Lecture 20: Bubbles and drops

- myriad applications in industry: ink-jet printing, digital microfluidics, aerosol sprays, fuel injectors
- critical role in environmental transfer of heat, chemistry, biology between atmosphere & ocean
  - biomaterial typically surface active, sticks to bubbles/drops

I. Bubbles

A. Birth
  - vigorous interfacial mixing (e.g., surf zone)
  - cavitation: vigorous fluid motions induce Bernoulli: low $p <$ cavitation $P$ e.g. near propellers
  - exsolution of dissolved gases e.g. open a coke
    - facilitated by nucleation sites on a rough solid e.g. champagne: bubbles form on scratched in glass
B. Life

Physical variables: \( U, J, \rho, V, a, \sigma, (\hat{\rho}, \hat{V}) \)

Fundamental units: \( MLT \)

\( \Rightarrow \) 3 dimensionless groups

\( Re = \frac{V a}{\nu} \), \( Bo = \frac{L g a^2}{\sigma} \), \( Ca = \frac{a v \sqrt{\nu}}{\sigma} \)

Reynolds 
Bond 
Capillary

or Ohnesorge number \( Oh = \left( \frac{Ca}{Re} \right)^{\frac{1}{2}} = \frac{\mu}{(\rho a \sigma)^{\frac{1}{2}}} \)

Air bubble in \( H_2O \)

- Grace, Clift & Weber

A few cases of interest...

\( Re \)

\( Bo \) Bond number
1. Small bubbles: \( \text{Re} \ll 1, \text{Bo} \ll 1 \)
   - dominance of \( \sigma \) (\( \text{Bo} \ll 1 \)) \( \Rightarrow \) spherical shape

Rise speed: \( \frac{\mu U_0}{\rho} \cdot \frac{a^2}{\Delta \rho g} \Rightarrow U \sim \frac{\rho a^2}{\Delta \rho g} \)

\[ U = \frac{2}{3} \frac{1+\lambda}{2+3\lambda} \frac{a^2 \Delta \rho g}{\mu} \]

where \( \lambda = \frac{\mu}{\Delta \rho g} \)

- \( \lambda \) varies very little between
  - \( \lambda = 0 \) (bubble) and \( \lambda \to \infty \) (rigid sphere) limits

2. \( \text{Bo} \gg 1, \text{Re} \gg 1 \)
   - oblate distortion due to stagnation pressures: Bernoulli highs at \( A \), lows at \( D \)
   - b.d. on bubble surface generates high \( \Rightarrow \) wake formation
3. High Re

Rise speed: \( \sqrt{\frac{\rho}{\sigma}} - \frac{\Delta p}{g' a^3} \Rightarrow U \sim \sqrt{g'a} \)

Dynamic P, buoyancy

where \( g' = \frac{g}{\rho} \)

Note: stagnation pressure will cleave bubble when
\[ \text{We} = \frac{\rho U^2 a^2}{\sigma} > 1 \Rightarrow \text{sub in } U \sim \sqrt{g'a} \]

Bubble will break when \( B_0 = \frac{4 \rho g a^2}{\sigma} > 1 \) i.e. \( a > l_0 \)

Exceptions

A. Spherical Cap

- for \( \text{Re} \gg 1, B_0 > 1 \) : on bubble surface, pressure remains const.
\[ \rho U^2 + p + \rho g \sigma = \frac{\rho U^2}{2} \]

\( \sigma \) not needed for integrity of bubble
B. Toroidal Bubbles

- Embedded within vortex, which stabilizes the bubble to both retraction to a sphere, and to $Re - P$ instability

A note on bubble transport (Darwin 1753)

At high $Re$, fluid displaced toward by a rising body is equal to the body's added mass

**Darwin Drift**:

\[ V_d = C_m V \]

* e.g. $C_m = \frac{3}{2}$ for a sphere
  * $C_m = 1$ for a cylinder

C. Death

- via dissolution of contained gas, rupture

Surface Bubble Rupture

Small Bubble
Large Bubble

- For $Oh = \left( \frac{\mu}{\sqrt{\rho \sigma}} \right)^2 \ll 1$, hole retracts at CPT speed $U_c = \left( \frac{2\sqrt{\rho h}}{\rho h} \right)^{\frac{1}{2}}$ ⇒ rim grows through engulfment

- Rim pinches off via Ra-P or Ra-T

- Retracting rim flings outwards by centripetal force $-pU_c^2/R$

- Are drops ejected? Depends on relative magnitude of timescales of retraction and Ra-P/Ra-T instabilities

II. Drops

- Vigorous flows e.g. fragmentation from volumes ⇒ sheets ⇒ filaments ⇒ drops via jetting, impact, etc.

- Condensation from moist air e.g. rain often produced by an updraft of moist air ⇒ cooling ⇒ condensation ⇒ clouds

  ⇒ assisted by vigorous flow in gas phase: Bernoulli
  law facilitates condensation e.g. jet contrails

  ⇒ assisted by solid phase providing nucleation sites
e.g. condensation of H$_2$O vapor into drops requires higher vapor p than found in clouds

$\Rightarrow$ prevalence of condensation nuclei (biogenic/anthropogenic) are key

$\Rightarrow$ feedback between water cycle + biosphere?

B. Life

- dynamics prescribed by $R_e$, $B_0$, $C_d$, $Oh$

  - low $R_e$, $B_0$: drops settle at
    \[ V = \frac{2}{9} \frac{a^2 \Delta \rho}{\mu} g \]

- as $R_e \uparrow$ weak oblate distortion $\Rightarrow$ wake instability $\Rightarrow$ weak deflection of path (since aerodynamic forces are weak)

- drop fracture via aerodynamic stresses when $B_0 \Rightarrow 1$ i.e. $\alpha \ll 1$
Drop Vibrations: for small distortions, large lip, Re

\[ \rho V^2 \sim \frac{\sigma}{R} \Rightarrow \rho w^2 R^2 \sim \frac{\sigma}{R} \]

\[ \Rightarrow \omega \sim \left( \frac{\sigma}{\rho R^3} \right)^{\frac{1}{2}} \]

- drop is an oscillator, a spring with
  natural frequency \( \omega \sim \left( \frac{\sigma}{m} \right)^{\frac{1}{2}} \)
  and natural period \( T \sim \left( \frac{m}{\sigma} \right)^{\frac{1}{2}} \)

**Drop Impact**

- drop will stick unless
  contact time is less than
  time required for air layer
  to drain

Small Deformation:

\[ \sigma x^2 \sim \rho a^3 V^2 \]

Deformation:

\[ x \sim \left( \frac{\rho a^3 V^2}{\sigma} \right)^{\frac{1}{2}} \]

Contact time:

\[ T \sim \frac{x}{V} \sim \left( \frac{m}{\sigma} \right)^{\frac{1}{2}} \text{ natural period} \]
Large Deformation

Drop Fracture (We > 1)

Moderate We > 1
- fracture created by Ra-P, Ra-T of rim, or by Ra-P of rebounding filaments

High We >> 1

< shock: \( p - \rho U^2 \)
C. Death

- coalescence cascade
- impacts on water surface

**Worthington Jet**

![Diagram of Worthington Jet]

*Filaments* → *drops* → *jet*

**Edgenton Crown**

![Images of Edgenton Crown]

\[ T = 0 \]
\[ T = 0.6 \]
\[ T = 1.2 \]
\[ T = 2.0 \]
\[ T = 3.5 \]

\[ \text{We} = 250 \] \[ \text{We} = 437 \] \[ \text{We} = 598 \]