18.S996 Hydrodynamic quantum analogs

Lecture 3: Quantum history, foundations
and hidden variable theories

ultracrepidarian [uhl-tr*uh*-krep-i-**dair**-ee-*uh* n]

pertaining to a person who criticizes, judges, or gives advice outside his area of expertise

Sutor, ne ultra crepidam!

References

1) The Age of Entanglement, by Louisa Gilder

2) Speakable and unspeakable in quantum mechanics, John S. Bell

3) Mike Towler's lecture notes on de Broglie-Bohm pilot-wave theory

4) History of Quantum Mechanics or the Comedy of Errors, by J. Bricmont

5) Quantum theory at the crossroads, by Bacciagaluppo & Valentini

Paradox

A statement or proposition that, despite apparently sound reasoning from acceptable premises, leads to a conclusion that seems senseless, logically unacceptable, or self-contradictory.

Paradoxes in science

An indication that workers in a field have insufficient perspective to solve a problem, that their premises are flawed.

Heisenberg, *On Quantum Mechanics (1930):*

"Both matter and radiation possess a remarkable duality of character, as they sometimes exhibit the properties of waves, at other times those of particles.

Flawed premise:

Now it is obvious that a thing cannot be a form of wave motion and composed of particles at the same time - the two concepts are too different."

Wave-particle duality at the macroscopic scale

We proceed by exploring the possibility that the quantum paradoxes are a consequence of this flawed premise.

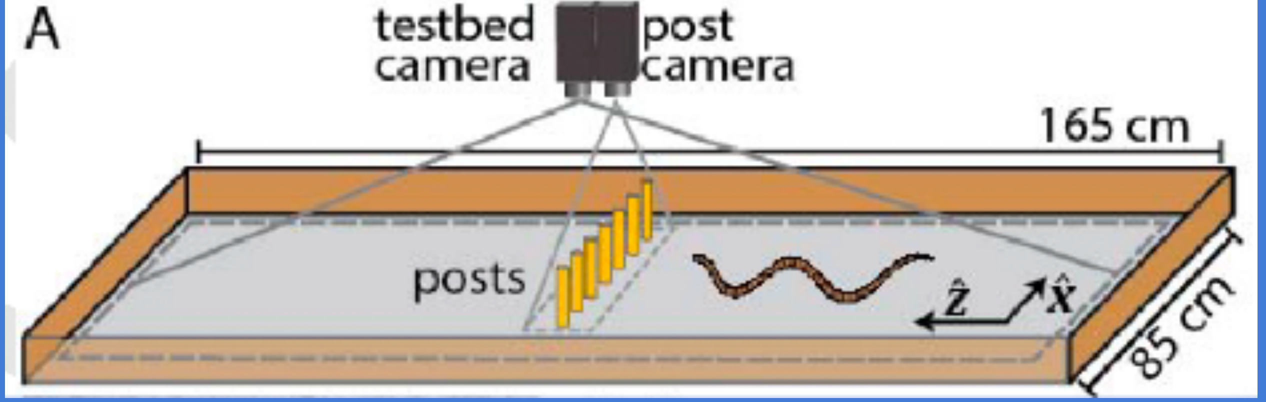
Principle of explosion

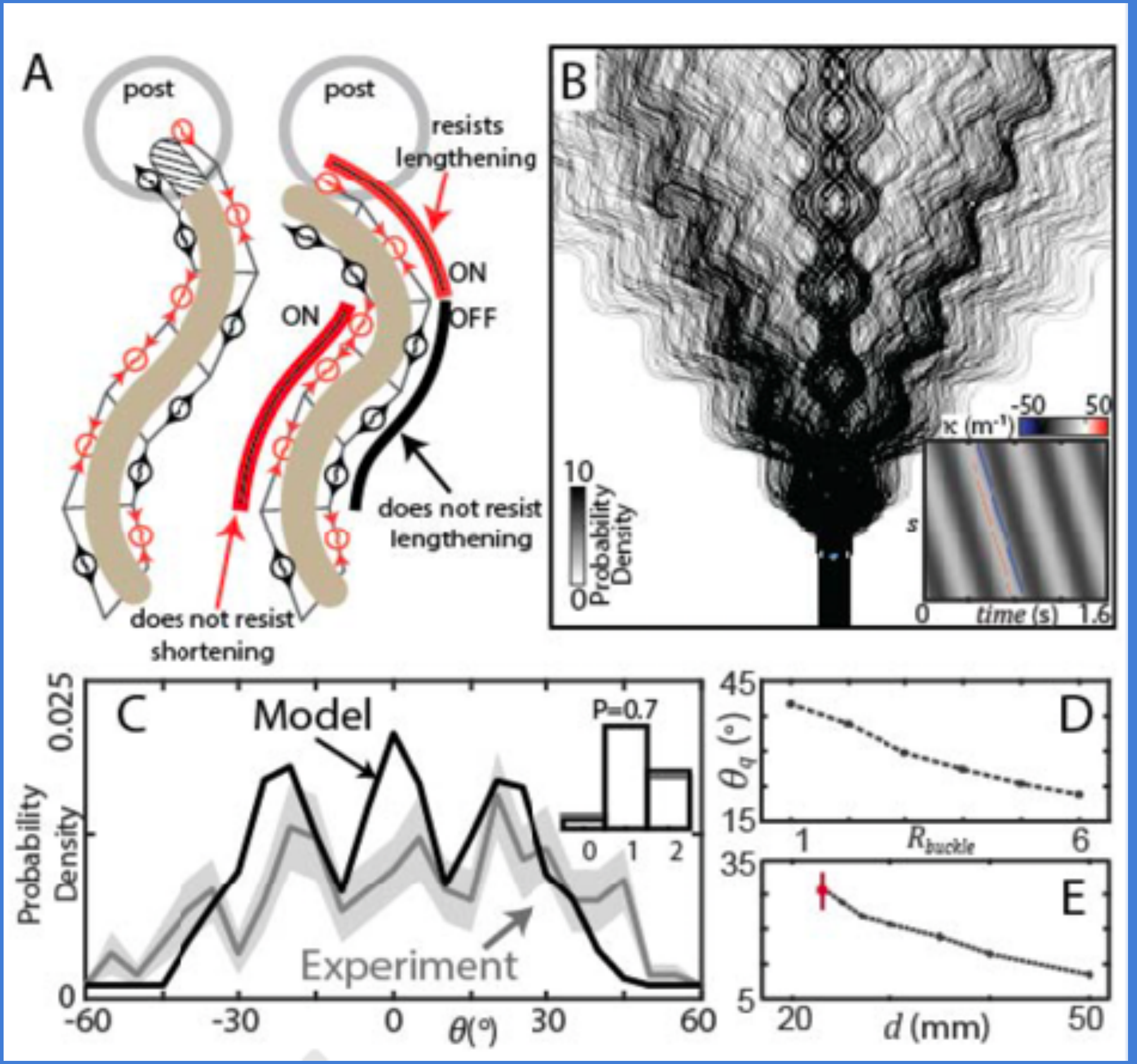
- in classical logic, the law that any proposition can be proven from a contradiction or falsehood
- `*ex falso sequitur quodlibet'* : from falsehood, anything follows

Wave-particle duality on the macroscopic scale…

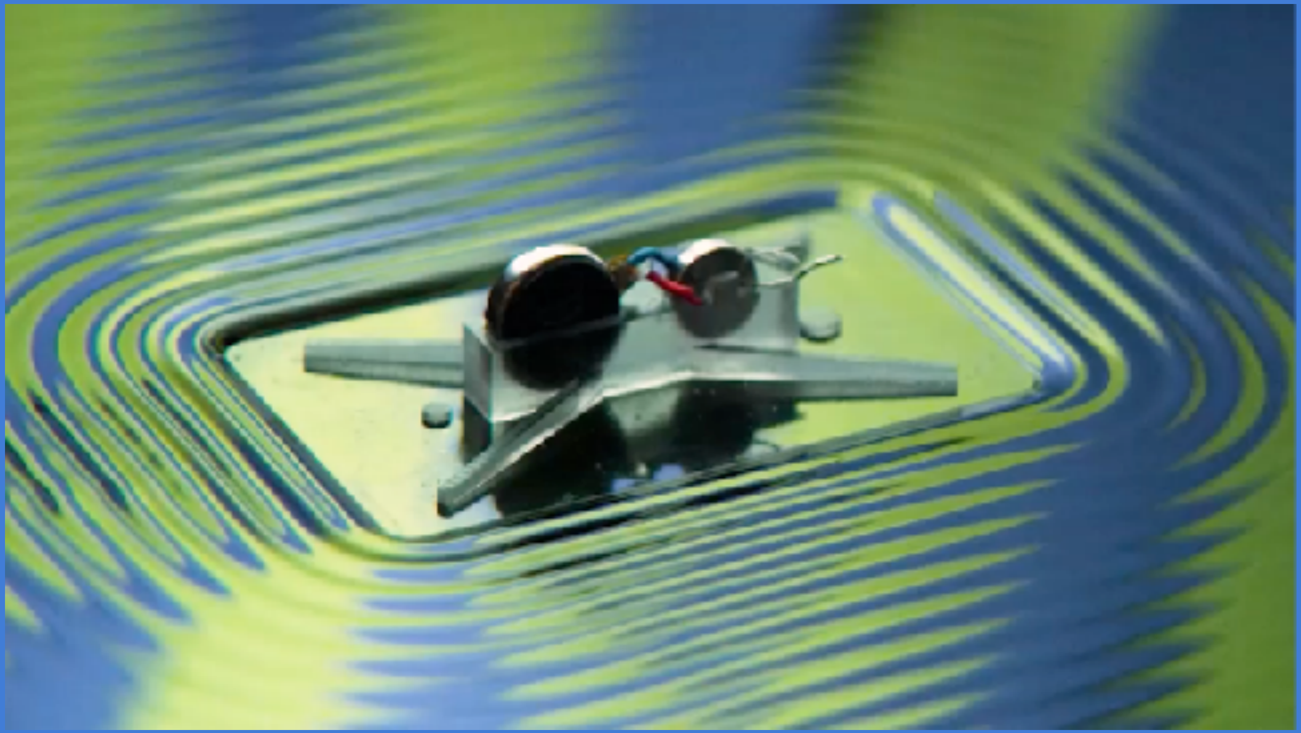
Diffracting snakes?

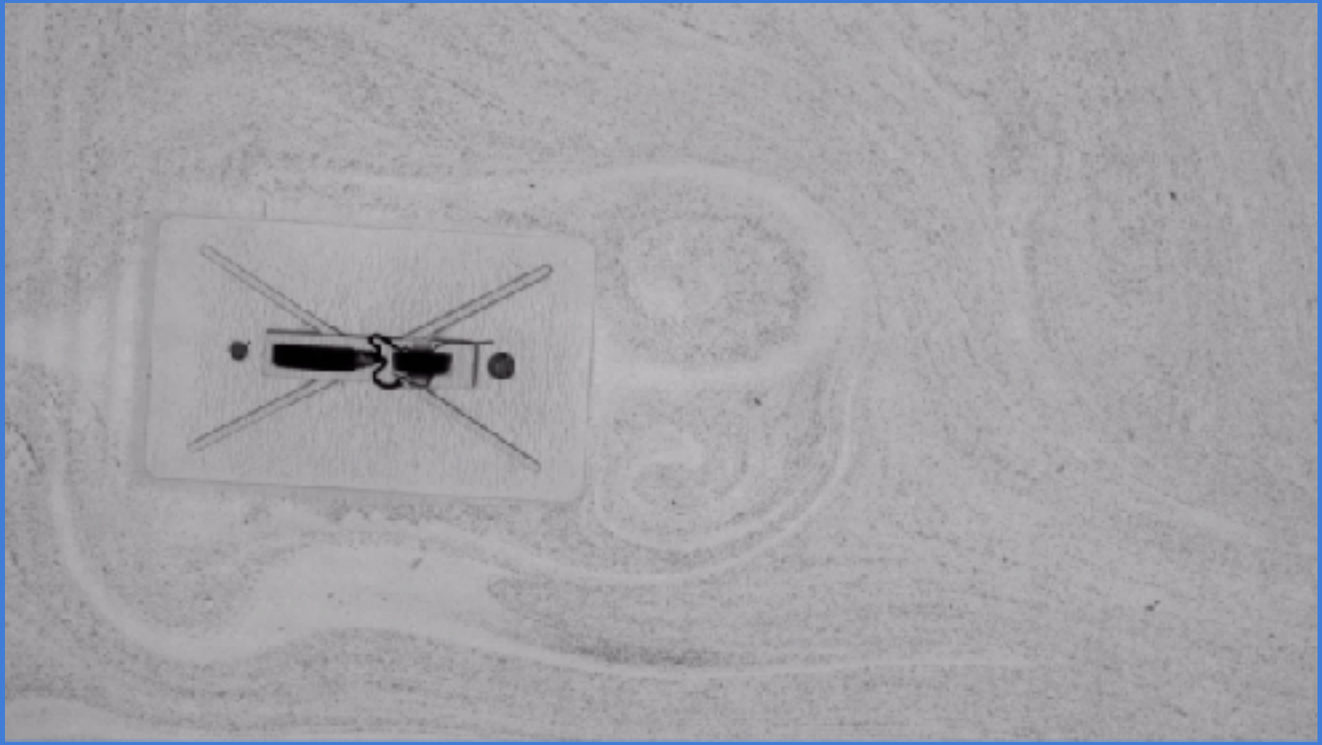
— Dan Goldman (2018)





