Some preliminaries

- all course materials posted on my webpage

  https://thales.mit.edu/bush/

- will send course announcements via email

- will plan course, problem set according to class demographics

Some leftovers from Lecture 1

Perspective

- at the time that pilot-wave theory was developed by de Broglie, there was no macroscopic analog to draw upon.

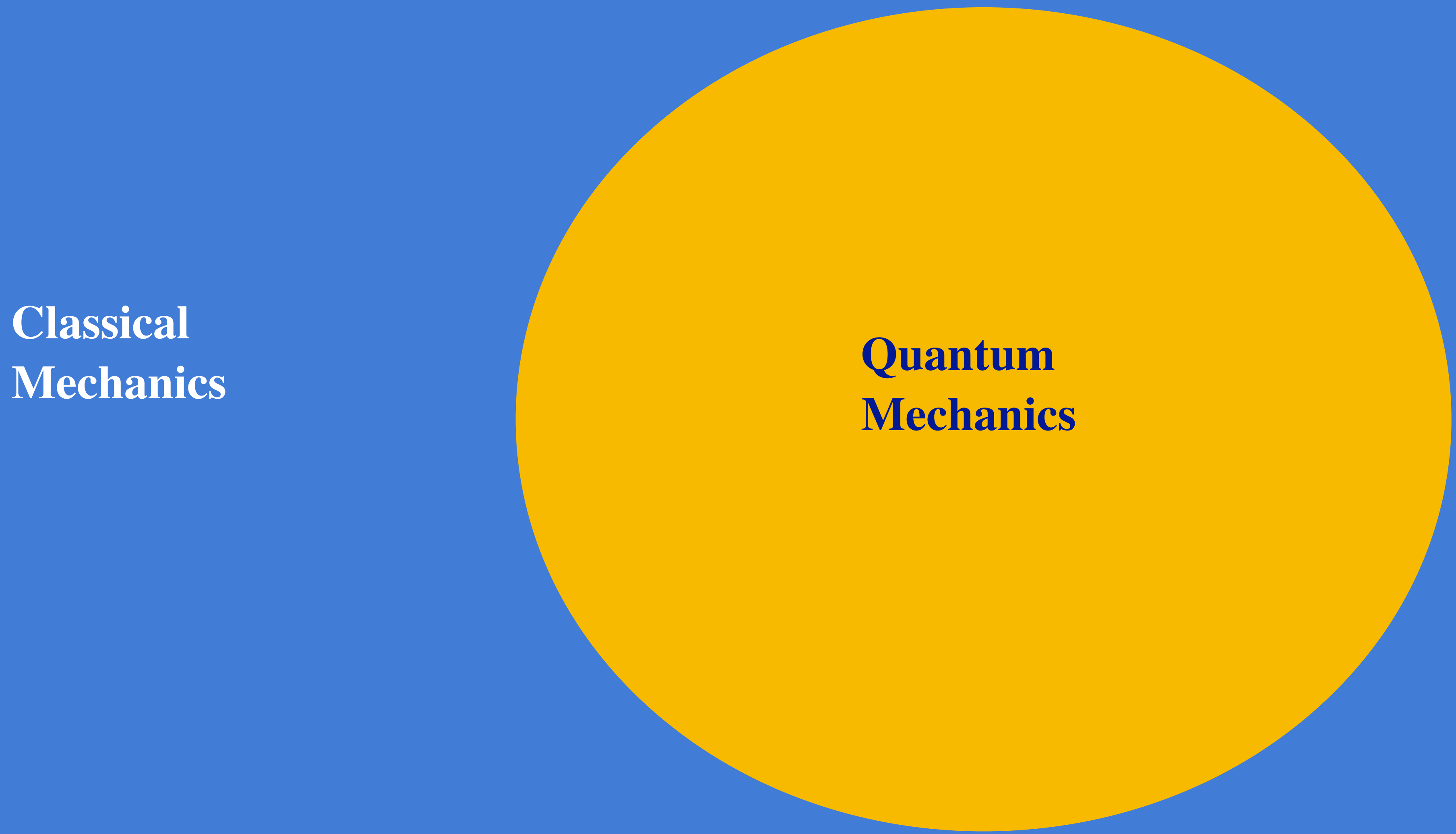
Now there is.

- pilot-wave dynamics can give rise to quantum-like behaviour on the macroscale.

So why not the microscale?

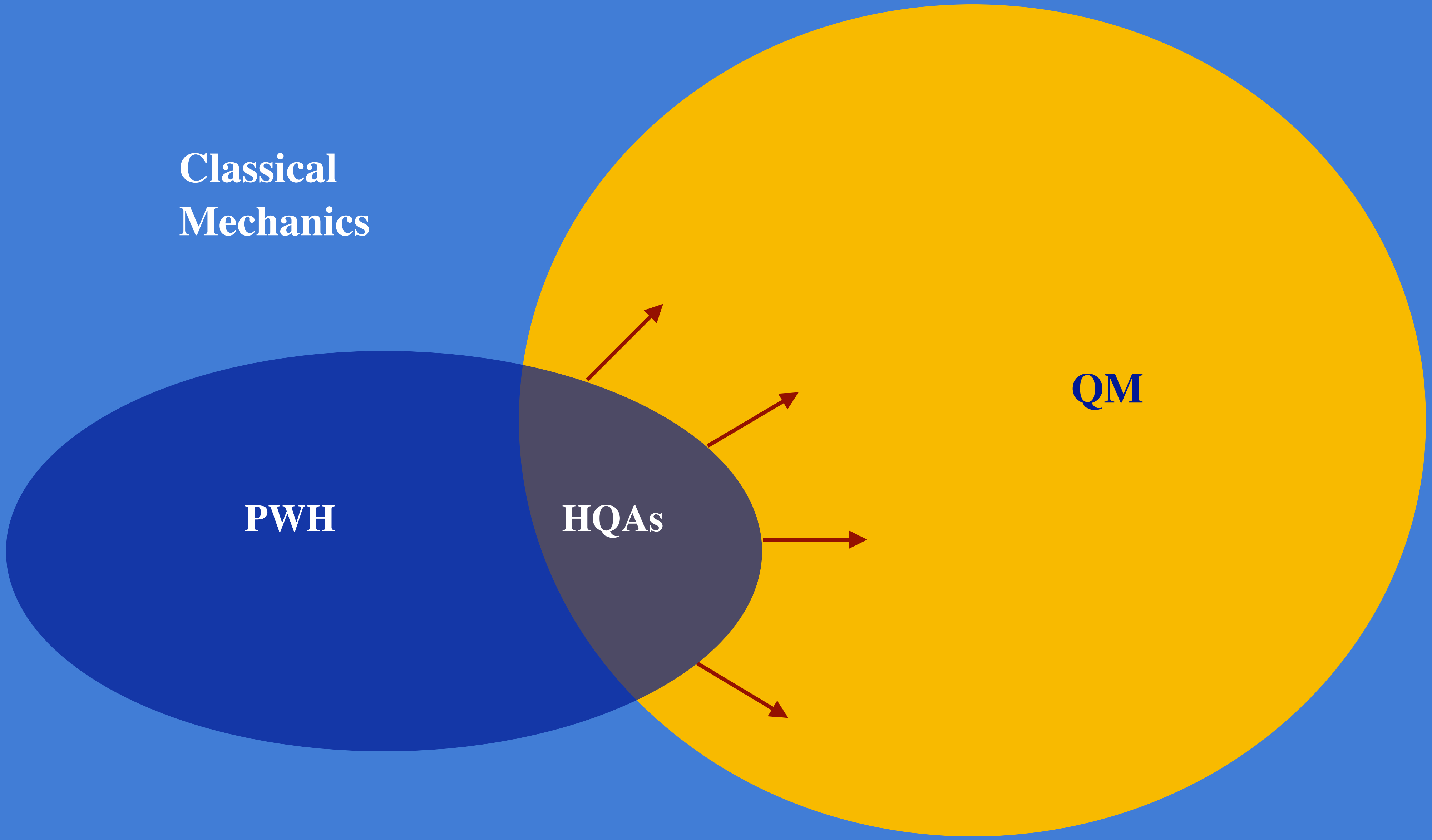
BIG PICTURE

- the landscape before PWH: classical mechanics and quantum mechanics



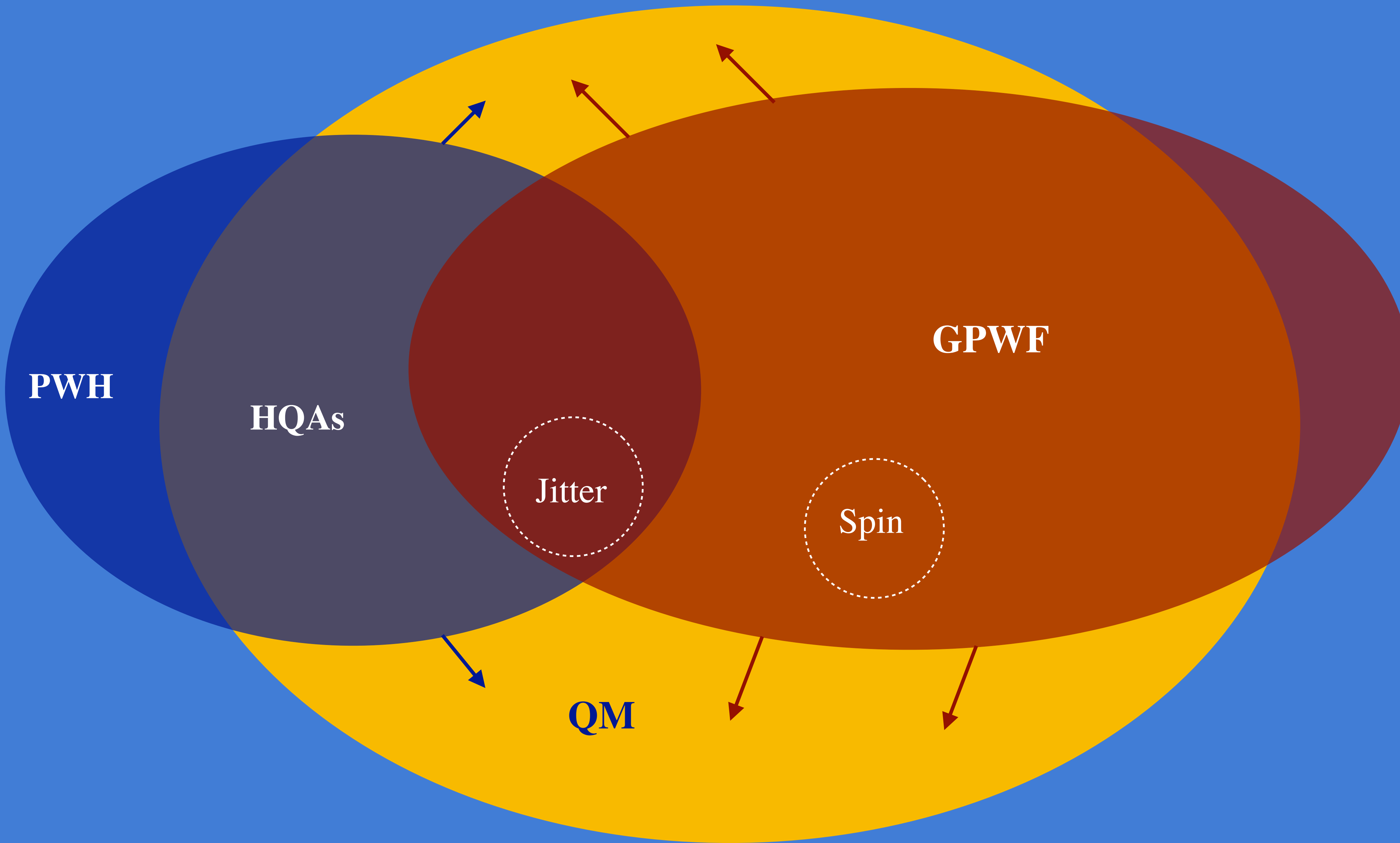
BIG PICTURE

- enter pilot-wave hydrodynamics



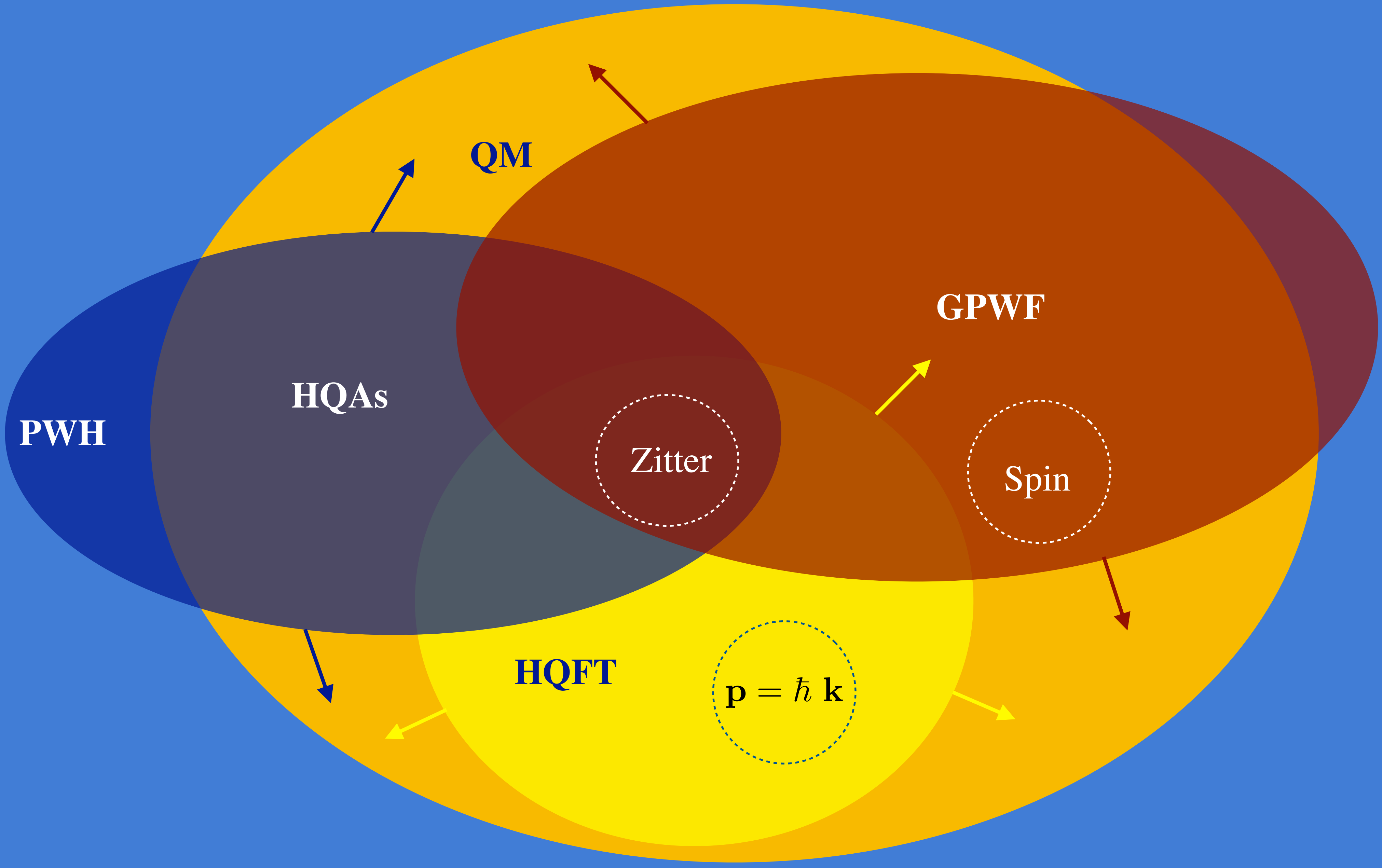
BIG PICTURE

- has motivated exploration of the Generalized Pilot-wave Framework



BIG PICTURE

- has inspired the development of a number of HQFTs



The appeal of walking droplets

- timescales of bouncing, lateral motion, statistical convergence all accessible

- a macroscopic example of a particle moving in response to its own wave field, the theoretical description of which is notoriously difficult on the microscale

  — *e.g.* the Lorentz-Abraham-Dirac equation of EM

- connects to a number of hidden-variable theories:

  — de Broglie's double-wave solution, Bohmian mechanics, Stochastic Mechanics (*Nelson 1964*), Stochastic Electrodynamics (*de la Peña et al. 2018*)

- spans wide range of disciplines: experimental & theoretical fluid mechanics, dynamical systems, connections to equations of EM and QM, history of QM, philosophy of science

- provides a platform for exploring the boundary between classical and quantum behaviour

COURSE OUTLINE

Lecture 1. Feb. 5 Introduction

- course survey, motivation and philosophy

Lecture 2. Feb 7. Analogical thinking

- degrees of similitude and modes of comparison
- metaphor, physical analogy, dynamic similarity, statistical similarity, philosophical similarity

Lecture 3. Feb 12. Quantum history and foundations

- quantum interpretations, impossibility proofs and paradoxes
- quantum pilot wave theories: from de Broglie to Bohm to stochastic electrodynamics

Lecture 4. Feb 14. The early HQAs of Yves Couder

- the discovery of walking droplets
- single droplet diffraction and interference
- orbital dynamics, tunneling, bound states

Lecture 5. Feb 20. (Monday schedule) Hydrodynamic preliminaries I

- continuum mechanics and Navier Stokes equations
- surface tension and interfacial phenomena

Lecture 6: Feb. 21 Hydrodynamic preliminaries II

- water waves: gravity and capillary waves
- Faraday waves on a vibrating bath

Lecture 7: Feb. 26. Hydrodynamic preliminaries III

- droplet impact and non coalescence events
- drops on a vibrating soap film

Lecture 8. Feb. 28. Bouncing droplets

- drops on a vibrating liquid bath
- the theoretical modeling of the drop dynamics and wave field

Lecture 9: March 4. Walking droplets (APS March?)

- the transition from bouncing to walking
- discrete and continuous models

Lecture 10: March 6. The stroboscopic model

- stability of the walking state
- stability of orbiting and promenading pairs
- energetics of pilot-wave hydrodynamics

Lecture 11: Mar. 11. Orbital pilot-wave dynamics

- walkers in a rotating frame: analog Landau levels
- walkers in a central spring force: analog particle in a SHO
- origins of quantization, chaos and emergent statistics

Lecture 12: Mar. 13. More recent theoretical models

- modeling boundaries (Luiz Faria)
- Faraday pilot-wave model
- discrete time model
- Rayleigh oscillator and Boost models

Lecture 13: March 18. Single-particle diffraction and interference

- historical context
- experimental and theoretical modeling
- comparison with Bohmian mechanics

Lecture 14: March 20. More boundary interactions

- scattering off a submerged pillar: the logarithmic spiral
- interaction with a submerged well: Friedel oscillations
- motion over sloping topography

SPRING BREAK March 25-29. NO CLASS

Lecture 15: Apr. 1. Non-resonant effects

- ratcheting pairs, orbital instability, tunneling
- stability of droplet pairs and rings
- erratic motion in the hydrodynamic corrals

Lecture 16: April 3. Crossing the threshold

- the Faraday-Talbot effect
- droplets walking above the Faraday threshold
- superradiant droplet emission
- ponderomotive effects in the corral and Kapitza-Dirac diffraction

Lecture 17: April 8. Corrals

- circular corrals: periodic and chaotic motion
- statistical projection effects in elliptical corrals
- the mean pilot wave potential and its relation to the quantum potential
- modeling attempts

Lecture 18: Apr. 10. Motion in 1D cavities

- conformal maps in HQA
- single-particle tunneling; superradiant tunneling pairs
- droplet correlations at a distance

Patriot's Day: April. 15. No class.

Lecture 19: April 17. Droplet lattices

- spin lattices: long range correlations and phase transitions
- Anderson localization

Lecture 20: April 22. Hydrodynamic interferometry

- real surreal trajectories
- the misinference of interaction free measurement
- the Elitzur-Vaidman bomb tester

Lecture 21: April 24. Generalized pilot-wave framework

- spin states, in-line oscillations and chaotic motion
- orbital dynamics in a Bessel potential
- 3D classical pilot-wave theory

Lecture 22: April 29. Hydrodynamically-inspired pilot-wave theory for the microscopic scale

- extending de Broglie's double solution pilot wave theory
- towards a relativistic pilot-wave theory

Lecture 23: May. 1. Bell's Theorem

- the implications of Bell violations in quantum mechanics
- towards hydrodynamic Bell tests

Lecture 24: May. 6. Analog gravity

- hydrodynamic analogs of GR
- walkers as a vehicle for single-particle GR analogs

Lecture 25: May. 8. STUDENT PRESENTATIONS

Lecture 26: May. 13. STUDENT PRESENTATIONS. **Course Projects Due**

18.996: Lecture 2

Analogical thinking

Means of comparison and degrees of similitude

I. Metaphor

II. Physical analogy

III. Dynamic similarity

IV. Mathematical equivalence

V. Statistical similarity

VI. Philosophical similarity

Degrees of similitude

I. Metaphor

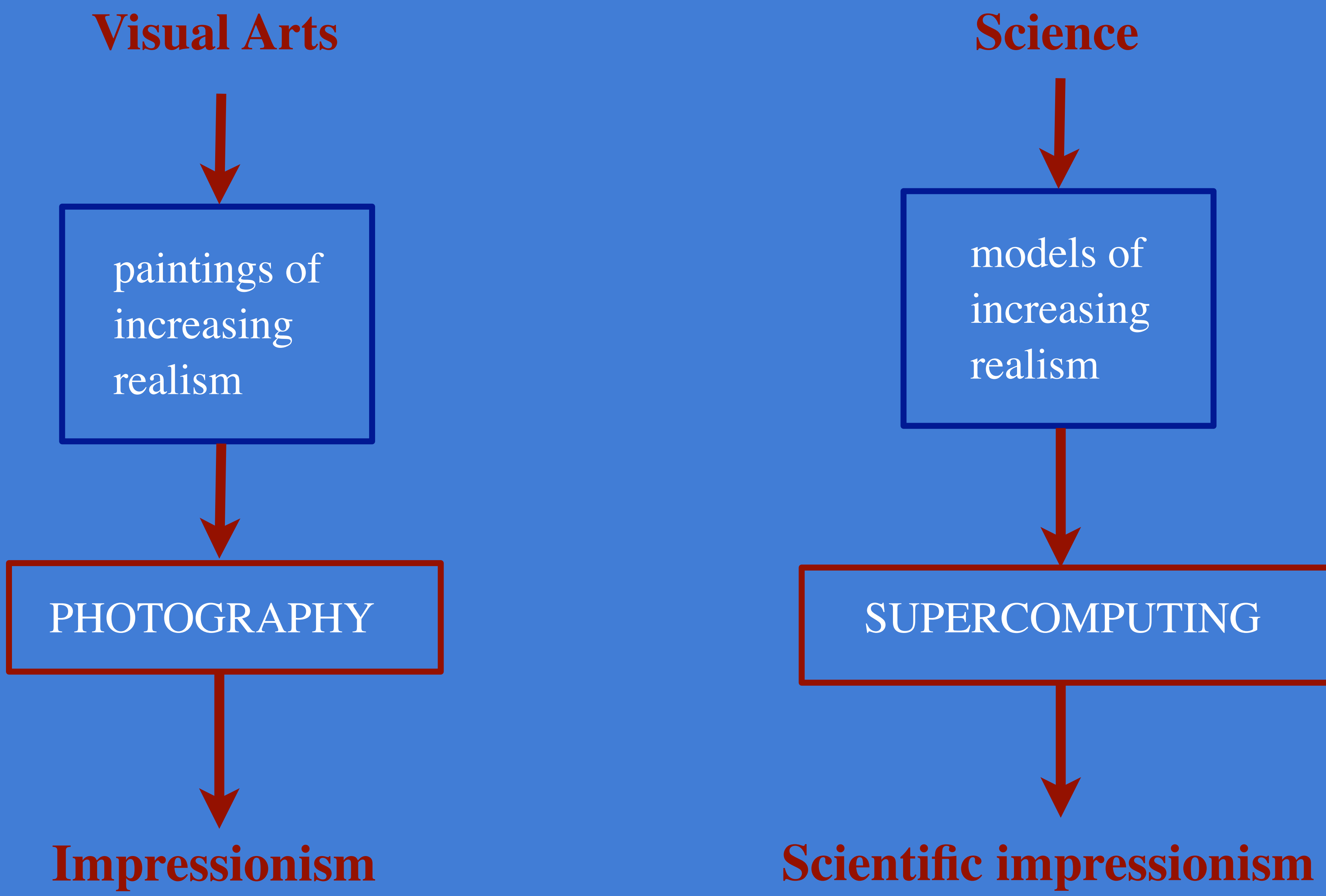
- an imprecise level of comparison that provides a means of using one system to gain insight into another

  e.g. the road unfurled before me

  e.g. the road was a ribbon of moonlight

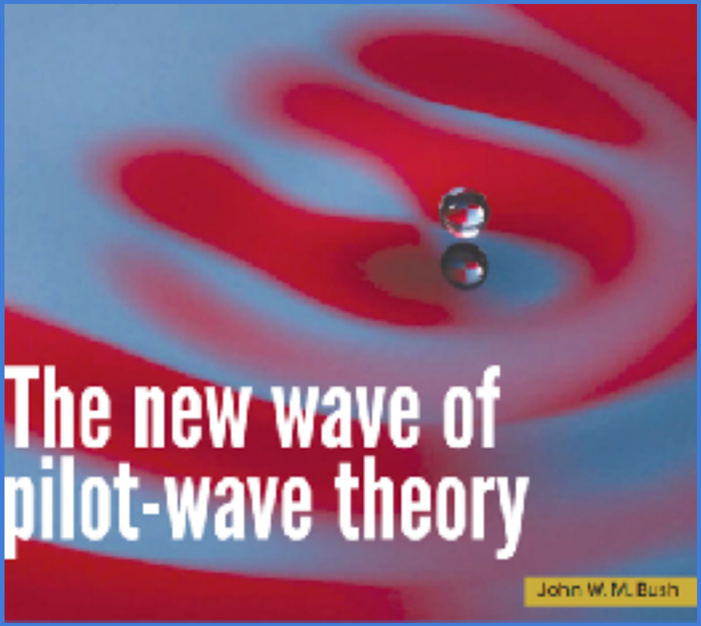
  e.g. the stars are blue and shiver in the distance

A METAPHOR ON STYLE IN SCIENCE



- capture the essential beauty/truth of a problem

A hydrodynamic metaphor



``If particle physics is the dazzling crown prince of science, fluid mechanics is the cantankerous queen mother: While her loyal subjects flatter her as being rich, mature, and insightful, many consider her to be démodé, uninteresting, and difficult. In her youth, she was more attractive. Her inconsistencies were taken as paradoxes that bestowed on her an air of depth and mystery. The resolution of her paradoxes left her less beguiling but more powerful, and marked her coming of age. She has since seen it all and has weighed in on topics ranging from cosmology to astronautics. Scientists are currently exploring whether she has any wisdom to offer on the controversial subject of quantum foundations."

