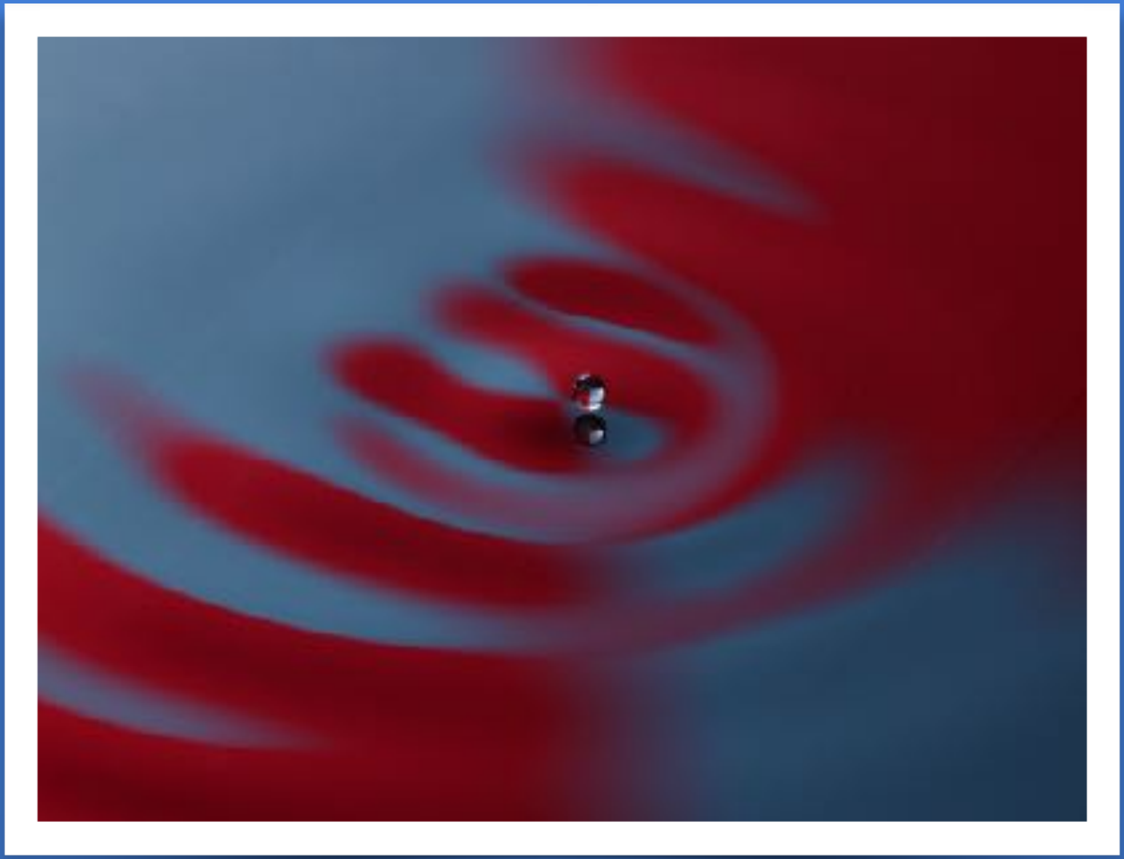
18.S996 Hydrodynamic quantum analogs

Lecture 1: Introduction



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HALF-TIME

Three paradigms for macroscopic quantum behavior

- depend critically on the quasi-monochromatic wave field
- the drop navigates the local, self-generated potential that is its pilot wave field

I. Orbital pilot-wave dynamics

- quantized orbits emerge from the dynamic constraint imposed on the droplet by its quasi-monochromatic wave field
- chaotic pilot-wave dynamics: intermittent switching between weakly unstable periodic orbits yields multimodal, quantum-like statistics

II. In-line oscillations

- speed fluctuations lead to correlation between position and speed, a statistical signature with the Faraday wavelength

III. Stochastic motion over mean-pilot-wave potential

- random walk characterized by effective diffusivity $D \sim U \lambda_F$
- evokes Nelson's Stochastic Mechanics: $D_Q \sim \frac{\hbar}{m} \sim \frac{\hbar k_B}{m k_B} \sim U \lambda_B$

Hydrodynamic quantum analogs

- orbital quantization: Larmor levels, SHO
- spin states, Zeeman splitting, spin lattices, Anderson localization
- statistical projection (`mirage') effects in confined geometries
- Friedel oscillations, corrals, interaction-free measurement
- tunneling, superradiant tunneling and emission
- single-particle diffraction and interference
- Uncertainty relations and Exclusion Principles
- boost factors, HOM effect, surreal trajectories, bomb testers
- optical effects: Talbot effect, Bragg scattering, optical ratcheting
- distant, two-particle and multi-particle correlations

Most significant limitations

- viscous damping of pilot wave: quantum features emerge at high memory
- drop inertia may dominate pilot-wave force

A generalized pilot-wave framework

(Bush ARFM 2015, *Durey & Bush, 2020*)

- retain key features of walker system

  (memory, resonance, quasi-monochromatic wave field)

- explore beyond the range of the hydrodynamic system

- discover new quantum-like features; e.g. stable spin states

- extended to 3D, where helical spin states have now been found

- connect to and inform quantum pilot-wave theories

(Bush, ARFM, 2015)

Classical pilot-wave dynamics: a parametric generalization

$$\kappa_0(1-\Gamma)\ddot{\mathbf{x}}_p + \dot{\mathbf{x}}_p = \frac{2}{(1-\Gamma)^2}\int_{-\infty}^{t} \frac{J_1(|\mathbf{x}_p(t)-\mathbf{x}_p(s)|)}{|\mathbf{x}_p(t)-\mathbf{x}_p(s)|}(\mathbf{x}_p(t)-\mathbf{x}_p(s))e^{-(t-s)}\,ds$$

INERTIA **DRAG** **WAVE FORCING**

where $$\Gamma = \frac{\gamma - \gamma_W}{\gamma_F - \gamma_W}\ ,\qquad \kappa_0 = (m/D)^{3/2} k_F \sqrt{gA/2T_F}$$

PROXIMITY TO THRESHOLD

$0 < \Gamma < 1$

CONTAINS ALL FLUID PARAMETERS: BOUNDED IN HYDRODYNAMIC SYSTEM

$0.8 < \kappa_0 < 1.6$ in lab

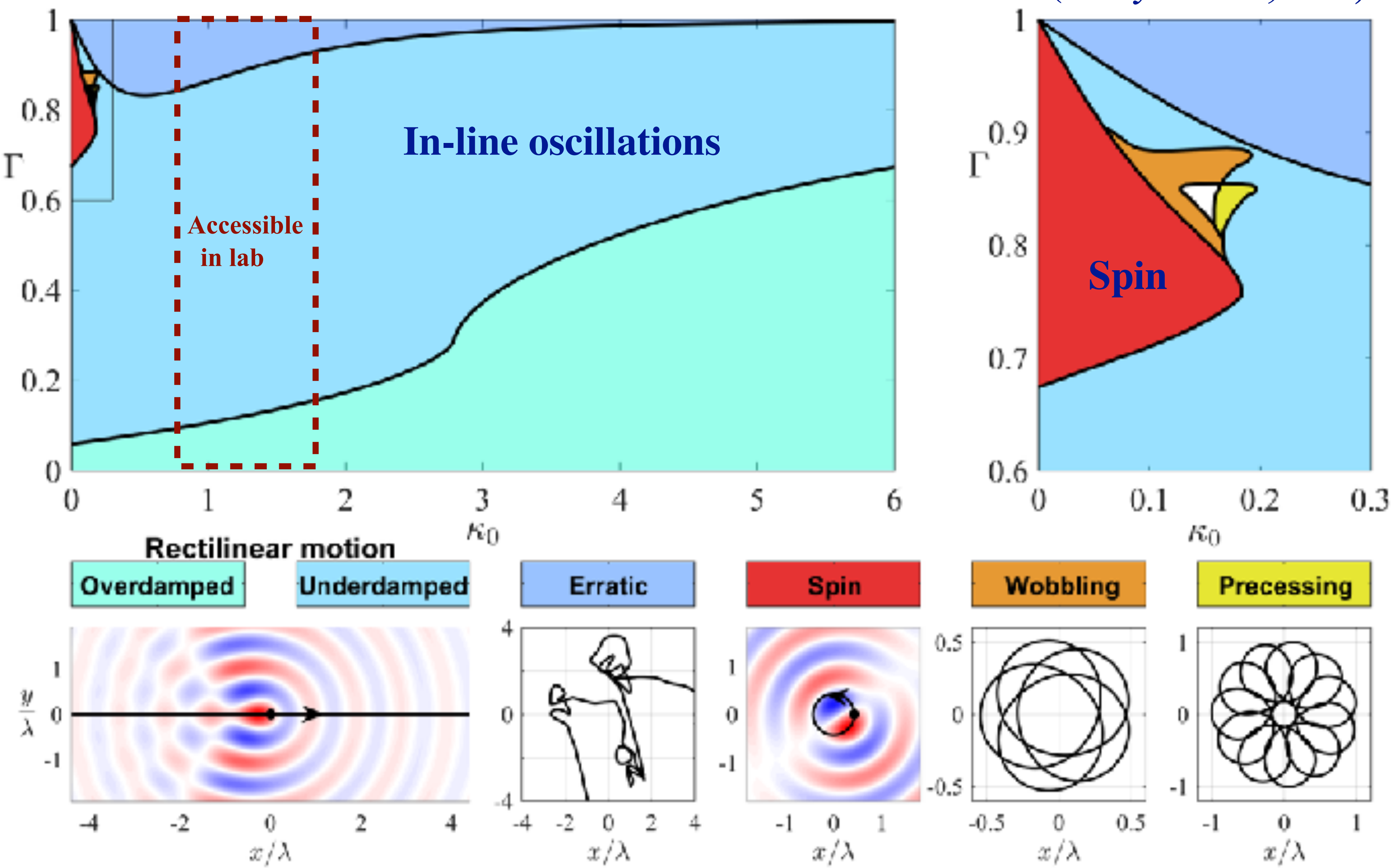
Question: For what values of (κ_0, Γ) does the system look most like QM?