The surferbot:
Dan Harris (2022)

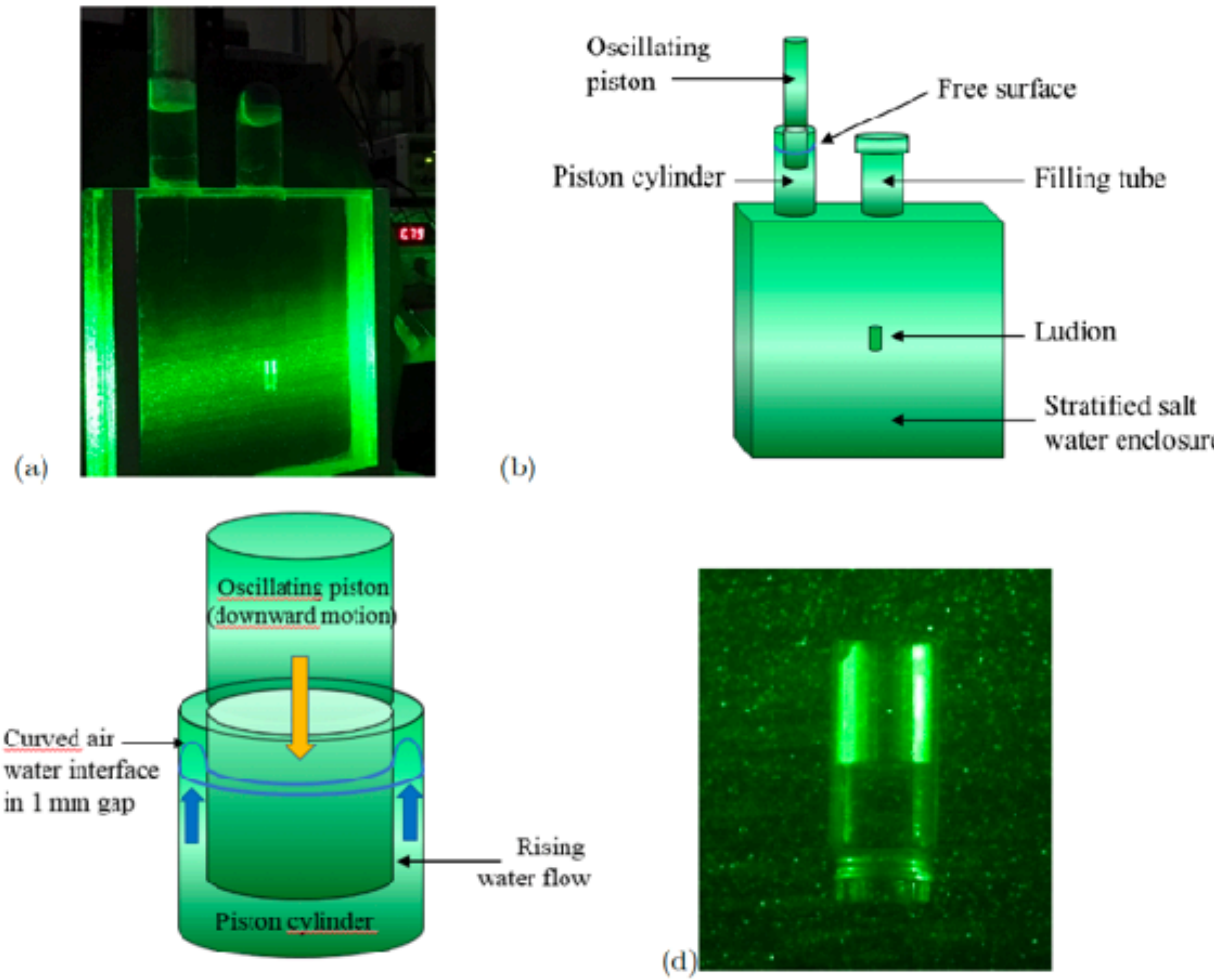




The ludon

Le Gal *et al.*, JFM (2021)

- pneumatically-forced floater in a stratified fluid self propels when forced at the fluid's Brunt-Vaisalla frequency



Gunwale bobbing

Neufeld *et al.*, JFM (2023)

A cautionary tale: On paradoxes in fluid mechanics

The resolution of the fluid mechanical paradoxes invariably arose through the elucidation of unimagined dynamics at an unanticipated scale.

D'Alembert's Paradox (1749)

- an object moving through an inviscid fluid experiences no drag
- at odds with experiments on ballistic flight in high-speed gas flows
- stood for 150 years, until Prandtl's resolution of the viscous boundary layer

A longstanding rift between theorists and experimentalists:

``experimentalists observed things that could not be explained, mathematicians explained things that could not be observed.''

A cautionary tale: **On paradoxes in fluid mechanics**

The resolution of the fluid mechanical paradoxes invariably arose through the elucidation of unimagined dynamics at an unanticipated scale.

The Paradox of the moving contact line

- the no-slip condition should preclude the motion of contact lines
- drops should not be able to advance along solid surfaces
- rolling drop experiments of Elizabeth Dussan showed the way

Resolution (de Gennes 1990s)

- the no-slip condition applies on the scale of the continuum hypothesis, specifically, for scales greater than 10 molecule
- molecular forces averaged over in the continuum hypothesis become dominant on the single-molecule scale of the contact line

Paradoxes in biolocomotion

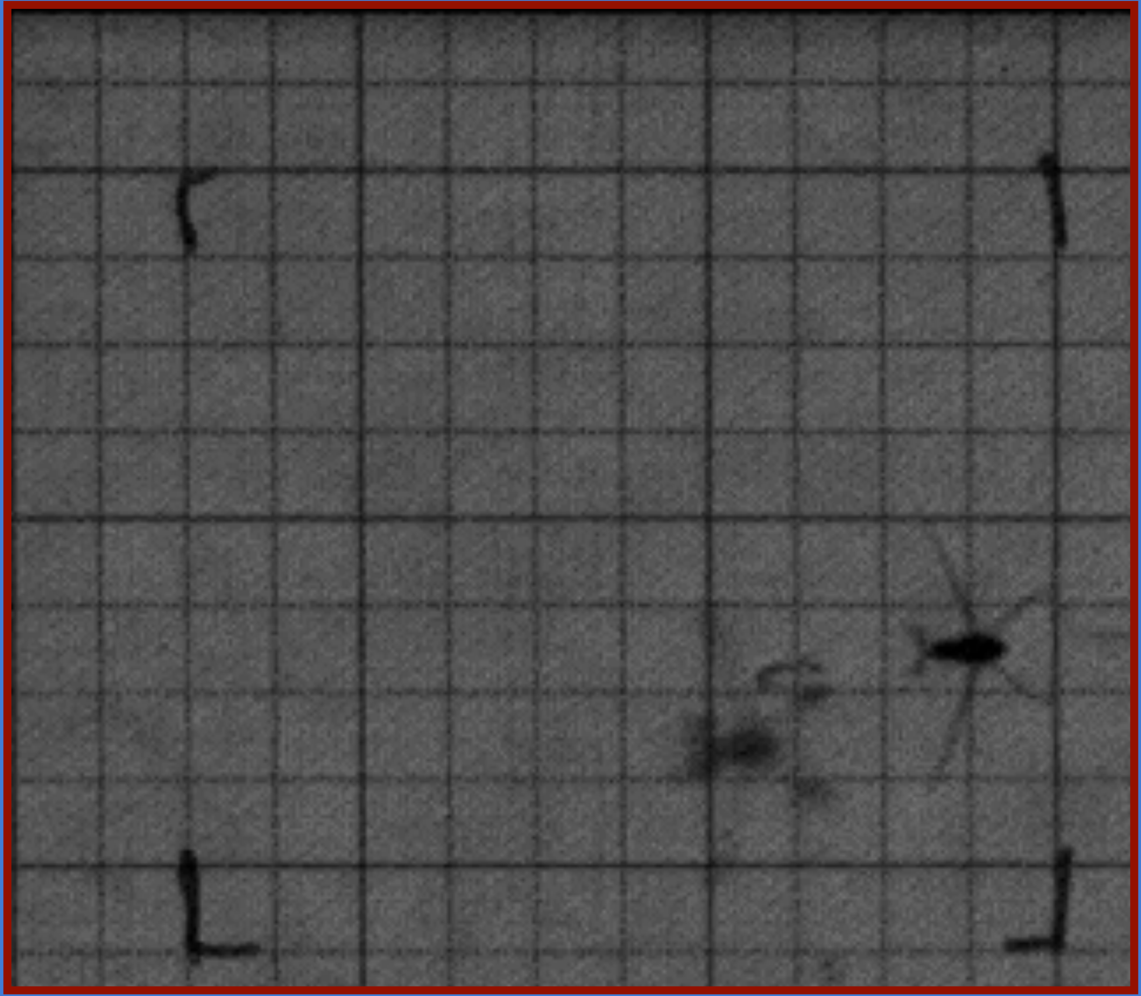
- a bumblebee shouldn’t be able to fly

Gray’s Paradox (1935-2000)

- a dolphin shouldn’t be able to swim as fast as it does

Denny's Paradox (1993)

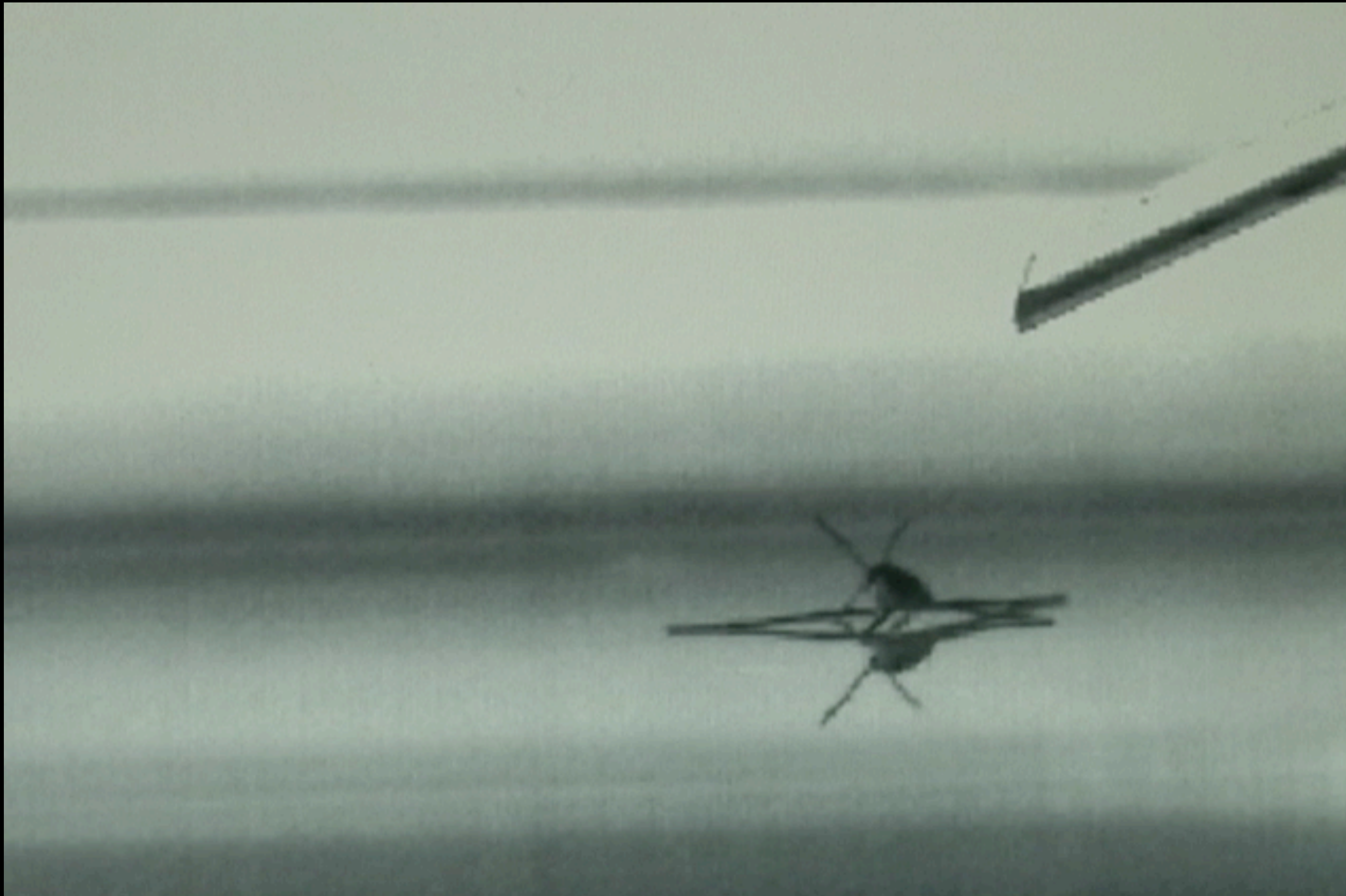
- infant water striders cannot swim:



> " Exactly how they manage to propel themselves across the water surface remains a mystery."