- JWMB, Physics Today (2015)

The dangers of colorful scientific writing

I was recently perusing my copy of the August 2015 issue of PHYSICS TODAY, looking in particular at the feature articles. To my surprise, I read the following as the opening of "The new wave of pilot-wave theory" by John Bush:

> "If particle physics is the dazzling crown prince of science, fluid mechanics is the cantankerous queen mother: While her loyal subjects flatter her as being rich, mature, and insightful, many consider her to be démodé, uninteresting, and difficult. In her youth, she was more attractive."

I trust that Bush was intending to be charismatic and appeal to his male readers. However, I was disappointed that he did not think through the sexist stereotypes that this writing reinforces. Invoking a metaphor that casts women as the "cantankerous queens" of science does not help us to be treated with respect in the workplace.

Leslie Kerby
(kerblesl@isu.edu)
Idaho State University
Pocatello

LETTERS

Walking droplets, pilot waves, and word choices

▶ **Bush replies:**
I feel obliged to point out to Leslie Kerby that in my opening paragraph, I was appealing not to my male readership but rather to the careful reader. The cantankerous queen mother was a metaphor for the field of fluid mechanics, not women in science. She was, moreover, cast as the heroine.

John W. M. Bush
Massachusetts Institute of Technology
Cambridge PT

Degrees of similitude

I. Metaphor

II. Physical analogy

- may be drawn between two systems comparable in significant respects owing to similarities in their essential physics and underlying mathematical structure
- fluid mechanics provides a framework for modeling a broader class of nonfluidic systems, including electromagnetic, optical, quantum and gravitational systems

Maxwell on physical analogy

``By a physical analogy I mean that partial similarity between the laws of one science and those of another which makes each of them illustrate the other. . . .. We find the same resemblance in mathematical form between two different phenomena."

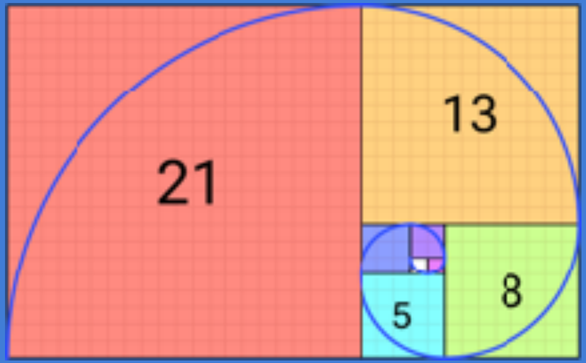
—James Clerk Maxwell, On Faraday's Lines of Force (1855).

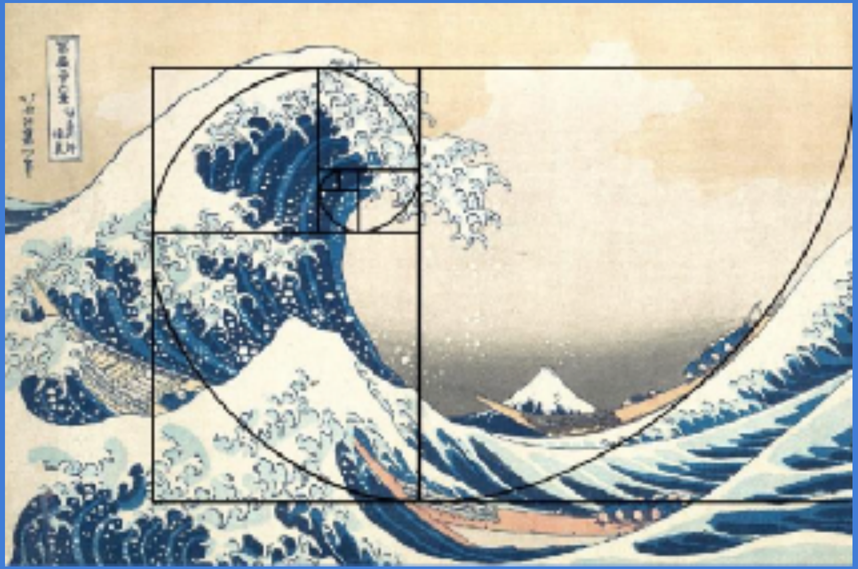
``Now, as in a pun two truths lie hid under one expression, so in an analogy one truth is discovered under two expressions. . . Every question concerning analogies is therefore a question concerning the reciprocal of puns, and the solutions can be transposed by reciprocation."

— James Clerk Maxwell, Are there real analogies in Nature? (1856).

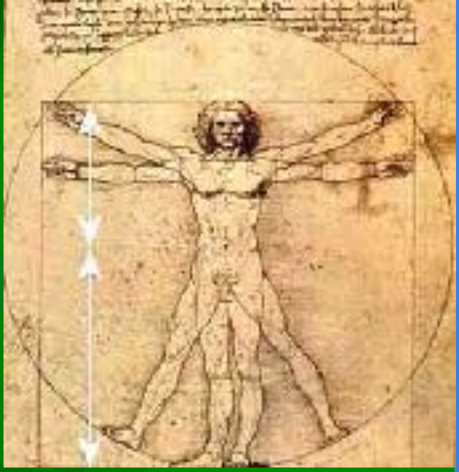
My favorite physical analogy

Fibonacci numbers in art —- ???





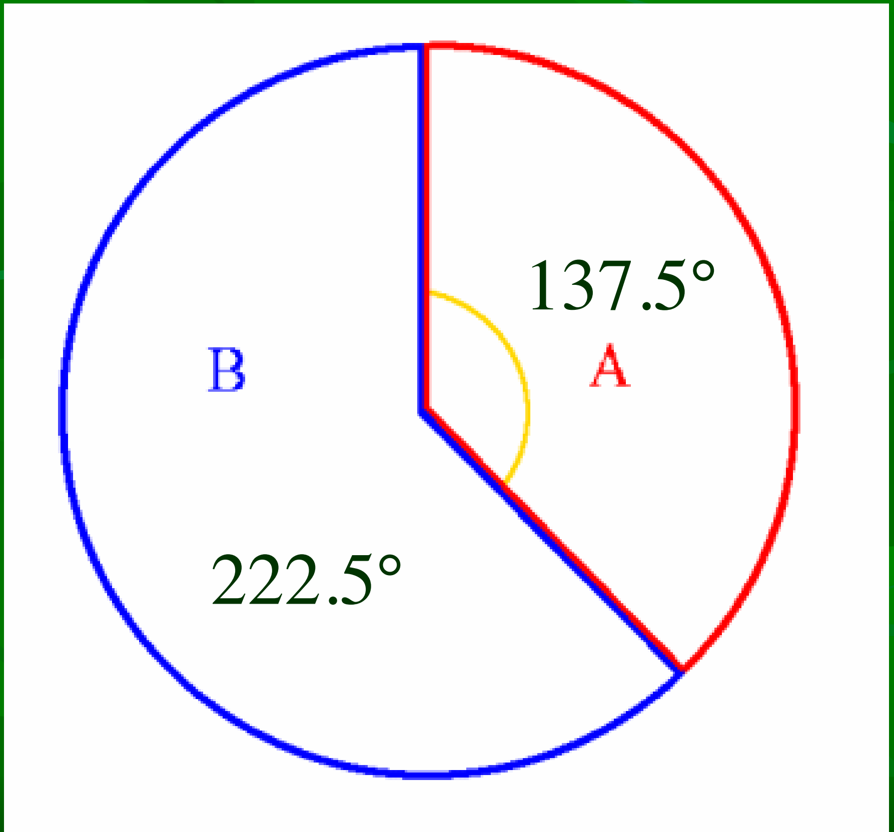
Φ ≈ 1.6180339887… is called the "Golden Ratio"



What do you think is called the "Golden Angle?"

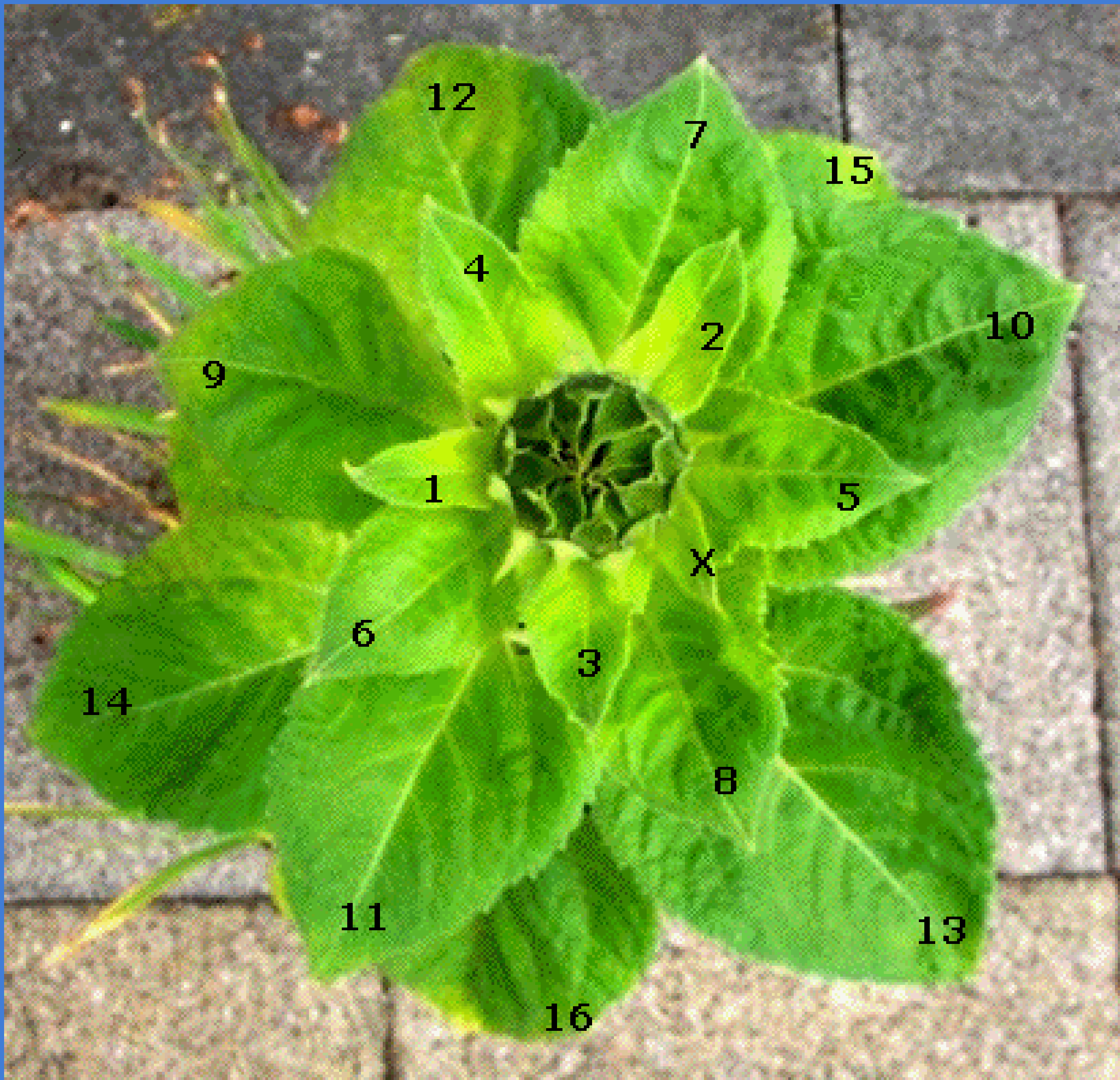
$$360° \times \Phi \approx 582.5°$$
$$= 222.5°$$

$$\Phi = \lim_{n \to \infty} \frac{F_{n+1}}{F_n}$$



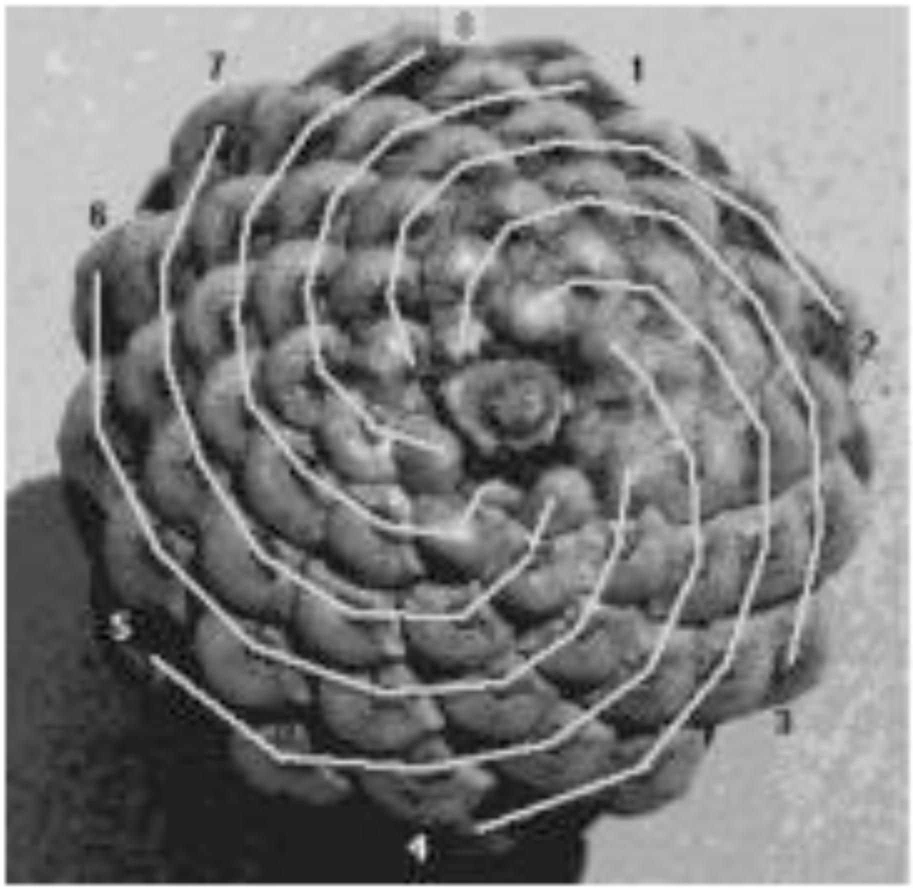
Fibonacci numbers in nature…

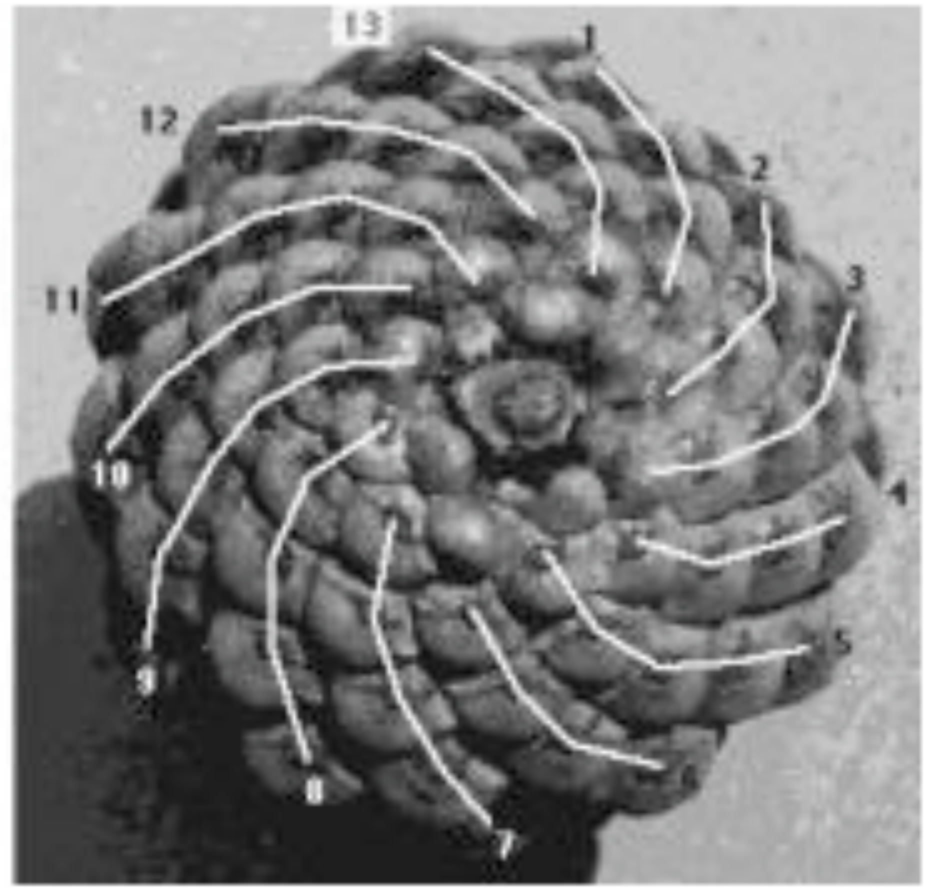
Plant leaves on stems emerge at the Golden Angle.



Why? To maximize the sunlight received.

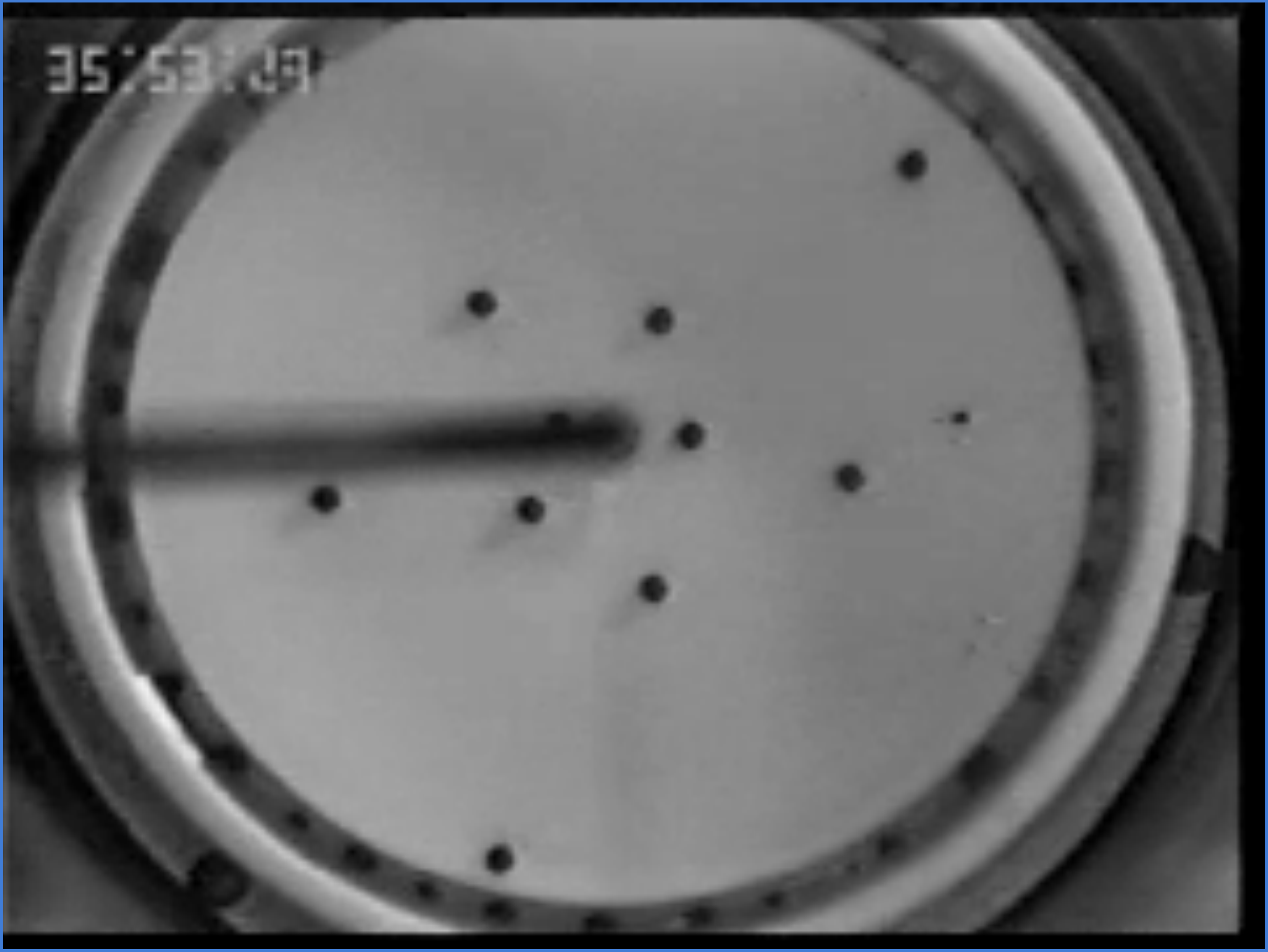
Fibonacci spirals in nature





“If you dig deeply enough into anything, you find mathematics.”

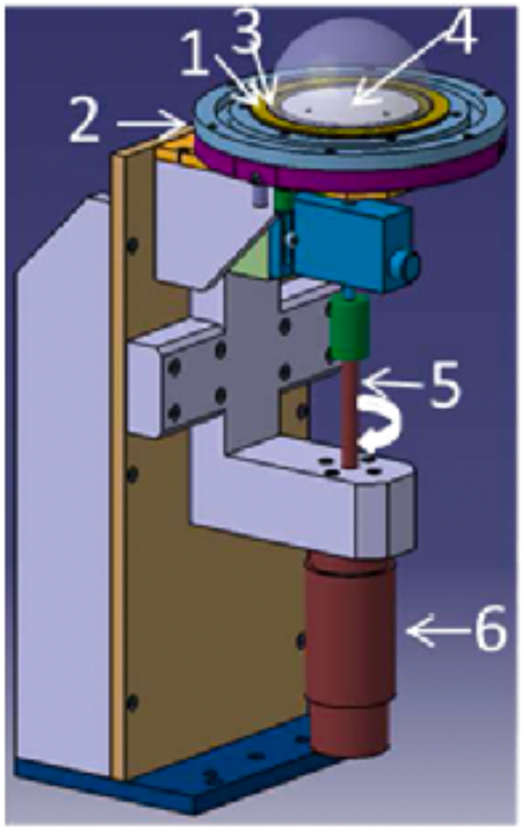
Douady & Couder (1991)

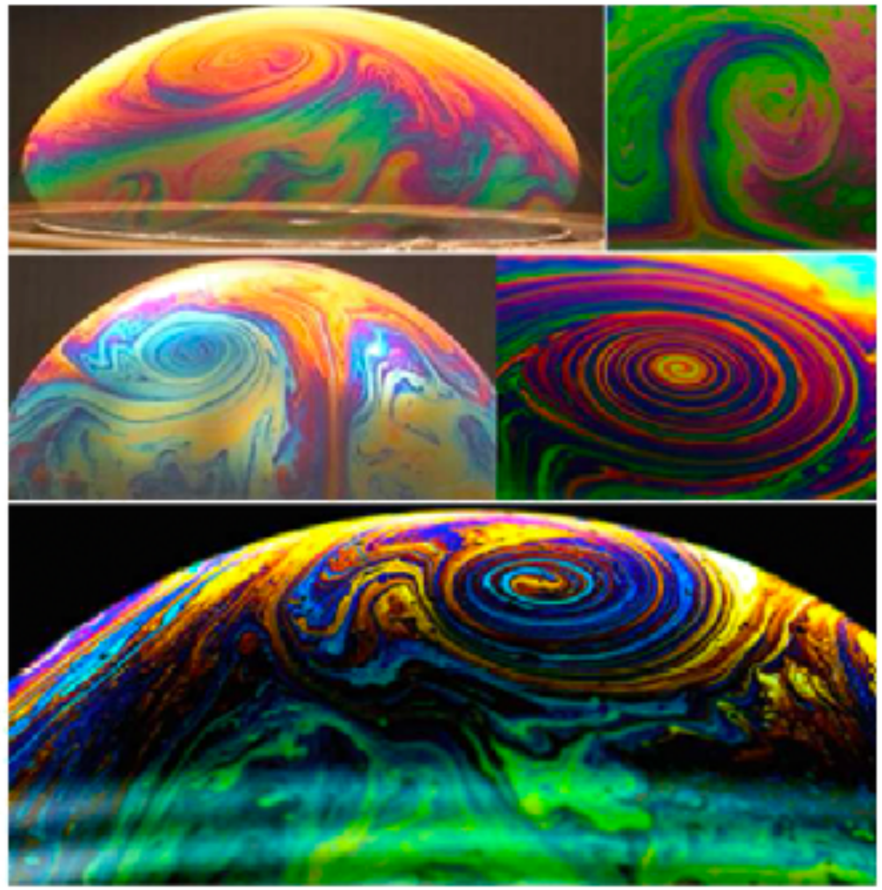


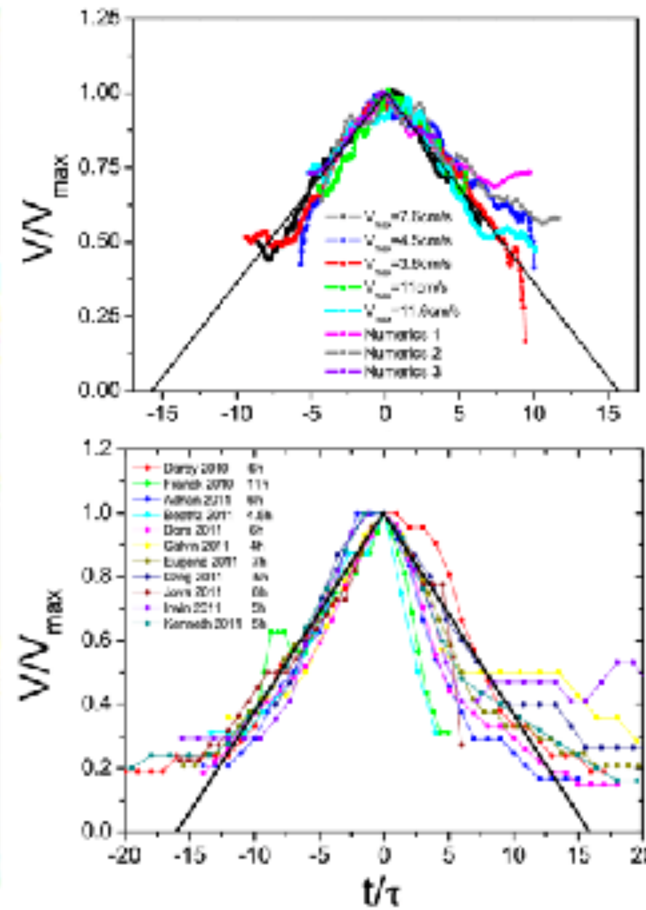
What makes this a good physical analogy?

