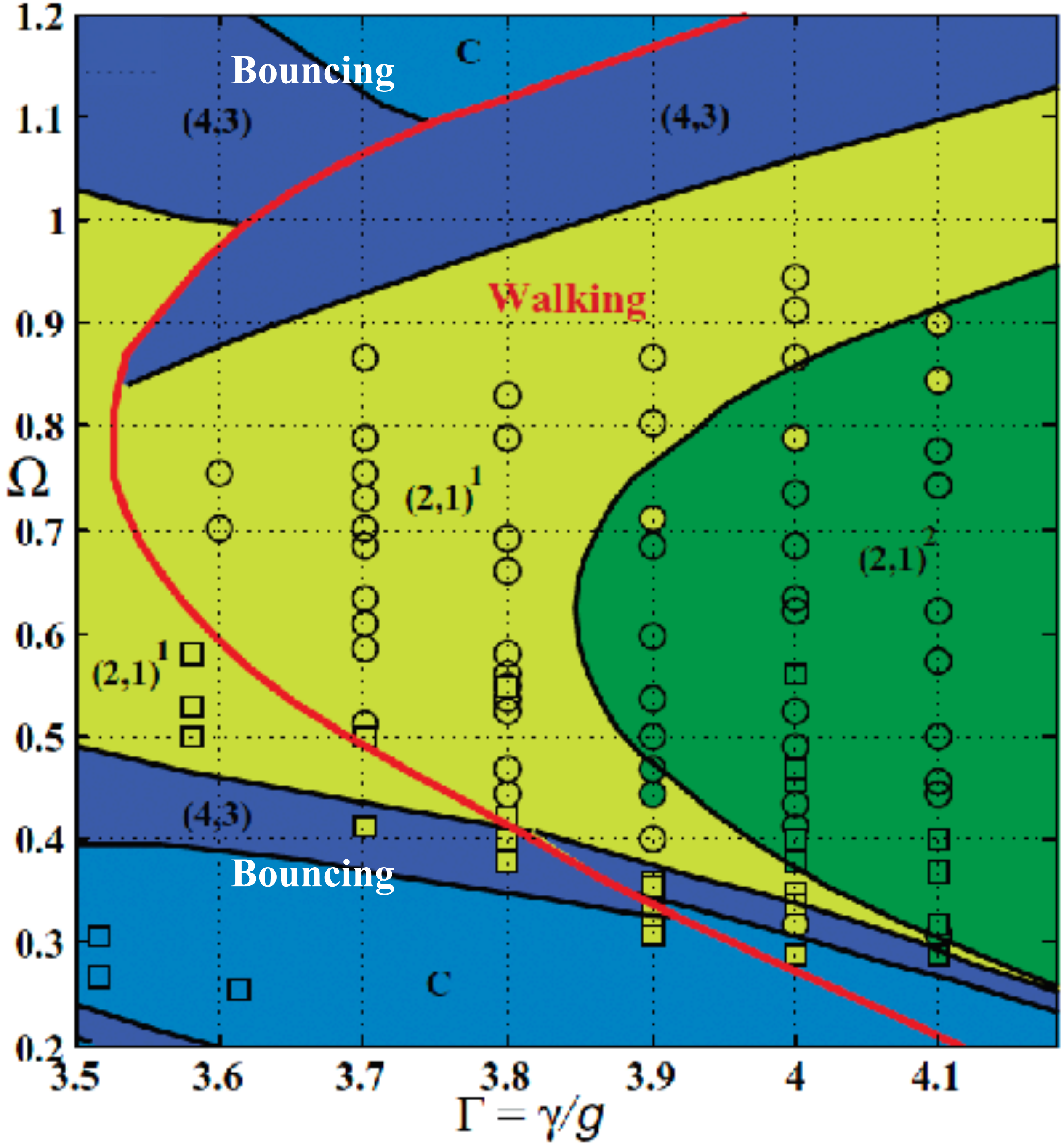
Regime diagram

(50cS, 50Hz)

