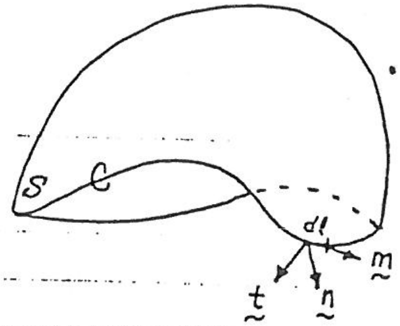


$\underset{\sim}{n} \equiv$ unit outward normal

$\underset{\sim}{t} = \underset{\sim}{m} \wedge \underset{\sim}{n} \equiv \underset{\sim}{\lambda}$ = TANGENT VECTOR

along the contour C, $d\underset{\sim}{l} = \underset{\sim}{m}\, dl$

Recall Stokes Theorem:

$$\oint_C \underset{\sim}{F} \cdot d\underset{\sim}{l} = \int_S \underset{\sim}{n} \cdot (\underset{\sim}{\nabla} \wedge \underset{\sim}{F})\, dS$$

so that

$$\oint_C \underset{\sim}{F} \cdot \underset{\sim}{m}\, dl = \int_S \underset{\sim}{n} \cdot (\underset{\sim}{\nabla} \wedge \underset{\sim}{F})\, dS$$

In order to develop a generalization of this theorem: let $\underset{\sim}{F} = \underset{\sim}{f} \wedge \underset{\sim}{b}$, where $\underset{\sim}{b}$ is an arbitrary constant vector

$$\Rightarrow \oint_C (\underset{\sim}{f} \wedge \underset{\sim}{b}) \cdot \underset{\sim}{m}\, dl = \int_S \underset{\sim}{n} \cdot \underbrace{\underset{\sim}{\nabla} \wedge (\underset{\sim}{f} \wedge \underset{\sim}{b})}\, dS$$

$(\underset{\sim}{f} \wedge \underset{\sim}{b}) \cdot \underset{\sim}{m} = -\underset{\sim}{b} \cdot (\underset{\sim}{f} \wedge \underset{\sim}{m})$

$\underset{\sim}{\nabla} \wedge (\underset{\sim}{f} \wedge \underset{\sim}{b}) = \underset{\sim}{f}(\underset{\sim}{\nabla} \cdot \underset{\sim}{b}) - \underset{\sim}{b}(\underset{\sim}{\nabla} \cdot \underset{\sim}{f}) + \underset{\sim}{b} \cdot \underset{\sim}{\nabla} \underset{\sim}{f} - \underset{\sim}{f} \cdot \underset{\sim}{\nabla} \underset{\sim}{b}$ (standard vector identity), where $\underset{\sim}{f}(\underset{\sim}{\nabla} \cdot \underset{\sim}{b}) = 0$ and $\underset{\sim}{f} \cdot \underset{\sim}{\nabla} \underset{\sim}{b} = 0$

$$\Rightarrow \underset{\sim}{b} \cdot \oint_C (\underset{\sim}{f} \wedge \underset{\sim}{m})\, dl = \underset{\sim}{b} \cdot \int_S \left[ \underset{\sim}{n}(\underset{\sim}{\nabla} \cdot \underset{\sim}{f}) - (\underset{\sim}{\nabla} \underset{\sim}{f}) \cdot \underset{\sim}{n} \right] dS$$

∴ Since $\underset{\sim}{b}$ is arbitrary,

$$\boxed{\oint_C (\underset{\sim}{f} \wedge \underset{\sim}{m})\, dl = \int_S \left[ \underset{\sim}{n}(\underset{\sim}{\nabla} \cdot \underset{\sim}{f}) - (\underset{\sim}{\nabla} \underset{\sim}{f}) \cdot \underset{\sim}{n} \right] dS}$$

→ If $\underset{\sim}{f} = \gamma \underset{\sim}{n}$, $\underset{\sim}{n} \wedge \underset{\sim}{m} = -\underset{\sim}{t}$,

$$-\oint_C \gamma \underset{\sim}{t}\, dl = \int_S \left[ \underset{\sim}{n}\, \underset{\sim}{\nabla} \cdot (\gamma \underset{\sim}{n}) - \underset{\sim}{\nabla}(\gamma \underset{\sim}{n}) \cdot \underset{\sim}{n} \right] dS$$

$$= \int_S \left[ \underset{\sim}{n}\, \underset{\sim}{\nabla}\gamma \cdot \underset{\sim}{n} + \gamma \underset{\sim}{n} (\underset{\sim}{\nabla} \cdot \underset{\sim}{n}) - \underset{\sim}{\nabla}\gamma - \gamma (\underset{\sim}{\nabla} \underset{\sim}{n}) \cdot \underset{\sim}{n} \right] dS$$

($\underset{\sim}{\nabla}\gamma \cdot \underset{\sim}{n} = 0$ since $\underset{\sim}{\nabla}\gamma$ tangent to S; $(\underset{\sim}{\nabla}\underset{\sim}{n}) \cdot \underset{\sim}{n} = \frac{1}{2} \underset{\sim}{\nabla}(\underset{\sim}{n} \cdot \underset{\sim}{n}) = \frac{1}{2} \underset{\sim}{\nabla}(1) = 0$)

$$\therefore \boxed{\oint_C \gamma \underset{\sim}{t}\, dl = \int_S \left[ \underset{\sim}{\nabla}\gamma - \gamma \underset{\sim}{n} (\underset{\sim}{\nabla} \cdot \underset{\sim}{n}) \right] dS}$$