Eg.1 When are hydrodynamic spin states stable?

Eg.2 When is walking state unstable to in-line oscillations?

Generalized pilot-wave theory: the free particle in 2D

(*Durey & Bush, 2020*)



- stable, wobbling and precessing spin states may obtain
- walking state may be unstable to in-line oscillations with wavelength λ_F
- aperiodic `*jittering*' gives rise to random walk with diffusivity $D \sim U\lambda_F$

Generalized pilot-wave dynamics

$$\kappa_0(1-\Gamma)\ddot{\mathbf{x}}_p + \dot{\mathbf{x}}_p = \frac{2}{(1-\Gamma)^2}\int_{-\infty}^{t} \frac{J_1(|\mathbf{x}_p(t)-\mathbf{x}_p(s)|)}{|\mathbf{x}_p(t)-\mathbf{x}_p(s)|}(\mathbf{x}_p(t)-\mathbf{x}_p(s))e^{-(t-s)}\,ds$$

INERTIA **DRAG** **WAVE FORCING**

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PROXIMITY TO THRESHOLD

CONTAINS ALL FLUID PARAMETERS: BOUNDED IN HYDRODYNAMIC SYSTEM

Question: For what values of (κ_0, Γ) does the system look most like QM?

Further generalizations

- consideration of alternative wave forms, spatio-temporal damping (Durey, *Chaos*, 2020; Valani *et al*., *PRE*, 2021)
- extend pilot-wave dynamics to three dimensions (3D spin states now found)
- include stochastic forcing, study hybrid pilot-wave stochastic dynamics
- connect to/inform quantum pilot-wave theories of Bohm and de Broglie and their modern extensions

The (Old) Hydrodynamic Interpretation of Quantum Mechanics

Schrodinger: $$i\hbar\ \Psi_t \ = \ -\frac{\hbar^2}{2m}\nabla^2\Psi \ + \ V\ \Psi$$

Madelung transformation (1928): $$\Psi = \sqrt{\rho}\ e^{iS/\hbar}$$

Continuity: $$\frac{D\rho}{Dt} + \rho\ \nabla\cdot\mathbf{u} = 0$$

Quantum Hamilton-Jacobi: $$\frac{\partial S}{\partial t} + \frac{1}{2}\mathbf{u}^2 - \frac{\hbar^2}{2m^2}\frac{1}{\sqrt{\rho}}\nabla^2\sqrt{\rho} + \frac{V}{m} = 0$$

QUANTUM POTENTIAL Q

where $\rho = |\Psi|^2$ is the probability density, S is the action,

$\mathbf{u} = \nabla S/m$ is the quantum velocity of probability,

$\mathbf{j} = \rho\ \mathbf{u}$ is the quantum probability flux.

Bohmian Mechanics (1952)

David Bohm

- equate quantum velocity of probability $\mathbf{u}$ and particle velocity $\dot{\mathbf{x}}_p$
- solve Schrodinger's equation for Ψ , from which Q is computed
- solve trajectory equation

$$m\,\ddot{\mathbf{x}}_p = -\nabla Q - \nabla V$$

NONLOCAL

Successes

- given initial conditions consistent with solution, it predicts emergent statistics consistent with those of the standard quantum formalism

FIGURE 1: An ensemble of trajectories for the two-slit experiment, uniform in the slits. (adapted by Gernot Bauer from Philippidis, Dewdney, & Hiley 1979: 23, fig. 3)

- a counterexample of the Impossibility Proofs that held sway at the time

Surreal trajectories

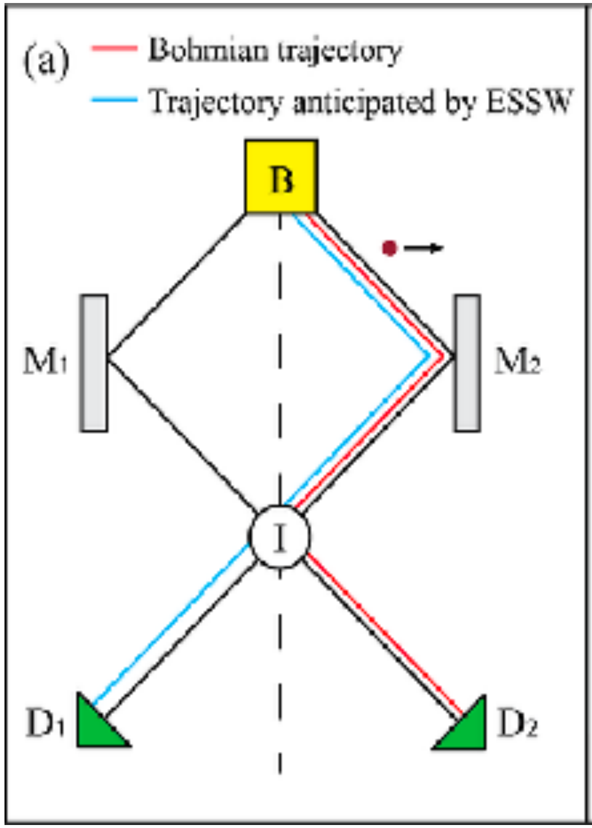
- Englert, Sully, Süssman and Walther (ESSW). 1992

- proposed an interference experiment intended to expose the shortcomings of Bohmian mechanics

`Bohmian trajectories are at odds with common sense: they are not real, they are surreal.'

- their reasoning was criticized by Aharanov & Vaidman (1996), who concluded:

` ESSW does not show that Bohmian mechanics is inconsistent, only that Bohmian trajectories behave differently from what one would expect classically.'



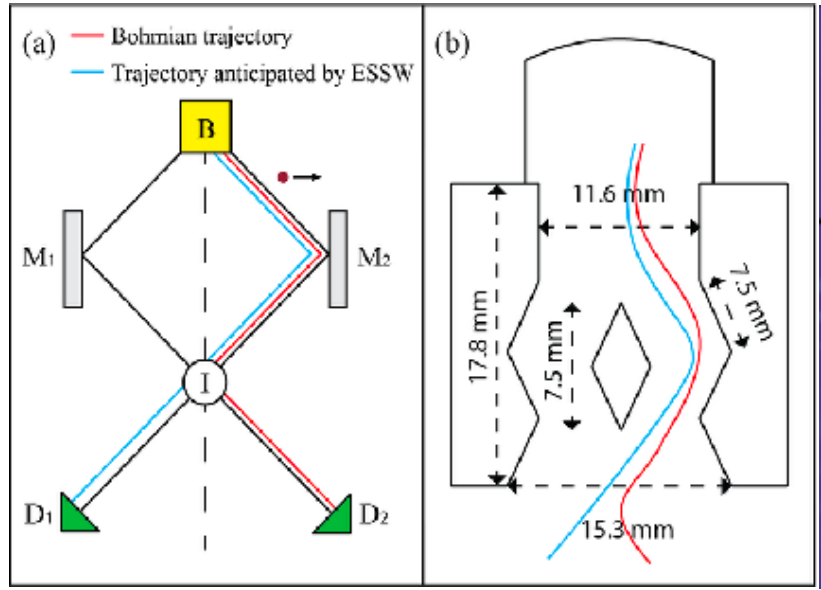
- experimental investigations using `weak measurement' found mean trajectories consistent with the surreal trajectories *(Mahler et al., 2016)*

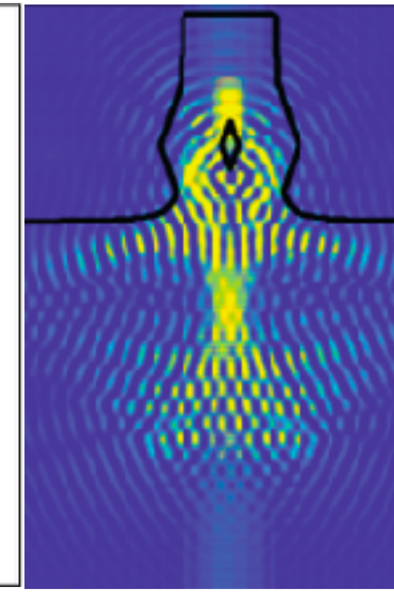
`We demonstrate that the trajectories seem surreal only if one ignores their manifest nonlocality.'

