Lecture 20: Bubbles and drops

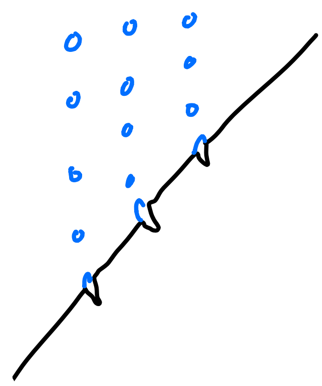
- myriad applications in industry: ink-jet printing, digital microfluidics, aerosol sprays, fuel injectors
- critical role in environmental transfer of heat, chemistry, biology between atmosphere + ocean
  - biomaterial typically surface active, sticks to bubbles / drops

I. Bubbles

A. Birth

- vigorous interfacial mixing (e.g. surf zone)
- cavitation: vigorous fluid motions induce Bernoulli low $p <$ cavitation P e.g. near propellers
- exsolution of dissolved gases e.g. open a coke

⇒ facilitated by nucleation sites on a rough solid
e.g. champagne: bubbles form on scratches in glass

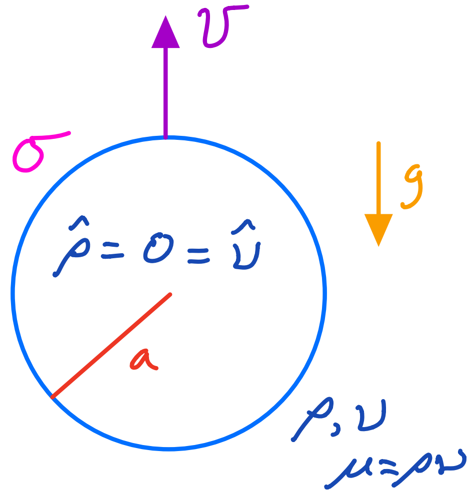


B. Life

$\rho = \hat{\rho} + \Delta\rho$

Physical variables:

