Monitoring carbon dioxide to quantify the risk of indoor airborne transmission of COVID-19

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Abstract
A new guideline for mitigating indoor airborne transmission of COVID-19 prescribes a limit on the time spent in a shared space with an infected individual (Bazant & Bush, Proceedings of the National Academy of Sciences of the United States of America, vol. 118, issue 17, 2021, e2018995118). Here, we rephrase this safety guideline in terms of occupancy time and mean exhaled carbon dioxide (CO2) concentration in an indoor space, thereby enabling the use of CO2 monitors in the risk assessment of airborne transmission of respiratory diseases. While CO2 concentration is related to airborne pathogen concentration (Rudnick & Milton, Indoor Air, vol. 13, issue 3, 2003, pp. 237–245), the guideline developed here accounts for the different physical processes affecting their evolution, such as enhanced pathogen production from vocal activity and pathogen removal via face-mask use, filtration, sedimentation and deactivation. Critically, transmission risk depends on the total infectious dose, so necessarily depends on both the pathogen concentration and exposure time. The transmission risk is also modulated by the fractions of susceptible, infected and immune people within a population, which evolve as the pandemic runs its course. A mathematical model is developed that enables a prediction of airborne transmission risk from real-time CO2 measurements. Illustrative examples of implementing our guideline are presented using data from CO2 monitoring in university classrooms and office spaces.

Impact Statement
There is mounting scientific evidence that COVID-19 is primarily transmitted through indoor airborne transmission, as arises when a susceptible person inhales virus-laden aerosol droplets exhaled by an infectious person. A safety guideline to limit indoor airborne transmission (Bazant & Bush, 2021) has recently been derived that complements the public health guidelines on surface cleaning and social distancing. We here recast this safety guideline in terms of total inhaled carbon dioxide (CO2), as can be readily monitored in most indoor spaces. Our approach paves the way for optimizing air handling systems by balancing health and financial concerns, and so informs policy for safely reopening schools and businesses as the pandemic runs its course. Finally, our approach may be applied quite generally in the mitigation of airborne respiratory illnesses, including influenza.
1. Introduction

Coronavirus disease 2019 (COVID-19) has caused a devastating pandemic since it was first identified in Wuhan, China, in December 2019 (Chen et al., 2020; Li et al., 2020). For over a year, public health guidance has focused on disinfecting surfaces in order to limit transmission through fomites (Van Doremalen et al., 2020) and maintaining social distance in order to limit transmission via large drops generated by coughs and sneezes (Bourouiba, Dehandschoewercker & Bush, 2014; Rosti, Olivieri, Cavaiola, Seminara & Mazzino, 2020). The efficacy of these measures has been increasingly called into question, however, since there is scant evidence for fomite transmission (Gandhi, Yokoe & Havlir, 2020; Lewis, 2021), and large-drop transmission is effectively eliminated by masks (Moghadas et al., 2020).

There is now overwhelming evidence that the pathogen responsible for COVID-19, severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), is transmitted primarily through exhaled aerosol droplets suspended in indoor air (Bazant & Bush, 2021; Greenhalgh et al., 2021; Jayaweera, Perera, Gunawardana & Manatunge, 2020; Morawska & Cao, 2020; Morawska & Milton, 2020; Prather et al., 2020; Stadnytskyi, Bax, Bax & Anfinrud, 2020; Zhang, Li, Zhang, Wang & Molina, 2020b). Notably, airborne transmission provides the only rational explanation for the so-called ‘superspreading events’, that have now been well chronicled and all took place indoors (Hamner, 2020; Hwang, Chang, Bumjo & Heo, 2020; Kwon et al., 2020; Miller et al., 2020; Moriarty, 2020; Nishiura et al., 2020; Shen et al., 2020). The dominance of indoor airborne transmission is further supported by the fact that face-mask directives have been more effective in limiting the spread of COVID-19 than either social distancing directives or lockdowns (Stutt, Retkute, Bradley, Gilligan & Colvin, 2020; Zhang et al., 2020b). Indeed, a recent analysis of spreading data from Massachusetts public schools where masking was strictly enforced found no statistically significant effect of social distance restrictions that ranged from three feet to six feet (van den Berg et al., 2021). A recent study of school transmissions in Georgia found that mask use and improvements in ventilation and filtration were the most effective mitigation strategies, while the imposition of barriers or six-foot distancing between desks had little effect (Gettings et al., 2021). Finally, the detection of infectious SARS-CoV-2 virions suspended in hospital room air as far as 5.5 m from an infected patient provides direct evidence for the viability of airborne transmission of COVID-19 (Lednicky et al., 2020; Santarpia et al., 2020).

With a view to informing public health policy, we proceed by developing a quantitative approach to mitigating the indoor airborne transmission of COVID-19, an approach that might be similarly applied to other airborne respiratory diseases. The canonical theoretical framework of Wells (1955) and Riley, Murphy & Riley (1978) describes airborne transmission in an indoor space that is well-mixed by ambient air flows, so that infectious aerosols are uniformly dispersed throughout the space (Beggs, Noakes, Sleigh, Fletcher & Siddiqi, 2003; Gammaitoni & Nucci, 1997; Nicas, Nazaroff & Hubbard, 2005; Noakes, Beggs, Sleigh & Kerr, 2006; Stilianakis & Drossinos, 2010). While exceptions to the well-mixed-room approximation are known to arise (Bhagat, Wykes, Dalziel & Linden, 2020; see supplementary information in Bazant & Bush, 2021), supporting evidence for the well-mixed approximation may be found in both theoretical arguments (Bazant & Bush, 2021) and computer simulations of natural and forced convection (Foster & Kinzel, 2021). The Wells–Riley model and its extensions have been applied to a number of superspreading events and used to assess the risk of COVID-19 transmission in a variety of indoor settings (Augenbraun et al., 2020; Buonanno, Morawska & Stabile, 2020a; Buonanno, Stabile & Morawska, 2020b; Evans, 2020; Miller et al., 2020; Prentiss, Chu & Berggren, 2020).

A safety guideline for mitigating indoor airborne transmission of COVID-19 has recently been derived that indicates an upper bound on the cumulative exposure time, that is, the product of the number of occupants and the exposure time (Bazant & Bush, 2021). This bound may be simply expressed in terms of the relevant variables, including the room dimensions, ventilation, air filtration, mask efficiency and respiratory activity. The guideline has been calibrated for COVID-19 using epidemiological data from the best characterized superspreading events and incorporates the measured dependence of expiratory droplet-size distributions on respiratory and vocal activity (Asadi, Bouvier, Wexler & Ristenpart, 2020a;
Asadi et al., 2019; Morawska et al., 2009). An online app has facilitated its widespread use during the pandemic (Khan, Bazant & Bush, 2020). The authors also considered the additional risk of turbulent respiratory plumes and jets (Abkarian, Mendez, Xue, Yang & Stone, 2020a, 2020b), as need be considered when masks are not worn. The accuracy of the guideline is necessarily limited by uncertainties in a number of model parameters, which will presumably be reduced as more data is analysed from indoor spreading events.