Hurricanes in a soap film

- H. Kellay (2017)



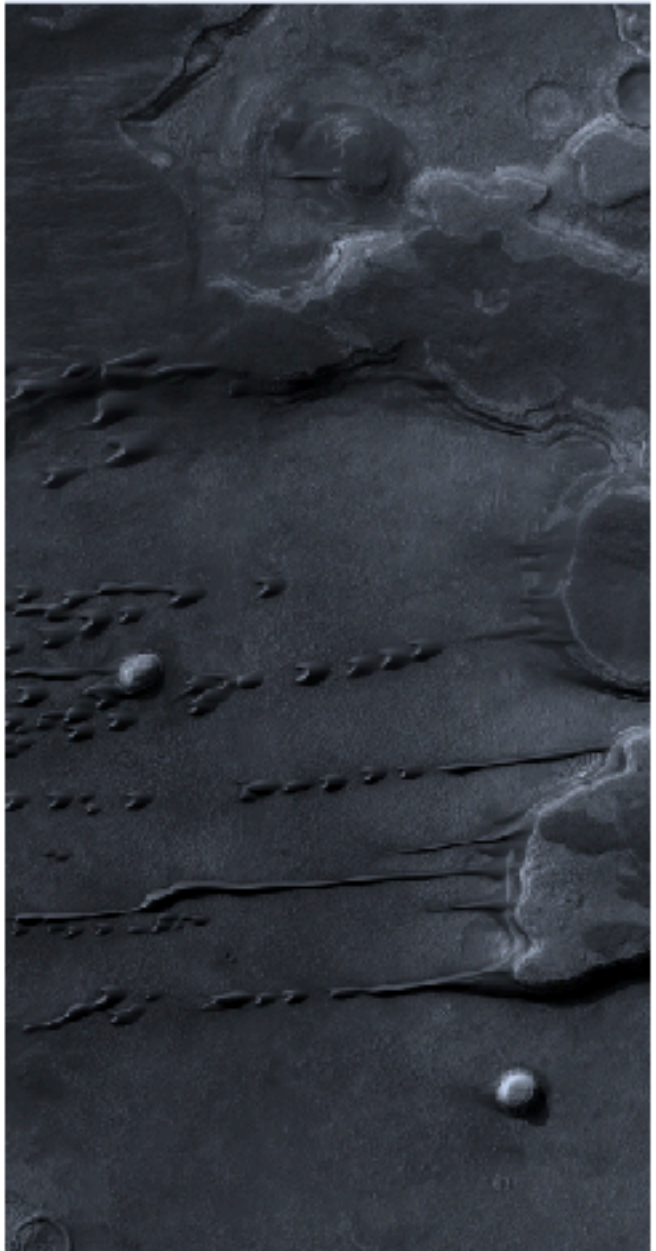




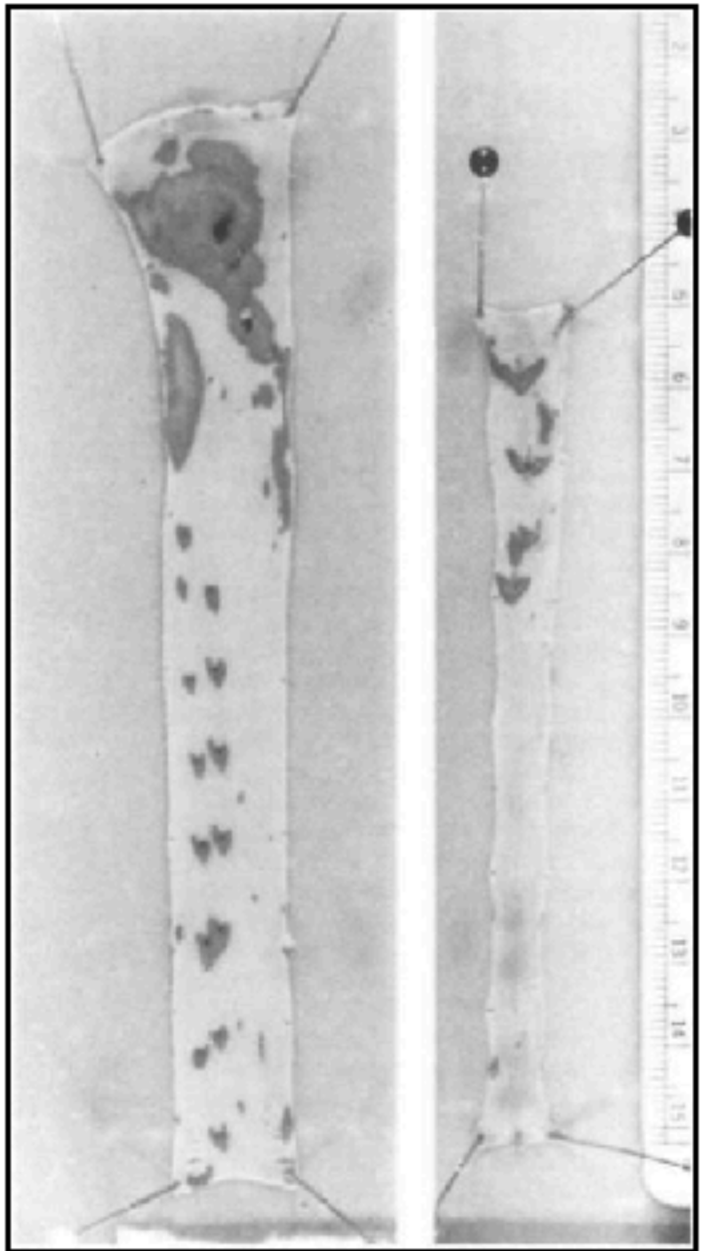
- heat and rotate base of a hemispherical soap film
- thermal plumes grow from the heated base of the soap film
- plumes generate long-lived, large-scale vortices
- intensification and relaxation of vortices matches that of hurricanes

Some physical analogies are better than others

Sand dunes



Rabbit artery



Hydrodynamic quantum analogues

- have to date primarily involved wave phenomena
- the walking drops represent the first analogs of single-particle QM

Hydrodynamic metaphor … or physical analogy?

"Light corpuscles generate waves in an Aethereal Medium, just like a stone thrown onto water generates waves. In addition, these corpuscles may be alternately accelerated and retarded by the waves."

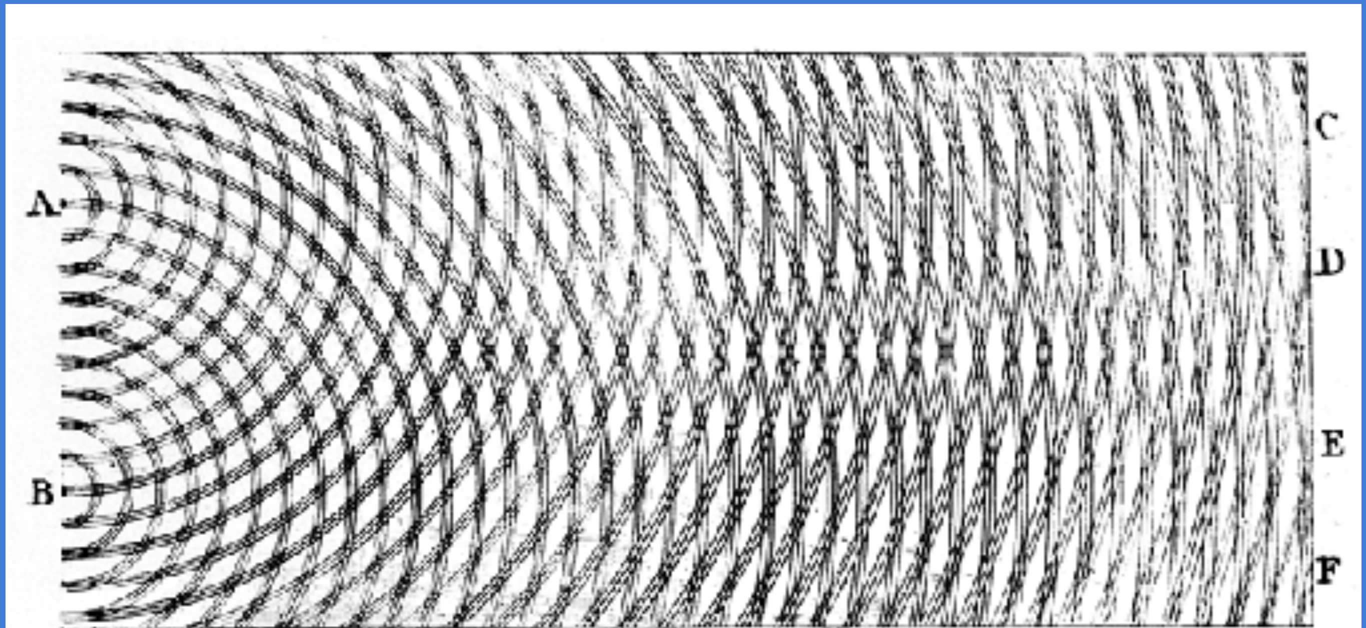
- Newton, Opticks (1704)

Thomas Young (1773-1829)

- contributed to theory of language, music harmony, solid mechanics, medicine, physiology, light, vision, surface tension

- deciphered the Rosetta stone

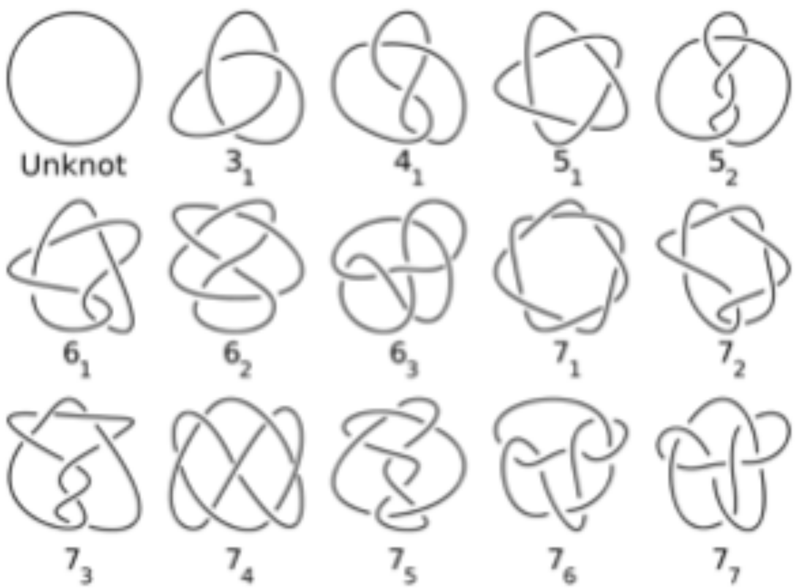
- felt that his greatest contribution was convincing the scientific community of the wave nature of light using ripple tank experiments

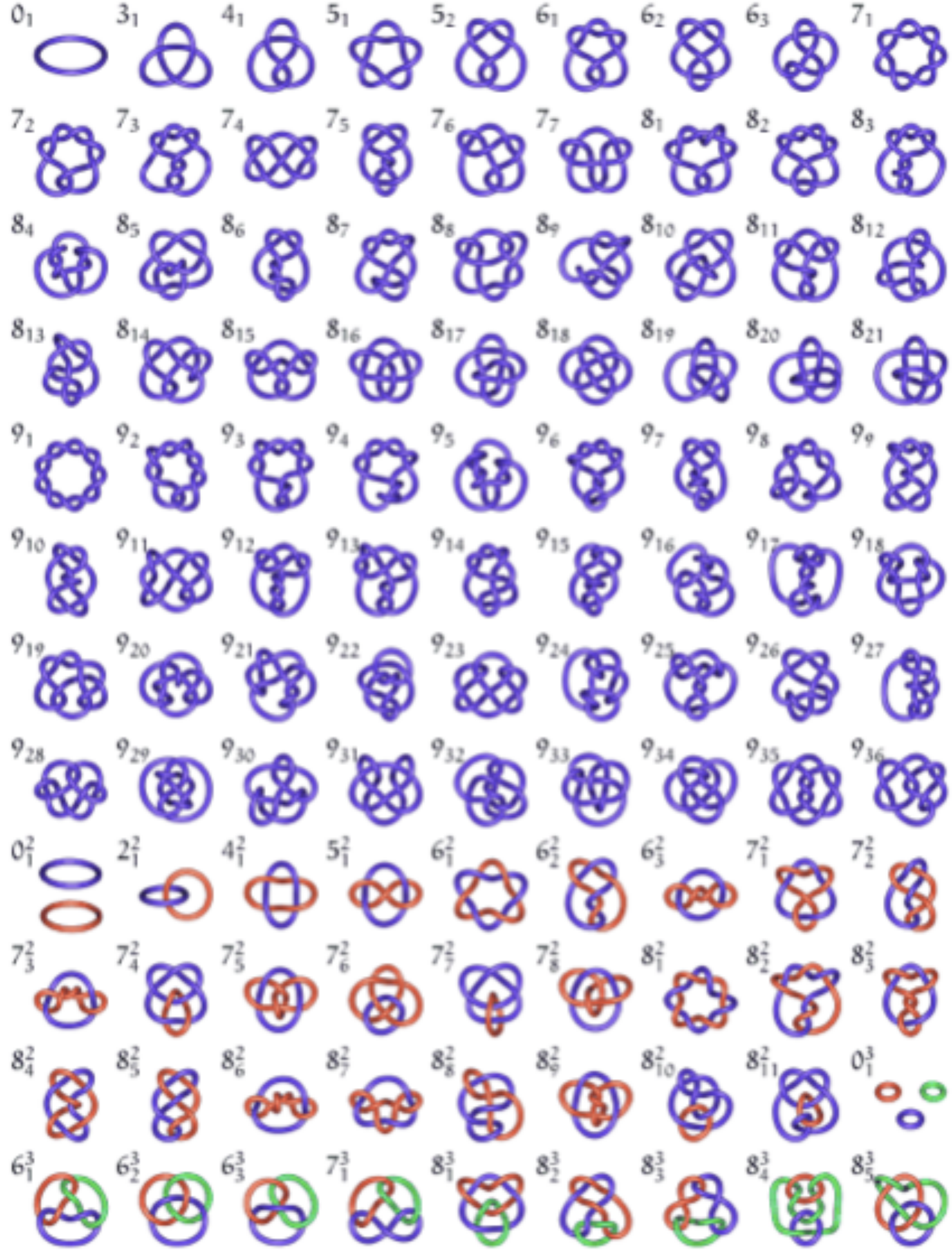


Young (1805)

Lord Kelvin (1880)

- proposed that subatomic particles were knots in the ether

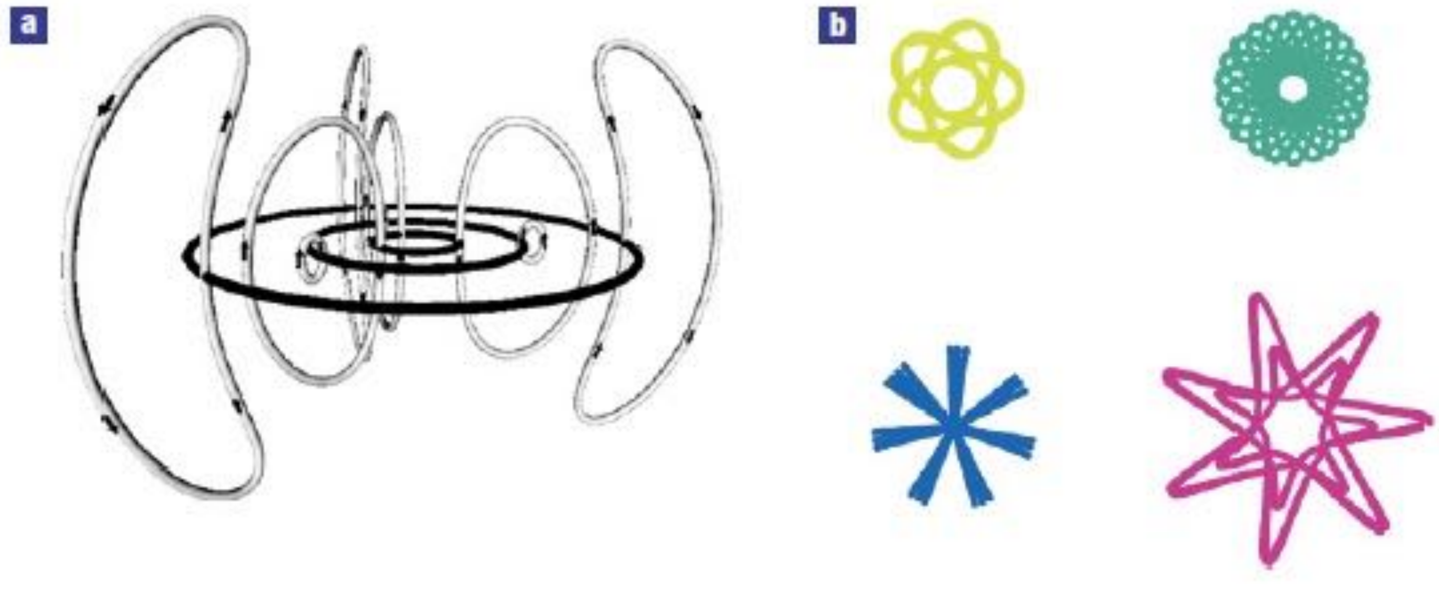


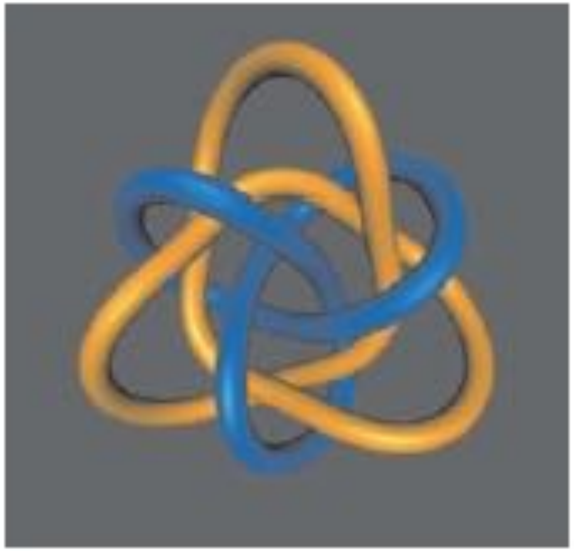


(2008)

Linked and knotted beams of light

WILLIAM T. M. IRVINE[1,2]* AND DIRK BOUWMEESTER[2,3]
[1]Center for Soft Condensed Matter Research, Department of Physics, New York University, New York 10003, USA
[2]Department of Physics, University of California, Santa Barbara, California 93106, USA
[3]Huygens Laboratory, Leiden University, PO Box 9504, 2300 RA Leiden, The Netherlands
*e-mail: wi1@nyu.edu



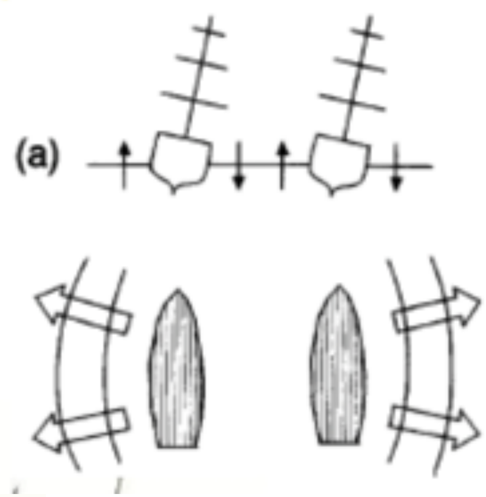


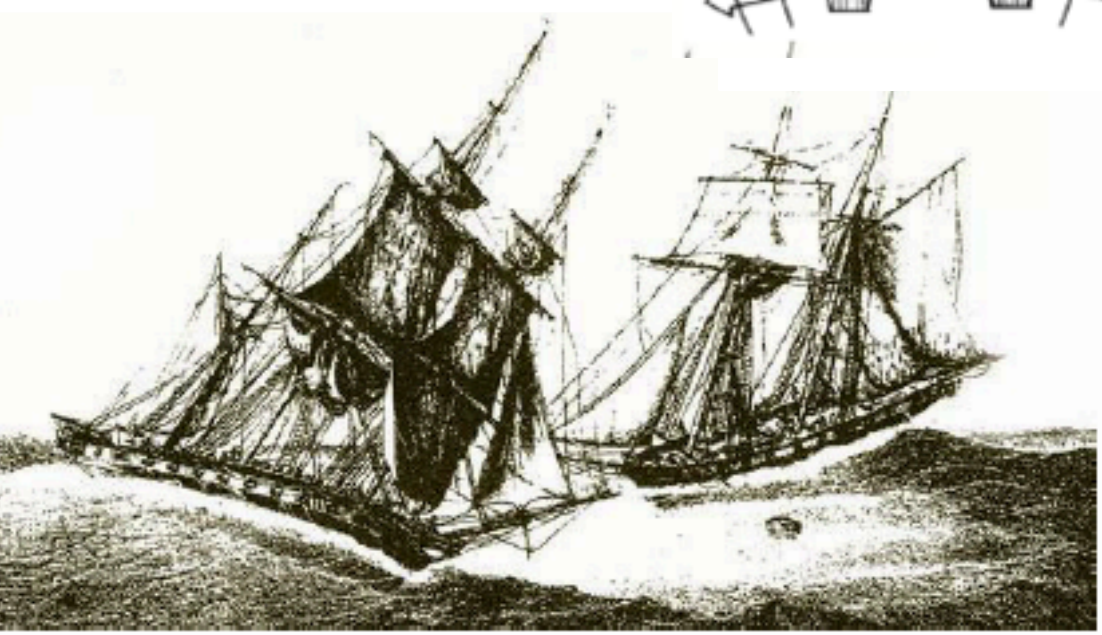
- have created such knots both optically and hydrodynamically

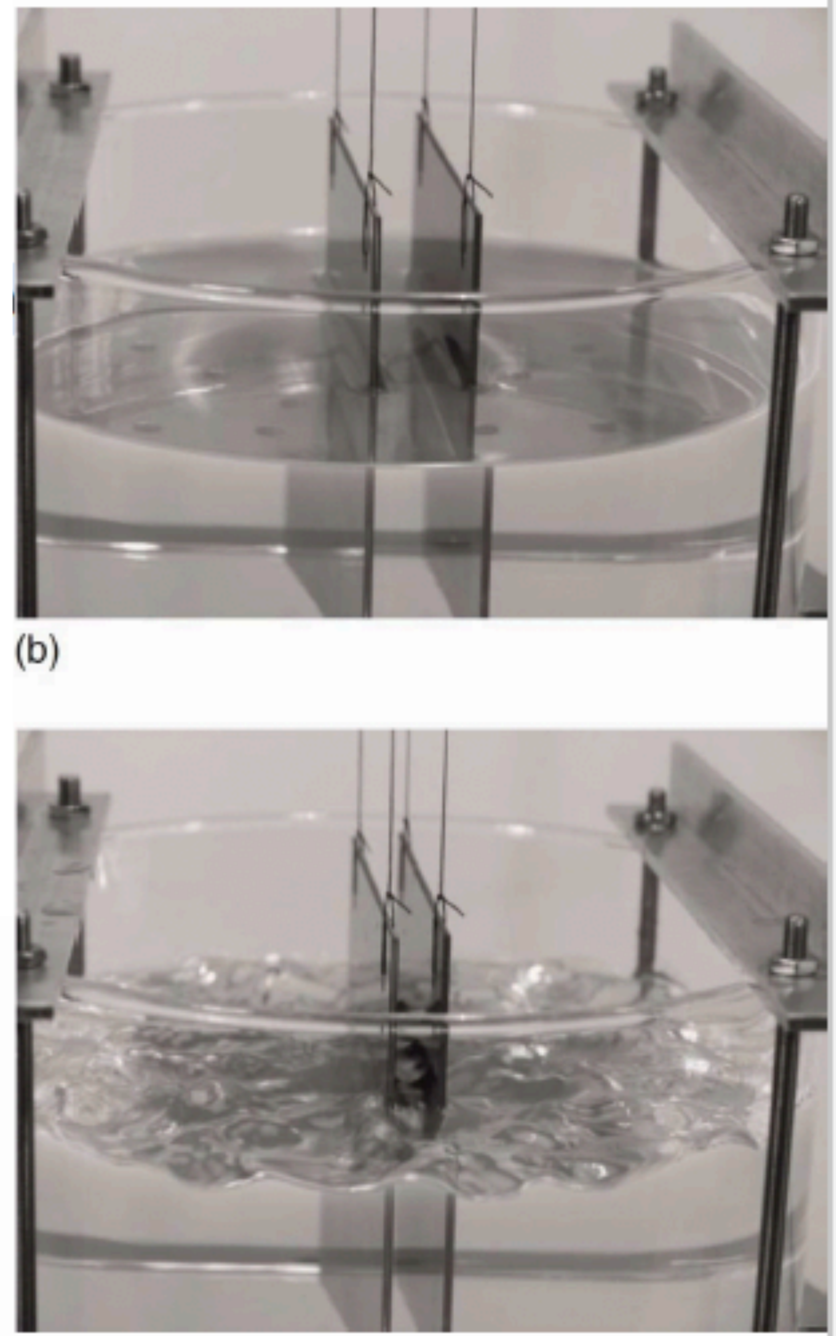
Hydrodynamic analogies of the Casimir effect

- in QM, the Casimir effect gives rise to forces between objects owing to geometric constraint on the EM quantum background field

Maritime analogy
(Boersma, AJP, 1996)