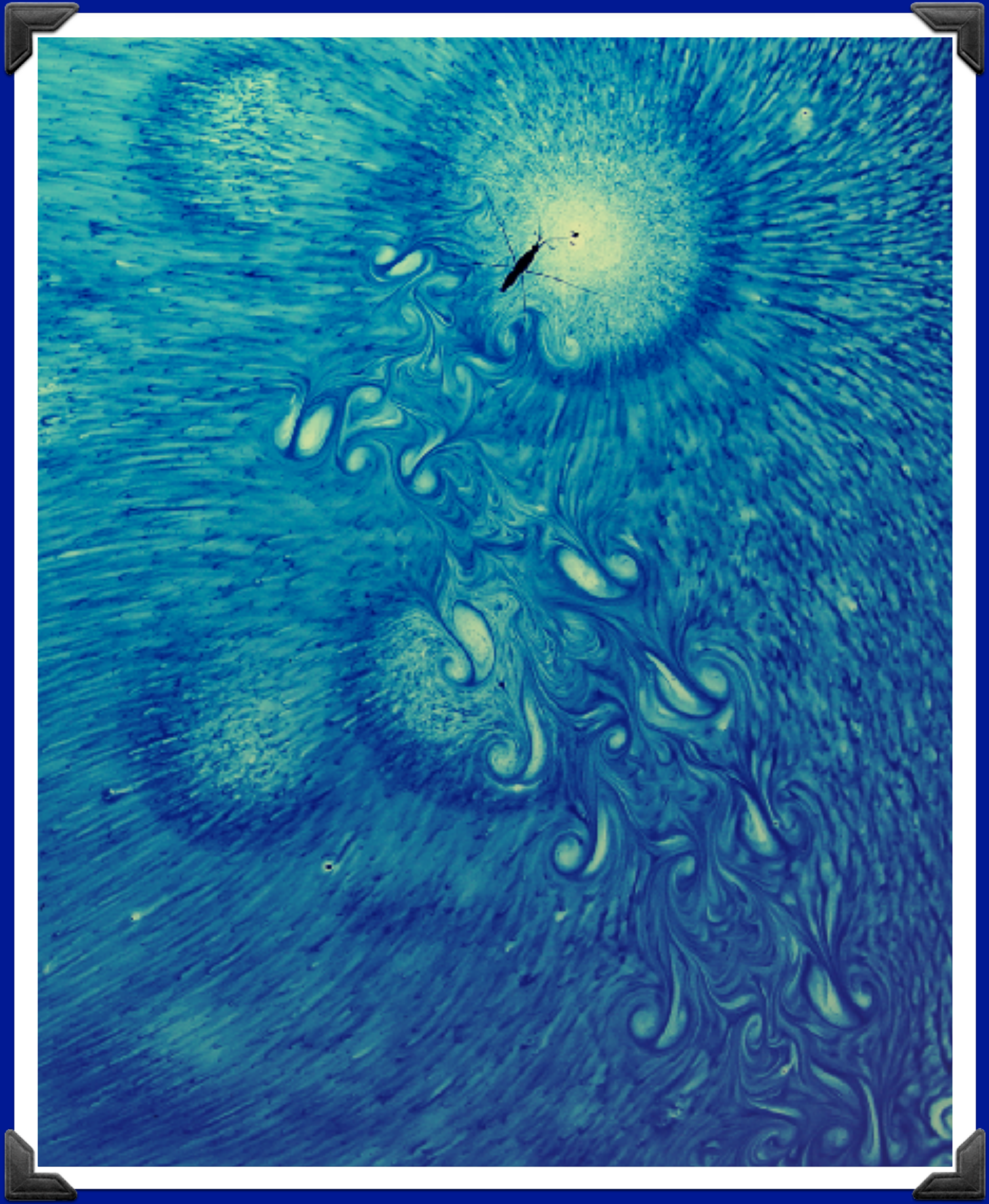
Reasoning

- assumed momentum transfer exclusively in waves
- to generate waves, legs must exceed $c_m = (4g\sigma/\rho)^{1/4} = 23$ cm/s
- infant leg speed $< c_m$



- infant strider (age 1 hour) propels itself across the surface

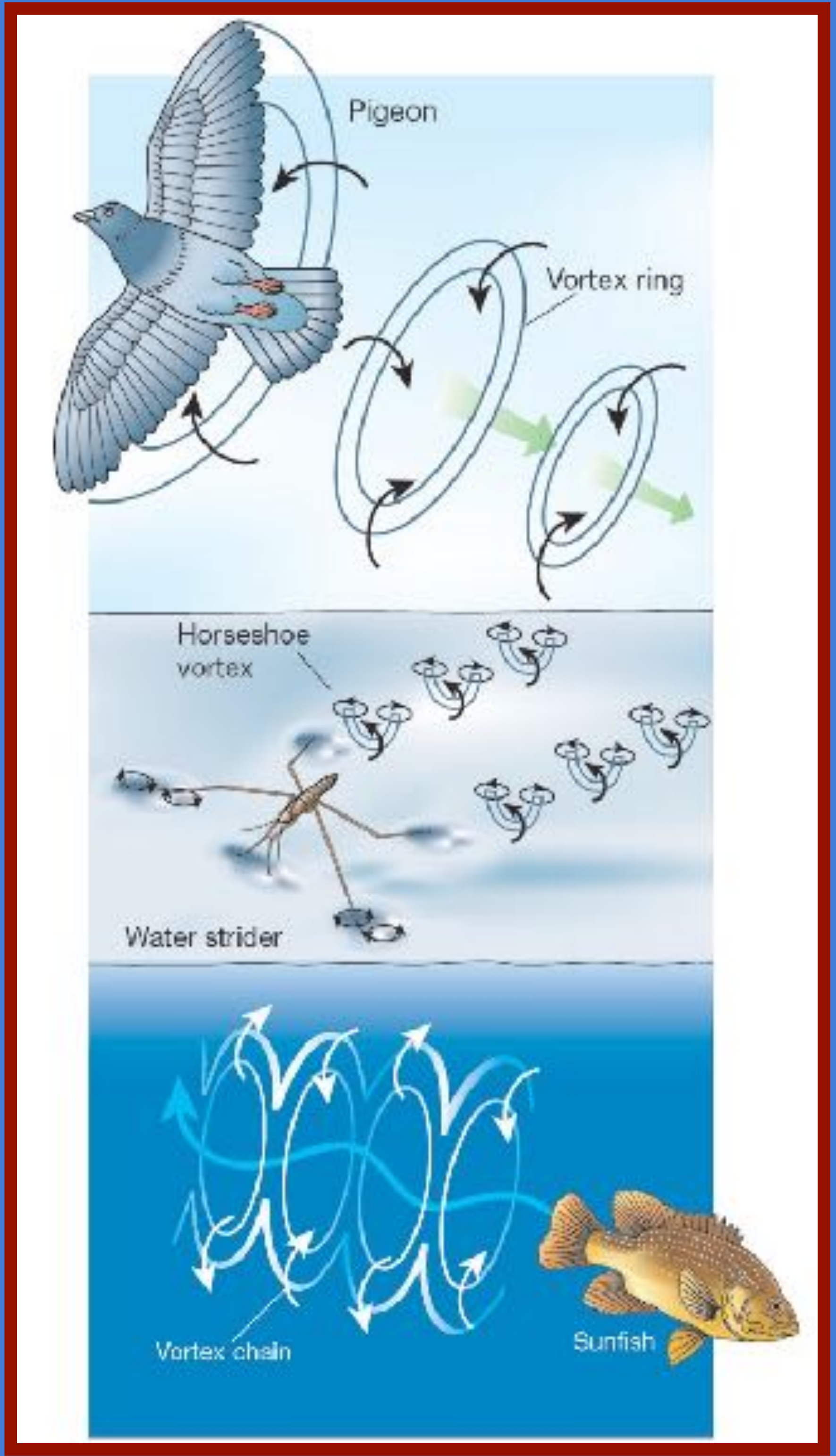
Icarus



Flying

Rowing

Swimming



Hu & Bush
Nature, 2003

Dickinson (2003)

Paradox

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Paradoxes in science

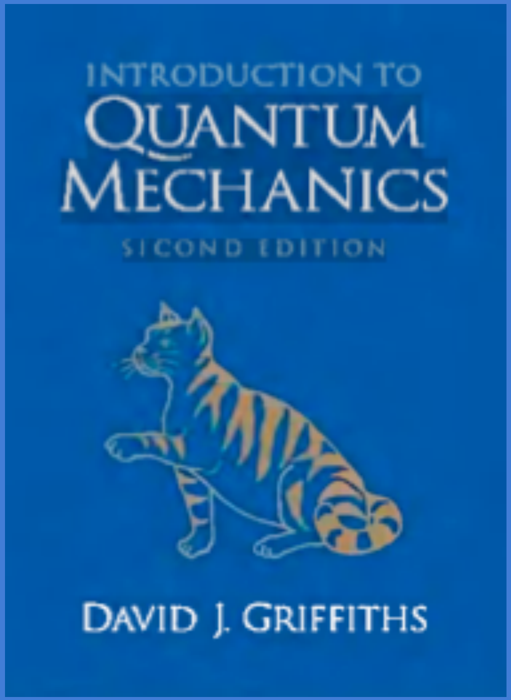
An indication that workers in a field have insufficient perspective to solve a problem, that their premises are flawed.

Paradoxes in fluid mechanics

- their resolution invariably arose through the elucidation of unimagined dynamics at an unanticipated scale

Walking droplets

- suggest that the quantum paradoxes may be resolved through the reinstatement of trajectories, the resolution of a pilot-wave dynamics on the Compton scale



"It is entirely possible that future generations will look back, from the vantage point of a more sophisticated theory, and wonder how we could have been so gullible."

- D. J. Griffiths

THE INHOMOGENEOUS UNIVERSE

Classical physics (Newton, c.a. 1700)

The macroscopic world is deterministic.

Initial conditions uniquely determine outcome.

Quantum physics (Bohr/Heisenberg, 1920s)

The microscopic world is intrinsically probabilistic.

Initial conditions determine only probability of outcome.

Some notable dissenters: " God does not play dice."

The Quantum Problem

(Holland 1993)

Individual events in QM are unpredictable, but a coherent statistics emerges when one considers a large number of such events.

Central puzzling phenomena

- ***self-interference*** of single particles; e.g. single- and double-slit
- ***tunneling***: a quantum particle can pass through a barrier forbidden to a classical particle
- ***stability of matter***: atoms and molecules exist only in certain discrete energy states, and do not collapse (as would their classical counterparts). During transition between states, a discrete quantum of energy is exchanged with the EM field.
- ***spin***: a novel type of nonclassical internal angular momentum, as is apparent in, for example, the Stern-Gerlach experiment
- ***nonlocal correlations***: the properties of one particle can appear to depend on those of an arbitrarily distant system with which it has interacted in the past

Schrodinger's equation: Origins

- in 1927, de Broglie's pilot-wave theory was criticized on the grounds that he did not have a theoretical description of his pilot wave
- Schrodinger was, like Einstein, a supporter of de Broglie's theory, and so derived an equation to describe the pilot wave

Ingredients

PLANCK RELATION

$$E = \hbar\ \omega$$

DE BROGLIE RELATION

$$p = \hbar\ k$$

$$E = \hbar\omega = \frac{1}{2}mv^2 = \frac{1}{2m}\hbar^2 k^2 \quad \longrightarrow \quad \omega = \frac{1}{2m}\ \hbar k^2$$

What equation has this dispersion relation?

