

## Problem Set 1

18.355, Fall 2021

Due Sept. 22

Apply some combination of dimensional analysis and scaling arguments to address the following.

1. An inviscid fluid drop of mass  $m$  and density  $\rho$  is bound by surface tension  $\sigma$ . What is its natural frequency? Express this frequency in terms of both the drop mass  $m$  and radius  $a$ .
2. A particle of mass  $m$  exists in a universe characterized by two physical constants, the speed of light  $c$  and Planck's constant,  $\hbar$ . What is the particle's natural frequency? Reexpress your answer to express the equality between two energies, the Einstein-de Broglie relation.
3. Identify and interpret the dimensionless groups that describe a drop of one fluid translating uniformly through another fluid. Assume an interfacial tension  $\sigma$  and an unbounded geometry.
4. What is the size of the largest pendant water drop, and the largest animal that can stand at rest on the water surface? (The surface tension at an air-water surface is approximately 70 dynes/cm).
5. Show that the dimensions of the kinematic viscosity  $\nu = \mu/\rho$  are those of a diffusivity. One can thus think of  $\nu$  as prescribing the rate of diffusion of momentum within a viscous fluid. If a fluid-filled sphere of radius  $a$  is made to spin, what is the characteristic time for the fluid inside to spin up, assuming low Reynolds number fluid motion?
6. Rationalize the size of raindrops via an appropriate scaling argument.
7. If an insect is water-repellent as a result of a rough surface coat, it may survive beneath the surface by virtue of an air layer trapped on its surface, which it uses as an external lung. If its roughness scale is  $1\mu\text{m}$ , deduce the depth to which it can safely submerge.
8. A water drop hits a soap film. Make an estimate for the critical speed at which it will break through the film.
9. A thin, two-dimensional planar fluid sheet of thickness  $a$ , density  $\rho$  and viscosity  $\mu$  retracts in response to the curvature pressure generated at its edge by its surface tension  $\sigma$ . Use scaling arguments to estimate its retraction speed in both the high and low Reynolds number limits.
10. A cornerstone of biomechanics (supported by the Olympic world records in power lifting) is that the force that can be generated or sustained by a creature scales as its cross-sectional area. Use this fact to deduce the dependence on body size of the height that may be achieved by a leaping animal.
11. Consider gait changes (*e.g.* from walking to trotting to running) in terrestrial creatures (*e.g.* dogs, horses, humans). Show that for most such creatures, air drag is negligible relative to the force of gravity; thus, the dominant scaling is between inertia and the force of gravity. Idealize the running creature as an inverted pendulum with mass  $M$ , speed  $U$  and pivot-arm/leg length  $L$ . By considering the condition for lift-off of this pendulum, show that one expects gait changes at critical values of a certain dimensionless group. What is this group?
12. Certain creatures are able to run on water, supporting their weight by thrusting water downwards as they run. The largest such animal, the basilisk lizard, is typically 2 feet long.
  - a) Deduce a criterion involving the creature's foot size, number of feet and foot speed that must be satisfied in order for it to walk on water.
  - b) Deduce the minimum foot size required for a human being to walk on water.
  - c) Comment on the relevance of water-repellent shoes for humans walking on water.