$V, g, \rho, \nu, a, \sigma, (\hat{\rho}, \hat{\nu})$ $\rightarrow 0$



Fundamental units: M, L, T

$\Rightarrow$ 3 dimensionless groups

$Re = \frac{Va}{\nu}$, Reynolds

$B_o = \frac{\rho g a^2}{\sigma}$, Bond

$Ca = \frac{\rho \nu V}{\sigma}$ Capillary

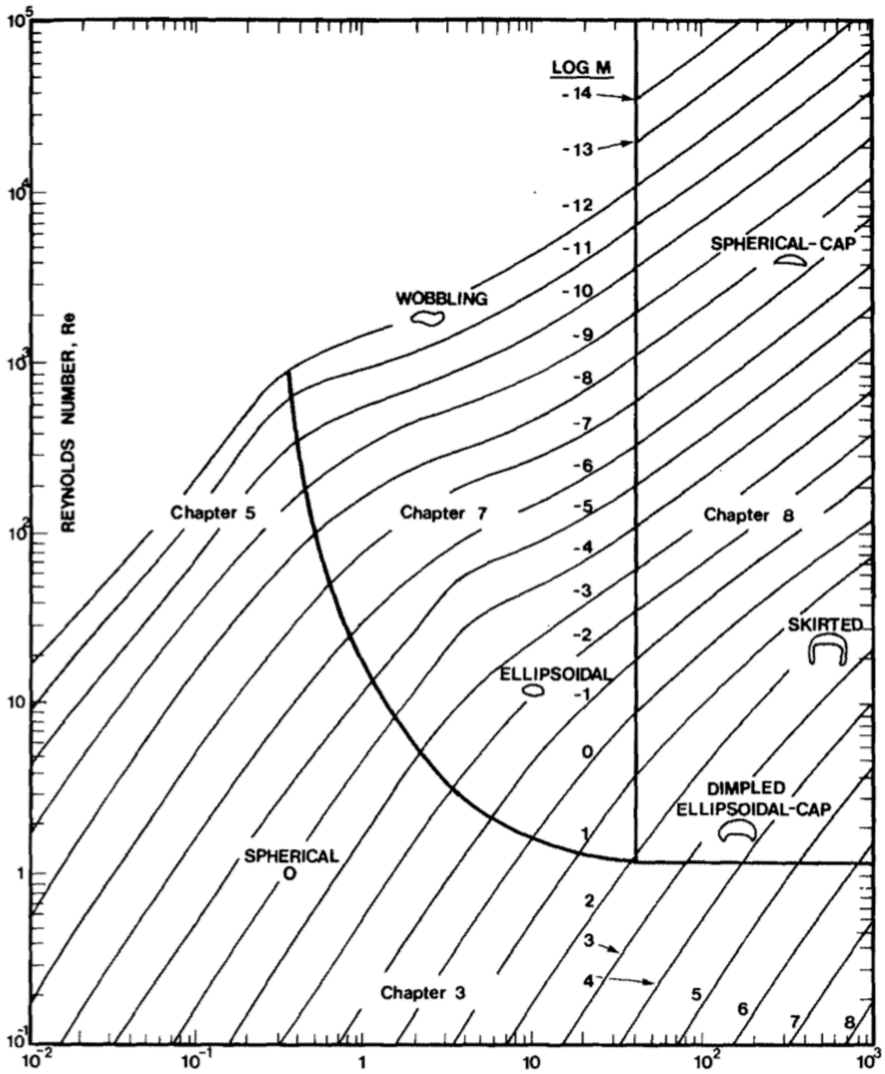
or Ohnesorge number $Oh = \left(\frac{Ca}{Re}\right)^{\frac{1}{2}} = \frac{\mu}{(\rho a \sigma)^{\frac{1}{2}}}$

Air bubble in H_2O

- Grace, Clift + Weber

A few cases of interest...

Re



Bo Bond number

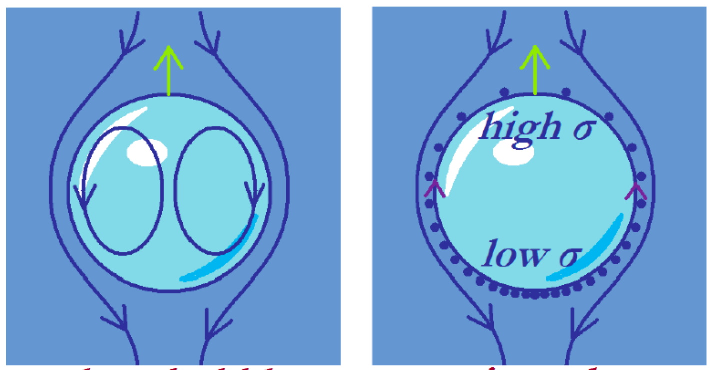
1. Small bubbles: $Re \ll 1$, $Bo \ll 1$

- dominance of σ ($Bo \ll 1$) $\Rightarrow$ spherical shape

Rise speed: $\underbrace{\frac{\mu U}{a} \cdot a^2}_{\text{Drag} \sim \mu U a} \sim \Delta \rho g a^3 \Rightarrow U \sim \frac{g a^2}{\nu}$