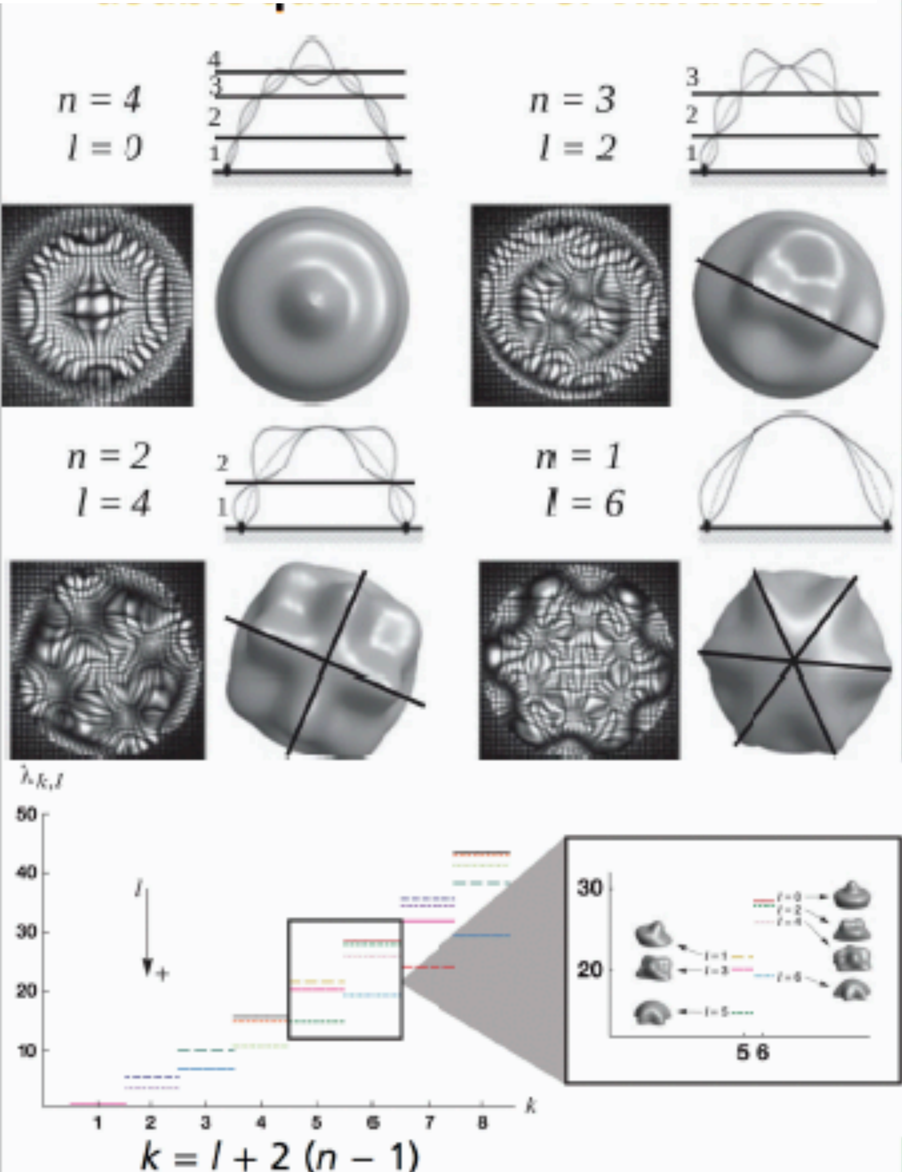


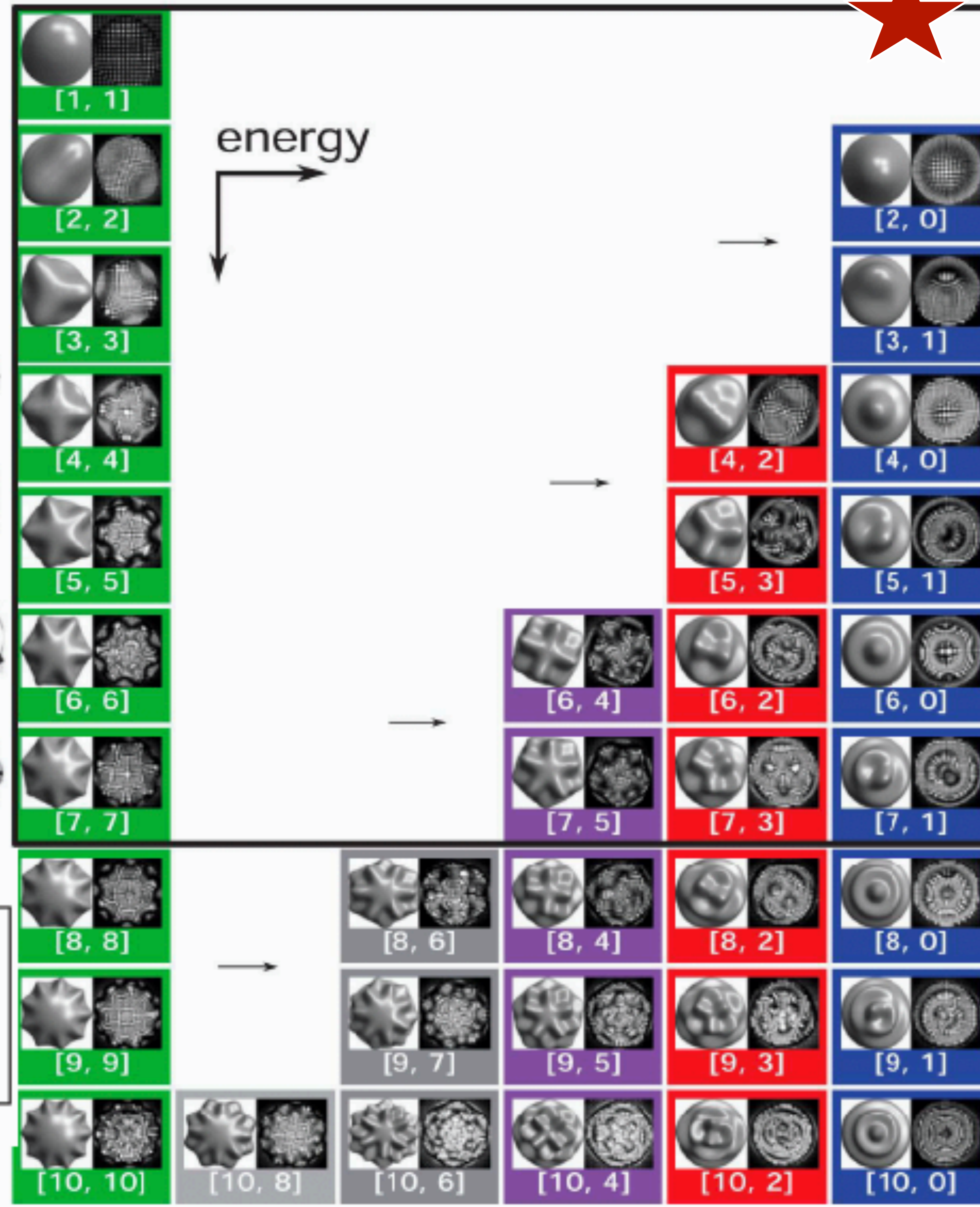


Water wave analogy
(Denardo et al. AJP, 2009)

Periodic table of drop vibrations

(Steen et al., PNAS, 2019)



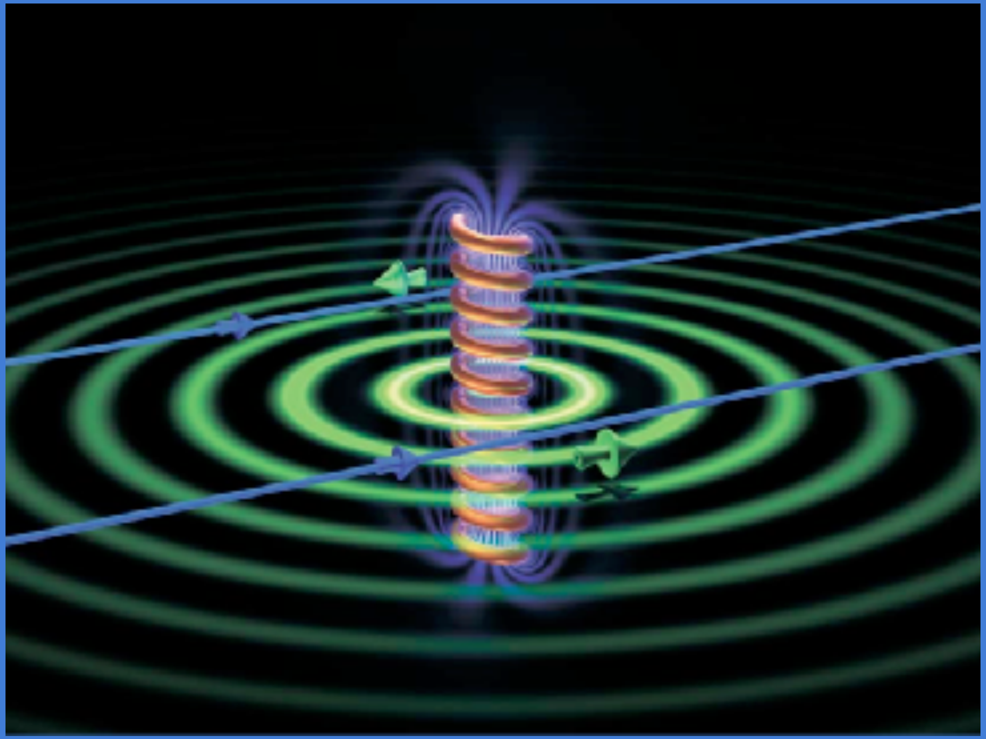


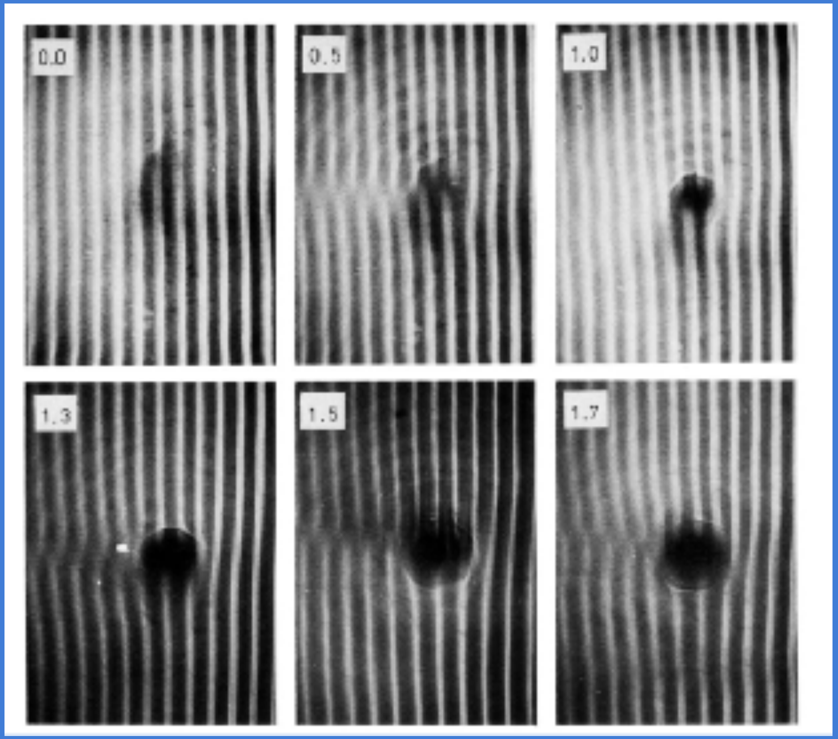
The Aharanov-Bohm effect

Michael Berry (1990)

- a quantum effect whereby a charged particle is effected by a magnetic potential **A** even though it moves exclusively in a domain where **B** = 0

- explored the Aharanov-Bohm effect via a hydrodynamic analogue: plane water waves pass over a potential vortex, for which vorticity vanishes





- phase dislocation in an amount proportional to the vortex circulation

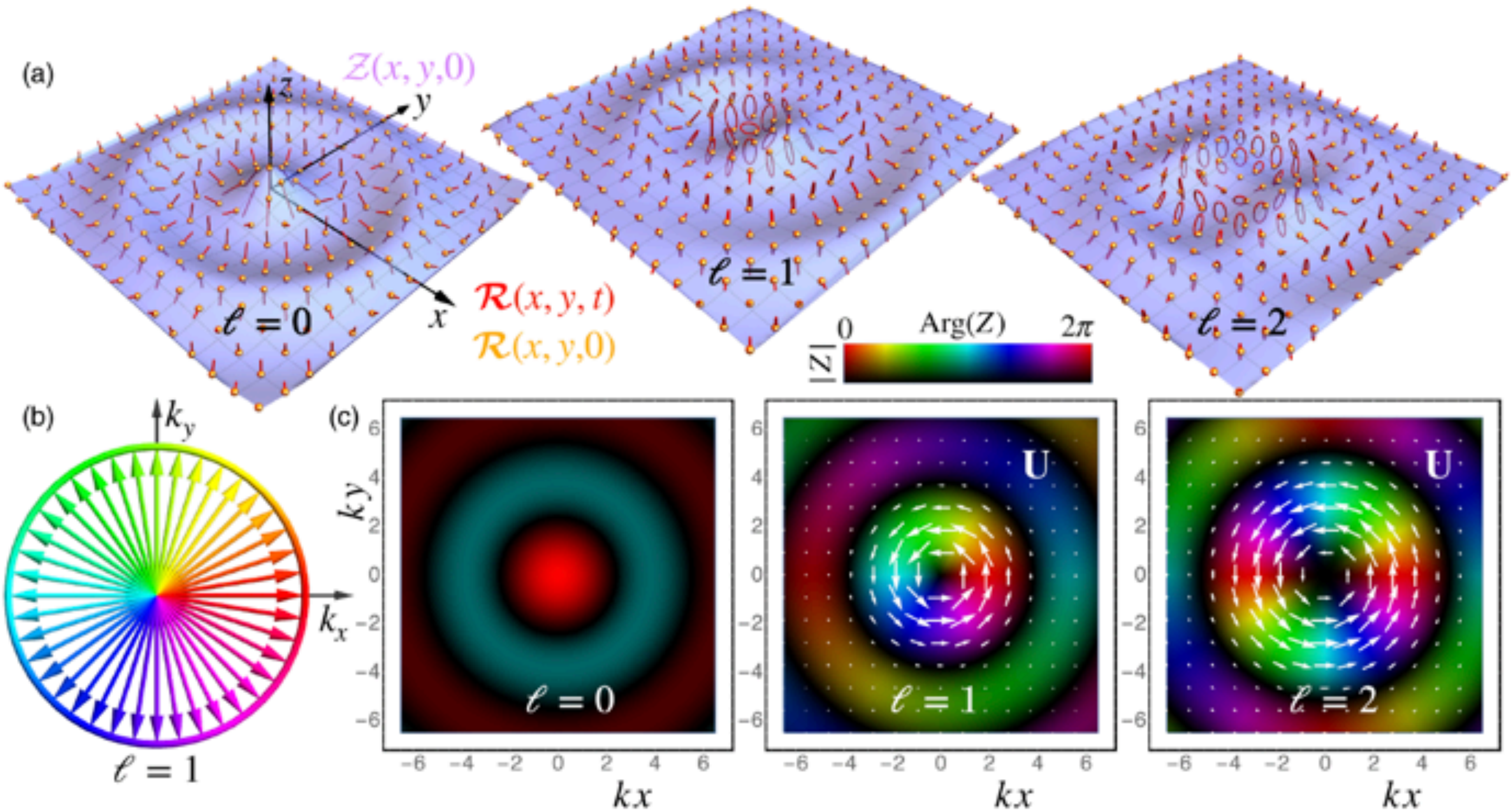
PHYSICAL REVIEW LETTERS **132,** 054003 (2024)

Editors' Suggestion Featured in Physics

Water-Wave Vortices and Skyrmions

Daria A. Smirnova,[1,2] Franco Nori,[1,3,4] and Konstantin Y. Bliokh[1,5,6]

- a Skyrmion is a topologically stable field configuration that arises in models of the neutron and also in string theory



PHYSICAL REVIEW LETTERS **132,** 054003 (2024)

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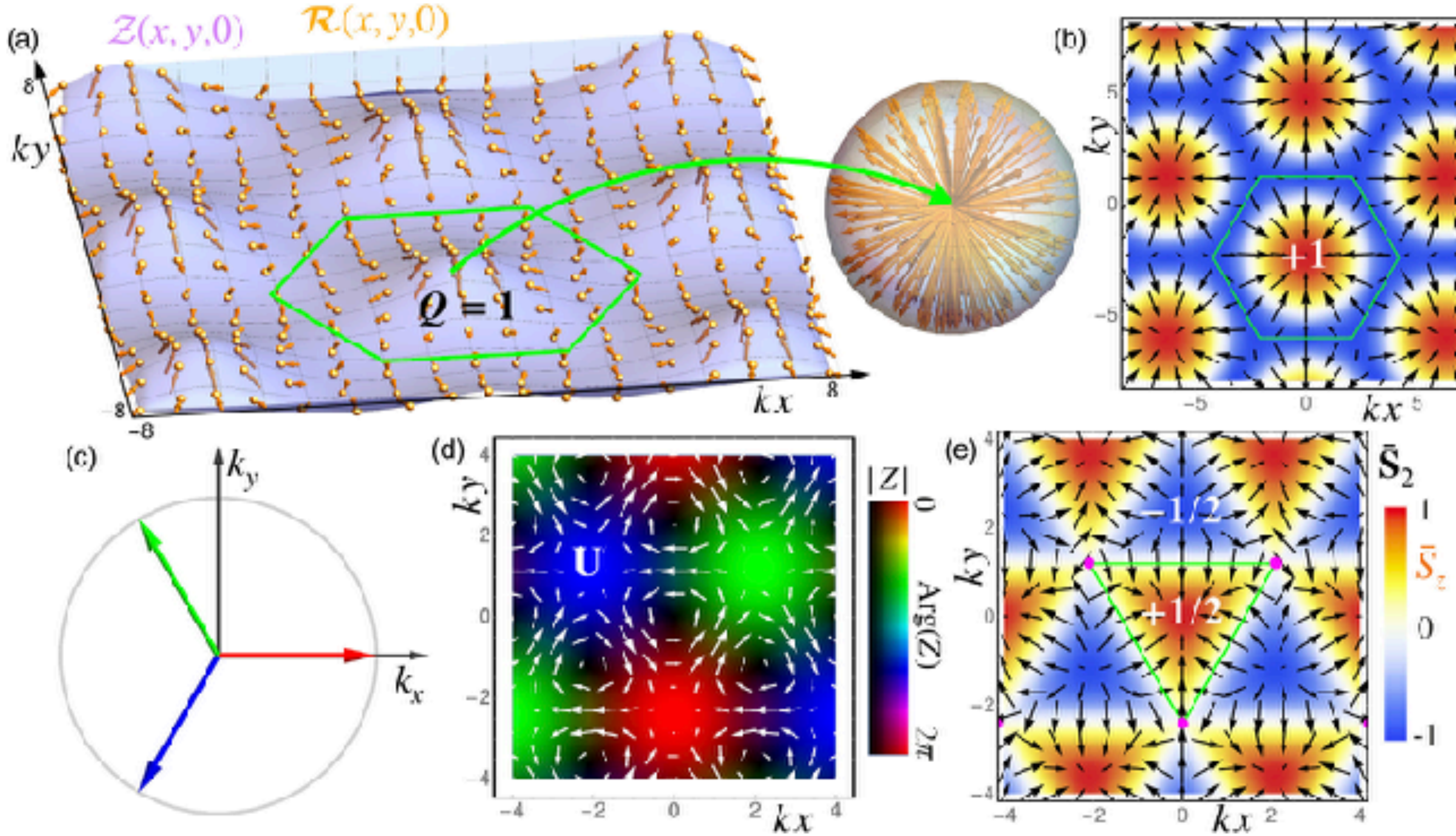
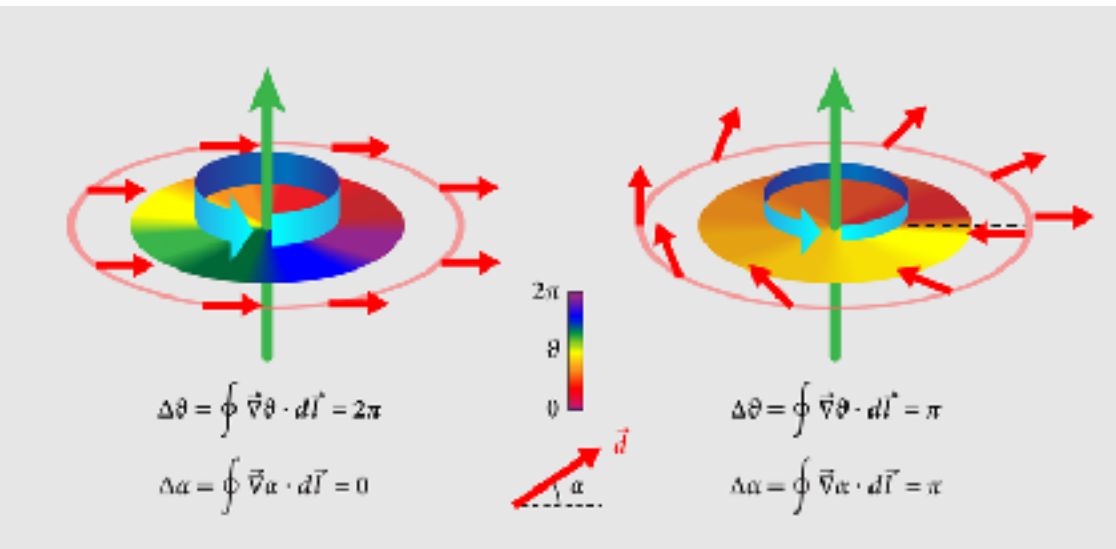
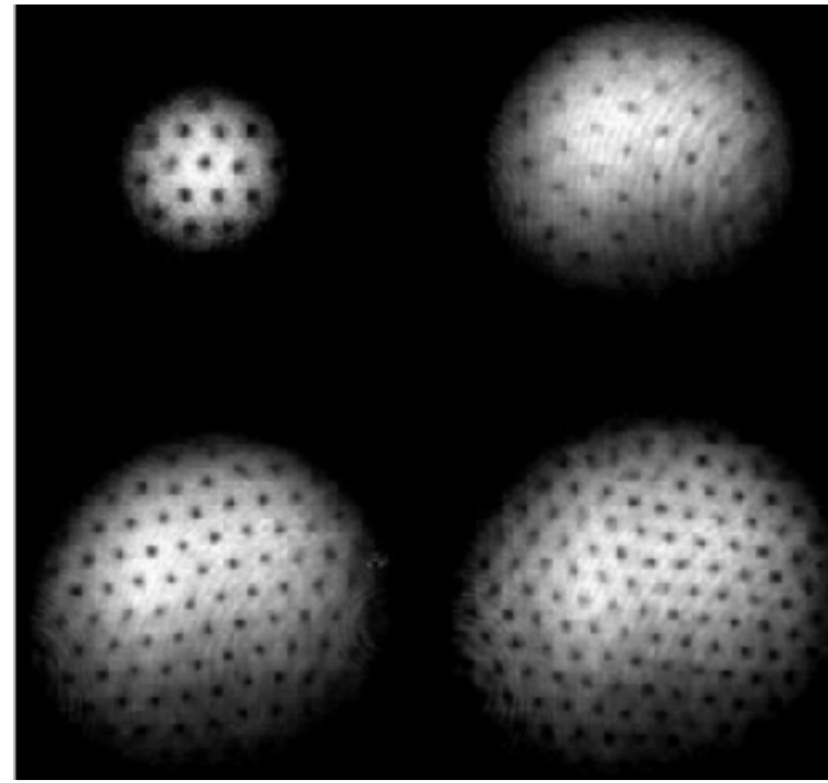


Superfluids (e.g. low temperature He)

- quantum phases of matter in which atoms condense into a single macroscopically occupied quantum state

- described in terms of inviscid fluid mechanics





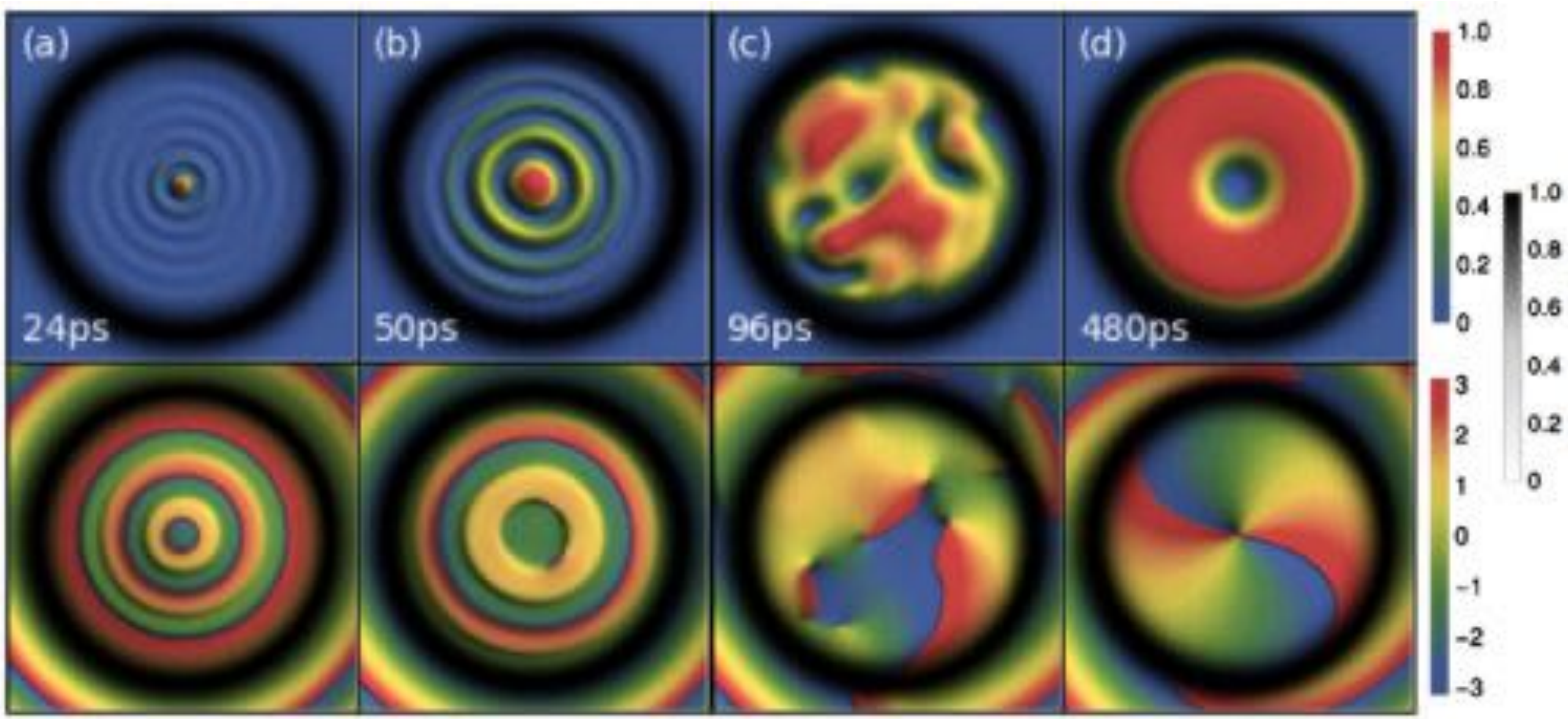
- circulation is quantized: vorticity appears discretely, and vortex lattices may form

