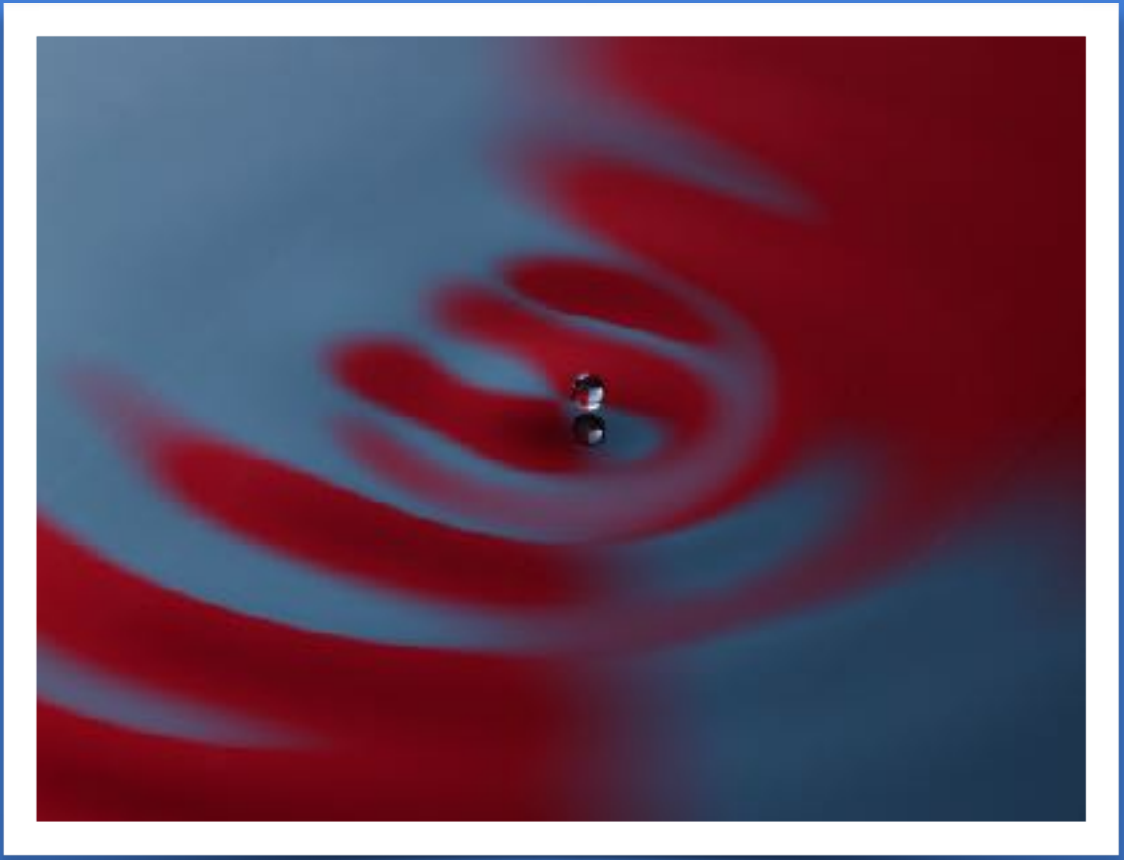
18.S996 Hydrodynamic quantum analogs

Lecture 1: Introduction



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18.S996 HYDRODYNAMIC QUANTUM ANALOGS

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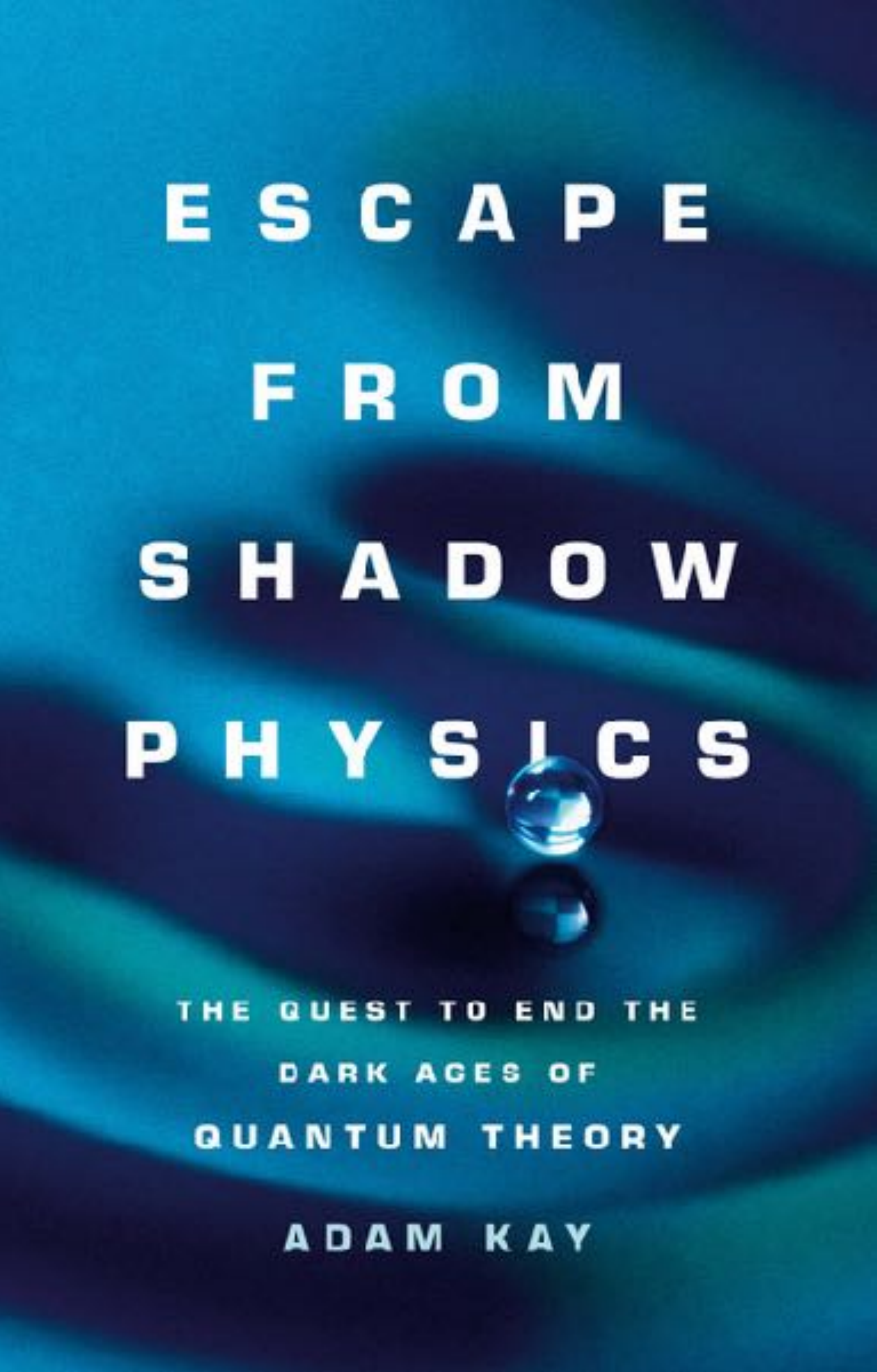
Spring 2024
MW 2:30-4
Room 2-143

GRADING SCHEME

- 50%: 2 problem sets (group discussion encouraged)
- 50%: course project on subject of your choosing
  - 30% based on final paper, 20% final presentation

There is **no required text** for the course, which will be based on the lecture notes; however, supporting material will be suggested throughout the course.

Coming out in April 2024…



CLASSICAL PHYSICS

Laplacian determinism

The macroscopic world is deterministic.

Initial conditions uniquely determine outcome.

Chaos theory (Poincaré 1900s, Lorenz 1960s)

There are practical limits to our predictive powers.

Complex systems are sensitive to initial conditions.

* Hereditary mechanics (Volterra 1920, Brillouin 1926)

Systems with memory, whose evolution is influenced by their past.

Predictive power requires knowledge of both ICs and history.

Pilot-wave hydrodynamics demonstrates how classical hereditary mechanics gives rise to behavior that, in QM, is taken as evidence of non-locality.

Quantum mechanics

- a wave theory that describes the statistics of microscopic particles
- fails to describe particle trajectories — indeed, some flatly deny that they exist

The free particle:

$$E = \hbar\,\omega \qquad\qquad \mathbf{p} = \hbar\,\mathbf{k}$$

- an association of a particle with a wave

But where is the particle, and why does it move?

- an insistence on the completeness of a trajectory-free quantum mechanics has lead to longstanding difficulties

  — the proliferation of quantum interpretations

  — an abundance of paradoxes and troubling language

`Hidden variable theories’

- seek a rational dynamics that underlies the theory of quantum statistics: seek to describe *particle trajectories*
- most involve a pilot-wave dynamics in which a particle is guided by a wave

A brief history

- de Broglie (1926) proposed a **double-wave** pilot-wave theory of quantum dynamics: quantum particles move in resonance with a monochromatic guiding wave
- Bohm (1952) presented a **single-wave** pilot-wave theory: a particle is guided by the standard quantum wavefunction
- Nelson (1966) proposed Stochastic Dynamics, that QM may be understood in terms of a diffusive, random-walk-like process
- de la Peña & Cetto (1996, 2015) have developed Stochastic Electrodynamics, in which the pilot-wave is sought in the electromagnetic quantum vacuum

Hydrodynamic quantum analogs

- in 2005, Couder and Fort discovered a hydrodynamic pilot-wave system in which a particle moves in resonance with a guiding wave
- the first macroscopic realization of the double-solution pilot-wave dynamics proposed by Louis de Broglie in the 1920s
- exhibits several features of quantum systems thought to be exclusive to the microscopic, quantum realm

THE QUESTIONS RAISED

What are the key dynamical features responsible for the quantum-like behavior?

What are the potential and limitations of this hydrodynamic system as a quantum analog?

Can it guide us towards a rational theory for quantum dynamics?

Wave-particle duality at the macroscopic scale

"Both matter and radiation possess a remarkable duality of character, as they sometimes exhibit the properties of waves, at other times those of particles.

Now it is obvious that a thing cannot be a form of wave motion and composed of particles at the same time - the two concepts are too different."

- Heisenberg, *On Quantum Mechanics (1930)*

TALK OUTLINE

I. Hydrodynamic quantum analogs

An experimental exploration of the potential and limitations of the walking-drop system as a quantum analog.

II. Pilot-wave hydrodynamics

Theoretical modeling of the walking droplet system.

III. A generalized pilot-wave framework

A mathematical bridge between hydrodynamic and quantum pilot-wave theories.

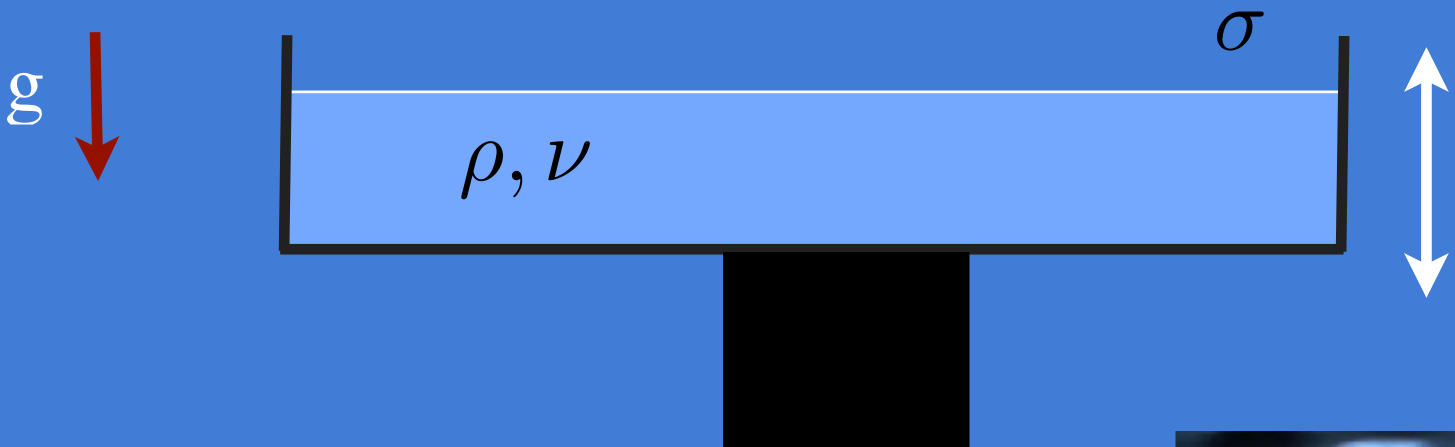
IV. Hydrodynamically-inspired quantum field theory

An extension of quantum pilot-wave theories, informed by the hydrodynamic system.

Faraday waves

Faraday (1831)

- surface undulations with twice the forcing period, a parametric instability
- arise above a threshold γ_F that depends on fluid depth, viscosity, surface tension

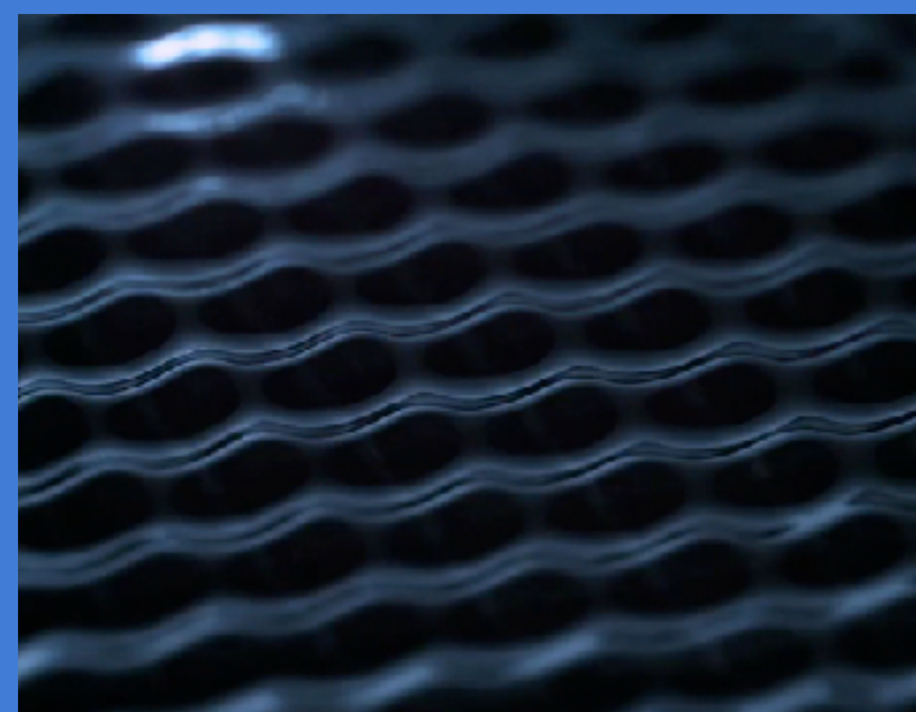


frequency f

amplitude A

$$\gamma > \gamma_F \approx 4g$$

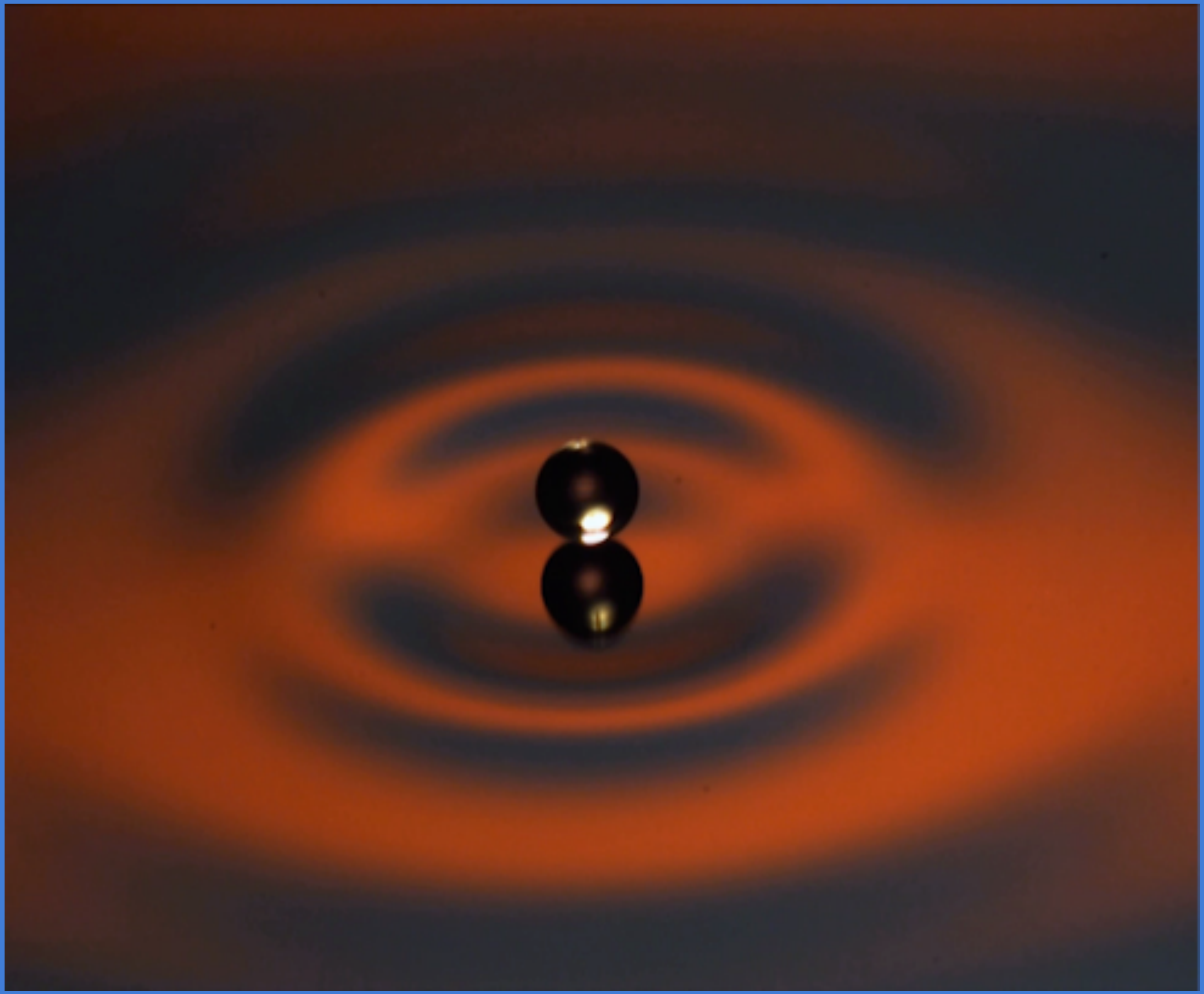
Vibrational acceleration: $\gamma = A(2\pi f)^2$