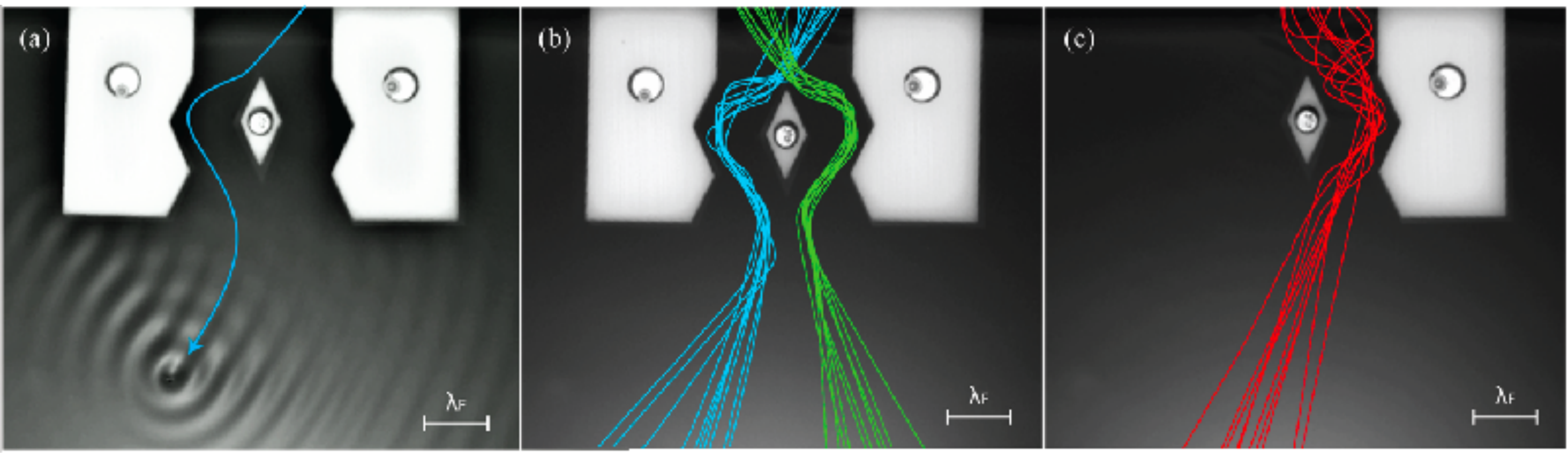
Real surreal trajectories

- *Frumkin, Struyve, Darrow, JB (PRA, 2022)*

- arise in the walker system at high memory







- `surreal' trajectories are not at odds with classical intuition informed by a familiarity with pilot-wave hydrodynamics

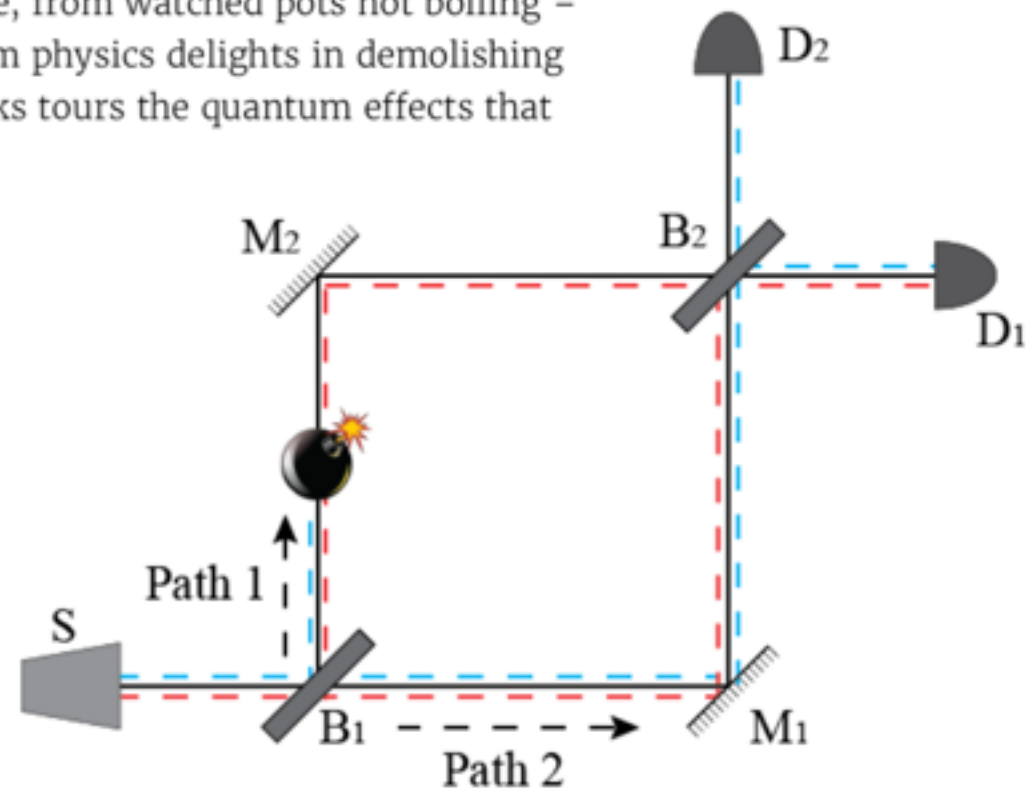
- may be readily understood as a manifestation of non-Markovian pilot-wave dynamics, with no need to invoke `quantum nonlocality'

NewScientist

Seven wonders of the quantum world

From undead cats to particles popping up out of nowhere, from watched pots not boiling – sometimes – to ghostly influences at a distance, quantum physics delights in demolishing our intuitions about how the world works. Michael Brooks tours the quantum effects that are guaranteed to boggle our minds.

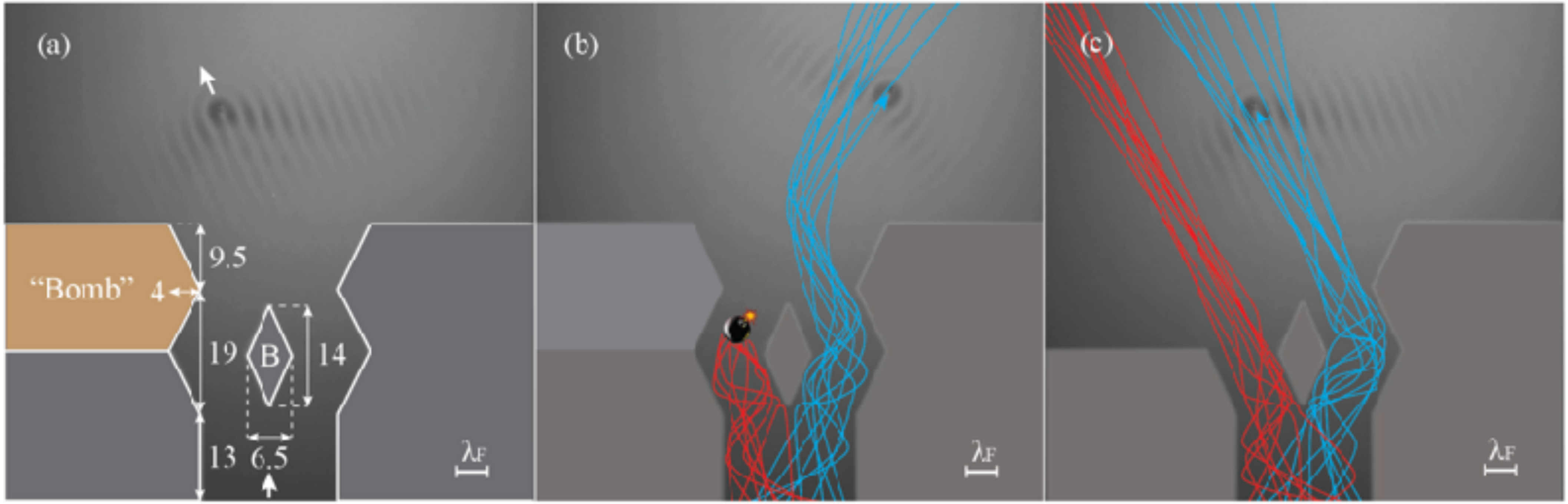
1. *Corpuscles and buckyballs*
2. *The Hamlet effect*
3. *Something for nothing*
4. *The Elitzur–Vaidman bomb tester*
5. *Spooky action at a distance*
6. *The field that isn't there*
7. *Superfluids and supersolids*



- in absence of bomb, interference always causes photon to arrive at D1
- with bomb, particle either detonates bomb (Path 1) or arrives at D2 or D1 with equal probability
- if bomb is present 50% of the time, then you can detect it 25% of the time via a particle that took Path 2, so never interacted with it

A hydrodynamic analog of the quantum Bomb tester

- *Frumkin & JB, PRA (2023)*



- submerged topography (orange) plays the role of the `bomb'
- in the absence of the bomb, all trajectories go to the left
- in the presence of the bomb, surreal trajectories may arise:
  - the droplet's pilot-wave interacts with the bomb, altering the droplet's path
- 25% of the time, the droplet detects a bomb along a path it didn't take

Bohmian Mechanics (1952)


David Bohm

- equate quantum velocity of probability $\mathbf{u}$ and particle velocity $\dot{\mathbf{x}}_p$
- solve Schrodinger's equation for Ψ , from which Q is computed
- solve trajectory equation

$$m\,\ddot{\mathbf{x}}_p = -\nabla Q - \nabla V$$

NONLOCAL

Shortcomings

- Einstein's objection: it is `*nonlocal'* by virtue of the quantum potential Q
- no mechanism for wave generation; no feedback of particle on field

Extensions (Bohm & Vigier 1954)

- invoke a stochastic forcing $\nabla \Phi_S$ from a `sub quantum realm':

$$m\,\ddot{\mathbf{x}}_p = -\nabla Q - \nabla V + \nabla \Phi_S$$

- particles jostle about $\mathbf{u}$ like Brownian motion of gas molecules about streamlines

	Bohmian mechanics	Walkers
WAVELENGTH	λ_B	λ_F
GUIDANCE	$m\,\ddot{\mathbf{x}}_p \;=\; -\nabla Q \;-\; \nabla V \;+\; \nabla\Phi_S$ NONLOCAL *AD HOC*	$m\,\ddot{\mathbf{x}}_\mathbf{p} \;=\; -D\,\dot{\mathbf{x}}_\mathbf{p} \;+\; \nabla\eta(\mathbf{x},t) \;-\; \nabla V$ LOCAL
NON-LOCAL WAVE POTENTIAL	$\mathbf{Q} = -\frac{\hbar^2}{m^2}\frac{1}{\sqrt{\rho}}\nabla^2\sqrt{\rho}$ QUANTUM POTENTIAL	$\bar{\eta}(\mathbf{x}) \;=\; \eta_B * \mu(\mathbf{x})$ MEAN WAVE FIELD
STOCHASTIC FORCING	$\nabla\Phi_S$ **ARBITRARY,** ***ad hoc***	$-\nabla\eta^*(\mathbf{x},t)$ PERTURBATION WAVE FIELD
WAVE ORIGIN	NONE	PARTICLE VIBRATION

Particle vibration on the Compton scale

- Frank Wilczek (*The Lightness of Being*, 2008): **`a poem in two lines'…**

RELATIVITY	**QM**
$E = m\,c^2$	$E = \hbar\,\omega$

Einstein-de Broglie relation: $mc^2 = \hbar\omega$

Natural frequency:

$$\omega_c = \frac{mc^2}{\hbar}$$

Compton frequency

- de Broglie (1926) suggested microscopic particles have an internal clock at ω_c that generates a wave that moves in concert with the particle

→ **particles move in resonance with a guiding or `pilot' wave field**

de Broglie's pilot-wave theory: The double-wave solution

" A freely moving body follows a trajectory that is orthogonal to the surfaces of an associated wave guide".