Formally (see 18.355) Hadamard–Rybczynski Formula

$$U = \frac{2}{3} \frac{1+\lambda}{2+3\lambda} \frac{a^2 \Delta \rho g}{\mu}$$



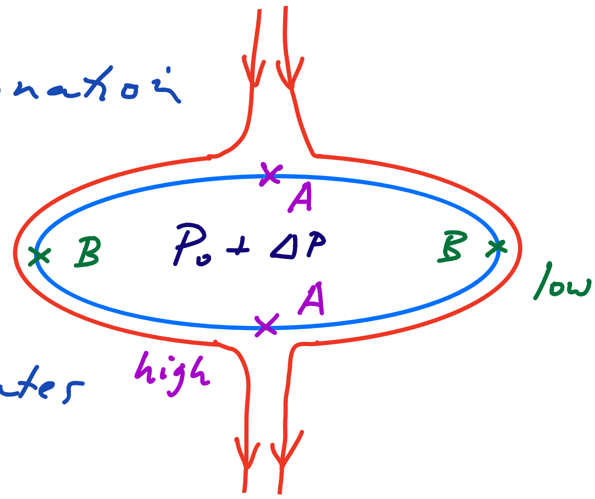
clean bubble   contaminated water

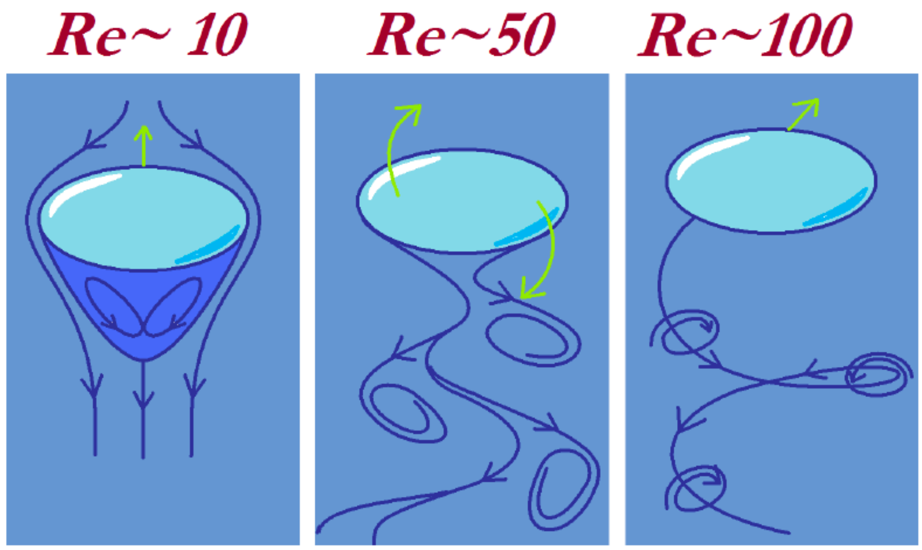
where $\lambda = \hat{\mu}/\mu$

- varies very little between $\lambda = 0$ (bubble) and $\lambda \to \infty$ (rigid sphere) limits

2. $Bo \lesssim 1$, $Re \gtrsim 1$

- oblate distortion due to stagnation pressures: Bernoulli highs at A, lows at B
- b.l. on bubble surface generates vorticity $\Rightarrow$ wake formation





stable wake

UNSTABLE WAKE

↓ ZIG-ZAG PATH ↓ HELICAL PATH

3. High Re

Rise speed: $\rho U^2 a^2 \sim \Delta\rho g a^3 \Rightarrow U \sim \sqrt{g' a}$

(dynamic p) (buoyancy) where $g' = \frac{g \Delta\rho}{\rho}$

Note: stagnation pressure will cleave bubble when

$$We = \frac{\rho U^2 a^2}{\sigma} > 1 \Rightarrow \text{sub in } U \sim \sqrt{g' a}$$

Bubble will break when $Bo = \frac{\Delta\rho g a^2}{\sigma} > 1$ i.e. $a > l_c$

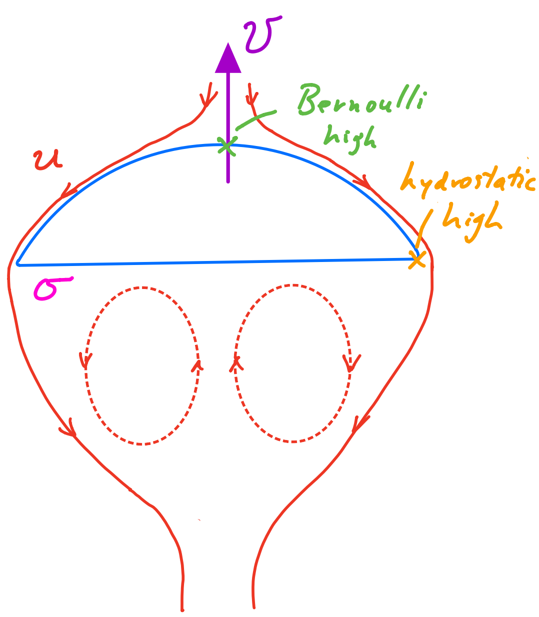
Exceptions

A. Spherical Cap

- for $Re \gg 1$, $Bo \gg 1$: on bubble surface, pressure remains const.