Noncoalescence on a vibrated fluid bath

Jearl Walker (1978), Protière et al. (2005)

- coalescence avoided provided impact time is less than time required for air layer between drop and bath to drain to ~100 nm



f ~ 50 Hz

50cS
Si oil

The Couder walker

Couder et al. (2005)

- ***resonance condition***: drop bounces at Faraday frequency
- resonant bouncing droplets may be destabilized by their wave field, walk
- spatially extended *walkers* consist of both droplet and guiding/pilot wave
- drop dynamics is ***non-Markovian, hereditary***: wave force depends on its history
- proximity to Faraday threshold prescribes longevity of waves, **`path memory'**

Eddi et al. (2011)

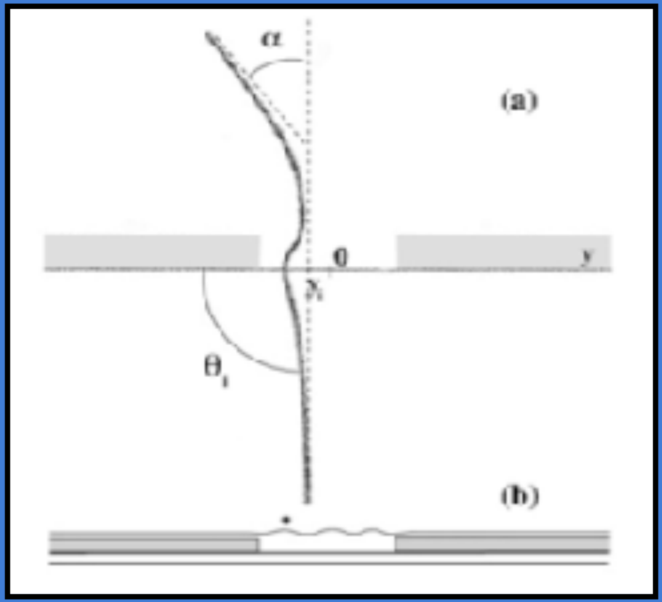
The early hydrodynamic quantum analogs (HQAs) of Yves Couder and Emmanuel Fort

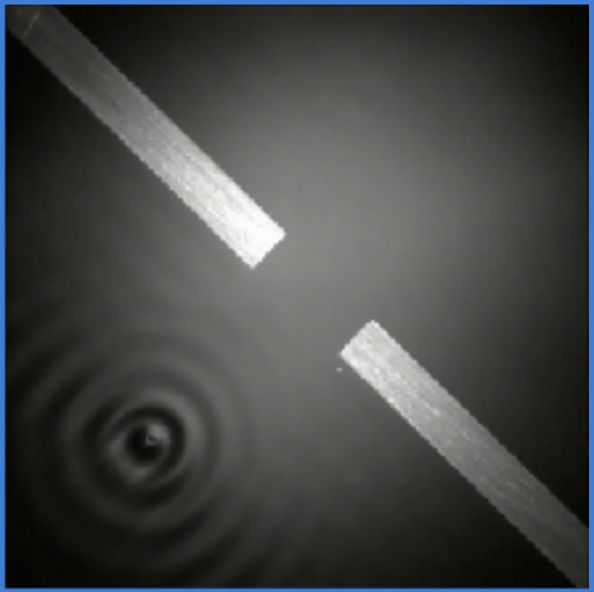
1. Single-particle diffraction and interference
2. Unpredictable tunneling
3. Quantized Landau orbits and Larmor levels
4. Doubly- quantized orbits in a simple-harmonic potential

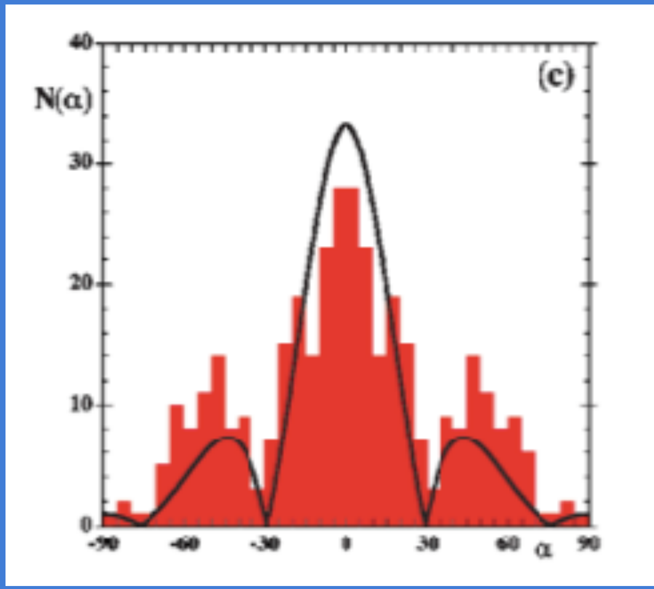
Single-walker diffraction and interference

Couder & Fort (2005)

Single slit



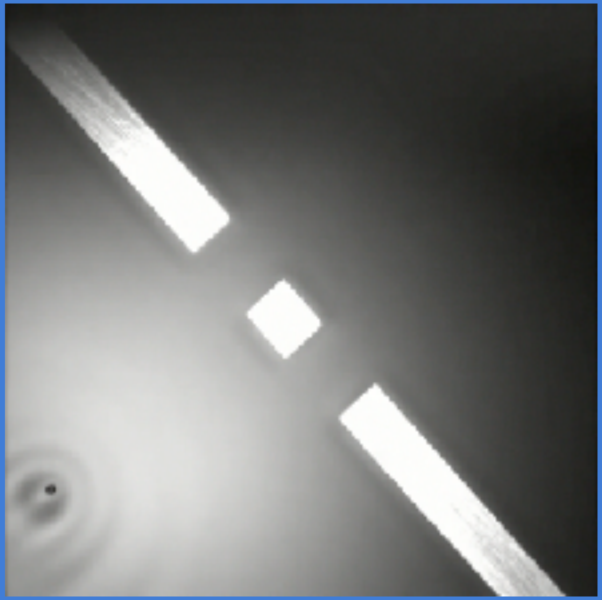


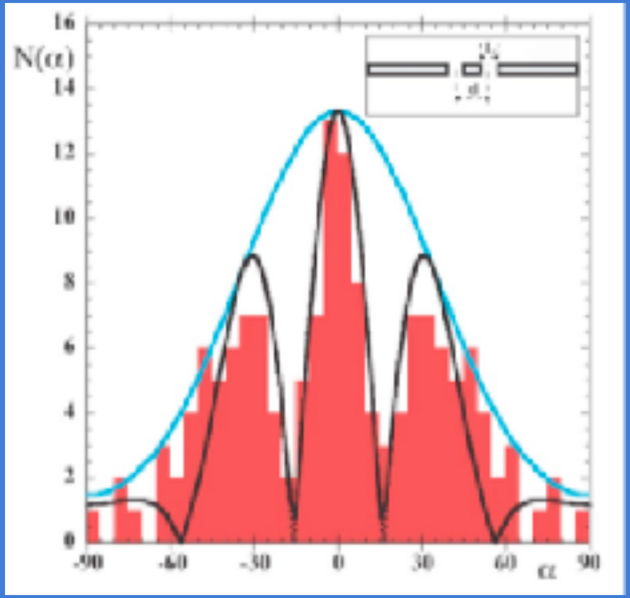


Double slit

“ A phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the only mystery.”

- Richard Feynman

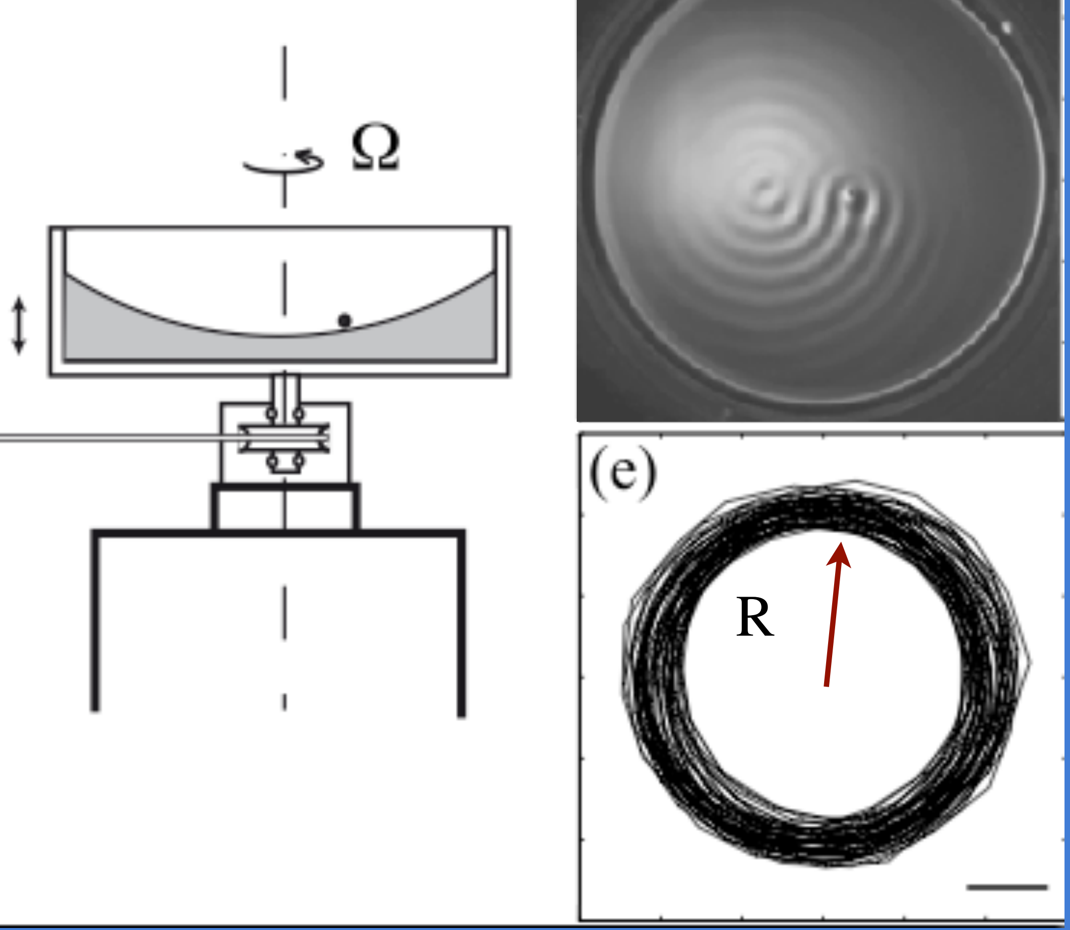




- **coherent, wave-like statistics emerge from chaotic pilot-wave dynamics**

Walkers in a rotating frame

Fort et al. (2010)



- execute circular orbits on which inertial, Coriolis forces balance:

$$\rho V^2/R = 2\rho\Omega V$$

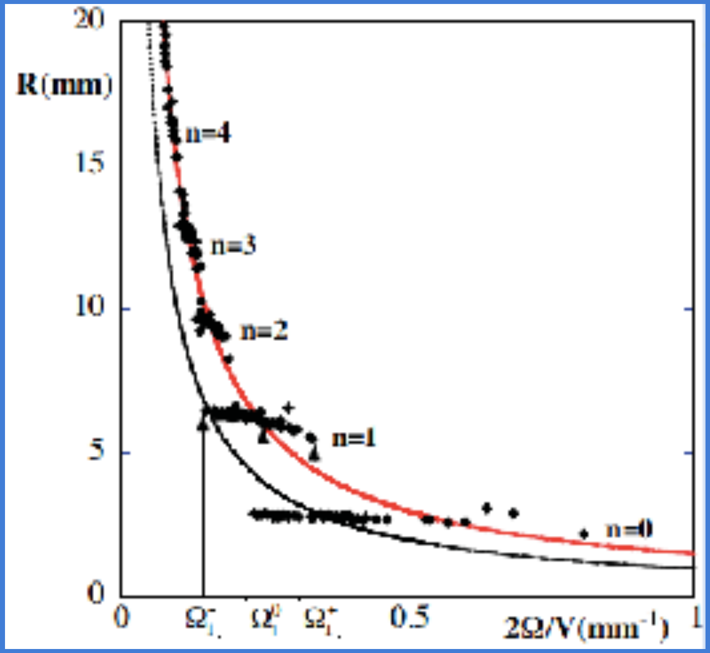
- one expects an orbital radius:

$$R = V/(2\Omega)$$

- in the long-memory limit, a 3rd force, the wave force, induces *orbital quantization*:

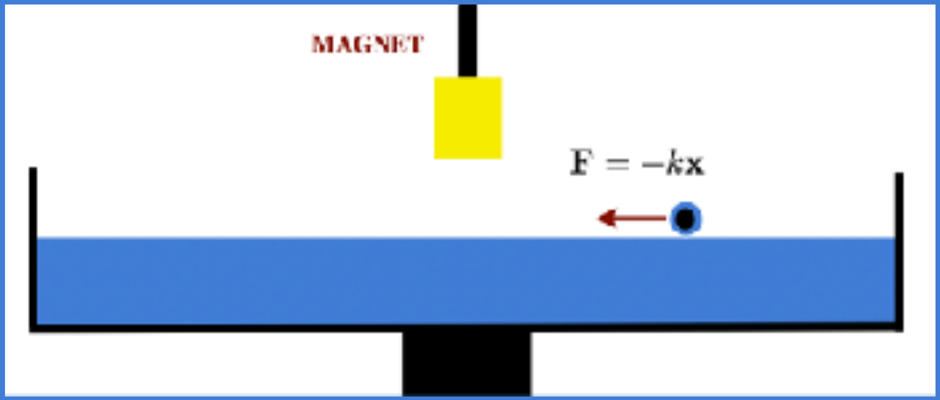
$$R_n \sim \frac{1}{2}(n+1/2)\lambda_F$$

- walker confined to move in the troughs of its associated wave field

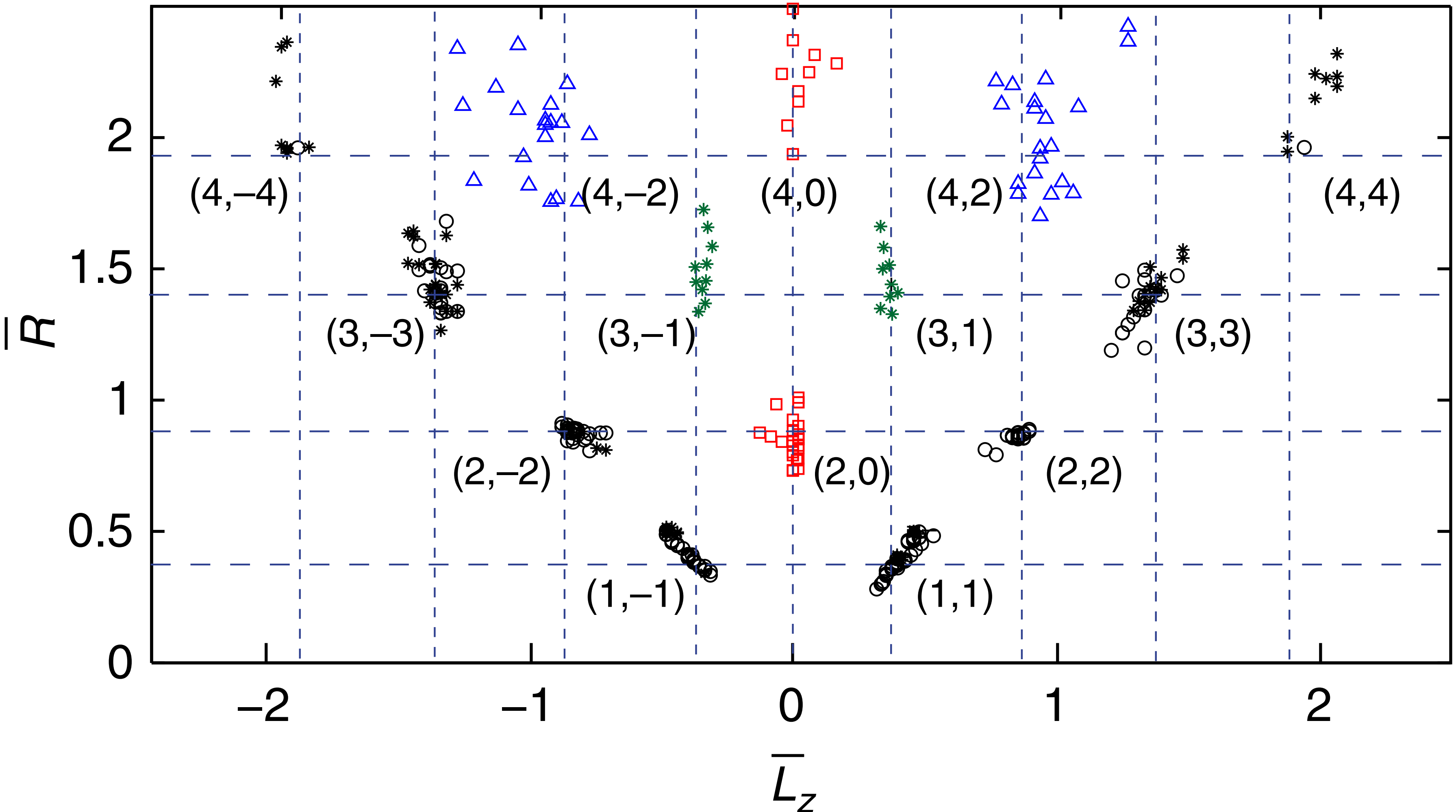


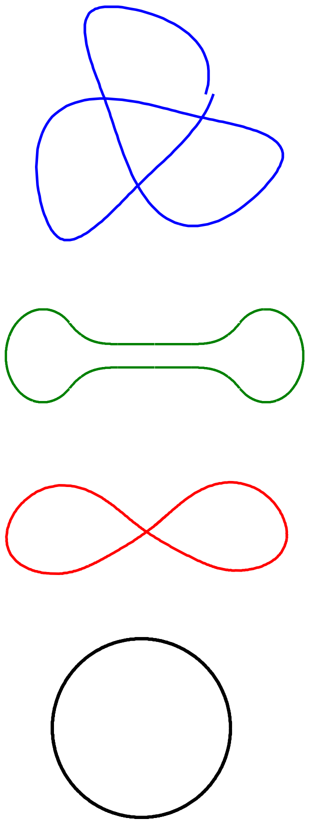
Landau orbits		Inertial orbits
• charge q of mass m orbits in a magnetic field $\mathbf{B}$		• walker of mass m orbits in a vortex $2\boldsymbol{\Omega}$
$\mathbf{F_B} = q(\mathbf{v} \wedge \mathbf{B})$	Force	$\mathbf{F_C} = -m(\mathbf{v} \wedge 2\boldsymbol{\Omega})$
$R_L = mv/(qB)$	Radius	$R_C = v/(2\Omega)$
$\tau_L = m/(qB)$	Period	$\tau_C = 1/(2\Omega)$
$R_n = \frac{1}{\pi}(n + 1/2)\lambda_{dB}$ Larmor levels	Orbit levels	$R_n \sim \frac{1}{2}(n + 1/2)\lambda_F$ Couder levels
λ_{dB} de Broglie wavelength	Quantum step	λ_F Faraday wavelength