- Louis de Broglie (1892-1987)

- Ψ is the probability wave, as prescribed by standard quantum theory
- $\phi = |\phi| \, e^{i\Phi/\hbar}$ is a real physical wave responsible for guiding the particle

according to his Guidance Equation: $$\dot{\mathbf{x}}_{\mathbf{p}} = \frac{\hbar}{m_0} Im \left[\frac{\nabla \phi}{\phi} \right]$$

- wave generated by internal particle vibration (*Zitterbewegung*) at the Compton frequency: $$\omega_c = \frac{m_0 c^2}{\hbar}$$
- a solution of Klein-Gordon equation triggered by oscillations in rest mass
- particle follows point of constant wave amplitude: his guidance equation yields

$\mathbf{p} = \gamma m_0 \dot{\mathbf{x}}_{\mathbf{p}} = \nabla \Phi = \hbar \mathbf{k}$ for a monochromatic wave $\Phi = \mathbf{k} \cdot \mathbf{x} - \omega t$

- **Harmony of Phases**: the particle oscillates in resonance with its guiding wave
- *incomplete*: wave generation mechanism, precise form of ϕ not specified

de Broglie's pilot-wave theory

- **fast** dynamics: mass oscillations at

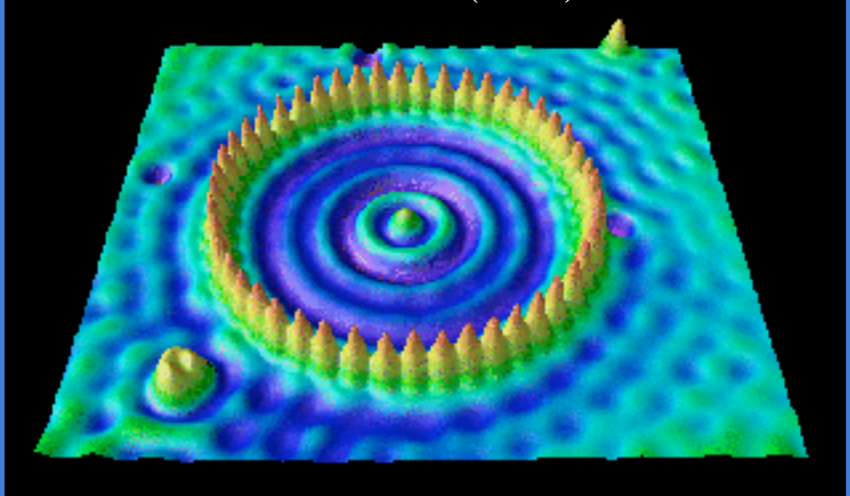
$$\omega_c = \frac{m_0 c^2}{\hbar}$$ create wave field centered on particle

- **intermediate** pilot-wave dynamics: particle rides its guiding wave field such that

$$\mathbf{p} = \hbar\ \mathbf{k}$$

- **long-term statistical** behaviour described by standard quantum theory

Fiete & Heller (2003)



	de Broglie	Walkers
WAVE TRIGGER	ZITTERBEWEGUNG	Bouncing
VIBRATION FREQUENCY	$\omega_c = \frac{m_o c^2}{\hbar}$	$\omega_d = \sqrt{\frac{\sigma}{m}}$
WAVES	Matter waves	Capillary Faraday
WAVE-PARTICLE RESONANCE	Harmony of phases	$\omega_d = \omega_F$
WAVE ENERGETICS	$mc^2 \longleftrightarrow \hbar\omega$	$mgH \longleftrightarrow$ Surface Energy
KEY PARAMETER	$\hbar$	σ
STATISTICAL WAVELENGTH	λ_B	λ_F
VIBRATION LENGTH	$\lambda_c = h/mc$	λ_F

Shortcomings of the quantum pilot-wave theories

- *no mechanism specified for pilot-wave generation*

Bohmian mechanics

- a dynamical reformulation of a statistical theory
- particle is piloted by a wave form Ψ of unspecified origins
- *nonlocal*: particle is guided by the non-local quantum potential

de Broglie's mechanics

- original double-solution theory distinguished between ϕ and Ψ
- form of pilot-wave ϕ unspecified: several options considered
- at one stage set $\phi \propto \Psi$: reduces to Bohmian mechanics

→ two theories conflated into `de Broglie-Bohm theory'

So, what is the matter wave field in QM?

- workers in Stochastic Electrodynamics (SED) suggest an EM pilot wave

(*de la Pena, Cetto, Valdes-Hernandes 2015*)

- de Broglie suggested that the field satisfies the Klein-Gordon equation, as describes the Higgs field and weak *gravitational* waves …

What might de Broglie have tried …
… had he had MATLAB?

Hydrodynamically-inspired quantum field theory

Dagan & Bush (2020)

- extend de Broglie's mechanics, informed by pilot-wave hydrodynamics

> ``*…we can envisage a more active role for the particle, something which is not even admitted as conceivable in the conventional view. This may, for instance, enter as a 'source' of the pilot-wave field through an inhomogeneous term in the wave equation…*"
>
> ***— Holland (1995)***

- model particle as wave source, an oscillation at twice the Compton frequency

Forced Klein-Gordon equation

$$\phi_{tt} \; - \; c^2 \; \phi_{xx} \; + \; \omega_c^2 \; \phi \; = \; \epsilon \; \sin(2\omega_c t) \; e^{-[(x-x_p)/\lambda_c]^2}$$

`particle': a localized excitation
in the Higgs field

- consider the zero-particle-inertia, no-wave-damping limit
- particle moves in response to gradients in wave amplitude

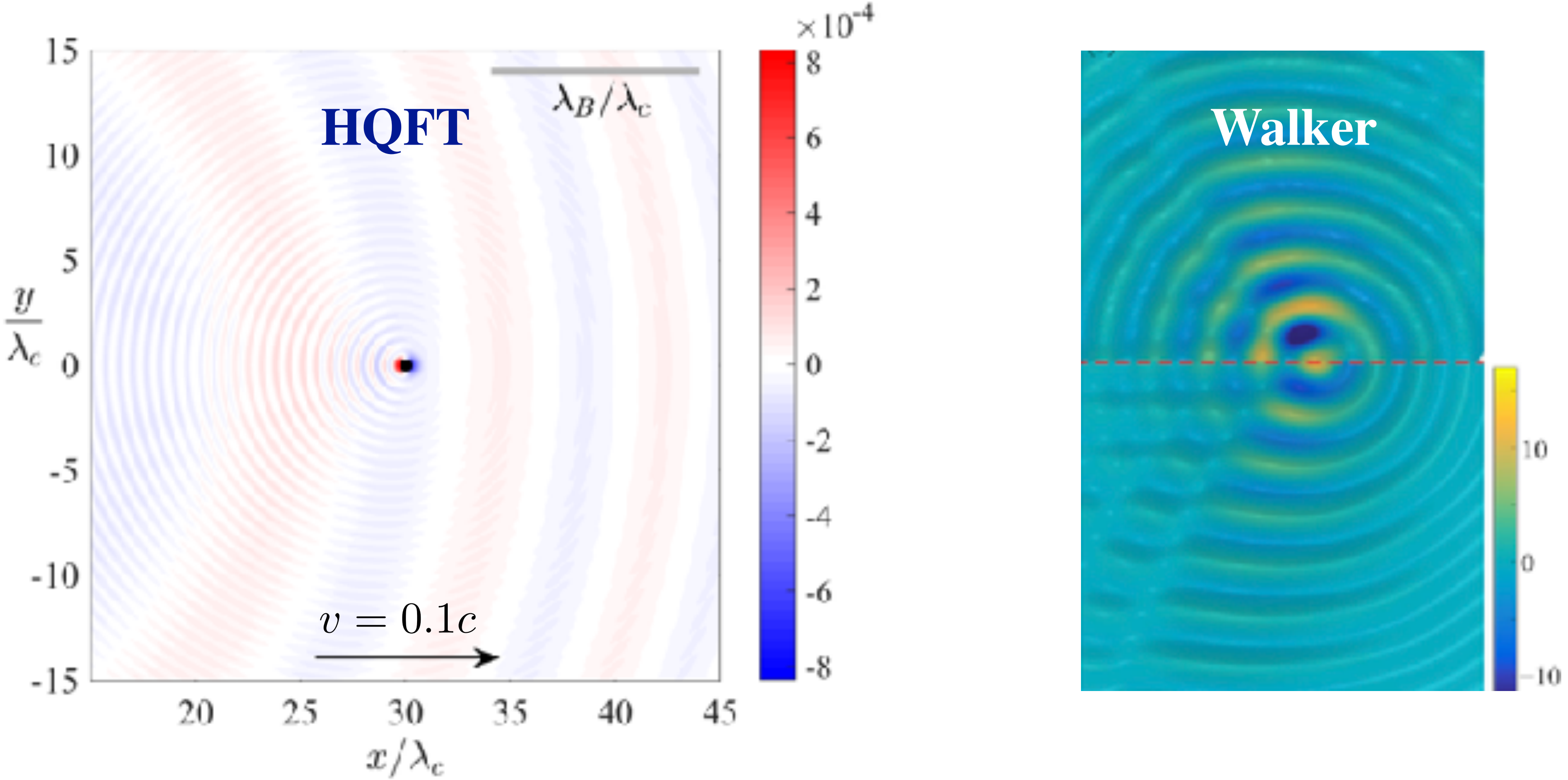
Coupling constant ↓

Guidance equation: $\gamma \; \dot{x}_p = -\alpha \; \dfrac{\partial \phi}{\partial x}$

