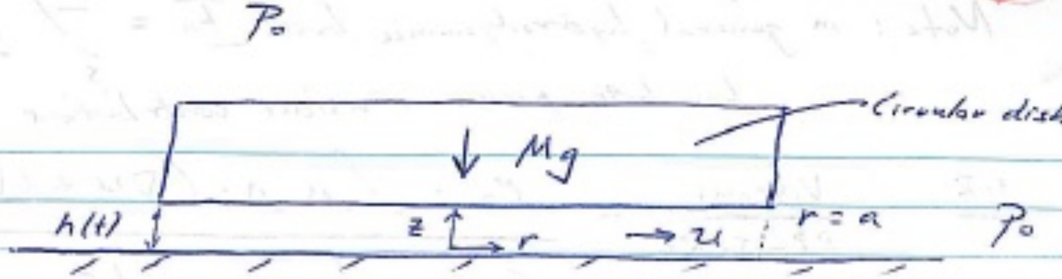
Squeeze Film



In the lubrication limit, the Navier Stokes equations yield

$$\frac{\partial p}{\partial z} = 0 \quad \text{and} \quad 0 = -\frac{\partial p}{\partial r} + \mu \frac{\partial^2 u}{\partial z^2}$$

$$\Rightarrow p = p(r) \text{ only}$$

B.C.s
1. $u = 0$ on $z = 0$
2. $u = 0$ on $z = h(t)$

Velocity Profile: $u = -\frac{1}{2\mu}\frac{dp}{dr} z(h - z)$

Vol Flux: $Q(r) = \int_0^h u(r) \cdot 2\pi r \, dz$

$$\Rightarrow Q(r) = -\frac{\pi r}{\mu}\frac{dp}{dr}\int_0^h zh - z^2 \, dz = -\frac{\pi r}{\mu}\frac{dp}{dr}\left[\frac{h^3}{2} - \frac{h^3}{3}\right]$$

$$Q(r) = -\frac{\pi r}{6\mu}\frac{dp}{dr} h^3$$

Conservation of Mass: $\pi r^2 \dot{h} = Q(r) = \frac{\pi r h^3}{6\mu}\frac{dp}{dr}$

$$\Rightarrow \frac{\partial p}{\partial r} = 6\mu \frac{\dot{h}}{h^3} r \Rightarrow p(r) = \frac{3\mu \dot{h}}{h^3}(r^2 - a^2) + P_o$$

Force Balance: (note again viscous contribution to F_N is negligible) * see over

$$Mg = \int_0^a 2\pi r(p(r) - P_o)\, dr = \frac{6\pi\mu\dot{h}}{h^3}\int_0^a (r^3 - a^2 r)\, dr$$

$$= \frac{6\pi\mu\dot{h}}{h^3}\left(\frac{a^4}{4} - \frac{a^4}{2}\right) = -\frac{3\pi\mu\dot{h}}{2h^3}a^4$$

$$\int_{h_o}^{h}\frac{dh}{h^3}\left(\frac{-3\pi\mu a^4}{2Mg}\right) = \int_0^t dt \Rightarrow t = \frac{3\pi\mu a^4}{4Mg}\left(\frac{1}{h^2} - \frac{1}{h_o^2}\right)$$

is time for disc to move from h_o to h

$$\Rightarrow \boxed{h(t) = \left(\frac{1}{h_o^2} + \frac{4}{3}\frac{Mgt}{\pi\mu a^4}\right)^{-\frac{1}{2}}}$$

Note: plate takes an infinite time for $h \to 0$