$$\Omega = 2\pi f(\rho R_0^3/\sigma)^{1/2}$$

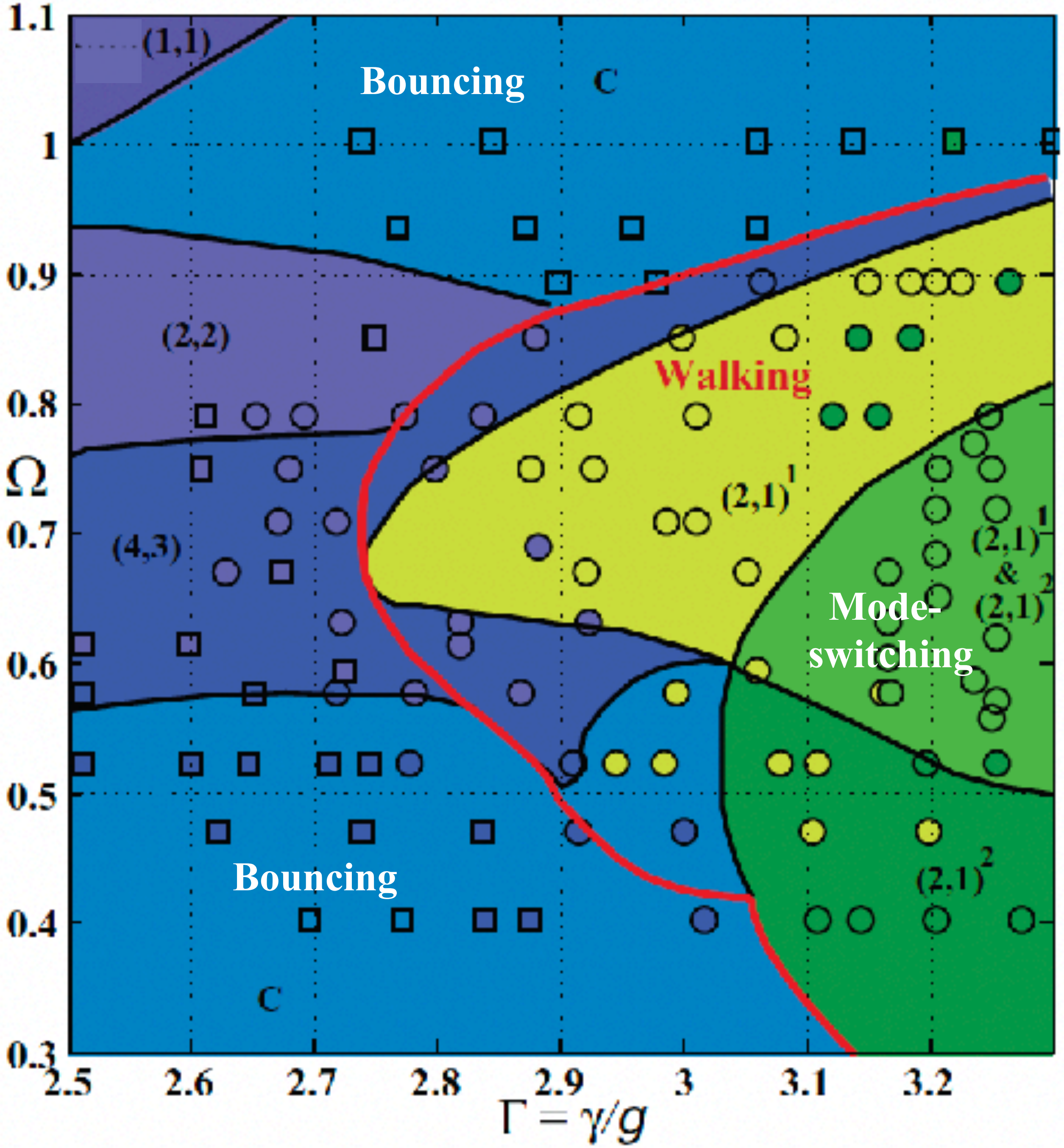