Walkers in a central force field: doubly quantized orbits



Perrard et al. (2014)
Labousse et al. (2015)





Pilot-wave hydrodynamics at MIT (2010 — present)

- refined experiments and theoretical modeling
- revisited original HQAs experimentally and/or theoretically
- discovered and explored new HQAs
- developed generalized pilot-wave theory
- elucidated links with and extended quantum pilot-wave theories

Molacek & Bush (2013)

Trajectory equation for resonant walkers

$$m\ddot{\mathbf{x}}_p + D\dot{\mathbf{x}}_p = -mg\nabla h(\mathbf{x}_p, t)$$

WAVE FORCE

Drag coefficient: $D = 6\pi\mu_a R + Cmg \cdot \sqrt{\frac{\rho R}{\sigma}}$

Wave field:

SPATIO-TEMPORAL DAMPING

$$h(\mathbf{x}, t) = A \sum_{k=-\infty}^{[t/T_F]} J_0(k_F|\mathbf{x} - \mathbf{x}_p(kT_F)|)\, e^{-\alpha|\mathbf{x}-\mathbf{x}_p(kT_F)|^2/(t-kT_F)}\, e^{-(t-kT_F)/(T_F M_e)}$$

Memory parameter: $M_e = \frac{T_d}{T_F(1-\gamma/\gamma_F)}$

Impact phase: Φ_I

Wave amplitude: $A = \frac{4\sqrt{2\pi}}{3}\frac{R^4 k_F^3 \mathbb{O}h_e^{1/2}}{3R^2k_F^2 + \mathbb{B}o} \cdot \frac{\mathbb{B}oT_F}{\sqrt{\rho R^3/\sigma}} \sin\Phi_I$

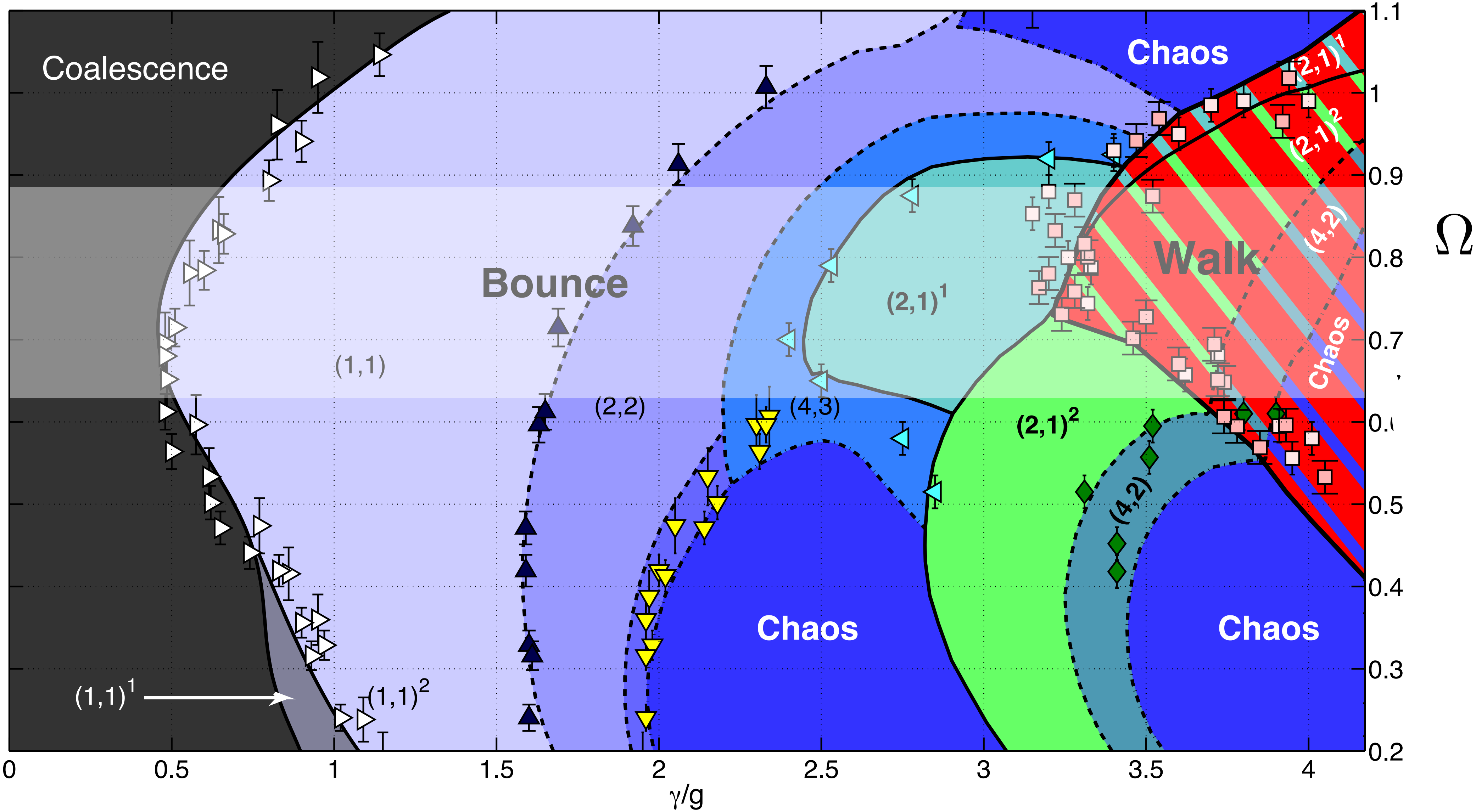
Bond number: $\mathbb{B}o = \frac{\rho g R^2}{\sigma}$

Ohnesorge number: $\mathbb{O}h_e \approx \frac{\mu}{\sqrt{\rho\sigma R}}$

Bouncing drop regime diagram (20cS, 80Hz)

Molacek & Bush (2013ab)

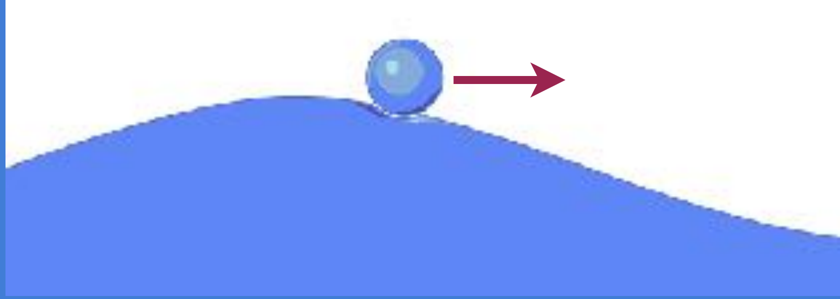
$$\Omega = 2\pi f(\rho R_0^3/\sigma)^{1/2}$$



In (m, n) mode, a drop bounces n times in m forcing periods.

- drops most readily bounce, walk when forced at their natural frequency

Strobed pilot-wave dynamics



- strobe the system once per bounce cycle
- conceals the vertical dynamics responsible for the guiding wave
- drop appears to surf on the interface, dressed by a quasi-monochromatic pilot-wave field that is stationary in the drop's frame of reference

The stroboscopic model

Oza, Rosales & Bush (2013)

$$m\ddot{\mathbf{x}}_p + D\dot{\mathbf{x}}_p = -mg\nabla h(\mathbf{x}_p, t)$$

MEMORY TERM

Approximate discrete sum as integral:

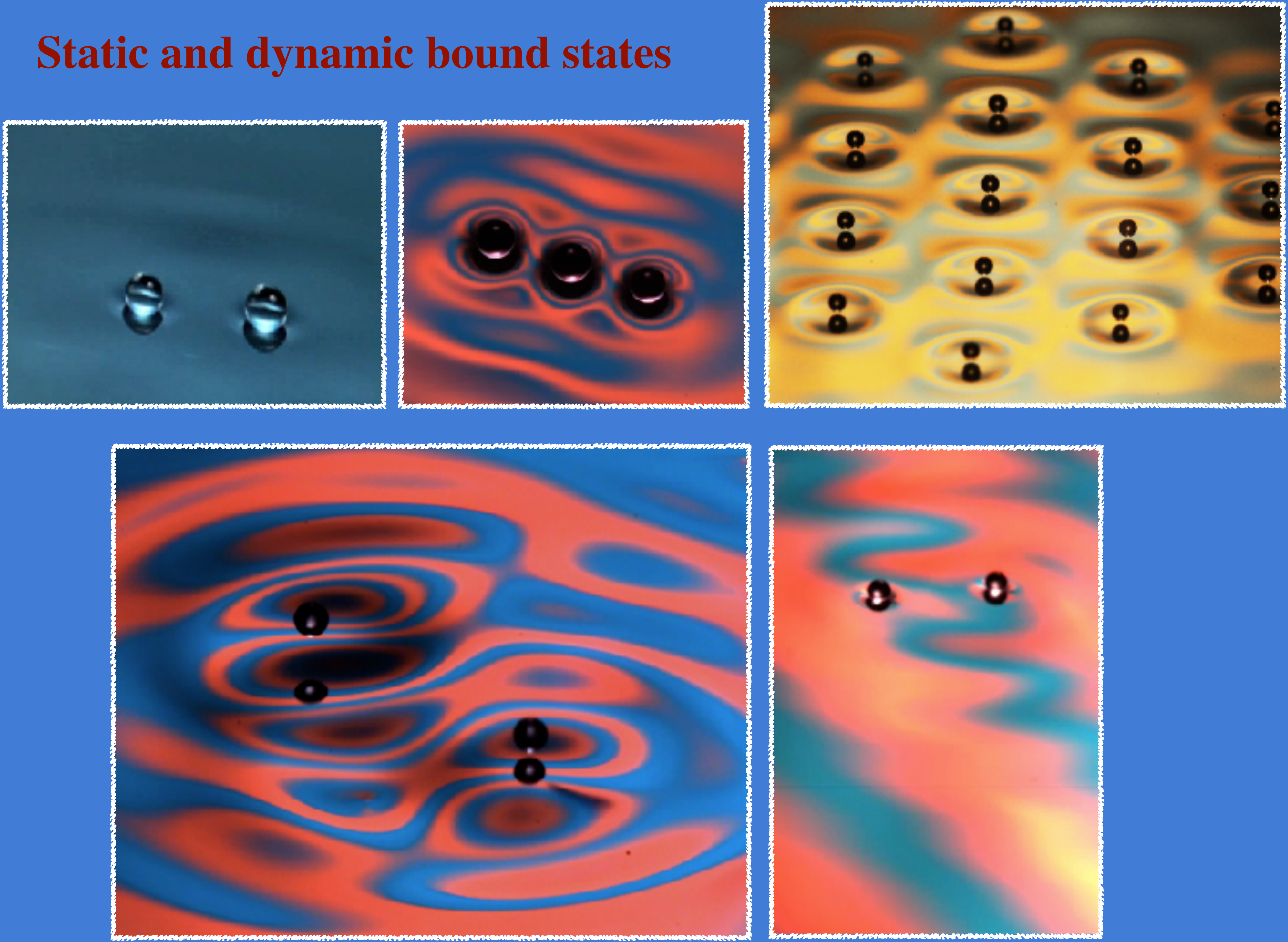
$$\nabla h(\mathbf{x}, t) = -Ak_F \int_{-\infty}^{t} \frac{J_1(k_F\,|\mathbf{x} - \mathbf{x}_p(s)|)}{|\mathbf{x} - \mathbf{x}_p(s)|}(\mathbf{x} - \mathbf{x}_p(s))e^{-(t-s)/(T_F M_e)}\,ds$$

$$F = mgAk_F$$

$$m\ddot{\mathbf{x}}_p + D\dot{\mathbf{x}}_p = \frac{F}{T_F}\int_{-\infty}^{t} \frac{J_1(k_F\,|\mathbf{x}_p(t) - \mathbf{x}_p(s)|)}{|\mathbf{x}_p(t) - \mathbf{x}_p(s)|}\,(\mathbf{x}_p(t) - \mathbf{x}_p(s))\,e^{-(t-s)/(T_F M_e)}\,ds$$

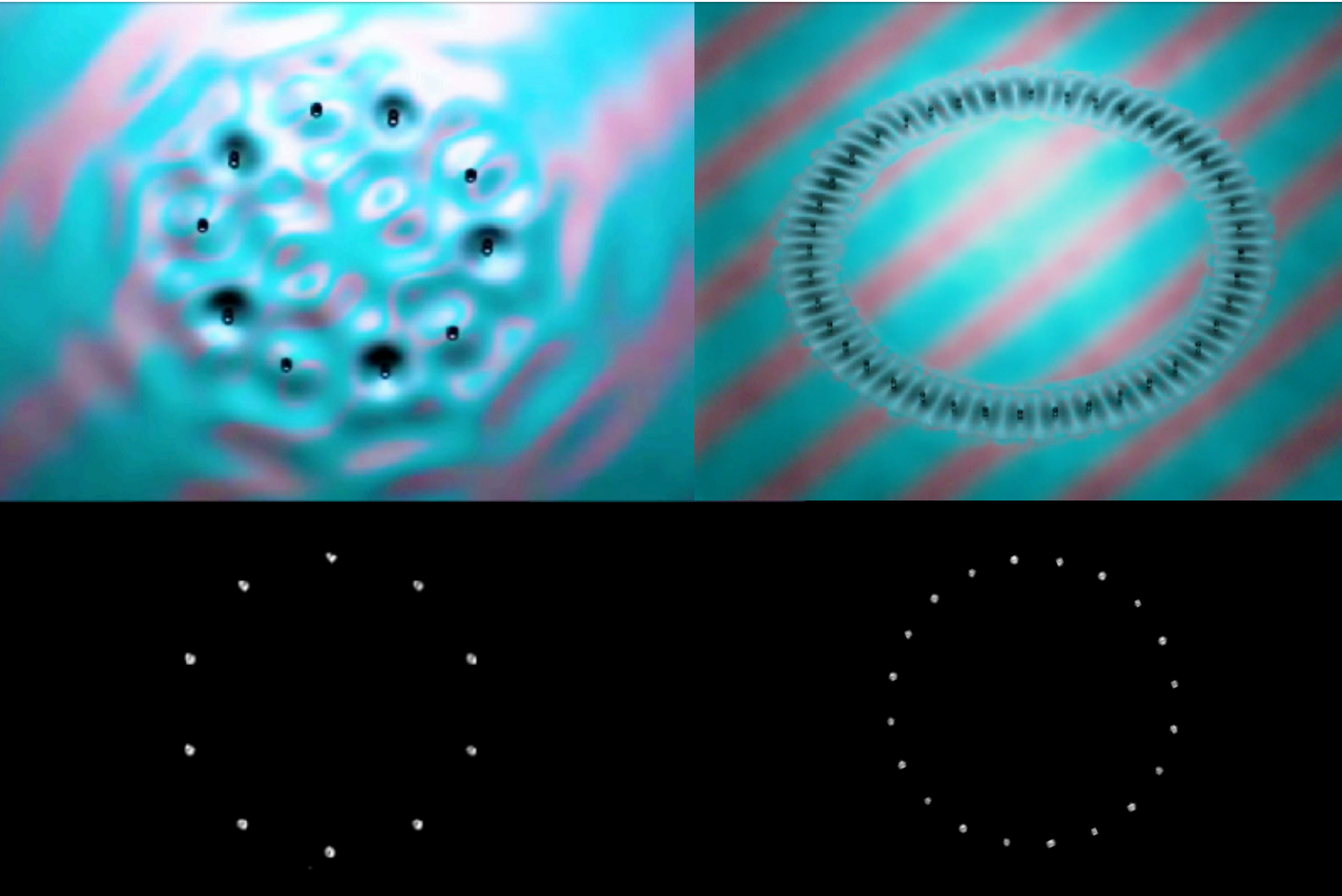
Integral-differential equation is amenable to analysis, provides rationale for transition from bouncing to walking, stability of various dynamical states.