Electron bubble

- when an electron is injected into superfluid He, the result is a bubble whose radius is set by a balance between the quantum pressure and surface tension

Polaritons

- a quantum hybrid of light and matter formed by shining light onto specially layered materials
- the resulting substance is a hybrid of light and matter: **`quantum fluids of light’**
- ring-pumped polarity condensate displays giant quantum vortices



Alperin & Berkoff (2021)

Quantum fluids of light: polaritons flow over a defect

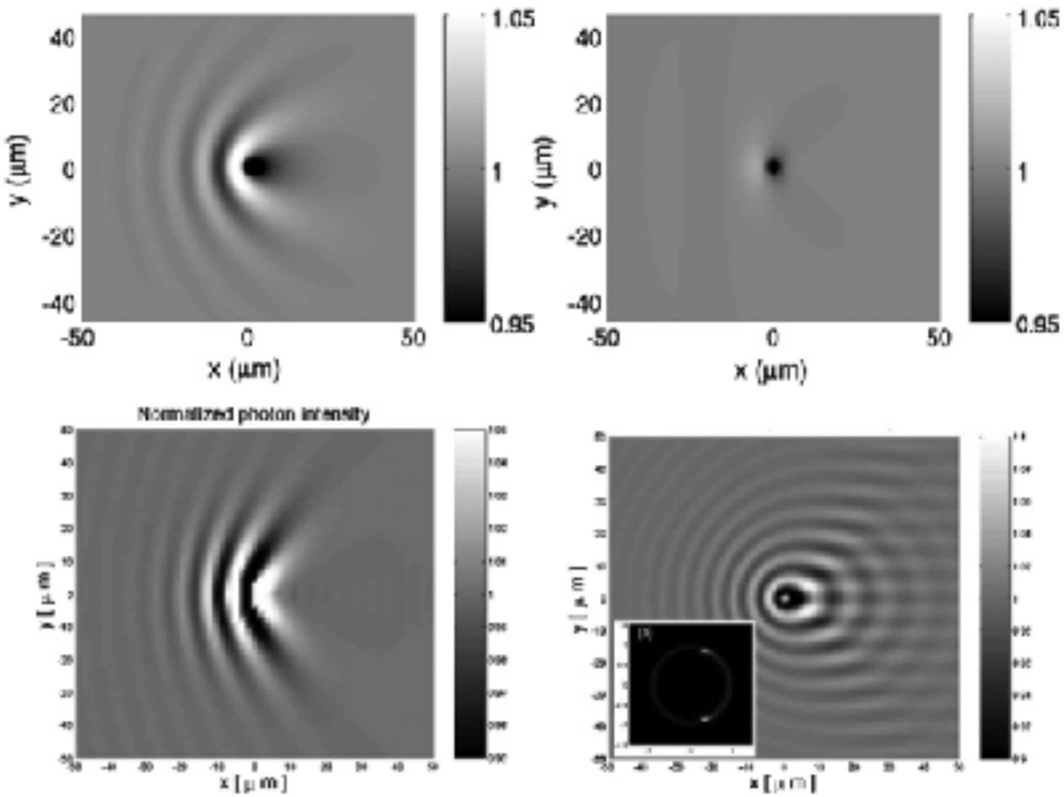
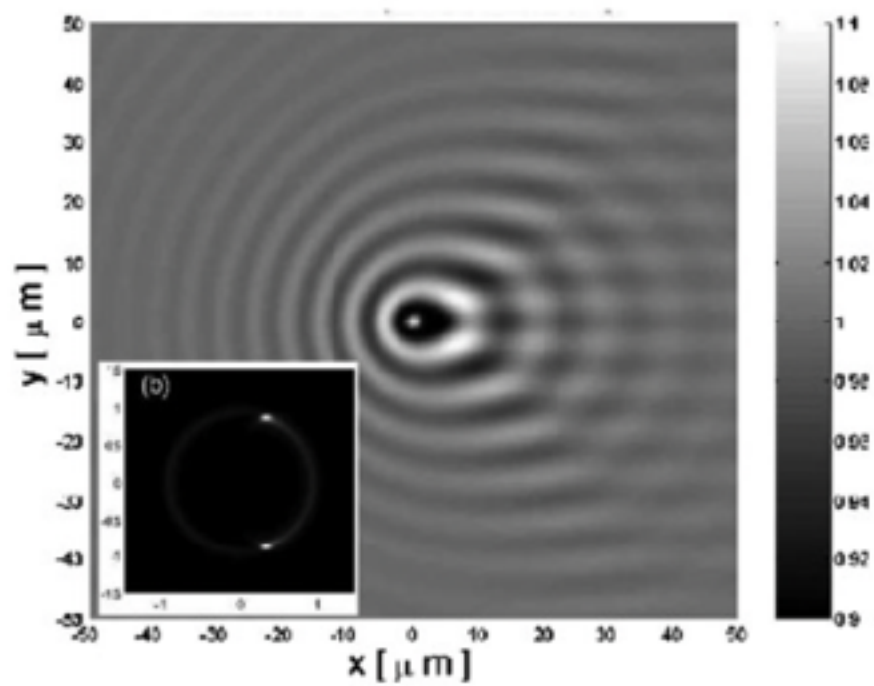
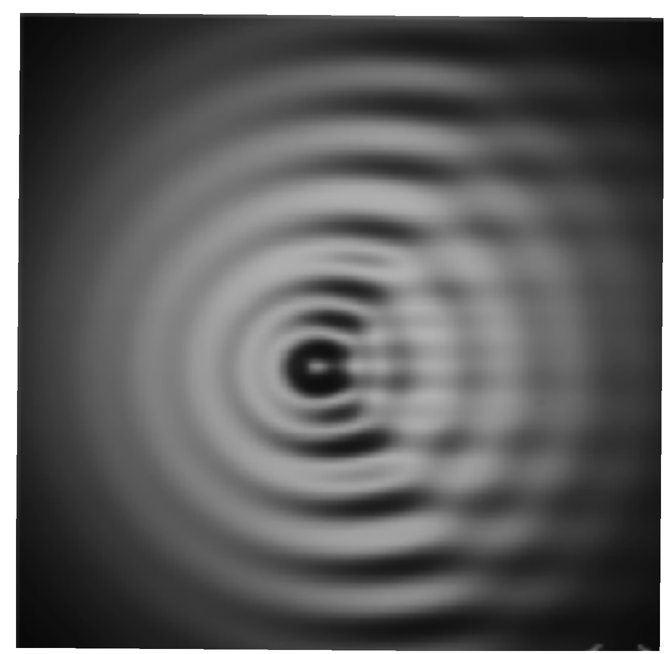


FIG. 19. Examples of real-space polariton density patterns for coherently pumped polariton condensates flowing in the rightward direction against a defect at rest. The defect is located at the center of each panel. Upper-left panel is for the noninteracting polariton regime of Figs. 18(a) and 18(b). Upper-right panel is for the superfluid regime of Figs. 18(e) and 18(f). From Carusotto and Ciuti , 2004. The lower-left panel is for a supersonic flow regime of Figs. 18(c) and 18(d). The lower-right panel illustrates the zebra-Cherenkov effect in the vicinity of a parametric instability; the corresponding k-space emission pattern is shown in the inset. From Ciuti and Carusotto, 2005.

Carusotto & Ciuti (Rev. Mod. Phys. 2013)





Zebra-Cherenkov radiation

A walking droplet

Time crystals

- a quantum system of particles in which the lowest energy state is one in which the particles are in repetitive motions
- proposed by Wilczek (2012) as a time-based analog of common crystals
- `no-go' theorems proved quantum space-time crystals in equilibrium are impossible
- various experiments (in BECs and spintronic systems) found time crystals

Time crystals

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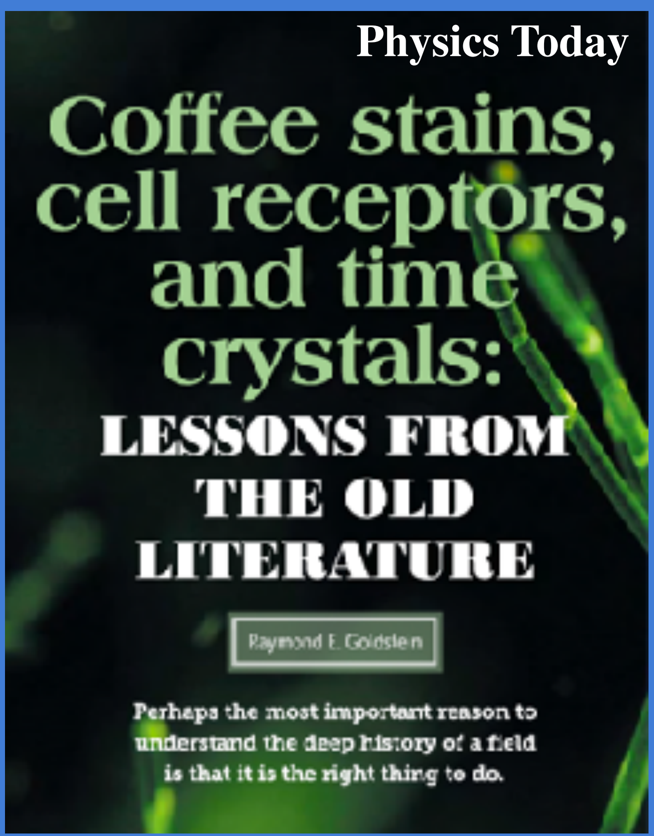
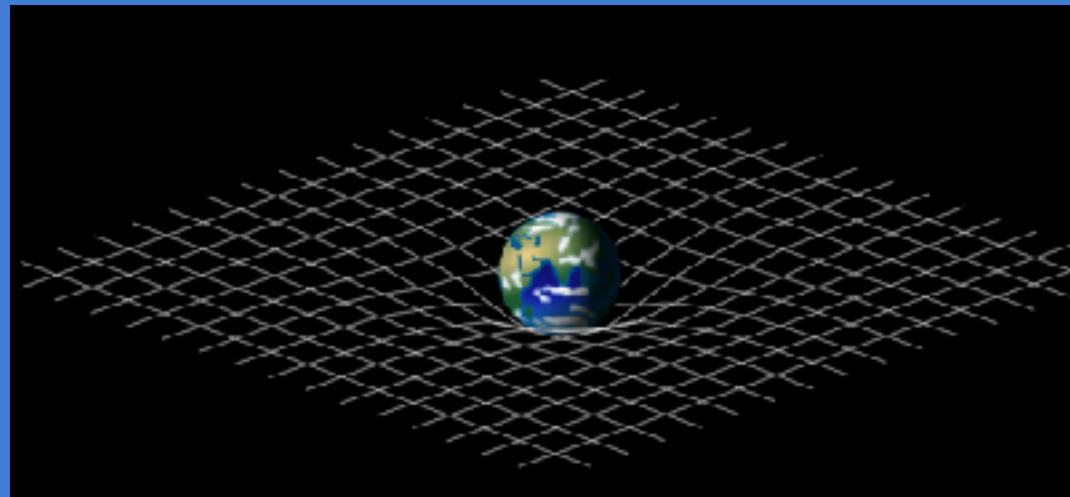
- various experiments (in BECs and spintronic systems) found time crystals

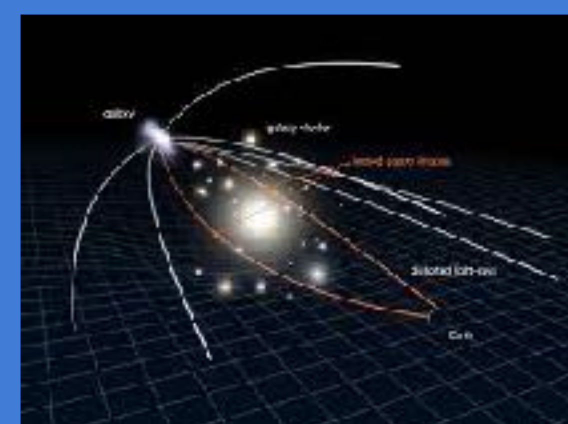
Historical precedent: Faraday waves



General relativity (Einstein 1915)



- describes gravity as the curvature of space-time caused by the presence of mass and energy
- massive objects like planets and stars curve the spacetime around them
- other objects, including photons, follow geodesics as they traverse this curved spacetime



- successful in rationalizing gravitational lensing

Analog gravity **(Unruh *et al.* 1980s —)**

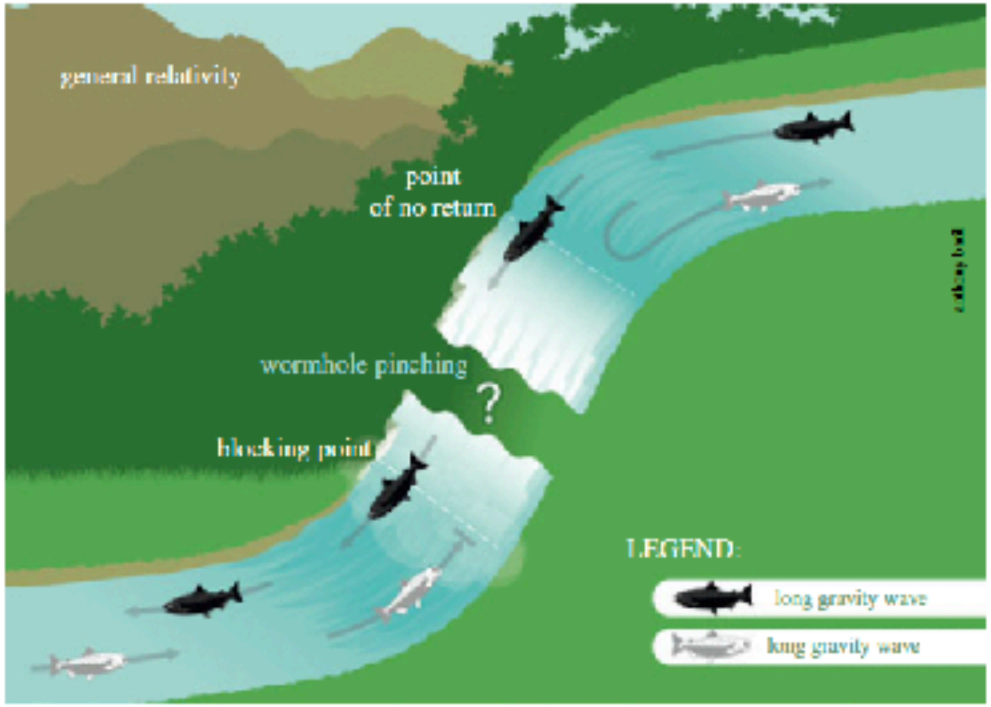
- attempts to model phenomena of GR (e.g. black holes) using fluidic systems,
  e.g. acoustics in moving fluids, gravity waves, superfluid He, BECs

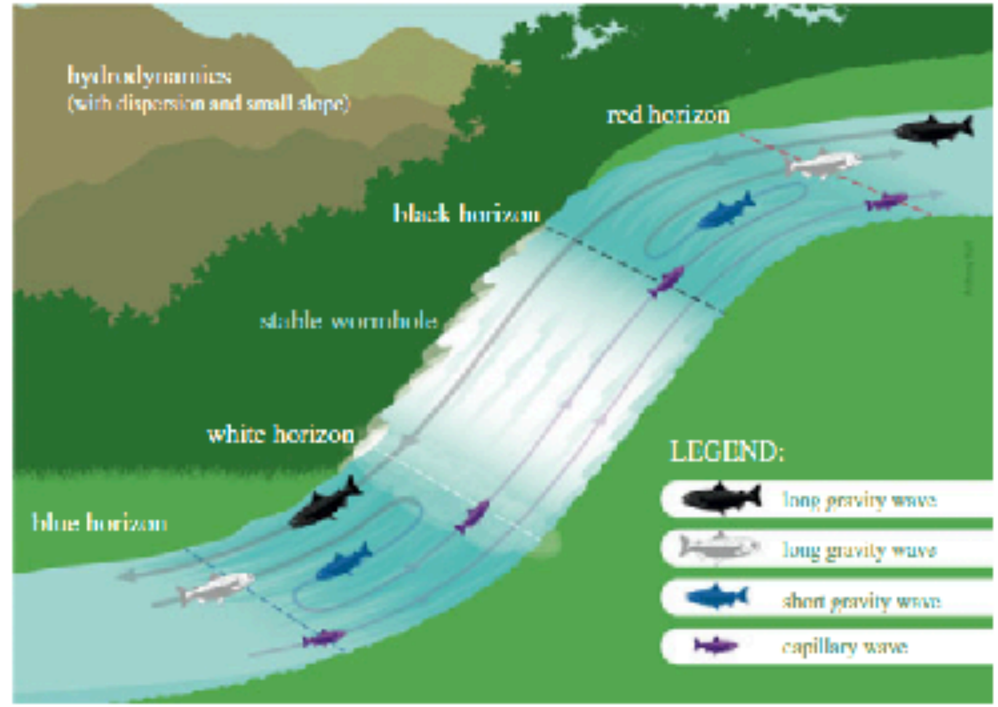
- in the hydrodynamics analogs, the interface plays the role of the fabric of space-time

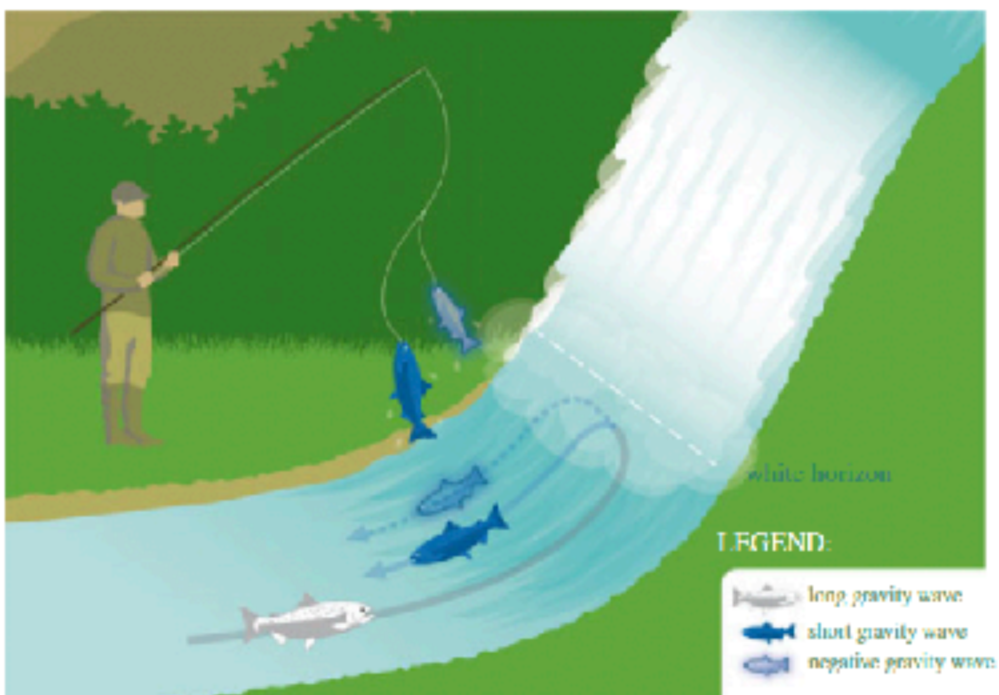
- established analogs between wave interactions with vortices and black holes
- established analogs between wave interactions with hydraulic jumps and white holes

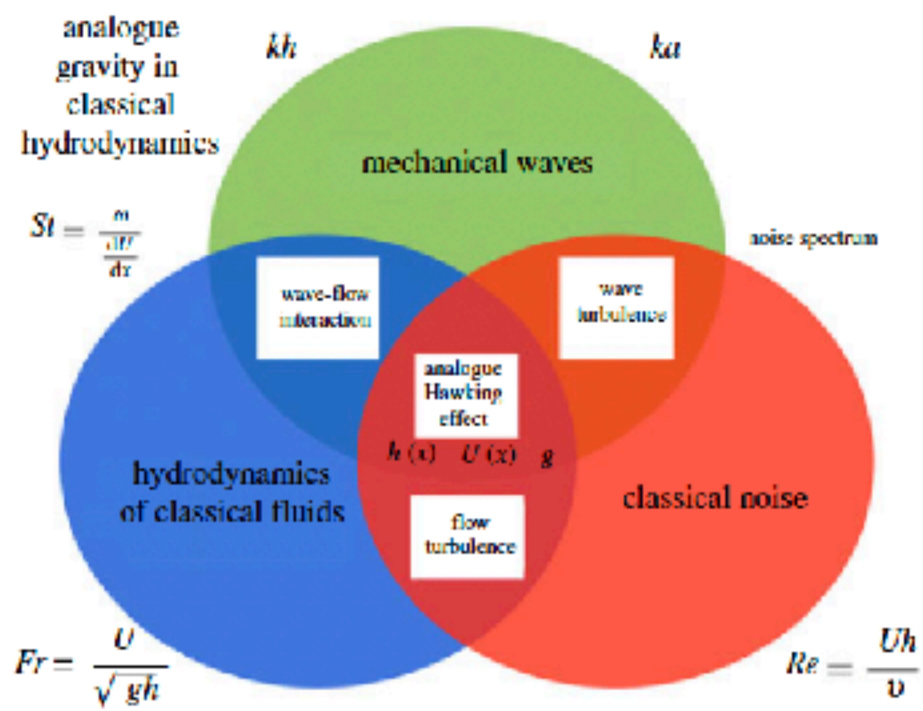
Analog Gravity

- Kellay & Rousseaux (2019)









Analog Gravity

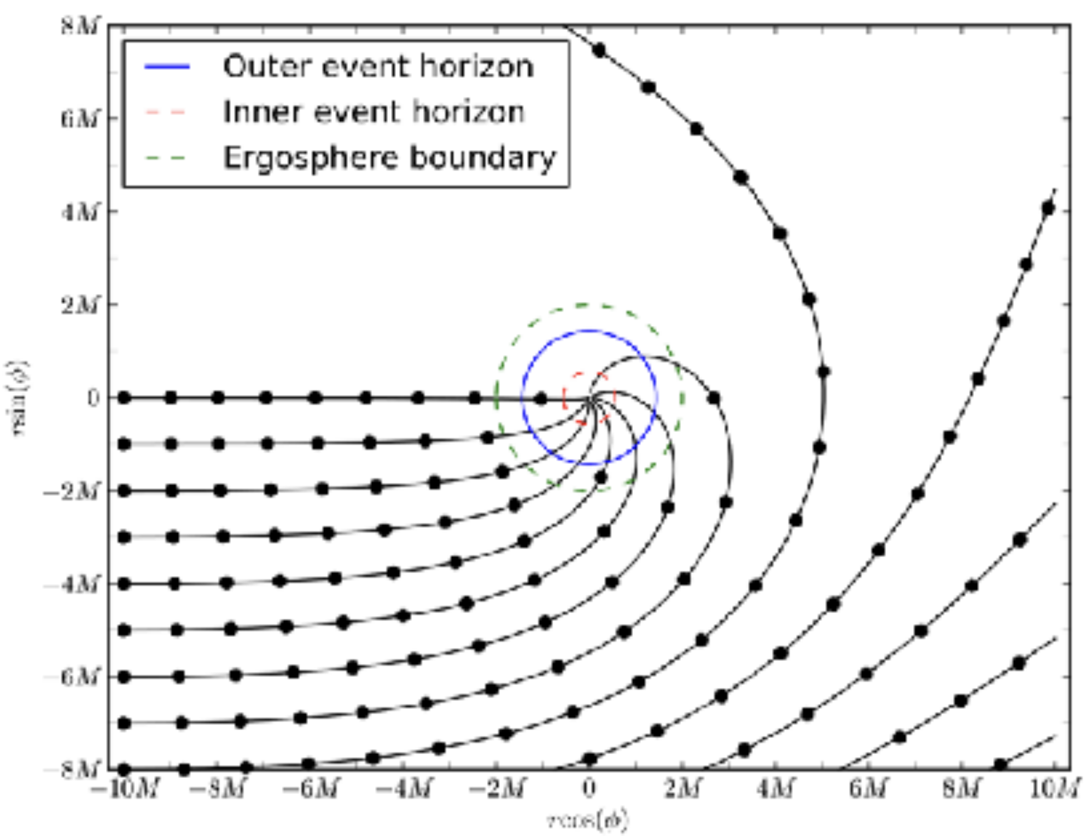
General Relativity	Condensed Matter Systems	Classical Hydrodynamics
Curved space-time	Moving/Changing Medium	Flow Current
Einstein Equations	Dynamical Equations	Navier-Stokes Equations
Poincaré-Lorentz Covariance	Poincaré-Lorentz or Galilean Covariance	Galilean Covariance
Painlevé-Gullstrand Metric for Light	Acoustic Metric for "Sound"	Hydraulic Metric for Water Waves
$V(r)$	$V(\mathbf{x})$	$U(x)$
$V(r) = -c\sqrt{\frac{2GM}{c^2 r} + \frac{\Lambda r^2}{3} - \frac{e^2}{r^2}}$	System-dependent (See the Dynamics)	$U(x) = \frac{q}{h(x)}$
$\omega^2 = c_L^2 k^2$	$\omega^2 = F(k)$	$\omega^2 = \left(gk + \frac{\gamma k^3}{\rho}\right)\tanh(kh)$
$c_L = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$	$c_{HydrodynamicLimit}$	$c_{kh<<1} \simeq \pm\sqrt{gh}$
$c_L = cst$	$c = c(x)$	$c = c(x) = \sqrt{gh(x)}$
Event Horizons	Group Velocity Horizons	Blocking Points
Surface Gravity	Medium and Waves Speed Gradients	Flow and Waves Speed Gradients
$\kappa = \left(\frac{dV}{dr}\right)_{r=r_H}$	$g_H = \left(\frac{d[V(x) - c(x)]}{dx}\right)_{x=x_H}$	$g_H = \left(\frac{d[U(x) - c(x)]}{dx}\right)_{x=x_H}$
Trans-Planckian Problems versus Unknown Physics (P-LIV) or Back Reaction Effects	Ultra-Violet Catastrophe versus Dispersive Regularizations or Back Reaction Effects	Dispersive Regularizations or Wave Breaking
Hawking Radiation	Super-Radiance (Non-WKB Scattering)	Over-Reflection
Negative Norm Mode (anti-particle)	Negative "Charge" Mode	Negative Relative Frequency Mode
$\|\beta\|^2 = \frac{1}{\exp(2\pi\frac{\omega}{\kappa}) - 1}$	$\|\beta\|^2 = \frac{1}{\exp(2\pi\frac{\omega}{g_H}) - 1}$	$\|\beta\|^2 = \frac{1}{\exp(2\pi\frac{\omega}{g_H}) - 1}$
Unitarity Condition	Norm/"Charge" Conservation	Wave Action Conservation
$\|\alpha\|^2 - \|\beta\|^2 = 1$	$\frac{\partial\rho}{\partial t} + \nabla.\mathbf{J} = 0$	$\frac{\partial}{\partial t}\left(\frac{E}{\omega}\right) + \nabla.\left(\frac{E}{\omega}\frac{d\omega}{dk}\right) = 0$
Unknown Physics	Zero Mode (Extra Scattering)	Stationary Undulation (Extra Scattering)

Figure 2. The Rosetta Stone of Analogue Gravity establishing the matching between General Relativity, Condensed Matter Systems and Classical Hydrodynamics [5,14,18,26–58]. (Online version in colour.)