- *Wind-Willassen et al. (2013)*

Regime diagram

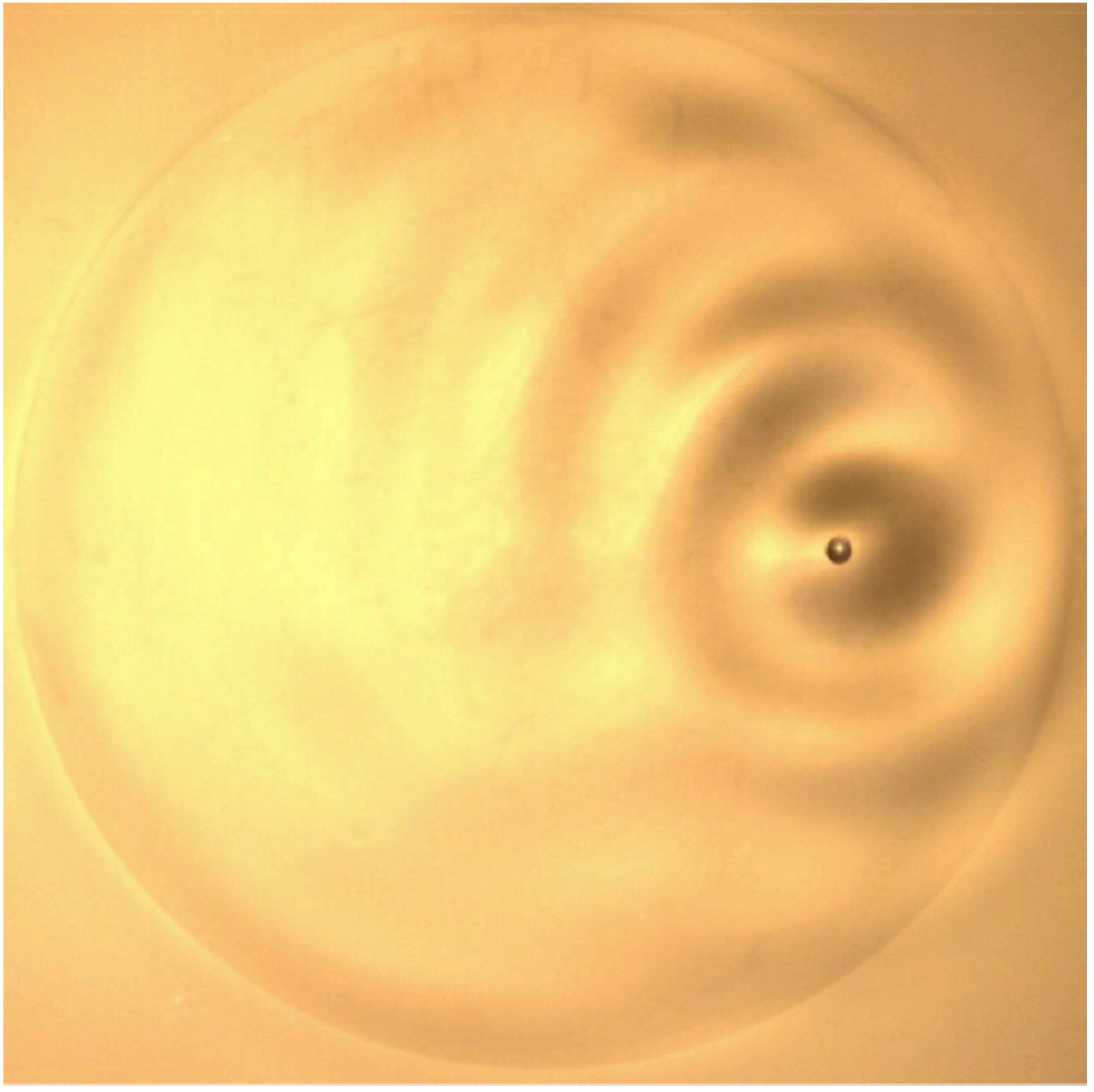
(20cS, 70Hz)