Static and dynamic bound states

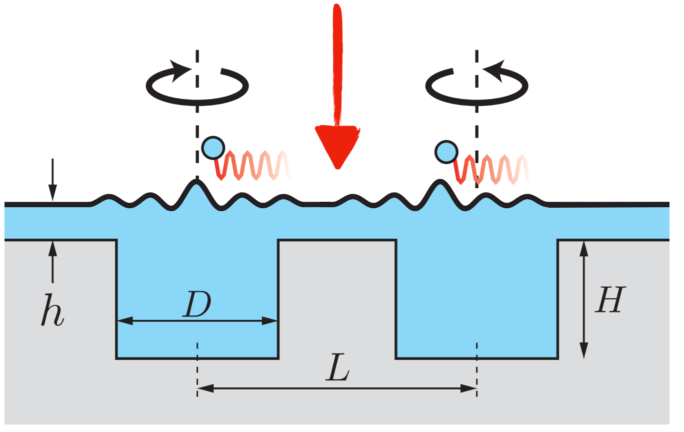


Rings of bouncing droplets

Couchman & Bush (2020)
Thomson, Couchman & Bush (2020)

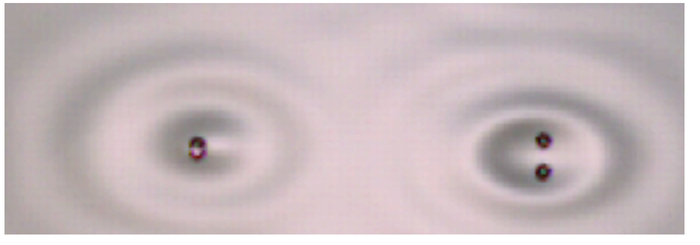


Hydrodynamic spin lattices



Control parameters

- Geometry h, L, D
- Vibrational acceleration γ/γ_F
- Drop size
- Frequency



Spin dynamics become coupled through **wave-mediated interactions** which may lead to **long-range correlations and spin ordering.**

Transitions between long-range analog ferro- and antiferromagnetic states can be controlled through memory, and system rotation.

Sáenz *et al.*, *Nature*, 2021

Pilot-wave dynamics in a rotating frame

Oza, Harris, Rosales & Bush (2013)

$$m\ddot{\mathbf{x}} + D\dot{\mathbf{x}} = \frac{F}{T_F}\int_{-\infty}^{t} \frac{J_1(k_F|\mathbf{x}(t)-\mathbf{x}(s)|)}{|\mathbf{x}(t)-\mathbf{x}(s)|}(\mathbf{x}(t)-\mathbf{x}(s))\, e^{-(t-s)/(T_F M_e)}\, ds - 2m\mathbf{\Omega}\times\dot{\mathbf{x}}$$

↑ Coriolis force

Seek orbital solutions: $r_p(t) = r_0\ ,\ \theta_p(t) = \omega t$

⬇

$$-mr_0\omega^2 = \frac{F}{T_F}\int_0^\infty J_1\left(2k_F r_0 \sin\frac{\omega z}{2}\right)\sin\frac{\omega z}{2} e^{-z/(M_e T_F)}\, dz + 2mr_0\Omega\omega$$

$$Dr_0\omega = \frac{F}{T_F}\int_0^\infty J_1\left(2k_F r_0 \sin\frac{\omega z}{2}\right)\cos\frac{\omega z}{2} e^{-z/(M_e T_F)}\, dz$$

nonlinear system of equations in *(r_0, ω)*

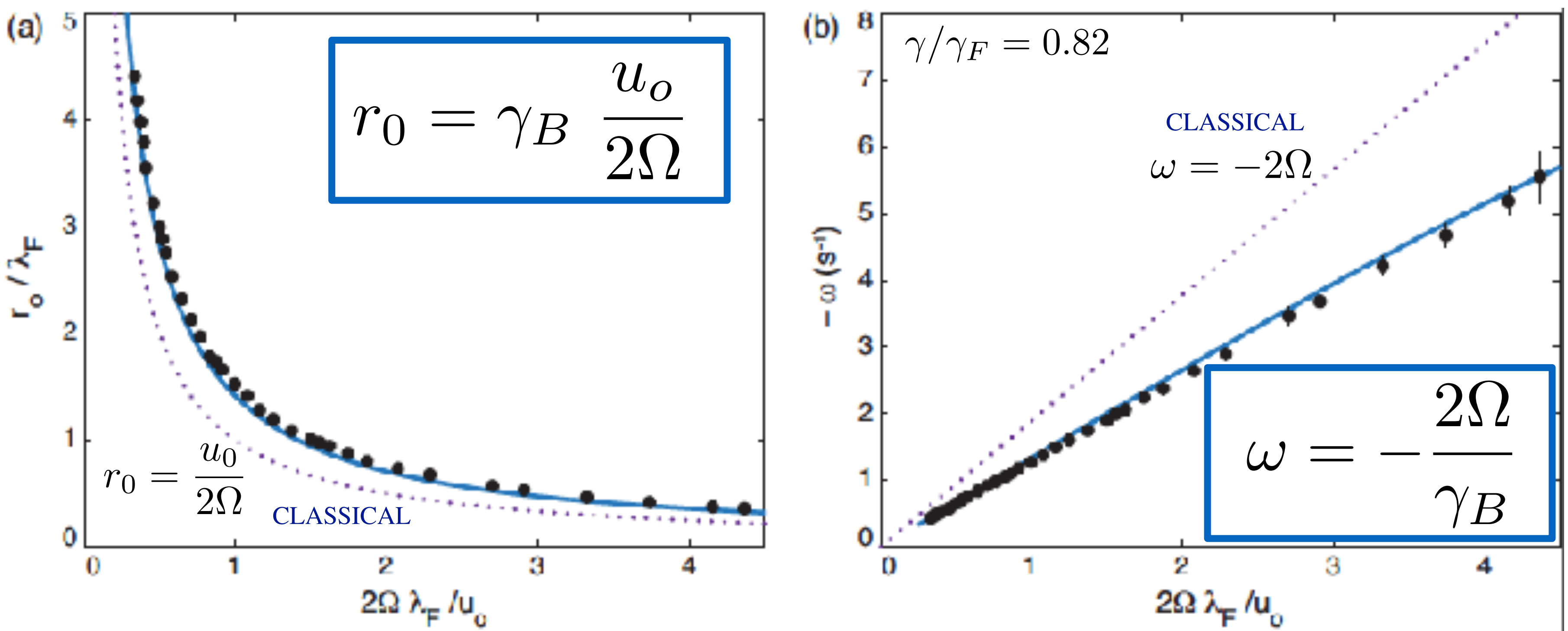
Inertial orbits of walking droplets


Harris & Bush, JFM (2014)

Coriolis force: $$\mathbf{f} = 2m_0\,\mathbf{v}\times\mathbf{\Omega}$$

Radial force balance:

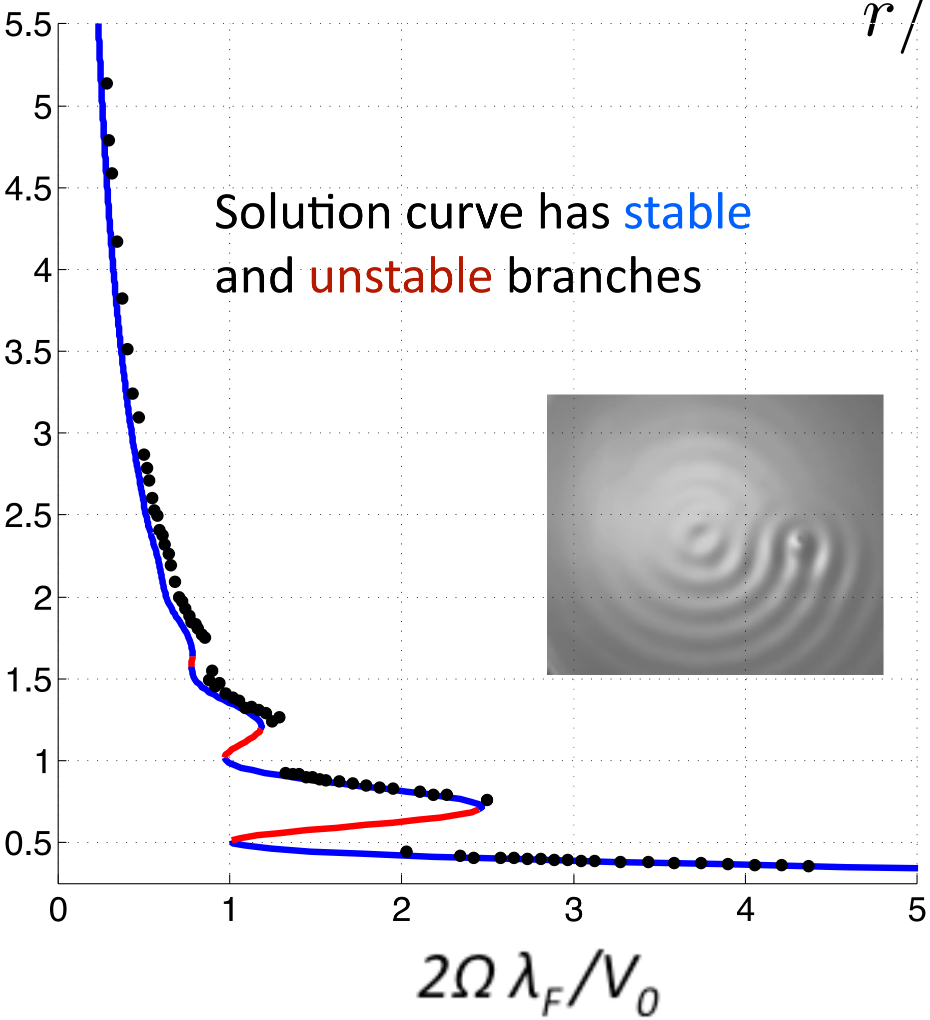
$$-m_0\gamma_B r_0\omega^2 = \mathbf{f}\cdot\mathbf{n} = 2m_0 r_0\omega\Omega$$



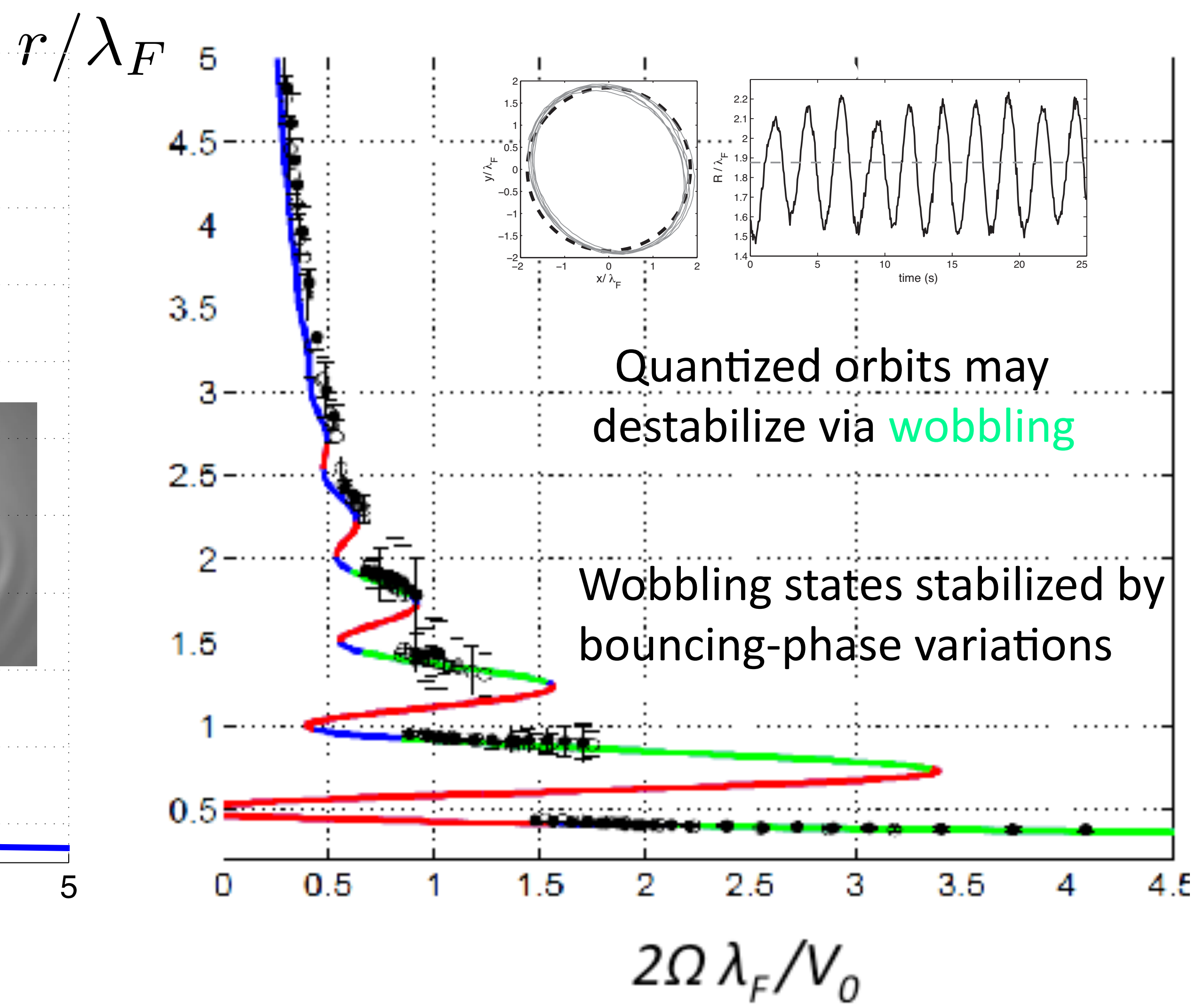
offset from `classical' results
due to hydrodynamic boost factor

Quantized orbits in a rotating frame

Mid memory



High memory

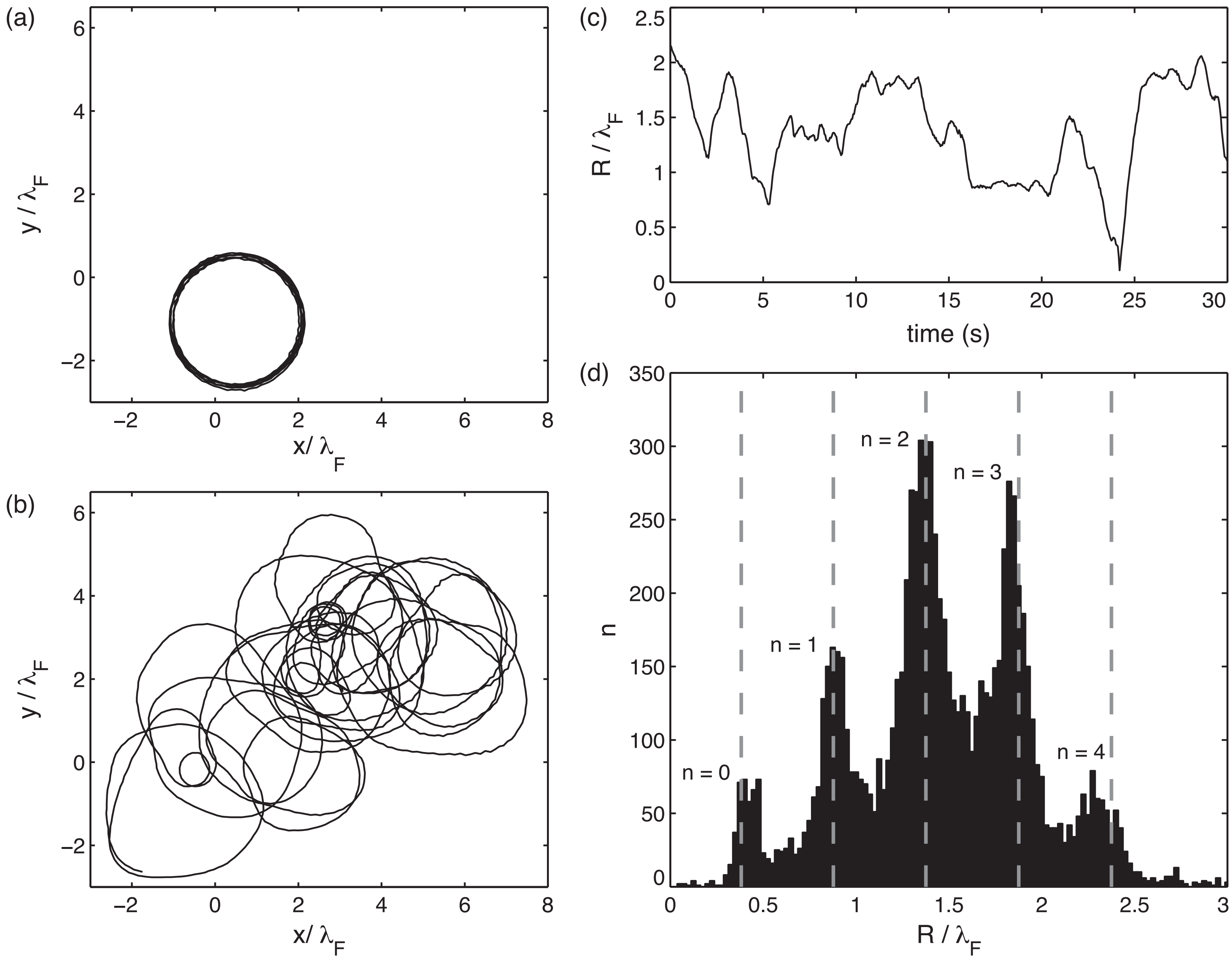


- **stable quantized orbits**

- **stable and unstable orbits**

Chaotic, orbital pilot-wave dynamics

Harris & Bush (2015)



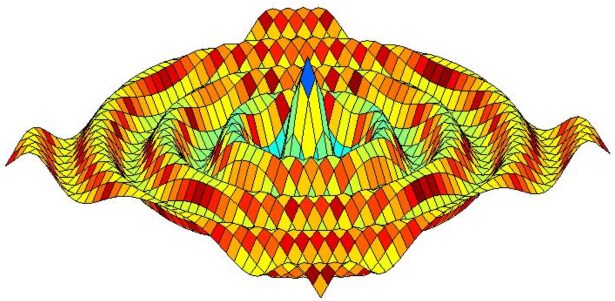
- **intermittent switching between weakly unstable periodic orbits**
- **coherent, wave-like statistics emerge from chaotic pilot-wave dynamics**

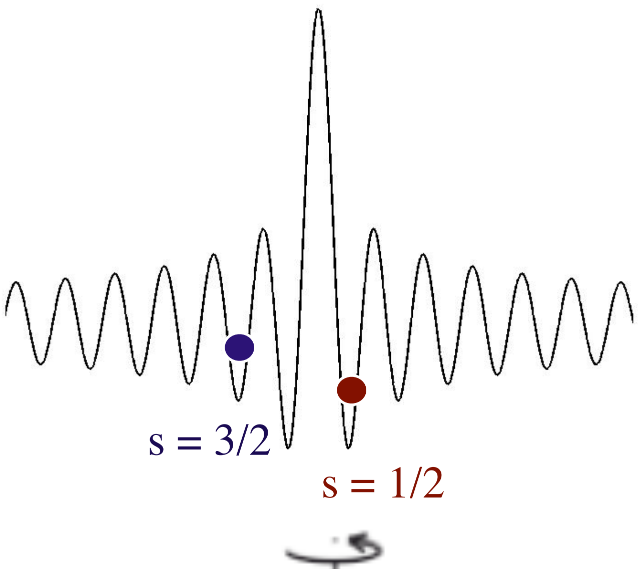
Hydrodynamic spin states at ultra-high memory?