Schrodinger's Equation:

$$i\hbar\ \frac{d\Psi}{dt} = -\frac{\hbar^2}{2m}\ \nabla^2\Psi$$

- despite his objections, Schrodinger's equation was adopted as a description of the statistics of quantum systems

Nonrelativistic, single-particle quantum mechanics

Wavefunction $\Psi(\mathbf{x}, t)$ evolves according to

Linear Schrodinger's equation :

$$i\hbar \, \Psi_t \; = \; - \frac{\hbar^2}{2m} \nabla^2 \Psi \; + \; V \, \Psi$$

Born's Rule: $\rho \; = \; |\Psi|^2$ prescribes the probability density

Measurement forces collapse of wavefunction in an unspecified way.

Copenhagen Interpretation: between measurements, the particles do not have positions

`It seems a little paradoxical to construct a configuration space with the coordinates of points that do not exist.'

— De Broglie

The instrumentalist view

- QM is the theory that describes matter on the microscopic scale
- predicts the probabilities of outcome of experimental measurements
- sufficient for the pragmatist interested only in applying the theory

But what is going on under the hood?

Quantum interpretations

- attempts to explain how QM informs our understanding of the physical world
- until the 1980s, one interpretation was preeminent…

The Copenhagen Interpretation

The Copenhagen Interpretation (c.a. 1920s)

`An intellectual deceit borne of despair.'   — Schrödinger

Nature us fundamentally probabilistic. Microscopic particles do not move on trajectories. Their dynamics are described entirely by a probability wave that evolves according to Schrodinger's equation. Until an observation is made, the particles exist only in a state of metaphysical limbo. Observation forces the collapse of the wave function, the emergence of the particle into reality.

Represents a drastic change of world view, an abandonment of determinism, realism and locality.   Suggests that classical and quantum physics do not have philosophical similarity.

Determinism: the evolution of a physical system can be determined from its initial conditions.

Realism: there is a reality that exists independent of human observation.

Locality: nothing travels faster than the speed of light.

Bohr vs Einstein: the debate over the Nature of Reality (*PRL*, 1935)

The Copenhagen Interpretation

The statistical predictions of QM provide a complete description of reality.

The Realist Stance

There is a hitherto unresolved dynamics underlying quantum statistics.

"I am, in fact, rather firmly convinced that the essentially statistical nature of contemporary quantum theory is solely to be ascribed to the fact that it operates with an incomplete description of physical systems."

"In a complete physical description, the statistical quantum theory would take an approximately analogous position to statistical mechanics within the framework of classical mechanics."

- Albert Einstein

The Copenhagen Interpretation

PRO: Bohr, Heisenberg

"It is clear that the double slit experiment can in no way be reconciled with the idea that electrons move in paths. In quantum mechanics, there is no such concept as the path of a particle."

- Landau and Lifshitz, 1977

" The idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees, independently of whether or not we observe them ... is impossible."

- Heisenberg, 1958

CON: Einstein, Schrodinger, Bell

"There is something rotten in the state of Denmark".

- John Bell

"Bohr brainwashed a whole generation of physicists into believing that the problem had been solved".

- Murray Gell-Mann

An epic take-down

``In the biological sciences, one takes it for granted that in addition to the condition, there must be some other causative factor, not yet identified. One searches for it, tracking down the assumed cause by a process of elimination of possibilities that is sometimes extremely tedious. But persistence pays off; over and over again... Most enzymes, vitamins, viruses, and other biologically active substances owe their discovery to this reasoning process.

What is done in quantum theory today is just the opposite; when no cause is apparent one simply postulates that no cause exists---ergo, the laws of physics are indeterministic and can be expressed only in probability form.

Biologists have a mechanistic picture of the world because, being trained to believe in causes, they continue to use the full power of their brains to search for them---and so they find them. Quantum physicists have only probability laws because for two generations we have been indoctrinated not to believe in causes---and so we have stopped looking for them. Indeed, any attempt to search for the causes of microphenomena is met with scorn and a charge of professional incompetence and 'obsolete mechanistic materialism."

E.T. Jaynes
Probability Theory: The Logic of Science
Cambridge University Press, 2003

An early objection

Are particle tracks left in a cloud chamber not trajectories?

Sealed chamber with supersaturated vapour kept near condensation point by regulating T. Ionizing radiation leaves trail of charged ions that serve as condensation centers. Vapour condenses around them. Radiation *path* thus indicated by tracks of tiny liquid droplets in supersaturated vapour.

Mike Towler

Quantum paradoxes and biolocomotion

Mike Towler

Environ. Sci. Techno

Differential Toxicity of Carbon Nanomaterials in *Drosophila*: Larval Dietary Uptake Is Benign, but Adult Exposure Causes Locomotor Impairment and Mortality

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- put fullerene molecules on the feet of fruit flies — and they can’t climb walls
- but fullerenes are quantum particles that exhibit single-particle diffraction and interference (Zeilinger et al., AJP, 2003)

Don’t look down!

The measurement problem

- wavefunction evolves deterministically according to Schrodinger's equation until a measurement is made
- measurement causes the collapse of quantum states, the potentialities expressed in the wavefunction, to a definite classical state
- subsequent evolution of the wavefunction follows from the state measured
- quantum mechanics provides no prescription for the collapse of the wavefunction accompanying measurement
- how can one establish a correspondence between quantum and classical reality?
- the measurement problem must be addressed by any quantum interpretation

Schrodinger's cat : a lampoon of quantum philosophy developed in correspondence between Schrodinger and Einstein

- prior to measurement, the cat is in a superposition of states, alive and dead

The EPR Paradox

Can quantum mechanical description of physical reality be considered complete?

Einstein, Podolsky & Rosen (PRL, 1935)

- consider 2 entangled particles that are space-like separated
- measuring the position/momentum/spin of particle 1 reveals that of particle 2
- no action on particle 1 can effect particle 2 without violating locality

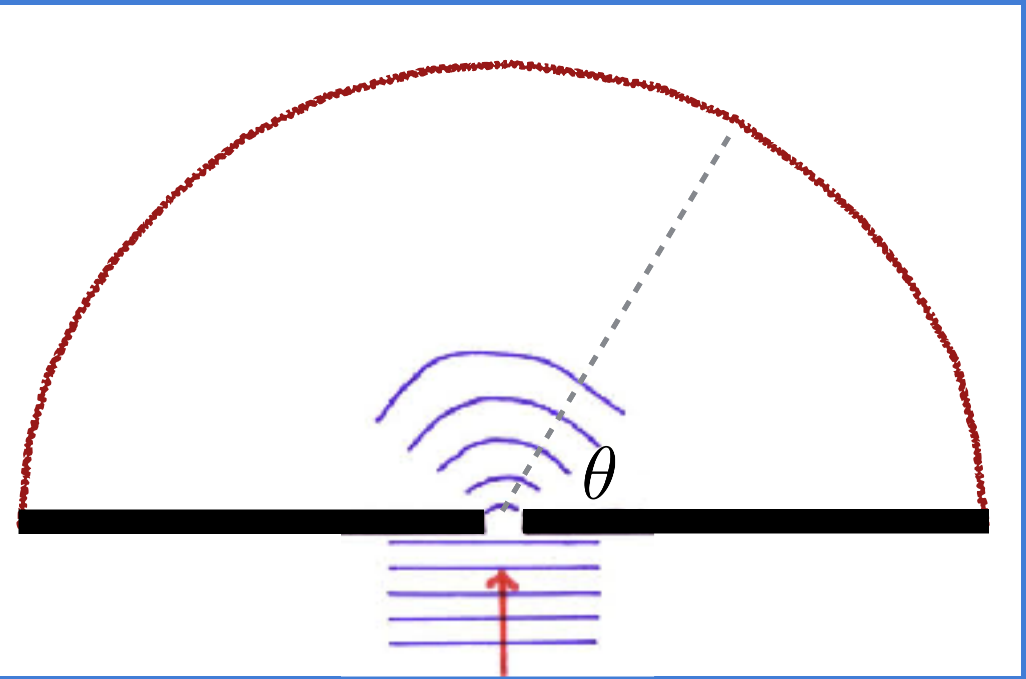
"If, without disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of reality corresponding to that quantity."

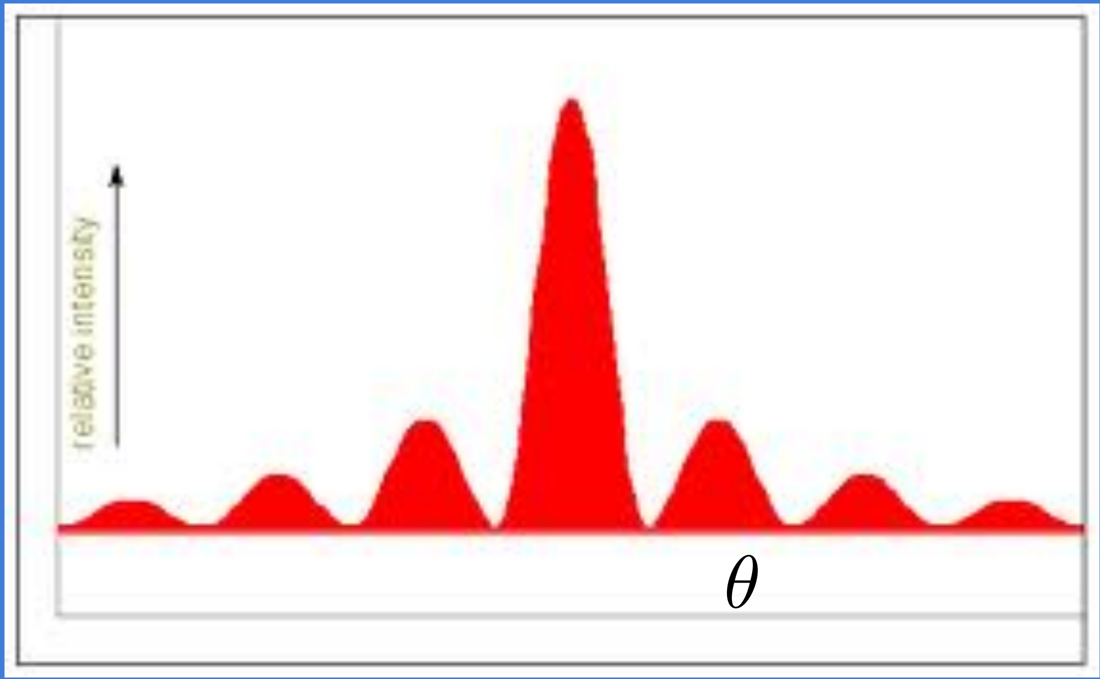
- if QM is complete, it must be non-local → **QM is incomplete**

Bohr's response

- baffling and incoherent, by all accounts
- many attributed to Bohr a knowledge too deep to express

A variant due to de Broglie (1967)