$$\Omega = 2\pi f(\rho R_0^3/\sigma)^{1/2}$$



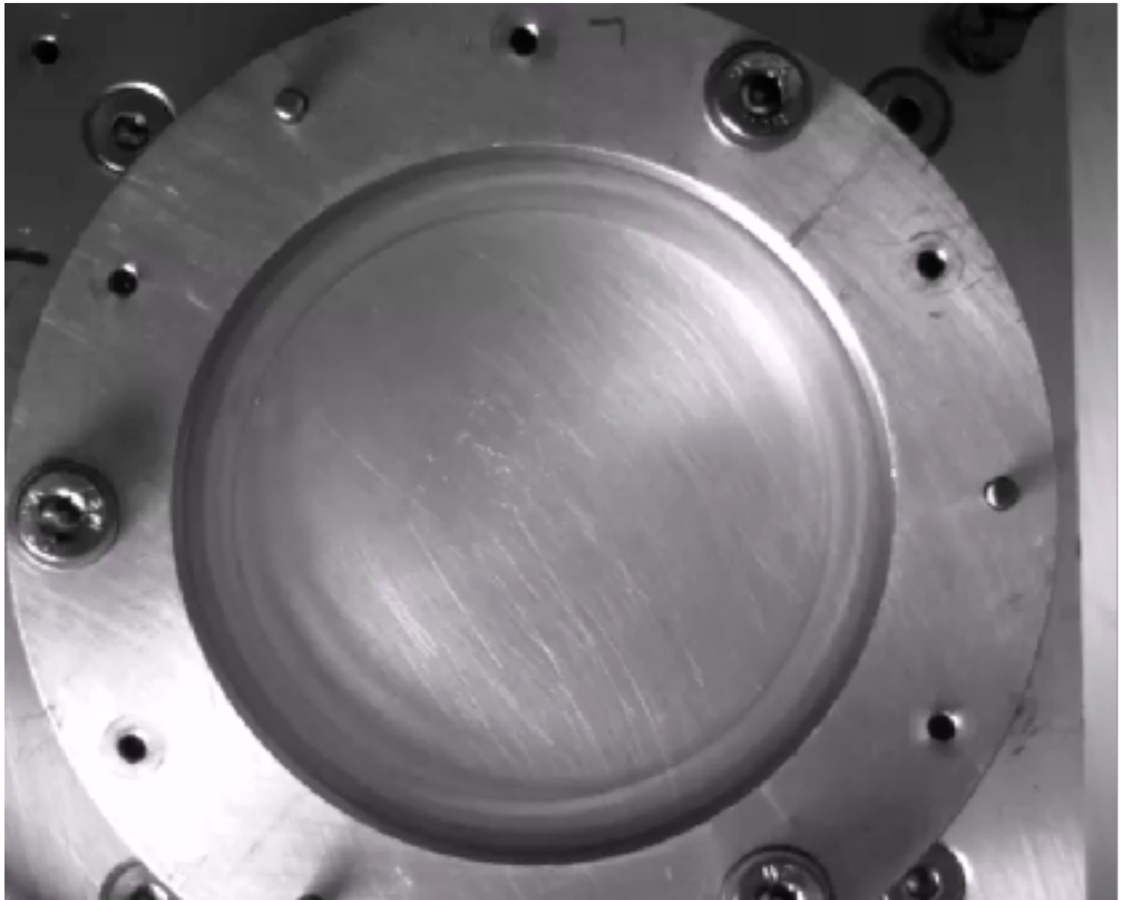
- *Wind-Willassen et al. (2013)*

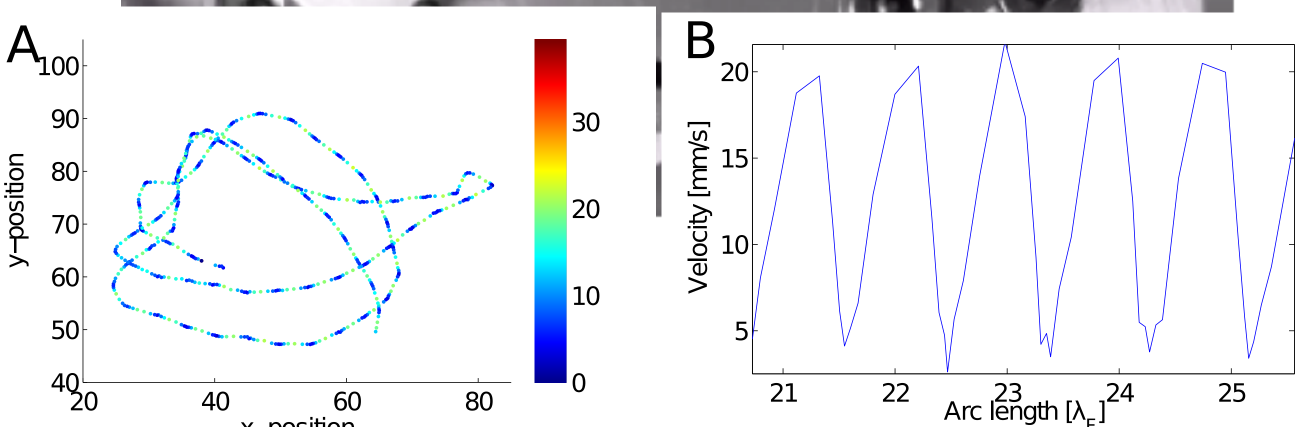
A walker in a mixed, mode-switching state