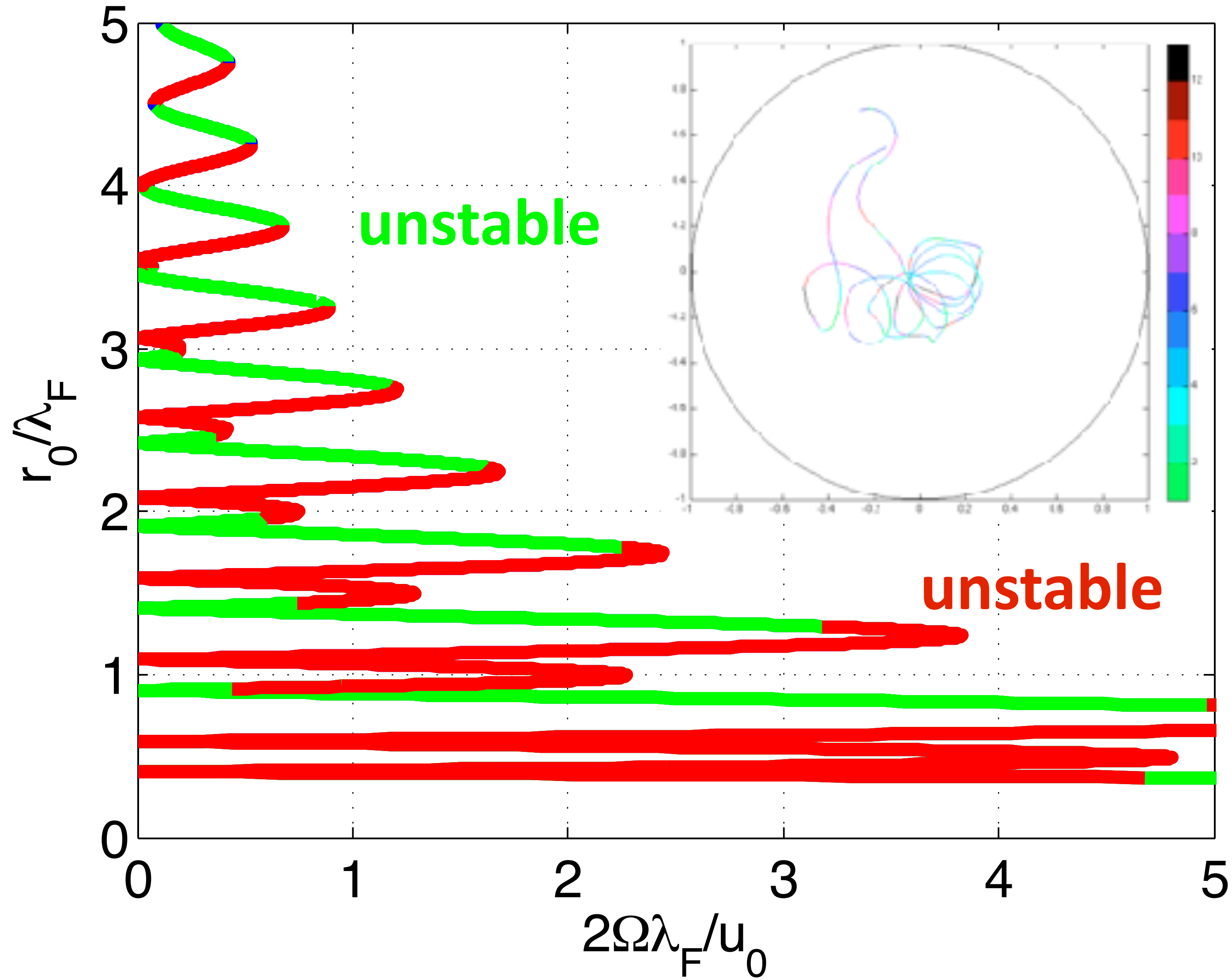
UNSTABLE!

Balance between inertial and wave force.

Orbital radii split by applied rotation.







Oza, Rosales & Bush (2018)

Walking drops

- a damped, driven, pilot-wave system
- work done by vibrational forcing balances viscous dissipation

What inferences might we make if we didn’t realize this ?

Q1: What is the mass of a walker?

Q2: What trajectory equation would be inferred?

The Boost equation

Bush, Oza & Molacek (JFM 2014)

In the weak-acceleration limit, the trajectory equation takes the form

$$\frac{d}{dt}\,\mathbf{p_w} + D_w\,\mathbf{v} = \mathbf{F}$$

where the walker mass $m_w = \gamma_B(v)\; m_0$ and momentum $\mathbf{p_w} = m_w \mathbf{v}$

depend on the ***hydrodynamic boost factor*:** $\gamma_B = 1 + \dfrac{\beta}{2\kappa(1+v^2)^{3/2}}$

and a nonlinear drag $D_w = D_0\left(\dfrac{\mathbf{v}^2}{u_0^2} - 1\right)$ drives it to its free walking speed.

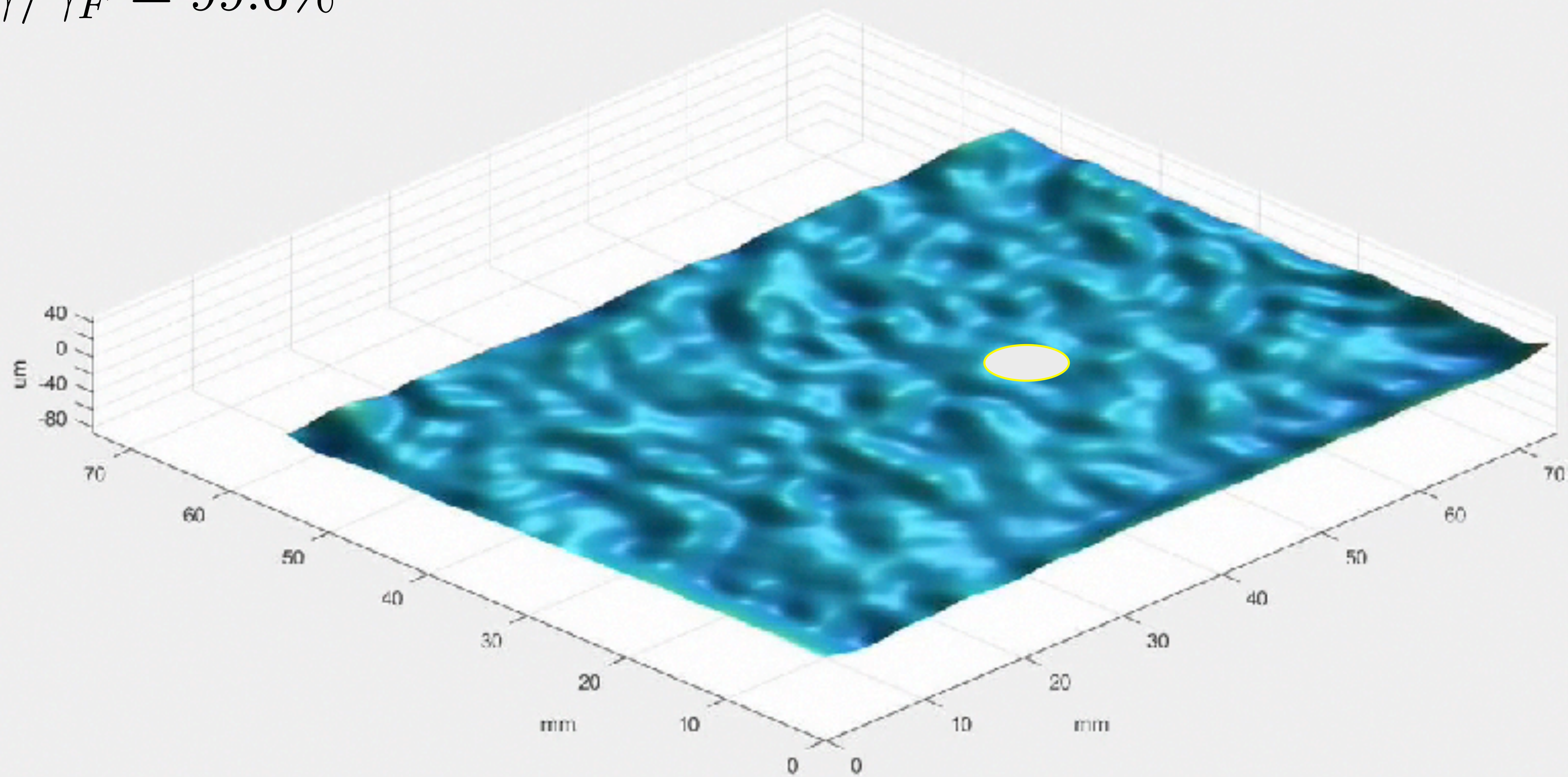
For motion at the free walking speed: $$\frac{d}{dt}\,\mathbf{p_w} = \mathbf{F}$$

- ***the inviscid dynamics of a particle with a speed-dependent mass***

Walker scattering from a submerged pillar

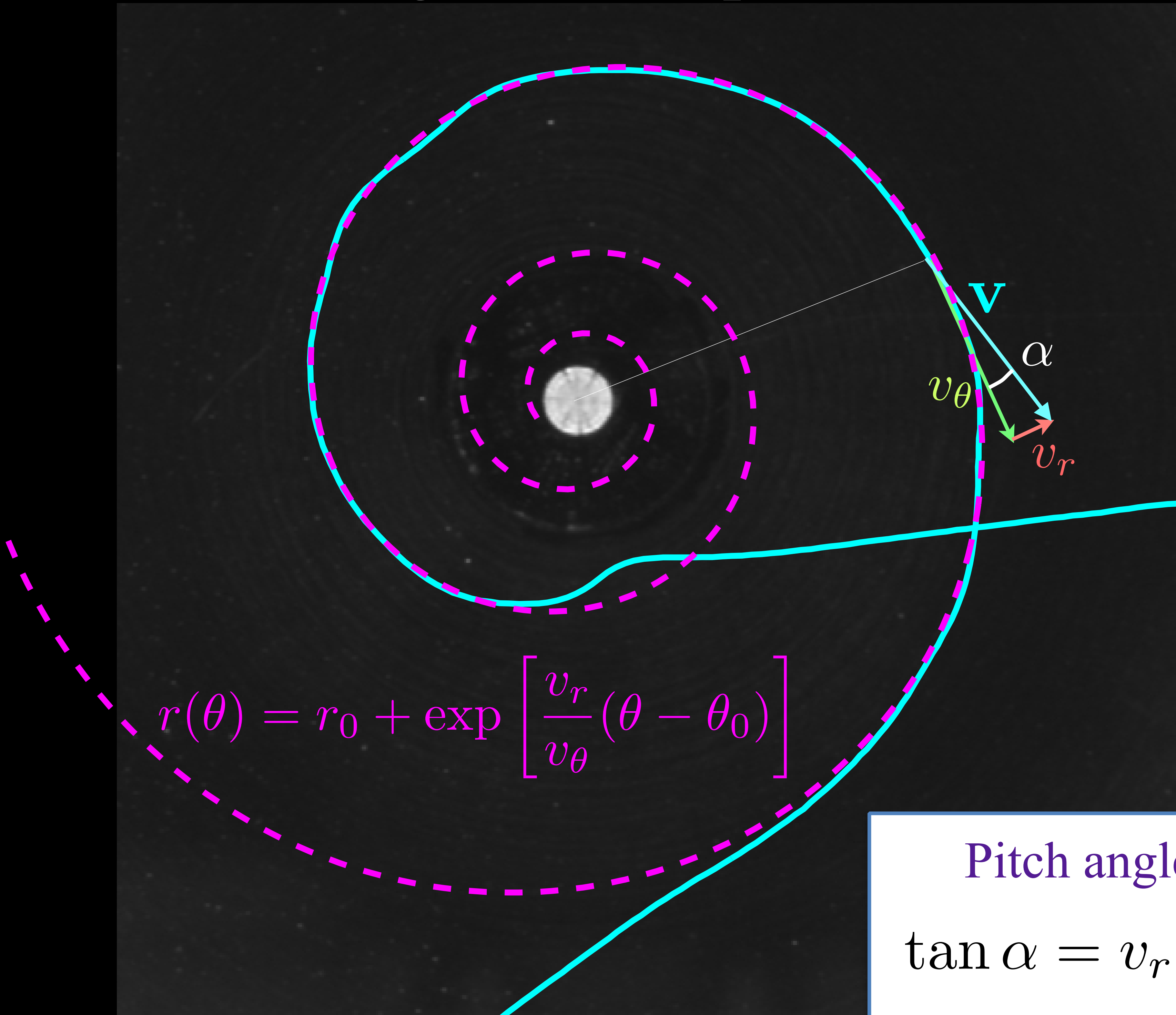
Damiano *et al*. 2016, Harris *et al*. 2018

$\gamma/\gamma_F = 99.6\%$



- pillar acts to locally suppress the walker-induced wave field

Logarithmic spiral



$$r(\theta) = r_0 + \exp\left[\frac{v_r}{v_\theta}(\theta - \theta_0)\right]$$

Pitch angle

$$\tan\alpha = v_r / v_\theta$$

Infer wave-mediated pillar-induced force from trajectory

$$\frac{d}{dt}\,\mathbf{p_w} = \mathbf{F_p}$$

Force required for a logarithmic spiral:

$$\mathbf{F}_p = 2\pi\,\gamma_B m\,\mathbf{v}\times\mathbf{\Omega}$$

where $\mathbf{\Omega} = \frac{v_\theta}{2\pi r}\,\hat{\mathbf{k}}$ is the walker's instantaneous angular velocity

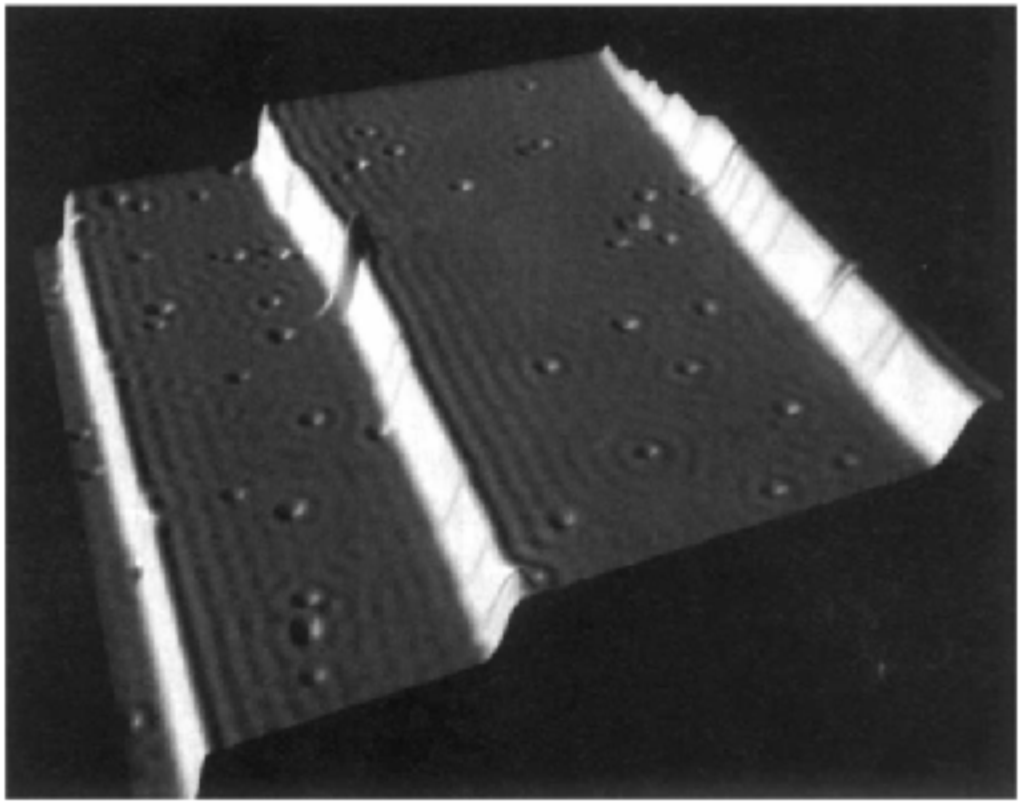
- identical forms of Coriolis force acting on a mass $\mathbf{F_C} = 2m(\mathbf{v}\wedge\mathbf{\Omega})$ and the Lorentz force acting on a charge $\mathbf{F_B} = q(\mathbf{v}\wedge\mathbf{B})$ was the basis for the analogy between inertial orbits and Larmor levels *(Fort et al. 2010)*
- here, it indicates that the walker is similar to a charge moving in the magnetic field associated with its own motion