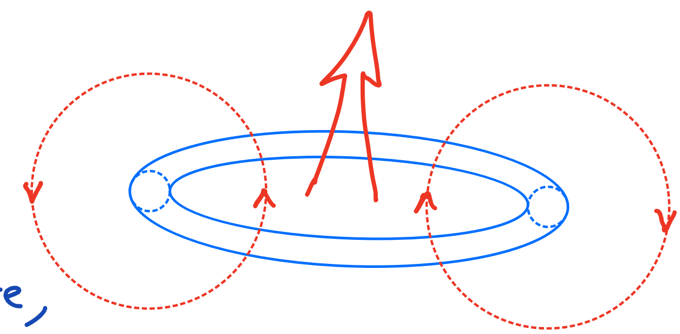
$$\underbrace{\rho u^2 + p + \rho g z}_{\text{const}} = \frac{\rho U^2}{2}$$

$\Rightarrow \sigma$ not needed for integrity of bubble



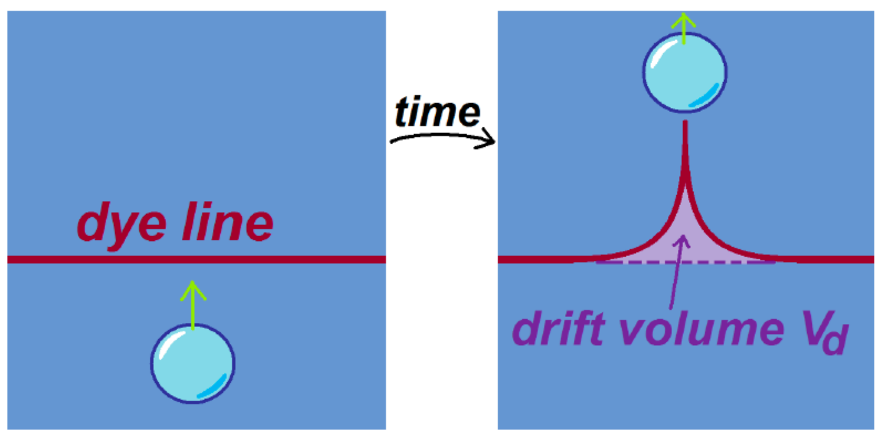
B. Toroidal Bubbles

- embedded within vortex, which stabilizes the bubble to both retraction to a sphere, and to Ra-P instability



A note on bubble transport (Darwin 1953)

At high Re, fluid displaced forward by a rising body is equal to the body's added mass



Darwin Drift:

$$\boxed{V_d = C_m V}$$

(added mass coeff.)

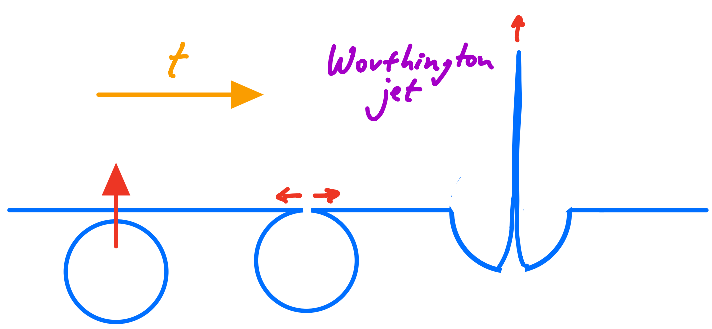
e.g. $C_m = \frac{1}{2}$ for a sphere
$= 1$ for a cylinder