Carbon dioxide measurements have been used for decades to quantify airflow and zonal mixing in buildings and so guide the design of heating, ventilation and air-conditioning (known as HVAC) systems (Fisk & De Almeida, 1998; Seppänen, Fisk & Mendell, 1999). Such measurements thus represent a natural source of data for assessing indoor air quality (Corsi, Torres, Sanders & Kinney, 2002), especially as they rely only on relatively inexpensive, widely available CO₂ sensors. Quite generally, high CO₂ levels in indoor settings are known to be associated with poor health and diminished cognitive function (Coley, Greeves & Saxby, 2007; Hung & Derossis, 1989; Salisbury, 1986; Seppänen et al., 1999). MacNaughton et al. (2015) examined the economic and environmental costs of increasing ventilation rates to improve indoor air quality, and concluded that the public health benefits of improved ventilation generally outweigh these costs. Statistically significant correlations between CO₂ levels and illness-related absenteeism in both the workplace (Milton, Glencross & Walters, 2000) and classrooms (Mendell et al., 2013; Shendell et al., 2004) have been widely reported (Li et al., 2007). Direct correlations between CO₂ levels and concentration of airborne bacteria have been found in schools (Liu et al., 2000). Correlations between outdoor air exchange rates and respiratory infections in dormitory rooms have also been reported (Bueno de Mesquita, Noakes & Milton, 2020; Sun, Wang, Zhang & Sundell, 2011). Despite the overwhelming evidence of such correlations and the numerous economic analyses that underscore their negative societal impacts (Fisk, 2000; Milton et al., 2000), using CO₂ monitors to make quantitative assessments of the risk of indoor disease transmission is a relatively recent notion (Li et al., 2007).

Rudnick and Milton (2003) first proposed the use of Wells–Riley models, in conjunction with measurements of CO₂ concentration, to assess airborne transmission risk indoors. Their model treats CO₂ concentration as a proxy for infectious aerosols: the two were assumed to be produced proportionally by the exhalation of an infected individual and removed at the same rate by ventilation. The current pandemic has generated considerable interest in using CO₂ monitoring as a tool for risk management of COVID-19 (Bhagat et al., 2020; Hartmann & Kriegel, 2020). The Rudnick–Milton model has recently been extended by Peng and Jimenez (2021) through consideration of the different removal rates of CO₂ and airborne pathogen. They conclude by predicting safe CO₂ levels for COVID-19 transmission in various indoor spaces, which vary by up to two orders of magnitude.

We here develop a safety guideline for limiting indoor airborne transmission of COVID-19 by expressing the safety guideline of Bazant and Bush (2021) in terms of CO₂ concentration. Doing so makes clear that one must limit not only the CO₂ concentration, but also the occupancy time. Our model accounts for the effects of pathogen filtration, sedimentation and deactivation in addition to the variable aerosol production rates associated with different respiratory and vocal activities, all of which alter the relative concentrations of airborne pathogen and CO₂. Our guideline thus quantifies the extent to which safety limits may be extended by mitigation strategies such as mask mandates, air filtration and the imposition of ‘quiet spaces’.

In § 2, we rephrase the indoor safety guideline of Bazant and Bush (2021) in terms of CO₂ concentration. In § 3, we present theoretical descriptions of the evolution of CO₂ concentration and infectious aerosol concentration in an indoor space, and highlight the different physical processes influencing the two. We then model the disease transmission dynamics, which allows for the risk of indoor airborne transmission to be assessed from CO₂ measurements taken in real time. In § 4, we apply our model to a pair of data sets tracking the evolution of CO₂ concentration in specific office and classroom settings. These examples illustrate how CO₂ monitoring, when coupled with our safety guideline, provides a means of assessing and mitigating the risk of indoor airborne transmission of respiratory pathogens.
2. Safety guideline for the time-averaged CO₂ concentration

2.1. Occupancy-based safety guideline

We begin by recalling the safety guideline of Bazant and Bush (2021) for limiting indoor airborne disease transmission in a well-mixed space. The guideline would impose an upper bound on the cumulative exposure time,

\[ N_T \tau < \frac{\epsilon \bar{\lambda} V}{Q_b C_q \bar{s} s_m}, \]

where \( N_T \) is the number of possible transmissions (pairs of infected and susceptible people) and \( \tau \) is the time in the presence of the infected person(s). The reader is referred to table 1 for a glossary of symbols and their characteristic values. Here \( Q_b \) is the mean breathing flow rate and \( V \) the room volume. The risk tolerance \( \epsilon < 1 \) is the prescribed bound on the probability of at least one transmission, as should be chosen judiciously according to the vulnerability of the population (Garg, 2020); for example, Bazant and Bush (2021) suggested \( \epsilon = 10\% \) for children and \( 1\% \) for the elderly.

The only epidemiological parameter in the guideline, \( C_q \), is the infectiousness of exhaled air, measured in units of infection quanta per volume for a given aerosolized pathogen. The notion of ‘infection quantum’, introduced by Wells (1955), is widely used in epidemiology to measure the expected rate of disease transmission, which may be seen as a transfer of infection quanta between pairs of infected and susceptible individuals. For airborne transmission, a suitable concentration of infection quanta per volume, \( C_q \), can thus be associated with exhaled air without reference to the microscopic pathogen concentration. Notably, \( C_q \) is known to depend on the type of respiratory and vocal activity (associated with an individual resting, exercising, speaking, singing, etc.), being larger for the more vigorous activities (Bazant & Bush, 2021; Buonanno et al., 2020b). The relative susceptibility \( s_r \), is introduced as a scaling factor for \( C_q \) that accounts for differences in the transmissibility of different respiratory pathogens, such as bacteria or viruses (Li et al., 2008; Rudnick & Milton, 2003) with different strains (Davies et al., 2020; Volz et al., 2021), and for differences in the susceptibility of different populations, such as children and adults (Riediker & Morawska, 2020; Zhang et al., 2020a; Zhu et al., 2020).

The mask penetration probability, \( p_m(r) \), is a function of drop size that is bounded below by 0 (for the ideal limit of perfect mask filtration) and above by 1 (as is appropriate when no mask is worn). Standard surgical masks at low flow rates allow only 0.04%–1.5% of the micron-scale aerosols to penetrate (Chen & Willeke, 1992), but these values should be increased by a factor of 2–10 to account for imperfect fit (Oberg & Brosseau, 2008). Cloth masks show much greater variability (Konda et al., 2020b). The mask penetration probability may also depend on respiratory activity (Asadi et al., 2020b) and direction of airflow (Pan, Harb, Leng & Marr, 2020). Here, for the sake of simplicity, we treat \( p_m(r) \) as being constant over the limited aerosol size range of interest, and evaluate \( \bar{p}_m = p_{m}(\bar{r}) \) at the effective aerosol radius \( \bar{r} \) to be defined below, above which drops tend to settle to the ground faster than they are swept away by ventilation. For this effective aerosol filtration factor, Bazant and Bush (2021) suggested \( \bar{p}_m = 1\%–5\% \) for surgical masks (Li et al., 2008; Oberg & Brosseau, 2008), \( \bar{p}_m = 10\%–40\% \) for hybrid cloth face coverings and \( \bar{p}_m = 40\%–80\% \) for single-layer fabrics (Konda et al., 2020b). Notably, even low quality masks can significantly reduce transmission risk since the bound on cumulative exposure time, (1), scales as \( \bar{p}_m^2 \).