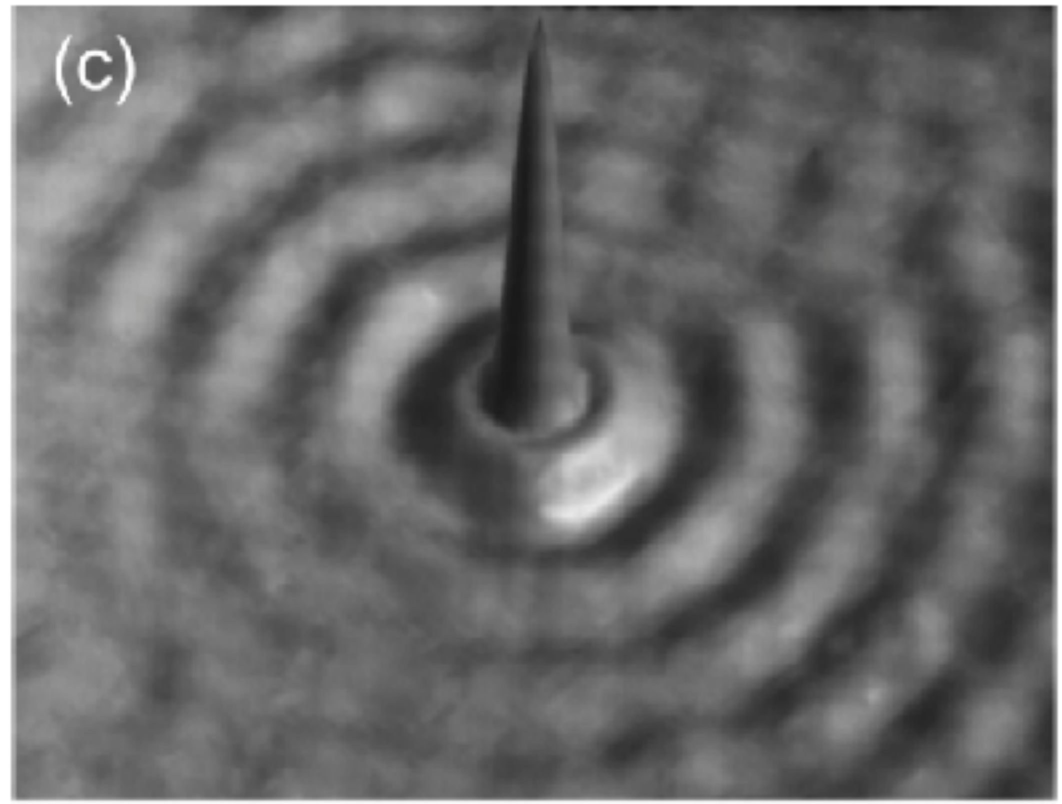
→ **hydrodynamic self-induction**

- a wave-mediated force gives rise to apparent `***spooky action at a distance***'

Friedel oscillations

- modulations of the probability density of the electron-sea on a substrate due to the presence of a scattering impurity
- taken as evidence of the finite size of an electron



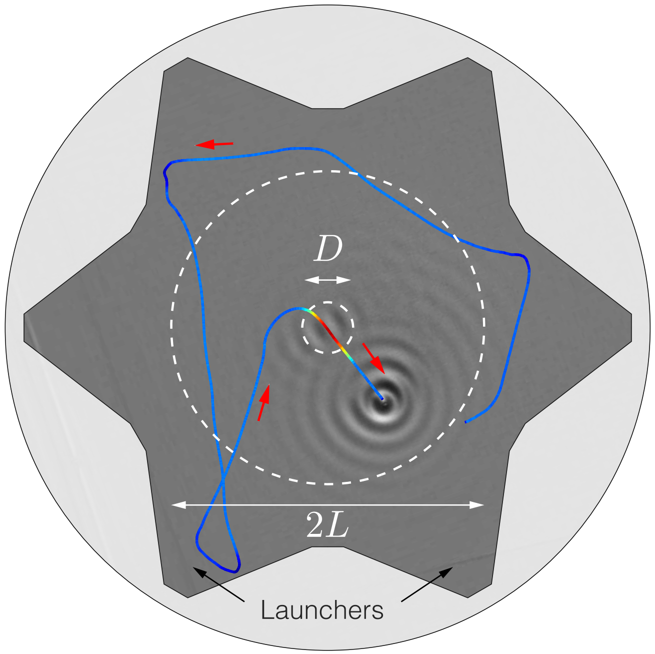


Unknown interaction mechanism

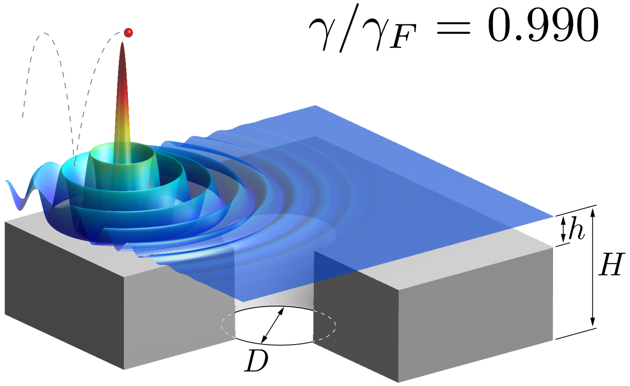
Modeled as localized scattering potentials

WALKER-WELL INTERACTION

Experimental Setup



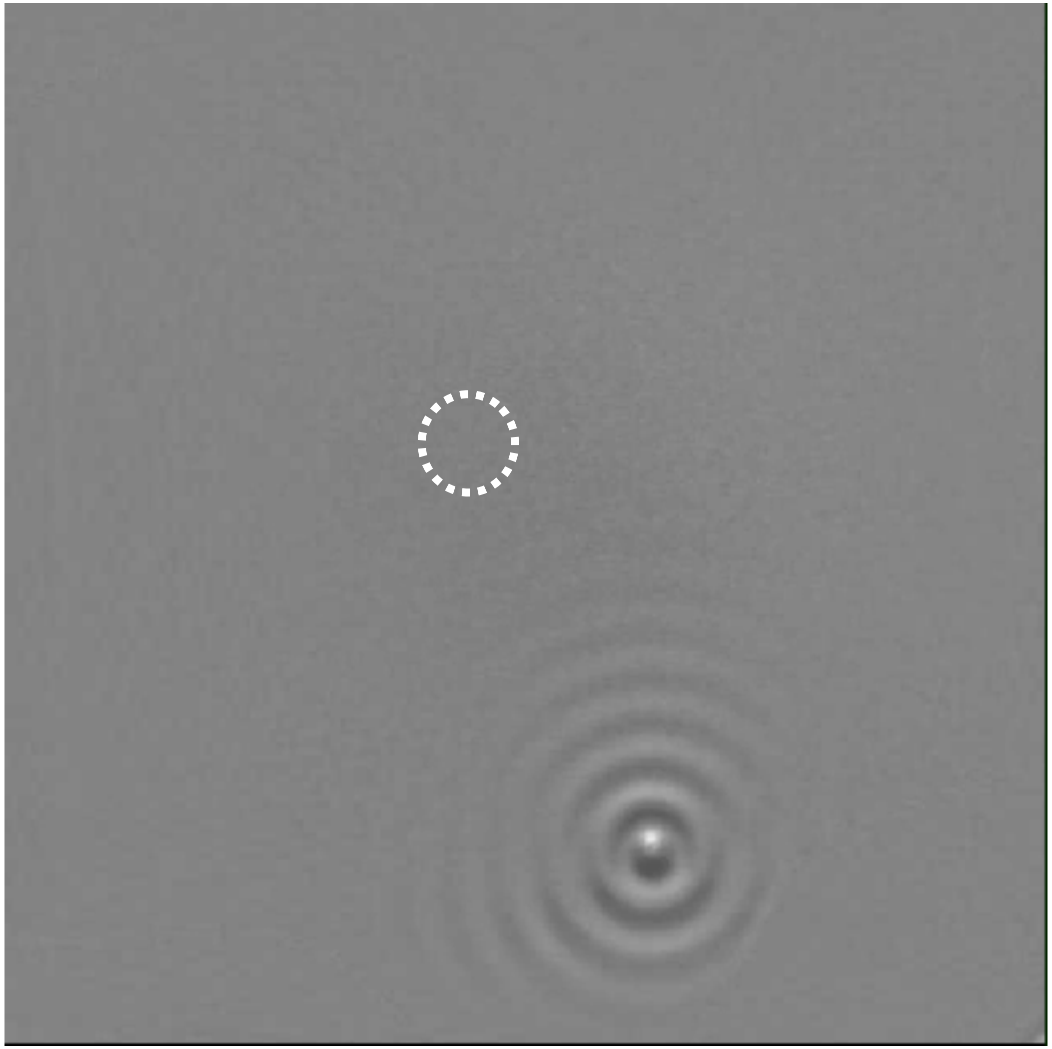
$\gamma/\gamma_F = 0.990$



Well

Region of high excitability

$\gamma_F^H < \gamma_F < \gamma_F^h$

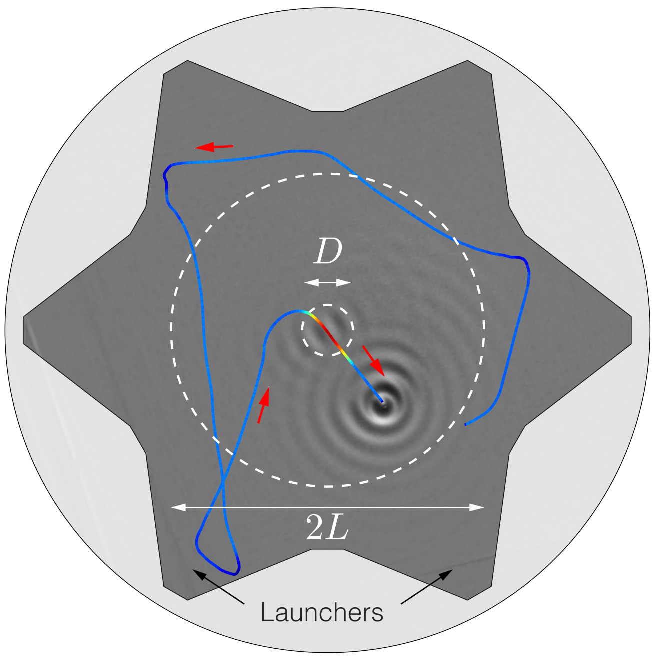


Drop drawn in along an Archimedean spiral

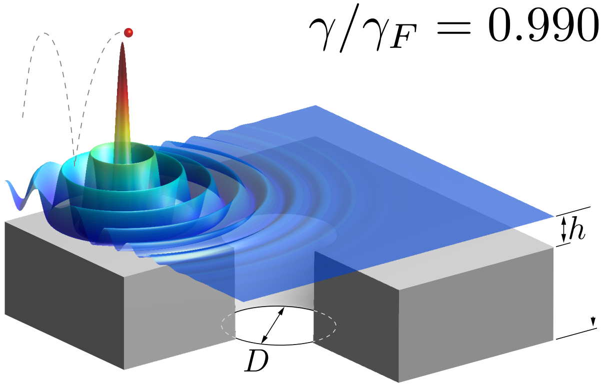
Speed modulations induced by interaction with waves generated above the well

WALKER-WELL INTERACTION

Experimental Setup



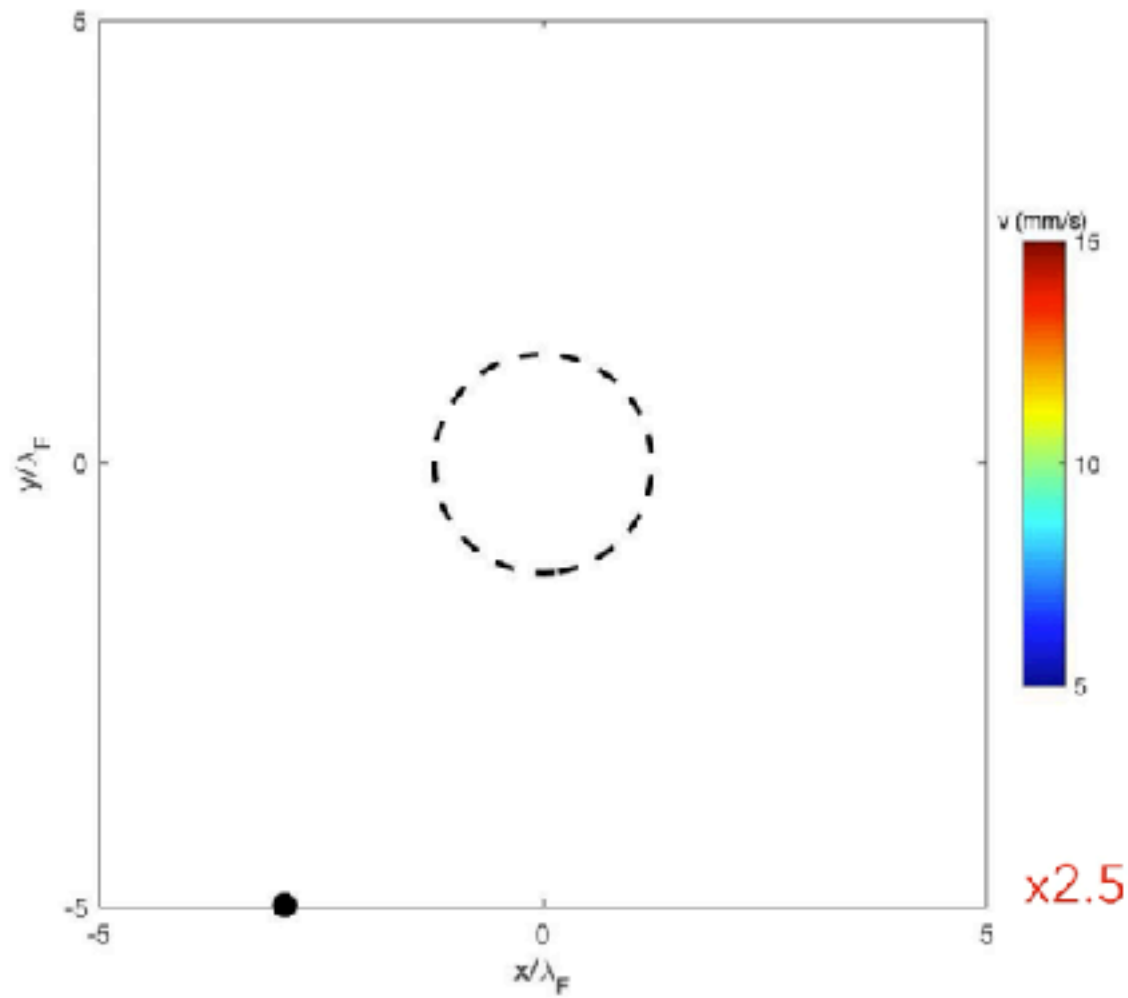
$$\gamma/\gamma_F = 0.990$$



Well

Region of high excitability

$$\gamma_F^H < \gamma_F < \gamma_F^h$$



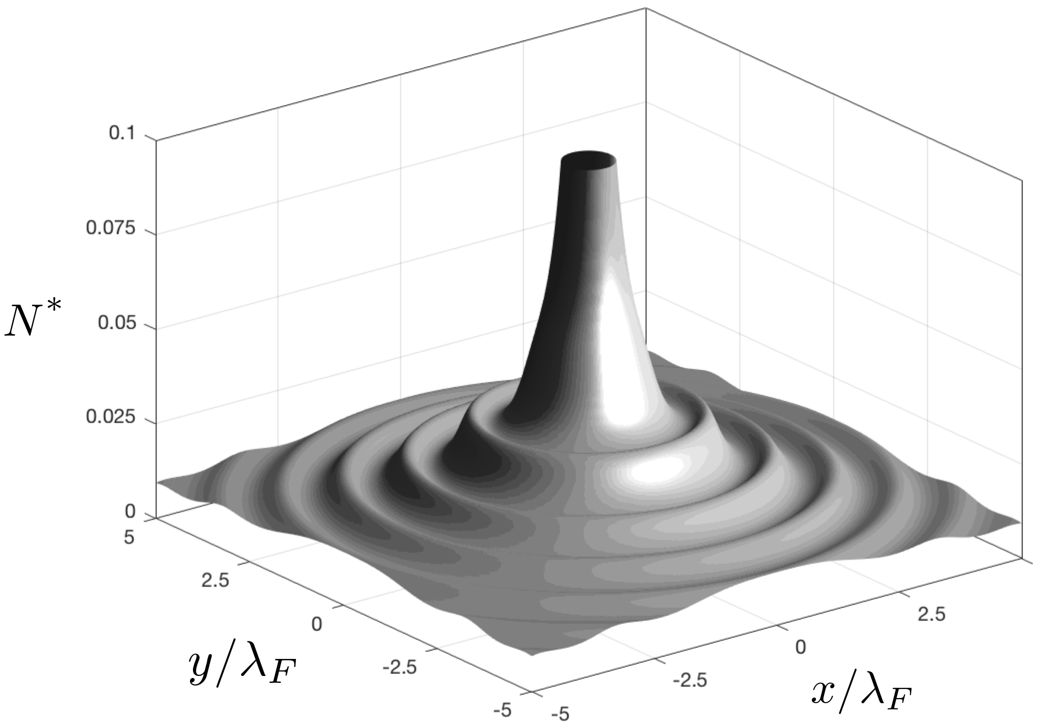
Drop drawn in along an Archimedean spiral

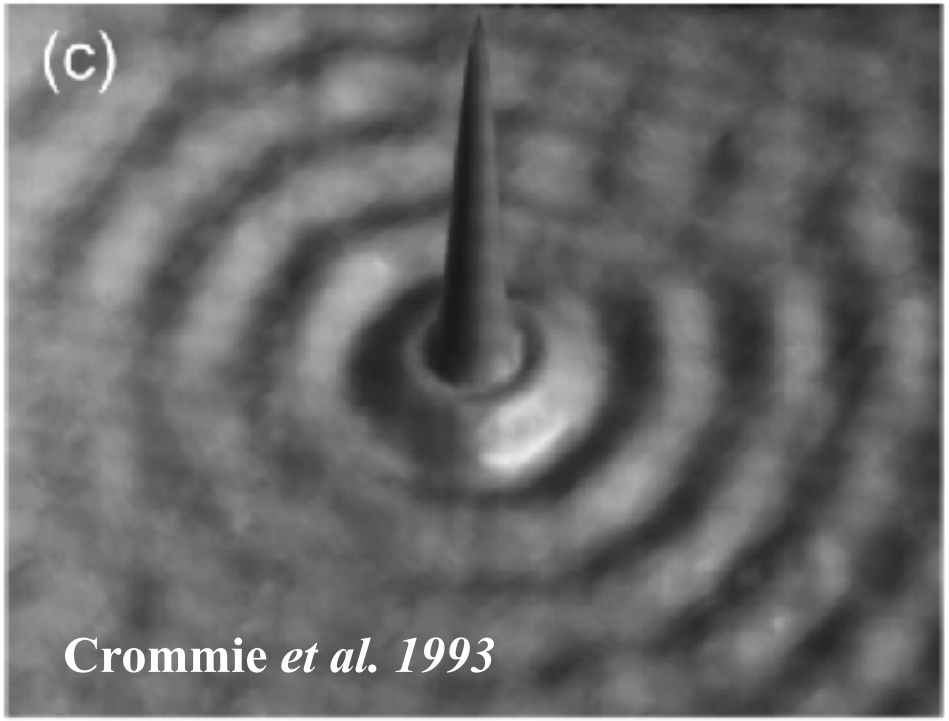
Speed modulations induced by interaction with waves generated above the well