A walker in a mixed, mode-switching state

(20cS, 70Hz)





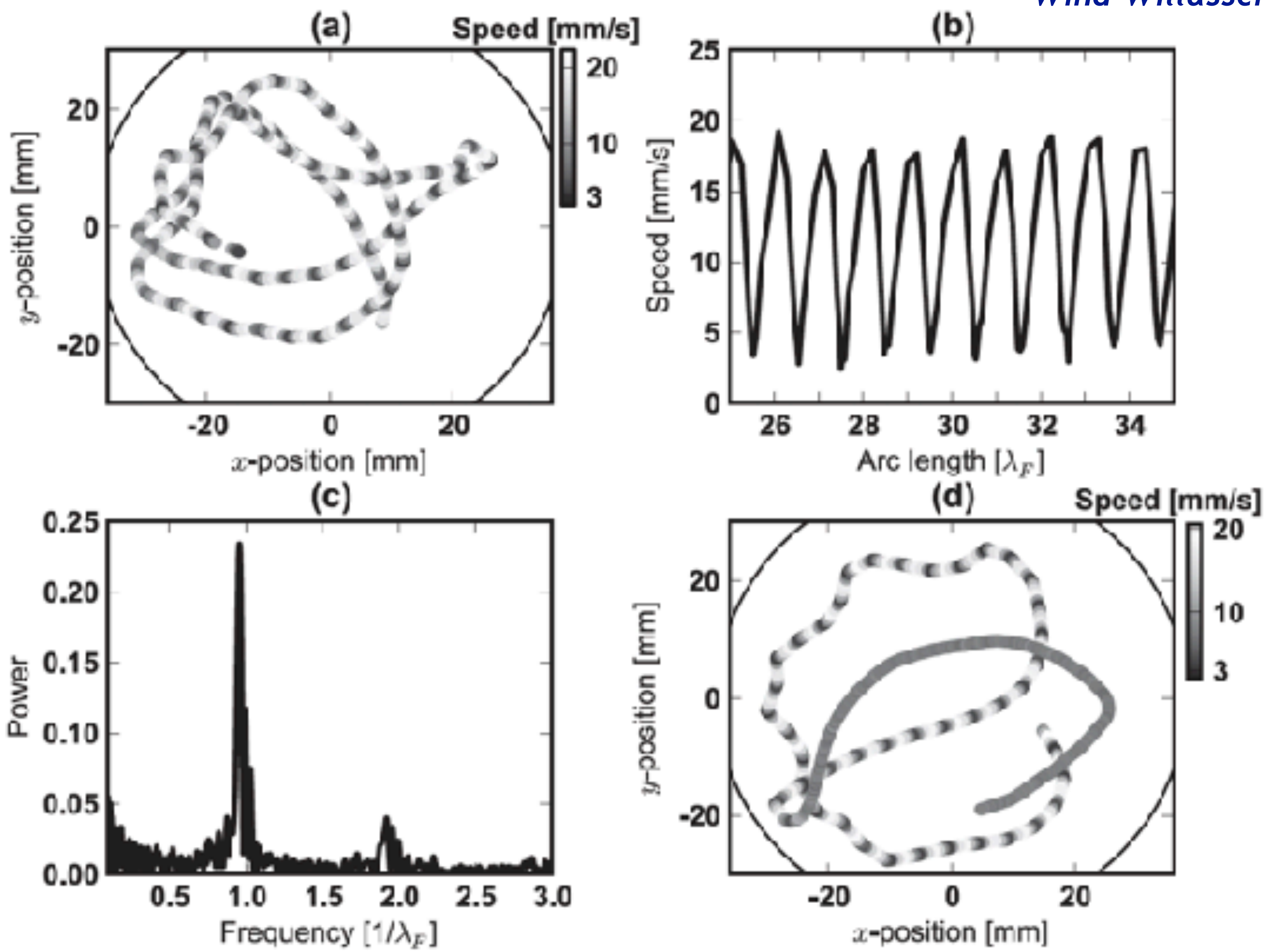
$(2,1)^2$

$(2,1)^1$

A walker in a mixed, mode-switching state

(20cS, 70Hz)