Finally, we define \( \lambda_c = \lambda_c(\bar{r}) \) as an effective relaxation rate of the infectious aerosol-borne pathogen concentration, \( C(r, t) \), evaluated at the effective aerosol radius \( \bar{r} \). The size-dependent relaxation rate of the droplet-borne pathogen has four distinct contributions:

\[ \lambda_c(r) = \lambda_a + \lambda_f(r) + \lambda_i(r) + \lambda_s(r). \]

Here, \( \lambda_a \) is the ventilation rate, specifically the rate of exchange with outdoor air; \( \lambda_f(r) = p_f(r) \lambda_r \) is the filtration rate, where \( p_f(r) \) is the droplet removal efficiency for air filtration at a rate \( \lambda_r \) (recirculated air changes per time); \( \lambda_i(r) = v_s(r) A/V \) is the net sedimentation rate for infectious droplets with the
Table 1. Glossary of symbols arising in our theory, their units and characteristics values.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Typical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Engineering parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>Number of people, room occupancy</td>
<td>1–1000</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Time since an infected person entered the room</td>
<td>0–1000 h</td>
</tr>
<tr>
<td>$V$</td>
<td>Room volume</td>
<td>$10^4$ m$^3$</td>
</tr>
<tr>
<td>$A$</td>
<td>Floor surface area</td>
<td>5–5000 m$^2$</td>
</tr>
<tr>
<td>$H$</td>
<td>Mean ceiling height, $V/A$</td>
<td>2–6 m</td>
</tr>
<tr>
<td>$Q_a$</td>
<td>Ventilation outflow rate</td>
<td>$10^5$ m$^3$ h$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_a$</td>
<td>Ventilation (outdoor air exchange) rate, $Q_a/V$</td>
<td>0.1–30 h$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_r$</td>
<td>Recirculation air exchange rate, $Q_r/V$</td>
<td>0.1–30 h$^{-1}$</td>
</tr>
<tr>
<td>$p_f$</td>
<td>Probability of droplet filtration via recirculation</td>
<td>0–1.0</td>
</tr>
<tr>
<td>$\lambda_f$</td>
<td>Filtration removal rate, $p_f\lambda_r$</td>
<td>0–30 h$^{-1}$</td>
</tr>
<tr>
<td>$p_m$</td>
<td>Mask penetration probability, $\overline{p}_m = p_m(\bar{r})$</td>
<td>0.01–0.1</td>
</tr>
<tr>
<td><strong>Physical parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>Respiratory drop radius</td>
<td>0.1–100 μm</td>
</tr>
<tr>
<td>$V_d$</td>
<td>Drop volume, $\approx \frac{4}{3}\pi r^3$</td>
<td>$10^3$–$10^6$ μm$^3$</td>
</tr>
<tr>
<td>$n_d$</td>
<td>Drop number density per radius</td>
<td>0.01–1.0 (cm$^{-3}$ μm)$^{-1}$</td>
</tr>
<tr>
<td>$v_s$</td>
<td>Drop settling speed</td>
<td>$10^{-5}$–$10^{-2}$ mm s$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_s$</td>
<td>Drop settling rate, $v_s(r)/H$</td>
<td>$10^{-5}$–$10^{-2}$ h$^{-1}$</td>
</tr>
<tr>
<td>$Q_b$</td>
<td>Mean breathing flow rate</td>
<td>0.5–3.0 m$^3$ h$^{-1}$</td>
</tr>
<tr>
<td>$C_0$</td>
<td>Background CO$_2$ concentration</td>
<td>250–450 p.p.m.</td>
</tr>
<tr>
<td>$C_2$</td>
<td>Exhaled CO$_2$ concentration</td>
<td>0–40 000 p.p.m.</td>
</tr>
<tr>
<td>$P_2$</td>
<td>Production rate of exhaled CO$_2$</td>
<td>0.02–10 m$^3$ h$^{-1}$</td>
</tr>
<tr>
<td><strong>Epidemiological parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S, I$</td>
<td>Number of susceptible and infected people</td>
<td></td>
</tr>
<tr>
<td>$\beta_a$</td>
<td>Airborne transmission rate per infected–susceptible pair</td>
<td>$10^{-6}$–10 quanta h$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_v$</td>
<td>Pathogen (virion) deactivation rate</td>
<td>0.01–10 h$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>Pathogen concentration relaxation rate, $\overline{\lambda}_c = \lambda_c(\bar{r})$</td>
<td>0.1–100 h$^{-1}$</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>Effective infectious drop radius</td>
<td>0.3–5 μm</td>
</tr>
<tr>
<td>$P$</td>
<td>Pathogen production rate / air volume / drop radius</td>
<td>$10^{-6}$–$10^9$ (h μm)$^{-1}$</td>
</tr>
<tr>
<td>$C$</td>
<td>Infectious pathogen concentration / air volume / radius</td>
<td>$10^{8}$–$10^{11}$ (μm)$^{-1}$</td>
</tr>
<tr>
<td>$c_v$</td>
<td>Pathogen (virion) concentration per drop volume</td>
<td>$10^4$–$10^{11}$ RNA copies mL$^{-1}$</td>
</tr>
<tr>
<td>$c_i$</td>
<td>Pathogen infectivity, 1/(infectious dose)</td>
<td>0.001–1.0</td>
</tr>
<tr>
<td>$C_q$</td>
<td>Infectiousness of breath, exhaled quanta concentration</td>
<td>1–1000 quanta m$^{-3}$</td>
</tr>
<tr>
<td>$\lambda_q$</td>
<td>Quanta emission rate, $Q_b C_q$</td>
<td>1–1000 quanta h$^{-1}$</td>
</tr>
<tr>
<td>$p_i$</td>
<td>Probability a person is infected (prevalence)</td>
<td>0–1</td>
</tr>
<tr>
<td>$p_{im}$</td>
<td>Probability a person is immune (by vaccination or exposure)</td>
<td>0–1</td>
</tr>
<tr>
<td>$p_s$</td>
<td>Probability a person is susceptible, $p_s = 1 - p_i - p_{im}$</td>
<td>0–1</td>
</tr>
<tr>
<td>$s_r$</td>
<td>Relative susceptibility (or transmissibility)</td>
<td>0.1–10</td>
</tr>
<tr>
<td>$N_i$</td>
<td>Expected number of infected–susceptible pairs</td>
<td>0–1000</td>
</tr>
<tr>
<td>$T_a$</td>
<td>Expected number of airborne transmissions, $N_t(\beta_a)\tau$</td>
<td>0–100</td>
</tr>
<tr>
<td>$R_{in}$</td>
<td>Indoor reproductive number, $(N - 1)(\beta_a)\tau$</td>
<td>0.001–100</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Risk tolerance, bound on $T_a$</td>
<td>0.005–0.5</td>
</tr>
</tbody>
</table>
Stokes settling speed \(v_s(r)\) sedimenting through a turbulent, well-mixed ambient in a room of height \(H = V/A\), volume \(V\) and floor area \(A\) (Corner & Pendlebury, 1951; Martin & Nokes, 1988); \(\lambda_s(r)\) is the deactivation rate of the aerosolized pathogen, which depends weakly on humidity and droplet size (Lin & Marr, 2019; Marr, Tang, Van Mullekom & Lakdawala, 2019; Yang & Marr, 2011), and may be enhanced by other factors such as ultraviolet (UV-C) irradiation (García de Abajo et al., 2020; Hitchman, 2021), chemical disinfectants (Schwartz et al., 2020) or cold plasma release (Filipić, Gutierrez-Aguirre, Princ, Mozetič & Dobnik, 2020; Lai, Cheung, Wong & Li, 2016).