HQFT

Durey & Bush (2020)

- above a critical coupling constant α_c, particle self propels
- deduced analytically the form of the 1D and 2D pilot-wave field by solving an IVP
- pilot wave and particle momentum related through: $\bar{p} = \gamma m v = \hbar k$



- superposition of radially propagating waves with λ_c and carrier wave with λ_B
- markedly different from the horseshoe-like form of the walker wave field
- for v << c, the 2D pilot-wave field takes the form of a plane wave with λ_B

Hydrodynamically-inspired quantum field theory II

David Darrow & JB

- combine particle and field Lagrangians at the level of actions

$$\mathcal{S} = \mathcal{S}_{\text{field}} + \mathcal{S}_{\text{particle}} + \mathcal{S}_{\text{interaction}}$$

↓

$$\mathcal{S}_{\text{field}} = \frac{1}{2}\int_{\Omega} d^4q \,\left(\partial^{\mu}\phi\partial_{\mu}\phi - \omega^2\phi^2\right)$$

$$\mathcal{S}_{\text{particle}} = -\int_0^{t'} dt\, mc^2\gamma^{-1}$$

$$\mathcal{S}_{\text{interaction}} = \int_0^{t'} dt\,\gamma^{-1}\left(a_{\tau}\phi(q_p) + b_{\tau}\gamma\dot{q}_p^{\mu}\partial_{\mu}\phi(q_p)\right)$$

Coupled wave and guidance equations

$$(\partial_{\mu}\partial^{\mu} + \omega^2)\phi = \gamma^{-1}(a_{\tau} - \dot{b}_{\tau})\delta^3(q - q_p)$$
$$\frac{d}{dt}\left((m - a_{\tau}\phi(q_p))\gamma\dot{q}_p\right) = \gamma^{-1}(a_{\tau} - \dot{b}_{\tau})\nabla\phi(q_p)$$

→

HQFT II

$$(\partial_{\mu}\partial^{\mu} + m^2)\phi = \gamma^{-1}b\delta^3(q - q_p)$$
$$\frac{d}{dt}(m\gamma\dot{q}_p) = \gamma^{-1}b\nabla\phi(q_p)$$

Coupling constants a, b

Single coupling constant b

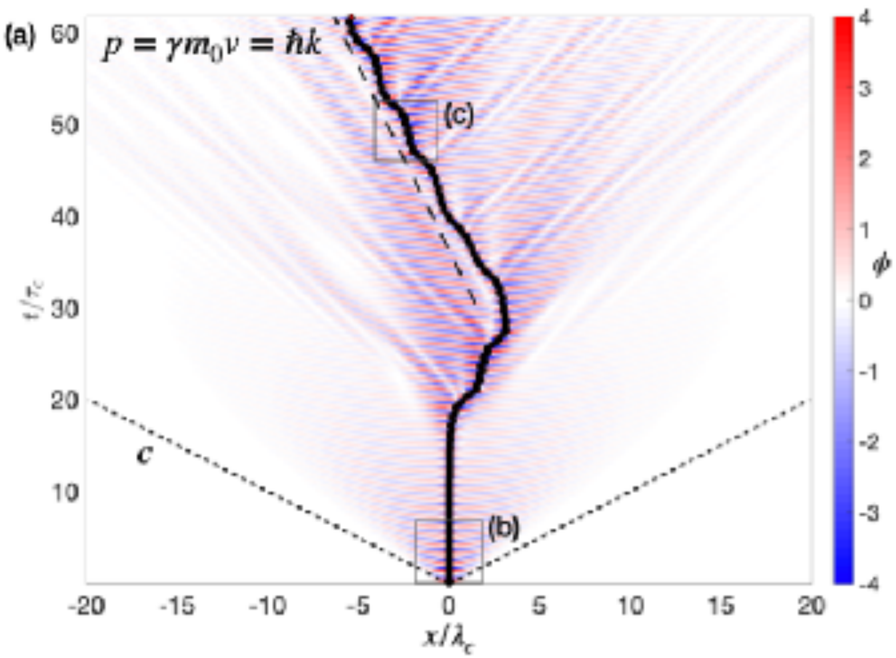
HQFT I

Dagan & Bush (2020)

$$(\partial_\mu \partial^\mu + \omega_c^2)\phi = -\sin(2\omega_c t)\delta^3(q - q_p)$$

$$\gamma \dot{q}_p = -\alpha \nabla \phi$$

1. Frame-dependent
2. Non-inertial dynamics
3. Forced oscillations at ω_c
4. No steady rectilinear state



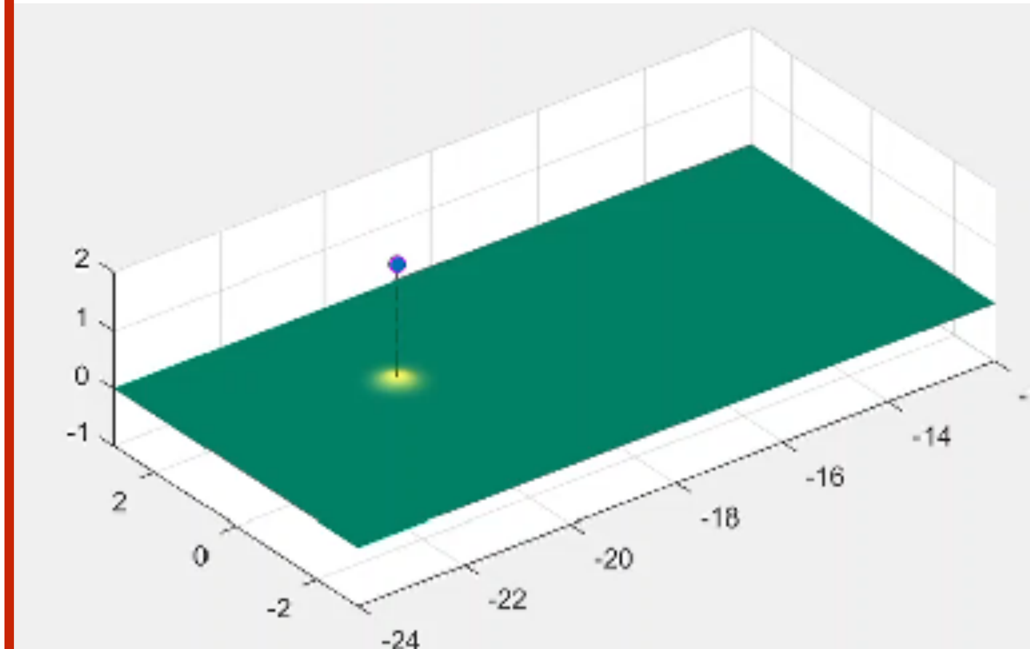
HQFT II

$$(\partial_\mu \partial^\mu + m^2)\phi = \gamma^{-1} b \delta^3(q - q_p)$$

$$\frac{d}{dt}(m\gamma \dot{q}_p) = \gamma^{-1} b \nabla \phi(q_p)$$

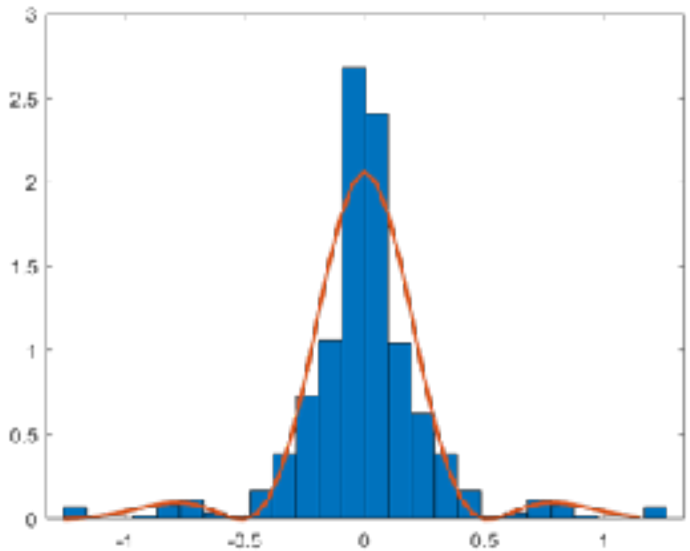
1. Lorentz invariant
2. Inertial dynamics
3. Time independent