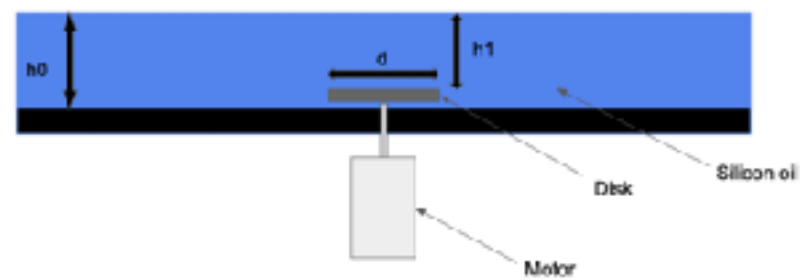
Walking droplets as single-particle GR analogs

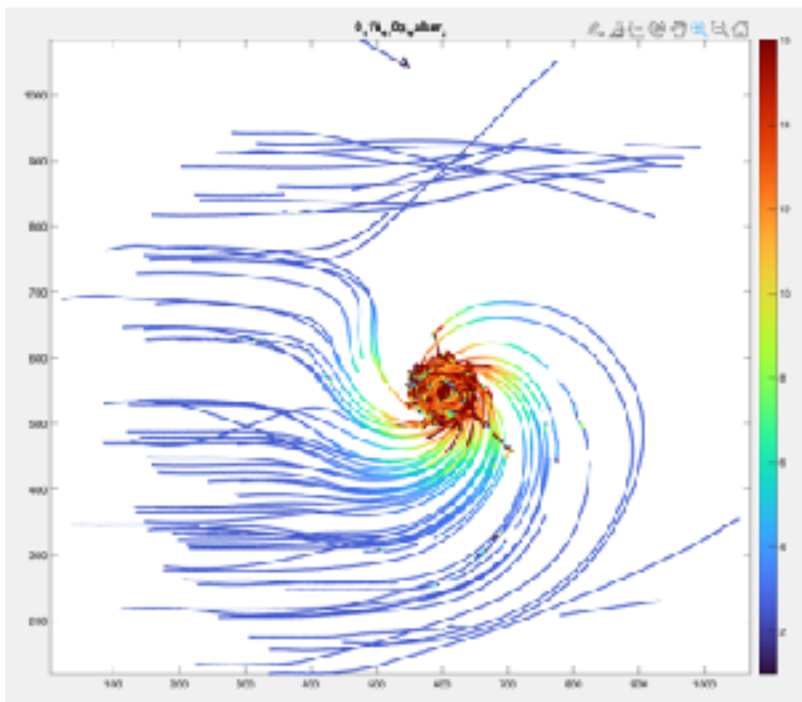
- have to date only involved wave phenomena, not single particle analogs

Photons near a rotating black hole



Walkers near a rotating meniscus





HALF TIME

Some physical analogies are better than others

How can you judge?

The similarity of the mathematical descriptions.

Establishing this similarity is easily done in fluid mechanics,
for which all systems are described by the Navier-Stokes equations.

Relies on assessment of relevant dimensionless groups.

Degrees of similitude

I. Metaphor

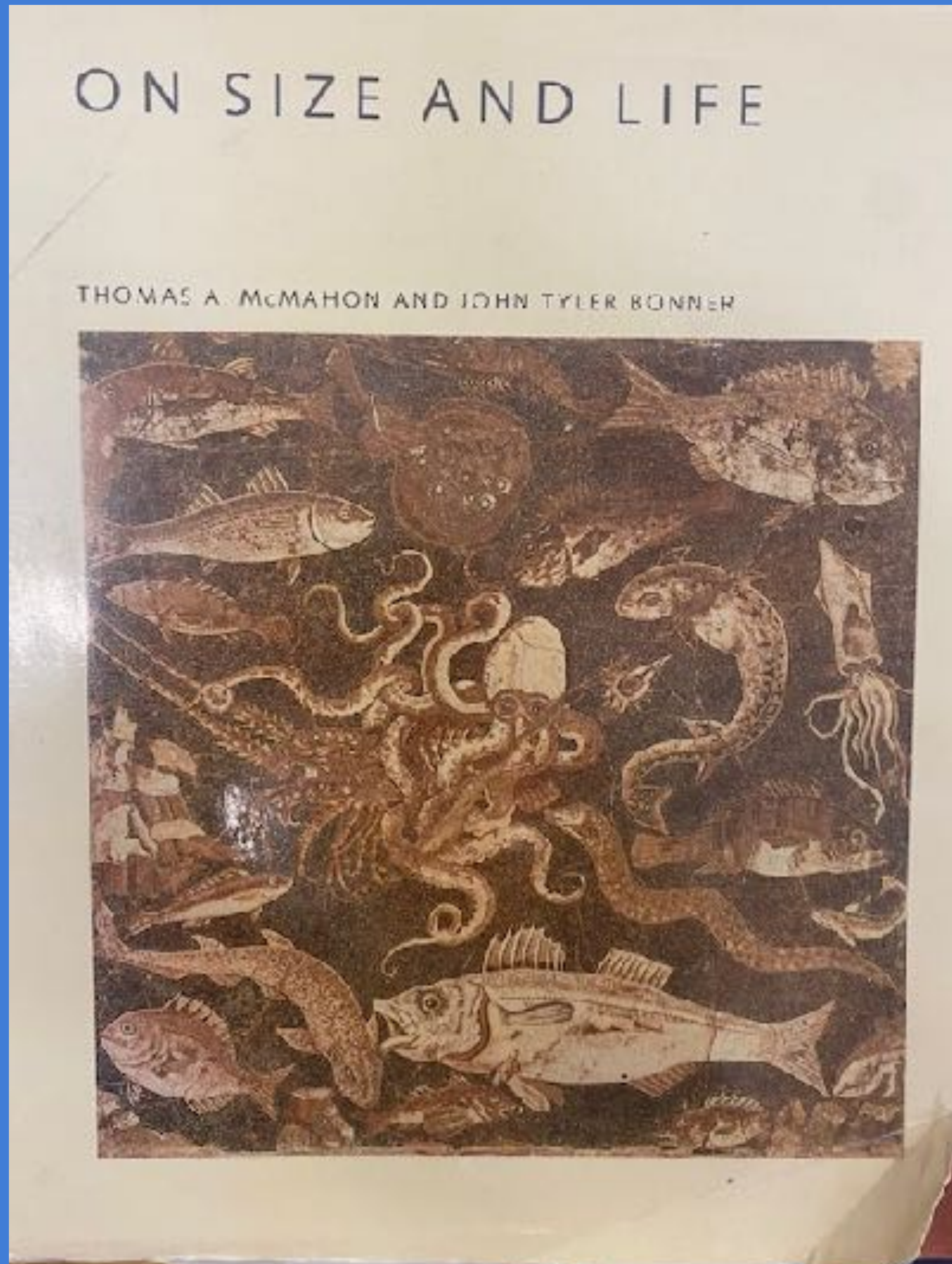
II. Physical analogy

III. Dynamic similarity

- the cornerstone of laboratory modeling in hydrodynamics
- arises between two physical systems when strict mathematical equivalence is achieved between them

… a digression into Dimensional Analysis

Dimensional analysis and scaling arguments



DIMENSIONAL ANALYSIS

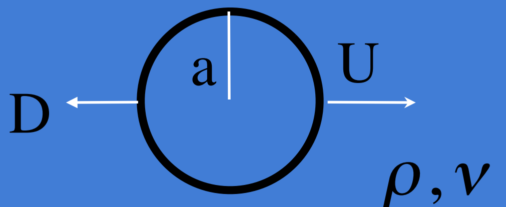
Fundamental Concept

The laws of Nature cannot depend on an arbitrarily chosen system of units.
A system is most succinctly described in terms of dimensionless variables.

Deduction of Dimensionless groups: **Buckingham's Theorem**

For a system with M physical variables (e.g. density, speed, length, viscosity) describable in terms of N fundamental units (e.g. mass, length, time, temperature), there are M - N dimensionless groups that govern the system.

E.g. Translation of a sphere



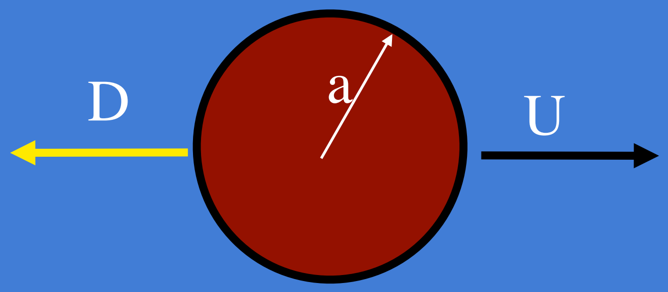
Physical variables: $U, \mathrm{a}, \nu, \rho, D \;\Rightarrow\;$ M = 5

Fundamental units: M , L , T $\;\Rightarrow\;$ N = 3

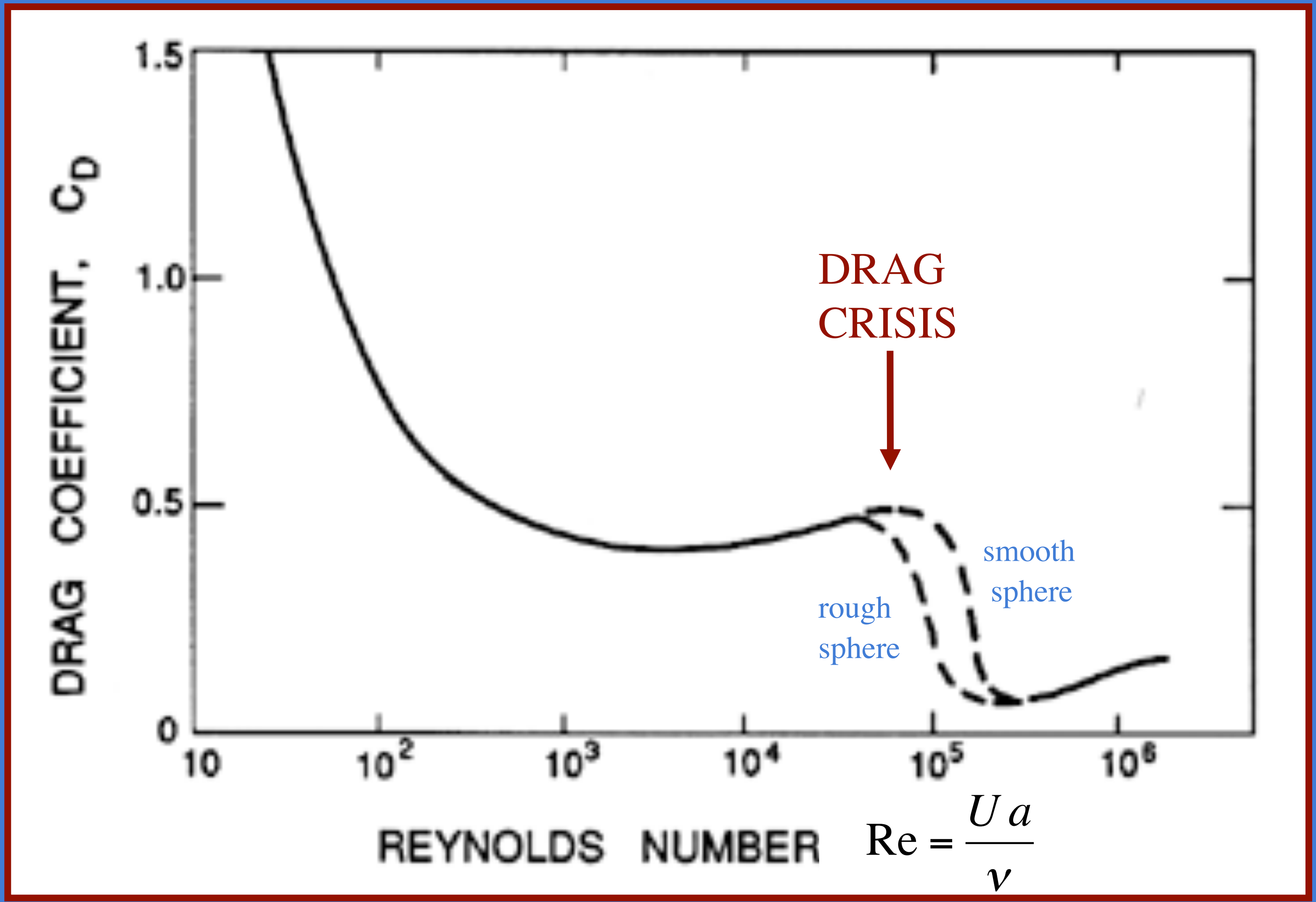
M - N = 2 dimensionless groups: $C_d = \dfrac{D}{\rho U^2}$, $\mathrm{Re} = \dfrac{U\mathrm{a}}{\nu}$

System uniquely determined by a single relation: $C_d = F(\mathrm{Re})$

Drag: $D = C_D(\mathrm{Re})\,\rho\, U^2\, a^2$



Variation of C_D with $\mathrm{Re} = \dfrac{U\,a}{\nu}$:



DIMENSIONAL ANALYSIS

- the deduction of the dimensionless groups governing a physical system

Value:

1) minimizes number of parameters governing a physical system (thus facilitating experimental modeling)

2) occasionally yields scaling of physical variables directly

Corollary to Buckingham's Theorem:

If there is only one dimensionless group, it must be constant.

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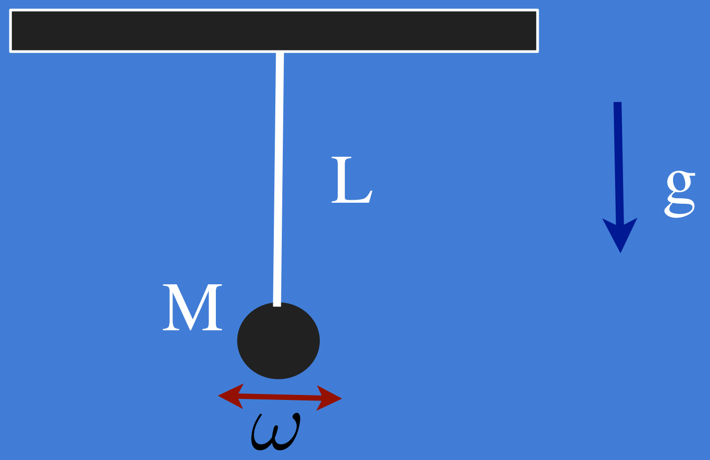
If there is only one dimensionless group, it must be constant.



E.g. Pendulum

Physical variables: M, L, g, ω

Fundamental units: M, L, T

→ system prescribed by one dimensionless group

$$\Pi = \frac{\omega^2 L}{g} = constant \longrightarrow \omega \sim \left(\frac{g}{L}\right)^{1/2}$$

Note: finite amplitude oscillations require consideration of θ

$$\longrightarrow \omega = f(\theta)\left(\frac{g}{L}\right)^{1/2}$$

Dimensional analysis and pilot-wave triggers

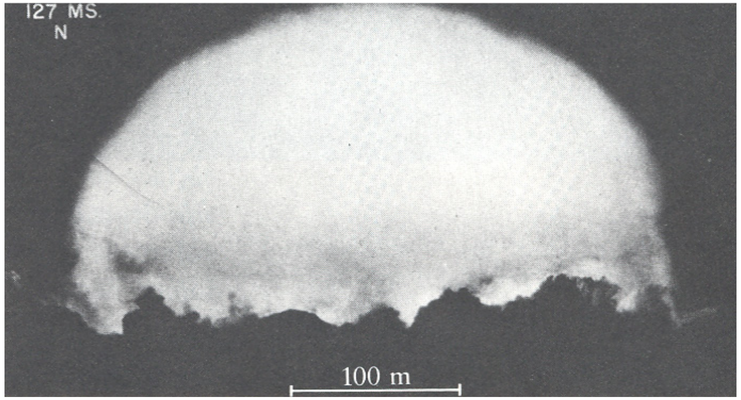
(see Problem Set 1)

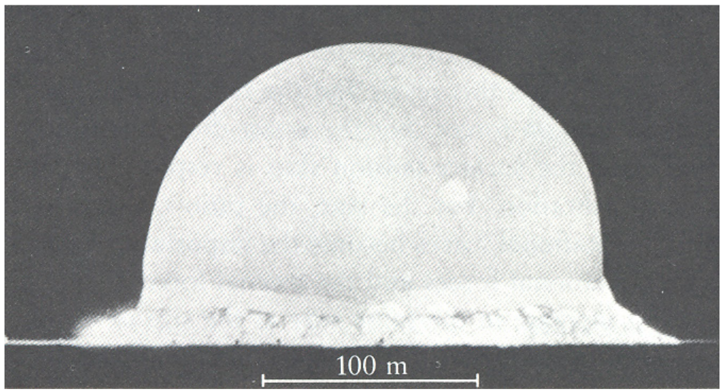
If there are two physical constants in the Universe, ρ and σ, what is the natural frequency of oscillation of a drop of radius a?

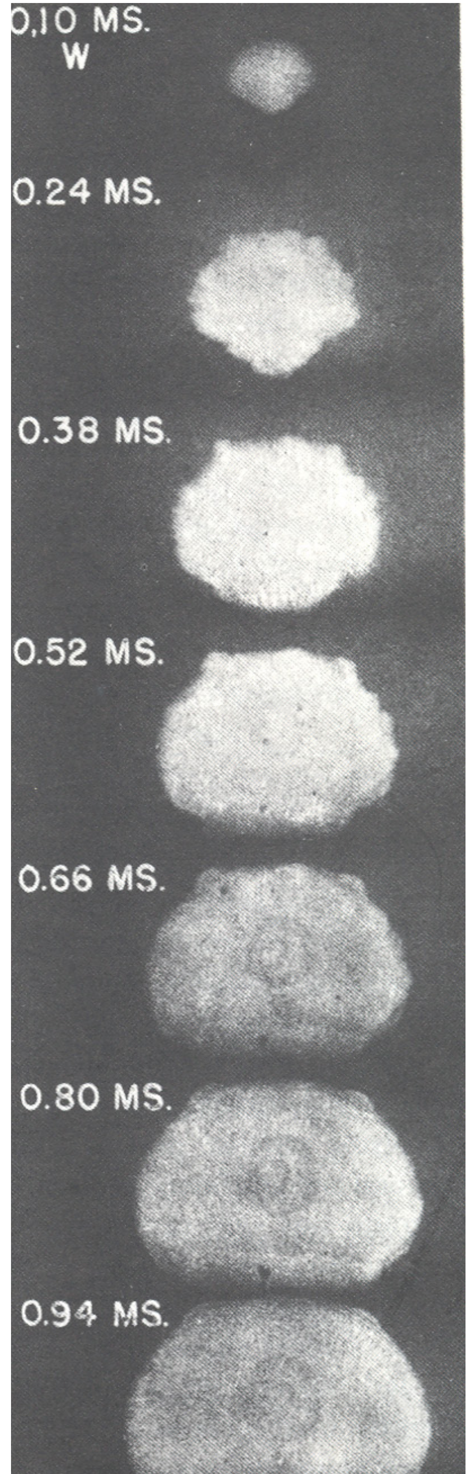
$$\omega_d = \sqrt{\frac{\sigma}{\rho a^3}}$$

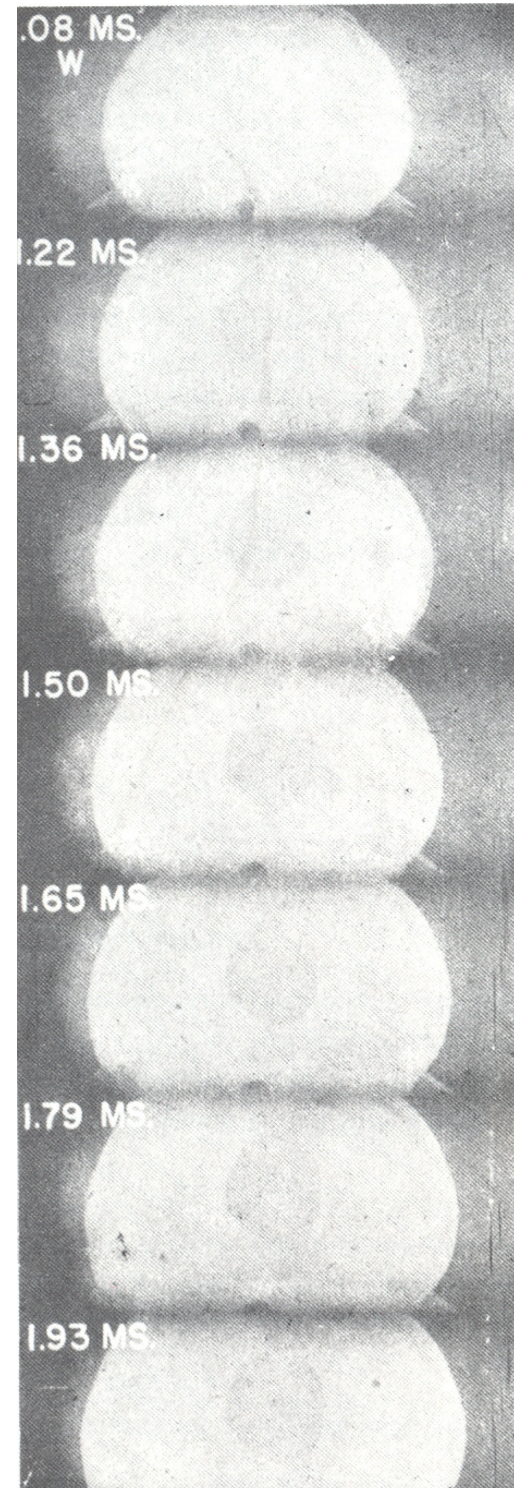
If there are two physical constants in the Universe, $\hbar$ and c, what is the natural frequency of oscillation of a particle of mass m?

$$\omega_c = \frac{mc^2}{\hbar}$$







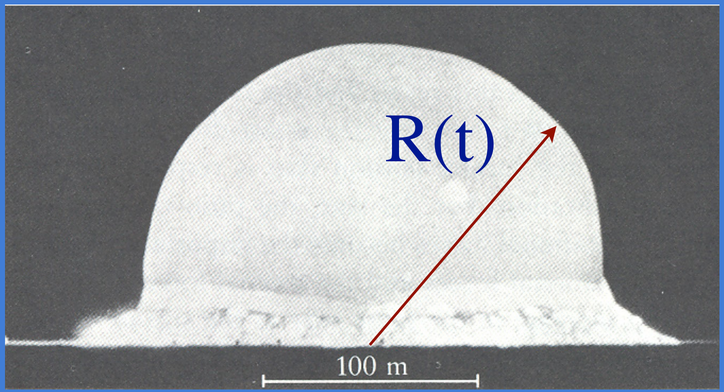


Can we predict R(t) ?

Given R(t), can we infer the energy released?