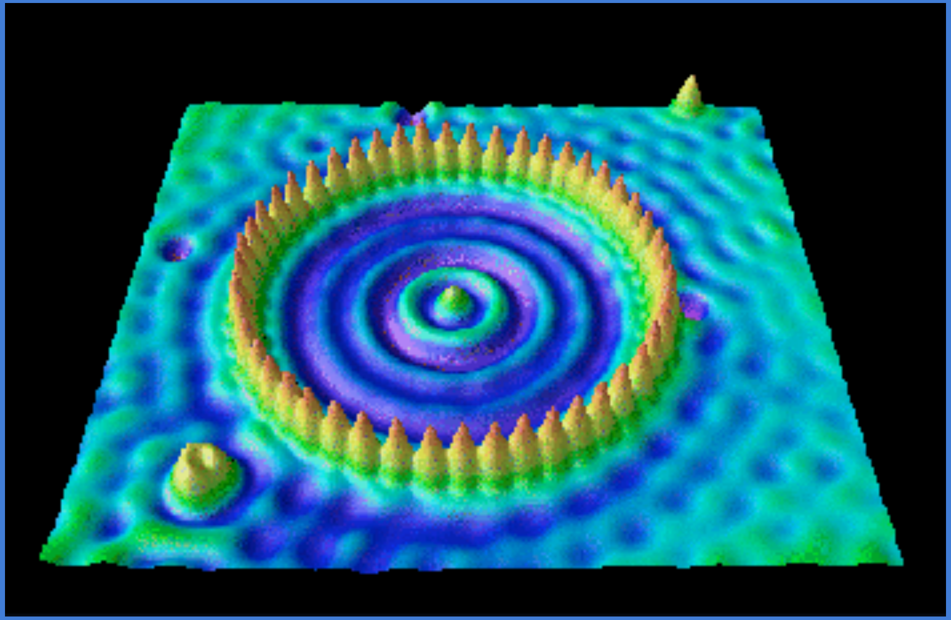


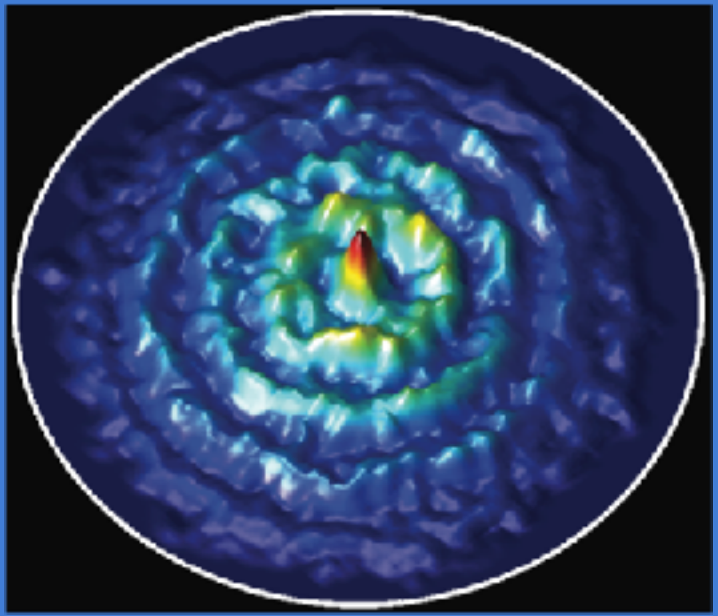
- if QM is complete, then diffracting particle is smeared out over space
- appearance of dot on screen requires instantaneous collapse of wave-function
- if QM is complete, it must be non-local → **QM is incomplete**

The EPR Paradox

- the statistical wave function, a spatially extended object, collapses instantaneously to a point when subjected to the act of human observation

But how can that possibly be?





``Is it possible to see this simple business as obscure and mysterious? We must try.”

- John S. Bell

Key points

- the Copenhagen Interpretation was adopted as a matter of theoretical preference, not forced upon us by experimental observation
- 90% of practicing physicists do not believe the Copenhagen Interpretation (most being agnostic pragmatists: "QM works, so let's get on with it")
- it has lowered the intellectual bar, invited the development of interpretations of ever-increasing philosophical extravagance

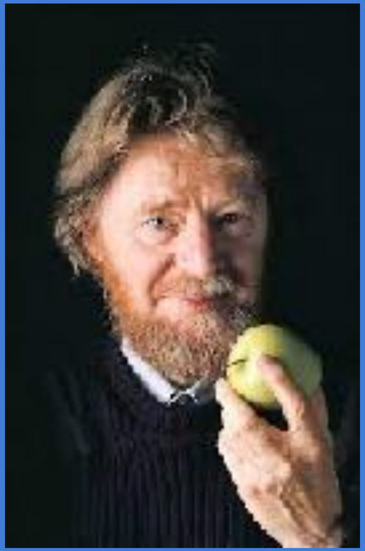
The bright side

- it has also encouraged the development of more sensible interpretations
- it has motivated the development of hidden variable theories, attempts to rationalize quantum statistics through an underlying dynamics
- it drew the fire of John Bell, quantum skeptic and debunker

The language of John Bell

- in `Speakable and Unspeakable', he lampoons the quantum orthodoxy
- develops an extravagant language that is now adopted by philosophers

Beable



- an object that would exist if observed

Counterfactual definiteness

- the notion than an object and its properties exist independent of human observation
- the notion that all objects are beables

The proliferation of Quantum Interpretations

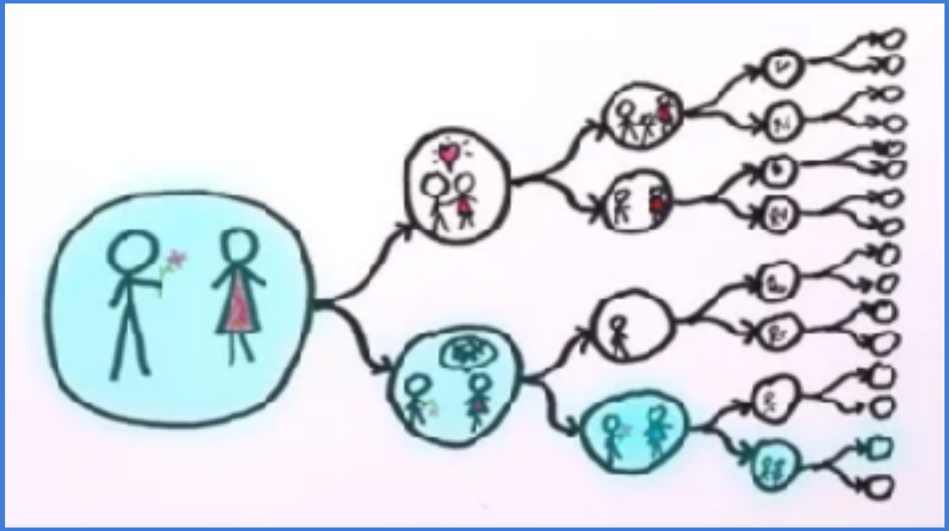
Interpretation	Author(s)	Deterministic?	Wavefunction real?	Unique history?	Hidden variables?	Collapsing wavefunctions?	Observer role?	Local?	Counterfactual definiteness?
Ensemble interpretation	Max Born, 1926	Agnostic	No	Yes	Agnostic	No	None	No	No
Copenhagen interpretation	Niels Bohr, Werner Heisenberg, 1927	No	No[1]	Yes	No	Yes[2]	None	No	No
de Broglie–Bohm theory	Louis de Broglie, 1927, David Bohm, 1952	Yes	Yes[3]	Yes[4]	Yes	No	None	No	Yes
von Neumann interpretation	von Neumann, 1932, Wheeler, Wigner	No	Yes	Yes	No	Yes	Causal	No	No
Quantum logic	Garrett Birkhoff, 1936	Agnostic	Agnostic	Yes[5]	No	No	Interpretational[6]	Agnostic	No
Many-worlds interpretation	Hugh Everett, 1957	Yes	Yes	No	No	No	None	Yes	No
Popper's interpretation[36]	Karl Popper, 1957[37]	No	Yes	Yes	Yes	No	None	Yes	Yes[13]
Time-symmetric theories	Yakir Aharonov, 1964	Yes	Yes	Yes	Yes	No	No	Yes	No
Stochastic interpretation	Edward Nelson, 1966	No	No	Yes	No	No	None	No	No
Many-minds interpretation	H. Dieter Zeh, 1970	Yes	Yes	No	No	No	Interpretational[7]	Yes	No
Consistent histories	Robert B. Griffiths, 1984	Agnostic[8]	Agnostic[8]	No	No	No	Interpretational[6]	Yes	No
Objective collapse theories	Ghirardi–Rimini–Weber, 1986	No	Yes	Yes	No	Yes	None	No	No
Transactional interpretation	John G. Cramer, 1986	No	Yes	Yes	No	Yes[9]	None	No	No
Relational interpretation	Carlo Rovelli, 1994	No	No	Agnostic[10]	No	Yes[11]	Intrinsic[12]	Yes	No

- encouraged by the philosophical extravagance of the Copenhagen Interpretation

HALF TIME

The Many Worlds Interpretation (Everett 1957)

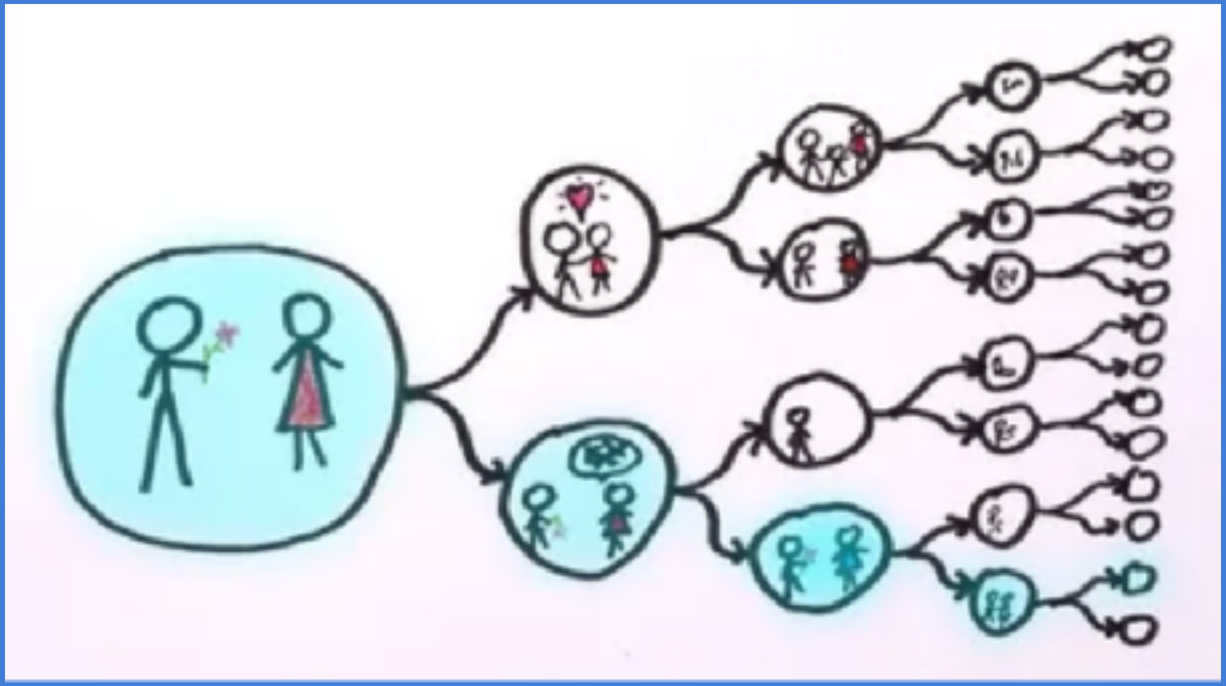
- there are many worlds that exist in parallel in the same time and space as our own
- there are myriad worlds in the Universe, in addition to the one we are aware of
- every time a quantum experiment is performed with different possible outcomes, every possible outcome is realized in a different, newly created world
- purports to eliminate problems of randomness, nonlocality, measurement
- introduces irrefutable nonsense



The Many Minds Interpretation (Zen, 1970)

- purports to eliminate problems of randomness, nonlocality, measurement
- the bifurcation arising in the Many Worlds Interpretation happens — in your mind
- every time a quantum experiment is performed with different possible outcomes, every possible outcome is realized in a different, newly created world

… but only in your mind …. ?



The von Neumann-Wigner Interpretation (1932)

- human consciousness is responsible for the collapse of the wavefunction
- the rules of QM are correct, but there are external observers that cannot be treated with QM, namely, human (and perhaps animal) minds, the actions of which cause wave function collapse
- our minds become parts of the quantum mechanically described physical universe
- purports to address the measurement problem

Karl Popper

- presented a philosophical repost to quantum metaphysics, entirely sensible
- properties are interpreted as objective, mind-independent outcomes of experiments
- distinguished scientific theory from interpretation: only the former is falsifiable
- a vocal critic of Copenhagen, supporter of the Ensemble Interpretation

Statistical/Ensemble Interpretation (Born 1954)

- the quantum state description applies only to an ensemble of similarly prepared systems, rather than supposing that it exhaustively represents an individual physical system

``The attempt to conceive the quantum theoretical description as the complete description of the individual systems leads to unnatural theoretical interpretations, which become immediately unnecessary if one accepts the interpretation that the description refers to ensembles of systems and not to individual systems."