A hydrodynamic analog of Friedel oscillations

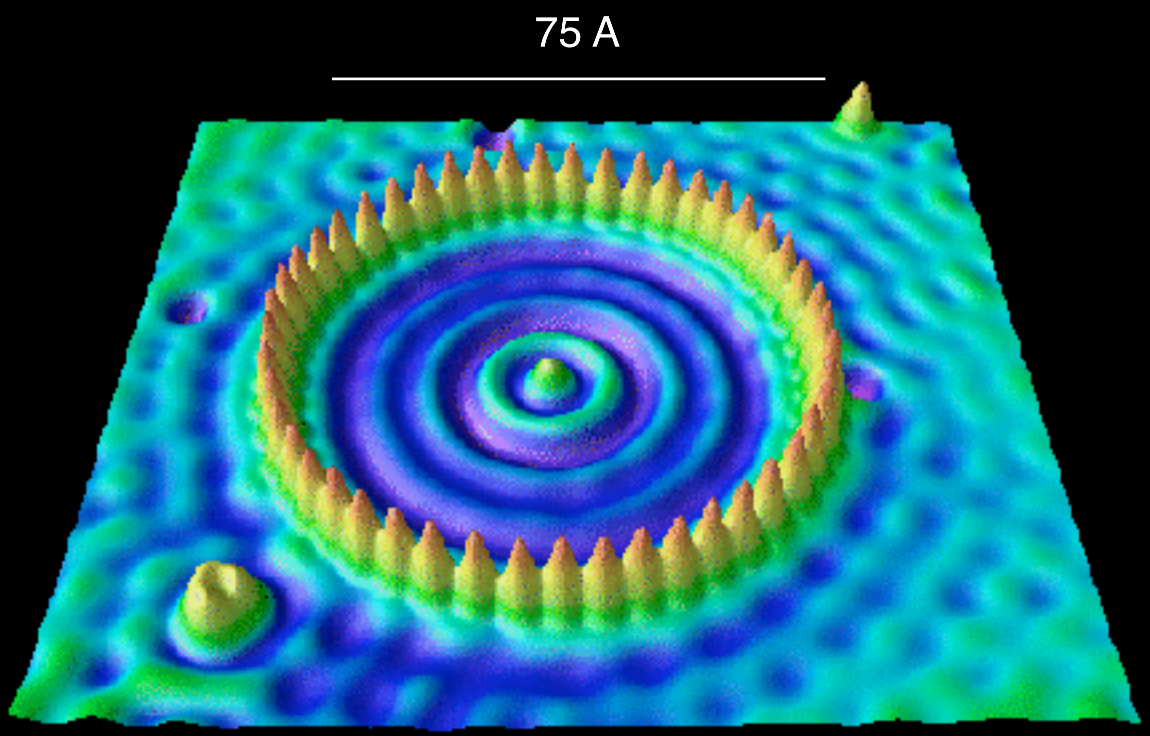
Sáenz, Cristea-Platon & Bush (Sci. Advances, 2020)

- arises due to wave-induced speed modulations in outgoing trajectory





Friedel-like oscillations are *not* inconsistent with the notion of particle trajectories

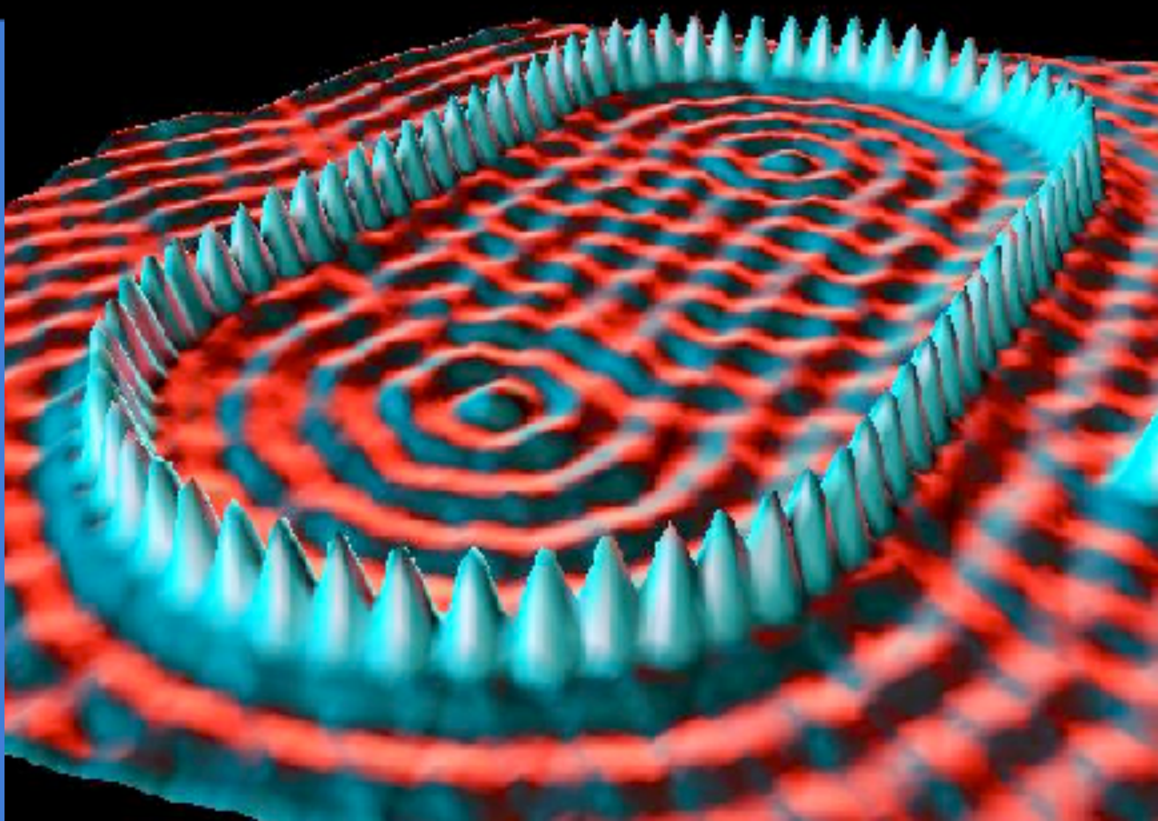


The quantum corral

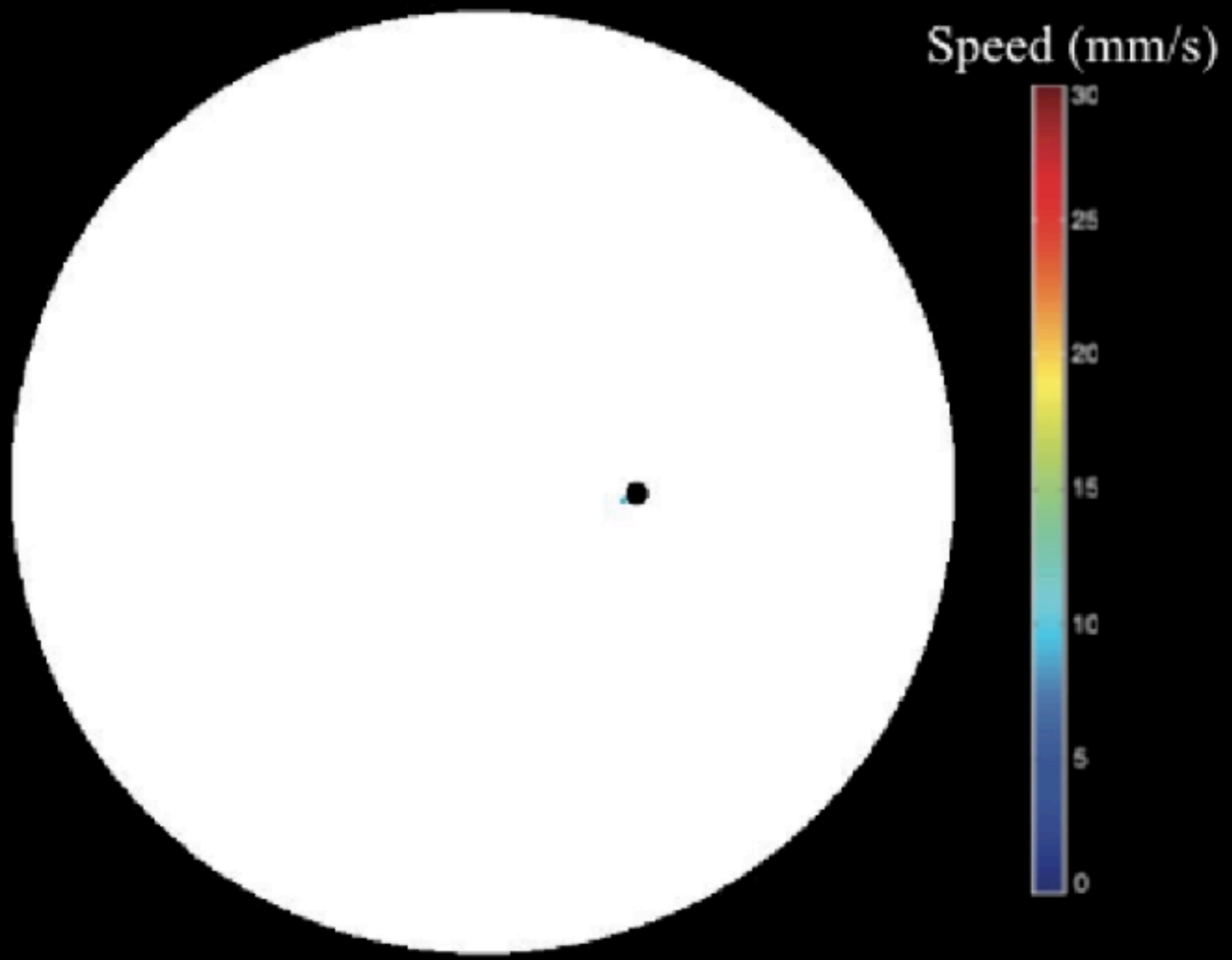
Crommie, Lutz & Eigler (1993)

Fiete & Heller (2003)

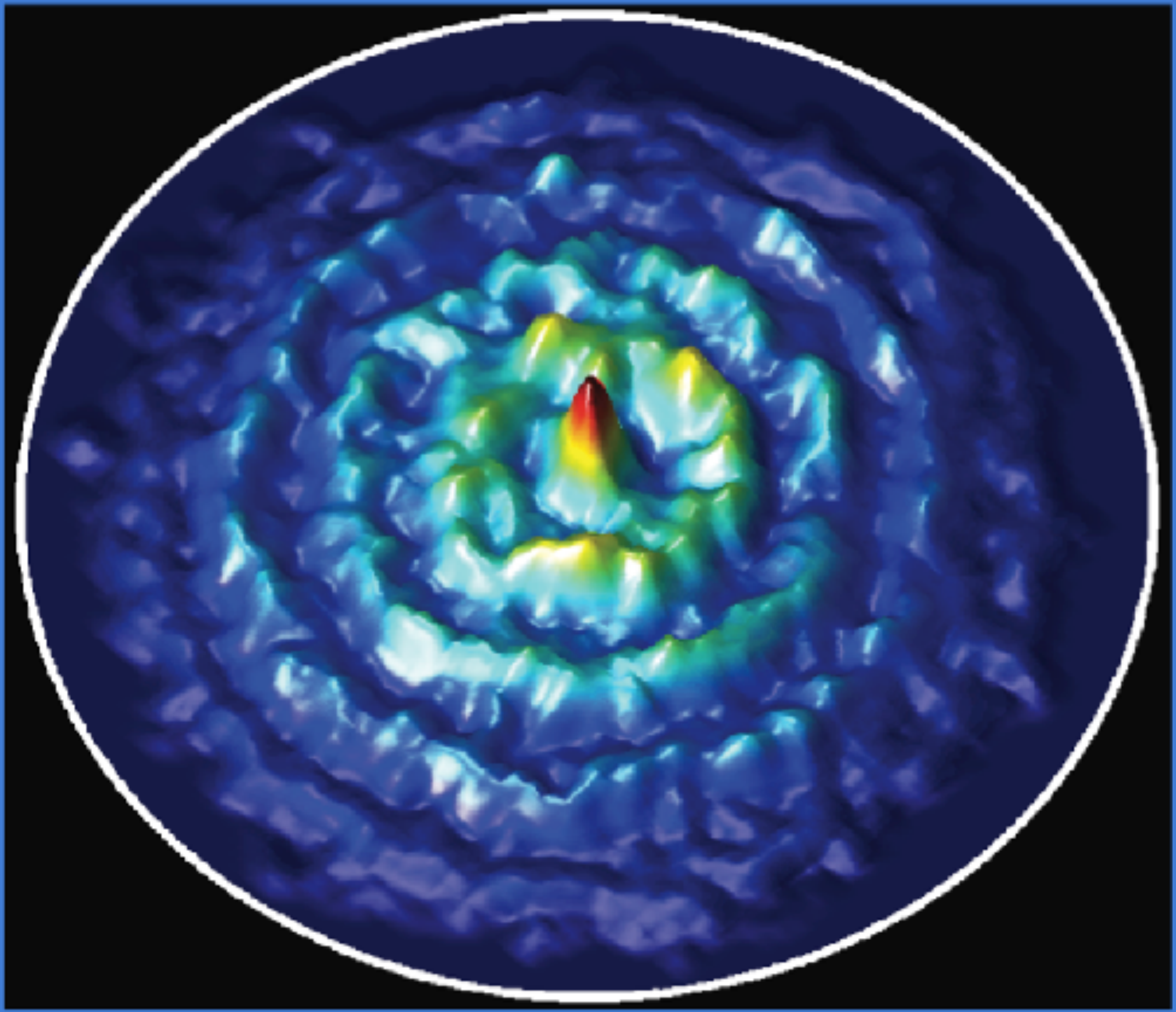
- de Broglie waves evident in the pdf of a sea of electrons trapped on a metal surface, excited by an SEM



Droplet walking in a circular corral



Probability density function

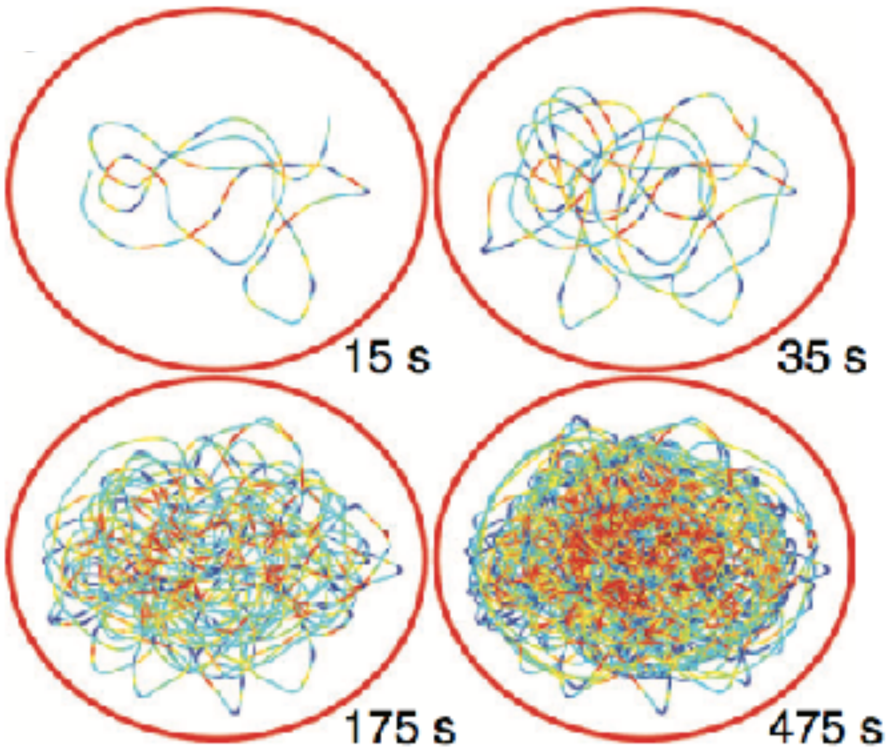


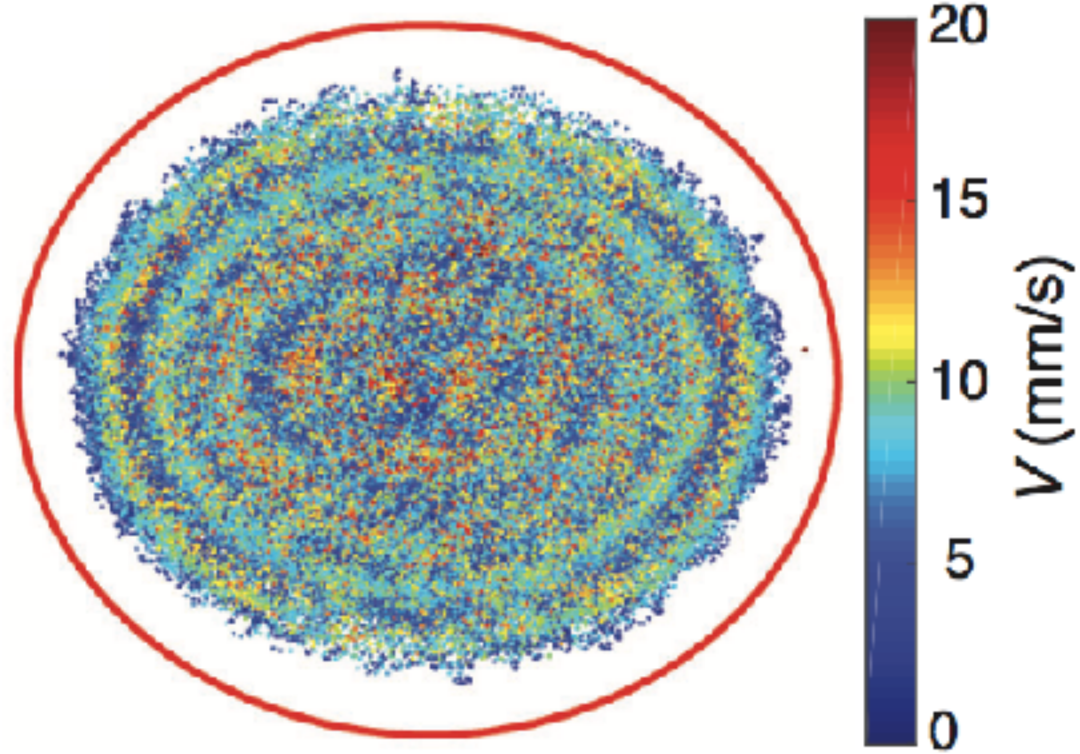
Harris, Moukhtar, Fort, Couder & Bush (PRE, 2013)