Notably, only the first of the four removal rates enumerated in (2) is relevant in the evolution of \(CO_2\); thus, the concentrations of \(CO_2\) and airborne pathogen may evolve independently. Specifically, the proportionality between the two equilibrium concentrations varies in different indoor settings (Peng & Jimenez, 2021), for example in response to room filtration (Hartmann & Kriegel, 2020). Moreover, when transient effects arise, for example, following the arrival of an infectious individual or the opening of a window, the two concentrations adjust at different rates. Finally, we note that there may also be sources of \(CO_2\) other than human respiration, such as emissions from animals, stoves, furnaces, fireplaces or carbonated drinks, as well as sinks of \(CO_2\), such as plants, construction materials or pools of water, which we neglect for simplicity. As such, following Rudnick and Milton (2003), we assume that the primary source of excess \(CO_2\) is exhalation by the human occupants of the indoor space.

In order to prevent the growth of an epidemic, the safety guideline should bound the indoor reproductive number, \(R_{in}\), which is the expected number of transmissions if an infectious person enters a room full of susceptible people. Indeed, the safety guideline, (1), corresponds to the bound \(R_{in} < \epsilon\) with the choice \(R = N - 1\), and so would limit to \(\epsilon\) the risk of an infected person entering the room of occupancy \(N\) transmitting to any other person during the exposure time \(\tau\). If the epidemic is well underway or subsiding, the guideline should take into account the prevalence of infection \(p_i\) and immunity \(p_{im}\) (as achieved by previous exposure or vaccination) in the local population. Assuming a trinomial distribution of \(N\) people who are infected, immune or susceptible, with mutually exclusive probabilities \(p_{i}, p_{im}\) and \(p_s = 1 - p_i - p_{im}\), respectively, the expected number of infected–susceptible pairs is \(N(N - 1)p_ip_s\).

It is natural to switch between these two limits \((N = N - 1\) and \(N = N(N - 1)p_ip_s\)) when one infected person is expected to be in the room, \(Np_i = 1\), and thus set

\[
N_i = (N - 1)\min\{1, Np_i(1 - p_i - p_{im})\}. \tag{3}
\]

One may thus account for the changing infection prevalence \(p_i\) and increasing immunity \(p_{im}\) in the local population as the pandemic evolves.

### 2.2. \(CO_2\)-based safety guideline

The outdoor \(CO_2\) concentration is typically in the range \(C_0 = 350–450\) p.p.m., with higher values in urban environments (Prill et al., 2000) and lower values in forests (Higuchi, Worthy, Chan & Shashkov, 2003). In the absence of other indoor \(CO_2\) sources, human occupancy in poorly ventilated spaces can lead to \(CO_2\) levels of several thousand p.p.m. People have reported headaches, slight nausea, drowsiness and diminished decision-making performance for levels above 1000 p.p.m. (Fisk, Satish, Mendell, Hotchi & Sullivan, 2013; Krawczyk, Rodero, Gladyszewska-Fiedoruk & Gajewski, 2016), while short exposures to much higher levels may go unnoticed. As an example of \(CO_2\) limits in industry, the American Conference of Governmental Industrial Hygienists recommends a limit of 5000 p.p.m. for an eight-hour period and 30 000 p.p.m. for 10 min. A value of 40 000 p.p.m. is considered to be immediately life-threatening.

We denote by \(C_0\) the background concentration that would arise in the room at zero occupancy, and by \(C_2\) the excess \(CO_2\) concentration (relative to \(C_0\)) associated with human respiration. The total rate of \(CO_2\) production by respiration in the room is given by \(P_2 = NQ_hC_{2,b}\), where \(C_{2,b}\) is the \(CO_2\) concentration of exhaled air, approximately \(C_{2,b} = 38 000\) p.p.m., although the net \(CO_2\) production rate, \(Q_hC_{2,b}\), varies considerably with body mass and physical activity (Persily & de Jonge, 2017). If the
production rate $P_2$ and the ventilation flow rate $Q = \lambda_a V$ are constant, then the steady-state value of the excess CO$_2$ concentration,

$$C_{2,s} = \frac{P_2}{Q} = \frac{Q_b C_{2,b} N}{\lambda_a V},$$  \hspace{1cm} (4)

is simply the ratio of the individual CO$_2$ flow rate, $Q_b C_{2,b}$, and the ventilation flow rate per person, $Q/N$.

The safety guideline, (1), was derived on the basis of the conservative assumption that the infectious aerosol concentration has reached its maximum, steady-state value. If we assume, for consistency, that the CO$_2$ concentration has done likewise, and so approached the value expressed in (4), then the guideline can be recast as a bound on the safe mean excess CO$_2$ concentration,

$$\langle C_2 \rangle \tau = \int_0^\tau C_2 \, dt < \frac{\epsilon C_{2,b}}{\lambda_q s P_m^2 \lambda_a N_t},$$ \hspace{1cm} (5)

where we replace the steady excess CO$_2$ concentration with its time average, $\langle C_2 \rangle \approx C_{2,s}$, and define the mean quanta emission rate per infected person, $\lambda_q = Q_b C_q$. For the early to middle stages of an epidemic or when $p_i$ and $p_{im}$ are not known, we recommend setting $N/N_t = 1 < N/(N - 1) \approx 1$, for a conservative CO$_2$ bound that limits the indoor reproductive number. In the later stages of an epidemic, as the population approaches herd immunity ($p_i \to 0$, $p_{im} \to 1$), the safe CO$_2$ bound diverges, $N/N_t \to \infty$, and so may be supplanted by the limits on CO$_2$ toxicity noted above, that lie in the range 5000–30 000 p.p.m. for eight hour and 10 minute exposures, respectively. Our simple CO$_2$-based safety guideline, (5), reveals scaling laws for exposure time, filtration, mask use, infection prevalence and immunity, factors that are not accounted for by directives that would simply impose a limit on CO$_2$ concentration. The substantial increase in safe occupancy times, as one proceeds from the peak to the late stages of the pandemic, is evident in the difference between the solid and dashed lines in figure 1, which were evaluated for the case of a typical classroom in the USA (Bazant & Bush, 2021). This example shows the critical role of exposure time in determining the safe CO$_2$ level, a limit that can be increased dramatically by efficient mask use and to a lesser extent by filtration. When infection prevalence $p_i$ falls below 10 per 100 000 (an arbitrarily chosen small value), the chance of transmission is extremely low, allowing for long occupancy times. The risk of transmission at higher levels of prevalence, as may be deduced by interpolating between the solid and dashed lines in figure 1, could also be rationally managed by monitoring the CO$_2$ concentration and adhering to the guideline.