— Einstein (1957)

- most widely believed Interpretation by physicists, perfectly sensible, but mute on the subject of dynamics

- (it motivates the development of a description of ensembles in the walking-droplet system)

Important distinction

Quantum interpretations

- attempts to understand the meaning of quantum theory in its present form
- can be neither proven nor disproven by experiment : *unfalsifiable*
- a matter of philosophical preference: an exercise in metaphysics

Hidden variable theories

- attempts to complement quantum theory with a description of quantum dynamics
- may be tested experimentally, so discarded if incorrect : *falsifiable*
- the great majority have involved wave-particle coupling
- include pilot-wave theories of de Broglie and Bohm, Nelson's Stochastic Mechanics and Stochastic Electrodynamics

Hidden variable theories (so named in an act of `historical silliness')

- purport that QM provides only an incomplete description of reality
- seek a rational dynamics that underlies the statistical quantum theory
- characterization of `hidden' variables would restore determinism, realism to QM

  e.g. the position and momentum of an electron would yield particle trajectories

A brief history

- de Broglie (1926) proposed a **double-wave** pilot-wave theory of quantum dynamics: quantum particles move in resonance with a guiding wave
- Von Neumann (1932) `*proved*' that there can be no hidden variable theory
- Grete Hermann (1935) pointed out flaw in von Neuman's `proof', was ignored
- Bohm (1952) presented a **single-wave** pilot-wave theory consistent with QM
- Bell (1958) discredited Von Neumann's and subsequent Impossibility Proofs

"The only thing proved by Impossibility Proofs is the author's lack of imagination."

\- On the Impossible Pilot Wave (Bell, 1982)

Bell's Inequality

- a means to assess the viability of hidden variable theories
- intended as a vehicle to investigate the EPR Paradox experimentally, to highlight the feature of quantum non-locality
- a mathematical statement bounding the correlations of particles separated from a common source
- based on three fundamental assumptions:

  1) realism: quantum particles exist independent of human observation
  2) locality: there is no superluminal signaling (in the sense discussed in EPR)
  3) measurement independence: hidden variables independent of measurements settings

Experiments of Alain Aspect (1982—)

- measured correlations of distant particles from a common source
- Bell's inequality violated: on the basis of which we can assume that assumption 1) or 2) or 3) must be abandoned

Common Inferences

A) Quantum mechanics is complete: there can be no hidden variable theories

- prevalent among the quantum orthodoxy: Bell's Theorem considered to be the Last Impossibility Proof
- does not follow logically (though adopted by the 2023 Nobel Committee)

B) Realism must be abandoned

- rare, but popular among the lunatic fringe

C) Quantum mechanics is inescapably non-local

- so concluded by Bell as well as most sensible, informed people
- favored among Bohmians, as Bohmian mechanics is implicitly nonlocal

D) Measurement independence does not apply in presence of background fields

- the hope of those invested in a local, realist hidden-variable theory

(Morgan 2006, Vervoort 2018)

A brief history of the Pilot-Wave Theory

- early form proposed by de Broglie in 1927 at Solvay Conference
- the theory was met with hostility
- Pauli's objection was later shown to be specious
  - Quantum theory at the crossroads, by Bacciagaluppo & Valentini
- it suffered another setback in 1932 with von Neumann 's bogus proof
- it lay dormant until 1952, when Einstein encouraged Bohm to pursue it
- Bohmian mechanics also received a tepid reception
- prompted Bell to revisit, debunk von Neumann's Impossibility Proof
- de Broglie returned to it thereafter, but its form was ever changing
- later versions invoked a stochastic background field
- it prompted a modern extension in the form of Stochastic Electrodynamics

de Broglie's matter waves (1925)

``History shows that there long has been dispute over two viewpoints on the nature of light: corpuscular and undulatory; perhaps, however, these two are less at odds with each other than heretofore thought."

" Wave mechanics is an essentially relativistic theory, as I perceived at its beginning; Schrodinger's equation, being non-relativistic, is insufficient to reveal its true nature."

- a fundamental symmetry argument: like light, matter must have both particle and waves aspects
- imagined a vibrating particle riding a monochromatic wave of its own creation
- predicted electron diffraction, the experimental confirmation of which by Davisson & Germer (1927) won de Broglie the 1929 Nobel Prize
- his physical picture provided a number of the cornerstones of modern QM, but was abandoned in favor of … ***no physical picture***

Some comments on de Broglie’s theory

“ While the founding fathers agonized over the question `particle’ or `wave’, de Broglie in 1925 proposed the obvious answer `particle’ and `wave’…. This idea seems so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored”.

— John S. Bell

``He has lifted a corner of the Great Veil.”

— Einstein

De Broglie's relativistic pilot-wave theory

- an attempt to reconcile relativity and QM through consideration of the wave nature of matter

Einstein-de Broglie relation:

$$E = m\,c^2 \qquad \text{RELATIVITY}$$

$$E = \hbar\,\omega \qquad \text{QM}$$

Natural frequency →

$$\omega_c = \frac{mc^2}{\hbar}$$

Compton frequency

But what is happening at this frequency?

- de Broglie suggested an exchange between rest mass energy and field energy

de Broglie's pilot-wave theory: A double-wave solution

" A freely moving body follows a trajectory that is orthogonal to the surfaces of an associated wave guide".

- **Louis de Broglie (1892-1987)**

- Ψ is the probability wave, as prescribed by standard quantum theory
- $\Psi^{dB} = |\Psi^{dB}|\ e^{i\phi/\hbar}$ is a real physical wave responsible for guiding the particle
- wave generated by internal particle vibration (Zitterbewegung) at the Compton frequency: $\omega_c = \dfrac{m_0 c^2}{\hbar}$

- a solution of Klein-Gordon equation triggered by oscillations in rest mass
- particle pushed perpendicular to surfaces of constant phase:

$$\mathbf{p} = m\dot{\mathbf{x}}_\mathbf{p} = \nabla\phi = \hbar\mathbf{k} \text{ for a monochromatic wave } \quad \Psi^{dB} = |\Psi^{dB}|e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$$

- a monochromatic standing wave in the particle frame of reference
- Harmony of Phases: the particle oscillates in resonance with its guiding wave

De Broglie's *double-solution* pilot-wave theory

- **fast** dynamics: internal vibration at

  $$\omega_c = \frac{m_0 c^2}{\hbar}$$ create real wave field

  ϕ centered on particle

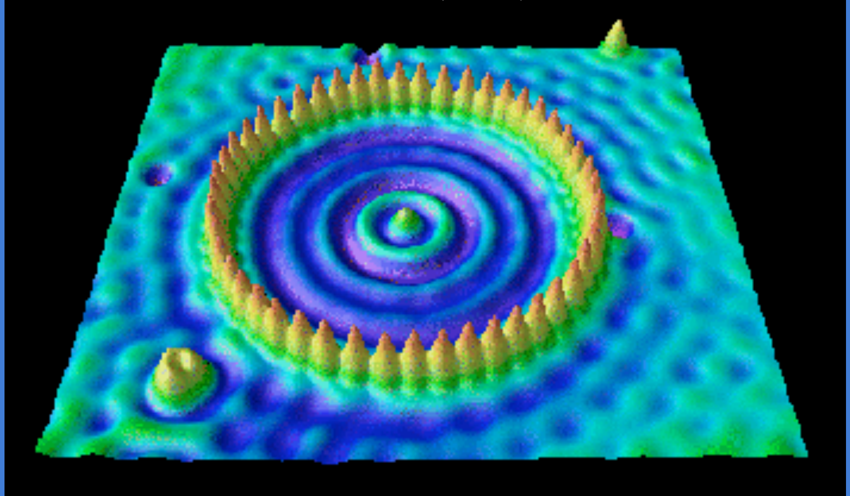
- **intermediate** pilot-wave dynamics: particle rides its guiding wave field

  $$\dot{\mathbf{x}}_{\mathbf{p}} = \frac{\hbar}{m_0} Im \left[\frac{\nabla \phi}{\phi} \right]$$, which for a monochromatic wave yields

  $$\mathbf{p} = \hbar \; \mathbf{k}$$

- emergent **long-term statistical** behaviour described by standard quantum theory, wave function Ψ

Fiete & Heller (2003)



The David Bohm story: A refugee of the Cold War

- grad student of Oppenheimer at Berkeley during WWII, thesis classified
- Oppenheimer secured him a faculty position at Princeton
- wrote the first textbook on QM, after which he was turned by Einstein
- Oppenheimer threw Bohm under the bus during his interrogation by McCarthy's House Committee on un-American Activities
- was obliged to plead the 5th Amendment during his interrogation
- despite his acquittal in 1951, Princeton sacked and blackballed him
- he took a faculty position in Brazil, where he published his seminal papers
- Einstein secured him a faculty position at Technion from 1955-57
- proceeded to Birkbeck College, University of London
- in his later life, he dabbled in philosophical and spiritual matters

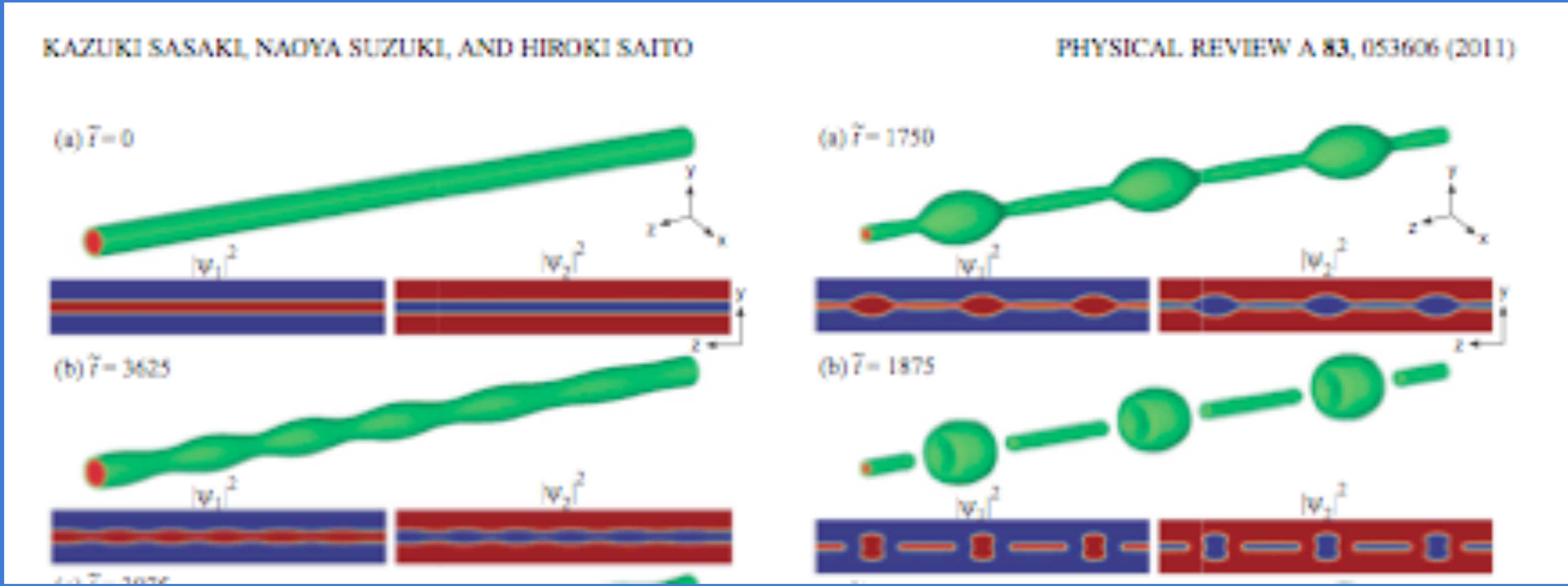
— e.g. *Wholeness and the Implicate Order*

The Hydrodynamic Interpretation of Quantum Mechanics

"in keiner Weise vorteilhaft" (in no way advantageous)

- Wolfgang Pauli

- the starting point for Bohmian mechanics
- has been revived and widely applied in the last 10 years
- the starting point for analysis of quantum fluids, BECs



The Madelung transformation (1928)

Schrodinger:
$$i\hbar\ \Psi_t = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\ \Psi$$

Transform:
$$\Psi = \sqrt{\rho}\ e^{iS/\hbar}$$

Continuity:
$$\frac{D\rho}{Dt} + \rho\ \nabla\cdot\mathbf{u} = 0$$

Quantum Hamilton-Jacobi:
$$\frac{\partial S}{\partial t} + \frac{1}{2}\mathbf{u}^2 - \frac{\hbar^2}{2m^2}\frac{1}{\sqrt{\rho}}\nabla^2\sqrt{\rho} + \frac{V}{m} = 0$$

QUANTUM POTENTIAL

where $\rho = |\Psi|^2$ is the probability density, S is the action,

$\mathbf{u} = \nabla S/m$ is the quantum velocity of probability,

$\mathbf{j} = \rho\ \mathbf{u}$ is the quantum probability flux.

