18.S996 Hydrodynamic quantum analogs: Problem Set 1 Due March 13

1. Dimensional analysis

Deduce via dimensional analysis the scaling of the following:

a) The natural frequency of a liquid droplet of mass m in a vacuum, where the additional relevant physical parameter is the surface tension σ.

b) The natural frequency of a microscopic particle of mass m, where the additional relevant physical variables are the speed of light c and Planck's constant $\hbar$.

c) The Planck length and time scales, at which quantum effects become comparable to gravitational effects. The relevant physical variables are $\hbar$, c and Newon's gravitational constant, G.

2. Scaling arguments

Consider a droplet of diameter 0.8mm bouncing on a liquid bath of viscosity 50cS at 50Hz. With each bounce, the drop flies approximately 2 diameters above the bath surface.

a) Use scaling arguments to justify the fact that the walking droplets deform very little during impact.

b) Assume that, during impact, the drop remains spherical. As a relatively crude approximation, assume further that the liquid bath conforms to the drop surface inside the contact area, beyond which the bath surface remains unperturbed. By calculating the rate of increase of surface energy with the drop's intrusion depth, show that the liquid interface acts like a linear spring. Estimate the associated spring constant and droplet contact time.

c) Use scaling arguments to justify the fact that the bouncing droplets don't coalesce. (Use the lubrication analysis for the boundary layer shown in class, plus the results of part b).

d) Compare the three characteristic frequencies in the problem, specifically, the drop's vibrational frequency, the bath's vibrational frequency, and the characteristic frequency of the drop impact (the inverse of the contact time).

3. Degrees of similitude in physical analogies

Choose **one** of the following. To assess the degree of similitude, discuss the relation between the associated mathematical descriptions.

a) Discuss the degree of similitude of the vibrating drop arrangement explored by Steen *et al.* (PNAS, 2019) and the periodic table.

b) Discuss the degree of similitude between the flow of a polariton fluid past a defect (Carusotto & Ciuti, *Rev. Mod. Phys.* 2013, Fig.19) and the wave field of a walking droplet.

c) Discuss the degree of similitude between water wave vortices and skyrmions (Smirnova *et al.* PRL, **203**, 2024).

d) Discuss the extent to which a standing field of subharmonic Faraday waves, or a crystal lattice of bouncing droplets, constitutes a time crystal. Reference: R. Goldstein (*Physics Today*, **32**, 2018).

4. Physical analogies

Describe your favorite physical analogy, which must be something other than those discussed in class. Assess its degree of similitude by giving a brief description of the mathematical models of both systems.