C. Death

- via dissolution of contained gas, rupture

Surface Bubble Rupture

Small Bubble



Large Bubble

- for $Oh = \left(\frac{\mu}{\rho a \sigma}\right)^{\frac{1}{2}} << 1$, hole retracts at Culick speed
  $$v_c = (2\sigma/\rho h)^{\frac{1}{2}} \Rightarrow \text{rim grows through engulfment}$$

- rim pinches off via Ra-P or Ra-T
- retracting rim flung outwards by centripetal force $\sim \rho v_c^2/R$
- are drops ejected? depends on relative magnitudes of timescales of retraction and Ra-P / Ra-T instabilities

II. Drops

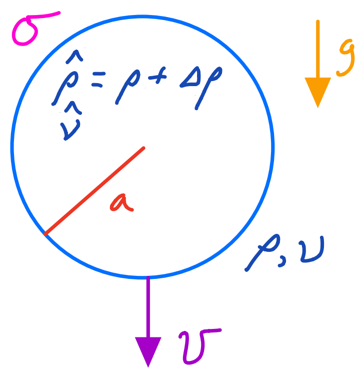
- vigorous flows e.g. fragmentation from volumes → sheets → filaments → drops via jetting, impact, etc.
- condensation from moist air e.g. rain often produced by an updraft of moist air → cooling → condensation → clouds
  - ⇒ assisted by vigorous flow in gas phase: Bernoulli low facilitates condensation e.g. jet contrails
  - ⇒ assisted by solid phase providing nucleation sites

e.g. condensation of H_2O vapor into drops requires higher vapor p than found in clouds

$\Rightarrow$ prevalence of condensation nucleii (biogenic/anthropogenic) are key

$\Rightarrow$ feedback between water cycle + biosphere?

B. Life



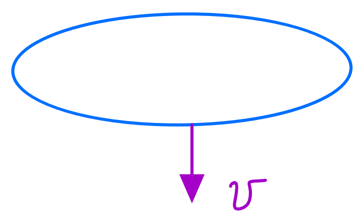
- dynamics prescribed by

  Re, Bo, Ca, Oh

- low Re, Bo : drops settle at

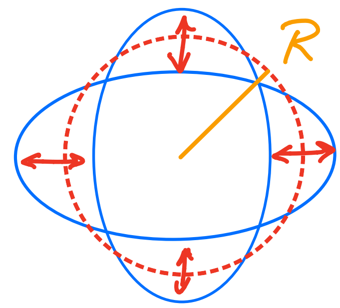
$$v = \frac{2}{9} \frac{a^2 \Delta\rho}{\mu} g$$

- as $Re \uparrow \Rightarrow$ weak oblate distortion $\Rightarrow$ wake instabilities $\Rightarrow$ weak deflection of path (since aerodynamic forces are weak)



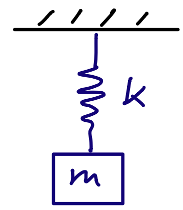
- drop fracture via aerodynamic stresses when $Bo \to 1$ i.e. $a \sim l_c$

Drop Vibration : for small distortions, large We, Re



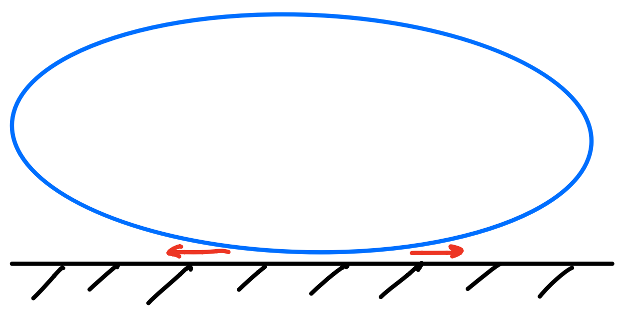
$$\rho v^2 \sim \frac{\sigma}{R} \Rightarrow \rho \omega^2 R^2 \sim \frac{\sigma}{R}$$

$$\Longrightarrow \boxed{\omega \sim \left(\frac{\sigma}{\rho R^3}\right)^{\frac{1}{2}}}$$