Inviscid fluid dynamics

Depth-averaged velocity: $\mathbf{u} = \nabla\phi$

Continuity: $\dfrac{Dh}{Dt} + h\,\nabla\cdot\mathbf{u} = 0$

Bernoulli at free surface:

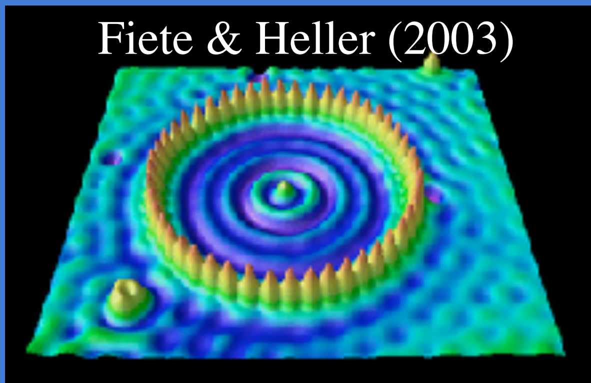
$$\frac{\partial\phi}{\partial t} + \frac{1}{2}|\mathbf{u}|^2 - \frac{\sigma}{\rho}\nabla^2 h + gh = 0$$

curvature

Taking gradient: $$\rho\frac{D\mathbf{u}}{Dt} = \nabla\left(\sigma\nabla^2 h - \rho g h\right)$$

- fluid moves in response to gradients in curvature and hydrostatic pressure

Madelunged Linear Schrodinger

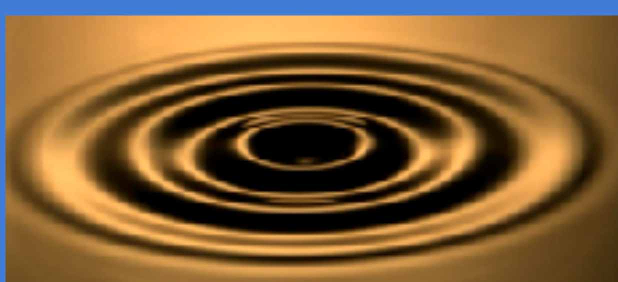


$$\frac{D\rho}{Dt} + \rho\,\nabla\cdot\mathbf{u} = 0$$

$$\frac{\partial S}{\partial t} + \frac{1}{2}\,\mathbf{u}^2 - \boxed{\frac{\hbar^2}{2m^2}\,\frac{1}{\sqrt{\rho}}\,\nabla^2\sqrt{\rho}} + \frac{V}{m} = 0$$

QUANTUM POTENTIAL

2D Euler equations

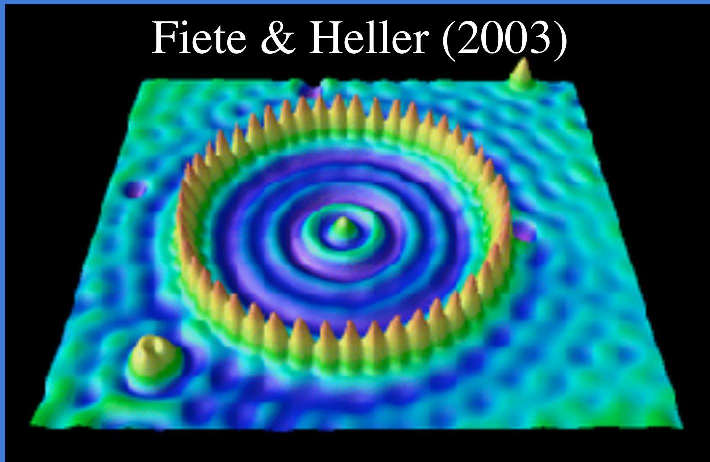


$$\frac{Dh}{Dt} + h\,\nabla\cdot\mathbf{u} = 0$$

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}\,\mathbf{u}^2 - \boxed{\frac{\sigma}{\rho}\,\nabla^2 h} + gh = 0$$

CURVATURE

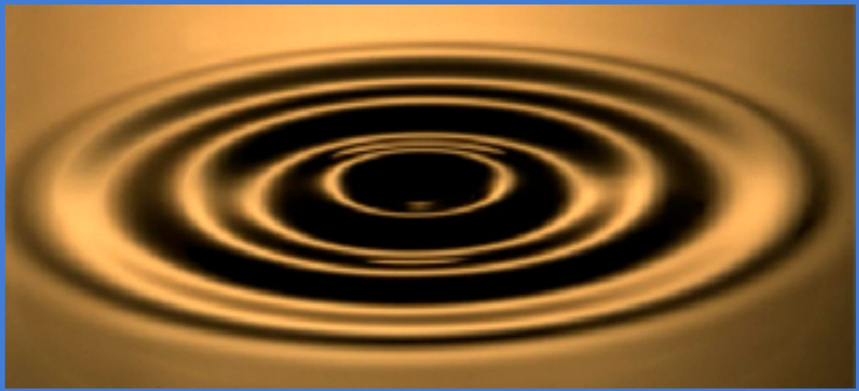
Free quantum particles



- particle energy prescribes $\omega = \dfrac{E_p}{\hbar}$
- dispersion relation:

$$\omega = \frac{\hbar}{2m} k^2 \quad \text{sets} \quad \lambda_{dB}$$

Shallow water capillary Faraday waves



- forcing frequency prescribes $\omega_F = \dfrac{\omega_0}{2}$
- dispersion relation:

$$\omega_F = \left(\frac{\sigma h_0}{\rho}\right)^{1/2} k^2 \quad \text{sets} \quad \lambda_F$$

- one can model the **statistics** of quantum particles hydrodynamically, i.e. probability waves with capillary Faraday waves
- does not inform dynamics of individual electrons in quantum corrals
- Note: **walker statistics** (in corral expts) also prescribed by amplitude of Faraday wave modes. *Might the underlying dynamics also be similar?*

Bohmian Mechanics (1952)

David Bohm

- equate quantum velocity of probability $\mathbf{u}$ and particle velocity $\dot{\mathbf{x}}_p$
- solve Schrodinger's equation for Ψ, from which Q is computed
- solve trajectory equation

$$m\,\ddot{\mathbf{x}}_p = -\nabla Q - \nabla V$$

Pros

- philosophical appeal: a successful Hidden Variable Theory
- a counterexample of the Impossibility Proofs that held sway at the time
- given a $|\Psi|^2$ distribution as initial conditions, results are equivalent to those of standard QM
- restores reality and determinism to QM

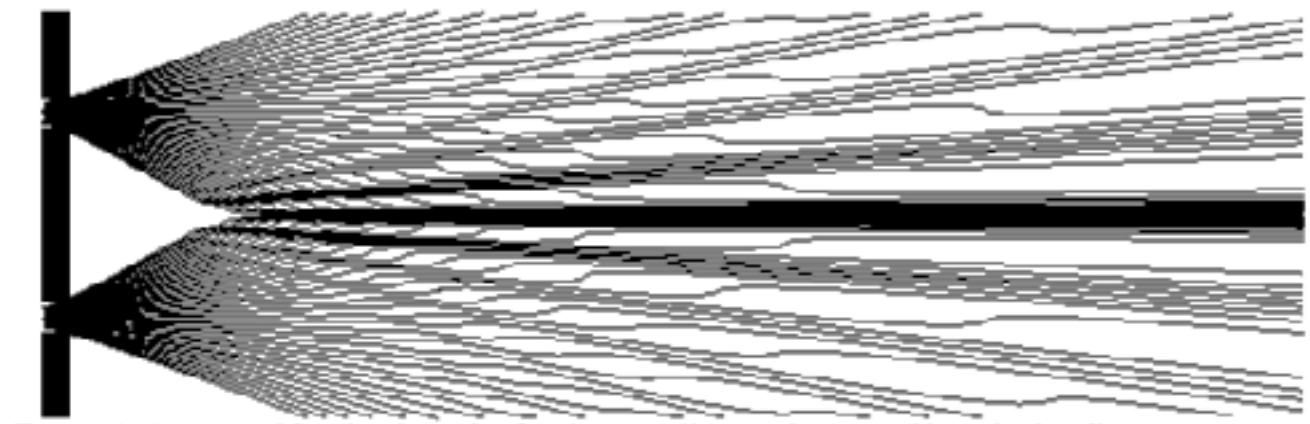


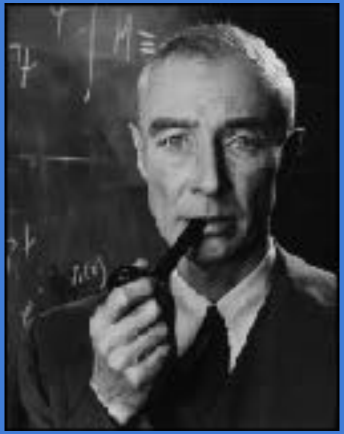
FIGURE 1: An ensemble of trajectories for the two-slit experiment, uniform in the slits. (adapted by Gernot Bauer from Philippidis, Dewdney, & Hiley 1979: 23, fig. 3)

Comments on David Bohm's Theory

"This isn't right. It isn't even wrong."

"Foolish simplicity, beyond all help."

— Pauli



"Juvenile deviationism."

"Bohm is a Trotskyist and a traitor."

"If we can't disprove him, we must agree to ignore him."

— Oppenheimer

"There is an air of contrivance about it that makes it unappealing. For example, the hidden wave has to satisfy a wave equation. Where does this equation come from? The frank answer is out of thin air or, more accurately, out of the mind of Schrödinger."

— Polkinghorne (2003)

Recall: Schrodinger derived the LSE to describe de Broglie's pilot wave

Bohmian Mechanics (1952)


David Bohm

- equate quantum velocity of probability $\mathbf{u}$ and particle velocity $\dot{\mathbf{x}}_p$
- solve Schrodinger's equation for Ψ , from which Q is computed
- solve trajectory equation

$$m\,\ddot{\mathbf{x}}_p = -\nabla Q - \nabla V$$

NONLOCAL

Shortcomings

- solution must be assumed to get the correct answer (Joe Keller 1956)
- Einstein's objection: it is `*nonlocal'* by virtue of the quantum potential Q
- no mechanism for wave generation; no effect of particle on field

Extensions (Bohm & Vigier 1954)

- invoke a stochastic forcing $\nabla\Phi_S$ from a `sub quantum realm':

$$m\,\ddot{\mathbf{x}}_p = -\nabla Q - \nabla V + \nabla\Phi_S$$

- particles jostle about $\mathbf{u}$ like Brownian motion of gas molecules about streamlines

Shortcomings of the quantum pilot-wave theories

- *no precise mechanism specified for pilot-wave generation*

Bohmian mechanics

- a dynamical reformulation of a statistical theory
- particle is piloted by a wave form Ψ of unspecified origins
- *nonlocal*: particle is guided by the non-local quantum potential

de Broglie's mechanics

- original double-solution theory distinguished between ϕ and Ψ
- form of pilot-wave ϕ unspecified: several options considered
- at one stage set $\phi \propto \Psi$: reduces to Bohmian mechanics

→ two theories conflated into `de Broglie-Bohm theory'

Common objections to pilot-wave theory

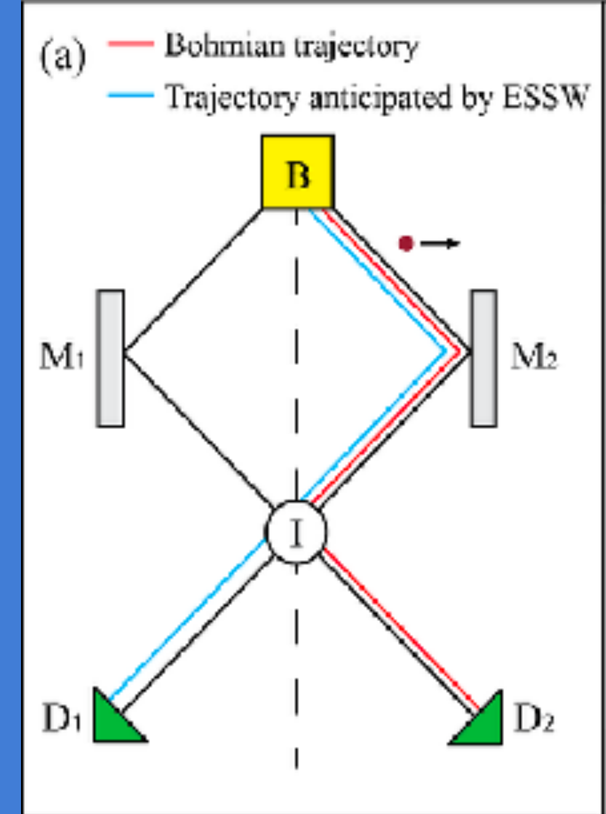
- Occam's razor: why add a trajectory equation, when you can just work with LSE
- what are the origins of the wave? In what medium do they propagate?
- too contrived and complicated (esp. de Broglie's): how could it ever work?