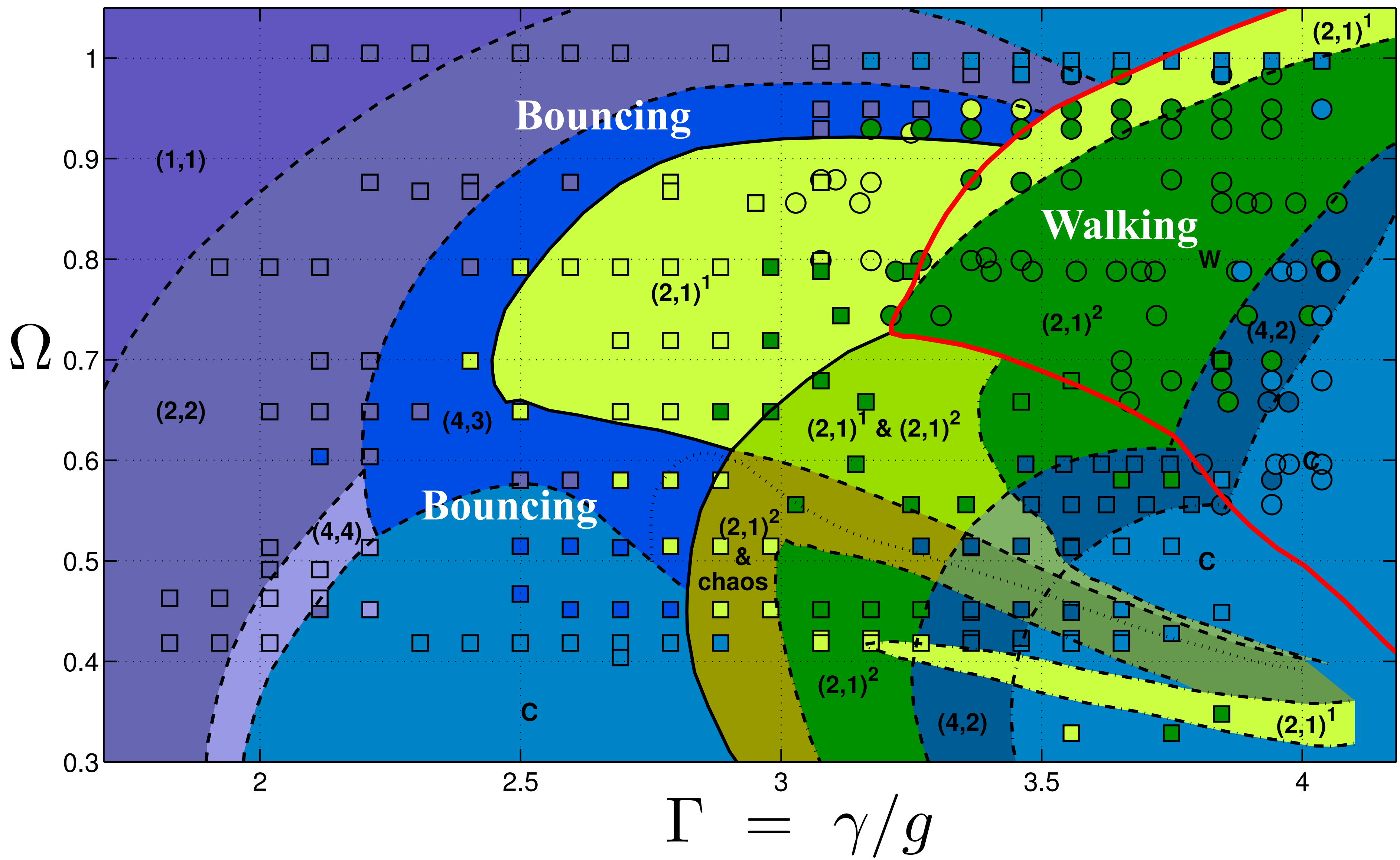
- *Wind-Willassen et al. (2013)*



- gives rise to in-line speed oscillations with the Faraday wavelength
- one of the three paradigms for the emergence of quantum-like statistics