- **coherent, wave-like statistics emerge from chaotic pilot-wave dynamics**
- **emergent statistics not inconsistent with the notion of particle trajectories**

The elliptical corral

Trajectories ***Mean speed***

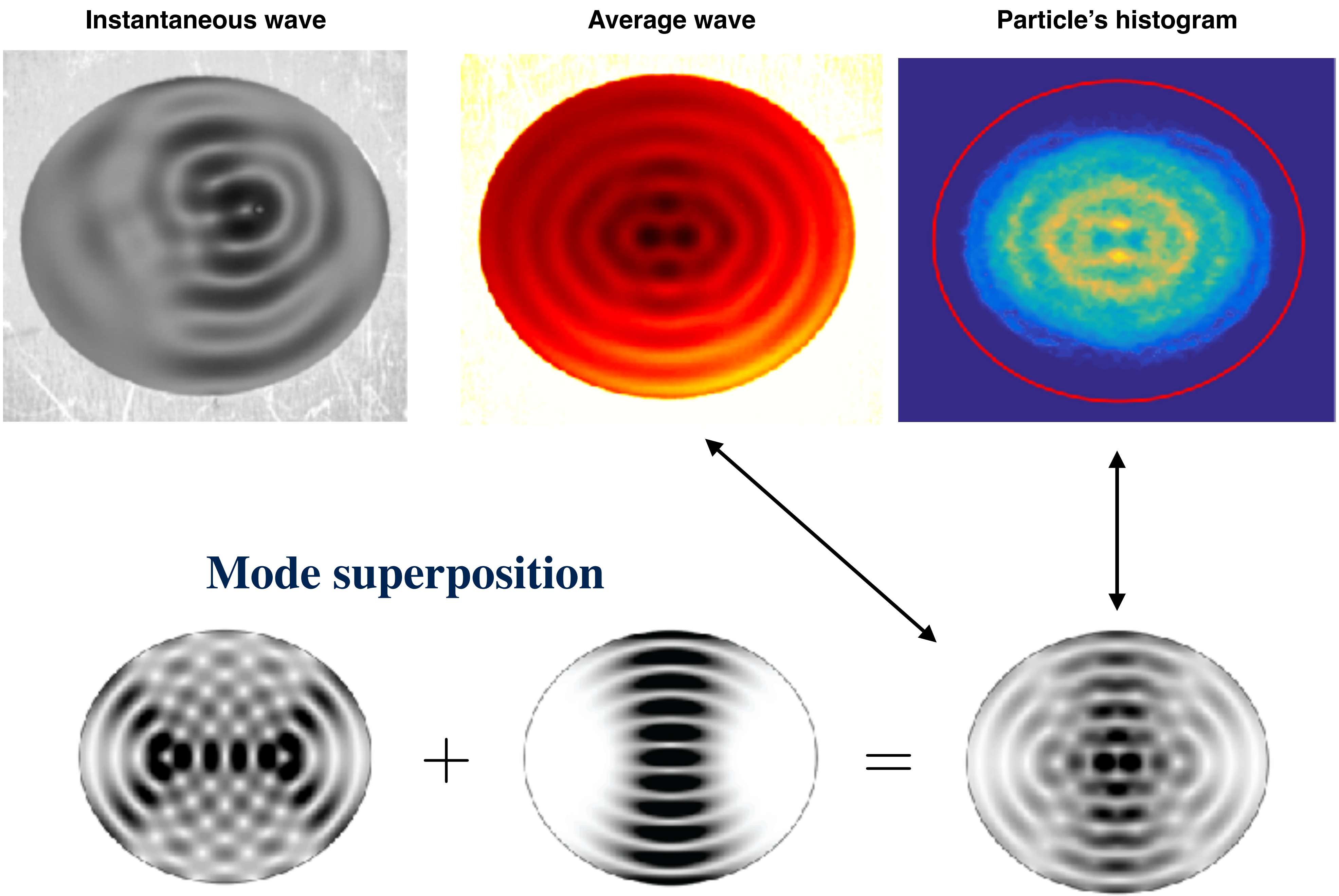




- correlation between position and speed, as in the circular corral
- exhibits statistical projection effects analogous to the `quantum mirage'

- Sáenz, Cristea-Platon & Bush, Nat. Phys. (2018)

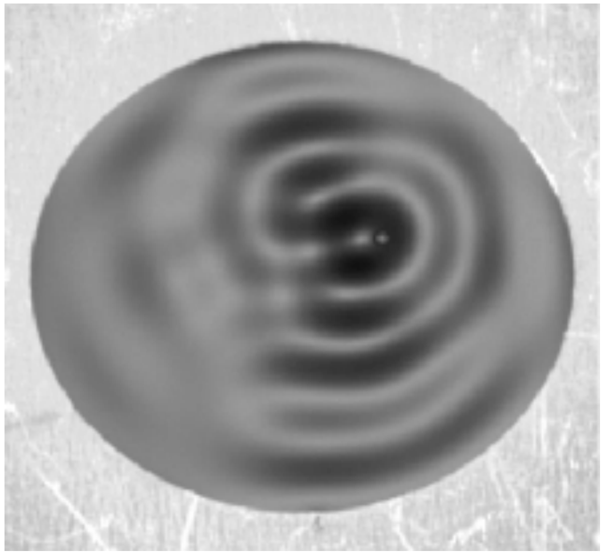
A striking equivalence



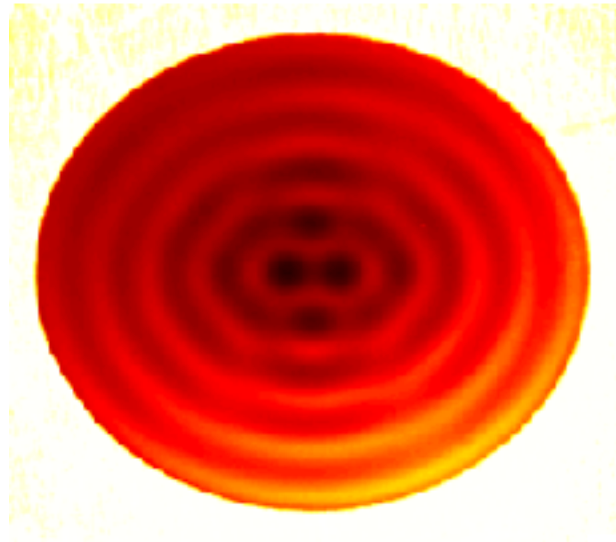
A superposition of statistical states

The mean pilot-wave field

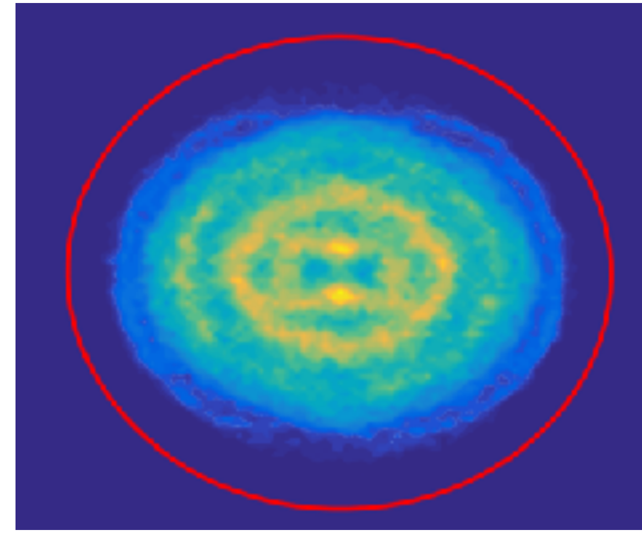
Instantaneous wave

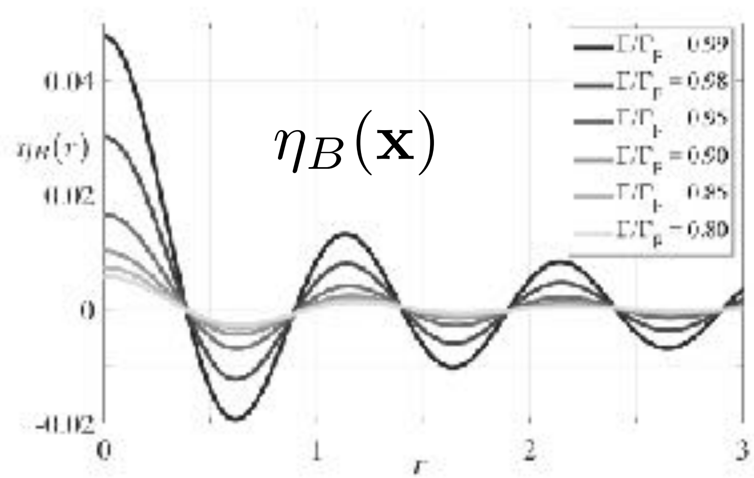


Average wave $\bar{\eta}(\mathbf{x})$



Particle's histogram $\mu(\mathbf{x})$





Theorem *(Durey, Milewski & JB, 2018)*

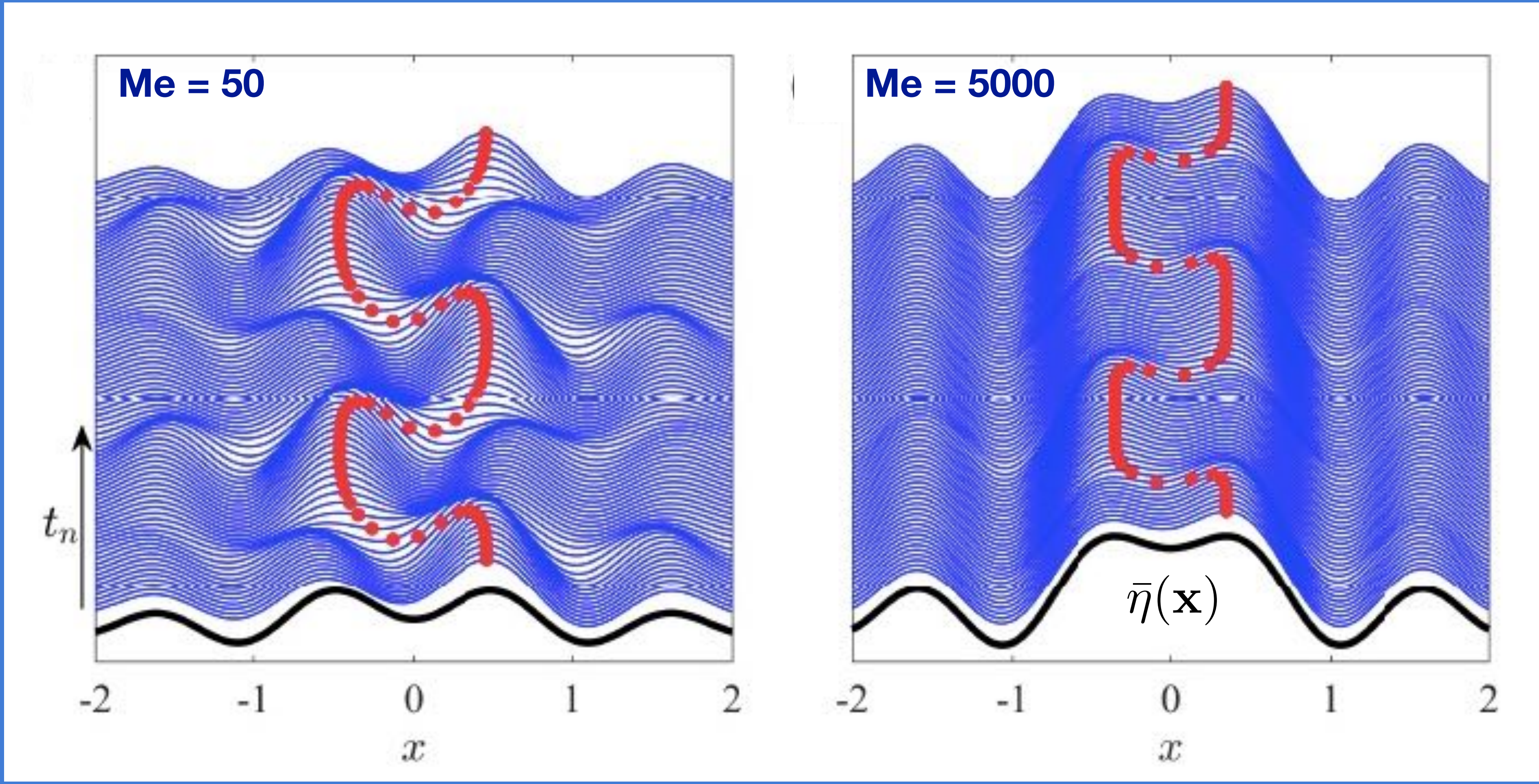
$$\bar{\eta}(\mathbf{x}) = \int_{\mathbf{R}^2} \eta_{\mathbf{B}}(\mathbf{x}-\mathbf{y})\ \mu(\mathbf{y})\ \mathbf{dy} = (\eta_{\mathbf{B}} * \mu)(\mathbf{x})$$

- the average wave field, $\bar{\eta}(\mathbf{x})$, corresponds to the convolution of the *pdf*, $\mu(\mathbf{x})$ and the wave field of a stationary bouncing droplet, $\eta_B(\mathbf{x})$
- allows one to associate waveforms with trajectories

Observation (from 1D simulations)

- when walker motion is confined by boundaries or applied forces, the instantaneous pilot wave approaches the mean wave field at high Me

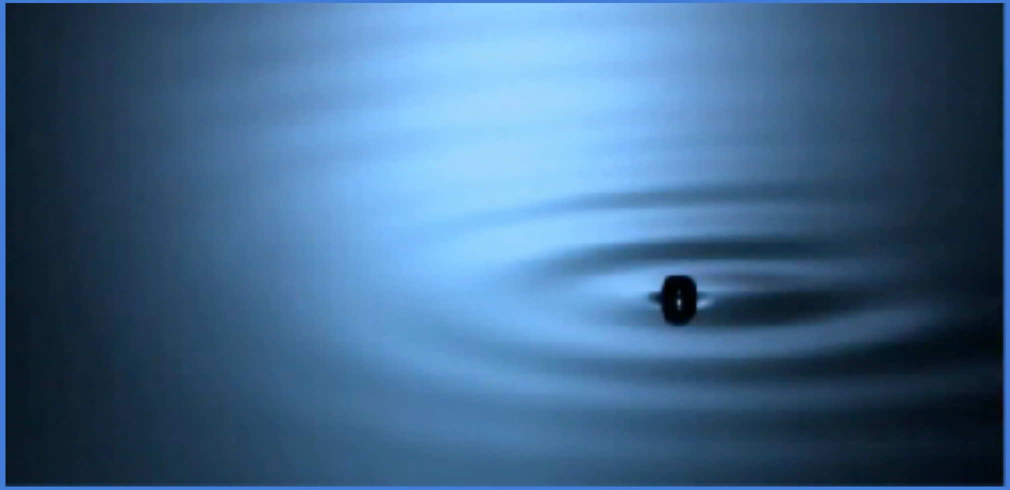
e.g. simulated 1D pilot-wave dynamics of a localized walker oscillation



- the drop moves in its self-induced `mean-pilot-wave' potential, as may be rationalized in terms of memory rather than nonlocality

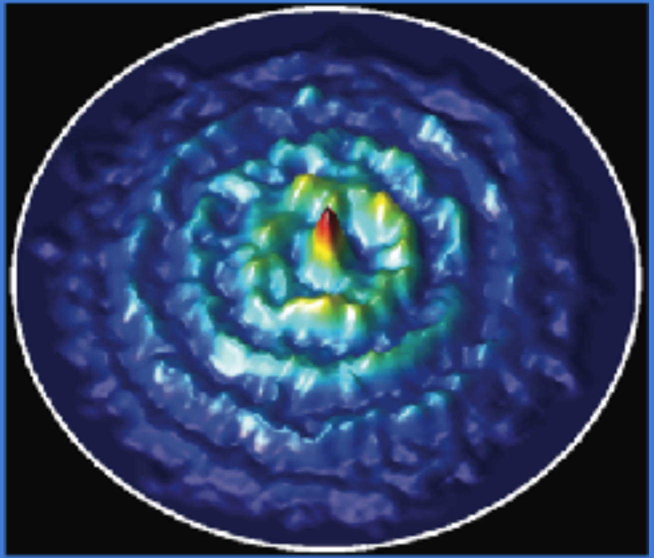
Durey, Milewski & JB (2018)

Emerging physical picture: 3 time scales



- **fast** dynamics: bouncing at resonance creates monochromatic wave field

- **intermediate** (strobed) pilot-wave dynamics: droplet rides its instantaneous guiding wave



- **long-term statistical** behaviour described by Faraday wave modes