A real beauty

"Attempts have been made by de Broglie, David Bohm and others to construct theories based on hidden variables, but the theories are very complicated and contrived. For example, the electron would definitely have to go through only one slit in the two-slit experiment. To explain that interference occurs only when the other slit is open, it is necessary to postulate a special force on the electron which exists only when that slit is open. Such artificial additions make hidden variable theories unattractive, and there is little support for them among physicists."

- Encyclopedia Britannica (2007)

Experimental support for pilot-wave theories



Pro-Bohm

- Mahler *et al*. (2016) used weak measurement to confirm that mean trajectories are *surreal*
- consistent with the Ensemble Interpretation: Bohm equates particle velocity to quantum velocity of probability

Pro-de Broglie

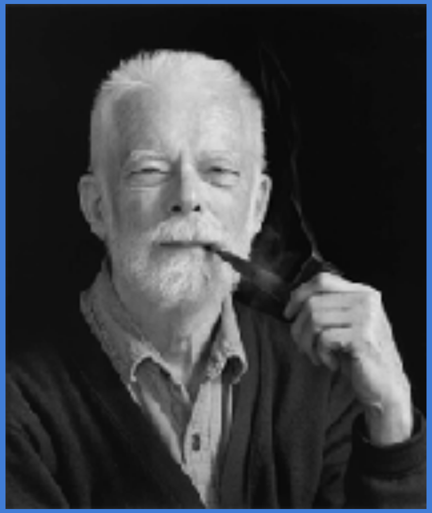
- evidence of ZTB frequency, de Broglie's internal clock, in electron channeling
- the quest goes on….

A Search for the de Broglie Particle Internal Clock by Means of Electron Channeling

P. Catillon · N. Cue · M.J. Gaillard · R. Genre · M. Gouanère · R.G. Kirsch · J.-C. Poizat · J. Remillieux · L. Roussel · M. Spighel

Received: 15 January 2008 / Accepted: 27 May 2008 / Published online: 8 July 2008

Ed Nelson



Stochastic Mechanics (1958)

- Bohm & Vigier (1954) posited a stochastic subquantum motion complementing the Madelung flow

- Nelson (1958) showed that LSE describes Brownian motion of a mass m with diffusivity $\hbar/m$

$$D \sim \frac{\hbar}{m} \sim \frac{\hbar k}{mk} \sim U\lambda_B$$

- a random walk with characteristic velocity U and length scale λ_B

- Surdin (1972) proposed the zero-point field as the source of the stochasticity

- this approach has been forwarded by the group of Gerhard Groessing (2013 onwards) who took inspiration from the walking droplets

The subquantum realm

In their later years, both de Broglie (1987) and Bohm appealed to a stochastic sub quantum realm, in what is now known as ….

The quantum vacuum

The quantum vacuum is seen as a turbulent sea, roiling with waves associated with a panoply of fields, including electromagnetic and Higgs fields, as well as those responsible for the weak and strong forces. Insofar as they interact with quantum particles, all such fields are candidates for de Broglie's pilot-wave.

Vacuum-based pilot-wave theories…

… seek de Broglie's pilot-wave theory in the quantum vacuum fields.

The Quantum Vacuum and Stochastic Electrodynamics

— Boyer 2011, Milonni 2013

- at zero temperature, there is electromagnetic (`zero-point') energy
- provides alternative explanations for Casimir forces, van der Waals forces, the blackbody radiation spectrum
- there is only one spectral form that is homogeneous, isotropic, scale invariant and Lorentz invariant

ENERGY PER NORMAL MODE: $U(\omega) = C\omega/c$ where C is a constant

- empirical fact deduced from experiments on Casimir effect: $C = \hbar c/2$

Zero point energy: $$U(\omega) = \hbar\omega/2$$

- provides a natural means of introducing $\hbar$ into a classical theory
- might provide the seed field for de Broglie's matter waves, which would thus be of electromagnetic origin (*de la Pena, Cetto & Valdes-Hernandes, 2015*)

The Zitterbewegung Interpretation of QM (Hestenes, 1990)

Zitterbewegung (ZTB)

- the internal vibration of subatomic particles at the Compton frequency $\omega_c = \frac{mc^2}{\hbar}$
- first proposed by Schrodinger (1930) in his analysis of solutions of the Dirac eqn
- interpreted by Hestenes (1990) as being the orbital frequency of electrons

  - electrons orbit a Compton radius $\lambda_c = \hbar/mc$ at the speed of light c

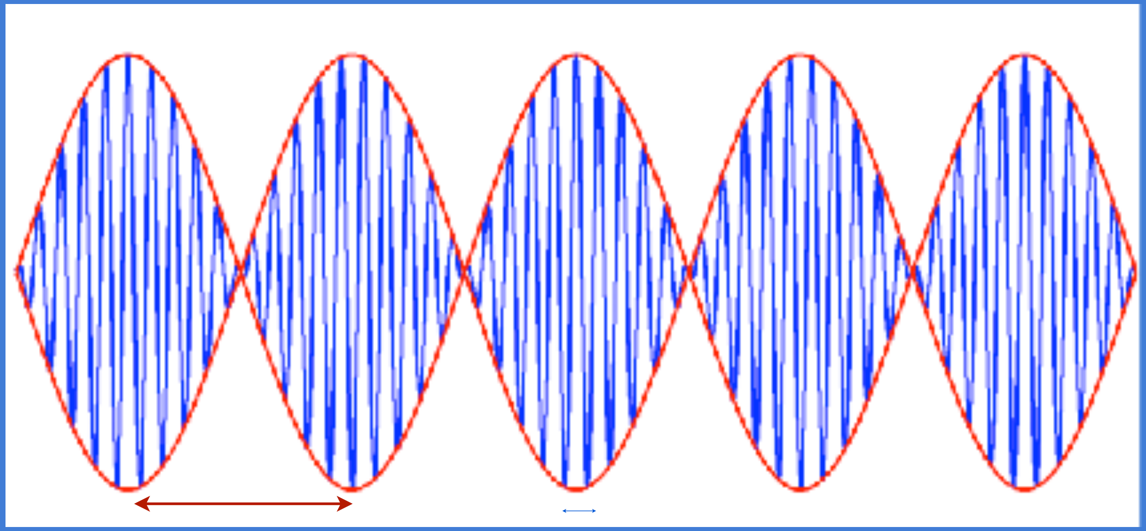
  - their associated frequency is $\omega_c = \frac{mc^2}{\hbar}$ and angular momentum $S = \hbar/2$

- the electron mass is associated with its electromagnetic self-interaction
- consistent with the Kerr-Newman model of the electron
- any deviation from a circular orbit will result in variations in mass

  - would provide the source of vibration in de Broglie's double wave solution

The Quantum Pilot Wave (according to SED)

- EM wave generated by resonant interaction between particle ZTB and the vacuum fluctuations



$$\lambda_B = \frac{h}{mv} \qquad \lambda_c = \frac{h}{m_o c}$$

"The de Broglie wave is the wave formed by the modulation of the Lorentz-transformed, Doppler-shifted superposition of the whole set of random, stationary EM waves with the Compton frequency with which the particle interacts."

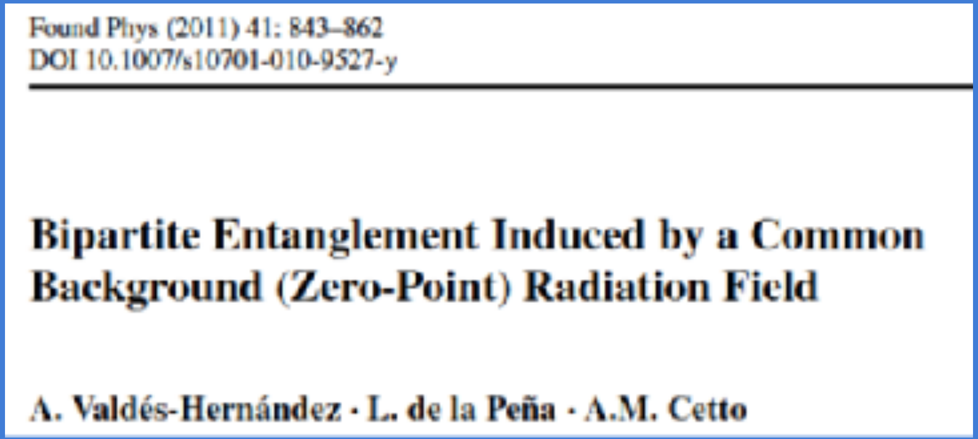
- De la Pena & Cetto (Quantum Dice, 1997)

- mass is simply a place holder for electromagnetic energy (Haesch & Rueda 2001)
- particle mass increases with speed due to increased interaction with vacuum field

Vacuum-based pilot-wave models

- modern extensions of de Broglie's pilot-wave theory, reminiscent of walkers
- suggests that QM paradoxes may be resolved by elucidating dynamics on the Compton scale

- in quantum field theory, the Compton frequency sets the time and length scales of particle pair production from the vacuum, which poses a challenge to experimental probing of such scales.



Found Phys (2011) 41: 843–862
DOI 10.1007/s10701-010-9527-y

Bipartite Entanglement Induced by a Common Background (Zero-Point) Radiation Field

A. Valdés-Hernández · L. de la Peña · A.M. Cetto

And quantum non-locality?

- identical particles interact through a common EM pilot wave
- rationalize entanglement in terms of classical, wave-induced correlations

Insights into quantum pilot-wave theories

- walking drops represent an example of a particle moving in its own wave field
- theoretical descriptions of such motions on a microscopic scale have been fraught with difficulties
- for example, the Lorentz-Dirac (LD) equation describing the trajectory of an electron in its own EM field has non-physical runaway solutions (Hammond 2010)

  - solutions fail at times short relative to the Compton scale

Interesting questions

- might there be asymptotic limits in which our trajectory equation resembles LD?

E.g. Avendano (2010) derived $\mathbf{v} \sim \frac{\nabla |E|}{|E|}$ which resembles PWH in no-inertia limit

- might the LD equation be an asymptotic expansion of limited validity, derivable from a trajectory equation with a pilot-wave form?

Relation between the hydrodynamic and QM pilot-wave theories

Table 1 A comparison among the walking droplet system, de Broglie's double-solution pilot-wave theory (de Broglie 1956, 1987), and its extension to stochastic electrodynamics (SED) (Kracklauer 1992, de la Peña & Cetto 1996, Haisch & Rueda 2000)

	Walkers	de Broglie	SED pilot wave
Pilot wave	Faraday capillary	Unspecified	Electromagnetic (EM)
Driving	Bath vibration	Internal clock	Vacuum fluctuations
Spectrum	Monochromatic	Monochromatic	Broad
Trigger	Bouncing	Zitterbewegung	Zitterbewegung
Trigger frequency	ω_F	$\omega_c = mc^2/\hbar$	$\omega_c = mc^2/\hbar$
Energetics	GPE $\leftrightarrow$ wave	$mc^2 \leftrightarrow \hbar\omega$	$mc^2 \leftrightarrow$ EM
Resonance	Droplet—wave	Harmony of phases	Unspecified
Dispersion $\omega(k)$	$\omega_F^2 \approx \sigma k^3/\rho$	$\omega^2 = \omega_c^2 + c^2k^2$	$\omega = ck$
Carrier λ	λ_F	λ_{dB}	λ_c
Statistical λ	λ_F	λ_{dB}	λ_{dB}

In the walker system, energy is exchanged at ω_F between the drop's gravitational potential energy (GPE) and the capillary Faraday wave field. Zitterbewegung denotes particle oscillations at the Compton frequency ω_c.

Bush, ARFM (2015)

Some final remarks on foundational issues…