The scaling of atomic blast clouds

- G.I. Taylor, Sedov



Physical variables: R, t, E, ρ

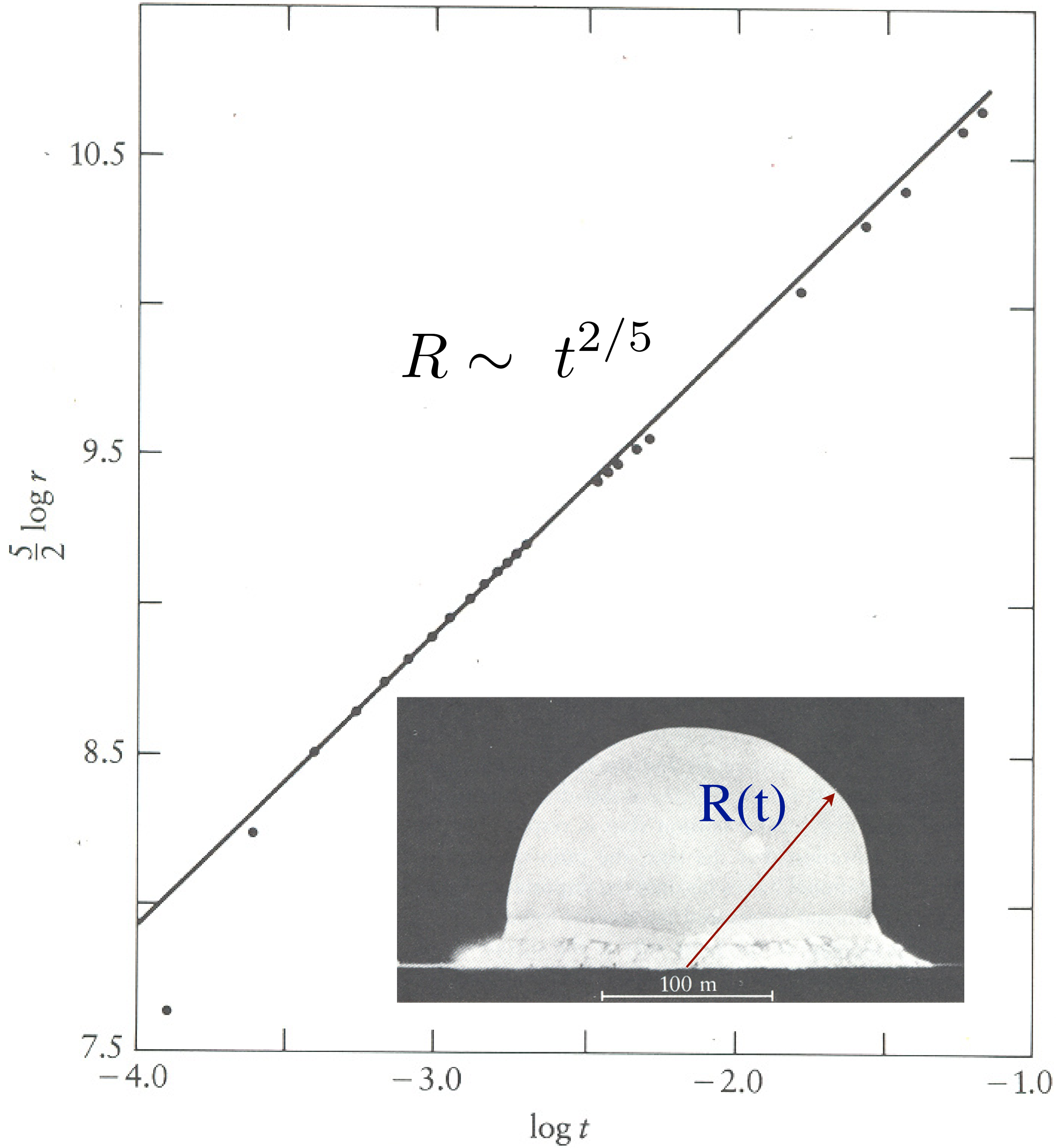
Fundamental units: M, L, T

→ system prescribed by one dimensionless group:

$$\Pi = \frac{Et^2}{\rho R^5} = constant$$

→ yields desired scaling for radius of the blast cloud:

$$R \sim \left(\frac{E}{\rho}\right)^{1/5} t^{2/5}$$

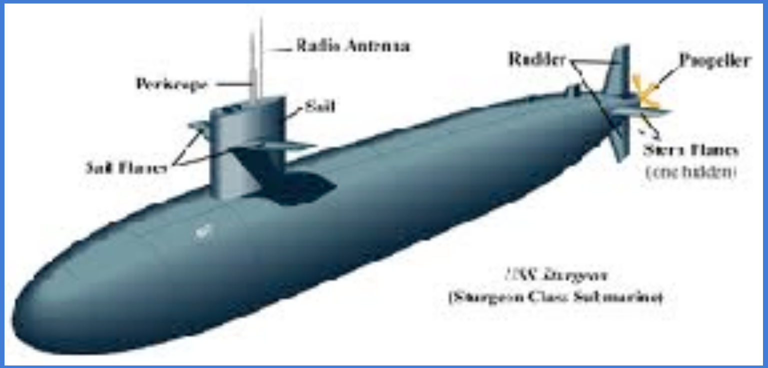


10.5
9.5
8.5
7.5
$\frac{5}{2}\log r$
$R \sim t^{2/5}$
R(t)
100 m
−4.0
−3.0
−2.0
−1.0
log t

III. Dynamic similarity

- arises between two physical systems when those systems are described by the same dimensionless parameters

- allows one to model (impractically large or small) physical systems in a laboratory setting



Eg. 1 Submarine building

Dimensional analysis yields

$$\frac{D}{\rho U^2 a^2} = F(Re, shape)$$

- $F(Re, shape)$ may be deduced from experimental modeling with miniature subs (and high U to match Re)

- results can be scaled up to deduce drag D and power required DU for a submarine to cruise submerged at a given speed

Eg. 2 Wind-tunnel testing

- one wants to know wind-induced pressure distribution around a building (e.g. the John Hancock tower)

$$\frac{P}{\rho U^2} = F(Re)$$ deduced from lab experiments

- must match Re for dynamic similarity: miniature models require the use of high laboratory wind speeds U

Eg. 3 Low Reynolds number swimming in water

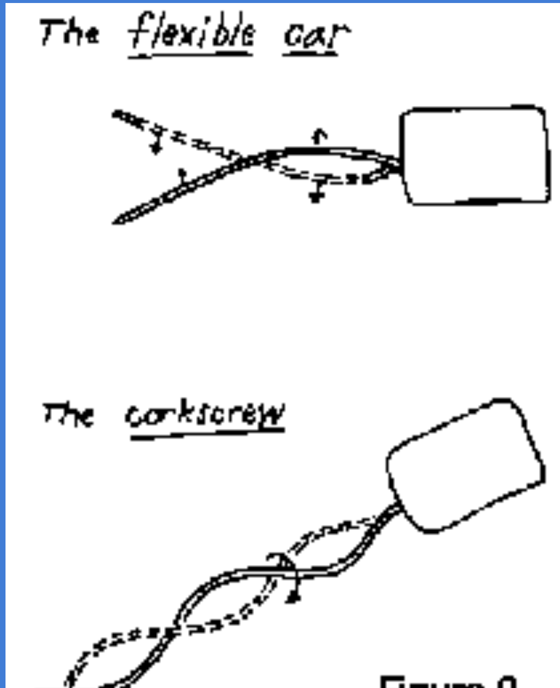


Figure 9

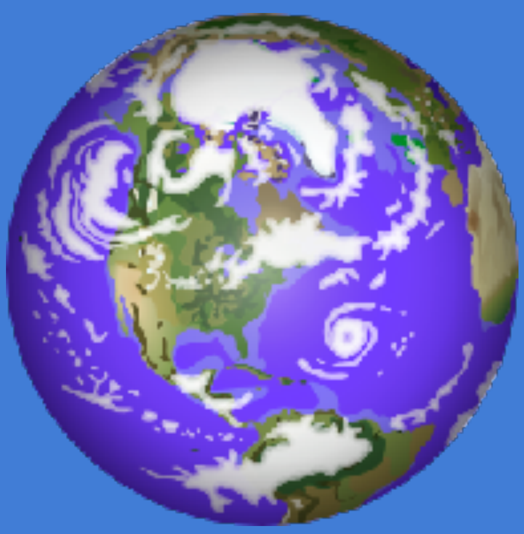
- bacteria swim with $Re = Ua/\nu \sim 10^{-5}$ since $a \sim 1\mu m$
- dynamic similarity can be achieved in a lab-scale setting (a ~ 5cm) by increasing fluid viscosity

Eg. 4 Geophysical flows

- motion of atmosphere and oceans influenced by earth's rotation Ω
- dimensionless groups are the Rossby and Ekman numbers:

$$R_o = \frac{U}{\Omega a} \quad , \quad E_k = \frac{\nu}{\Omega a^2}$$

- dynamic similarity in lab experiments (a ~1m) requires high $\Omega \sim 1/s$



Means of comparison and degrees of similitude

I. Metaphor

II. Physical analogy

III. Dynamic similarity

IV. Mathematical equivalence

- arises when two systems have precisely the same mathematical description
- one system may be understood directly in terms of the other
- arises when dynamic similarity is achieved in fluid systems, but also arises more broadly…

Observation of Bohm trajectories and quantum potentials of classical waves

Physica Scripta (2022)

Georgi Gary Rozenman[1,2], Denys I Bondar[3], Wolfgang P Schleich[4,5], Lev Shemer[6] and Ady Arie[2]

Table 1. Bohmian mechanics of classical surface gravity water waves motivated by its quantum counterpart. Here t and x denote time and space, whereas τ and ξ are dimensionless transverse and propagation coordinates. In this transition [27] from a quantum wave to a classical surface gravity wave we make the replacements $\hbar \to 1$, $i \to -i$ and $m \to 1/2$.

Quantity	Quantum mechanics	Surface gravity water waves
Complex-valued function	wave function $\psi(x, t)$	surface envelope $A(\tau, \xi)$
Propagation coordinate	t	ξ
Transverse coordinate	x	τ
Wave equation	$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}$	$i\frac{\partial A}{\partial \xi} = \frac{\partial^2 A}{\partial \tau^2}$
Guiding equation	$\frac{d}{dt}\overline{x}(t) = \frac{\hbar}{m} Im\left\{\frac{1}{\psi(\overline{x}(t), t)}\frac{\partial}{\partial \overline{x}}\psi\right\}$	$\frac{d}{d\xi}\overline{\tau}(\xi) = 2\, Im\left\{\frac{1}{A(\xi, \overline{\tau}(\xi))}\frac{\partial}{\partial \overline{\tau}}A\right\}$
Quantum potential	$Q = -\frac{\hbar^2}{2m}\frac{1}{\mid\psi\mid}\frac{\partial^2}{\partial x^2}\mid\psi\mid$	$Q = -\frac{1}{\mid A\mid}\frac{\partial^2}{\partial \tau^2}\mid A\mid$

Observation of Bohm trajectories and quantum potentials of classical waves

Georgi Gary Rozenman[1,2], Denys I Bondar[3], Wolfgang P Schleich[4,5], Lev Shemer[6] and Ady Arie[2]

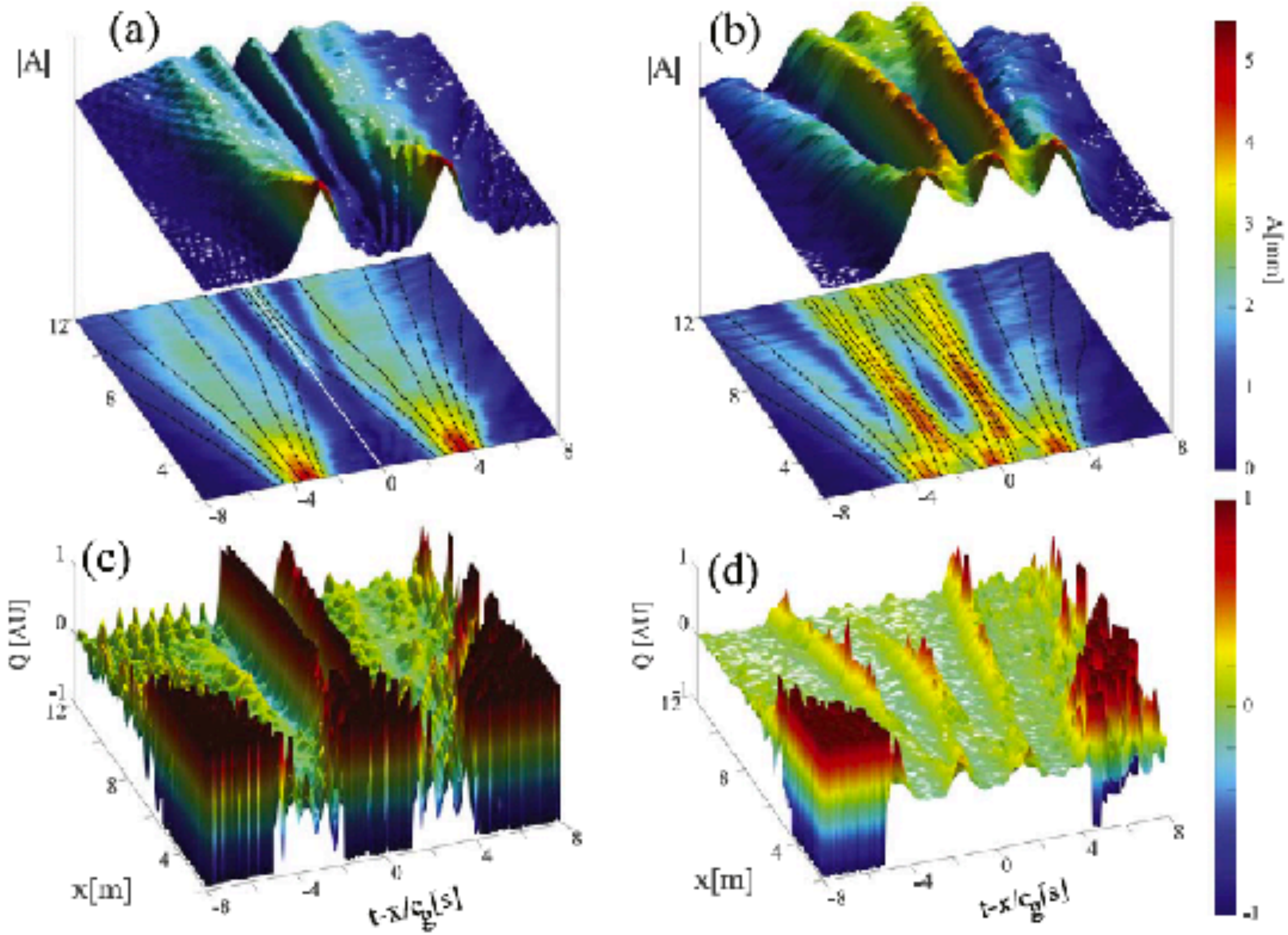


Figure 2. Experimentally obtained surface gravity water wave evolutions (top) and corresponding quantum potentials (bottom) of two-slit (a), (c) and three-slit (b), (d) envelopes $|A|$. Bright and dark colors reflect high and low values, and the color bar units are millimeters. The Bohm trajectories indicated by dashed lines in the 2D-intensity plot underneath (a), (b) and following from equation (2), run in the valleys of the landscape formed by the quantum potentials (c), (d). The tall mountain ranges lead to low amplitudes $|A|$. A key difference between the two-slit and three-slit results is the dark island apparent in the 2D plot of (b) which the Bohm trajectories seem to avoid, and the corresponding crater in the envelope, both of which result from an additional mountain at the center of the quantum potential (d).

Bose-Einstein condensates

- a gas of Bosons in the same quantum state, describable with the same wavefunction
- evolves according to the Gross-Pitaevskii or non-linear Schrödinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi + g_0|\psi|^2\psi$$

Madelung Transformation (1928): $\Psi = \sqrt{\rho}\; e^{iS/\hbar}$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\nabla S) = 0$$

$$\frac{\partial S}{\partial t} + \frac{1}{2}\|\nabla S\|^2 - \left(\frac{\hbar}{m}\right)^2\frac{1}{2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} + \frac{V}{m} + \frac{g_0}{m}\rho = 0$$

QUANTUM POTENTIAL Q NONLINEARITY

- nonlinearity gives rise to term analogous to gravity in shallow-water hydrodynamics

PRL 98, 095301 (2007) PHYSICAL REVIEW LETTERS week ending 2 MARCH 2007

Observation of Faraday Waves in a Bose-Einstein Condensate

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(Received 29 November 2006; published 26 February 2007)

Faraday waves in a cigar-shaped Bose-Einstein condensate are created. It is shown that periodically modulating the *transverse* confinement, and thus the nonlinear interactions in the BEC, excites small amplitude *longitudinal* oscillations through a parametric resonance. It is also demonstrated that even without the presence of a continuous drive, an initial transverse breathing mode excitation of the condensate leads to spontaneous pattern formation in the longitudinal direction. Finally, the effects of strongly driving the transverse breathing mode with large amplitude are investigated. In this case, impact-oscillator behavior and intriguing nonlinear dynamics, including the gradual emergence of multiple longitudinal modes, are observed.

Faraday waves in BECs

$$\omega^2 = \left[\frac{g_0}{m}k + \frac{1}{R}\left(\frac{\hbar}{2m}\right)^2 k^3\right] kR$$

Dispersion relation

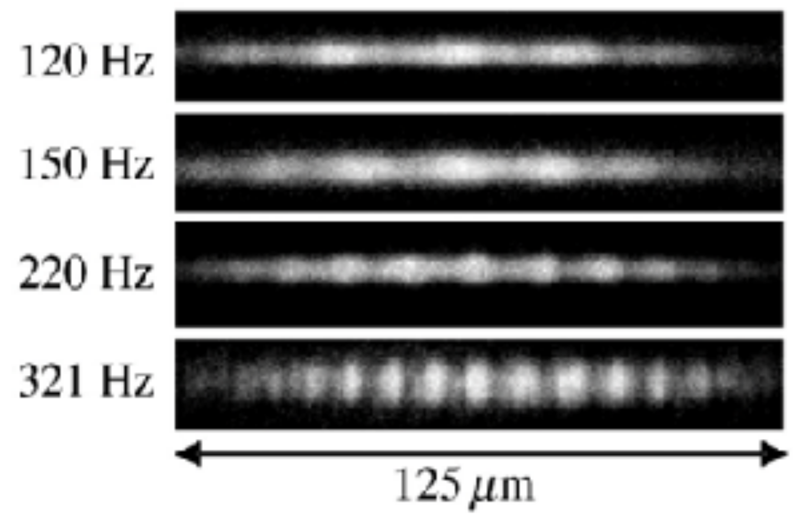


FIG. 1. In-trap absorption images of Faraday waves in a BEC. Frequency labels for each image represent the driving frequency at which the transverse trap confinement is modulated.

Vortices in rotating BECs and stratified flows

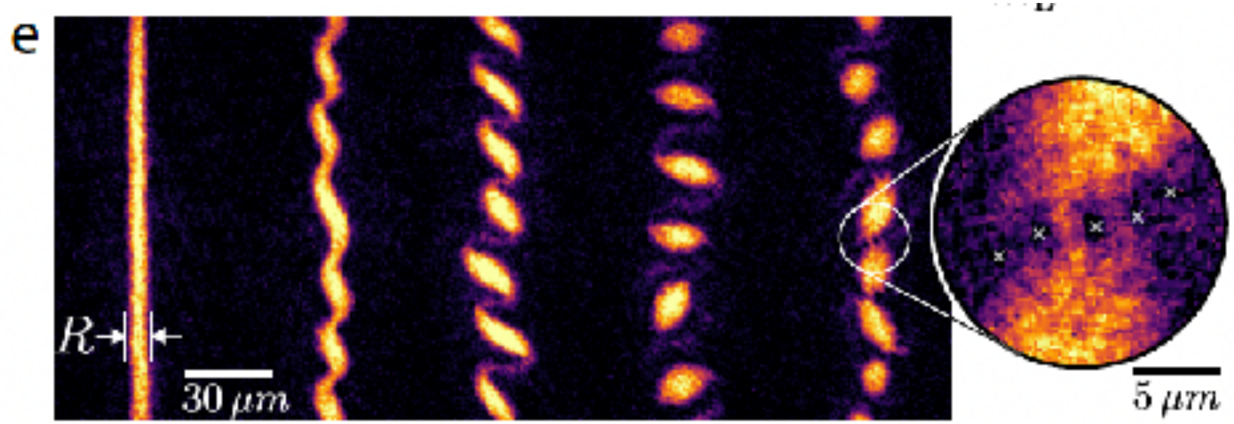
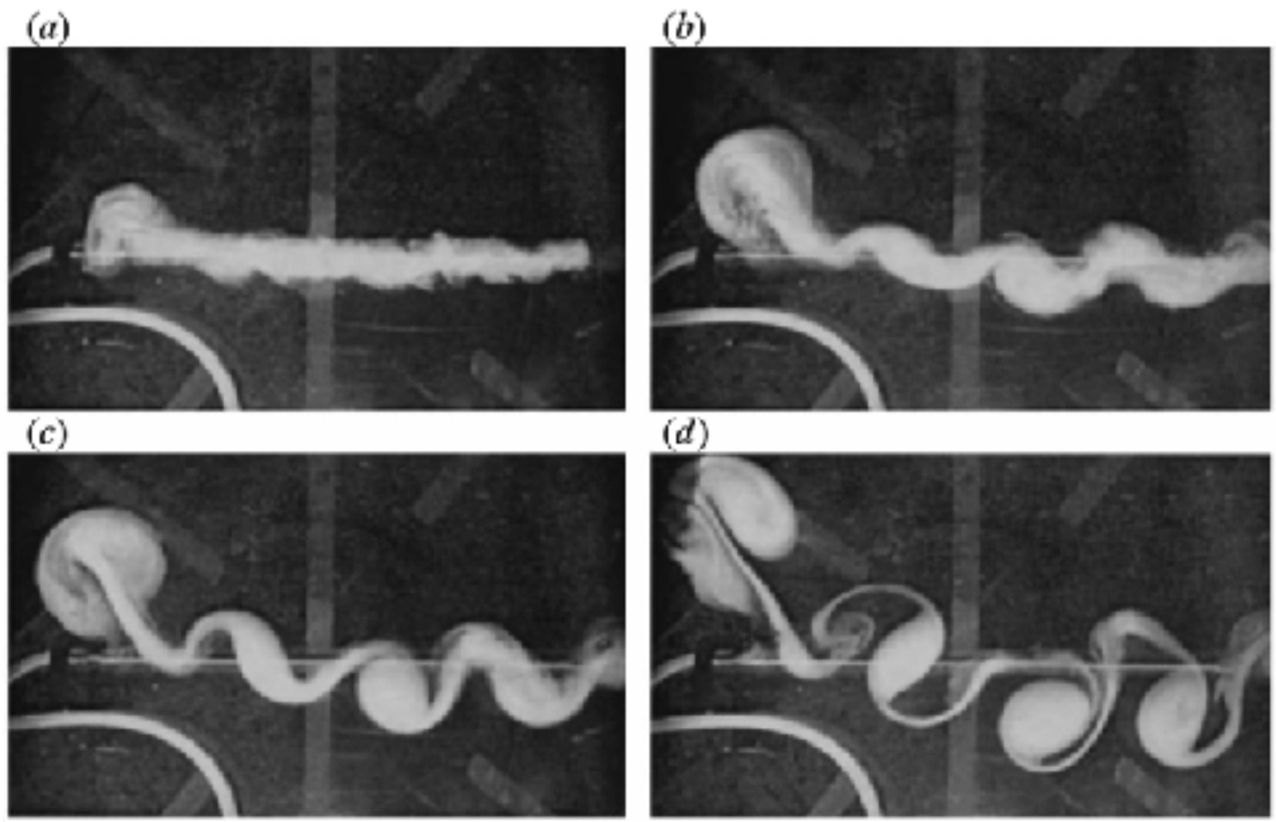


FIG. 1. **Spontaneous crystallization of an interacting Bose-Einstein condensate in an artificial magnetic field.**

Zwierlein (Nature, 2021)



Bush & Woods (JFM, 1999)

Inviscid, incompressible flows | **Electrostatics**

$$\nabla^2\Phi = 0$$

$$\mathbf{u} = \nabla\Phi$$

$$\mathbf{E} = \nabla\Phi$$

- **source-sink flows**
- **fields from point charges**

- **flows near walls identical to electric fields near insulating boundaries**

$$\mathbf{n}\cdot\nabla\Phi = 0$$

Mathematical methods directly transferable

- **method of images, superposition, Gauss's Law, complex potentials**

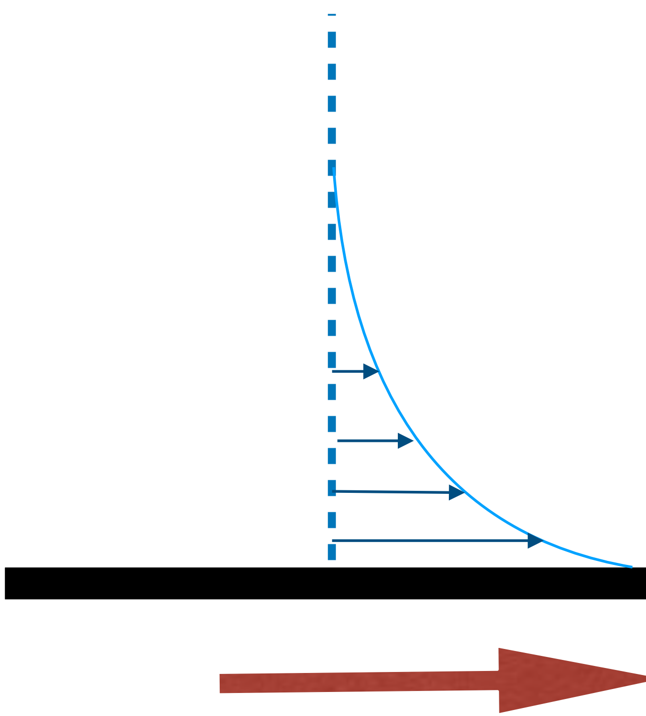
Static heat equation

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T$$

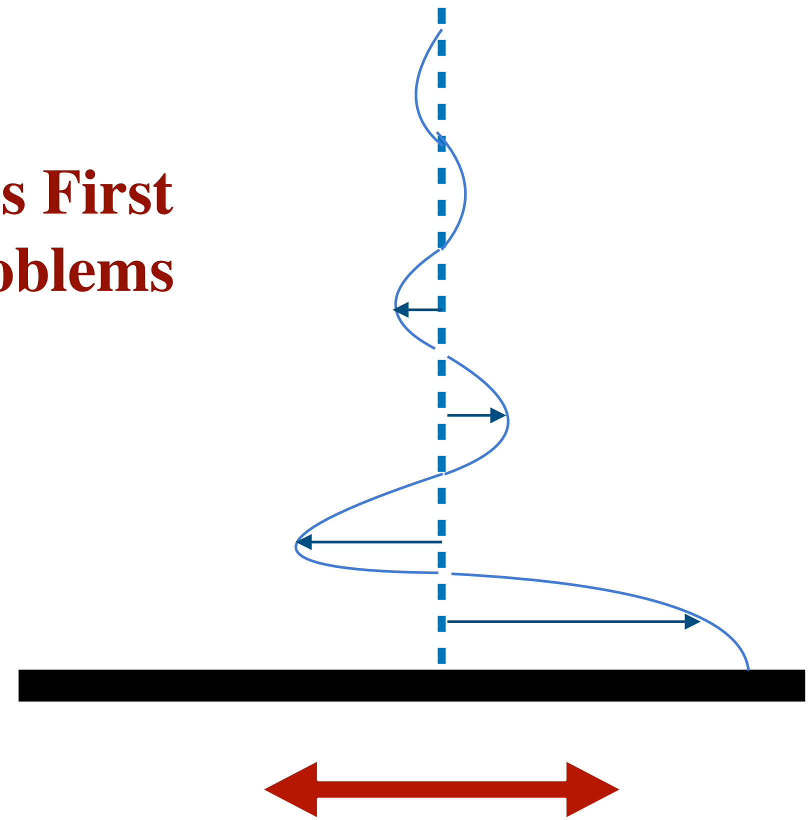
Vorticity equation for unidirectional flows

$$\frac{\partial \omega}{\partial t} = \nu \nabla^2 \omega$$

- **vorticity evolves as a passive scalar**



Rayleigh-Stokes First and Second Problems



Vortex dynamics

$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = \omega \cdot \nabla \mathbf{u} + \nu \nabla^2 \omega$$