$\nrightarrow$ Emergent oscillations at ω_c

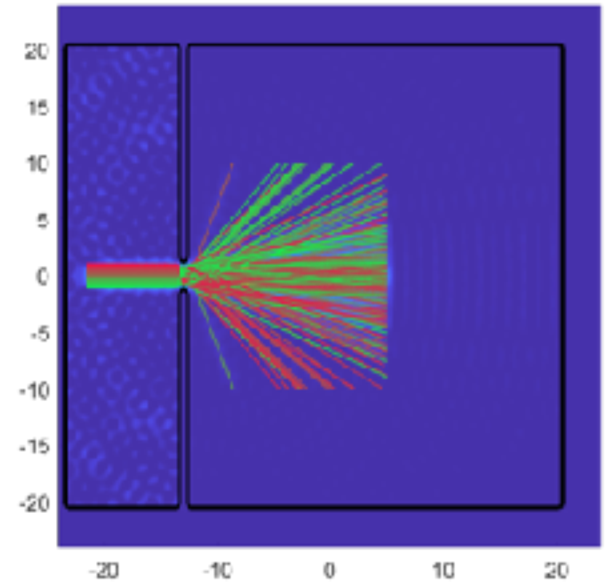


Slit diffraction with HQFT II

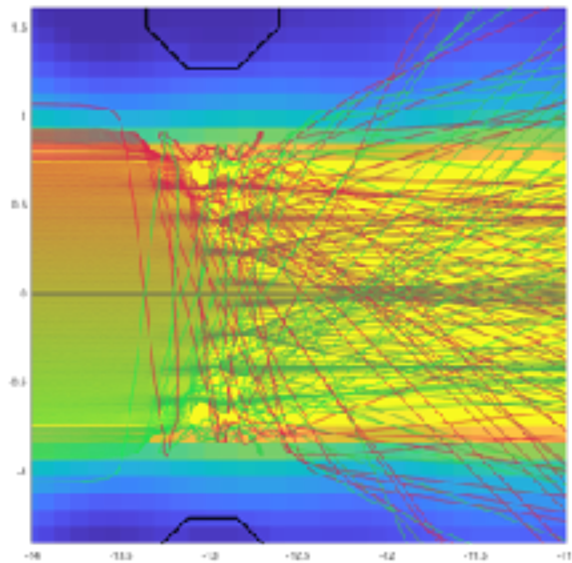
Single slit diffraction pattern



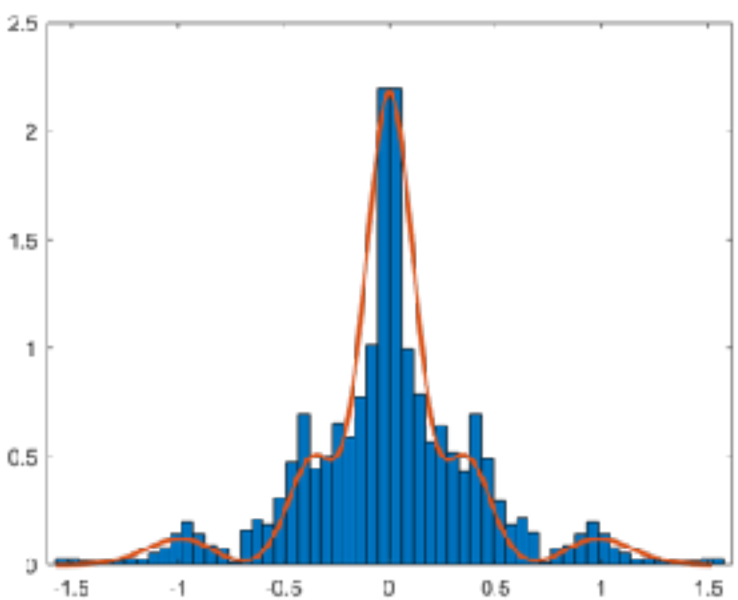
Trajectories



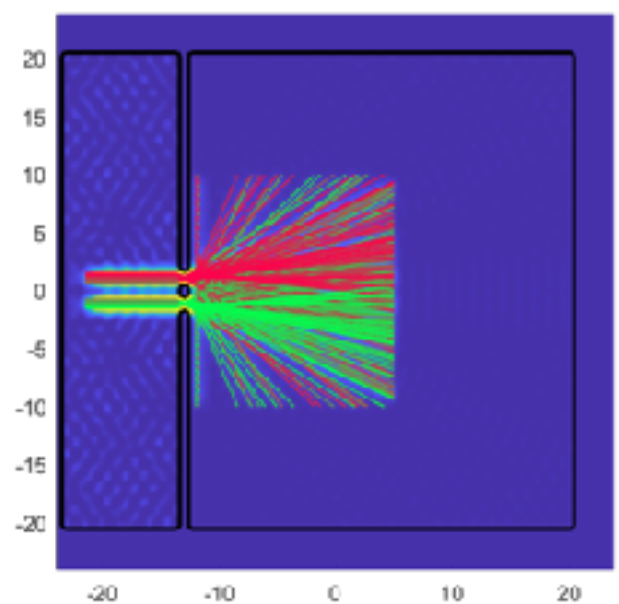
Trajectories inside slit



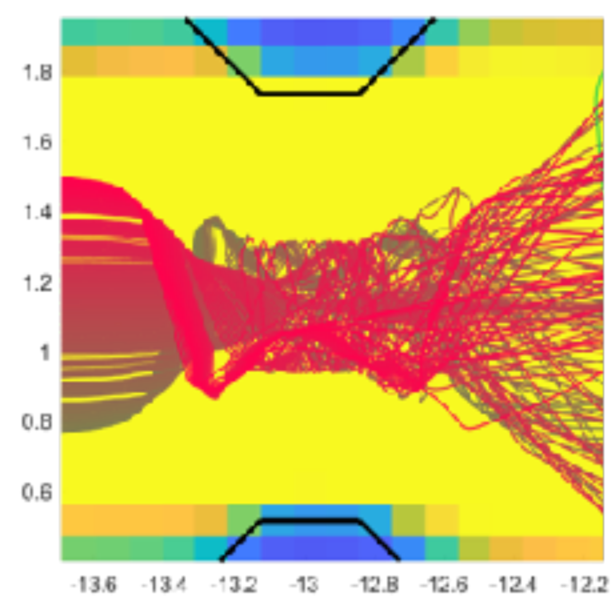
Double slit pattern



Trajectories



Trajectories inside slit



The New Hydrodynamic Interpretation of QM

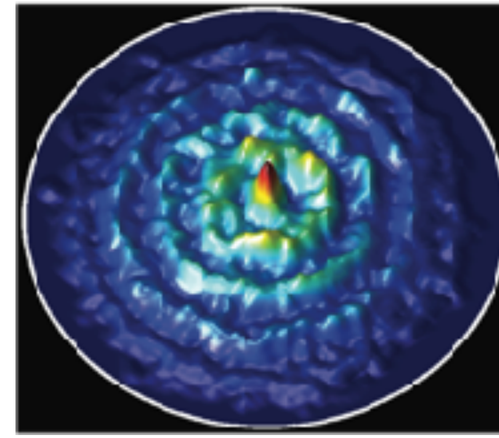
- an amalgam of de Broglie's pilot-wave theory and the walker system

˵Bush (2015) has further explored this sort of possibility for the emergence of a Bohmian version of quantum mechanics from something like classical fluid dynamics. A serious obstacle to the success of such a program is the quantum entanglement and nonlocality characteristic of many-particle quantum systems."

— S. Goldstein, *Bohmian Mechanics (Stanford Encyclopedia of Philosophy, 2021)*

Nonnonlocality: misinferences of non-locality from local hereditary pilot-wave dynamics

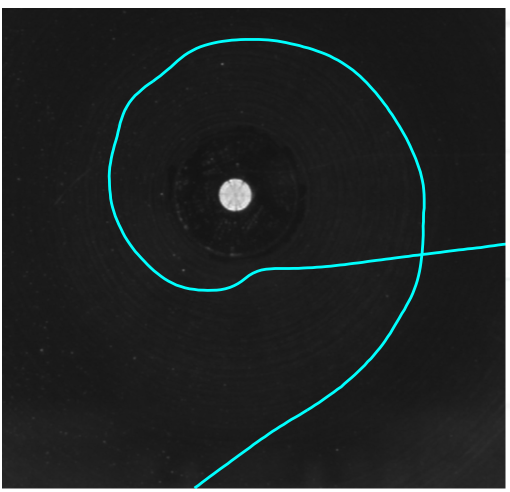
Harris et al. (2013)

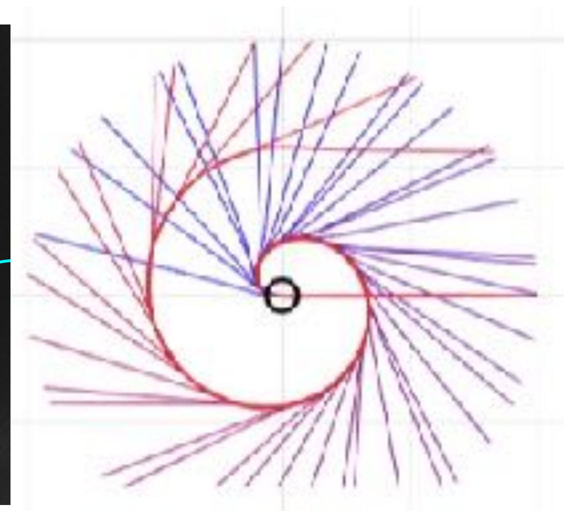


1. Wave function collapse

- act of observation causes instantaneous collapse of statistical wave form

2. Spooky action at a distance

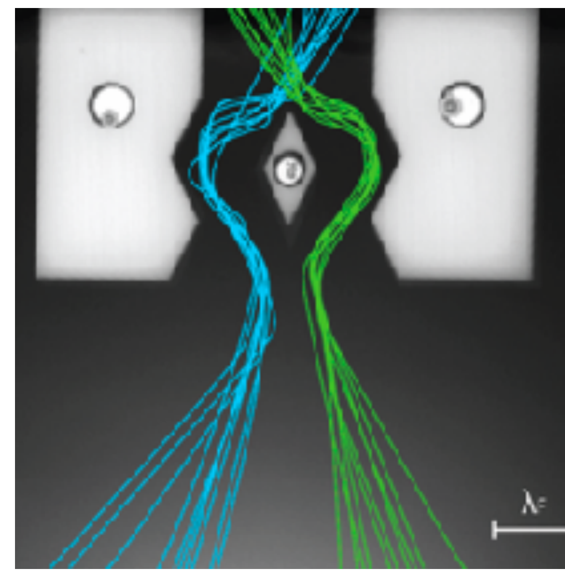




Harris et al. (2018) *Sáenz et al. (2020)*

- interaction with pillars and well: wave-mediated local forces give rise to apparently non-local lift forces