Regime diagram (20cS, 80Hz)

$$\Omega = 2\pi f(\rho R_0^3/\sigma)^{1/2}$$



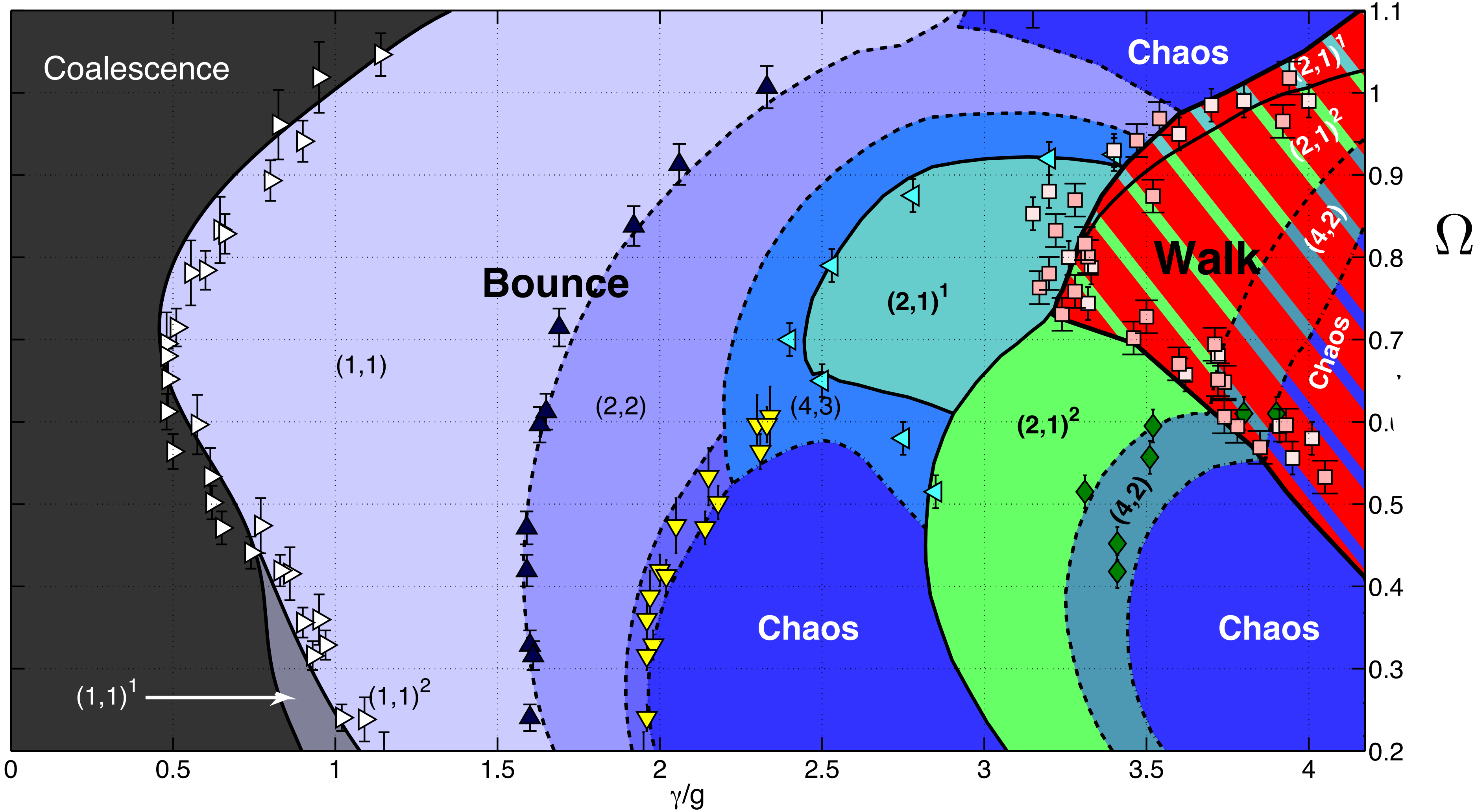
- *Wind-Willassen, Molacek, Harris & Bush, Phys. Fluids (2013)*

Molacek model of walking drops

(20cS, 80Hz)

Molacek & Bush (2013ab)

$$\Omega = 2\pi f(\rho R_0^3/\sigma)^{1/2}$$



In (m, n) mode, a drop bounces n times in m forcing periods.

- rationalized bouncing, walking behavior, but is cumbersome

Summary

In (the extremely limited) parameter regime of interest for walkers:

- drop deformation negligible
- interface behaves like a linear spring
- horizontal drag primarily due to momentum transferred to bath
- aerodynamics effects negligible in vertical dynamics, appreciable in horizontal
- drop impact generates Bessel function waves with spatiotemporal damping
- have rationalized regime diagrams, observed walking speeds
- formulation ungainly for non-resonant walkers, involving discrete sums
- time-averaging will yield the analytically tractable **stroboscopic model**