Magnetic induction equation

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{u} + \nu_m \nabla^2 \mathbf{B}$$

Rapidly rotating flows

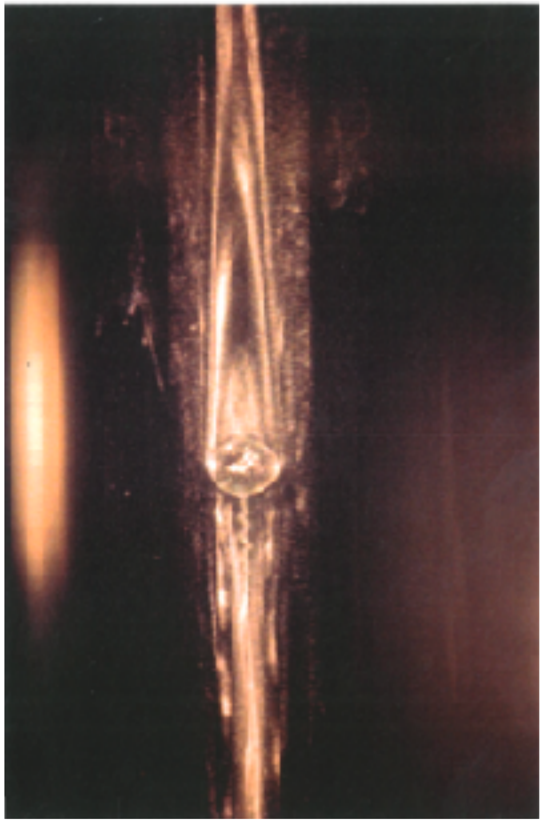
$$\Omega \cdot \nabla \mathbf{u} = 0$$

Strong background field

$$\mathbf{B_0} \cdot \nabla \mathbf{u} = 0$$

Taylor-Proudman Thm

- flow constrained to be 2D



Taylor column

Magnetic column

Gravitoelectromagnetism

- in limit of weak spacetime curvature (weak gravitational fields)

GEM equations	Maxwell's equations
$\nabla \cdot \mathbf{E}_g = -4\pi G \rho_g$	$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$
$\nabla \cdot \mathbf{B}_g = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E}_g = -\frac{\partial \mathbf{B}_g}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{B}_g = -\frac{4\pi G}{c^2}\mathbf{J}_g + \frac{1}{c^2}\frac{\partial \mathbf{E}_g}{\partial t}$	$\nabla \times \mathbf{B} = \frac{1}{\epsilon_0 c^2}\mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$

Lorentz force

GEM equation	EM equation
$\mathbf{F}_\mathbf{g} = m\left(\mathbf{E}_g + \mathbf{v} \times 4\mathbf{B}_g\right)$	$\mathbf{F}_\mathbf{e} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$

Means of comparison and degrees of similitude

I. Metaphor

II. Physical analogy

III. Dynamic similarity

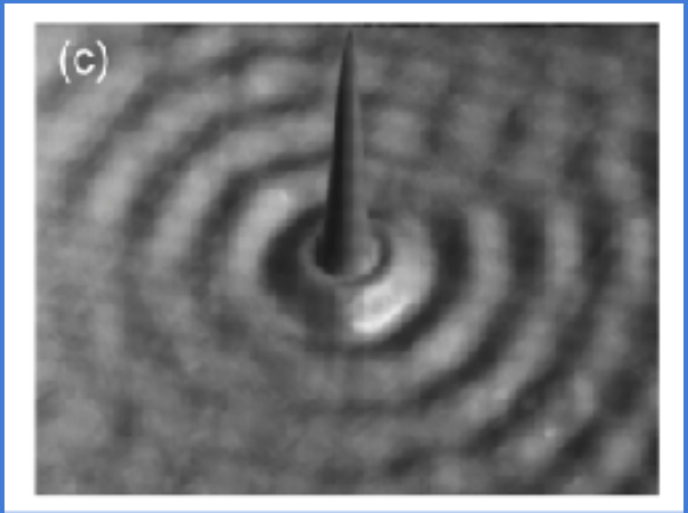
IV. Mathematical equivalence

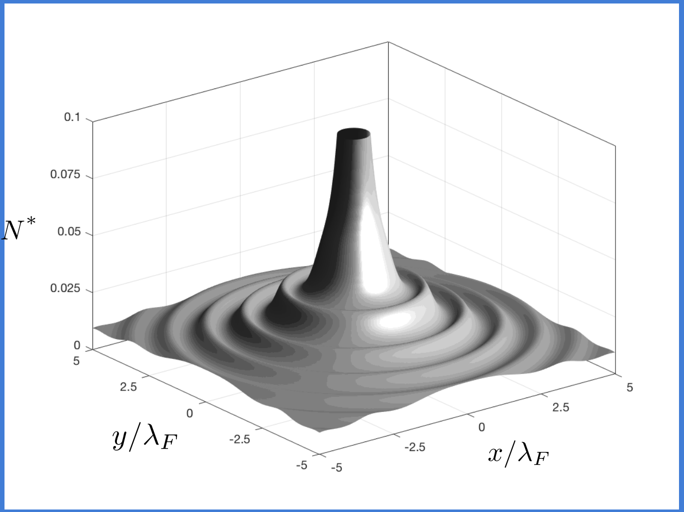
V. Statistical equivalence

- exists when two systems exhibit identical statistics
- the underlying dynamics need not be the same

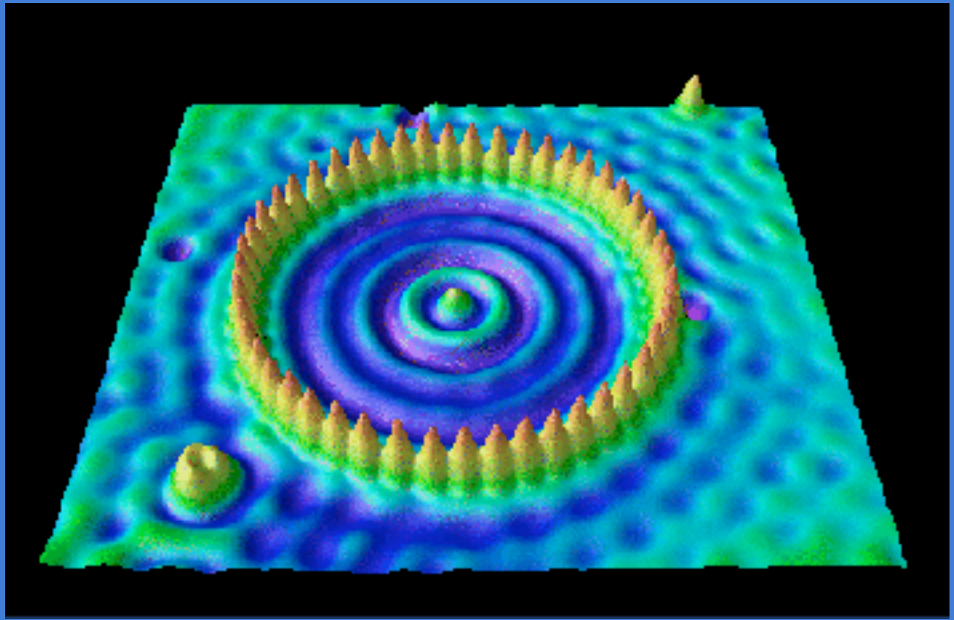
Statistical similarity in HQA

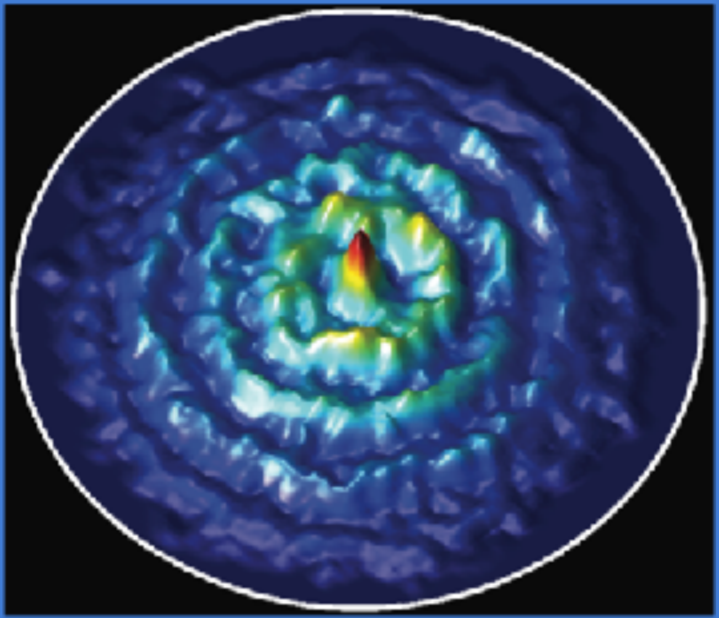
Eg. 1 Friedel oscillations





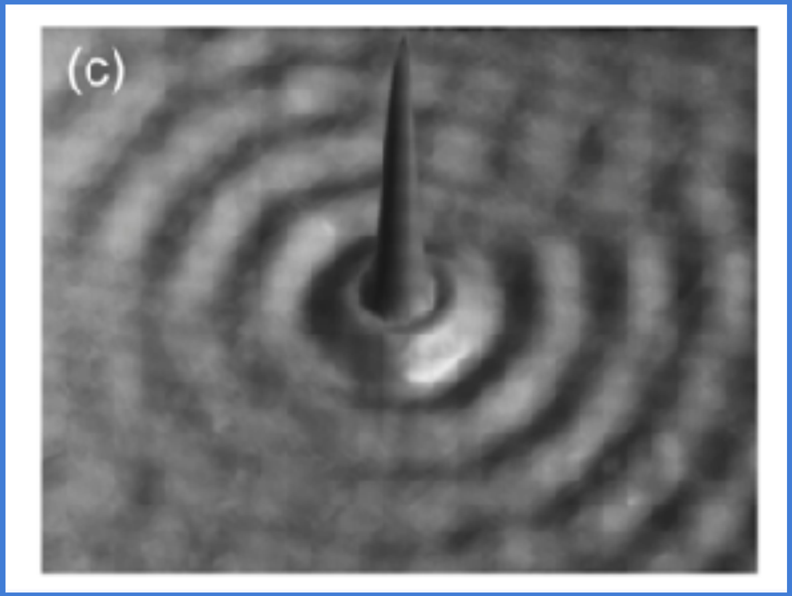
Eg. 2 Corrals





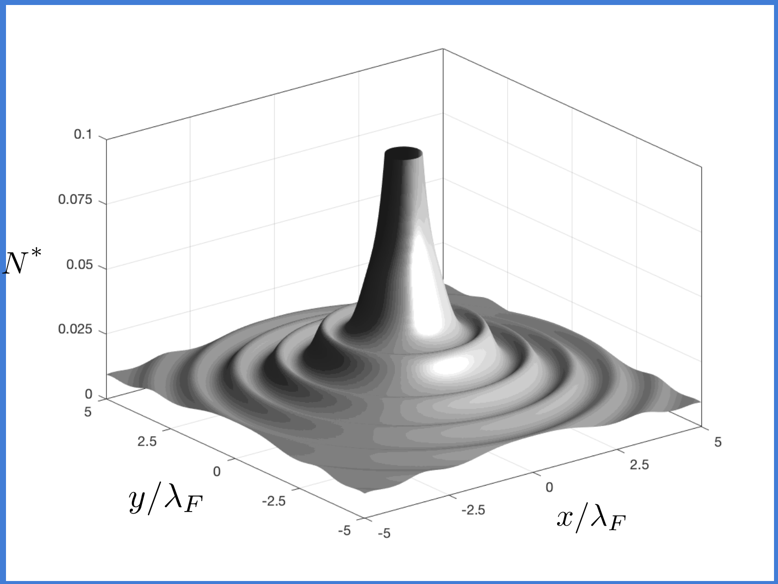
Friedel oscillations

Quantum particles

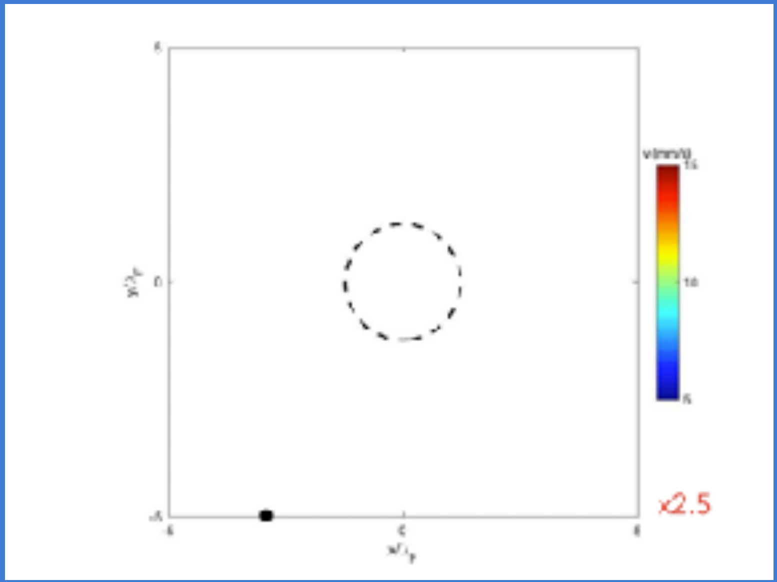


Crommie et al. (2003)

Bouncing droplets

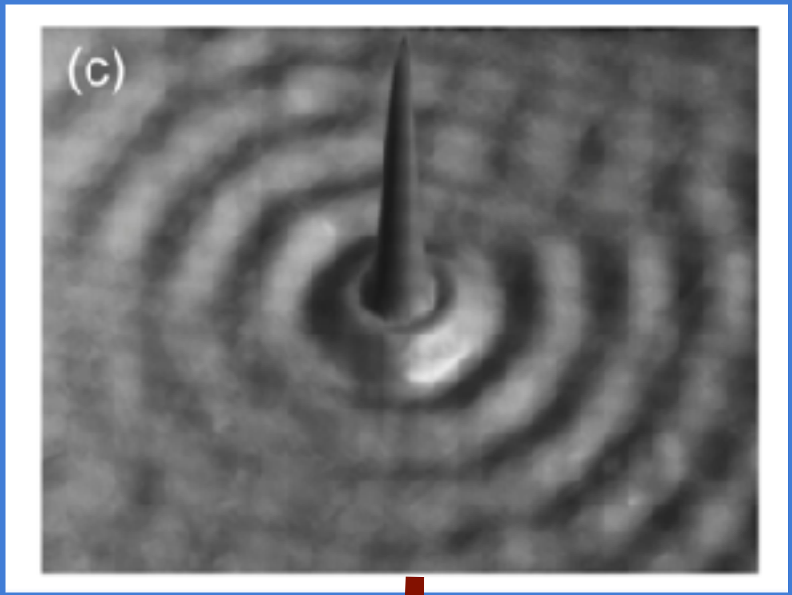


Saenz et al. (2018)



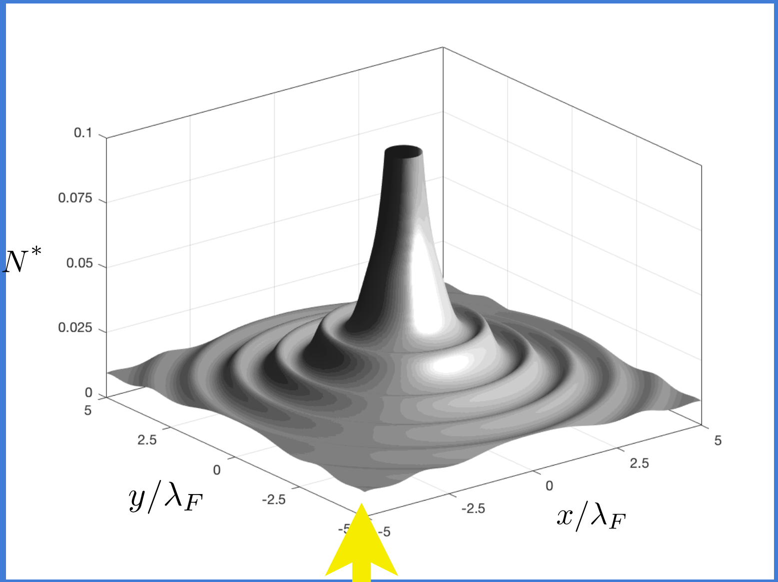
Friedel oscillations

Statistical description



Crommie et al. (2003)

Statistical description



Saenz et al. (2018)

Underlying dynamics

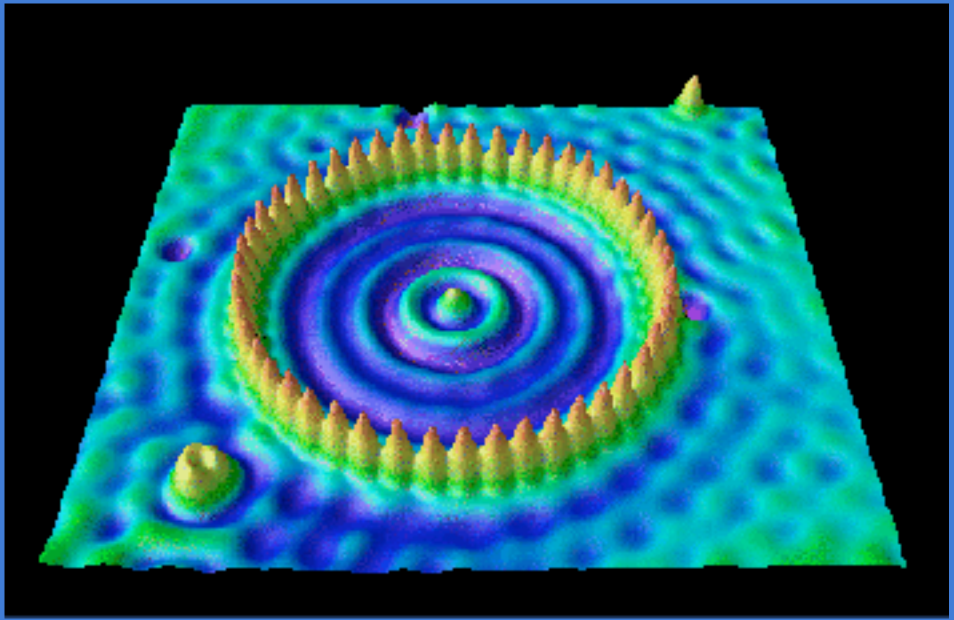
An Exposed Variable Theory

Underlying dynamics

Hidden Variable Theory

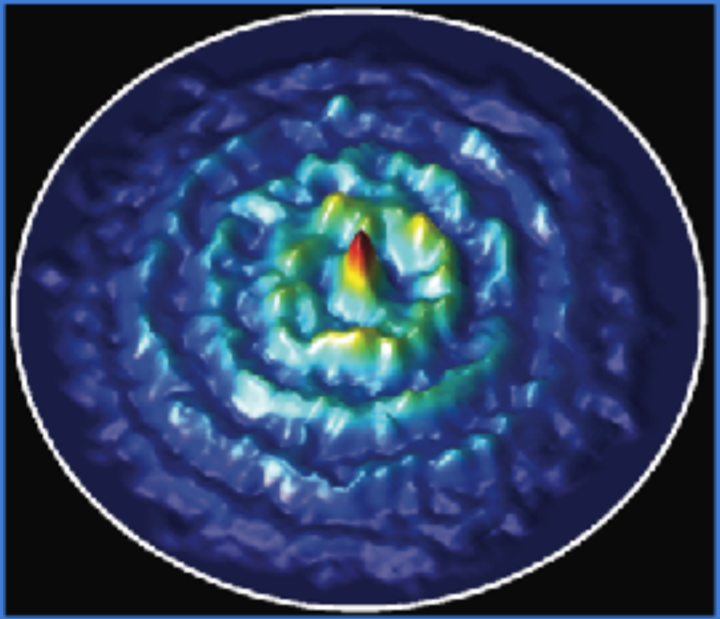
Corrals

Quantum particles

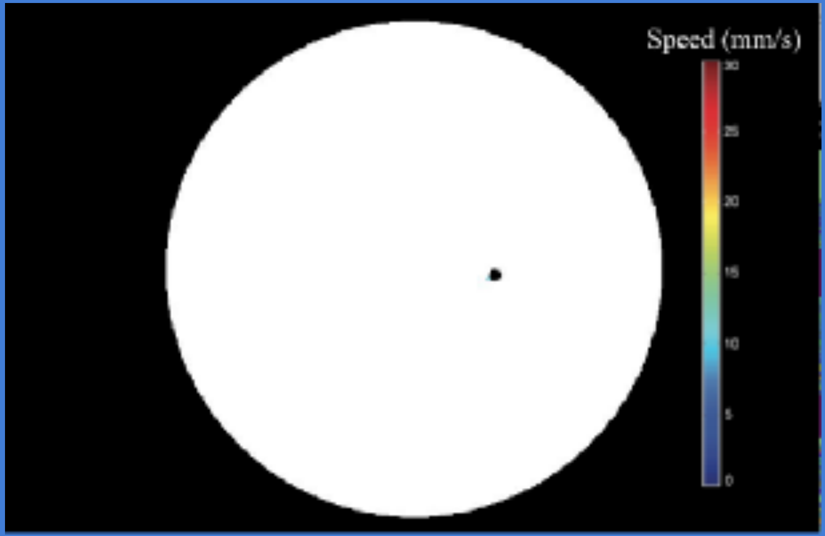


Fiete & Heller (2003)

Bouncing droplets



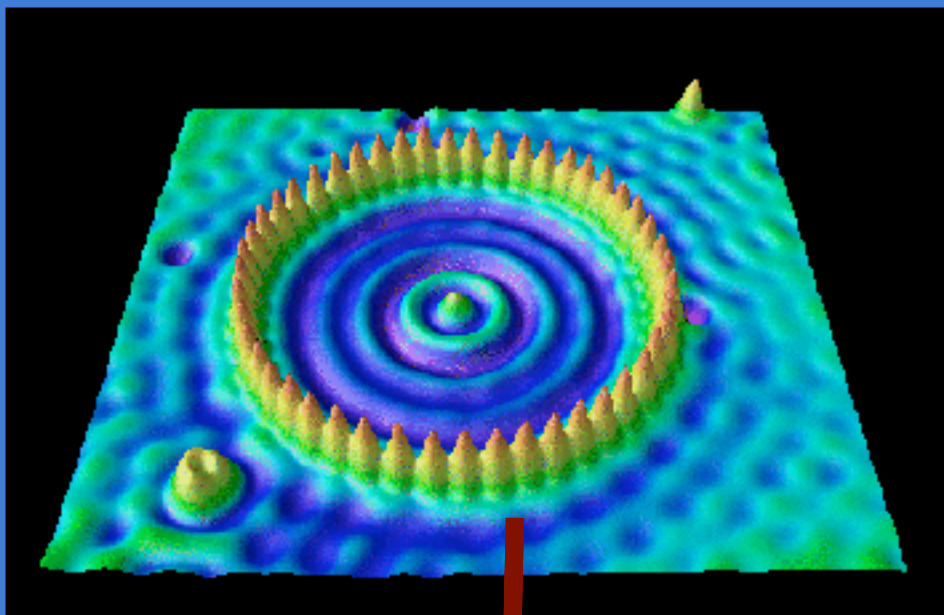
Harris et al. (2012)



Quantum particles	Bouncing droplets

Quantum particles

Statistical description



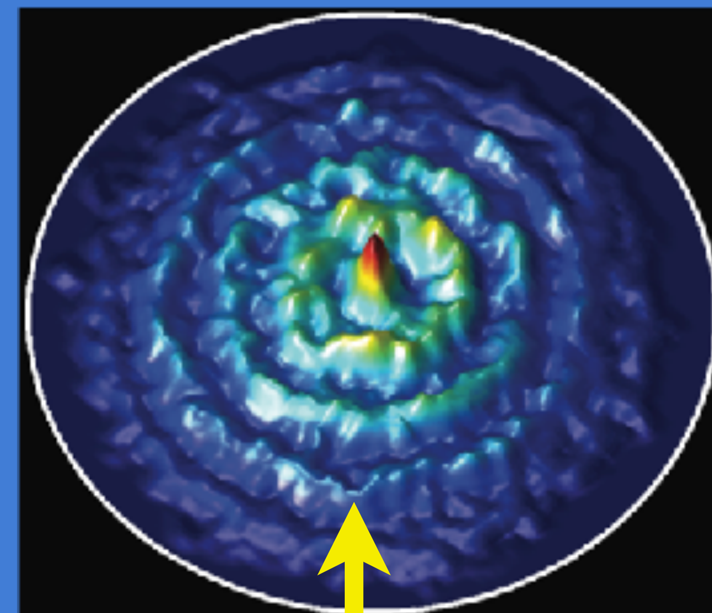
Fiete & Heller (2003)

Underlying dynamics

Hidden Variable Theory

Bouncing droplets

Statistical description



Harris et al. (2012)

Underlying dynamics

An Exposed Variable Theory

Means of comparison and degrees of similitude

I. Metaphor

II. Physical analogy

III. Dynamic similarity

IV. Mathematical equivalence

V. Statistical equivalence

VI. Philosophical similarity

- exists between any physical systems whose dynamics and statistics may be described rationally

e.g. all classical systems

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  e.g. all classical systems

- precludes the need for philosophical extravagance; for example, a revision of our notions of reality (see Lecture 3)

Important points

- the bar for success is very low for the HQA venture: we need only convince ourselves that QM is *philosophically similar* to classical mechanics

- but our demonstrations of *statistical similarity* (in addition to the historical precedent of QM pilot-wave theories) would seem to suggest the possibility of there being a *physical analogy* with pilot-wave hydrodynamics

- as the walking droplets provide physical analogies for both QM and GR, they may be the analog system with the greatest range of scales

One last metaphor

``One is invited to believe that the macroscopic and microscopic worlds are separated by a philosophical chasm so deep that there is no hope of ever finding one's way across. In the depths of the abyss, profound and obscure, lurk the Impossibility Proofs. To those who cannot see beyond strict mathematical equivalence, one must concede that bouncing droplets are not quantum particles. To those with the relatively modest goal of establishing philosophical equivalence through physical analogy, the walking-droplet system has extended a speculative bridge from the macroscopic towards the microscopic, and linked with a structure emerging from the other side, the modern extensions of de Broglie's pilot-wave theory. Rickety though this bridge may be, under construction, it offers striking new views to workers from both sides."