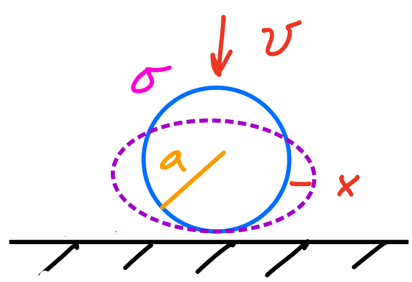
- drop is an oscillator, a spring with natural frequency $\omega \sim \left(\frac{\sigma}{m}\right)^{\frac{1}{2}}$ and natural period $\tau \sim (m/\sigma)^{\frac{1}{2}}$

Drop Impact



- drop will stick unless contact time is less than time required for air layer to drain

Small Deformation :

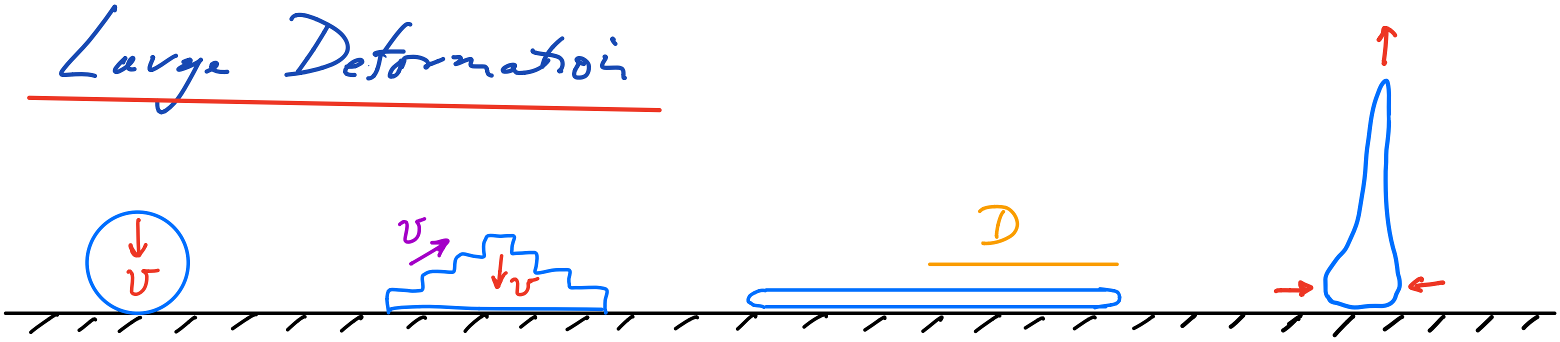


$$\sigma x^2 \sim \rho a^3 v^2$$

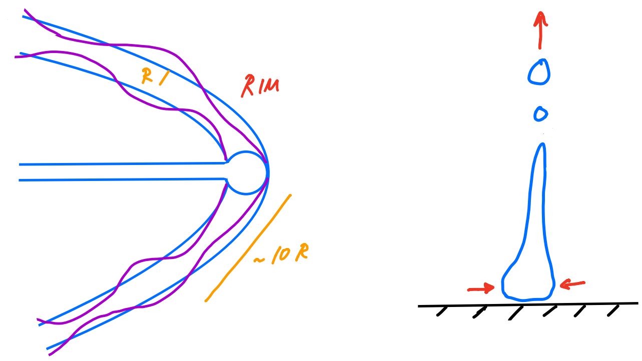
Deformation : $x \sim \left(\frac{\rho a^3 v^2}{\sigma}\right)^{\frac{1}{2}}$

Contact time : $\tau \sim \frac{x}{v} \sim \left(\frac{m}{\sigma}\right)^{\frac{1}{2}}$ natural period