3. Mathematical model of CO$_2$ monitoring to predict airborne disease transmission risk

3.1. CO$_2$ dynamics

We follow the traditional approach of modelling gas dynamics in a well-mixed room (Shair & Heitner, 1974), as a continuous stirred tank reactor (Davis & Davis, 2012). Given the time dependence of occupancy, $N(t)$, mean breathing flow rate, $Q_b(t)$, and ventilation flow rate, $Q_a(t) = \lambda_a(t)V$, one may express the evolution of the excess CO$_2$ concentration $C_2(t)$ in a well-mixed room through

$$V \frac{dC_2}{dt} = P_2(t) - Q_a(t)C_2,$$ \hspace{1cm} (6)

where

$$P_2(t) = N(t)Q_b(t)C_{2,b}$$ \hspace{1cm} (7)

is the exhaled CO$_2$ production rate. The relaxation rate for excess CO$_2$ in response to changes in $P_2(t)$ is precisely equal to the ventilation rate, $\lambda_a(t) = Q_a(t)/V$. For constant $\lambda_a$, the general solution of (6)
Figure 1. Illustration of the safety guideline, (5), which bounds the safe excess CO₂ (p.p.m.) and exposure time $\tau$ (hours). Here, we consider the case of a standard US classroom (with an area of 83.6 m² and a ceiling height of 3.6 m) with $N = 25$ occupants, assumed to be children engaging in normal speech and light activity ($\lambda_{qs} = 30$ quanta h⁻¹) with moderate risk tolerance ($\epsilon = 10\%$). Comparison with the most restrictive bound on the indoor reproductive number without any precautions (red line) indicates that the safe CO₂ level or occupancy time is increased by at least an order of magnitude by the use of face masks (blue line), even with relatively inconsistent use of cloth masks ($p_m = 30\%$). The effect of air filtration (green line) is relatively small, shown here for a case of efficient HEPA filtration ($p_f = 99\%$) with 17% outdoor air fraction ($\lambda_f = 5\lambda_a$). All three bounds are increased by several orders of magnitude (dashed lines) during late pandemic conditions ($p_{ps} = 10$ per 100,000), when it becomes increasingly unlikely to find an infected–susceptible pair in the room. The other parameters satisfy $(\lambda_v + \lambda_s)/\lambda_a = 0.5$, as could correspond to, for example, $\lambda_v = 0.3$, $\lambda_s = 0.2$ and $\lambda_a = 1$ h⁻¹ (1 ACH).

for $C_2(0) = 0$ is given by

$$C_2(t) = \int_0^t \exp(-\lambda_a(t - t')) \frac{P_2(t')}{V} \, dt',$$

which can be derived by Laplace transform or using an integrating factor. The time-averaged excess CO₂ concentration can be expressed as

$$\langle C_2 \rangle = \frac{1}{\tau} \int_0^\tau \left( 1 - \exp(-\lambda_a(\tau - t)) \right) \frac{P_2(t)}{Q_a} \, dt$$

by switching the order of time integration. If $P_2(t)$ is slowly varying over the ventilation time scale $\lambda_a^{-1}$, the time-averaged CO₂ concentration may be approximated as

$$\langle C_2 \rangle \simeq \frac{\langle P_2 \rangle}{Q_a} - \left( 1 - e^{-\lambda_a \tau} \right) \frac{P_2(\tau)}{Q_a},$$

where the excess CO₂ concentration approaches the ratio of the mean exhaled CO₂ production rate to the ventilation flow rate at long times, $\tau \gg \lambda_a^{-1}$, as indicated in (4).
3.2. Infectious aerosol dynamics

Following Bazant and Bush (2021), we assume that the radius-resolved concentration of infectious aerosol-borne pathogen, \( C(r, t) \), evolves according to

\[
V \frac{\partial C}{\partial t} = P(r, t) - \lambda_c(r, t) V C,
\]

where the mean production rate,

\[
P(r, t) = I(t) Q_d(t) n_d(r, t) V_d(r) p_m(r) c_v(r),
\]

depends on the number of infected people in the room, \( I(t) \), and the size distribution \( n_d(r, t) \) of exhaled droplets of volume \( V_d(r) \) containing a pathogen (i.e. virion) at microscopic concentration, \( c_v(r) \). The droplet size distribution is known to depend on expiratory and vocal activity (Asadi et al., 2019, 2020c; Morawska et al., 2009). Quite generally, the aerosols evolve according to a dynamic sorting process (Bazant & Bush, 2021): the drop-size distribution evolves with time until an equilibrium distribution obtains.

Given the time evolution of excess CO2 concentration, \( C_2(t) \), one may deduce the radius-resolved pathogen concentration \( C(r, t) \) by integrating the coupled differential equation

\[
\frac{\partial C}{\partial t} + \lambda_c(r, t) C = \frac{P(r, t)}{P_2(t)} \left( \frac{dC_2}{dt} + \lambda_a(t) C_2 \right).
\]

This integration can be done numerically or analytically via Laplace transform or integrating factors if one assumes that \( \lambda_a, \lambda_c, P \) and \( P_2 \) all vary slowly over the ventilation (air change) time scale, \( \lambda_a^{-1} \). In that case, the general solution takes the form

\[
C(r, t) \approx \frac{P}{P_2} \left( C_2(t) + (\lambda_a - \lambda_c(r)) \int_0^t \exp(-\lambda_c(r)(t-t')) C_2(t') \, dt' \right),
\]

where we consider the infectious aerosol build-up from \( C(r, 0) = 0 \).

3.3. Disease transmission dynamics

According to Markov’s inequality, the probability of at least one airborne transmission taking place during the exposure time \( \tau \) is bounded above by the expected number of airborne transmissions, \( T_a(\tau) \), and the two become equal in the (typical) limit of rare transmissions, \( T_a(\tau) \ll 1 \). The expected number of transmissions to \( S(t) \) susceptible people is obtained by integrating the mask-filtered inhalation rate of infection quanta over both droplet radius and time:

\[
T_a(\tau) = \int_0^\tau S(t) Q_b(t) s_r \left( \int_0^\infty C(r, t) c_i(r) p_m(r) \, dr \right) \, dt,
\]

where \( c_i(r) \) is the infectivity of the aerosolized pathogen. The infectivity is measured in units of infection quanta per pathogen and generally depends on droplet size. One might expect pathogens contained in smaller aerosol droplets with \( r < 5 \, \mu m \) to be more infectious than those in larger drops, as reported by Santarpia et al. (2020) for SARS-CoV-2, on the grounds that smaller drops more easily penetrate the respiratory tract, absorb and coalesce onto exposed tissues, and allow pathogens to escape more quickly by diffusion to infect target cells. The mask penetration probability \( p_m(r) \) also decreases rapidly with increasing drop size above the aerosol range for most filtration materials (Chen & Willeke, 1992; Konda et al., 2020b; Li et al., 2008; Oberg & Brousseau, 2008), so the integration over all radii in (15) gives the most weight to the aerosol size range, roughly \( r < 5 \, \mu m \). We note that this range includes the maxima in exhaled droplet size distributions (Asadi et al., 2019, 2020c; Morawska et al., 2009) as defined in
terms of either number or total volume in the aerosol range (Bazant & Bush, 2021); nevertheless, larger droplets may also contribute to airborne transmission (Tang et al., 2021).