3. Real surreal trajectories



Frumkin et al. (2022)

- wave-mediated forces play role of non-local quantum potential in Bohmian mechanics

Bell Tests: *the acid test of quantumness*

- can be performed on any probabilistic system consisting of two subsystems (1, 2) on which one measures a stochastic process with outcomes X = +1 or -1

$$S = |M(a,b)+M(a',b)+M(a,b')-M(a',b')| \leq 2 \quad \forall(a,a',b,b')$$

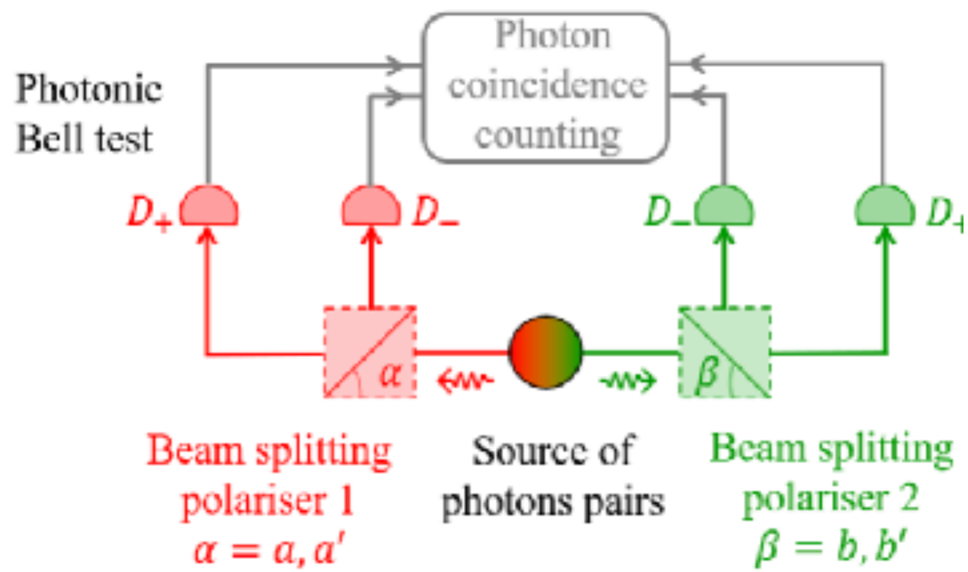
where $M(\alpha,\beta)$ is the average product:

$$M(\alpha,\beta) =< X_1X_2 >_{\alpha,\beta}= \sum_{X_1,X_2} X_1X_2P(X_1,X_2|\alpha,\beta)$$

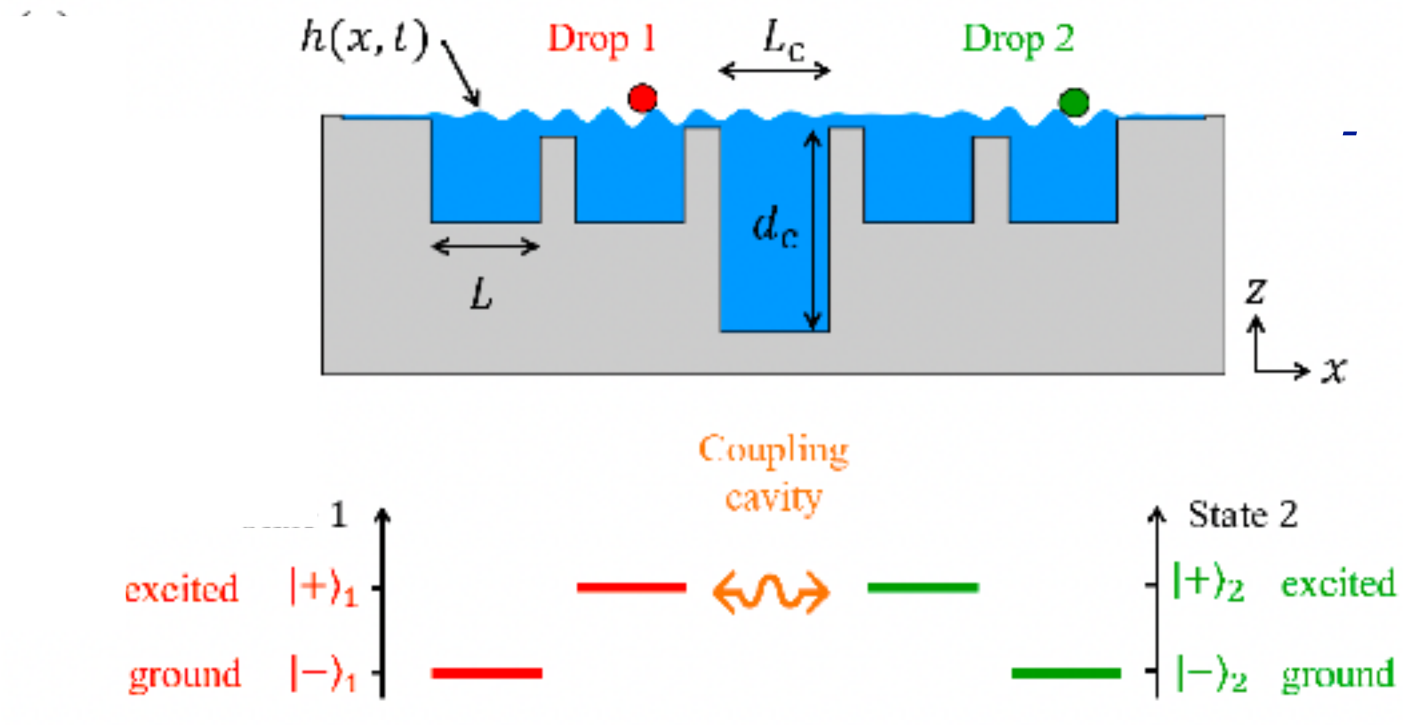
and $P(X_1,X_2|\alpha,\beta)$ is the joint probability of (X_1,X_2) when the left and right analyzers are set to $(\alpha,\ \beta)$.