Large Deformation



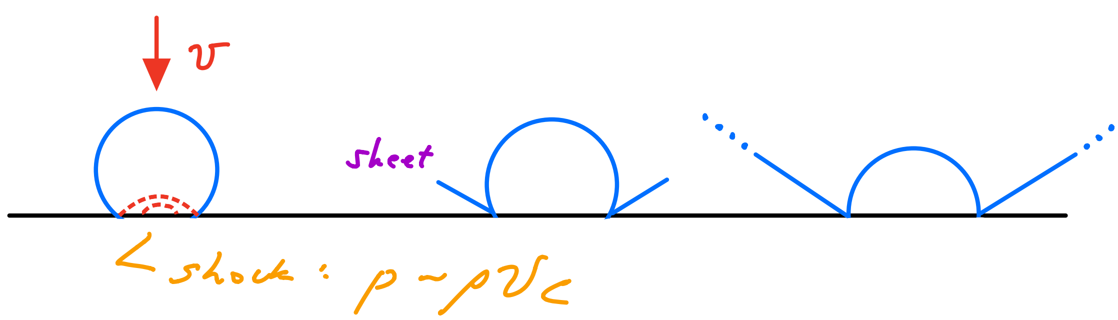
Drop Fracture (We > 1)



Moderate We > 1

- fracture created by Ra-P, Ra-T of rim, or by Ra-P of rebounding filaments

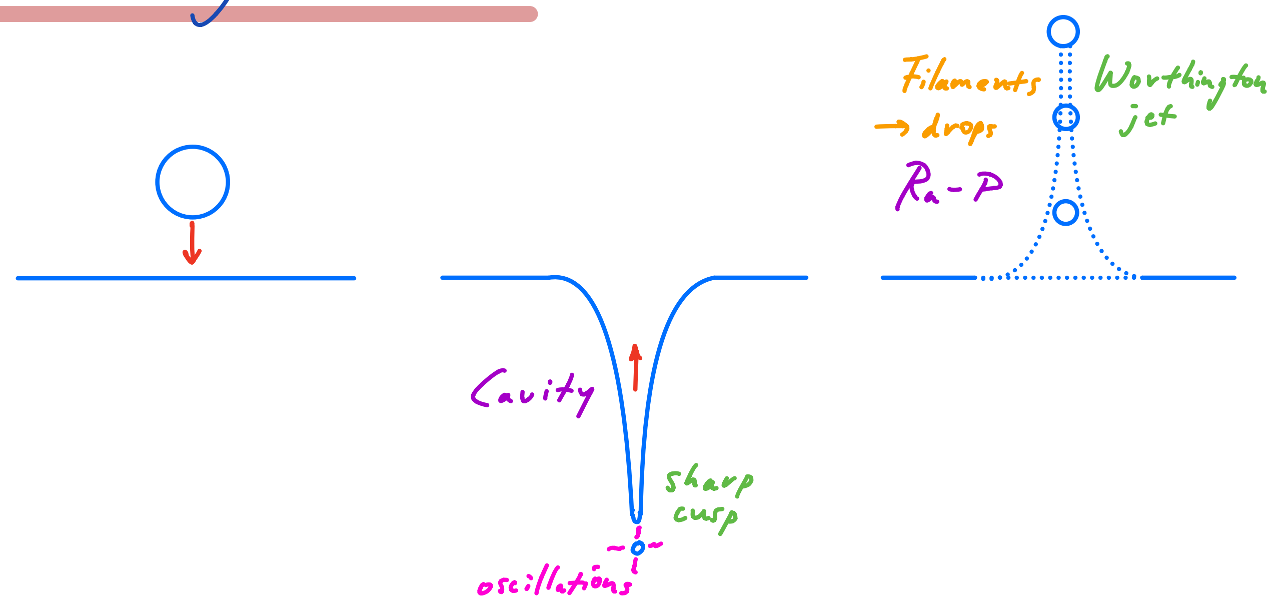
High We >> 1



C. Death

- coalescence cascade
- impacts on water surface

Worthington Jet



Edgerton Crown