The inverse of the infectivity, $c_r^{-1}$, is equal to the ‘infectious dose’ of pathogens from inhaled aerosol droplets that would cause infection with probability $1 - (1/e) = 63\%$. Bazant and Bush (2021) estimated the infectious dose for SARS-CoV-2 to be of the order of 10 aerosol-borne virions. Notably, the corresponding infectivity, $c_r \approx 0.1$, is an order of magnitude larger than previous estimates for SARS-CoV (Buonanno et al., 2020b; Watanabe, Bartrand, Weir, Omura & Haas, 2010), which is consistent with only COVID-19 reaching pandemic status. The infectivity is known to vary across different age groups and pathogen strains, a variability that is captured by the relative susceptibility, $s_r$. For example, Bazant and Bush (2021) suggest assigning $s_r = 1$ for the elderly (over 65 years old), $s_r = 0.68$ for adults (aged 15–64) and $s_r = 0.23$ for children (aged 0–14) for the original Wuhan strain of SARS-CoV-2, based on a study of transmission in quarantined households in China (Zhang et al., 2020a). The authors further suggested multiplying these values by 1.6 for the more infectious Alpha variant of concern of the lineage B.1.1.7 (VOC 202012/01), which emerged in the UK with a reproductive number that was 60\% larger than that of the original strain (Davies et al., 2020; Volz et al., 2021). Based on the latest epidemiological data (Abdool Karim & de Oliveira, 2021), we likewise suggest multiplying $s_r$ by 1.5 for the South African variant, 501 Y.V2, and by 1.2 for the Californian variants, B.1.427 and B.1.429. Appropriate factors multiplying $s_r$ for more recent strains are provided on our online app (Khan, Bazant & Bush, 2020), such as 1.5 for the Beta variant B.1.351 from South Africa, 2.0 for the Gamma variant P.1 from Brazil and 2.5 for the Delta variant B.1.617.2 from India that has become dominant at the time of publication.

### 3.4. Approximate formula for the airborne transmission risk from CO2 measurements

Equations (14) and (15) provide an approximate solution to the full model that depends on the exhaled droplet size distribution, $n_d(r, t)$, and mean breathing rate, $Q_b(t)$, of the population in the room. Since the droplet distributions $n_d(r)$ have only been characterized in certain idealized experimental conditions (Asadi et al., 2020a, 2020c; Morawska et al., 2009), it is useful to integrate over $r$ to obtain a simpler model that can be directly calibrated for different modes of respiration using epidemiological data (Bazant & Bush, 2021). Assuming $Q_b(t)$, $I(t)$, $S(t)$, $N(t)$ and $n_d(r, t)$ vary slowly over the relaxation time $\tau_c$, we may substitute (14) into (15) and perform the time integral of the second term to obtain

$$T_a(\tau) \approx \frac{s_r \lambda_a}{C_{2,b}} \int_0^\tau \int_0^\infty \frac{n_d(r, t)p_m(r)^2}{\lambda_c(r)} \frac{Q_b(t)I(t)S(t)C_2(t)}{N(t)} \left[ 1 + \frac{\lambda_c(r)}{\lambda_a} - 1 \right] \exp\left(-\lambda_c(r)(\tau - t)\right) \, dr \, dt,$$

(16)

where $n_d(r, t) = n_d(r) V_d(r) c_i(r) c_i(r)$ is the radius-resolved exhaled quanta concentration.

Following Bazant and Bush (2021), we define an effective radius of infectious aerosols $\tau$ such that

$$\int_0^\infty \frac{n_d(r, t)p_m(r)^2}{\lambda_c(r)} \, dr = \frac{C_q(t) p_m(\tau)^2}{\lambda_a(\tau)},$$

(17)

where $C_q(t) = \int_0^\infty n_d(r, t) \, dr$ is the exhaled quanta concentration, which may vary in time with changes in inspiratory activity, for example, following a transition from nose breathing to speaking. In principle, the effective radius $\tau$ can be evaluated, given a complete knowledge of the dependence on drop radius of the mask penetration probability, $p_m(r)$, and of all the factors that determine the exhaled quanta concentration, $n_d(r, t)$ and pathogen removal rate, $\lambda_c(r)$. While these dependencies are not readily characterized, typical values of $\tau$ are at the scale of several microns, based on the size dependencies of $n_d(r, t)$, $c_i(r)$ and $p_m(r)$ noted above.

Further simplifications allow us to derive a formula relating CO2 measurements to transmission risk. By assuming that $C_q(t)$ varies slowly over the time scale of concentration relaxation, one may
approximate the memory integral with the same effective radius \( r \). Thus, accounting for immunity and infection prevalence in the population via

\[
\frac{I(t)S(t)}{N(t)} \approx \frac{N_I}{N} = \left( 1 - \frac{1}{N} \right) \min\{1, Np_s\},
\]

we obtain a formula for the expected number of airborne transmissions,

\[
T_a(\tau) \approx \frac{s_i P_m^2 \lambda_a N_I}{C_{2,b} \Lambda_c} \int_0^\tau \lambda_q(t) C_2(t) \left[ 1 + \left( \frac{\Lambda_c}{\Lambda_a} - 1 \right) \exp(-\Lambda_c(\tau - t)) \right] dt,
\]

in terms of the excess CO\(_2\) time series, \( C_2(t) \), where \( \lambda_q(t) = C_q(t) \frac{Q_b(t)}{N} \) is the mean quanta emission rate. It is also useful to define the expected transmission rate,

\[
\frac{dT_a}{dt}(\tau) = N_i \beta_a(\tau) = \frac{s_i P_m^2 \lambda_a N_I}{C_{2,b} \Lambda_c} \left[ \lambda_q(\tau) C_2(\tau) + (\Lambda_a - \Lambda_c) \int_0^\tau \lambda_q(t) C_2(t) \exp(-\Lambda_c(\tau - t)) dt \right],
\]

which allows for direct assessment of airborne transmission risk based on CO\(_2\) levels. A pair of examples of such assessments will be presented in §4.

Notably, the mean airborne transmission rate expected per infected–susceptible pair, \( \beta_a(t) \), reflects the environment’s memory of the recent past, which persists over the pathogen relaxation time scale, \( \Lambda_c^{-1} \). Likewise, the CO\(_2\) concentration defined in (9) has a memory of recent changes in CO\(_2\) sources or ventilation, which persists over the air change time scale. Notably, the airborne pathogen concentration equilibrates more rapidly than CO\(_2\); specifically, \( \lambda_a^{-1} > \Lambda_c^{-1} \), since CO\(_2\) is unaffected by the filtration, sedimentation and deactivation rates enumerated in (2). The time delays between the production of CO\(_2\) and infectious aerosols by exhalation and their build-up in the well-mixed air of a room shows that CO\(_2\) variation and airborne transmission are inherently non-Markovian stochastic processes. As such, any attempt to predict fluctuations in airborne transmission risk would require stochastic generalizations of the differential equations governing the mean variables, (6) and (11), and so represent a stochastic formulation of the Wells–Riley model (Noakes & Sleigh, 2009).