Assumptions

1. Realism
2. Locality
3. Measurement independence



2-particle correlations: towards hydrodynamic Bell tests

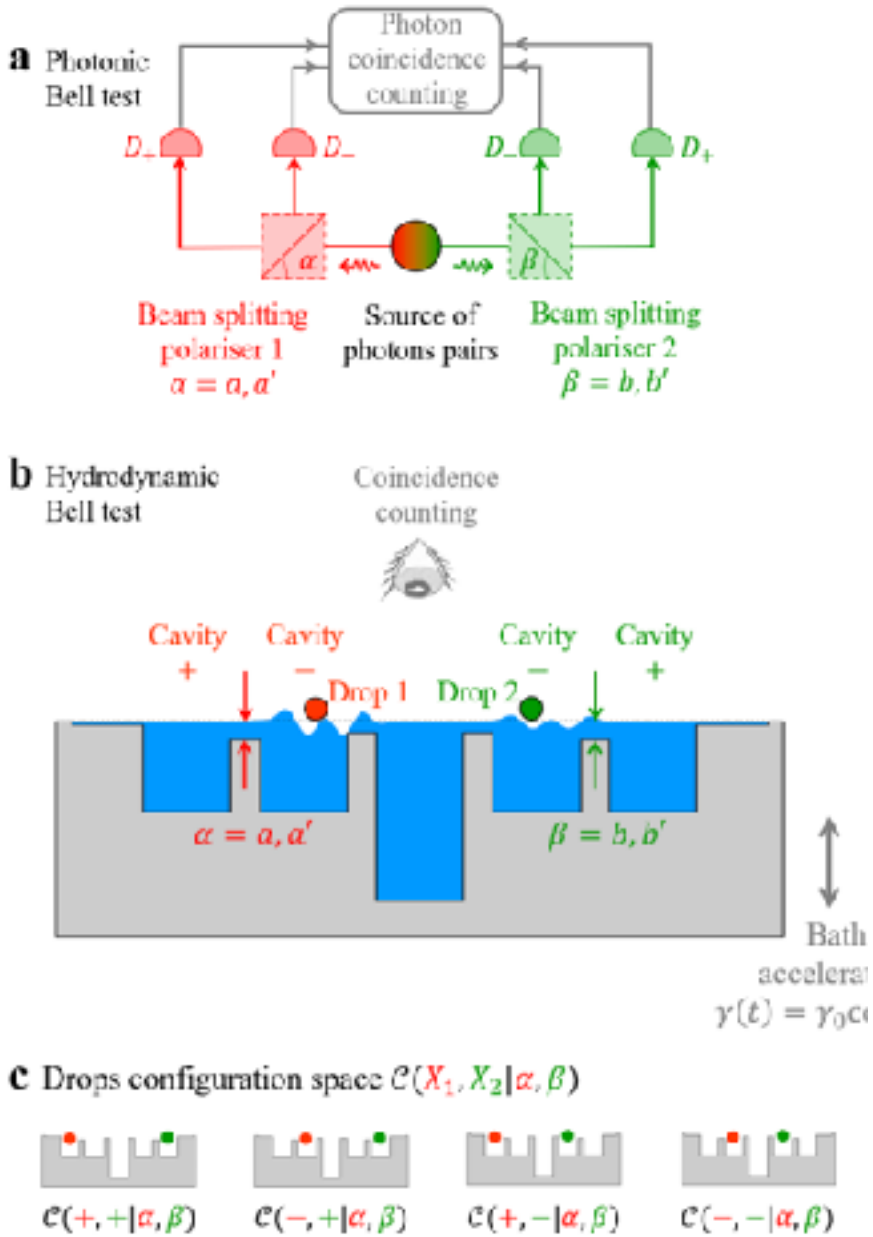


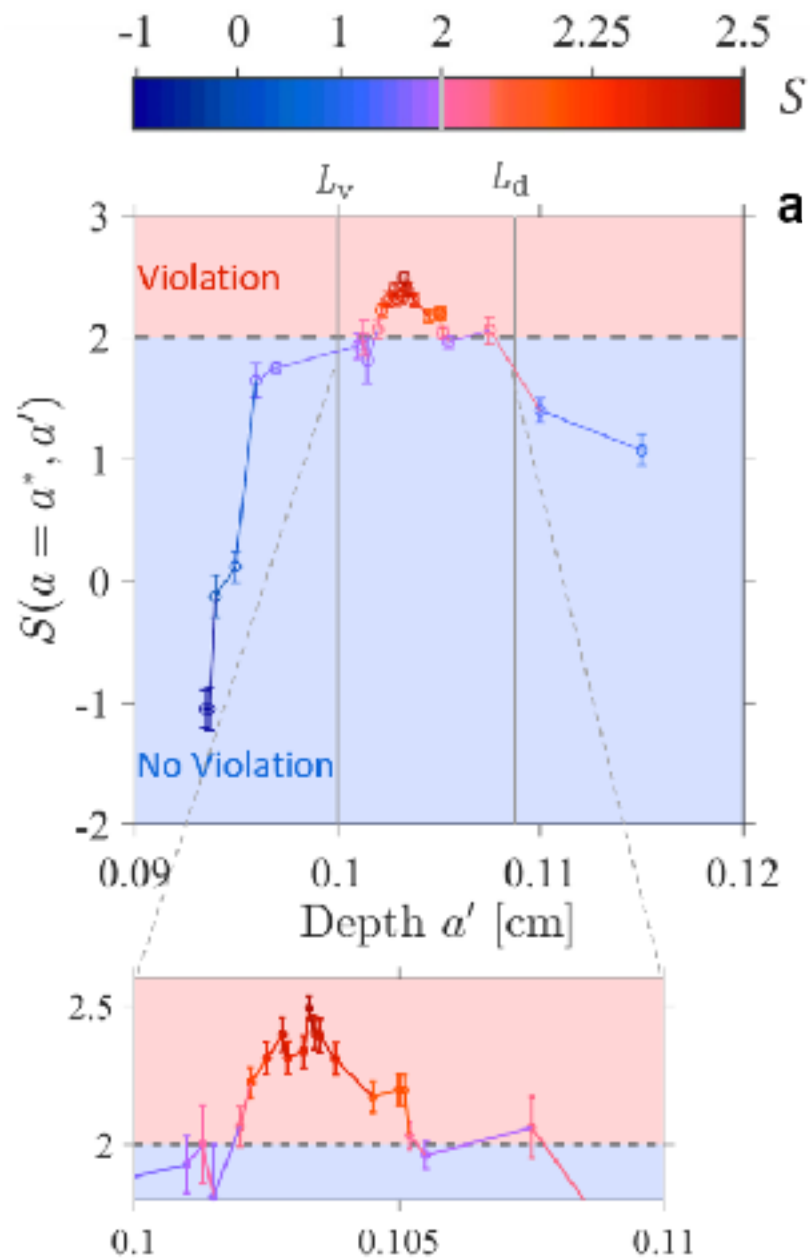
- *Papatryfonos, Nachbin, Labousse, Vervoort, JB*

Approach

- drops confined to a pair of wells across which they tunnel unpredictably between the ground (preferred) state and the excited state
- identify +/- (excited/ground) states with inner/outer cavities
- identify well geometries with measurement settings
- explore various measurement settings, measure emergent statistics
  - seek violations of Bell's Inequality

Violating Bell's Inequality: *static* Bell test



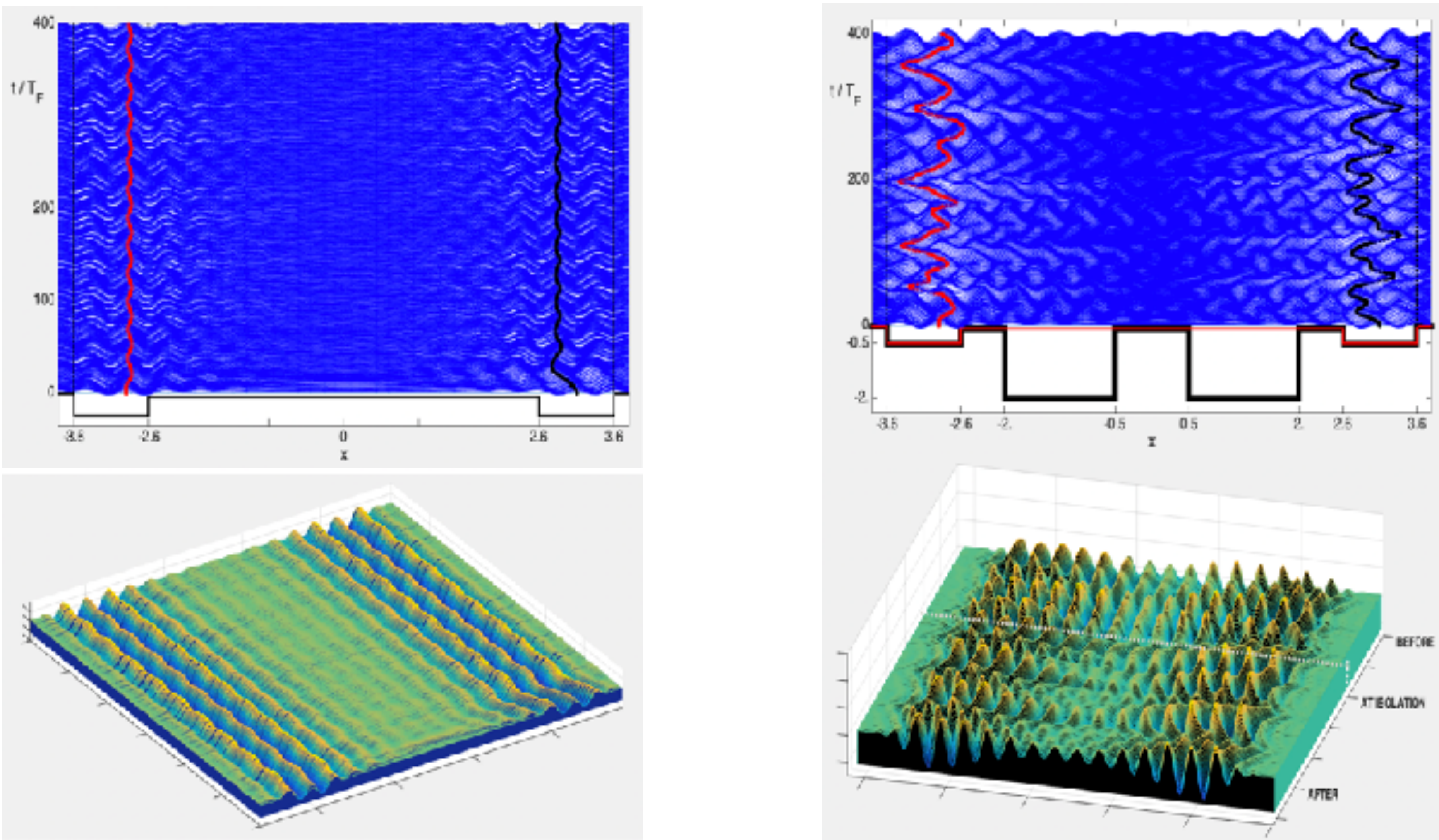


- assumption of Measurement Independence not valid: the geometry of both cavities influences the pilot wave, the probability of outcomes in both cavities…

Towards a dynamic Bell test

- Nachbin (2022)

- use dynamic topography to isolate the two subsystems



- excited, correlated state survives topographically-induced isolation
- might the static Bell violations likewise survive dynamic changes in measurement settings? ***Might memory account for entanglement?***

Overview

The hydrodynamic pilot-wave system

- provides a vehicle to explore the boundaries between classical and QM
- extends the range of classical systems to include features previously thought to be exclusive to the microscopic, quantum realm
- provides a conceptual framework, a progressive approach, for understanding QM

Pilot-wave hydrodynamics demonstrates how classical hereditary mechanics gives rise to behavior that is taken as evidence of non-locality in QM.

- is reminiscent of the de Broglie's pilot-wave theory of quantum dynamics
- suggests the shortcomings of quantum pilot-wave theories of de Broglie and Bohm
- has motivated a new class of local theoretical models of quantum dynamics
- suggests that the quantum paradoxes may be resolved through the elucidation of pilot-wave dynamics on the Compton scale

Perspective

- at the time that pilot-wave theory was developed by de Broglie, there was no macroscopic analog to draw upon.

Now there is.

- pilot-wave dynamics can give rise to quantum-like behaviour on the macroscale.

So why not the microscale?