3.5. Reduction to the CO\(_2\)-based safety guideline

Finally, we connect the general result, (19), with the CO\(_2\)-based safety guideline derived above, (5). Since \( C_2(t) \) varies on the ventilation time scale \( \lambda_a^{-1} \), which is necessarily longer than the relaxation time scale of the infectious aerosols \( \Lambda_c^{-1} \), we may assume that \( \lambda_q(t) C_2(t) \) is slowly varying and evaluate the integral in (19). We thus arrive at the approximation

\[
T_a(\tau) \approx \frac{s_i P_m^2 \lambda_a N_I}{C_{2,b} \Lambda_c} \left[ \langle \lambda_q C_2 \rangle \tau + \frac{\lambda_q(\tau) C_2(\tau)}{\Lambda_c} \left( \frac{\Lambda_c}{\Lambda_a} - 1 + e^{-\Lambda_c \tau} \right) \right].
\]

Since \( \lambda_q(t) C_2(t) \) is slowly varying, the second term in brackets is negligible relative to the first for times longer than the ventilation time, \( \tau \gg \lambda_a^{-1} > \Lambda_c^{-1} \). In this limit, the imposed bound on expected transmissions, \( T_a(\tau) < \epsilon \), is approximated by

\[
\langle \lambda_q C_2 \rangle \tau = \int_0^\tau \lambda_q(t) C_2(t) dt < \frac{\epsilon C_{2,b} \Lambda_c N}{s_i P_m^2 \lambda_a N_I}.
\]

This formula reduces to the safety guideline, (5), in the limit of constant mean quanta emission rate, \( \lambda_q \), which confirms the consistency of our assumptions.
4. Examples of implementation

We proceed by illustrating the process by which the guideline, (5), can be coupled to real data obtained from CO2 monitors. Specifically, we consider time series of CO2 concentration gathered in classroom and office settings at the Massachusetts Institute of Technology using an Atlas Scientific EZO-CO2 Embedded NDIR CO2 Sensor controlled with an Arduino Uno and an Aranet4, respectively. The measurements were collected at desk level. Social distancing guidelines were adhered to, and masks were worn by all participants. We assume a constant exhaled CO2 concentration of 38 000 p.p.m., and use the global minimum of the CO2 series as the background CO2 level $C_0$ from which the excess concentration $C(t)$ is deducted. Notably, the relatively small fluctuations in the CO2 measurements recorded in a variety of settings support the notion of a well-mixed room.

The sensors yield a time series of CO2 concentration that we use, along with (19) and (20), to calculate the transmission rate, assuming that there is a single infected person in the room and that all others are susceptible ($N_i = N - 1$). In this case, the expected number of transmissions is equal to the indoor reproductive number, $T_a(t) = \mathcal{R}_{in}(t)$, and the transmission rate is $(dT_a/dt)(t) = N_i \beta_a(t)$. The approximations made in the derivation above are valid in these examples, so a direct numerical solution of (13) would yield an identical result. In particular, the ‘slowly varying’ assumptions are satisfied, since we keep $N$ and $I$ constant, and any time-dependence of $Q_b$ cancels in the ratio $P/P_2$. Moreover, there was no indication that the air change rate varied over the relatively short time periods considered. Finally, the droplet distributions $n_d(\tau, t)$ may vary in time, but no significant changes in mean respiratory activity were observed or evidenced in the CO2 measurements.

We choose realistic values of the parameters that fall within the typical ranges estimated by Bazant and Bush (2021). The mean breathing rate is set to a value, $Q_b = 0.5 \text{ m}^3 \text{ h}^{-1}$, appropriate for light respiratory activity, and we choose an effective settling speed of $\bar{v}_s = 0.108 \text{ m} \text{ h}^{-1}$ appropriate for $\bar{r} = 0.5 \mu \text{m}$. Larger values of $Q_b$ would be appropriate for more vigorous respiratory activity, as would accompany, for example, a workout at a gym. Our estimate for $\bar{v}_j$ may be revised in response to additional information concerning the dependence on size of the infectivity of exhaled aerosol droplets (Stadnytskyi et al., 2020). The viral deactivation rate for SARS-CoV-2 is set to $\lambda_v = 0.3 \text{ h}^{-1}$, an estimate appropriate for 50% relative humidity. We plot $N_i \beta_a(t)$ and $\mathcal{R}_{in}(t)$ for cases where masks are and are not worn, and choose a mask penetration probability of $p_m \approx 0.3$, as is roughly appropriate for a cloth mask (Konda et al., 2020a). For more effective masks, such as surgical masks or N95s, a smaller $p_m$ value is appropriate, as noted above. The parameters $\bar{L}_e$, $\lambda_v$, $H$ and $N$ are chosen according to the specific scenario presented. Exhaled COVID-19 quanta concentrations $C_q$ for various expiratory activities are estimated from figure 2 of Bazant and Bush (2021). In order to be conservative, we assume that $s_r = 1$, suitable for the most susceptible individuals exposed to the Wuhan strain of SARS-CoV-2. When applying our model to less susceptible populations, such as children (Zhang et al., 2020a; Zhu et al., 2020) or vaccinated individuals exposed to new variants (Sheikh et al., 2021; Nasreen et al., 2021), a smaller $s_r$ value may be used.

4.1. Small office with two workers

Figure 2a shows CO2 measurements taken in an office of length $L = 4.2 \text{ m}$, width $W = 3 \text{ m}$ and height $H = 3 \text{ m}$. Initially, a single worker is present, but at 19:00, a second worker arrives at the office. The workers exit the office at 21:09, return at 21:39 and exit again at 22:30. As the participants were speaking, we use $C_q = 72$ quanta m$^{-3}$ (Bazant & Bush, 2021). We compute the room’s ventilation rate, $\lambda_v \approx 2.3 \text{ h}^{-1}$, from the exponential relaxation that follows the occupants’ exit from the office (as indicated by the orange curve in figure 2). The office was equipped with moderate ventilation, which we characterize with $p_f = 0.99$ and $\lambda_v = 6 \text{ h}^{-1}$. The background concentration $C_0 = 420 \text{ p.p.m.}$ is evident both before and after occupancy.
Figure 2. Measured CO$_2$ concentration and calculated transmission rate in a two-person office. 
(a) Black dots represent the concentration of CO$_2$. The solid blue, dashed magenta and dash–dot green curves represent the transmission rate, as calculated from (20) for three different scenarios, two of which were hypothetical: (blue) the pair are not wearing masks and there is no filtration present; (magenta) the pair are not wearing masks and there is filtration present; (green) the pair are wearing masks and there is no filtration present. The orange solid curve denotes the period of exponential relaxation following the exit of the room’s occupants, from which one may infer both the room’s ventilation rate, $\lambda_a = 2.3$ h$^{-1}$, and the background CO$_2$ concentration, $C_0 = 420$ p.p.m. (b) Corresponding blue, magenta and green curves, deduced by integrating (20), indicate the total risk of transmission over the time of shared occupancy. If the pair were not wearing masks, the safety limit $R_{in} < 0.1$ would be violated after approximately an hour.

As shown in figure 2b, if the office workers were not wearing masks, the safety guideline of expected transmissions $T_a < 10\%$ would be violated after approximately an hour together. However, office workers wearing cloth masks would not violate the safety guideline during the 2.5 h spent together. Filtration without masks also extends the occupancy time limits; however, $R_{in}$ approaches the safety limit after approximately 2 h. While this example should not be taken as a definitive statement of danger or safety in this setting, it does serve to illustrate how our CO$_2$ guideline can be implemented in a real-life situation.

4.2. University classroom adhering to social distancing guidelines

We next monitor CO$_2$ levels during a university lecture. There were $N = 12$ participants in a lecture hall of length $L = 13$ m, width $W = 12$ m and height $H = 3$ m. The lecture started at 13:05 and finished at 13:50. Four people remained in the room for 30 min after the class. The classroom has mechanical ventilation, which is characterized in terms of $\lambda_a \approx 5.2$ h$^{-1}$ (corresponding to a 12 min outdoor air change time; here, we estimate $\lambda_a$ by fitting the build-up of CO$_2$, see figure 3). During the lecture, the professor spoke while the students were quiet and sedentary; thus, we assume $C_q = 30$ quanta m$^{-3}$ (Bazant & Bush, 2021). The maximum CO$_2$ concentration reached in the classroom was $\approx 550$ p.p.m., the excess level no more than 100 p.p.m. Thus, the safety limit was never exceeded, and would not have been even if masks had not been worn. We note that this lecture hall was particularly large, well-ventilated and sparsely populated, and so should not be taken as being representative of a classroom setting. A more complete assessment of safety in schools would require integrating CO$_2$
Figure 3. Measured CO₂ concentration and calculated transmission rate for 12 masked students in a university lecture hall. (a) Black dots represent the concentration of CO₂. The dash–dot green and solid blue curves represent the transmission rate, as calculated from (20), when the occupants are wearing masks, and in the hypothetical case where they are not, respectively. The gold curve indicates a fit to the transient build-up of CO₂ from which we infer an air change rate of $\lambda_a = 5.2/\text{h}$. (b) The dash–dot green and solid blue curves indicate the total risk of transmission with and without masks, respectively, as deduced by integrating (20) over time. Even had masks not been worn, the safety guideline would not have been violated during the lecture.

concentrations over a considerably longer time interval. For example, if students are tested weekly for COVID-19, then one should assess the mean CO₂ concentration in class during the course of an entire school week. Nevertheless, this second example further illustrates the manner in which our model may be applied to real-world settings and suggests that precautions such as ventilation, filtration and mask use, can substantially increase safe occupancy times.

5. Conclusion

Mounting evidence suggests that COVID-19 is spread primarily via indoor airborne transmission. Such an inference is no surprise, as such is also the case for many other respiratory illnesses, including influenza, tuberculosis, measles and severe acute respiratory syndrome (as is caused by a precursor to SARS-CoV-2, the coronavirus SARS-CoV). More than a year into the pandemic, public health guidance continues to emphasize the importance of social distancing and surface cleaning, despite evidence that mask directives are much more effective than either in limiting airborne transmission. We have here illustrated the manner in which CO₂ monitoring may be used in conjunction with the safety guideline of Bazant and Bush (2021) to assess the risk of indoor airborne respiratory disease transmission, including that of COVID-19. Morawska et al. (2021) emphasize the need for improved air quality standards to combat airborne respiratory diseases, similar in spirit to food safety standards for restaurants. We hope that our study might prove useful in defining such standards, and in informing personal and policy decisions about closing and reopening indoor spaces, such as schools and businesses.

We have here reformulated the COVID-19 indoor safety guideline of Bazant and Bush (2021), expressing it in terms of cumulative exposure to CO₂, which can be readily monitored in real time for most indoor spaces. In so doing, we have built upon the seminal work of Rudnick and Milton (2003), as was recently extended and applied to COVID-19 by Peng and Jimenez (2021). The guideline of Bazant...
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and Bush (2021) makes clear that, since the risk of indoor airborne infection is determined by the total volume of pathogen inhaled, safety limits intended to protect against it must be expressed in terms of occupancy time. Likewise, in the context of CO₂ measurements, safety limits cannot be expressed solely in terms of a limit on CO₂ levels, but must also depend on occupancy time. As we have demonstrated with our two case studies, the safety guideline, (5), when coupled with CO₂ monitors, allows for real-time assessment of risk of airborne disease transmission $\mathcal{R}_{in}$ in indoor spaces. Moreover, this approach has the distinct advantage that one can assess certain key model parameters, including the background concentration of CO₂ and the room’s ventilation rate, directly from the CO₂ measurements.

Within a well-mixed space, CO₂ is effectively a passive scalar that tracks the ambient flow, and is removed only through exchange with outdoor air. Aerosol-borne pathogens are subject to additional removal mechanisms, including filtration (by face masks and internal circulation), sedimentation and deactivation. Thus, the concentration of CO₂ cannot be taken as a proxy for those of an aerosol-borne pathogen without resolving the proportionality constant between the two that results from these additional removal mechanisms. We stress that the effects of face mask use are dramatic in reducing the ratio of aerosol-borne pathogens to CO₂ concentration, and in reducing the risk of indoor transmission. Finally, we note that the additional removal mechanisms acting on the droplet-borne pathogen alter not only its equilibrium concentrations relative to that of CO₂, but their relaxation times in transient situations, as may be treated using the mathematical formalism of Bazant and Bush (2021).

We emphasize that the caveats enumerated by Bazant and Bush (2021) concerning the limitations of their safety guideline apply similarly here. First, there is considerable uncertainty in a number of model parameters, including the critical viral load and relative susceptibility. While our inferences are consistent with available data, we hope that these uncertainties will be reduced as more COVID-19 spreading events are characterized and analysed. Second, when masks are not worn, there is substantial additional risk of short-range airborne transmission from respiratory jets and plumes, as accompany breathing, speaking (Abkarian et al., 2020a; Abkarian & Stone, 2020), coughing and sneezing (Bourouiba et al., 2014). Third, while the assumption of the well-mixed room is widely applied and represents a reasonable first approximation, it is known to have limitations (Bhagat et al., 2020; Linden, Lane-Serff & Smeed, 1990; Linden, 1999). Notably, measured fluctuations in CO₂ levels provide a direct means of assessing the validity of the well-mixed-room hypothesis, especially when several sensors are used simultaneously at different locations in the same space. Indeed, CO₂ monitoring has been used for decades to assess the quality of air handling and zonal mixing in buildings (Cheng et al., 2011; Fisk & De Almeida, 1998; Hung & Derossis, 1989; Salisbury, 1986; Seppänen et al., 1999), and can now be repurposed to assess the risk of indoor airborne disease transmission.

Our transmission theory and safety guideline provide a quantitative basis for the use of CO₂ monitors in assessing the risk of indoor airborne disease transmission. Specifically, the simple guideline, (5), and mathematical formulae connecting CO₂ data to the evolving transmission risk, (19)–(20), pave the way for real-time assessment of personal risk in indoor spaces. Our safety guideline might also be implemented in spaces where not only CO₂ is monitored, but also other relevant model parameters. For example, changes in occupancy could be monitored at entrances and exits, while monitoring decibel levels and type of vocalization would serve to inform the infectious aerosol production rate (Asadi et al., 2019, 2020c; Morawska et al., 2009). One could further envision feeding all such data into air regulation controls in order to ensure that our CO₂-based indoor safety limit is never violated. We note that such a prospect would be most easily achieved in quasi-steady circumstances in which a fixed population is behaving in a predictable fashion over the course of an event of known duration, for example, students in a lecture hall or passengers on a charter bus. In more complicated situations, our model provides a framework for optimizing sensor-based, demand-controlled ventilation (Fisk & De Almeida, 1998), with a view to limiting transmission risk while reducing energy consumption and system costs (Risbeck et al. 2021a, 2021b). Finally, our model provides a general framework for using CO₂ monitors to mitigate the indoor airborne transmission of other respiratory illnesses, including the seasonal flu.

In order to facilitate the application of our safety guideline, in the supplementary material we provide a link to an online app that computes the safety guideline in terms of both room occupancy and CO₂.
levels (Khan et al., 2020). The CO2-based guideline, available in the app’s advanced mode, may be used in conjunction with CO2 monitors to formulate safe reopening strategies for indoor spaces in the later stages of the pandemic.

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**Supplementary Material.** Supplementary material are available at https://indoor-covid-safety.herokuapp.com.

**References**


